Asia Pacific Center for Theoretical Physics (APCTP)

Crete Center for Theoretical Physics

Geometry and Holography for Quantum Criticality

Universal bounds on transport To be or not to be?

Matteo Baggioli

UOC & Crete Center for Theoretical Physics

with B. Gouteraux, E. Kiritsis, W.Li, A.Amoretti

Holography 2013
:Gauge/gravity duality and strongly correlated systems

June 13 (Thu) ~ June 22 (Sat), 2013
APCTP, Pohang, Korea

Main Page
Registration/Participants
Program
Accommodation
Travel Information

Main Page

Overview

This focus workshop is to promote research oriented discussions between the world class experts on holography and strongly interacting and strongly correlated system. This year, however, we will also have a school session for students and postdocs.

Organizers

Sang-Jin Sin (Hanyang Univ.,Chair)
Nick Evans (U. of Southampton)
Deog-Ki Hong (Pusan Nat'l Univ.)
Keun-Young Kim (GIST)
Piljin Yi (KIAS)

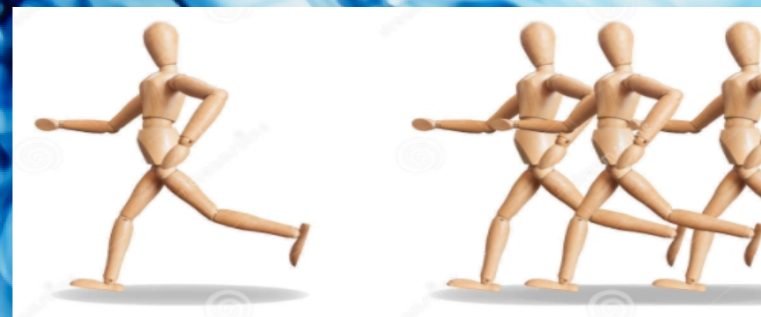
Geometry and Holography for Quantum Criticality

August 18 (Fri), 2017 ~ August 26 (Sat), 2017

Organizers

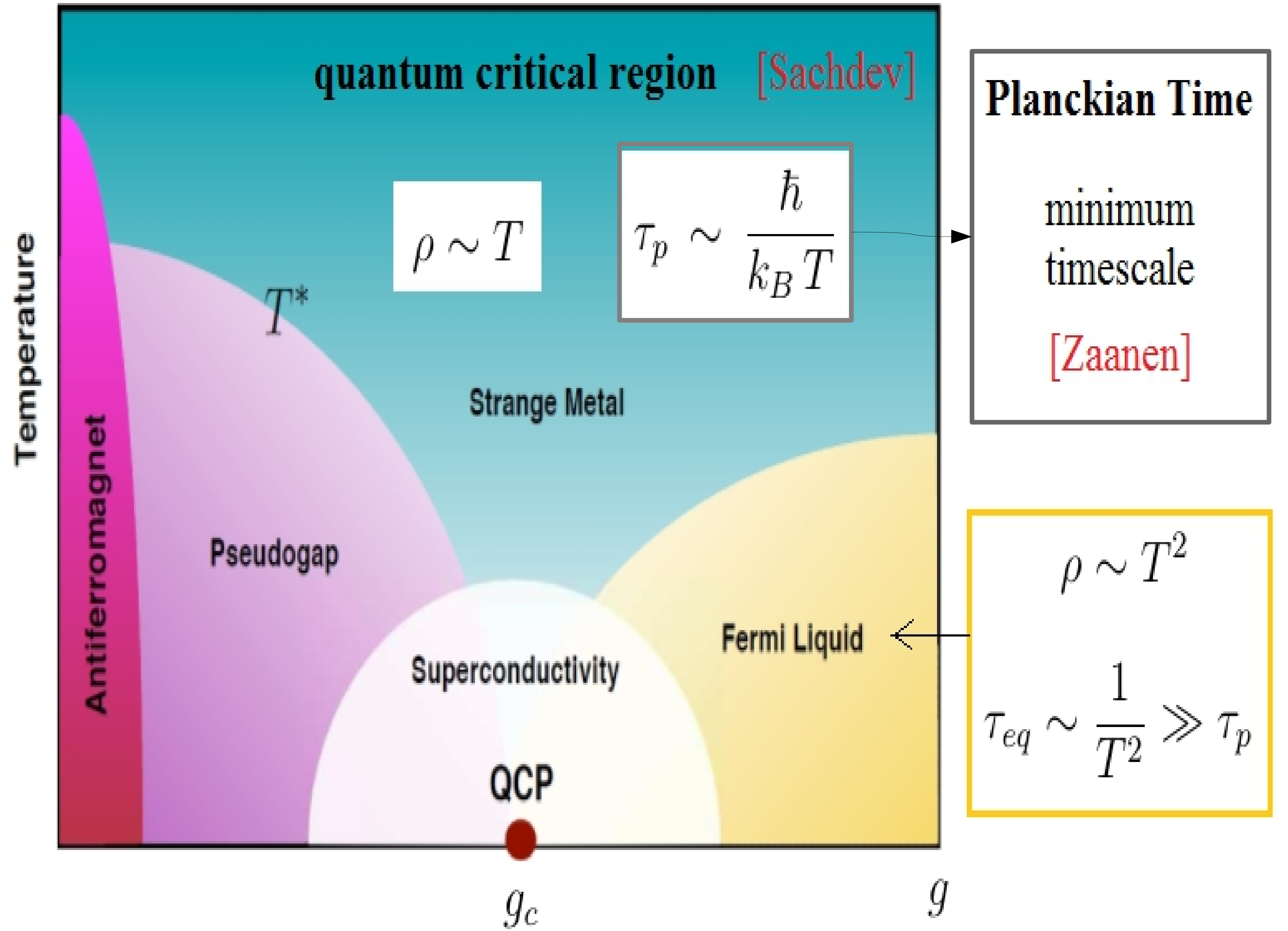
Koji Hashimoto (Osaka University)
Deog-Ki Hong (Pusan National University)
Keun-Young Kim (GIST) - *Cochair**
Ki-Seok Kim (Postech)
Nakwoo Kim (Kyunghee University)
Sang-Jin Sin(Hanyang University) - *Chair**
Piljin Yi (KIAS)

Is there a miminum (Planckian) timescale in nature ?

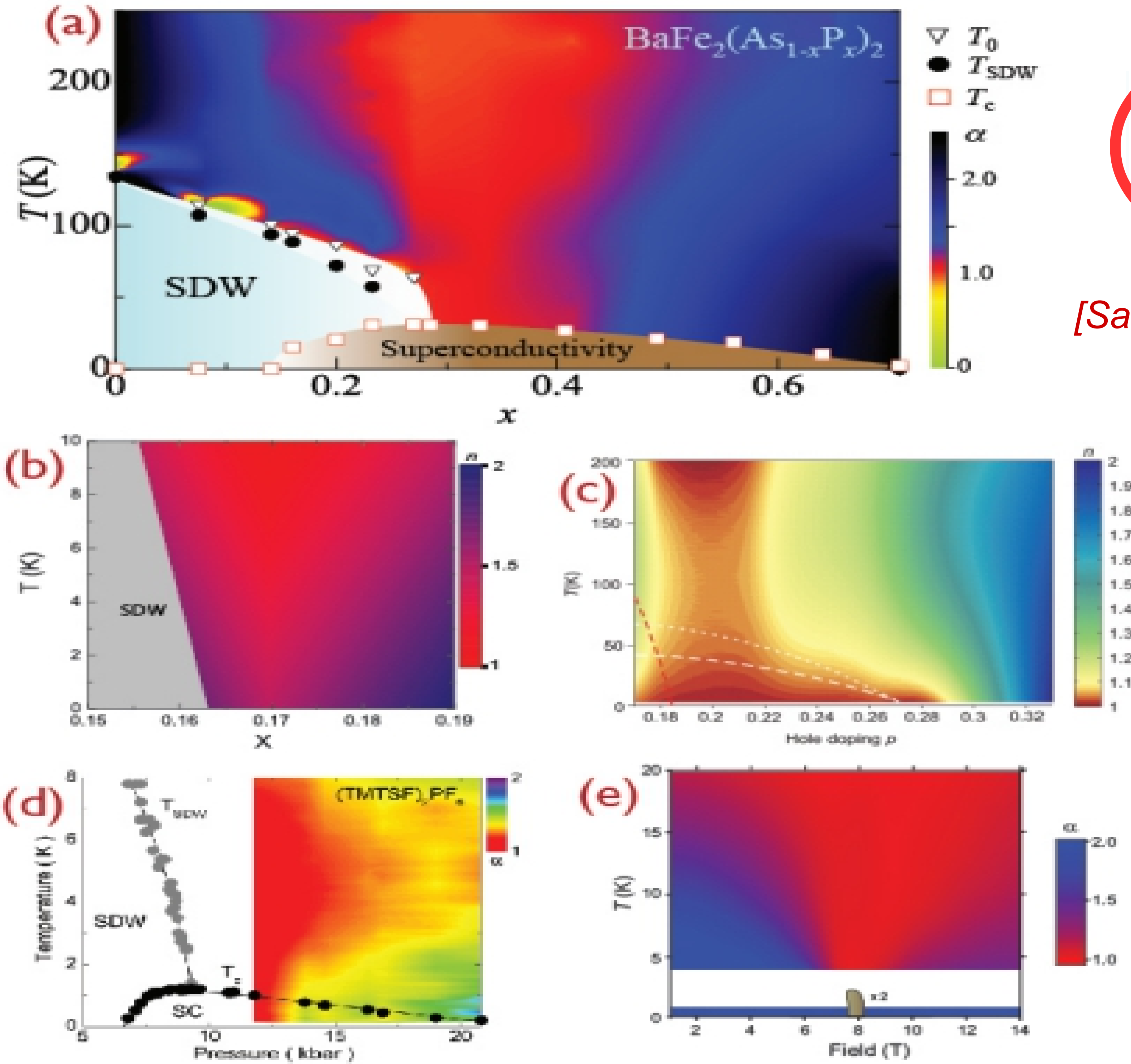


Imprints on transport ??

