# Theory of metallic transport in strongly coupled matter

## 4. Magnetotransport

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$$\partial_t T^{ti} + \partial_j T^{ji} = F^{i\mu} J_\mu = \rho E^i + B_{ij} J_j.$$

- in $d = 2$ spatial dimensions, $B_{ij} = B\epsilon_{ij}$; as $\omega \to 0$:

$$\int \frac{\mathrm{d}^d\mathbf{x}}{V_d} \partial_j T^{ji} = 0 = \int \frac{\mathrm{d}^d\mathbf{x}}{V_d} \left[\rho E^i + B_{ij} J_j\right]$$

which gives

$$\langle J_i \rangle = -B^{-1}_{ij} \rho E_j = \frac{\langle \rho \rangle}{B} \epsilon_{ij} E_j.$$

## The Hall Conductivity

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  - what happens at weak vs. strong magnetic fields?
  - interplay of magnetic fields and disorder? dissipative magnetotransport?
  - generalize memory matrix formalism, hydrodynamics?

- let's return to our toy Drude model ($d = 2$):

$$-\mathrm{i}\omega\mathcal{M}v_i = \rho E_i - \frac{\mathcal{M}}{\tau}v_i + B\epsilon_{ij}J_j,$$

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$$\sigma_{xx} = \frac{\rho^2}{\mathcal{M}}\frac{\tau^{-1} - \mathrm{i}\omega}{(\tau^{-1} - \mathrm{i}\omega)^2 + \omega_\mathrm{c}^2}, \quad \sigma_{xy} = \frac{\rho^3 B}{\mathcal{M}^2[(\tau^{-1} - \mathrm{i}\omega)^2 + \omega_\mathrm{c}^2]},$$

with $\sigma_{yx} = -\sigma_{xy}$, $\sigma_{yy} = \sigma_{xx}$, and **cyclotron frequency**

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$$\omega_\mathrm{c} \equiv \frac{\rho B}{\mathcal{M}}.$$

- what about $d = 3$? $\sigma_{xx}, \sigma_{xy}$ etc. unchanged, and

$$\sigma_{zz} = \frac{\rho^2\tau}{\mathcal{M}}, \quad \sigma_{xz} = 0, \ldots$$

- the Drude model gives simpler formula for resistivity:

$$\rho_{xx} = \rho_{yy} = \frac{\mathcal{M}}{\rho^2}\left(\frac{1}{\tau} - \mathrm{i}\omega\right), \quad \rho_{xy} = -\rho_{yx} = -\frac{B}{\rho}.$$

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- $\omega = 0$: if $\omega_{\mathrm{c}}\tau \gg 1$, we have

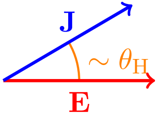
$$\sigma_{xx} = \frac{\mathcal{M}}{\tau B^2} = \frac{\rho^2}{B^2}\rho_{xx}$$

and so $\sigma_{xx}$ and $\rho_{xx}$ are *proportional*.

- the Drude model makes a very simple prediction:

$$\tan\theta_{\mathrm{H}} \equiv \left.\frac{\sigma_{xy}}{B\sigma_{xx}}\right|_{\omega\to 0, B\to 0} = \frac{\rho\tau}{\mathcal{M}} = \left.\frac{\sigma_{xx}}{\rho}\right|_{B=0,\omega=0}.$$

where $\theta_{\mathrm{H}}$ is called the **Hall angle**



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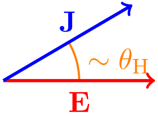


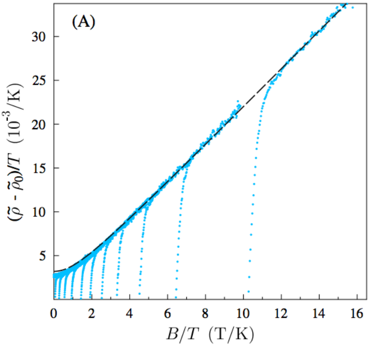

- this relation is violated in the strange metal phase of cuprates [Chien, Wang, Ong (1991)] among other materials:

$$\sigma_{xx} \sim \frac{1}{T}, \qquad \tan\theta_{\mathrm{H}} \sim \frac{1}{T^2}$$

- the magnetic field may give 'universal' corrections to dissipative transport in certain strange metals:

[Hayes *et al*; 1412.6484]

$$\rho_{xx} = \frac{\mathcal{M}}{\rho^2 \tau_{\text{Drude}}} \sim \sqrt{\left(\frac{T}{T_0}\right)^2 + \left(\frac{B}{B_0}\right)^2}.$$



