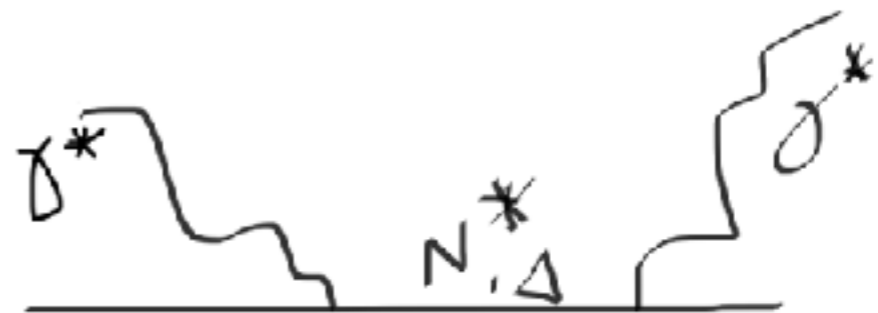
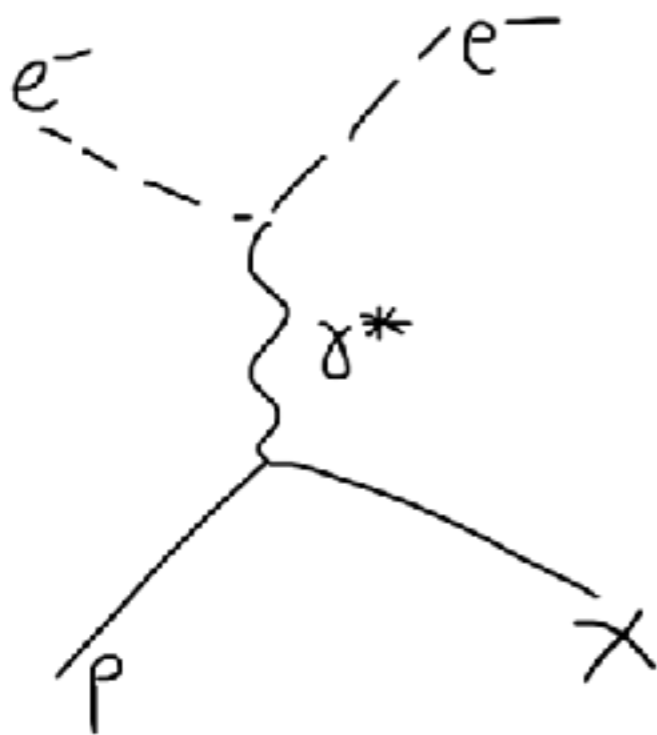


APCTP Workshop - JLAB physics in the 12-GeV era

FESR constraints on electron and Compton scattering

2018-07-01

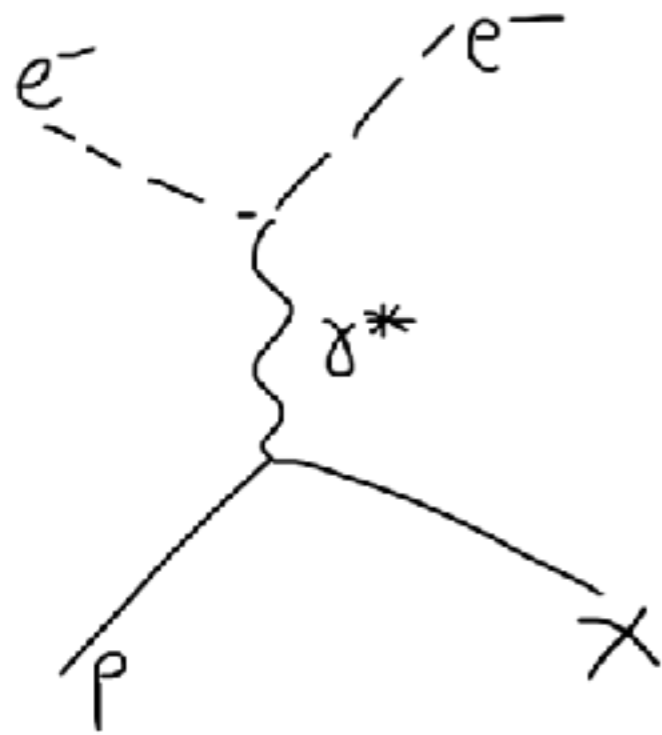


Astrid N. Hiller Blin

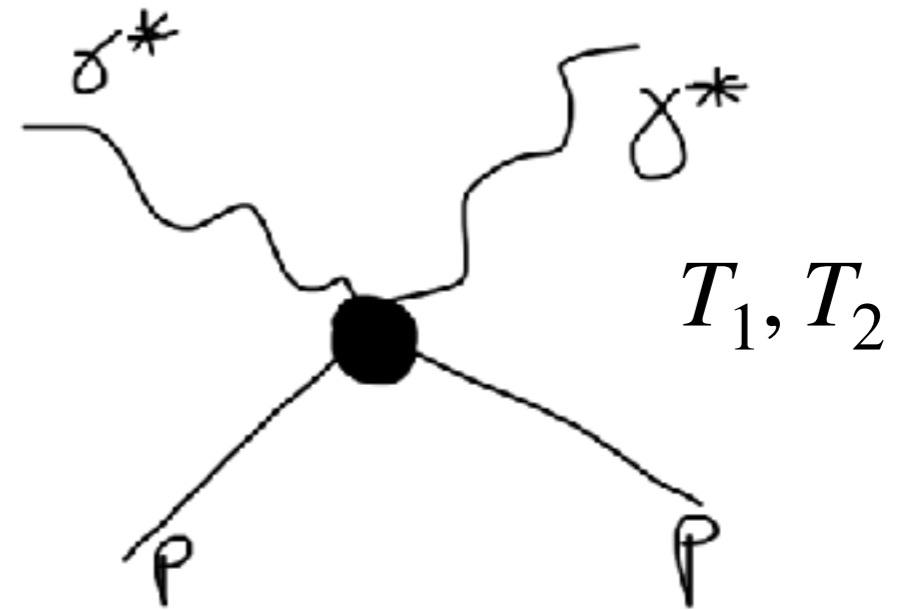
JGU Mainz

In collaboration with M. Vanderhaeghen and 

e^- scattering and VVCS: connection



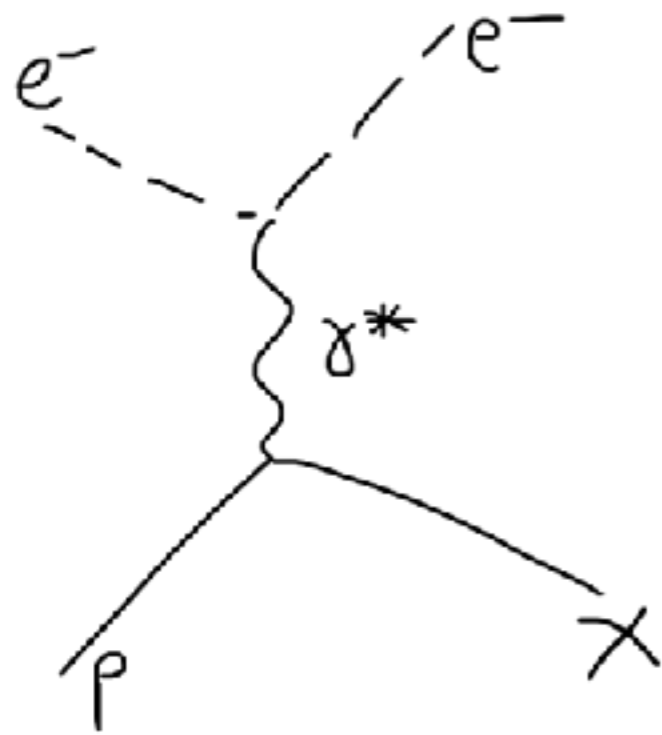
F_1, F_2
 σ_T, σ_L



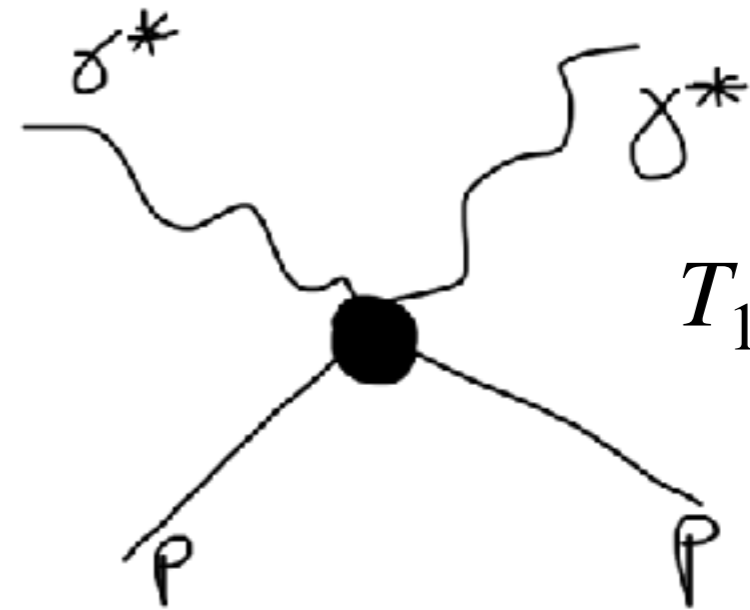
$$F_1(\nu, q^2) \propto \sigma_T(\nu, q^2)$$

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e⁻ scattering and VVCS: connection



