
Parton distributions in hadrons: nucleon and pion

Wally Melnitchouk



JLab Angular Momentum collaboration



CTEQ-Jefferson Lab collaboration

Outline

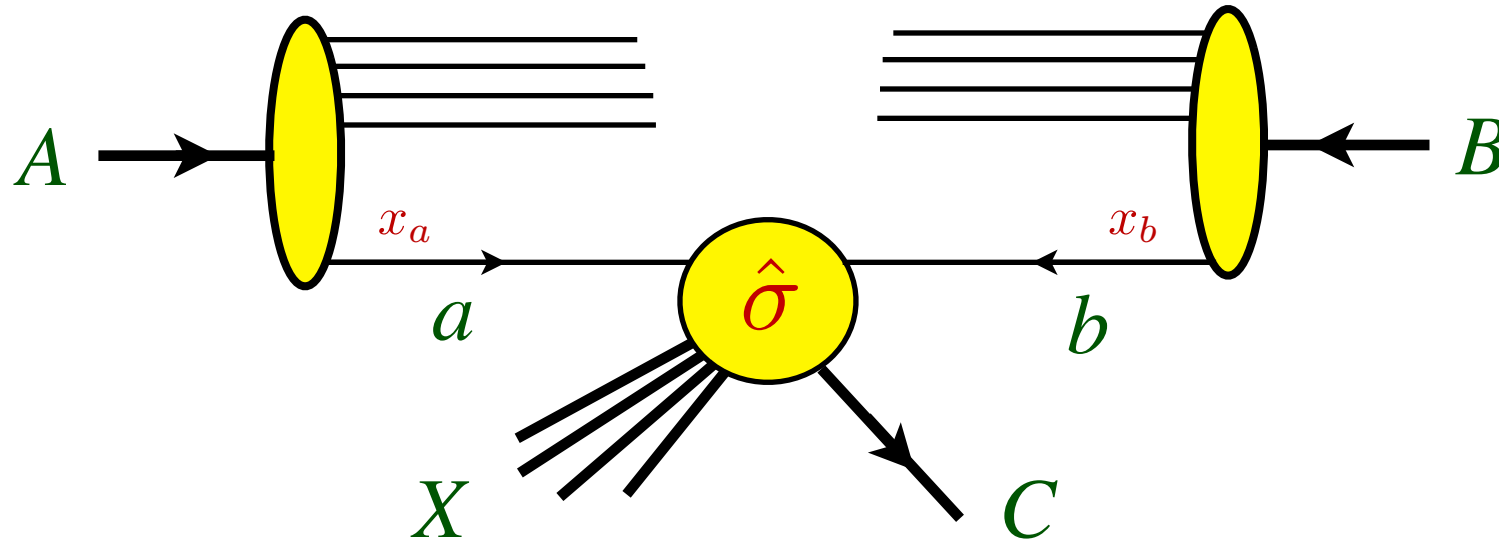
- *Aim*: understand internal quark-gluon structure of hadrons
- *Method*: extract parton distribution functions (PDFs) from global QCD analysis, using new Monte Carlo-based methods

Recent highlights:

- Constraints from Fermilab & JLab data on unpolarized PDFs at high x
- First combined analysis of polarized DIS + SIDIS + SIA data, with *simultaneous* extraction of PDFs & fragmentation functions
- First MC analysis of nucleon's transversity PDFs + lattice QCD
- First extraction of pion PDFs from Drell-Yan and HERA leading neutron production data

Parton distributions in hadrons

- Generic process: inclusive particle production $AB \rightarrow CX$



$$\sigma_{AB \rightarrow CX}(p_A, p_B) = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, \mu) f_{b/B}(x_b, \mu) \times \sum_n \alpha_s^n(\mu) \hat{\sigma}_{ab \rightarrow CX}^{(n)}(x_a p_A, x_b p_B, Q/\mu)$$

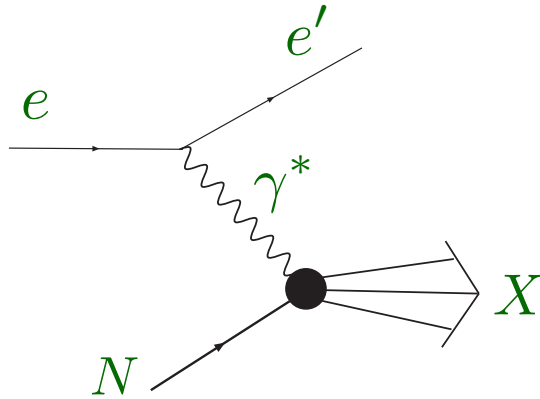
“factorization”

Collins, Soper, Sterman (1980s)

→ universal functions $f_{a/A}$ characterize internal structure of bound state A

Parton distributions in hadrons

- Most information on parton distribution functions obtained from inclusive deep-inelastic scattering (DIS)

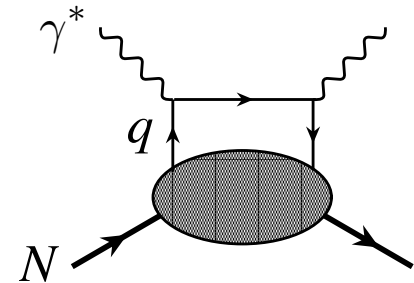


$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{2M} + \frac{F_2}{\nu} \right)$$

$$\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^2 - \nu^2 \\ W^2 &= M^2 + Q^2 \frac{(1-x)}{x} \end{aligned} \quad x = \frac{Q^2}{2M\nu}$$

- At leading order (LO) in pQCD, structure functions given in terms of charge-weighted sums of PDFs

$$F_2(x, Q^2) = x \sum_q e_q^2 q(x, Q^2)$$

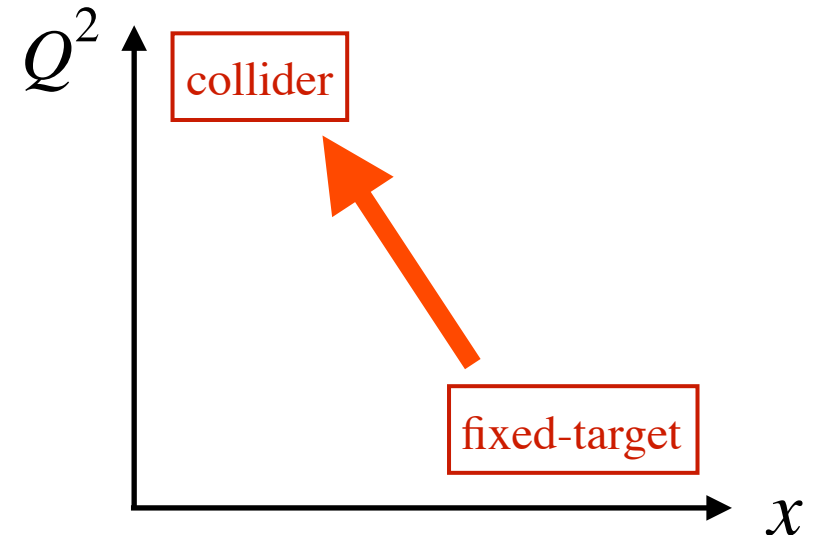


Parton distributions in hadrons

• Precision PDFs needed to

- (1) understand basic structure of QCD bound states
- (2) compute backgrounds in searches for BSM physics

→ Q^2 evolution feeds
low x , high Q^2 (“LHC”)
from high x , low Q^2 (“JLab”)



• Information on PDFs obtained from

- (1) nonperturbative approaches (low-energy models, DSE, χ EFT)
- (2) lattice QCD
- (3) global QCD analysis