Bounds on transport !??



quantum critical region
[Sachdev]
Planckian Time
$\rho \sim T$
$\tau_p \sim \frac{\hbar}{k_B T}$
minimum
timescale
[Zaanen]
Temperature
Antiferromagnet
T^*
Strange Metal
Pseudogap
Superconductivity
Fermi Liquid
QCP
$\rho \sim T^2$
$\tau_{eq} \sim \frac{1}{T^2} \gg \tau_p$
g_c
g

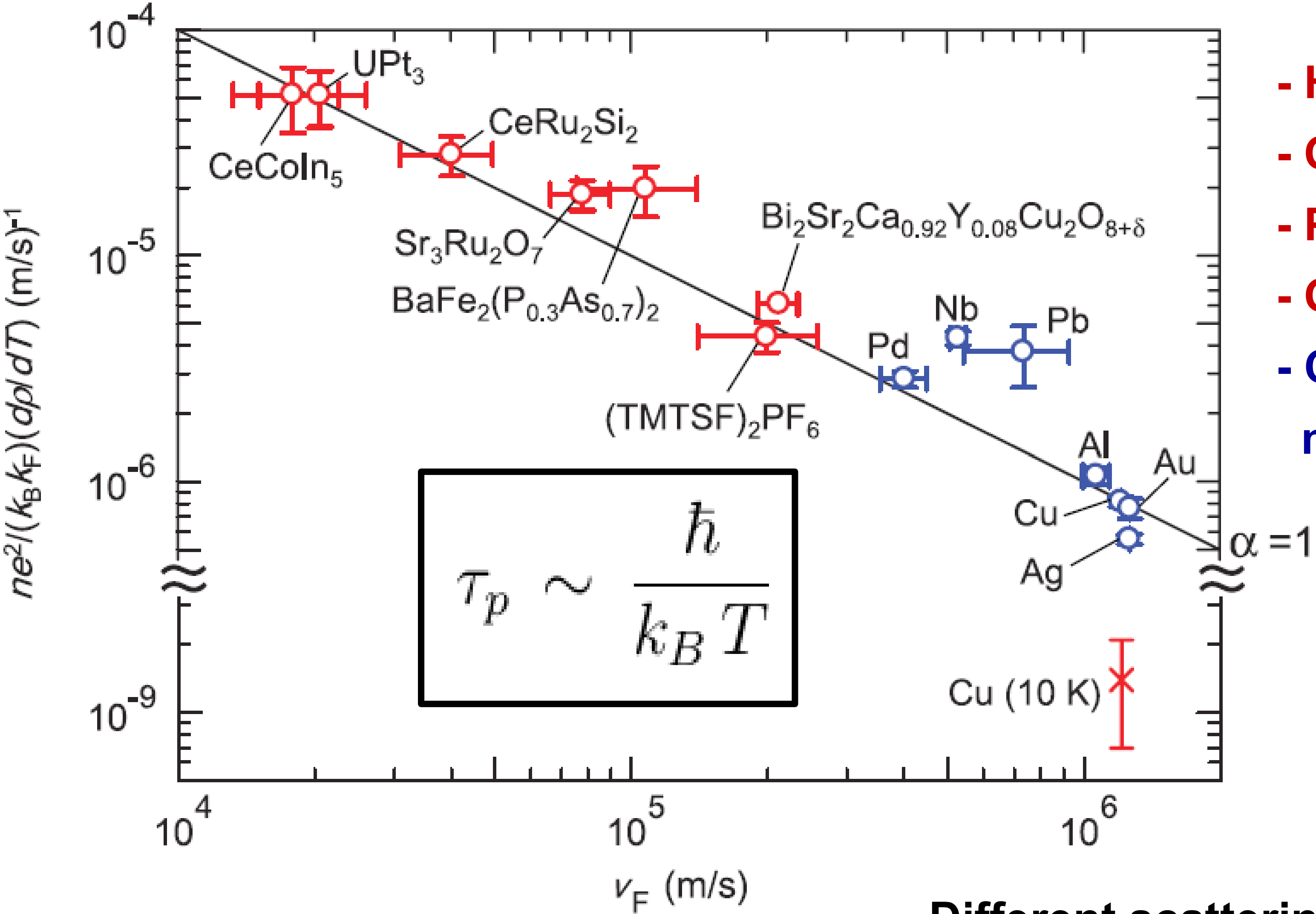


$$\rho \sim T$$

[Sachdev, Keimer]

UNIVERSAL PLANCKIAN SCATTERING TIME

[Bruin JA, Sakai H, Perry RS, Mackenzie AP., Science 2013]



- **Heavy fermions**
- **Oxides**
- **Pnictides**
- **Organic metals**
- **Conventional metals**

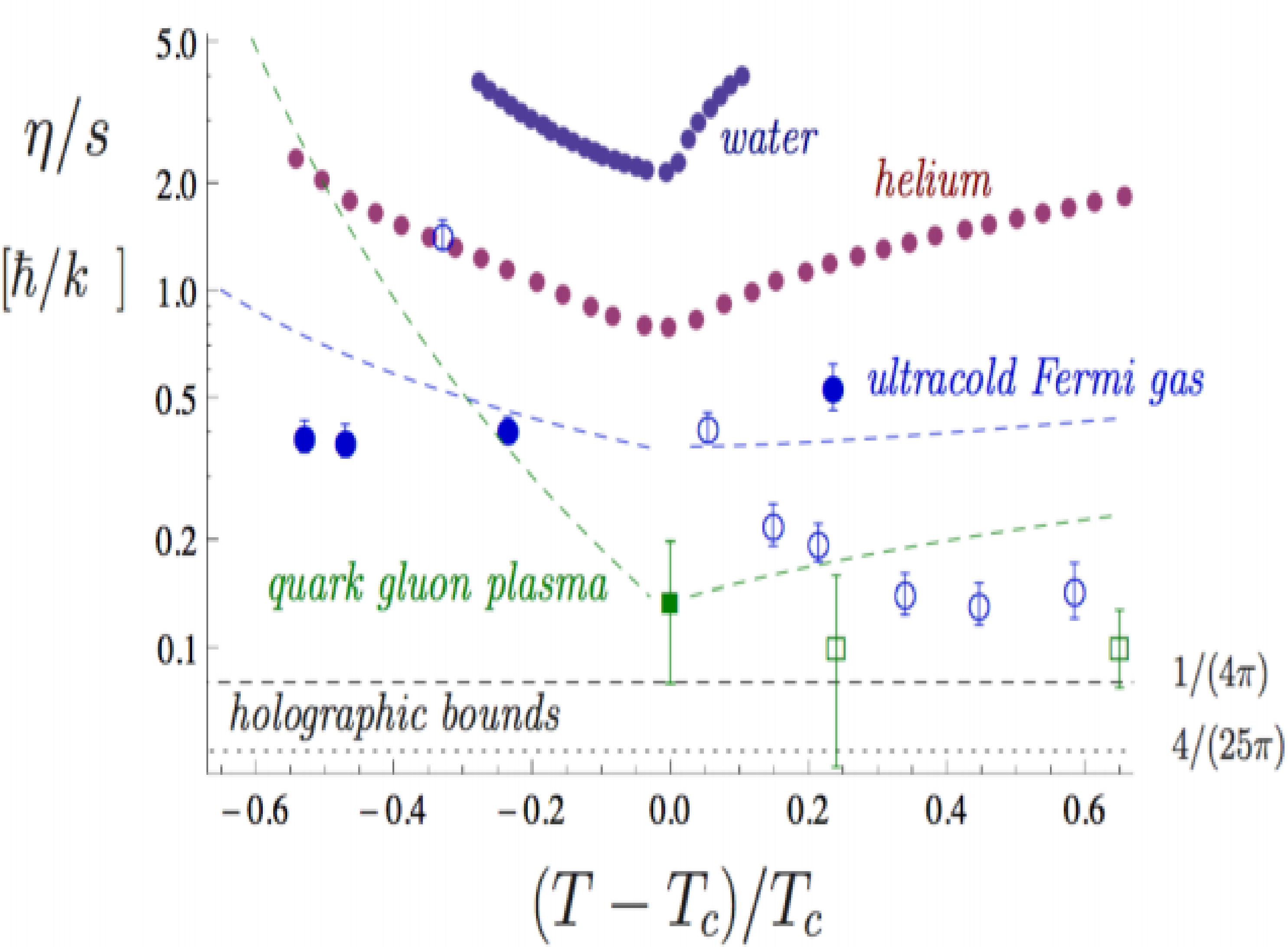
$$\frac{1}{\tau} = \frac{e^2\rho}{hd}\sum_i k_{Fi} v_{Fi} \qquad (T\tau)^{-1} \sim \alpha \frac{k_B}{\hbar}$$