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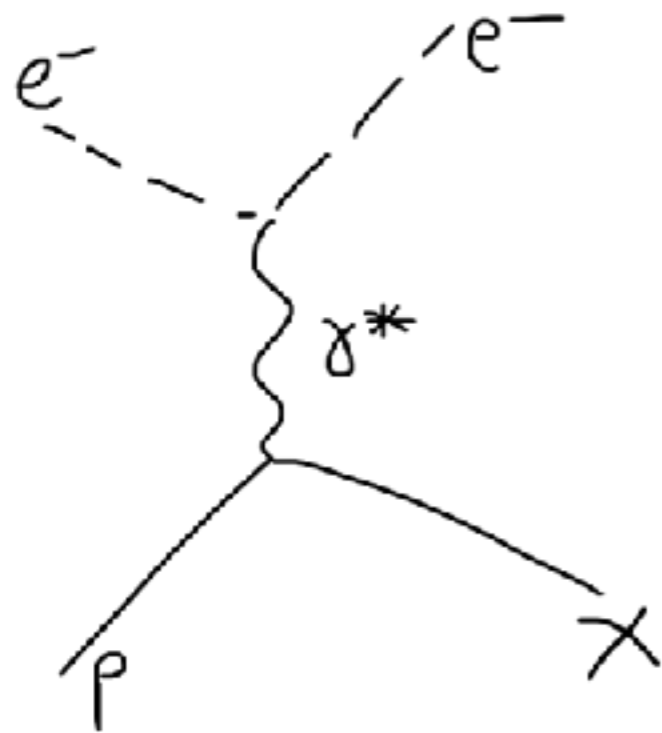
T_1, T_2

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, q^2) + \frac{1}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, q^2)$$

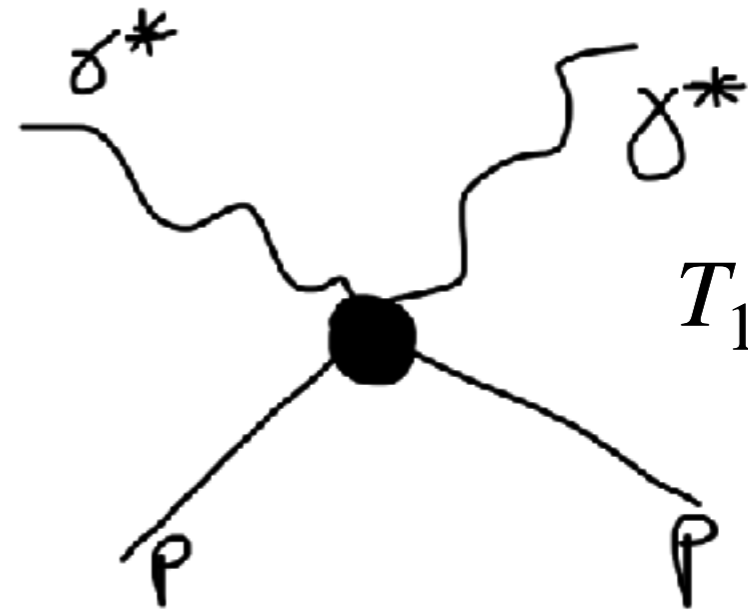
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$$\text{Im}T_1(\nu, q^2) = \frac{e^2}{4M} F_1(\nu, q^2)$$

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Interest in electron-scattering structure functions

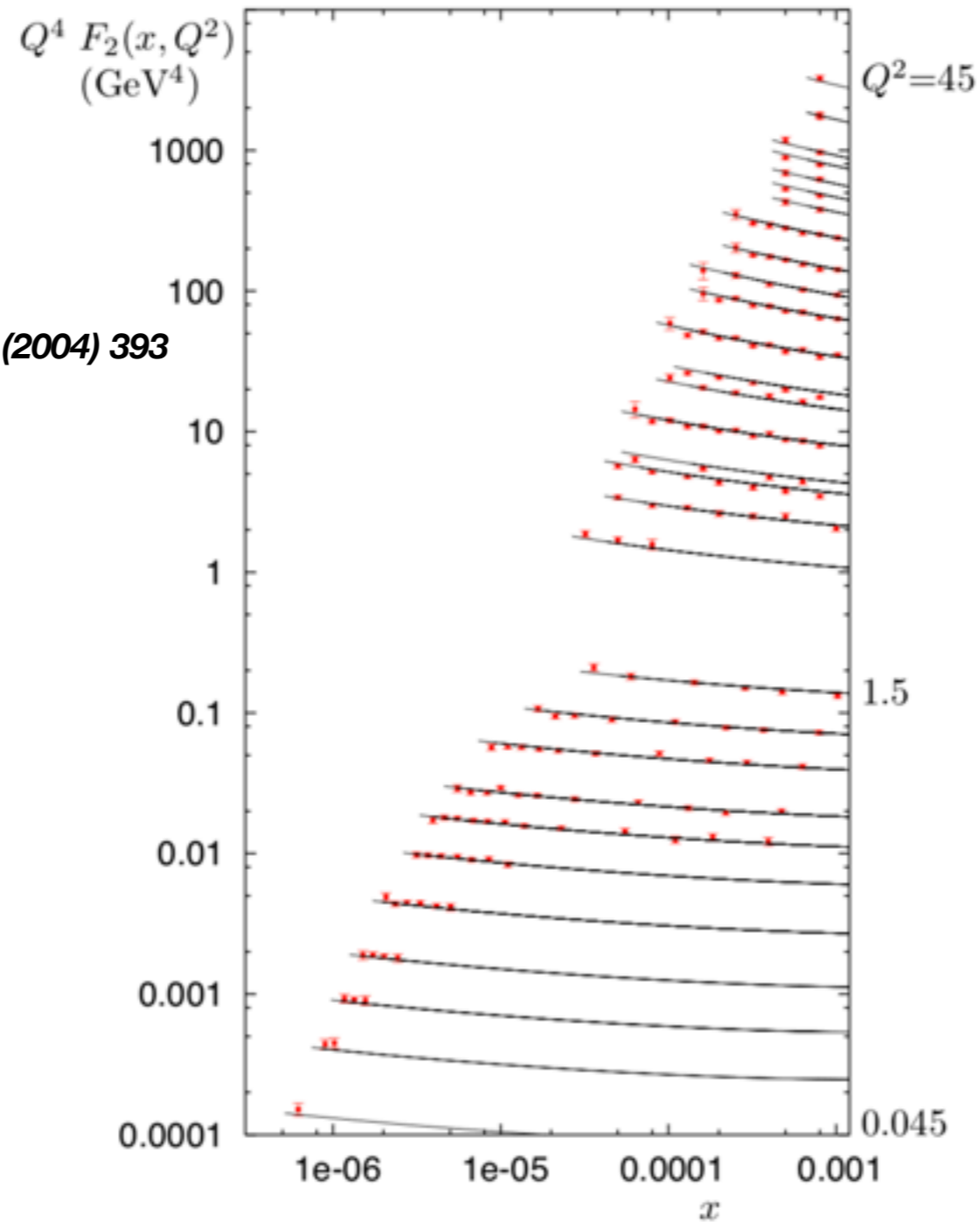
- Tests on quark-hadron duality
- Comparison with CLAS(12) [F₂ data](#):
Existing narrow binning in Q^2 and W ;
Experiments at higher Q^2 in **12-GeV era**
- Connection with CLAS(12) [electrocouplings](#)
(see talk by V. Mokeev)

The interest in T_1

- **Lamb shift** and **Cottingham formula**
- Determining T_1 is intricate:
unlike T_2 , **subtracted** dispersion relation
- Want: parametrization of T_1 at all W and Q^2