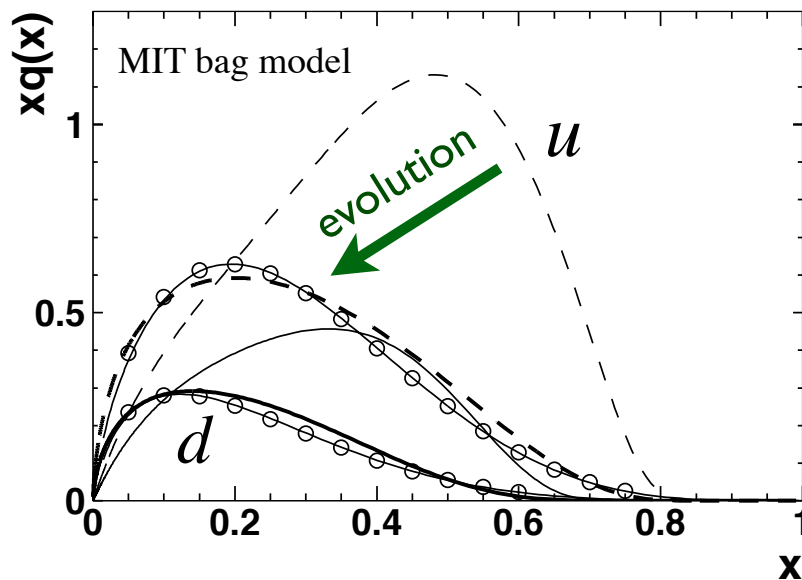
Parton distributions in hadrons

- Compute matrix element of leading twist operator at a low scale μ ; evolve PDF to higher Q^2 using DGLAP

$$q(x, \mu) = \frac{M}{(2\pi)^3} \sum_n \int d^3 p_n |\langle n | \gamma_- \gamma_+ \psi(0) | N \rangle|^2 \delta((1-x)M - p_n^+)$$

Jaffe (1975, 1983)

Parisi, Petronzio (1976)



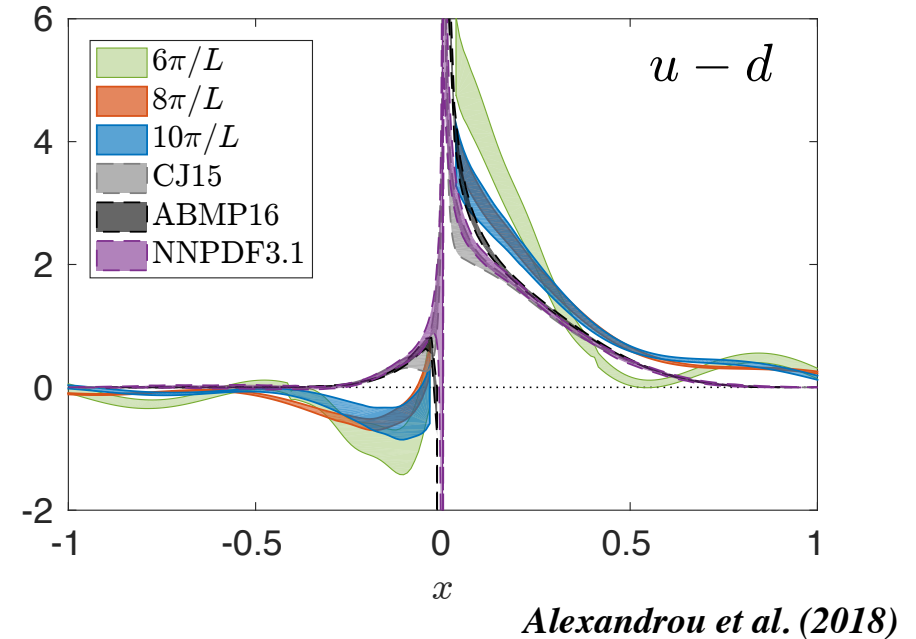
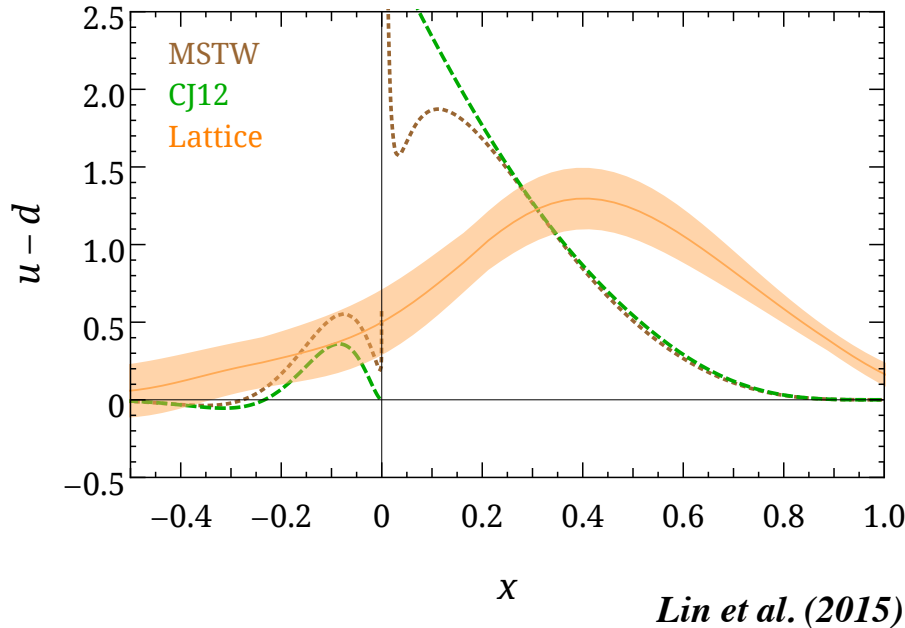
Boros, Thomas (1999)

- reasonable phenomenology, but evolution (questionable at low scales $\sim 0.1-0.5 \text{ GeV}^2$??) “washes out” many differences between models

Parton distributions in hadrons

More recently, significant progress made towards computing x dependence of PDFs directly on lattice

*X. Ji (2013); Ma, Qiu (2014)
Radyushkin (2017)*



$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C(\xi, \mu/xP_z) \tilde{q}(x/\xi, \mu, P_z)$$

$$\tilde{q}(x, P) = \int \frac{dz}{4\pi} e^{-ixPz} \langle P | \bar{\psi}(0, z) \gamma_z W(z) \psi(0, 0) | P \rangle$$

“quasi-PDF”

Global PDF analysis

- Universality of PDFs allows data from different processes (DIS, SIDIS, jet production, Drell-Yan ...) to be analyzed simultaneously
- Several dedicated global efforts to extract PDFs using factorization theorems + pQCD at a given order in α_s
 - CTEQ, MRS/MMHT, HERAPDF, DSSV, ...
use standard maximum likelihood methods (χ^2 minimization)
 - NNPDF, JAM
use Monte Carlo methods (neural networks, nested sampling)
- Typically PDF parametrizations are nonlinear functions of PDF parameters, e.g. $f(x, \mu) = N x^\alpha (1-x)^\beta P(x)$ where P is a polynomial, neural net, ...
 - *multiple local minima* present in the χ^2 function
 - thoroughly scan over sufficiently large parameter space

Global PDF analysis

- A major challenge has been to characterize PDF *uncertainties*, especially in the presence of *tensions* among data sets
- Previous attempts sought to address tensions in data sets by introducing
 - “tolerance” factors (artificially inflating PDF errors)
 - “neural net” parametrization (instead of polynomial parametrization), together with MC techniques
- However, to address the problem in a more statistically rigorous way, one requires going *beyond* the standard χ^2 minimization paradigm
 - utilize modern techniques based on Bayesian statistics

Bayesian approach to global analysis

- Analysis of data requires estimating expectation values E and variances V of “observables” \mathcal{O} (functions of PDFs) which are functions of parameters

$$E[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|\text{data}) \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|\text{data}) [\mathcal{O}(\vec{a}) - E[\mathcal{O}]]^2$$

“Bayesian master formulas”

- Using Bayes’ theorem, probability distribution \mathcal{P} given by

$$\mathcal{P}(\vec{a}|\text{data}) = \frac{1}{Z} \mathcal{L}(\text{data}|\vec{a}) \pi(\vec{a})$$

in terms of the likelihood function \mathcal{L}

Bayesian approach to global analysis

Likelihood function

$$\mathcal{L}(\text{data}|\vec{a}) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

is a Gaussian form in the data, with χ^2 function

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\text{data}_i - \text{theory}_i(\vec{a})}{\delta(\text{data})}\right)^2$$

with priors $\pi(\vec{a})$ and “evidence” Z

$$Z = \int d^n a \mathcal{L}(\text{data}|\vec{a}) \pi(\vec{a})$$

→ Z tests if *e.g.* an n -parameter fit is statistically different from $(n+1)$ -parameter fit

