Renormalization Group Transformation of One-dimensional Ising spins by linear-weight projector

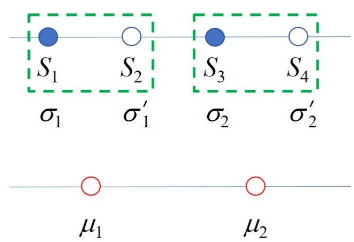
In this project we consider one-dimensional N Ising spins with the Hamiltonian

$$\bar{K}[S] \equiv -\beta\mathcal{H}[S] = K\sum_{i=1}^{N} S_i S_{i+1} + h\sum_{i=1}^{N} S_i.$$

Note that $S_{N+1} \equiv S_1$. We use a projector operator with linear weight defined by

$$P(\mu,\sigma) = \prod_{I=1}^{N/2} \frac{1}{2}[1 + f\mu_I(\sigma_I + \sigma_I')],$$

where the schematic block spins are represented in the figure.



1. Find the constants a_1, a_2, a_3 with the identification

$$\frac{1}{2}[1 + f\mu_I(\sigma_I + \sigma_I')] = a_1 \exp[a_2\mu(\sigma + \sigma') - a_3\sigma\sigma']$$

2. Use the above result to express $e^{\bar{K}'[\mu]}$. Take the trace over $\{\sigma_I, \sigma_I'\}$. In the process use the value $f = [1 - e^{-4K}]^{1/2}/2$ and explain why the value is convenient.

3. Find the recursion relations between (K, h) and (K', h'). Express them in terms of $x \equiv e^{-4K}$ and $y \equiv e^{-2h}$.

4. Find all fixed points. Near ferromagnetic fixed point $(T^* = 0, H^* = 0)$ find linearized renormalization group transformation and compare the scaling behaviors with the results by the decimation projector.

5. Draw renormalization group flows in the entire parameter space.