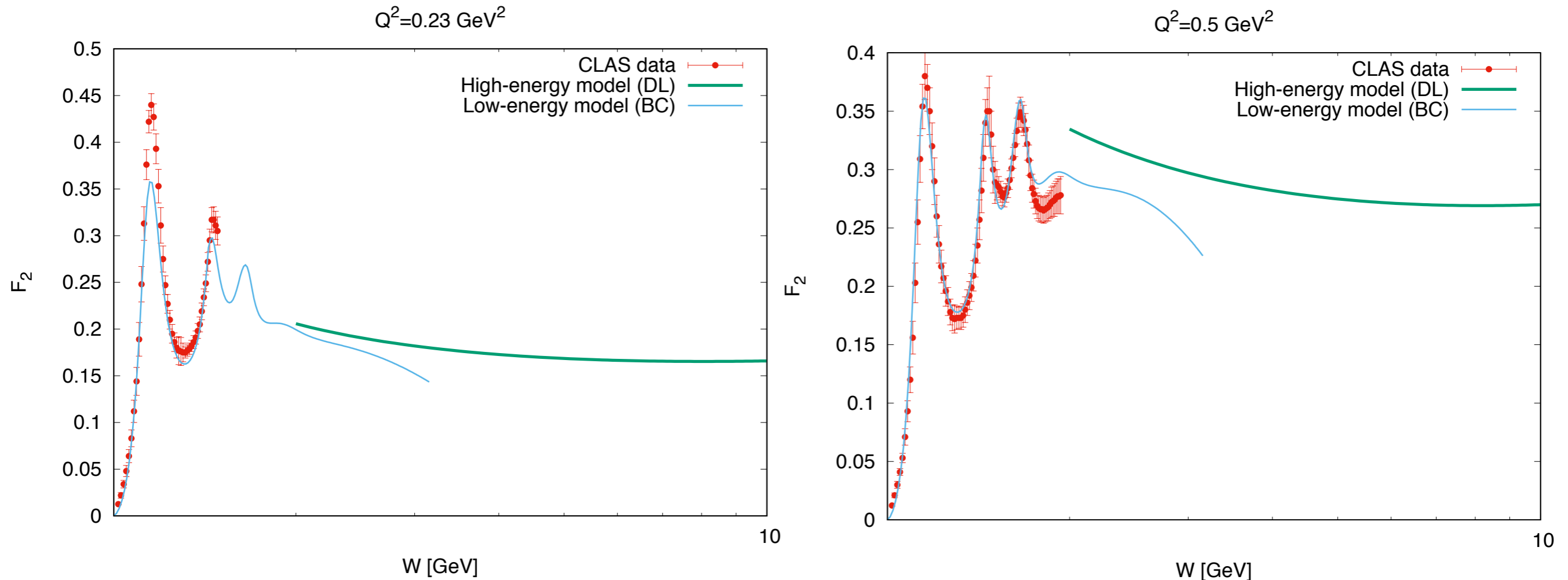
Data & model status: high energies

Donnachie and Landshoff, PLB 595 (2004) 393
HERA data



Data & model status: low energies

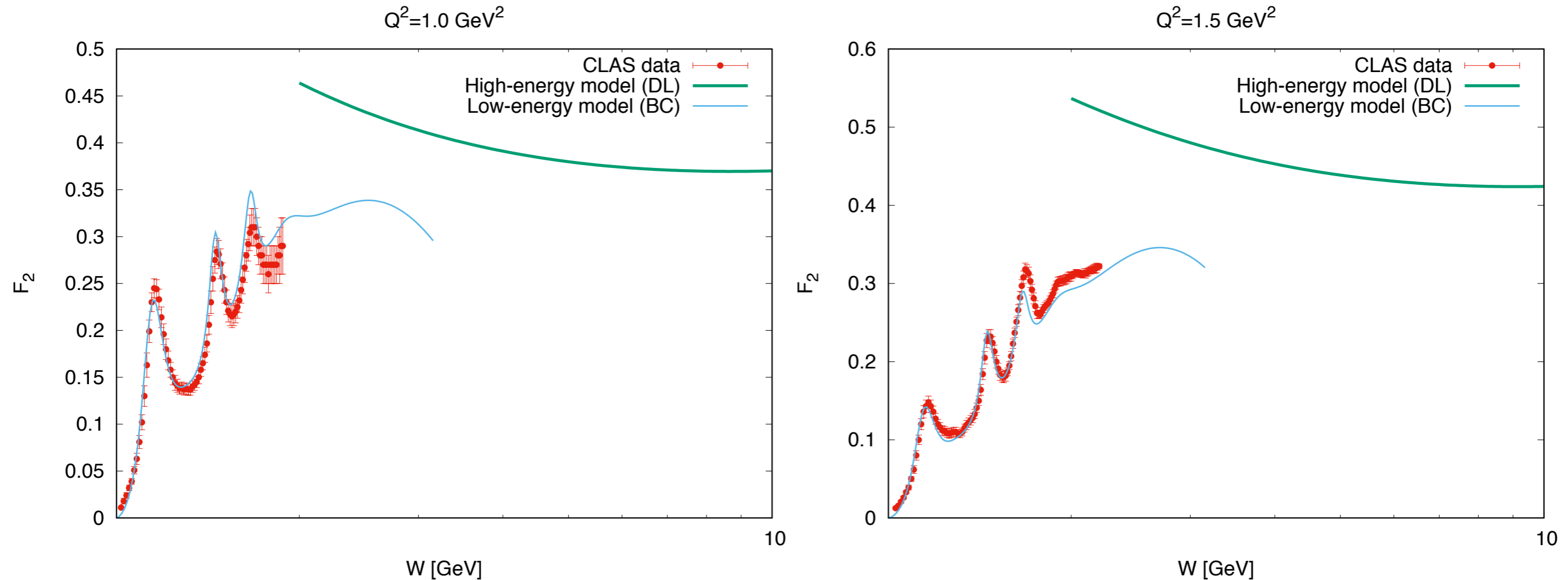
BC: Christy and Bosted, PRC 86 (2010) 055213 DL: Donnachie and Landshoff, PLB 595 (2004) 393 CLAS data: <http://clas.sinp.msu.ru/strfun/>



At low Q^2 : high- and low-energy theories compatible in overlap region

Low energies: status

BC: Christy and Bosted, PRC 86 (2010) 055213 DL: Donnachie and Landshoff, PLB 595 (2004) 393 CLAS data: <http://clas.sinp.msu.ru/strfun/>



For larger Q^2 : huge gap between high and low-energy models in overlap region

Low energies: describing the resonant region

Our tool: finite-energy sum rules

High-energy model

Goals: high and low-energy connection

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Background: status

Moiseev et al., PRC 86 (2012) 035203

$$\sigma_{T,L}(W, Q^2) = \sigma_{T,L}^R(W, Q^2) + \sigma_{T,L}^{NR}(W, Q^2)$$

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The non-resonant background

$$\sigma_T^{NR}(W, Q^2) = x'_T \sum_{i=1}^2 \frac{\sigma_T^{NR,i}(0)}{(Q^2 + a_i^T)[b_i^T + c_i^T Q^2 + d_i^T Q^4]} (W - m_\pi - M_N)^{i+1/2}$$

$$\sigma_L^{NR}(W, Q^2) = \frac{(1 - x'_L)^{[a_1^L t + b_1^L]} x'_L^{[d_1^L + e_1^L t]}}{1 - x} \frac{\sigma_L^{NR,1}(0)(Q^2)^{c_1^L}}{(Q^2 + Q_0^{2L})^{[1+c_1^L]}}$$

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- Do not fulfill dispersion relations/analyticity
- Need to be revisited/fixed by FESR

Resonance model

Moiseev et al., PRC 86 (2012) 035203

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Breit-Wigner resonance model

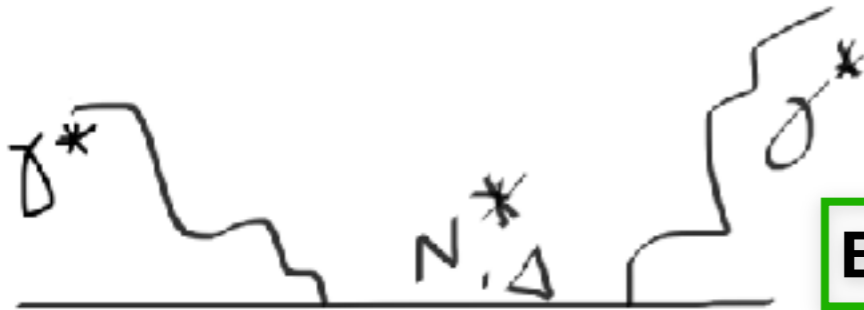
$$\sigma_{T,L}^R(W, Q^2) = \frac{\pi}{q_\gamma^2} \sum_{N^*, \Delta^*} (2J_r + 1) \frac{M_r^2 \Gamma_{\text{tot}}(W) \Gamma_\gamma^{T,L}(M_r)}{(M_r^2 - W^2)^2 + M_r^2 \Gamma_{\text{tot}}^2(W)}$$

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Electrocouplings from CLAS data and fits