Bayesian approach to global analysis

- Standard method for evaluating E, V via maximum likelihood

→ maximize probability distribution

$$\mathcal{P}(\vec{a}|\text{data}) \rightarrow \vec{a}_0$$

→ if \mathcal{O} is linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0), \quad V[\mathcal{O}(\vec{a})] \rightarrow \text{Hessian} \quad H_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2(\vec{a})}{\partial a_i \partial a_j} \Big|_{\vec{a}=\vec{a}_0}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method often fails

→ need more robust (Monte Carlo) approach

$$E[\mathcal{O}] \approx \frac{1}{N} \sum_k \mathcal{O}(\vec{a}_k), \quad V[\mathcal{O}] \approx \frac{1}{N} \sum_k [\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]]^2$$

Monte Carlo methods

- First group to use MC for global PDF analysis was NNPDF, using neural network to parametrize $P(x)$ in

Forte et al. (2002)

$$f(x) = N x^\alpha (1 - x)^\beta P(x)$$

— α, β are fitted “preprocessing coefficients”

- Iterative Monte Carlo (IMC), developed by JAM Collaboration, variant of NNPDF, tailored to non-neutral net parametrizations

N. Sato et al. (2016)

- Markov Chain MC (MCMC) / Hybrid MC (HMC)

— recent “proof of principle” analysis, ideas from lattice QCD

Gbedo, Mangin-Brinet (2017)

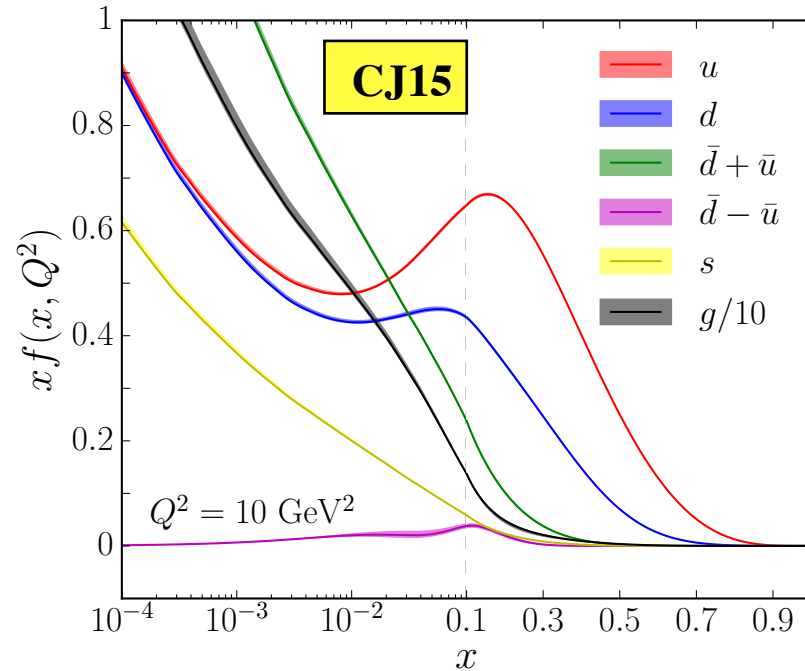
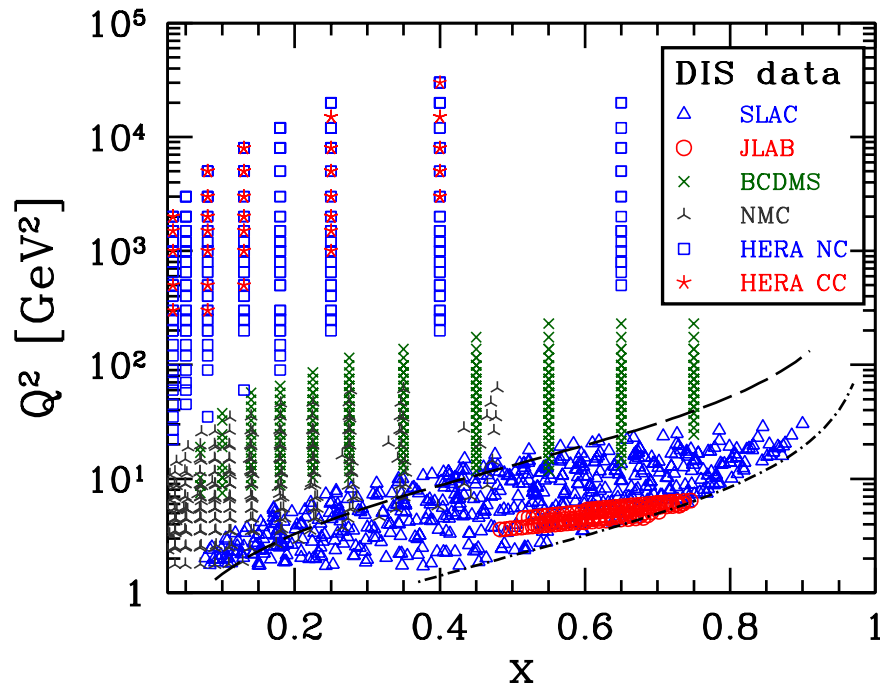
- Nested sampling (NS) — computes integrals in Bayesian master formulas (for E, V, Z) explicitly

Skilling (2004)

Unpolarized Nucleon PDFs

Unpolarized PDFs

- Ubiquity of proton F_2 data (SLAC, BCDMS, NMC, HERA, JLab, ...) provides strong constraints on u -quark PDF over large x range



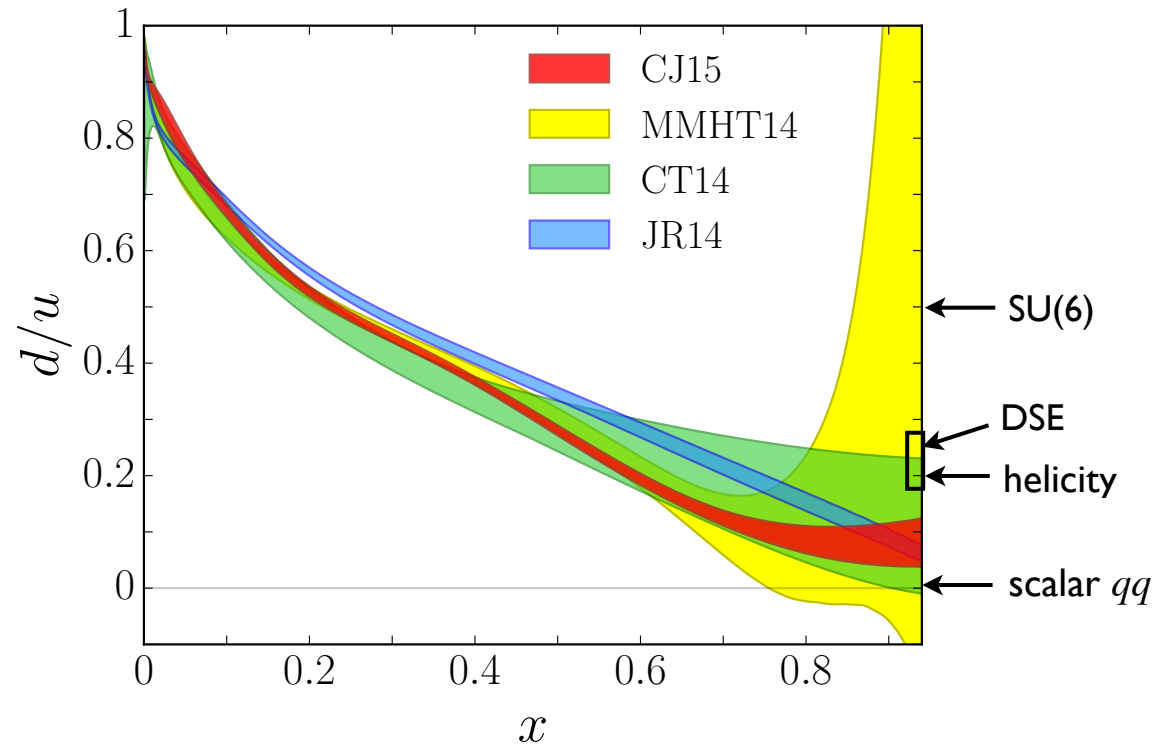
- Absence of free-neutron data and smaller $|e_q|$ of d quarks limit precision of d -quark PDF, especially at high x
 - nuclear effects in deuterium obscure free-neutron structure

