Theory of metallic transport in strongly coupled matter

1. Introduction

Andrew Lucas

Stanford Physics

Geometry and Holography for Quantum Criticality; Asia-Pacific Center for Theoretical Physics

August 18-19, 2017

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These lectures loosely follow Chapter 5 of

Hartnoll, Lucas, Sachdev: "Holographic quantum matter",
`arXiv:1612.07324`

Sean Hartnoll
Stanford Physics

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- These lectures will not focus on holography. I will mostly keep discussion to where holographic input can and has been useful.

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Drude's model and shortcomings. Experimental puzzles. Formal definitions.

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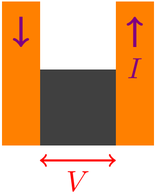
4. Magnetotransport (5.7)

Experimental puzzles in magnetic field. Memory matrix and hydrodynamics.

- Ohm's law – the "simplest" experiment:



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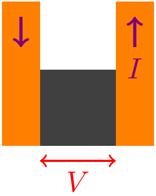
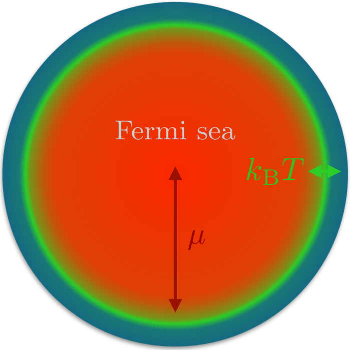


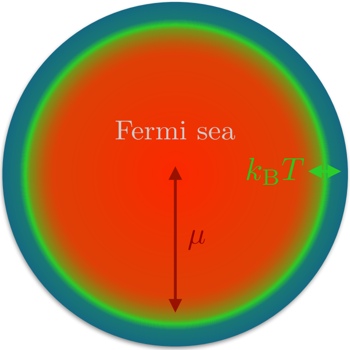
$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, thermoelectric transport:

$$\begin{pmatrix} \mathbf{J} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ T\bar{\alpha} & \bar{\kappa} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}$$



- describes electrons in ordinary metals



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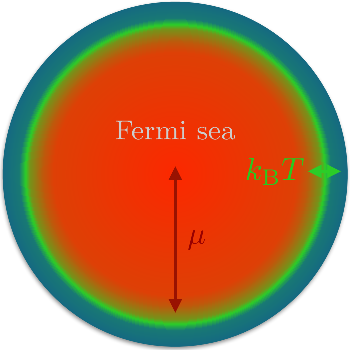
- interaction time constrained by near-Fermi surface phase space:

$$t_{ee} \sim \frac{\hbar\mu}{(k_B T)^2}$$



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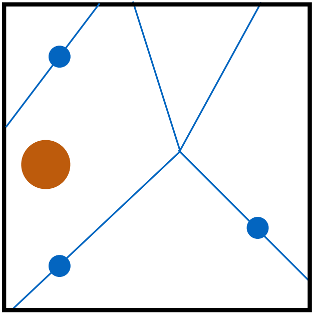
- **long-lived quasiparticles**; (quantum) kinetic theory

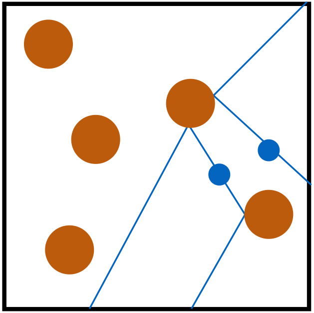
- in ordinary metals, the effects of electron-electron interactions are negligible:

$t_{\text{ee}} \ll t_{\text{imp}}$



ultraclean metal (GaAs, graphene?)

$t_{\text{ee}} \gg t_{\text{imp}}$

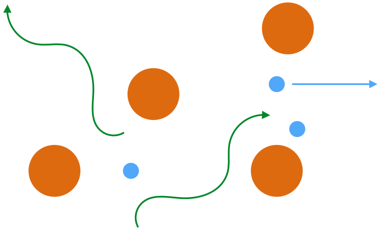


ordinary metal (iron etc.)

$t_{\text{ee}} \sim 10^{-11}$ s $\quad t_{\text{imp}} \sim 10^{-14}$ s

- ρ governed by scattering?