<https://userweb.jlab.org/~isupov/couplings/>

$$\Gamma_\gamma^T(M_r, Q^2) \sim |A_{1/2}(Q^2)|^2 + |A_{3/2}(Q^2)|^2$$

$$\Gamma_\gamma^L(M_r, Q^2) \sim 2 \frac{\nu^2}{Q^2} |S_{1/2}(Q^2)|^2$$

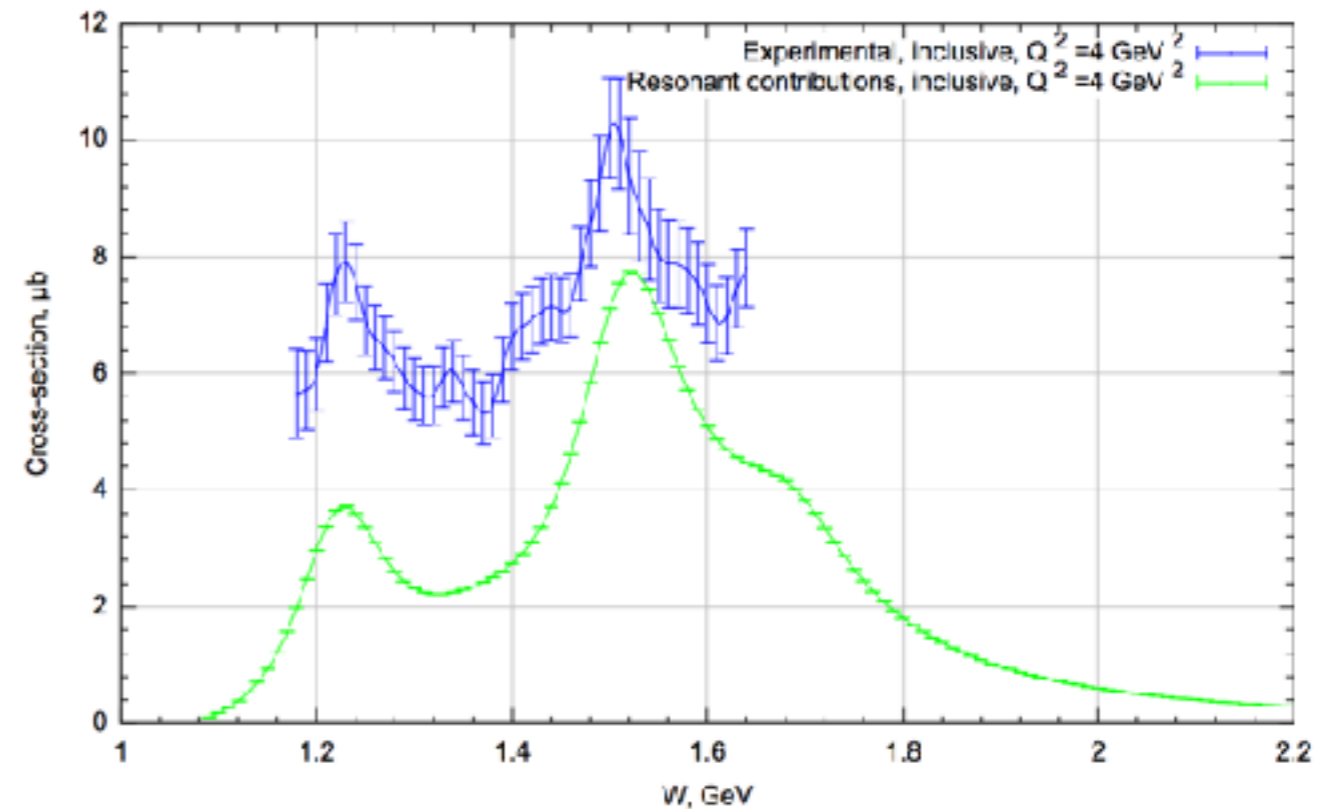
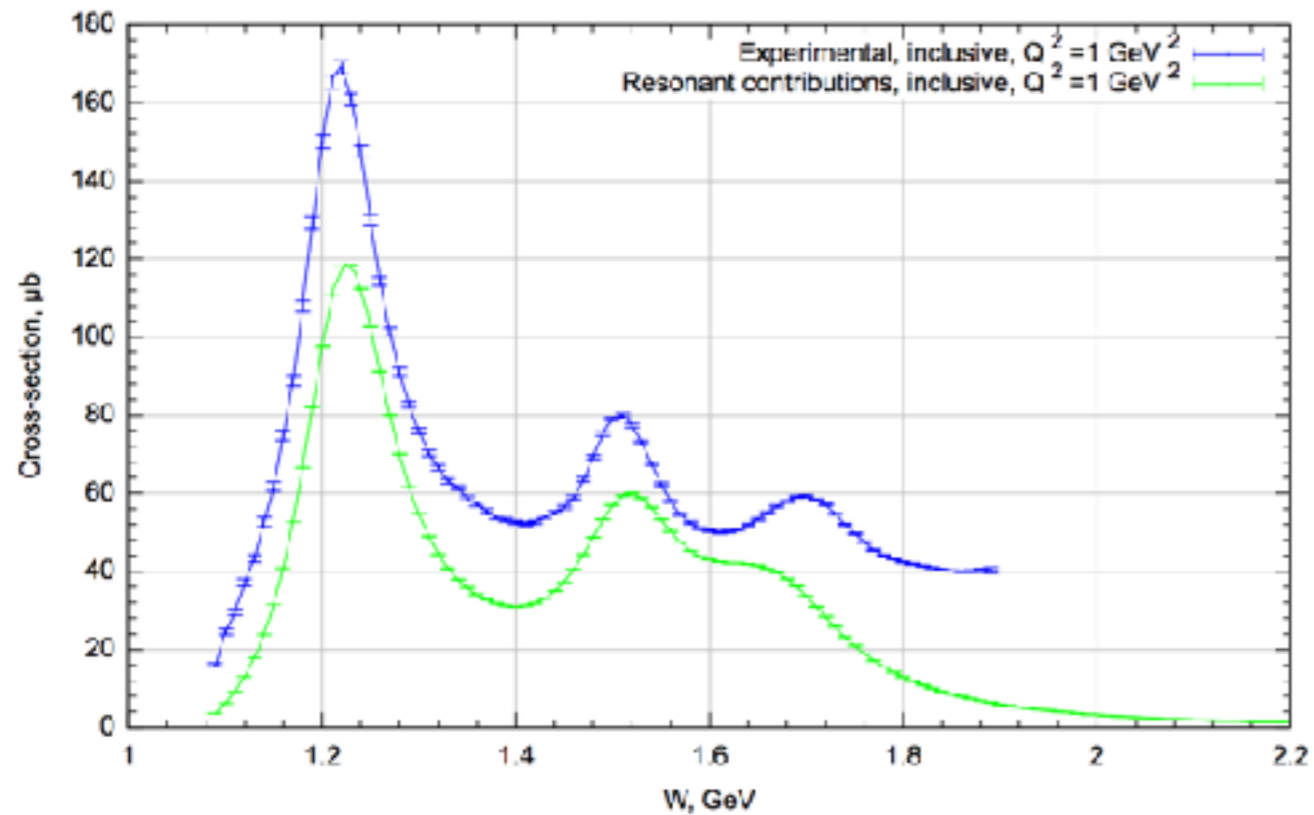
$$\Gamma_{\text{tot}}(W) = \Gamma_{\pi N}(W) + (1 - \text{BF}_{\pi N}) \Gamma_r$$

Energy-dependence from decays into πN and $\pi\pi N$

Aznauryan, PRC 67 (2003) 015209

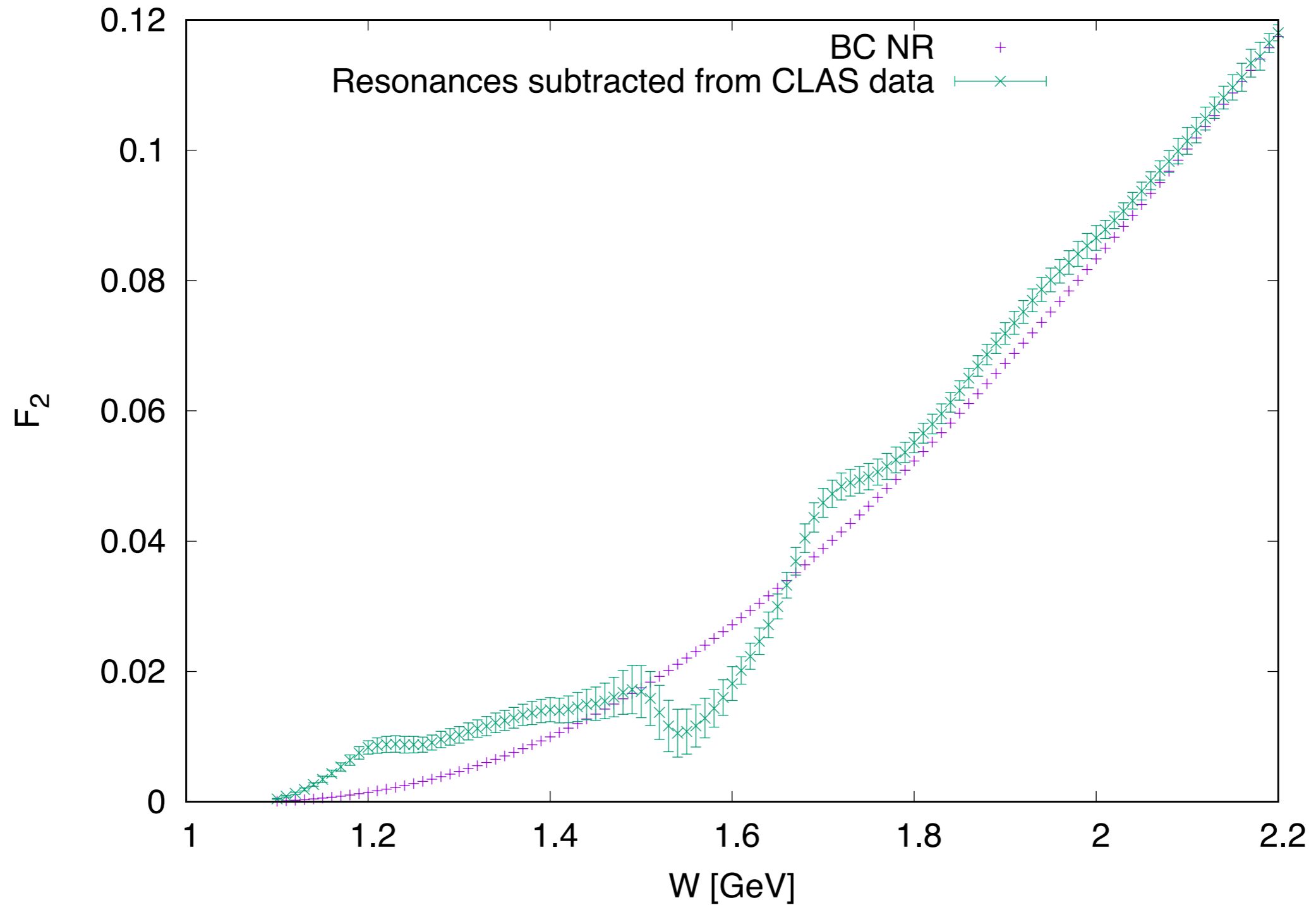
Resonances compared to CLAS data

CLAS structure functions and cross sections: user choice of W , Q^2 and beam energy
<http://clas.sinp.msu.ru/strfun/>