Unpolarized PDFs

Valence d/u ratio at high x of particular interest

→ testing ground for nucleon models in $x \rightarrow 1$ limit

- $d/u \rightarrow 1/2$
SU(6) symmetry
- $d/u \rightarrow 0$
 $S = 0$ qq dominance
(color-hyperfine interaction)
- $d/u \rightarrow 1/5$
 $S_z = 0$ qq dominance
(perturbative gluon exchange)
- $d/u \rightarrow 0.18 - 0.28$
DSE with qq correlations



→ considerable uncertainty at high x from deuterium corrections (no free neutrons!)

$$F_2^d(x, Q^2) = \int_x^1 dy f(y, \gamma) F_2^N(x/y, Q^2)$$

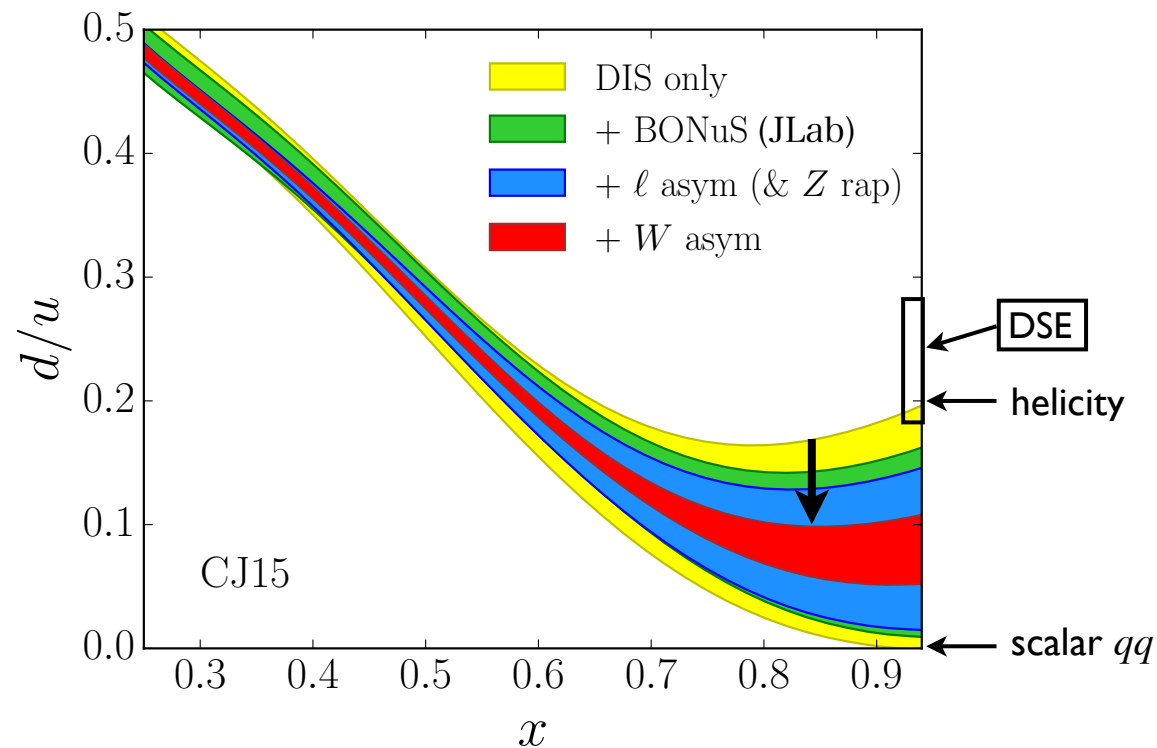
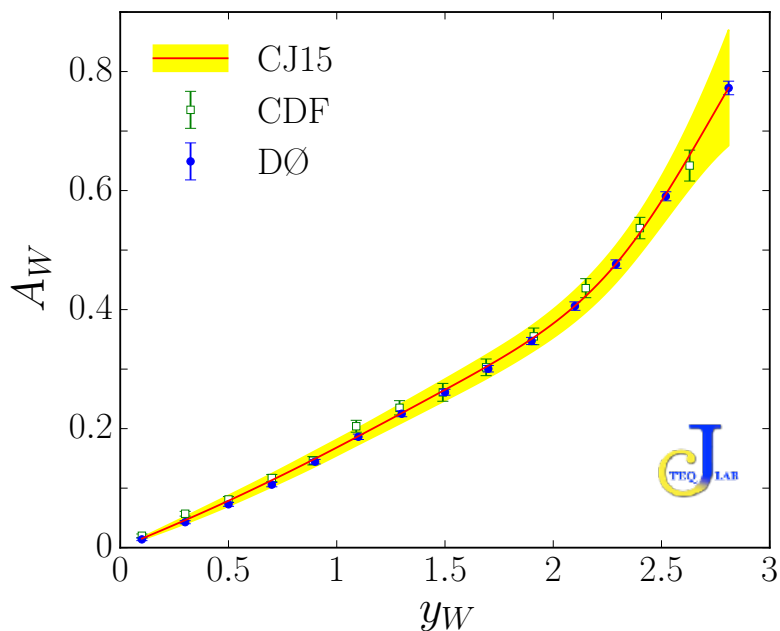
$$f(y, \gamma) = \int \frac{d^3p}{(2\pi)^3} |\psi_d(p)|^2 \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right) \times \frac{1}{\gamma^2} \left[1 + \frac{\gamma^2 - 1}{y^2} \left(1 + \frac{2\varepsilon}{M} + \frac{\vec{p}^2}{2M^2} (1 - 3\hat{p}_z^2) \right) \right]$$

Unpolarized PDFs

Valence d/u ratio at high x of particular interest

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL W -asymmetry data

$$d + \bar{u} \rightarrow W^- \rightarrow \ell^- + \bar{\nu}$$



→ extrapolated ratio at $x = 1$

$$d/u \rightarrow 0.09 \pm 0.03$$

does not match any model...

→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine d/u up to $x \sim 0.85$

Nucleon Helicity PDFs

Proton spin structure

- Question of how proton spin decomposed into its q & g constituents has engrossed community for > 30 years

→ in nonrelativistic quark model, spin of proton is carried entirely by quarks $\Delta\Sigma = \Delta u^+ + \Delta d^+ + \Delta s^+ = 1$ while early data suggested that

$$\Delta q^+ \equiv \Delta q + \Delta \bar{q}$$

$$\Delta\Sigma \approx 0! \quad \Delta s^+ \approx -(0.1 - 0.2)$$

EMC (1988)