Different scattering mechanisms

2 order of magnitude variations in the Fermi velocity

KSS BOUND

[Kovtun,Son,Starinets,Policastro, 2004]



Minimum Planckian Time

↓

UNIVERSAL BOUND(S)

$$\frac{\eta}{s} \geq \boxed{\frac{1}{4\pi}} \frac{\hbar}{k_B}$$

⟷

$$\tau_p \sim \frac{\hbar}{k_B T}$$

BOUNDS ON DIFFUSIVITIES

[Hartnoll, 2014]

$$[D] \equiv [v^2] * [t]$$

Diffusion can't be arbitrarily fast

$$D \geq v^2 \tau_p = v^2 \frac{\hbar}{k_B T}$$

Generically

$$\tau_{eq} \gg \tau_p$$

Fast(est) dissipation
Strong coupling

KSS story
(v=c)

$$D_{\mathcal{P}} \equiv \frac{\eta}{s T} \geq \left(\frac{1}{4\pi}\right) \frac{\hbar}{k_B T}$$

Charge and Energy
Diffusivities (??)

$$D_c = \frac{\sigma}{\chi} \qquad D_e = \frac{\kappa}{c_v}$$

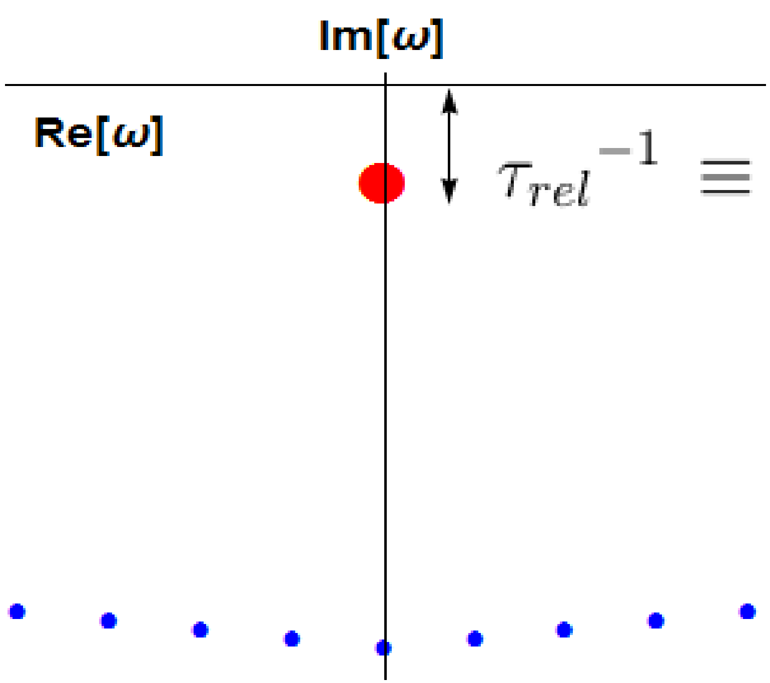
COHERENT METALLIC TRANSPORT

$$\text{electric current } \mathcal{J} \; + \; \text{momentum } \mathcal{P}$$

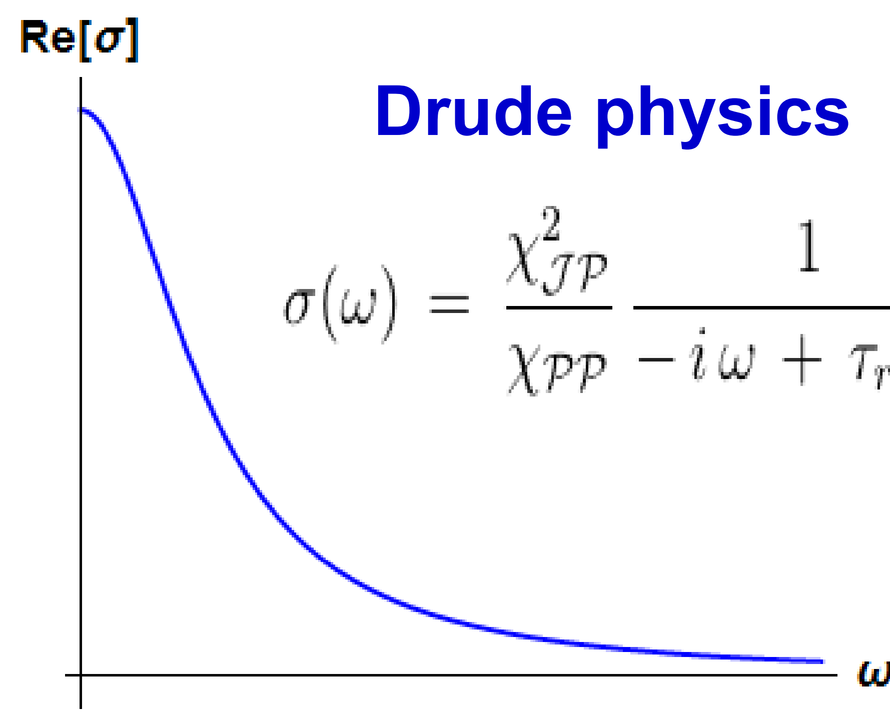
They overlap because a finite charge density !!

Momentum relaxation : $\langle \mathcal{P}(t) \rangle \sim e^{-t/\tau_{rel}}$

Weak momentum relaxation (long lived momentum) $\Gamma \ll E \sim k_B T$



$${\tau_{rel}}^{-1} \equiv \Gamma$$



Drude physics

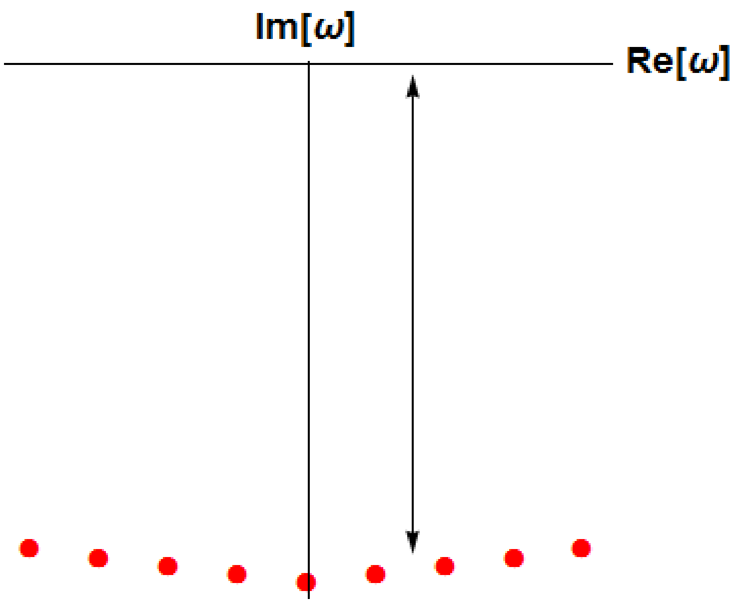
$$\sigma(\omega) = \frac{\chi^2_{\mathcal{J}\mathcal{P}}}{\chi_{\mathcal{P}\mathcal{P}}} \frac{1}{-i\omega + {\tau_{rel}}^{-1}}$$

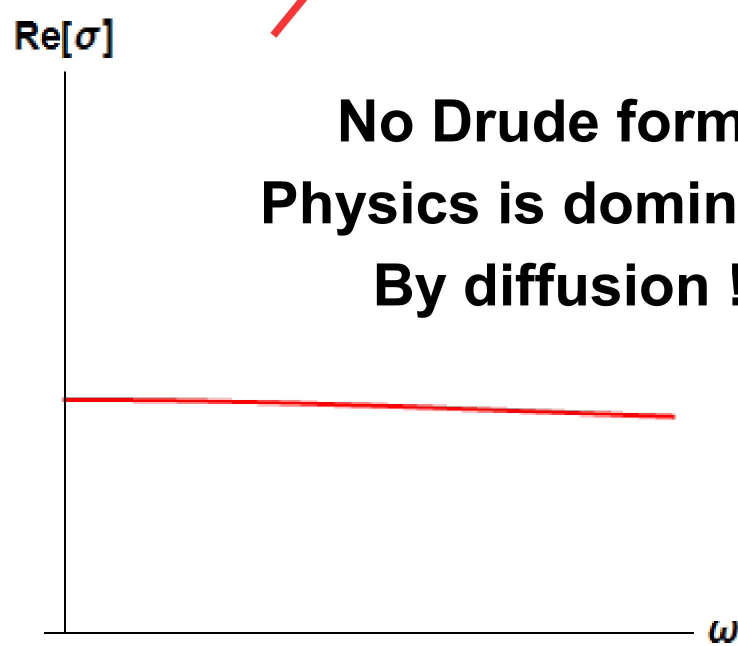
INCOHERENT METALLIC TRANSPORT

no long-lived quantities overlapping with the current operators.