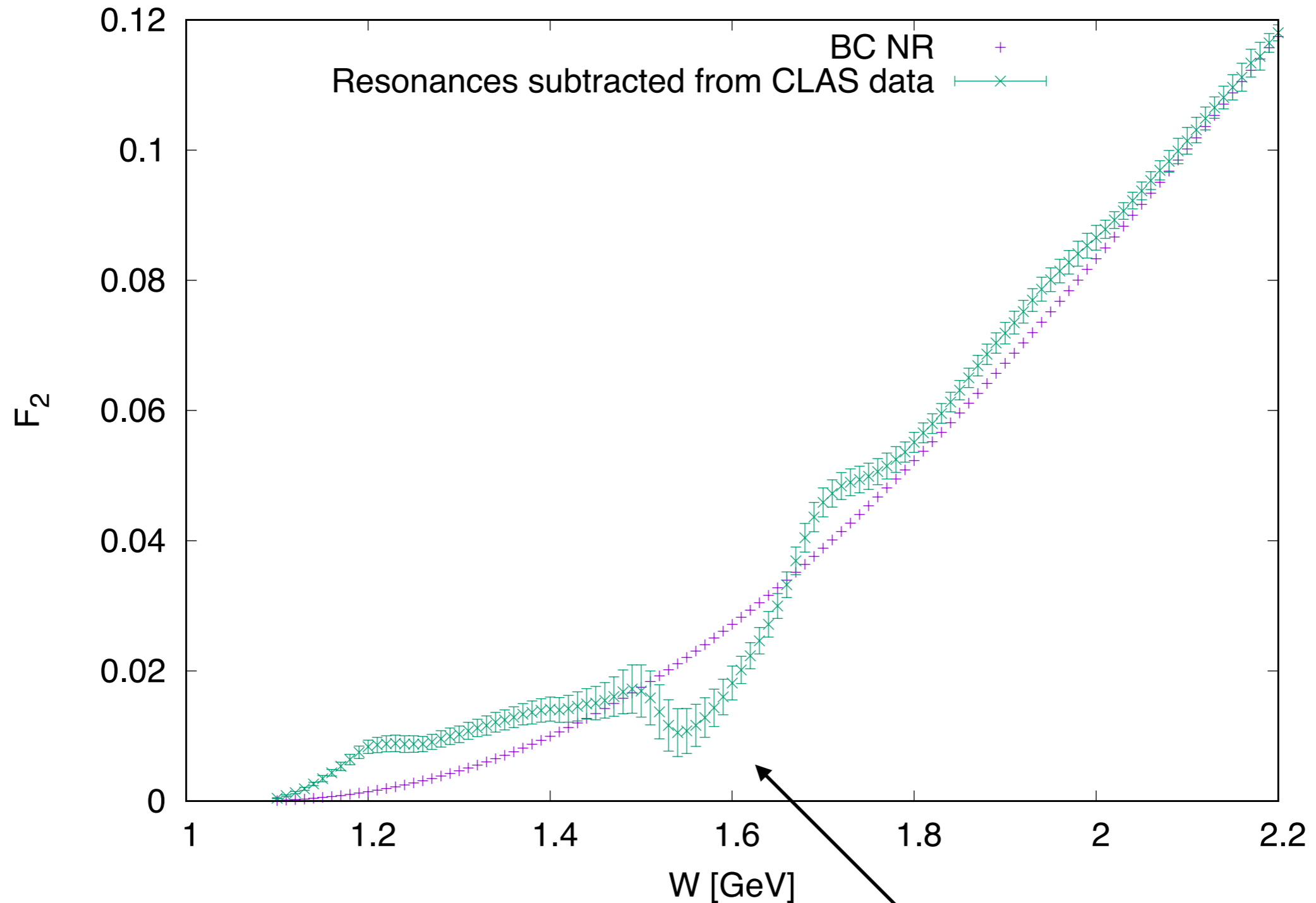
Duality in CLAS F_2 data

$Q^2=5.0 \text{ GeV}^2$



Duality in CLAS F_2 data

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The dip is expected from opening of meson-production channels!

Low energies: describing the resonant region

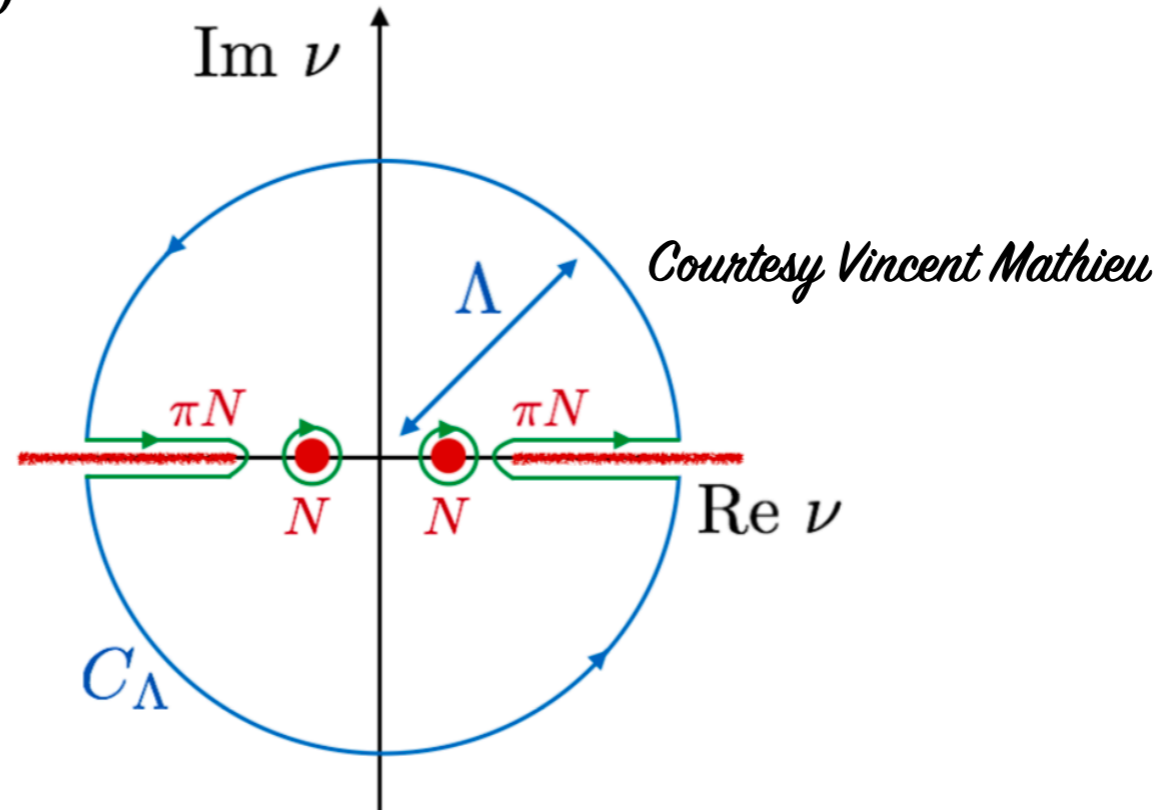
Our tool: finite-energy sum rules

High-energy model

Goals: high and low-energy connection

Cauchy's Theorem

$$\oint T_i(\nu, Q^2) d\nu = 0$$



Cauchy's Theorem

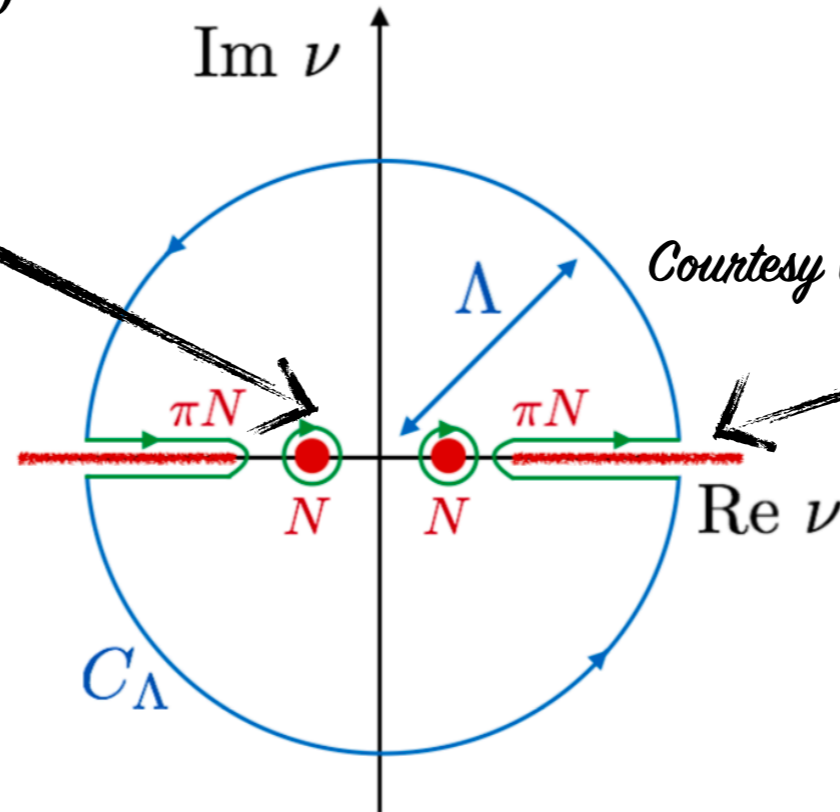
$$\oint T_i(\nu, Q^2) d\nu = 0$$



Poles: Born terms



Cuts: meson production



Courtesy Vincent Mathieu

Cauchy's Theorem

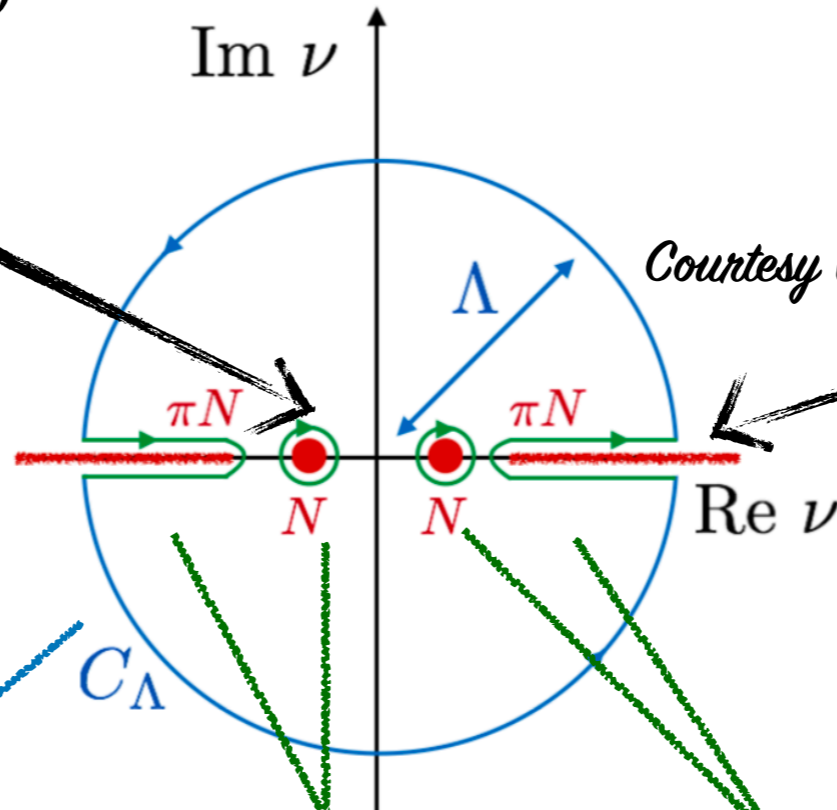
$$\oint T_i(\nu, Q^2) d\nu = 0$$



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In absence of fixed poles:

$$0 = \int_{C_\Lambda} T_i(\nu, Q^2) \nu^k d\nu + \int_0^\Lambda [(-1)^k D_{i,L}(\nu, Q^2) + D_{i,R}(\nu, Q^2)] \nu^k d\nu$$

$$D_{i,R}(\nu, Q^2) = \lim_{\epsilon \rightarrow 0} [T_i(\nu + i\epsilon, Q^2) - T_i(\nu - i\epsilon, Q^2)]$$

FESR: the idea

- Amplitudes with correct analytic behaviour:
Cauchy's theorem fulfilled exactly
- Equivalently: FESR fulfilled

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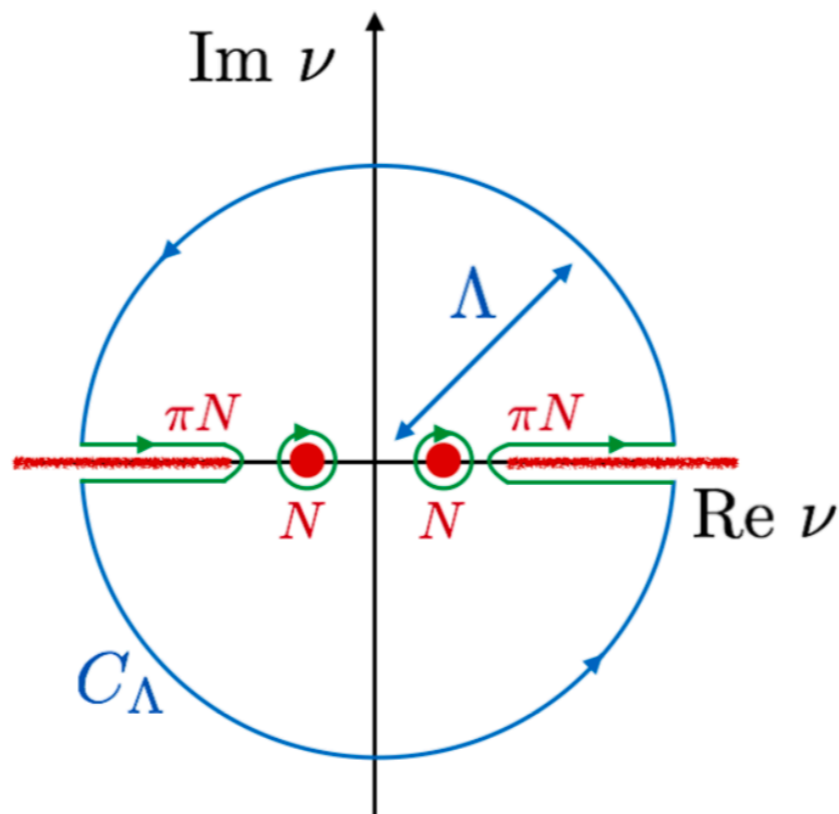
- We want to generalize this to:

High-energy model = LHS **Low-energy model = RHS**

Appropriate **cutoff Λ** needed! Larger than resonance region, but still sensitive to it.

RHS: low energies

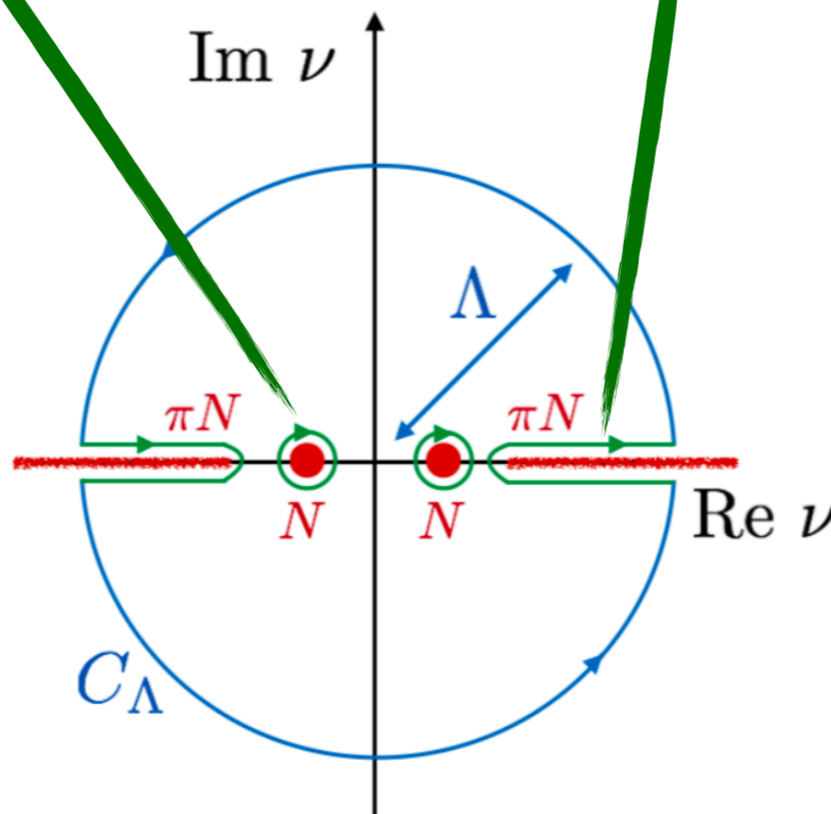
$$\begin{aligned} \text{RHS} &= \int_0^\Lambda [D_{i,R}(\nu, Q^2) + (-1)^k D_{1,L}(\nu, Q^2)] \nu^k d\nu \\ &= -2i \left(\pi B_i(Q^2) \nu_N^k + \int_{\nu_0}^\Lambda \text{Im} T_i(\nu, Q^2) \nu^k d\nu \right) (1 + \tau_i (-1)^k) \end{aligned}$$



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Form factors from A1 collaboration
Bernauer et al., PRC 90 (2014) 015206



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LHS: high-energy model

Donnachie and Landshoff, PLB 595 (2004) 393

$$F_2(x, Q^2) = f_h(Q^2)x^{-\epsilon_h} + f_s(Q^2)x^{-\epsilon_s} + f_r(Q^2)x^{-\epsilon_r}$$

$$x = \frac{Q^2}{2M_N\nu}$$



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↓

Hard pomeron

$\epsilon_h = \alpha_h - 1 = 0.452$

↓

Soft pomeron

$\epsilon_s = \alpha_s - 1 = 0.0667$

↓

Meson exchange (a_2)

$\epsilon_r = \alpha_r - 1 = -0.476$



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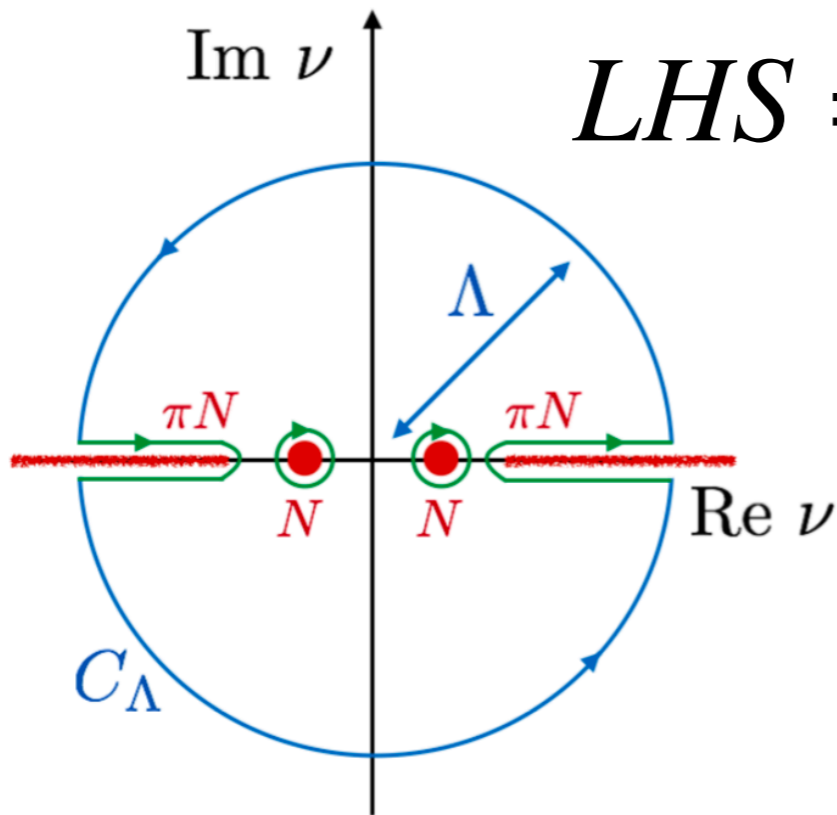
$$F_1(\nu, Q^2) = \frac{M_N\nu}{Q^2} \frac{F_2(\nu, Q^2)}{1 + R}$$

$$R \propto \frac{\sigma_L}{\sigma_T} \quad \text{Ricco et al., NPB 555 (1999) 306}$$