→ proton spin sum requires

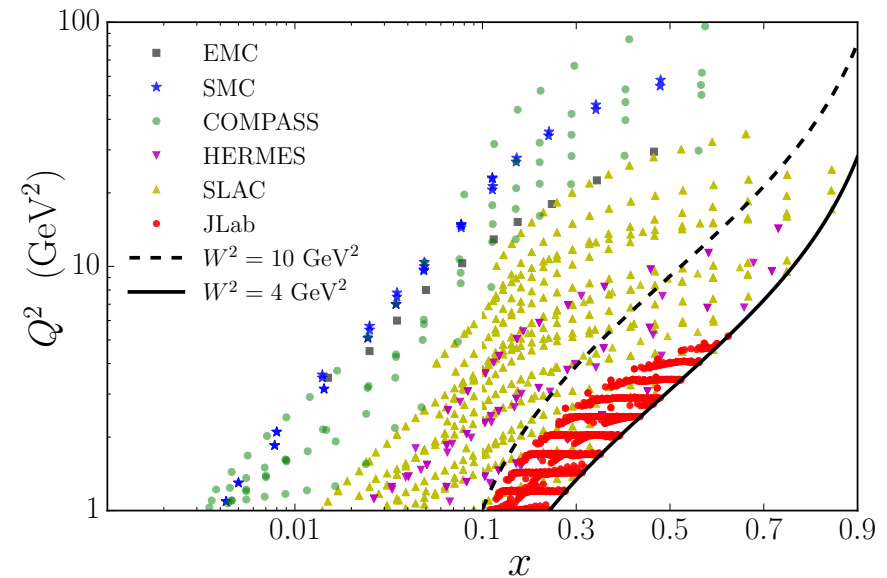
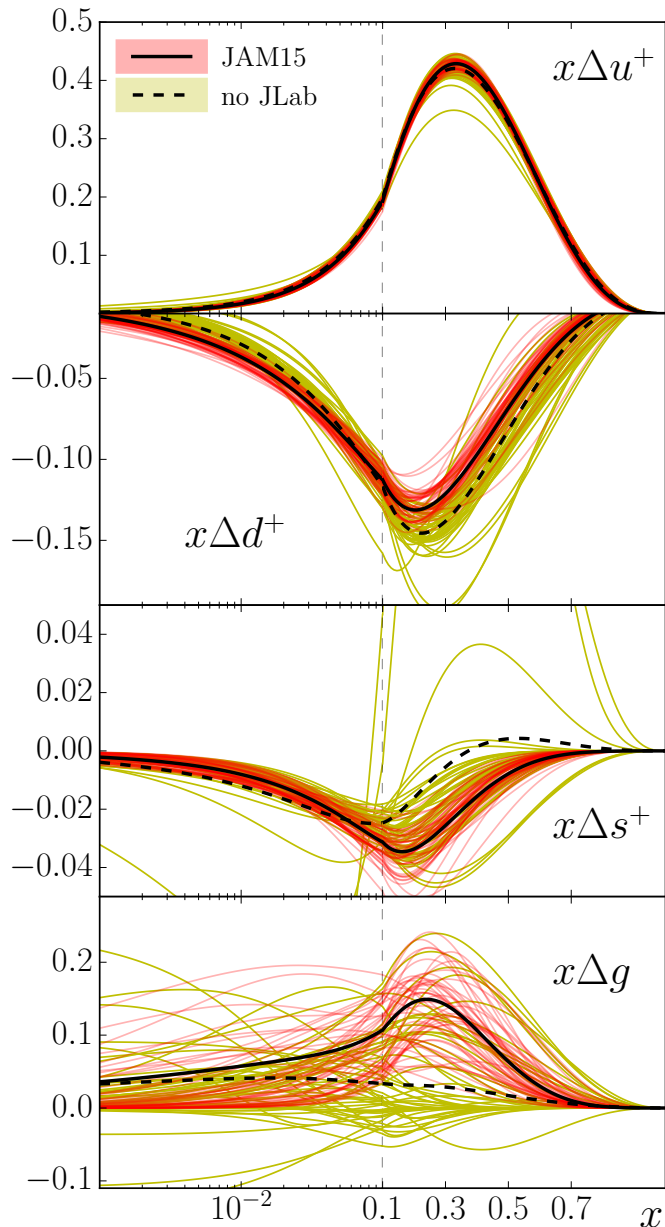
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

... does remaining spin come from large *gluon* polarization or *orbital* angular momentum?

- stimulated many advances in theory, experiment & analysis
 - recent JAM global analyses, including JLab 6 GeV data

Proton spin structure

Impact of JLab data



- inclusion of JLab data increases # data points by factor ~ 2
- reduced uncertainty in Δs^+ , Δg through Q^2 evolution
- s -quark polarization *negative* from inclusive DIS data (assuming SU(3) symmetry)

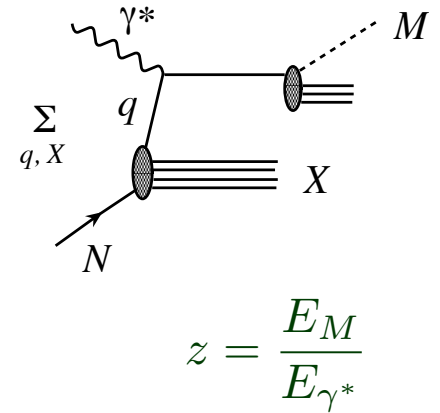
Polarization of quark sea?

- Inclusive DIS data cannot distinguish between q and \bar{q}

→ semi-inclusive DIS sensitive to Δq & $\Delta \bar{q}$

$$\sim \sum_q e_q^2 [\Delta q(x) D_q^h(z) + \Delta \bar{q}(x) D_{\bar{q}}^h(z)]$$

→ but need fragmentation functions!



- Global analysis of DIS + SIDIS data gives different *sign* for strange quark polarization for different fragmentation functions!

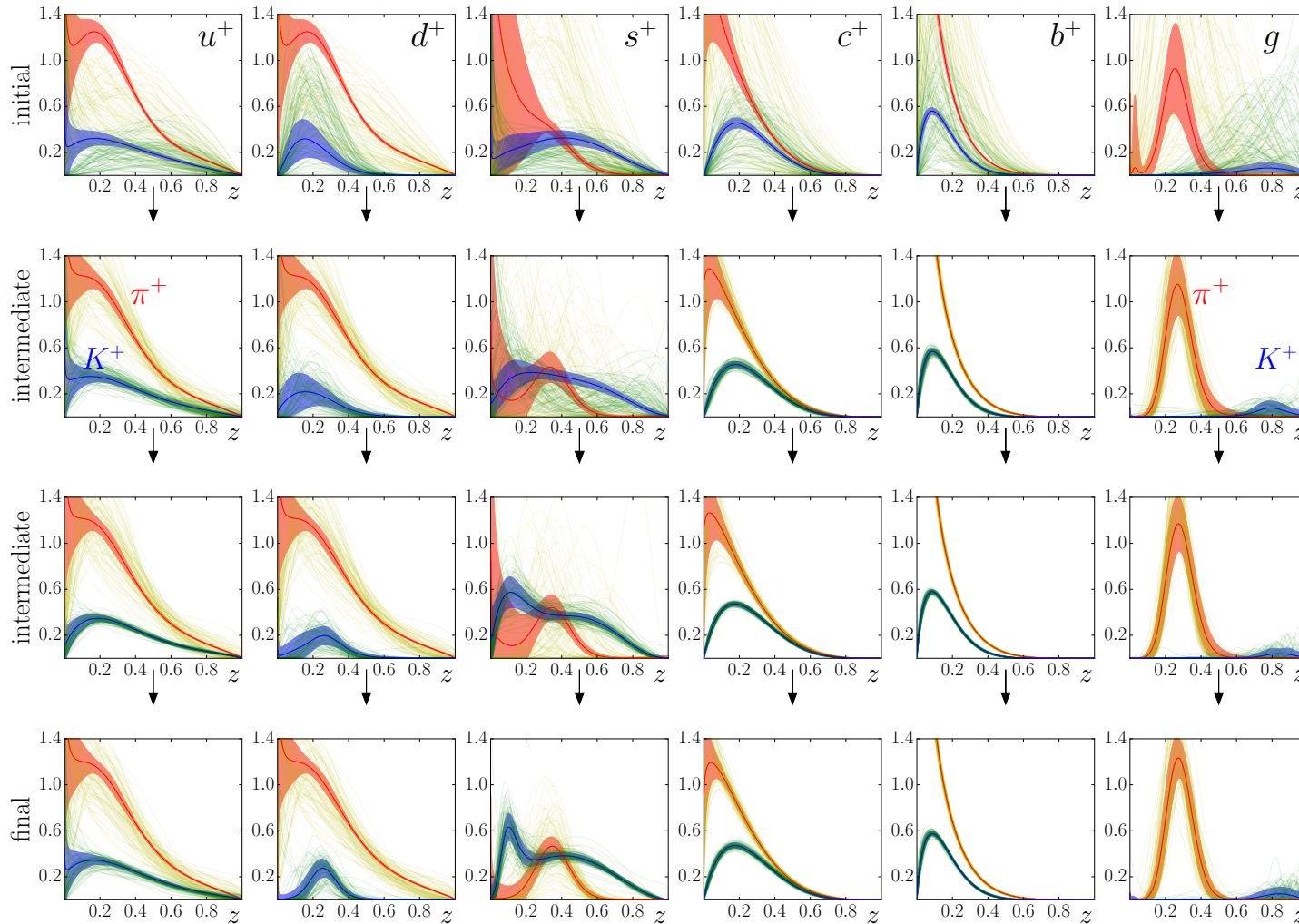
→ $\Delta s > 0$ for “DSS” FFs *de Florian et al. (2007)*

$\Delta s < 0$ for “HKNS” FFs *Hirai et al. (2007)*

→ need to understand origin of differences in fragmentation!