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- for "slow" operators $A$, $B$, the conductivity is

$$\sigma_{AB} = \chi_{AC}(M(\omega) + N - \mathrm{i}\omega\chi)^{-1}_{CD}\chi_{DB},$$

with $M$ the memory matrix (we've discussed), and

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- with long lived momentum: 4 "slow" operators (in $d = 2$): $P_x$, $P_y$, $J_x$, $J_y$

## The $N$ Matrix

- the most important elements of $N$ are

$$-N_{P_y P_x} = N_{P_x P_y} = \frac{1}{T}(P_x|\dot{P}_y) = -\frac{B}{T}(P_x|J_x) = -B\rho$$

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- recall that if $H = H_0 + \epsilon H_{\mathrm{imp}}$ with $[H_0, P_i] = 0$, then $M_{P_iP_j} \sim \epsilon^2$; and if $B \sim \omega \sim \epsilon^2$:

$$\sigma_{J_iJ_j} = \chi_{J_iP_k}(M(\omega) + N - \mathrm{i}\omega\chi)^{-1}_{P_kP_l}\chi_{P_lJ_j} + \mathrm{O}(\epsilon^0),$$

  or

$$\sigma_{ij} = \rho^2 \begin{pmatrix} \mathcal{M}(\tau^{-1} - \mathrm{i}\omega\chi) & -B\rho \\ B\rho & \mathcal{M}(\tau^{-1} - \mathrm{i}\omega\chi) \end{pmatrix}^{-1}$$

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- recover Drude formula when $\tau^{-1} \sim B \sim \omega \sim \epsilon^2$
- $M_{P_iP_j}$, and thus $\tau$, not affected by $B$ (to leading order in $\epsilon$)

[Lucas, Sachdev; 1502.04704]

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- we also have

$$N_{J_iP_j} = \frac{(J_i|\dot{P}_j)}{T} = \epsilon_{jk}\chi_{J_iJ_k}, \quad N_{P_iP_j} = \frac{(P_i|\dot{P}_j)}{T} = \epsilon_{jk}\chi_{P_iJ_k}$$

[Lucas; in progress]

- (assuming isotropy) the electrical conductivity is

$$\sigma = \begin{pmatrix} \chi_{JP} & \chi_{JJ} \end{pmatrix} \begin{pmatrix} -B\chi_{JP}\epsilon & -B\chi_{JJ}\epsilon \\ -B\chi_{JJ}\epsilon & M_{JJ} + N_{JJ} \end{pmatrix}^{-1} \begin{pmatrix} \chi_{JP} \\ \chi_{JJ} \end{pmatrix}$$

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- use block matrix inversion identities:

$$\sigma = \chi_{JP}(-B\chi_{JP}\epsilon)^{-1}\chi_{JP} + X[M_{JJ}+N_{JJ} - B\chi_{JJ}\epsilon(-B\chi_{JP}\epsilon)^{-1}B\chi_{JJ}\epsilon]^{-1}X$$

where we find

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- thus we find the Hall conductivity

$$\sigma_{ij} = \frac{\chi_{JP}}{B}\epsilon_{ij} = \frac{\rho}{B}\epsilon_{ij}.$$

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  with $\delta N_{PP}$ arising from disorder corrections to $(P_x|\dot{P}_y)$.
- dissipative magnetotransport given by (e.g.)

$$\sigma_{xx} \approx \frac{M_{yy}}{B^2},$$

  and with $H - H_0 = \epsilon H_{\mathrm{imp}} \sim \epsilon \int \mathrm{d}^d\mathbf{x} h(\mathbf{x})\mathcal{O}(\mathbf{x})$:

$$\begin{aligned} M_{ij} &= \frac{\mathrm{i}}{T}(\dot{P}_i|\mathfrak{q}(z - \mathfrak{q}L\mathfrak{q})^{-1}\mathfrak{q}|\dot{P}_j) \\ &\approx \int \frac{\mathrm{d}^d\mathbf{k}}{(2\pi)^d} k_i k_j |h(\mathbf{k})|^2 \underbrace{\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(\mathbf{k},\omega)\right)}_{\text{evalauted at } B \neq 0} \end{aligned}$$