Fast momentum relaxation

$$\Gamma \ll E \sim k_B T$$





No Drude form !

Physics is dominated

By diffusion !

In this limit we should approach the minimum timescale $\dfrac{\hbar}{k_B T}$

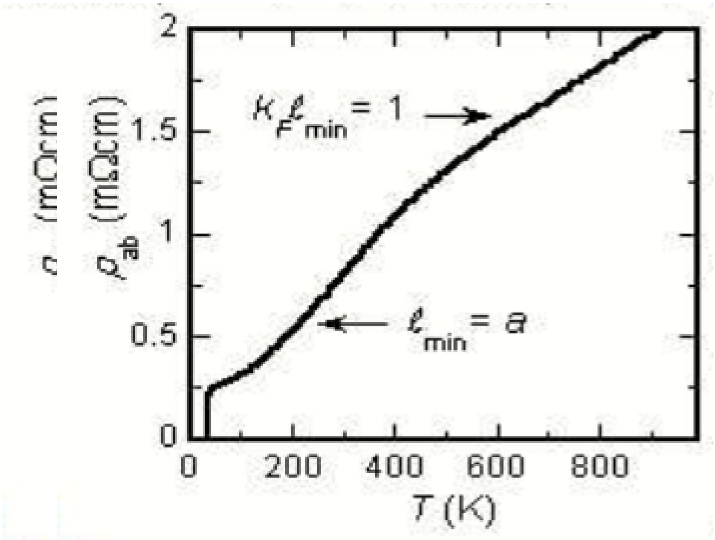
In this limit $D \geq v^2 \tau_p$ **should be saturated !!**

TRANSPORT WITHOUT QUASIPARTICLES

$\ell_{min} \sim a.$

Mott-Ioffe-Regel bound
Is brutally violated in
Bad metals

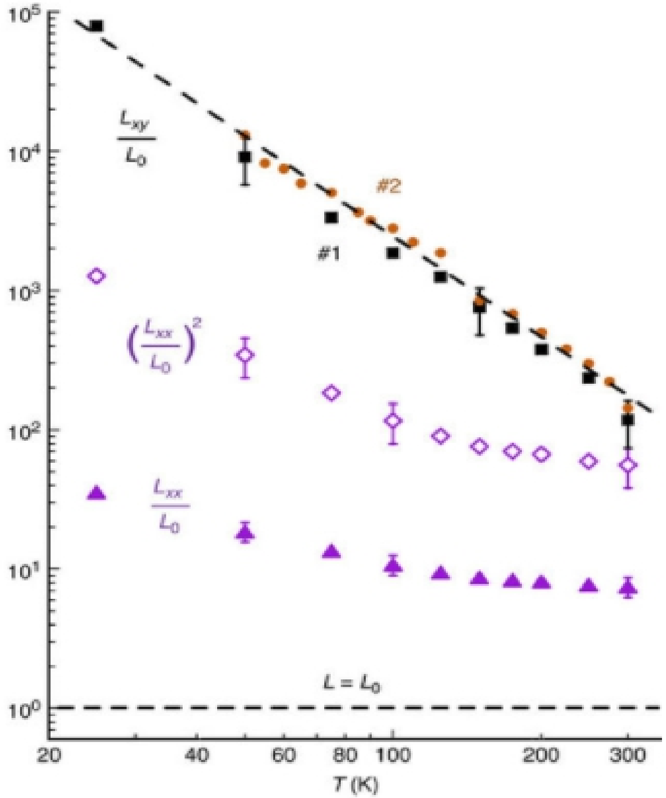
[Hussey, Takenaka, Takagi]



Wiedemann – Franz law
Is violated as well

[many authors ...]

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$



Concept of Landau quasiparticles breaks down !!!
Strong coupling / correlations

The butterfly velocity and quantum chaos

Who is the velocity v ??

It can't be the Fermi velocity !!

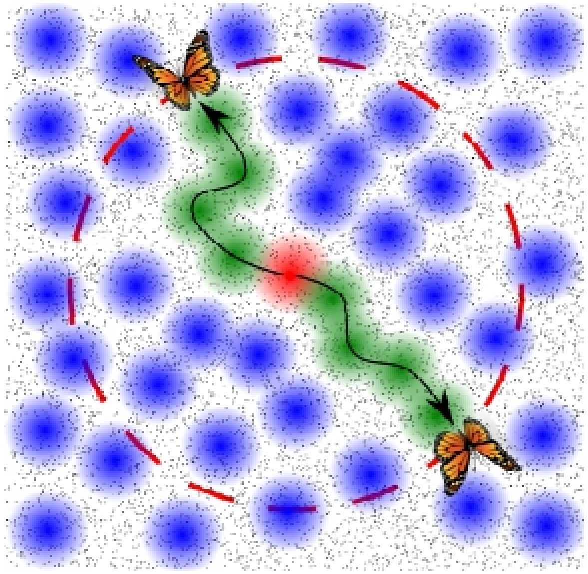
v IS THE BUTTERFLY VELOCITY = SPEED OF INFORMATION PROPAGATION

QUANTUM CHAOS
SCRAMBLING

[Shenker, Stanford, Susskind, Swingle, Maldacena, Blake, Roberts, Douglas, ...]

OUT-OF-TIME CORRELATOR

$$\langle [\mathcal{V}(x,t)\,\mathcal{W}(0,0)]^2 \rangle_\beta \sim e^{\lambda_L (t - t^* - |x|/v_B)},$$



PROPOSAL

$$\frac{D}{v_B^2} \geq C\,\frac{\hbar}{k_B T}$$

[Blake, 2016]

PHYSICS AT STRONG COUPLING

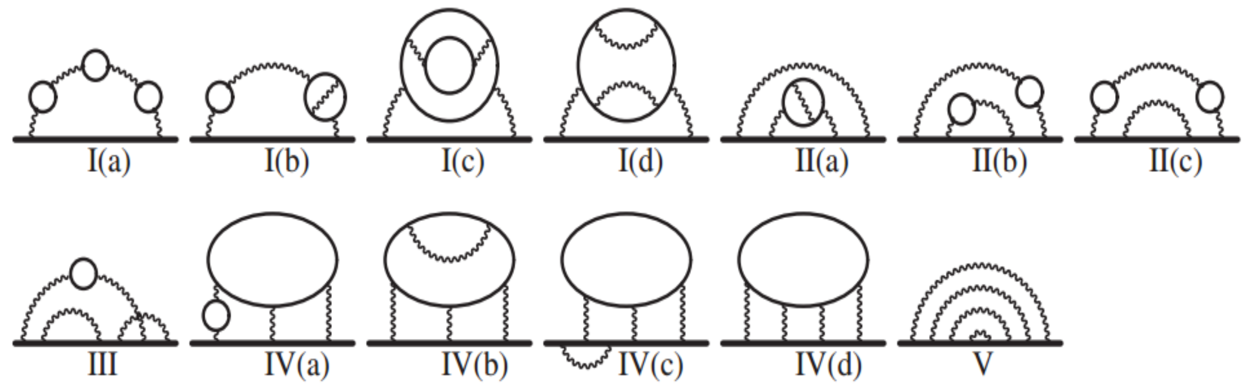
We are <u>very good</u> at computing

$$\blacksquare(g) = \mathcal{O}\left(g^0\right) + \mathcal{O}\left(g\right) + \mathcal{O}\left(g^2\right) + \ldots$$

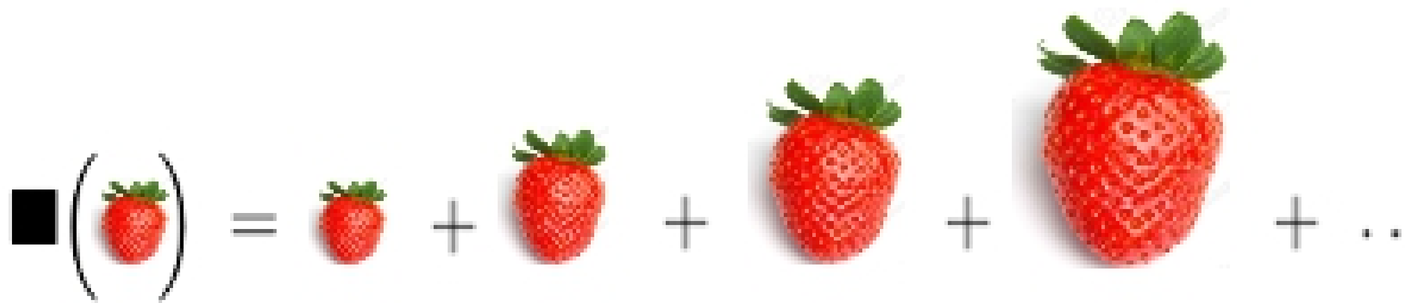
Best example

$$g_e = 2\left(1 + \frac{\alpha}{2\pi} + \cdots\right),$$

$$g_e = 2.0023193043617(15).$$



But what if the coupling is not small ??