Polarization of quark sea?

● First MC analysis of fragmentation functions



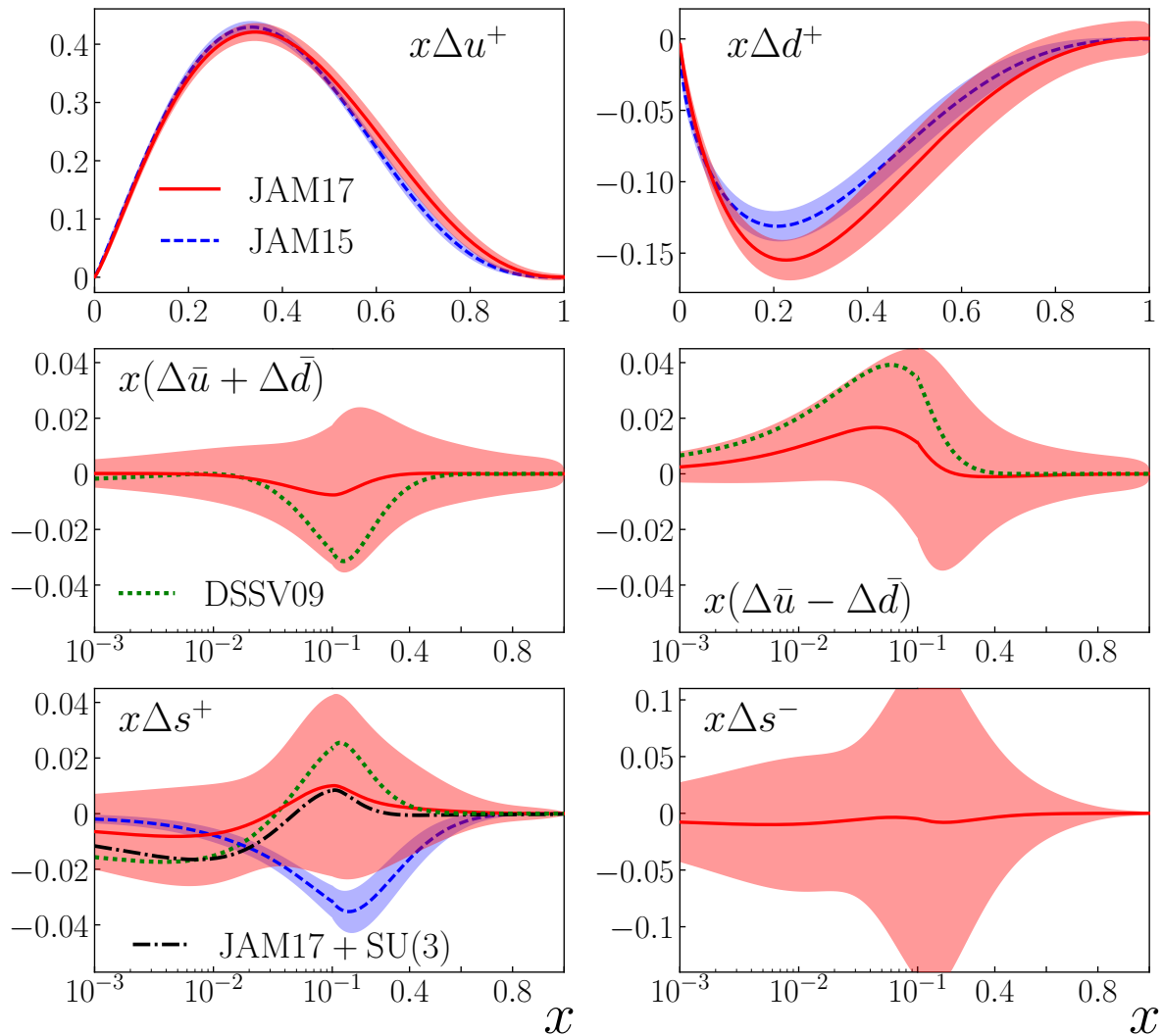
$e^+e^- \rightarrow h X$
single-inclusive
annihilation (SIA)

*Sato, Ethier, WM, Hirai,
Kumano, Accardi (2016)*

→ convergence after ~ 20 iterations

Polarization of quark sea?

- Simultaneous determination of spin PDFs and FFs, fitting to DIS, SIA and polarized SIDIS (HERMES, COMPASS) data



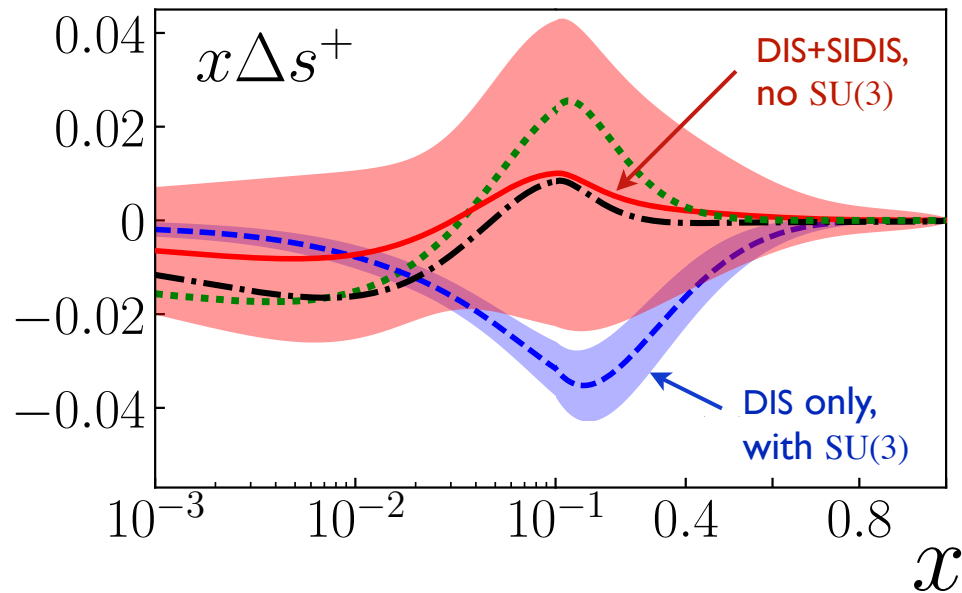
→ no assumption of SU(3) symmetry

→ Δ_s slightly > 0 at high x , consistent with zero

→ $\Delta_s - \Delta_{\bar{s}}$ & $\Delta_{\bar{u}} - \Delta_{\bar{d}}$ consistent with zero

Simultaneous analysis

- Polarized strangeness in previous, DIS-only analyses was negative at $x \sim 0.1$, induced by SU(3) and parametrization bias



Ethier, Sato, WM (2017)

- weak sensitivity to Δs^+ from DIS data & evolution
 - SU(3) pulls Δs^+ to generate moment ~ -0.1
 - negative peak at $x \sim 0.1$ induced by fixing $b \sim 6 - 8$
- less negative $\Delta s = -0.03(10)$ gives larger total helicity $\Delta\Sigma = 0.36(9)$

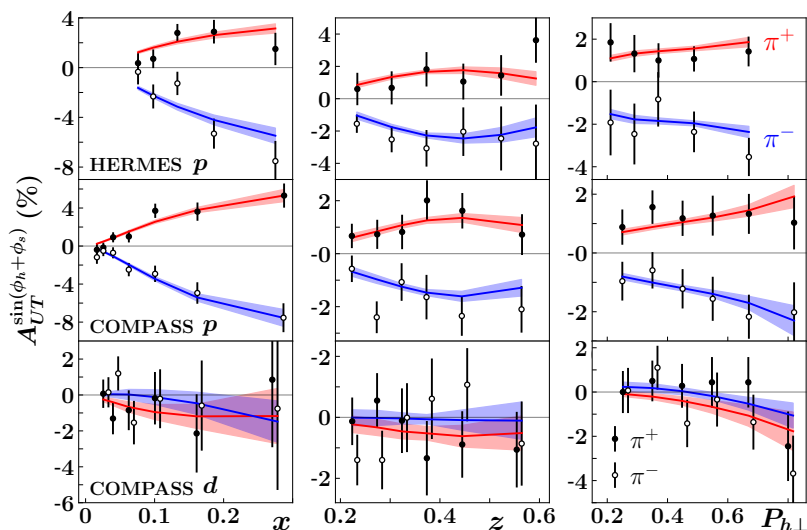
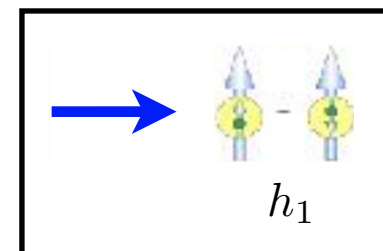
Nucleon Transversity PDFs