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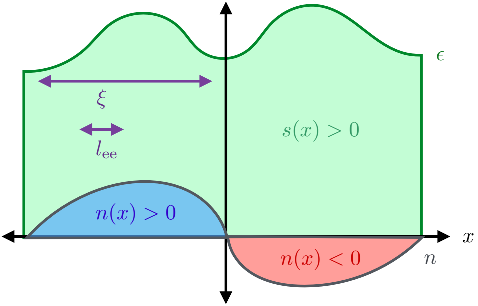
- for stronger inhomogeneity, let us again resort to a hydrodynamic picture:



- recall: linearized generalized hydrodynamics:

$$\partial_i J_i^A = 0 = \partial_i \left(n^A v_i + \Sigma^{AB}\left(E_i^B - \partial_i \mu^B\right)\right)$$
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- solve equations in inhomogeneous background, compute $\int \mathrm{d}^d\mathbf{x}\ J_i^A$

- in $d = 2$: current conservation implies $\partial_k \epsilon_{ki}(B\epsilon_{ij}J_j) = 0$:

$$\epsilon_{ki}\partial_k n^A(\partial_i \mu^A - E_i^A) = \epsilon_{ki}\partial_i\partial_j(\eta_{ijmn}\partial_m v_n) \sim \eta\partial^2 \epsilon_{ki}\partial_i v_k$$

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- transport problem ill-posed without viscosity!

$$\frac{\mathcal{M}}{\tau} \sim \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{k^2}{2}|\mu(\mathbf{k})|^2 \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left(G^{\mathrm{R}}_{\dot{P}\dot{P}}(\mathbf{k},\omega)\right)$$
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- conductivity may provide a good viscometer?

$$\sigma_{xx} \sim \frac{1}{\eta}\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\frac{|\mu(\mathbf{k})|^2}{k^2} \sim \frac{1}{\eta}\underbrace{\log\frac{L}{\xi}}_{\text{IR divergent!}}$$

[Patel, Davison, Levchenko; 1706.03775] [Lucas; in progress]

- in $d = 3$, the current constraint is less severe:

$$\partial_k \epsilon_{nki}(\epsilon_{ijm} J_j B_m) = B_k \partial_k J_n \neq 0$$

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- for viscous dominated transport:

$$\sigma_{ij} \sim \frac{1}{\eta} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} |\mu(\mathbf{k})|^2 \frac{(3 + \cos^2\theta) B^2 \rho^2 k_i k_j}{\underbrace{B^2\rho^2\cos^2\theta + k^4}_{\text{very anisotropic } \sigma_{ij}!}}$$

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and transport problem is well-posed without viscosity

- for viscous dominated transport:

$$\sigma_{ij} \sim \frac{1}{\eta} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} |\mu(\mathbf{k})|^2 \frac{(3+\cos^2\theta)B^2\rho^2 k_i k_j}{\underbrace{B^2\rho^2\cos^2\theta + k^4}_{\text{very anisotropic } \sigma_{ij}!}}$$

- for diffusion dominated transport (generic situation):

$$\sigma_{ij} \sim \Sigma \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} |\mu(\mathbf{k})|^2 \frac{k_i k_j}{\underbrace{k^2(b_1 - b_2\cos^2\theta)}_{\text{minor anisotropy in } \sigma_{ij}}}$$

[Baumgartner, Karch, Lucas; 1704.01592] [Lucas; in progress]

## Accounting for $\sigma_{\mathrm{Q}}$

- less rigorous: employ relaxation time approximation

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- poles in the conductivity at

$$\omega = \underbrace{\pm\frac{\rho B}{\mathcal{M}}}_{\text{cyclotron pole}} \quad \underbrace{-\mathrm{i}\frac{B^2\sigma_{\mathrm{Q}}}{\mathcal{M}}}_{\text{violation of Kohn's theorem}} \quad -\frac{\mathrm{i}}{\tau}$$

[Hartnoll, Kovtun, Müller, Sachdev; 0706.3215]

- this relaxation time model resolves our Hall angle puzzle if

$$\tan\theta_{\mathrm{H}} \equiv \left.\frac{\sigma_{xy}}{B\sigma_{xx}}\right|_{B\to 0} \sim \frac{\rho\tau}{\mathcal{M}} \sim \frac{1}{T^2}, \qquad \tau \sim \frac{1}{T^2}$$

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- similar formulas hold in holographic models, even beyond the hydrodynamic regime

[Blake, Donos; 1406.1659]