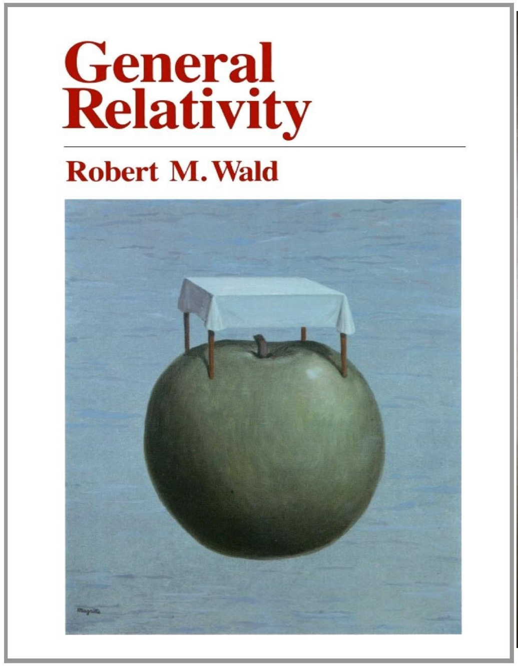


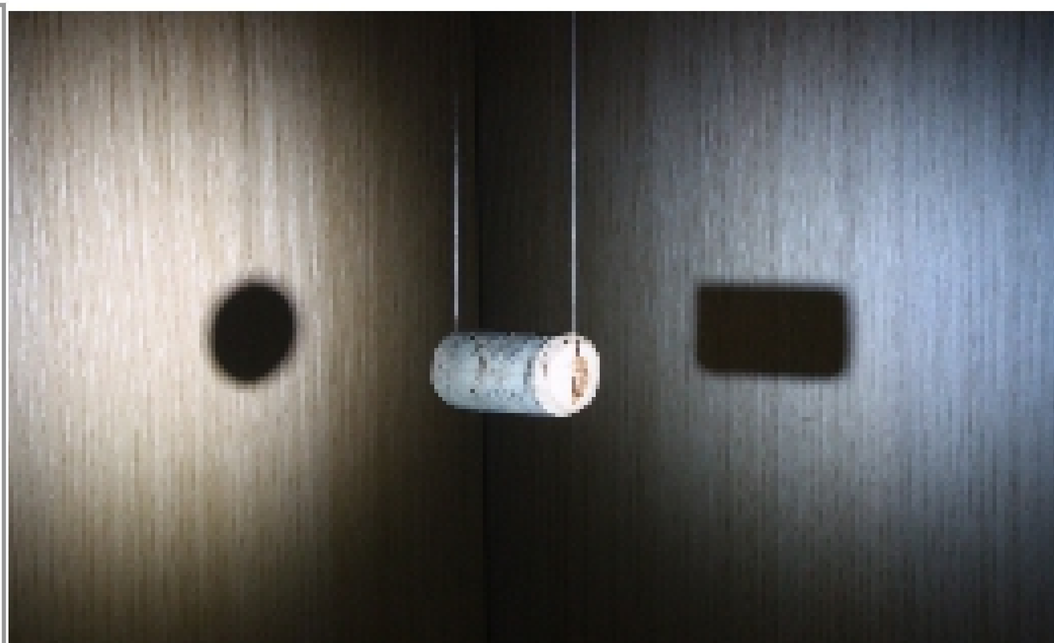
$$g \not\ll 1$$

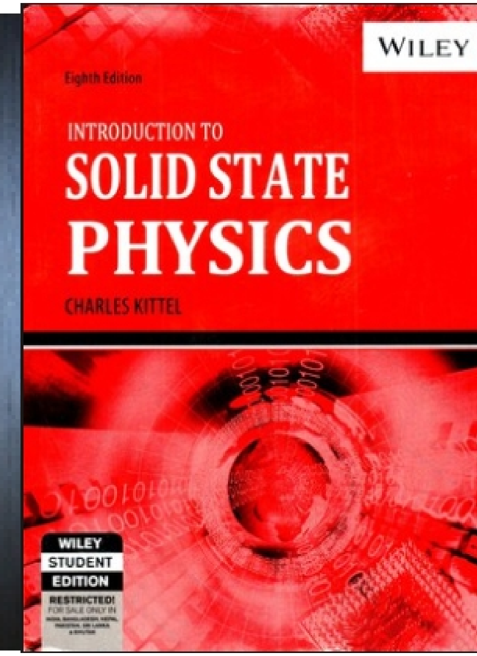
<u>STRONG COUPLING</u>

Standard (usually very efficient) methods are not useful anymore !!

IDEA : DUALITY (AdS-CMT)







Use a dual description in terms of different d.o.f.
where the theory is weakly coupled
and the computations are doable

BOTTOM – UP HOLOGRAPHY

In some limits :

STRONGLY COUPLED (LARGE N) QFT ⟺ **CLASSICAL GR + BULK FIELDS**

d | **d + 1**

$$Z_{CFT}\left[\phi_0\left(\vec{x}\right)\right] = \left\langle e^{\int d^d x \phi_0(\vec{x})\mathcal{O}(\vec{x})}\right\rangle_{CFT} = e^{-S_{gravity}[\phi(\vec{x},z=0)=\phi_0(\vec{x})]}$$

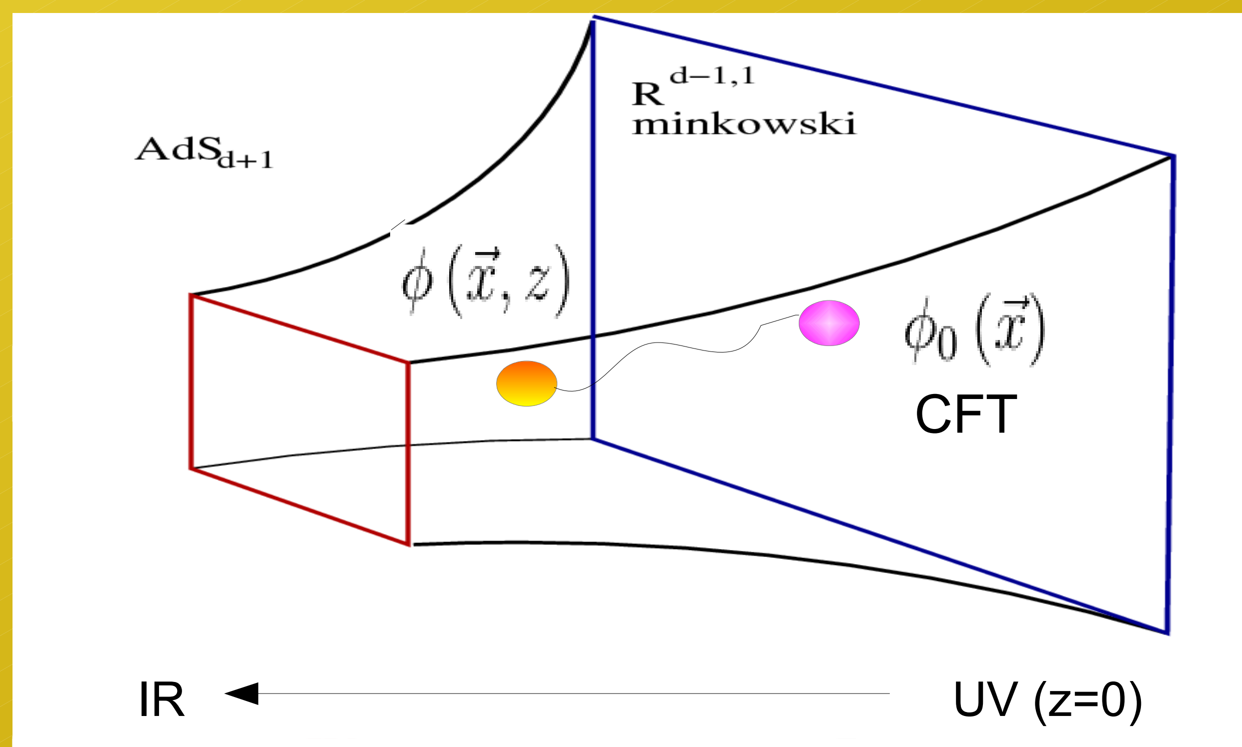
Everything !!