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$



impurities	phonons	electron interactions (umklapp)
$\rho \sim T^0$	$\rho \sim T^{d+2}$ (low T) $\rho \sim T$ (high T)	$\rho \sim T^2$

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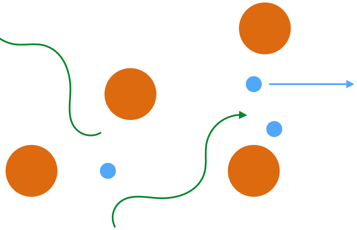

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- scattering rates add (Mattheisen's "rule"):

$$\rho = \rho_{\text{e,imp}} + \rho_{\text{e,ph}} + \rho_{\text{ee}}?$$

- thermal conductivity κ; electrical conductivity σ:

$$\mathbf{Q}|_{\mathbf{J}=\mathbf{0}} \equiv -\kappa \nabla T, \quad \mathbf{J}|_{\nabla T=\mathbf{0}} = \sigma \mathbf{E}.$$

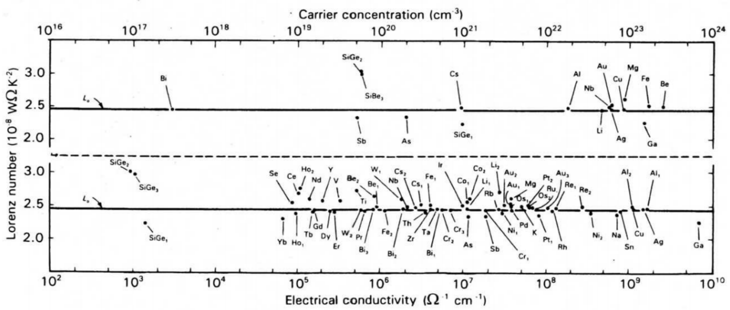
Wiedemann-Franz Law

- thermal conductivity κ; electrical conductivity σ:

$$\mathbf{Q}|_{\mathbf{J}=\mathbf{0}} \equiv -\kappa\nabla T, \quad \mathbf{J}|_{\nabla T=\mathbf{0}} = \sigma\mathbf{E}.$$

- Wiedemann-Franz law in a Fermi liquid:

$$\mathcal{L} \equiv \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_{\mathrm{B}}^2}{3e^2} \approx 2.45\times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



[Kumar, Prasad, Pohl; (1993)]

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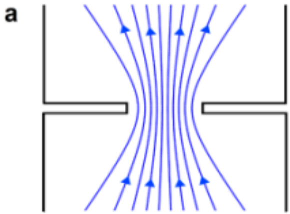
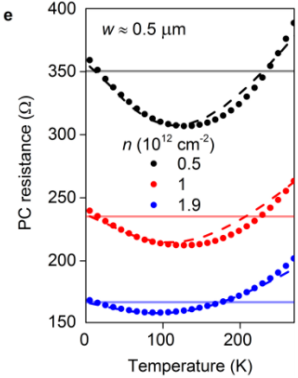
- $\dfrac{\partial\rho}{\partial T} < 0$ in metallic graphene constrictions:

  [Kumar *et al*; 1703.06672]





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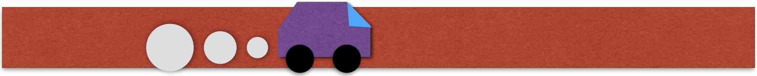

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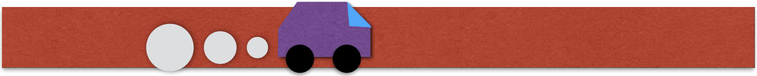

- if there is translation invariance:

$$\sigma = \infty$$

(surprisingly, boost invariance is not needed)

- Ward identity for momentum conservation in QFT in an external electric field E^i:

$$\partial_t T^{ti} + \partial_j T^{ji} = F^{i\mu} J_\mu = \underbrace{\rho E^i}_{\text{Lorentz force}} .$$

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- transport problem is ill-posed so far!

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- compare with before: $\mathcal{M} = nm$, $\rho = -ne$

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- thus we find

$$\alpha = \frac{J}{-\partial T} = \frac{Q}{TE} = \frac{s\rho\tau}{\mathcal{M}}, \quad \bar{\kappa} = \frac{Q}{-\partial T} = \frac{Ts^2\tau}{\mathcal{M}}.$$

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- *all* transport coefficients are linked to the conservation of momentum

- apply a time-dependent electric field $E_i \mathrm{e}^{-\mathrm{i}\omega t}$:

$$\sigma(\omega) = \frac{\rho^2}{\mathcal{M}} \times \frac{1}{\tau^{-1} - \mathrm{i}\omega}$$

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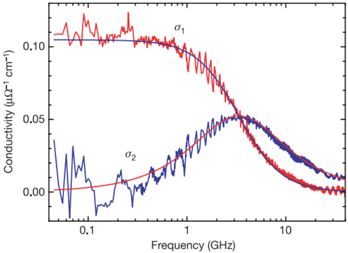
- this **Drude peak** can be seen experimentally in exceptionally pure metals: [Scheffler *et al* (2005)]



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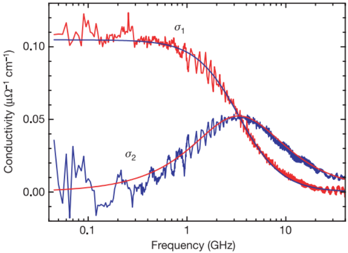

- often there is no sharp Drude peak: τ^{-1} is too large, and so there are competing effects: interband transitions, etc.