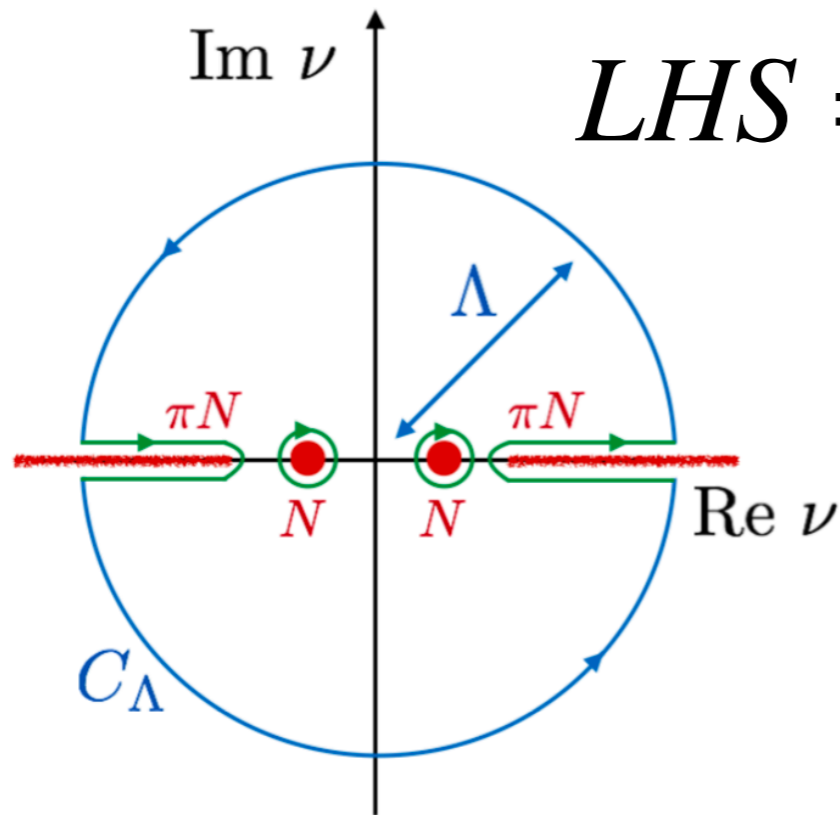


LHS: high energies

$$LHS = - \int_{C_\Lambda} T_i(\nu, Q^2) \nu^k d\nu$$



LHS: high energies

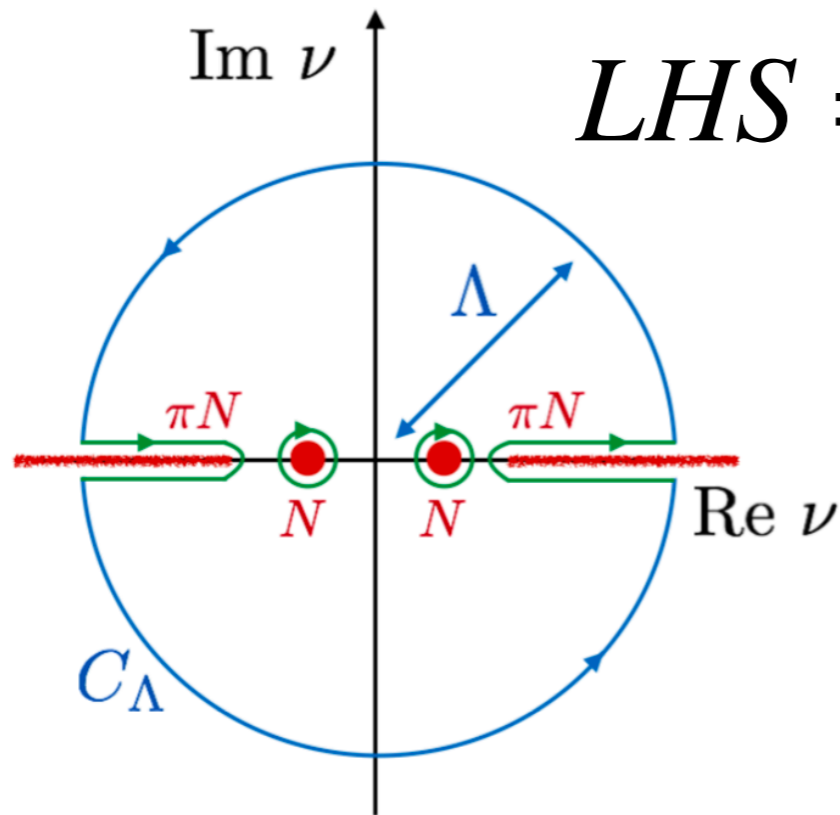


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$$\frac{\nu}{\pi \alpha} \text{Im } T_2 = F_2$$

$$F_2(\nu, Q^2) = \sum_{i=h,s,r} f_i(Q^2) \left(\frac{Q^2}{2M_N \nu} \right)^{1-\alpha_i}$$

LHS: high energies



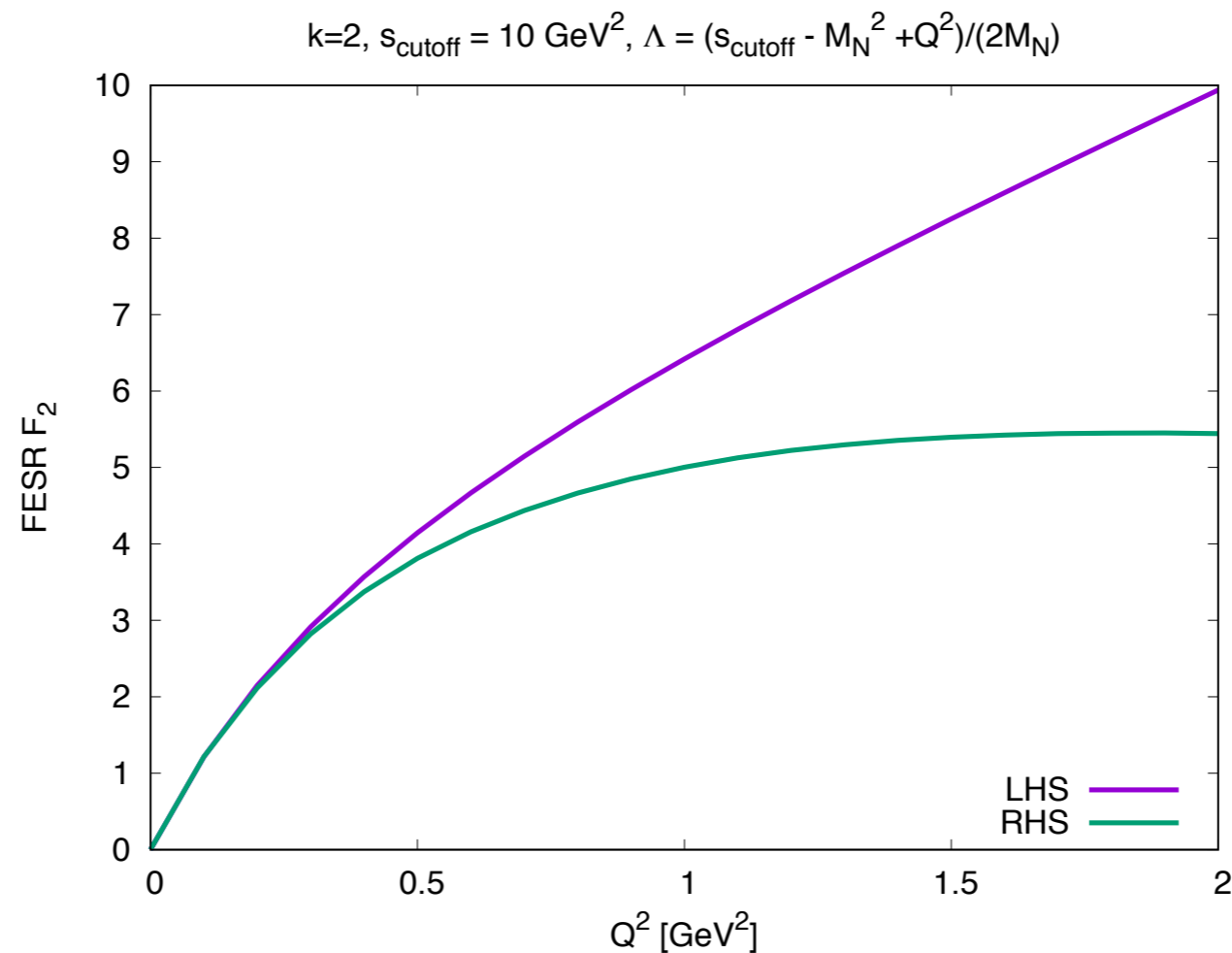
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$$T_2 = \frac{\pi\alpha}{\nu} f_i(Q^2) \frac{\tau_i \left(\frac{2M_N\nu}{Q^2} \right)^{\alpha_i-1} + \left(-\frac{2M_N\nu}{Q^2} \right)^{\alpha_i-1}}{\sin(\pi\alpha_i)}$$