Transversity distributions

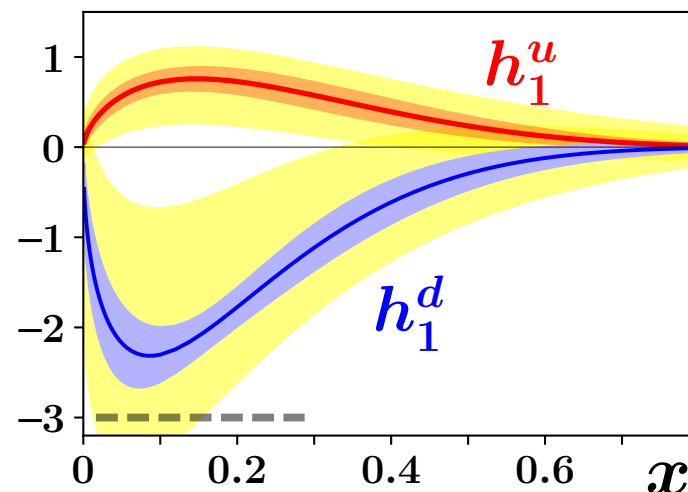
- Extraction of transversity (TMD) PDF from SIDIS data + isovector moment $g_T = \int dx (h_1^u - h_1^d)$ from lattice QCD

→ Collins asymmetry

$$A_{UT}^{\sin(\phi_h + \phi_s)} \propto \frac{h_1 \otimes H_1^\perp}{f_1 \otimes D_1}$$



$$g_T^{\text{latt}} = 1.01(6)$$

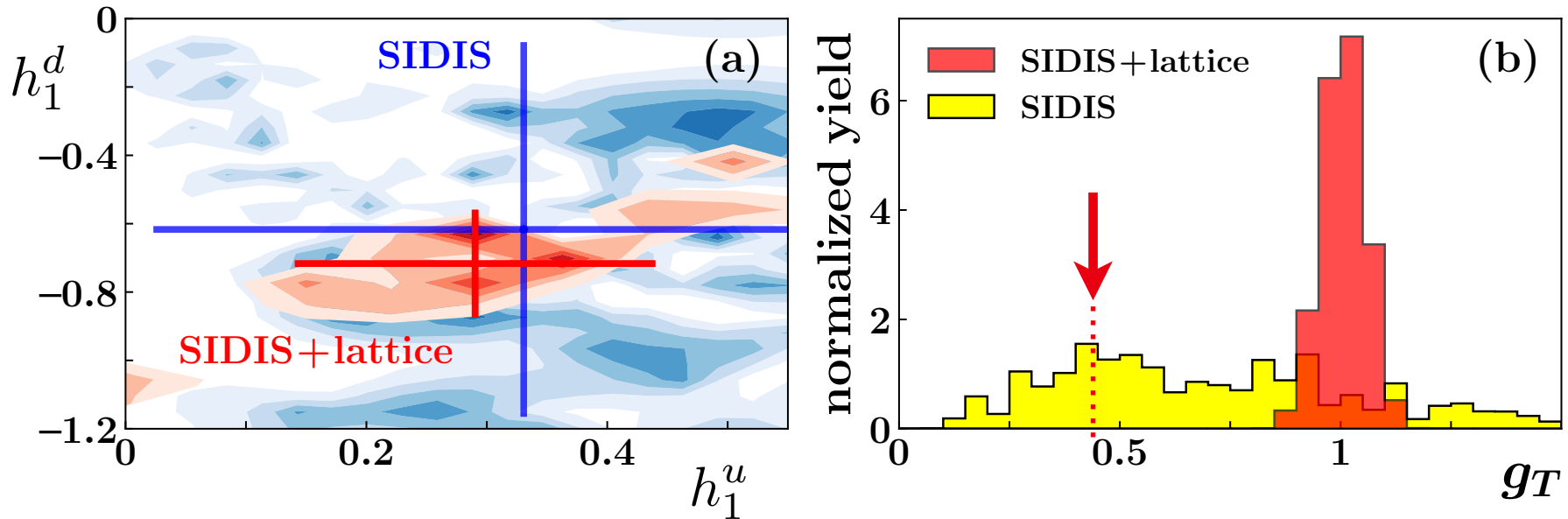


Lin, WM, Prokudin, Sato, Shows (2018)

→ significantly reduced uncertainties with lattice constraint

Transversity distributions

- Extraction of transversity (TMD) PDF from SIDIS data + isovector moment $g_T = \int dx (h_1^u - h_1^d)$ from lattice QCD



Lin, WM, Prokudin, Sato, Shows (PRL, 2018)

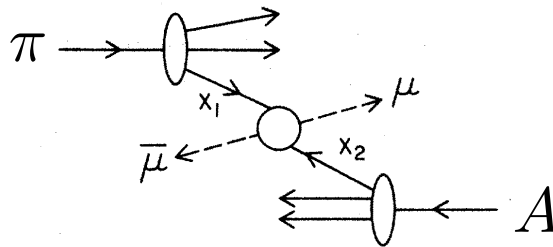
- distributions do not look very Gaussian!
- MC analysis gives $g_T = 1.0 \pm 0.1$
- maximum likelihood analysis would have given $g_T \approx 0.5$

Pion PDFs

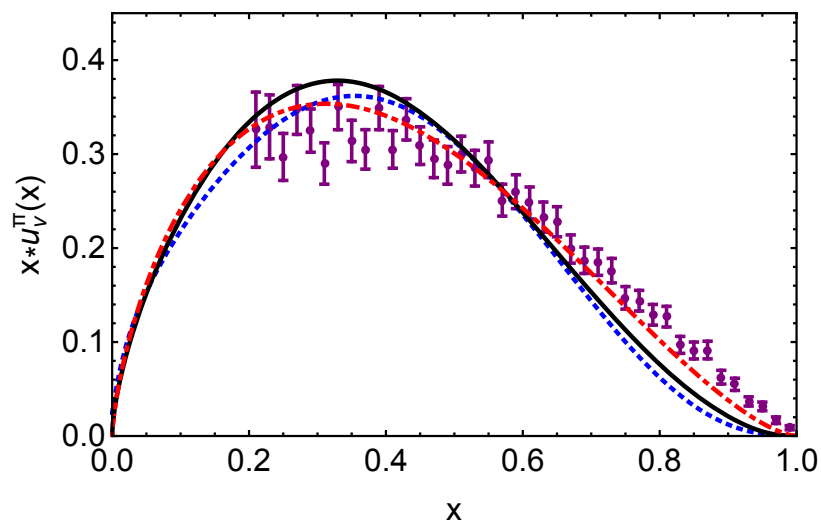
PDFs in the pion

PDFs in the pion (in principle) simpler to compute than baryons, but are more difficult to study experimentally

→ most information has come from pion-nucleus (tungsten) Drell-Yan data (CERN, Fermilab)



→ constrains valence PDFs at $x \gg 0$ (uncertainty from gluon resummation)



Shi, Mezrag, Zong (2018)