- recall that experimentalists often measure κ, not $\bar{\kappa}$:

$$\bar{\kappa} = \left. \frac{Q_x}{-\partial_x T} \right|_{E=0}, \quad \kappa = \left. \frac{Q_x}{-\partial_x T} \right|_{J=0},$$

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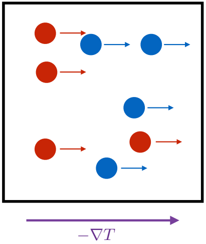
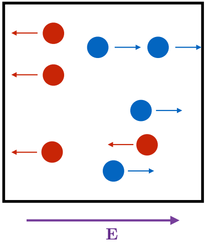
- when $\rho \neq 0$, κ is *finite* even if momentum is conserved:

Wiedemann-Franz Revisited: Zero Density

- when $\rho = 0$:

$$\kappa \to \infty, \quad \sigma = \text{finite}$$





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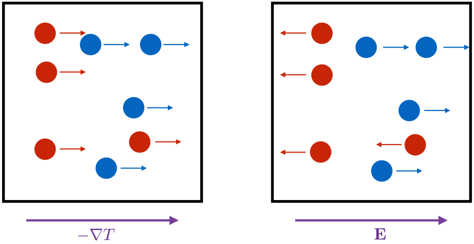

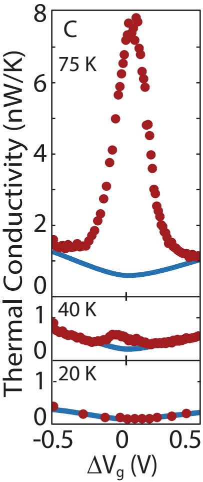

- effect seen in charge neutral graphene:

[Crossno *et al*; 1509.04713]

- in a Fermi liquid:

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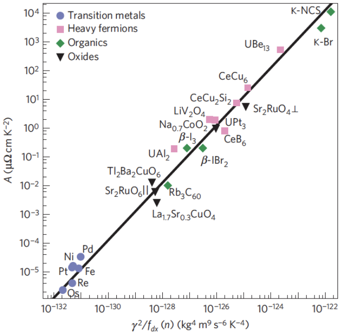
- B depends on thermodynamics (not disorder?):

[Jacko, Fjaerestad, Powell; 0805.4275]



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Linear Resistivity: A Challenge

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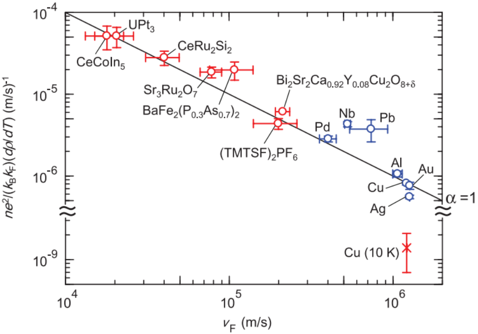
- "Drude" $\rho = \frac{m}{ne^2}\frac{1}{\tau_{\rm ee}} \sim \frac{m}{ne^2}\frac{k_{\rm B}T}{\hbar}$:

[Bruin, Sakai, Perry, Mackenzie (2013)]



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$$\begin{pmatrix} \sigma & \alpha \\ T\alpha & \bar{\kappa} \end{pmatrix} = \begin{pmatrix} \rho^2 & \rho s \\ T\rho s & Ts^2 \end{pmatrix} \frac{\tau}{M}$$

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  - when does this formula apply?
  - how do we compute τ?

How to Compute the Relaxation Time?

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- open questions (I will speculate on):
  - why do we see a universal

$$\rho \sim \frac{1}{\tau_{\mathrm{ee}}},$$

    with τ_{ee} a *momentum-conserving* scattering time?
  - how to think about transport beyond relaxation time approximation (partial theory in Lecture 3)

Formally Applying an Electric Field

it is now time to be rigorous.