$J_\mu\left(\vec{x}\right)$ — $A_\mu\left(\vec{x}, z\right)$

$T_{\mu\nu}\left(\vec{x}\right)$ — $g_{\mu\nu}\left(\vec{x}, z\right)$

$$\left\langle \mathcal{O}\left(\vec{x}\right)\mathcal{O}\left(\vec{y}\right)\right\rangle$$

At strong coupling



MOMENTUM DISSIPATION

$$T_{\mu\nu}(\vec{x}) \;—\; g_{\mu\nu}(\vec{x}, z)$$

Global symmetry ⟷ **Gauged symmetry**

Translational symmetry of the CFT ⟷ **Diffeomorphism invariance Of the bulk theory**

In order to break translational invariance in the CFT
We need to break (spatial) diffeomorphisms in the bulk

[Vegh,'13]

Generic effective holographic theory With momentum dissipation → **MASSIVE GRAVITY**

[Vegh, Tong, Blake]

Translations broken Energy conserved → **LORENTZ VIOLATING MASSIVE GRAVITY**

Massive gravity and phenomenology

c

[Davison]

$$\partial_i T^{ij} = -\frac{1}{\tau_{rel}} T^{tj} \neq 0 \qquad \frac{1}{\tau_{rel}} \sim \mathcal{M}_h^2 (T,\, k,\, q,\, g_i\,, \dots)$$

[cf. MEMORY MATRIX FORMALISM , Andy's lectures]

$$\sigma = \sigma^{\mathcal{I}} + \frac{q^2}{\mathcal{M}_h^2}, \qquad \alpha = \bar{\alpha} = \frac{s\,q}{\mathcal{M}_h^2}, \qquad \bar{\kappa} = \frac{s^2\,T}{\mathcal{M}_h^2}.$$

[Donos, Gauntlett, ...]

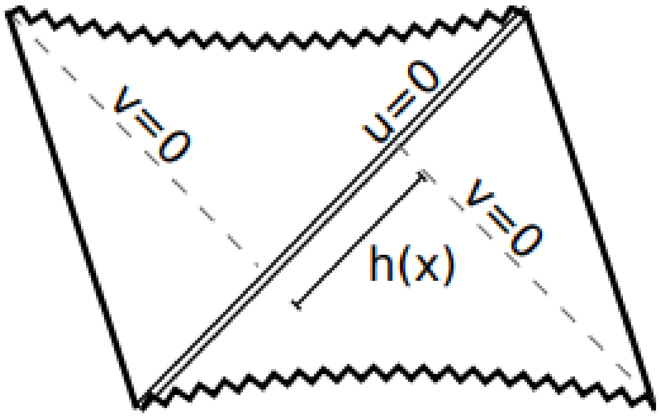
- $\mathcal{M}_h^2 \longrightarrow 0$ **in the case of translational invariance**
- $\mathcal{M}_h^2$ **is the effective, model dependent, graviton mass !**
- $\mathcal{M}_h^2 \to \infty$ **: incoherent limit**

Quantum chaos and shockwaves

c

[Susskind, Maldacena, Stanford, Douglas, Shenker, Blake, ...]

$$\langle [\mathcal{V}(x,t)\mathcal{W}(0,0)]^2 \rangle_\beta \sim e^{\lambda_L (t - t^* - |x|/v_B)},$$



Horizon shockwave $\delta T_{uu} \sim E e^{\frac{2\pi}{\beta} t_w} \delta(u)\delta(\vec{x})$

$$(\partial_i \partial_i - m^2) h(x) \sim \frac{16\pi G_N V(0)}{A(0)} E e^{\frac{2\pi}{\beta} t_w} \delta(\vec{x})$$

Time shift : $h(x) \sim \dfrac{E e^{\frac{2\pi}{\beta}(t_w - t_*) - m|x|}}{|x|^{\frac{d-1}{2}}}$

RESULTS:

$$ds^2_{d+2} = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r) d\vec{x}_d^{\,2}$$

$$m^2 = d\pi T V'(r_0)$$

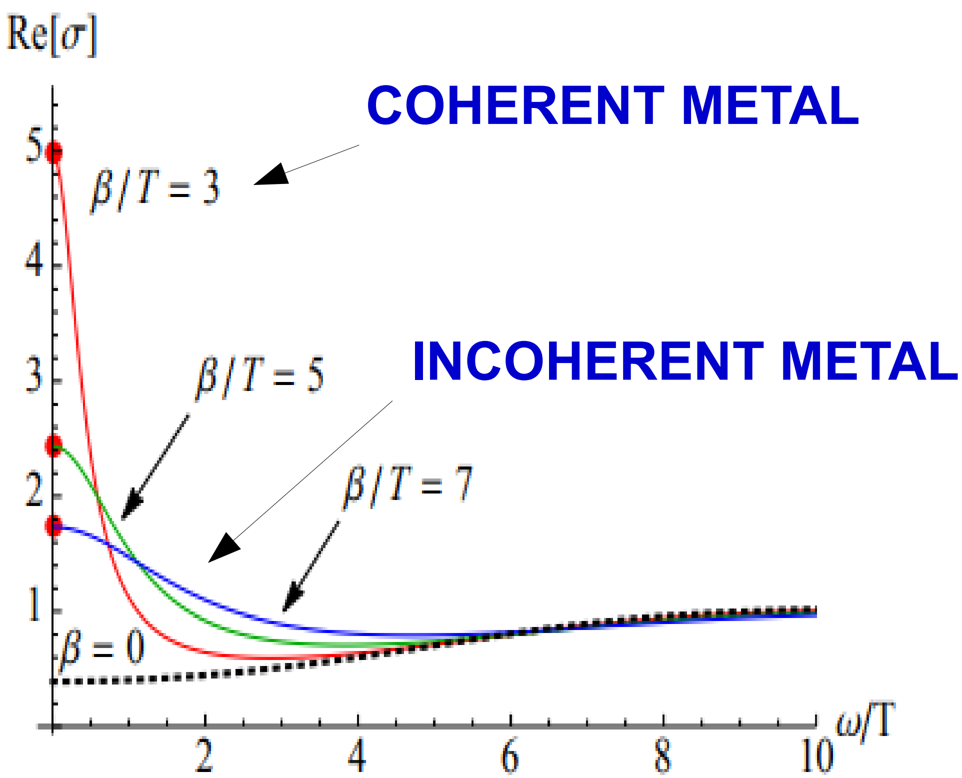
$$\lambda_L = \frac{2\pi}{\beta} \qquad v_B = \frac{2\pi}{\beta m}$$

THE SIMPLEST THEORY

[Andrade, Withers,'13] **LINEAR (~~AXIONS~~) STUECKELBERGS**

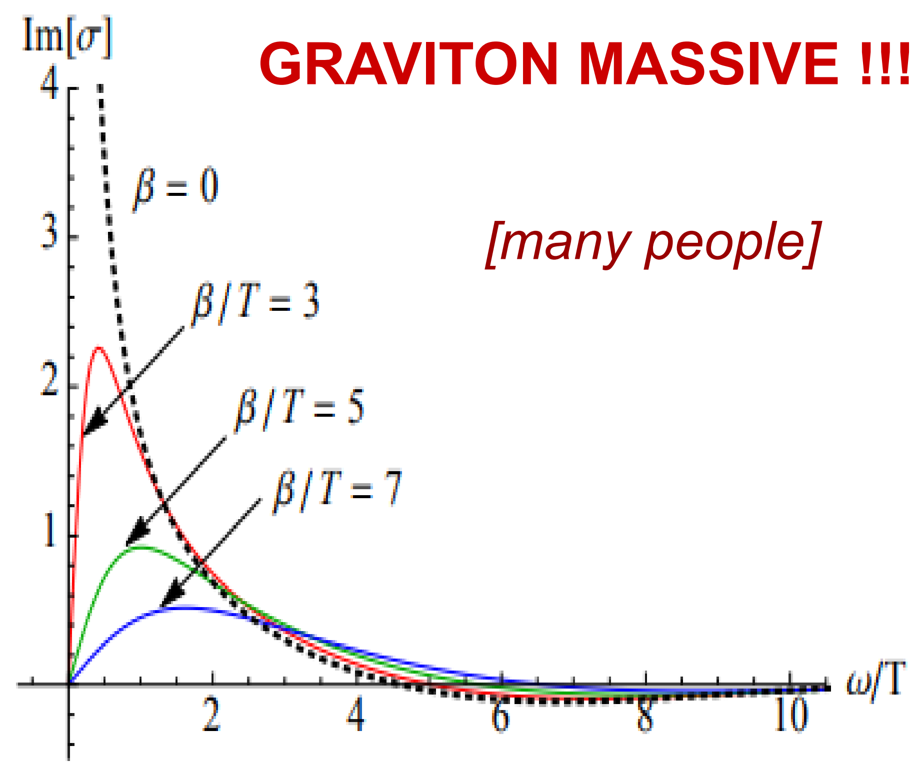
$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x$$

Massless scalars with x-dependent sources : $\psi^I = \beta\, x^I$



GRAVITON MASSIVE !!!