→ pion sea quark & gluon PDFs at small x mostly unconstrained

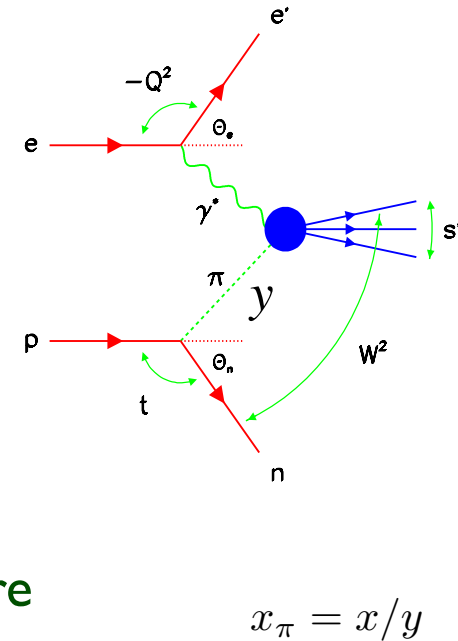
PDFs in the pion

- Recently a new (Monte Carlo-based) global analysis used chiral effective field theory to include also leading neutron electroproduction from HERA

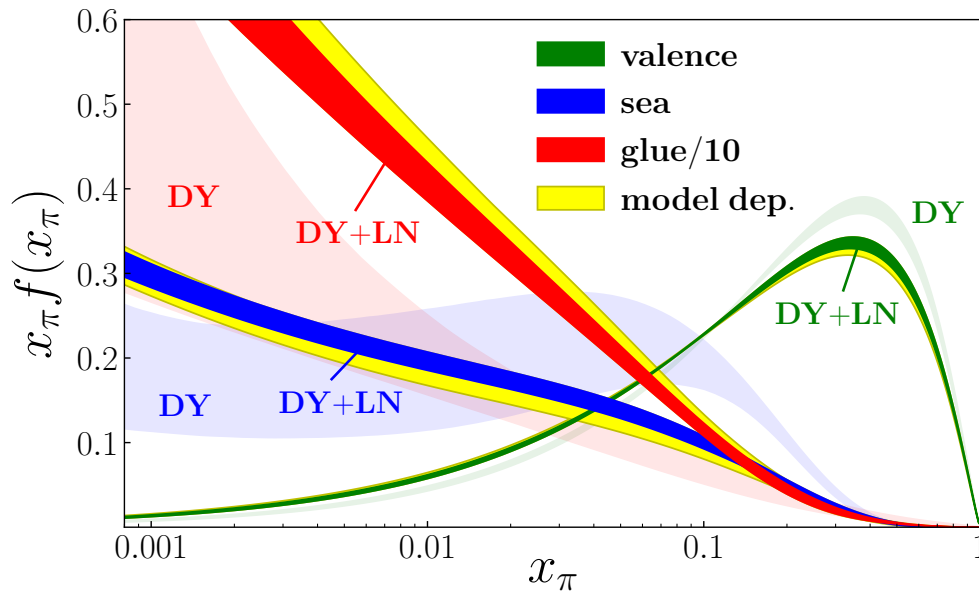
$$\frac{d^3 \sigma^{\text{LN}}}{dx dQ^2 dy} \sim 2f_{\pi/N}(y) F_2^\pi(x_\pi, Q^2)$$

$N \rightarrow N + \pi$
splitting function
(computed from χEFT)

pion structure
function



McKenney, Sato, WM, C.-R. Ji (2018)

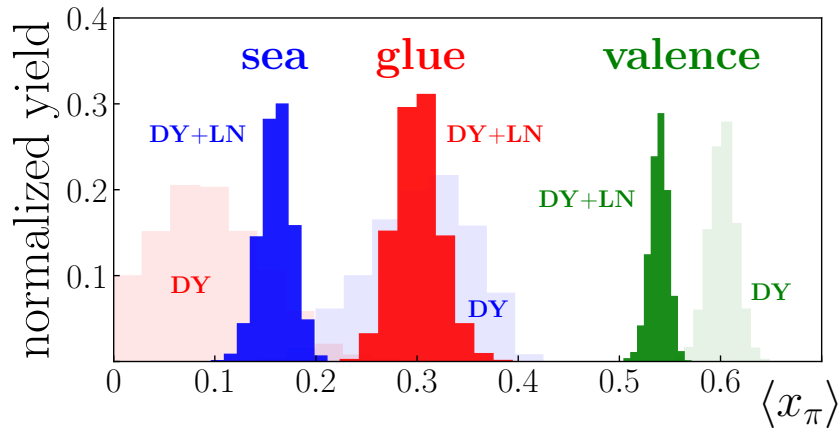


Barry, Sato, WM, C.-R. Ji (2018)

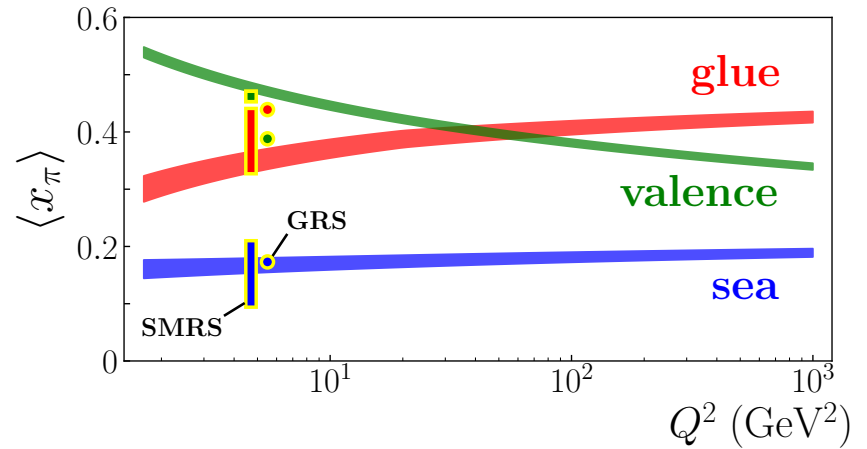
→ first constraints on
pion PDFs at low x

PDFs in the pion

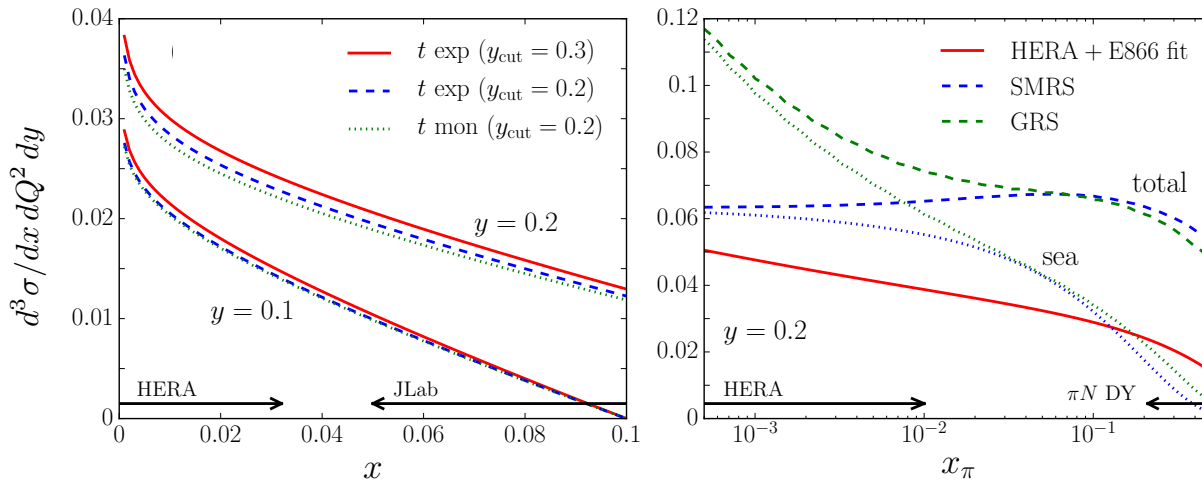
• Larger gluon fraction in the pion than without LN constraint



Barry, Sato, WM, C.-R. Ji (2018)



• Tagged DIS experiment at JLab ($e n \rightarrow e' p X$) will probe pion structure at intermediate x values (between DY and LN)



→ extension to hyperon final state will probe kaon structure

Outlook

- New paradigm in global analysis — *simultaneous* determination of collinear distributions using MC sampling of parameter space
→ providing new insights into quark/gluon structure of hadrons
- *Short-term*: “universal” QCD analysis of all observables sensitive to collinear (unpolarized & polarized) PDFs and FFs
- *Longer-term*: technology developed will be applied to global analysis of transverse momentum dependent (TMD) distributions to map out full 3-d image of hadrons
→ vital interplay between theory & experiment at JLab



대단히 감사합니다.