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- add a *background, non-dynamical, gauge field* A_μ that couples to conserved current operator J^μ:

$$Z[A^\mu] \equiv \left\langle \exp\left[\mathrm{i}\int \mathrm{d}^{d+1}x J^\mu(x) A_\mu(x)\right]\right\rangle_{\mathrm{QFT}}$$

here and forever, d is the number of *spatial* dimensions

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- to get electric fields, choose

$$A = -E_i t \mathrm{d}x^i, \quad \text{or} \quad \boxed{A = \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{\mathrm{i}\omega} E_i \mathrm{d}x^i, \quad (\omega \to 0)}$$

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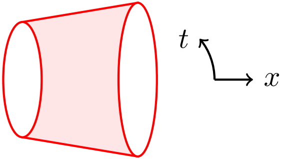
- temperature gradient $\implies$ "cone-like" space with metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 + \mathrm{d}x_i^2, \quad t \sim t + \beta\zeta_i x^i$$



- change coordinates to $\tilde{t} = t/\beta(x)$:

$$\mathrm{d}s^2 \approx \beta(x)^2 \mathrm{d}\tilde{t}^2 + \mathrm{d}x_i^2 \approx \left(\beta^2 + 2\beta^2 \mathrm{e}^{-\mathrm{i}\tilde{\omega}\tilde{t}} \zeta_i x^i\right) \mathrm{d}\tilde{t}^2 + \mathrm{d}x_i^2$$

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- using transformation rules for $g_{\mu\nu}$ and A_μ:

$$g_{\tilde{t}\tilde{t}} \approx \beta^2, \quad g_{\tilde{t}i} \approx \zeta_i \beta^2 \frac{\mathrm{e}^{-\mathrm{i}\tilde{\omega}\tilde{t}}}{\mathrm{i}\tilde{\omega}}, \quad A_i = -\zeta_i \frac{\mathrm{e}^{-\mathrm{i}\tilde{\omega}\tilde{t}}}{\mathrm{i}\tilde{\omega}} A_{\tilde{t}}$$

and converting back to physical coordinates:

$$g_{ti} = \zeta_i \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{\mathrm{i}\omega}, \quad A_i = -\mu\zeta_i \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{\mathrm{i}\omega}$$

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is *flat*: $R_{\alpha\beta\mu\nu} = 0$. (diffeomorphic to warped $\mathrm{S}^1 \times \mathbb{R}^d$)

Is a Temperature Gradient a Gravitational Field?

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  is *flat*: $R_{\alpha\beta\mu\nu} = 0$. (diffeomorphic to warped $\mathrm{S}^1 \times \mathbb{R}^d$)
- but consistency of QFT on curved space *does* give deep relations between 'gravity' and ∇T...

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IBM researchers used an exotic material known as a Weyl semimetal to confirm the existence of a gravitational anomaly predicted in equations that describe the...

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- temperature gradients relate to metric perturbations, and

$$Z[g_{\mu\nu}, A_\mu] = \left\langle \exp\left[\mathrm{i}\int \mathrm{d}^{d+1}x\sqrt{-g}\left(\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + J^\mu \delta A_\mu\right)\right]\right\rangle$$

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- response to perturbations:

$$\underbrace{\frac{\delta Z}{\delta E_i} \to J^i}_{\text{spatially averaged charge current}} \qquad \underbrace{\frac{\delta Z}{\delta \zeta_i} \to T^{ti} - \mu J^i \equiv Q^i}_{\text{spatially averaged \textbf{heat current}}}$$

(though note possible subtleties with "contact terms")

- for most purposes, it suffices to define:

$$\begin{pmatrix} J^i \\ Q^i \end{pmatrix} = \begin{pmatrix} \sigma^{ij} & T\alpha^{ij} \\ T\bar{\alpha}^{ij} & T\bar{\kappa}^{ij} \end{pmatrix} \begin{pmatrix} E_j \\ \zeta_j \end{pmatrix}$$

$$\sigma^{ij}(\omega) = \frac{G^{\mathrm{R}}_{J^iJ^j}(\omega)}{\mathrm{i}\omega}, \quad T\alpha^{ij} = \frac{G^{\mathrm{R}}_{J^iQ^j}(\omega)}{\mathrm{i}\omega}, \quad T\bar{\kappa}^{ij} = \frac{G^{\mathrm{R}}_{Q^iQ^j}(\omega)}{\mathrm{i}\omega}$$

Formal Definitions of Thermoelectric Conductivity Matrix

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- with time reversal symmetry, **Onsager reciprocity**

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- these "simple" two-point functions are very hard to compute: remaining lectures will describe various techniques and simplifying limits in interacting QFTs