[many people]



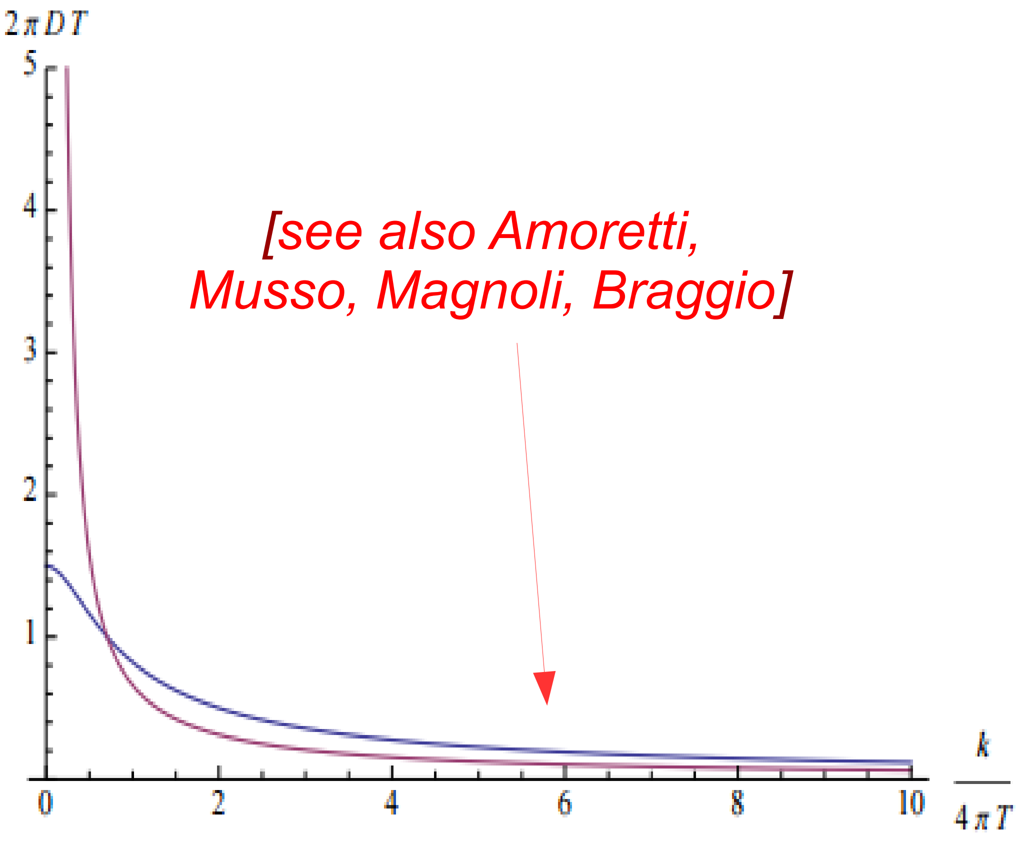
Checking the "bound" : part I

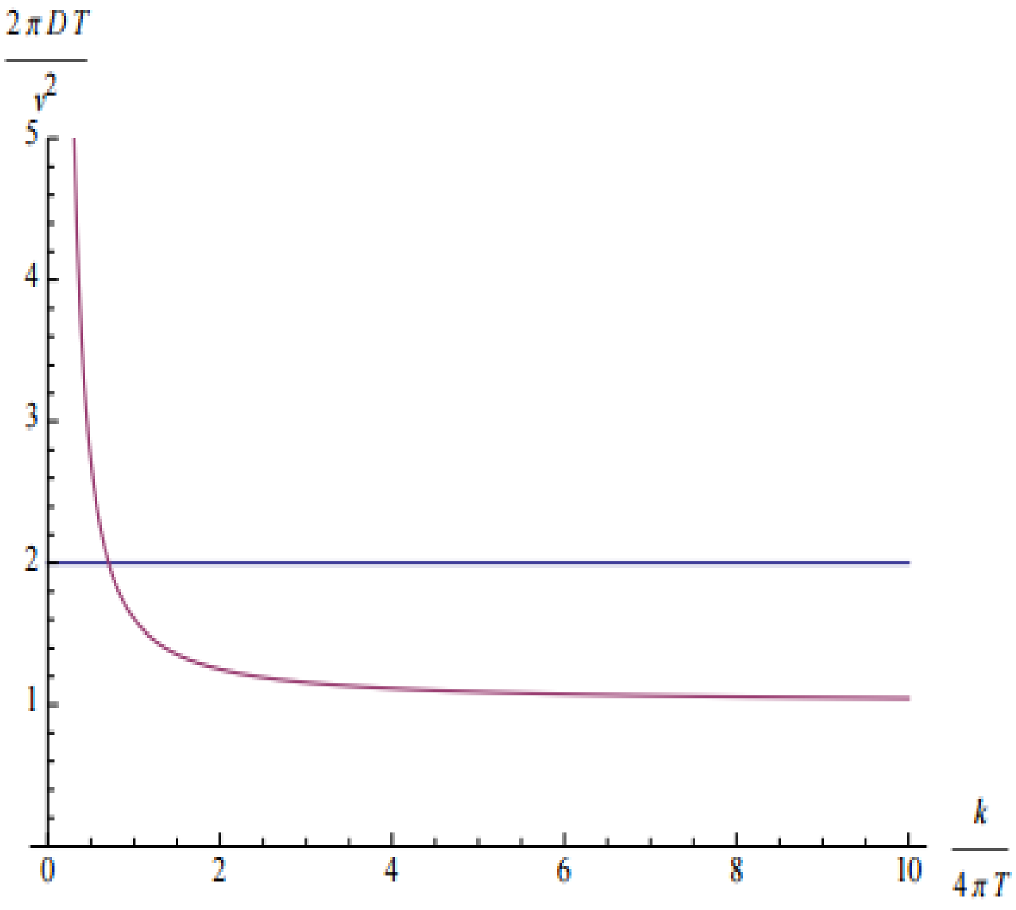
In the simplest holographic theories

[Blake]

$$D_c = \frac{v_B^2}{\pi T} \qquad D_e \approx \frac{v_B^2}{2\pi T}$$

[see also Amoretti, Musso, Magnoli, Braggio]





HOW GENERAL IS IT ?

[M.B., Kiritsis, Gouteraux, Li]

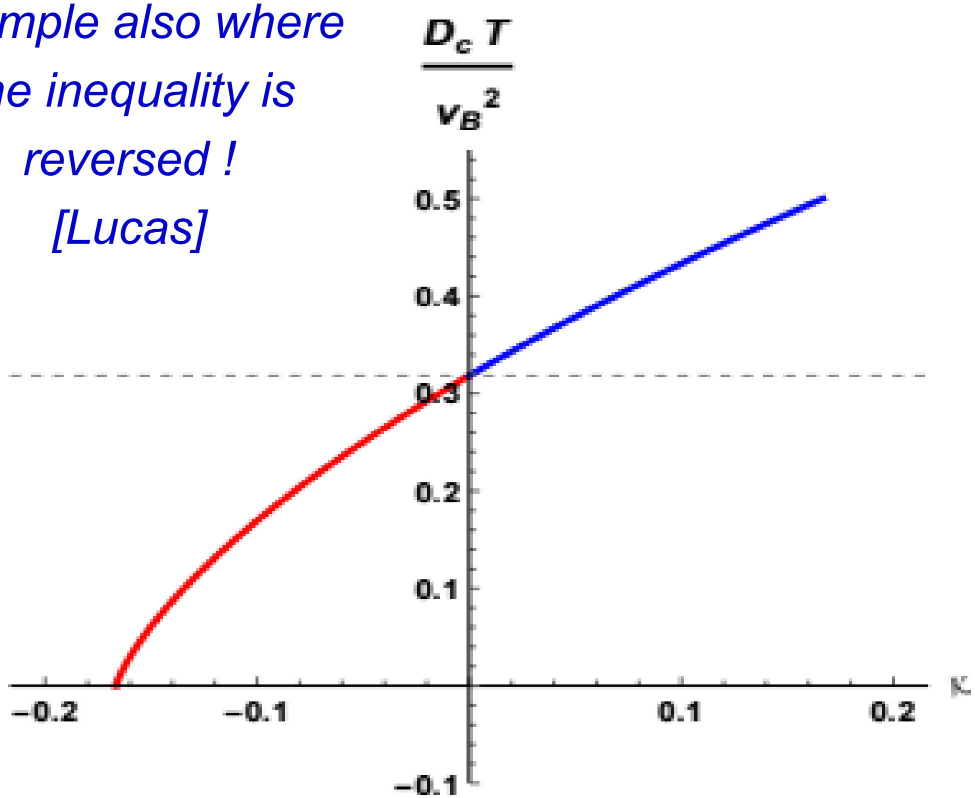
$$\mathcal{X}^{\mu}{}_{\nu} \equiv \frac{1}{2} \sum_{I=x,y} \partial^{\mu}\phi^{I}\partial_{\nu}\phi^{I}$$

HIGHER DERIVATIVES CHECKS

$$\mathcal{L} \;=\; \ldots \;-\; \frac{\mathcal{J}}{4} Tr\left[\mathcal{X}\, F^2\right]$$

$$\mathcal{L} \;=\; \ldots \;-\; \frac{F^2}{4}\left(1 + \mathcal{K}\, Tr\left[\mathcal{X}\right]\right)$$

Example also where The inequality is reversed ! [Lucas]