- ‘incoherent’ conductivities

$$\Sigma = \begin{pmatrix} \sigma_{\mathrm{Q}} & T\alpha_{\mathrm{Q}} \\ T\alpha_{\mathrm{Q}} & T\bar{\kappa}_{\mathrm{Q}} \end{pmatrix}$$

together with relaxation time approximation give:

$$\alpha_{xx} = \frac{J_x}{-\partial_x T} = \frac{(\tau^{-1}\mathcal{M} + B^2\sigma_{\mathrm{Q}})\alpha_{\mathrm{Q}} + \rho s}{\rho^2 B^2 + [\tau^{-1}\mathcal{M} + B^2\sigma_{\mathrm{Q}}]^2}\frac{\mathcal{M}}{\tau}$$

$$\alpha_{xy} = \frac{J_x}{-\partial_y T} = \frac{\tau^{-1}\mathcal{M}\rho\alpha_{\mathrm{Q}} + (\rho^2 + B^2\sigma_{\mathrm{Q}}^2 + \sigma_{\mathrm{Q}}\mathcal{M}\tau^{-1})s}{\rho^2 B^2 + [\tau^{-1}\mathcal{M} + B^2\sigma_{\mathrm{Q}}]^2} B$$

$$\bar{\kappa}_{xx} = \frac{Q_x}{-\partial_x T} = \bar{\kappa}_{\mathrm{Q}} + \frac{(s^2 - B^2\alpha_{\mathrm{Q}}^2)(B^2\sigma_{\mathrm{Q}} + \tau^{-1}\mathcal{M}) - 2s\rho\alpha_{\mathrm{Q}}B^2}{\rho^2 B^2 + [\tau^{-1}\mathcal{M} + B^2\sigma_{\mathrm{Q}}]^2} T,$$

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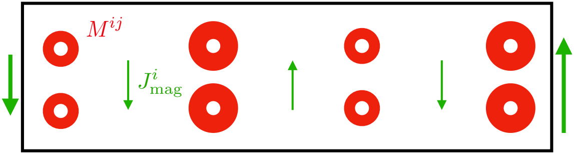
- to compute $J^i$ and $Q^i$ microscopically, we must subtract out **magnetization currents**:



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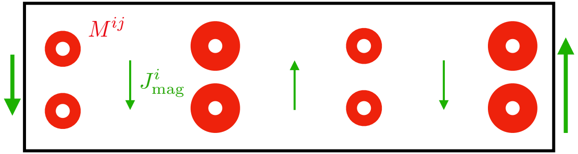


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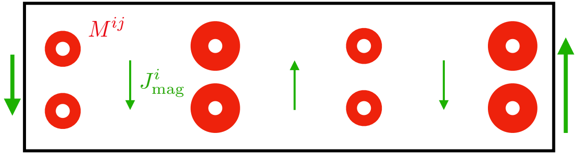


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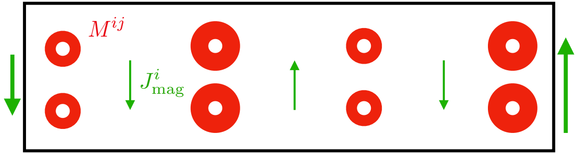


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- for $\alpha_{ij}$, $\bar{\kappa}_{ij}$: cannot ignore magnetization currents

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$$M_{ij} = \frac{\partial P}{\partial B_{ij}}, \quad P = P(\mu, T, X)$$

$$X \equiv \frac{1}{8} \left(\epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma}\right) \left({\epsilon_\mu}^{\nu'\rho'\sigma'} u_{\nu'} F_{\rho'\sigma'}\right) \sim \frac{1}{2} B^2$$

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- $X$ shifts in background gauge field $F_{\mu\nu} + \zeta_i$:

$$\delta X \subset -B^{ij} \zeta_i t \partial_t \delta A_j$$

- the thermodynamic generating functional is

$$Z = \exp[W[g_{\mu\nu}, A_\mu]] = \left\langle \exp\left[\mathrm{i}\int \mathrm{d}^d\mathbf{x}\frac{P}{T}\right]\right\rangle$$

## Magnetization Currents for $\alpha$

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- these magnetization currents are *thermodynamic*; need to be subtracted out

the past 5 years have seen large advances in transport theory:

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  - kinetic and hydrodynamic treatment of spectral weight in quasiparticle systems
  - scaling theories (many holographic) in non-quasiparticle theories

- when the inhomogeneity length scale $\xi$ is large compared to $\ell_{\mathrm{ee}}$, use hydrodynamics to compute transport:

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- ‘universal’ semiclassical transport bound in kinetic theory [Lucas, Hartnoll; 1706.04621]

- generic magnetotransport theory at weak disorder
  [Lucas; in progress]

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## What is Left to Do?

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  - origin of the $T$-linear resistivity?
  - 'practical' applications: good viscous conductors