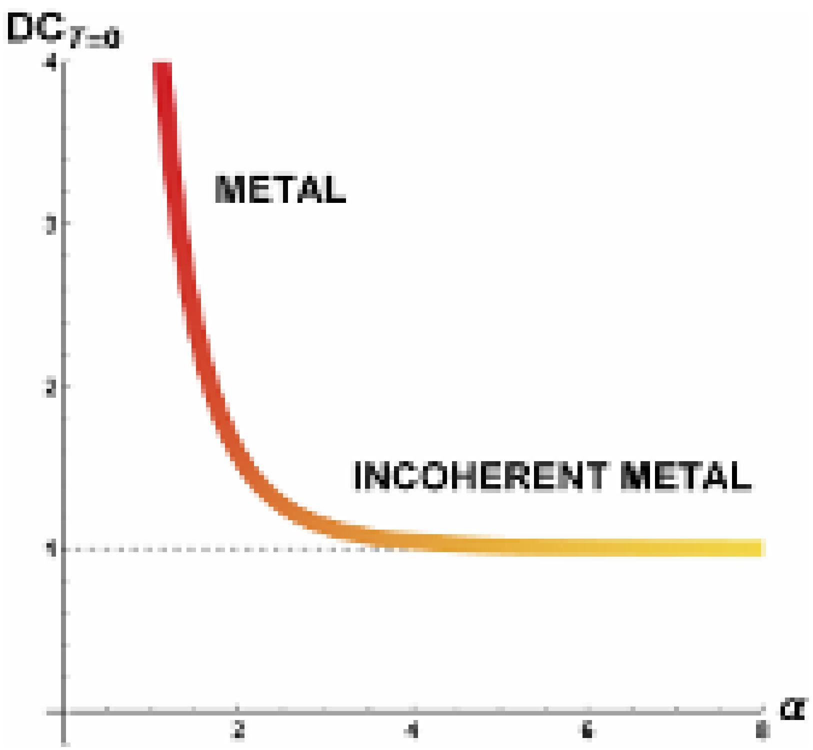
CHARGE SECTOR

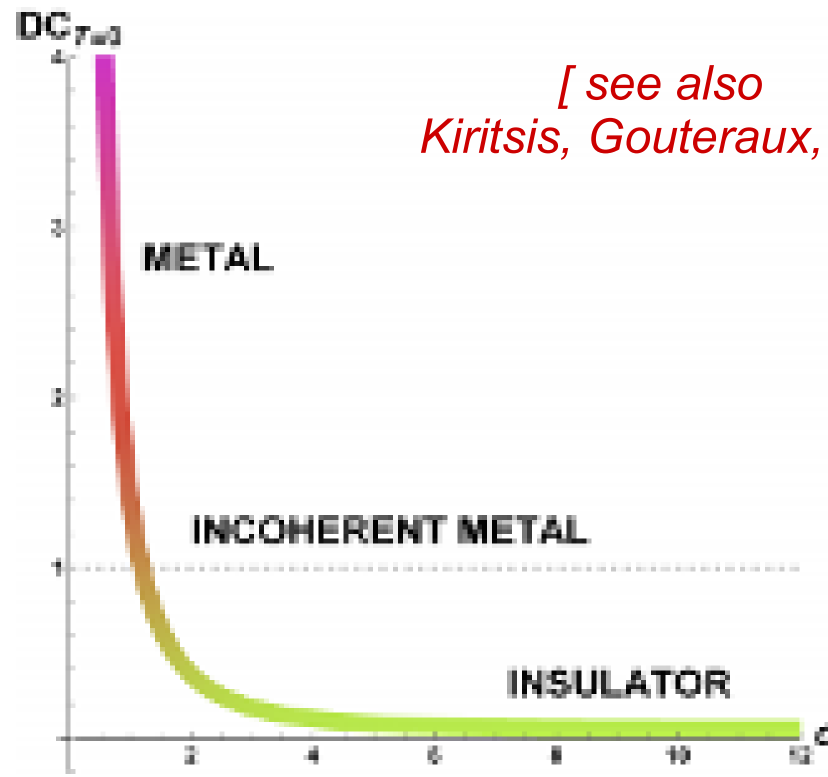
ENERGY SECTOR

[M.B. , Pujolas]

ANOTHER BOUND ✝

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{R}{2} - \Lambda - \frac{1}{4e^2} Y(X)\, F^2 - m^2 V(X)\right].$$





[see also Kiritsis, Gouteraux, Li]

$$\sigma_{DC} \not> \sigma_{min}$$

The electric conductivity Is not bounded !

STATUS AND (MY) INTUITION

$$D_c \geq v_B^2 \tau_l \qquad \sigma_{DC} > \sigma_{min}$$

D_c **Maxwell sector** $\neq$ v_B^2 **Gravitational sector**

$$D_c = \frac{\sigma}{\chi}$$

→ **The charge susceptibility is not IR data**

→ $\sigma \to 0$

$$D_e \geq v_B^2 \tau_l \qquad \frac{\kappa}{T} \geq \mathcal{C}_{min}$$

Higher derivatives check part II

Higher derivative couplings Charge sector - Momentum relaxing sector → ~~CHARGE BOUND~~

[MB, Gouteraux, Kiritsis, Li, 2016]

What about Gravity – Momentum Relaxing sector ??

$$S = \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F^2 - \frac{1}{2} (g^{\mu\nu} - \gamma G^{\mu\nu}) \sum_{i=1}^{2} \partial_\mu \phi^i \partial_\nu \phi^i \right)$$

Holographic Horndeski theories

[MB, Li, 2017]

Results: →

$$\frac{D_e}{v_B^2} \geq \mathcal{C} \frac{\hbar}{k_B T}$$

More and more checks

Lifshitz – Hyperscaling FP (dilaton couplings) ✓
[Blake, Sachdev, Davison]

AdS2 horizons ✓
[Blake, Donos]

Gauss- Bonnet ✓ → $L^2 \rightarrow L^2_{eff}\,(\lambda_{GB})$
[Wu,Wang,Ge,Tian]

Charge and magnetic field → ✓
[Blake, Sachdev, Davison] *[Kim, Niu]*

SYK ✓ → $D_2 = \frac{v_B^2}{2\pi T}$.
[Davison,Fu, Gu, Georges, Sachdev, Jensen]

Weakly coupled Fermi Liquids ✓
[Aleine, Faoro, Ioffe]

Bose-Hubbard models ✓
[Bohrdt, Endrel, Mendes, Knap]

Diffusive metals ✓
[Swingle, Chowdhury]

Electron-Phonon bad metals ✓
[Werman, Kivelson, Berg]

Critical Fermi Surfaces ✓ → $D_T \sim v_B^2\,\lambda_L^{-1}$
[Patel, Sachdev]

Beyond the incoherent limit : I

[Blake, Donos]

ENERGY DIFFUSION

$$D = \kappa / c_\rho$$

BUTTERFLY VELOCITY

$$v_B$$

$$ds^2_{d+2} = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + h(r)dx^2_A$$

$$h(r) = h_0 + c_h^0(\rho)r + \dots$$

SAME IRRELEVANT DEFORMATIONS OF AdS_2

$$D = E\frac{v_B^2}{2\pi T} \qquad 1/2 < E \leq 1$$

Where E is Related to the Conformal Dimension of The (dilaton) Deformation At the IR fixed point

No incoherent limit taken !!!

Just a property of the IR fixed point !!

Beyond the incoherent limit : II

[Blake, Davison, Sachdev]

$$ds^2_{d+2} = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + h(r)dx_i^2,$$

Generic theories :
Dilaton couplings, matter, magnetic field
(but Einstein gravity)

$$\kappa = 4\pi \frac{f' h^{d-2}}{(f' h^{d/2-1})'}\Big|_{r_0},$$

Thermal conductivity just in
Terms of metric data at the horizon !

UNIVERSAL RELATION

$$D_T \sim \frac{f' h^{d/2-1}}{(f' h^{d/2-1})'} \frac{h'}{h}\Big|_{r_0} v_B^2 \tau_L.$$

For generic
Hyperscaling - Lifshitz
IR fixed points

$$D_T = \frac{z}{2z-2} v_B^2 \tau_L,$$

Conclusions

$$\frac{D_e}{v_B^2} \geq C \, \frac{\hbar}{k_B \, T}$$

UNIVERSAL RELATION VERY ROBUST

HOLOGRAPHY + CONDENSED MATTER

Just one possible violation (inhomogeneous SYK chain) [Lucas, Gu, Qi]

[Gouteraux, Blake, Davison, Sachdev, Donos, Kiritsis,Patel,Li ,Kim, Ling,Wu, Jensen, Tian, Wang, Swingle]

[Hartnoll, Grozdanov, Lucas, Gentle, Shenker, Liu, Stanford, Phillips, Ge,Niu, Amoretti, Jin-Sin Musso, Magnoli, ...]

Holography, Condensed Matter, Quantum Chaos, Hydrodynamics, Quantum Information, Random matrix theories, Black Holes ….

Future

IS THIS BOUND REALLY UNIVERSAL ???

Higher derivative corrections

[IN PROGRESS !! M.B. , A. Amoretti]

Gravity dual of inhomogeneous SYK chain ??
KSS bound with momentum dissipation ??

CAN WE PROVE IT ?? UNDERSTAND IT BETTER !!
vB? Strong coupling ↔ Chaos?

IF UNIVERSAL, CAN WE MEASURE IT ?? CAN WE TEST IT ?

감사합니다
[kamsahamnida]