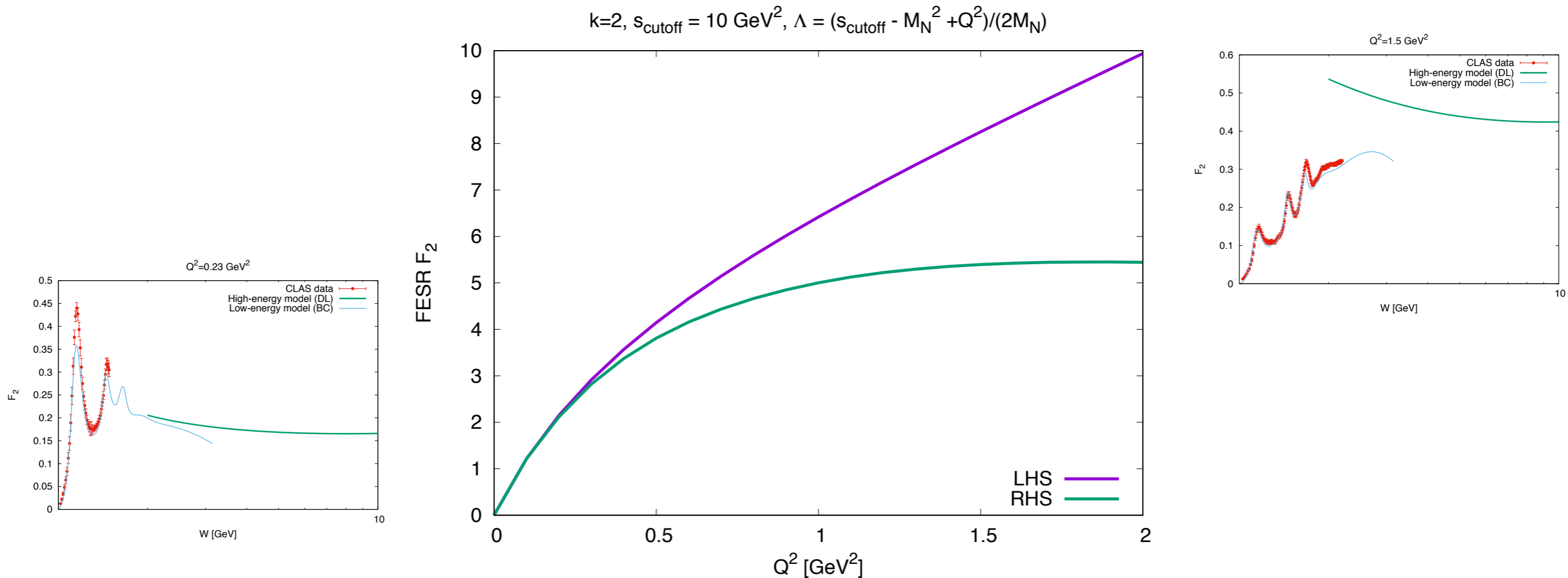
Example FESR evaluation



Result for the current Bosted&Christy model vs. Donnachie&Landshoff

Good match at $Q^2 = 0$ as expected, less compatible the larger Q^2 !

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- **a' and b': fit to F_2 data**

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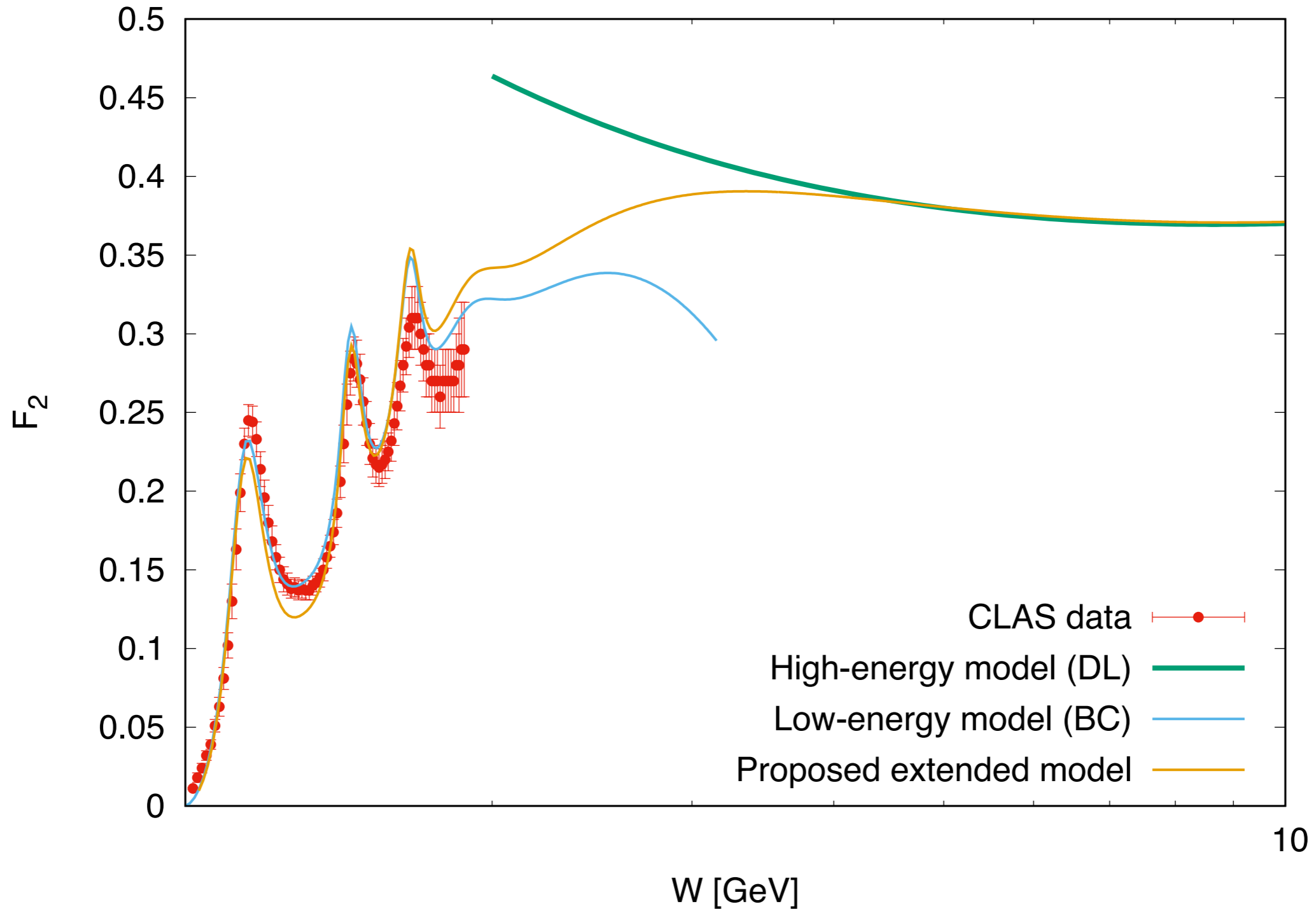
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Preliminary: overlap region

$$Q^2=1.0 \text{ GeV}^2$$



Summary

- New resonant model:
Full information on **N* electrocouplings** from CLAS data;
No fits needed!
- High-energy data described well by Regge models
- FESR to fix overlap-region behaviour for any Q^2 :
Modified Regge-like amplitude with threshold parameters

N^*	M_r [MeV]	Γ_r [MeV]	J_r	L_r	$\text{BF}_{\pi N}$
1440	1430	350	$\frac{1}{2}$	1	0.65
1520	1515	115	$\frac{3}{2}$	2	0.60
1535	1535	150	$\frac{1}{2}$	0	0.45
1650	1655	140	$\frac{1}{2}$	0	0.60
1675	1675	150	$\frac{5}{2}$	2	0.40
1680	1685	130	$\frac{5}{2}$	3	0.68
1710	1710	100	$\frac{1}{2}$	1	0.13
1720	1720	250	$\frac{3}{2}$	1	0.11
2190	2190	500	$\frac{7}{2}$	4	0.15
2220	2250	400	$\frac{9}{2}$	5	0.20
2250	2280	500	$\frac{9}{2}$	4	0.10

All (**) resonances included**

$A_{1/2}(Q^2), A_{3/2}(Q^2), S_{1/2}(Q^2)$

Δ	M_r [MeV]	Γ_r [MeV]	J_r	L_r	$\text{BF}_{\pi N}$
1232	1232	117	$\frac{3}{2}$	1	0.99
1620	1630	140	$\frac{1}{2}$	0	0.25
1700	1700	300	$\frac{3}{2}$	2	0.15
1905	1880	330	$\frac{5}{2}$	3	0.12
1910	1890	280	$\frac{1}{2}$	1	0.23
1950	1930	285	$\frac{7}{2}$	3	0.40
2420	2420	400	$\frac{11}{2}$	5	0.10

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1520	1515	115	$\frac{3}{2}$	2	0.60
1535	1535	150	$\frac{1}{2}$	0	0.45
1650	1655	140	$\frac{1}{2}$	0	0.60
1675	1675	150	$\frac{5}{2}$	2	0.40
1680	1685	130	$\frac{5}{2}$	3	0.68
1710	1710	100	$\frac{1}{2}$	1	0.13
1720	1720	250	$\frac{3}{2}$	1	0.11
2190	2190	500	$\frac{7}{2}$	4	0.15
2220	2250	400	$\frac{9}{2}$	5	0.20
2250	2280	500	$\frac{9}{2}$	4	0.10

All (**) resonances included**

$A_{1/2}(Q^2), A_{3/2}(Q^2), S_{1/2}(Q^2)$

<https://userweb.jlab.org/~isupov/couplings/>

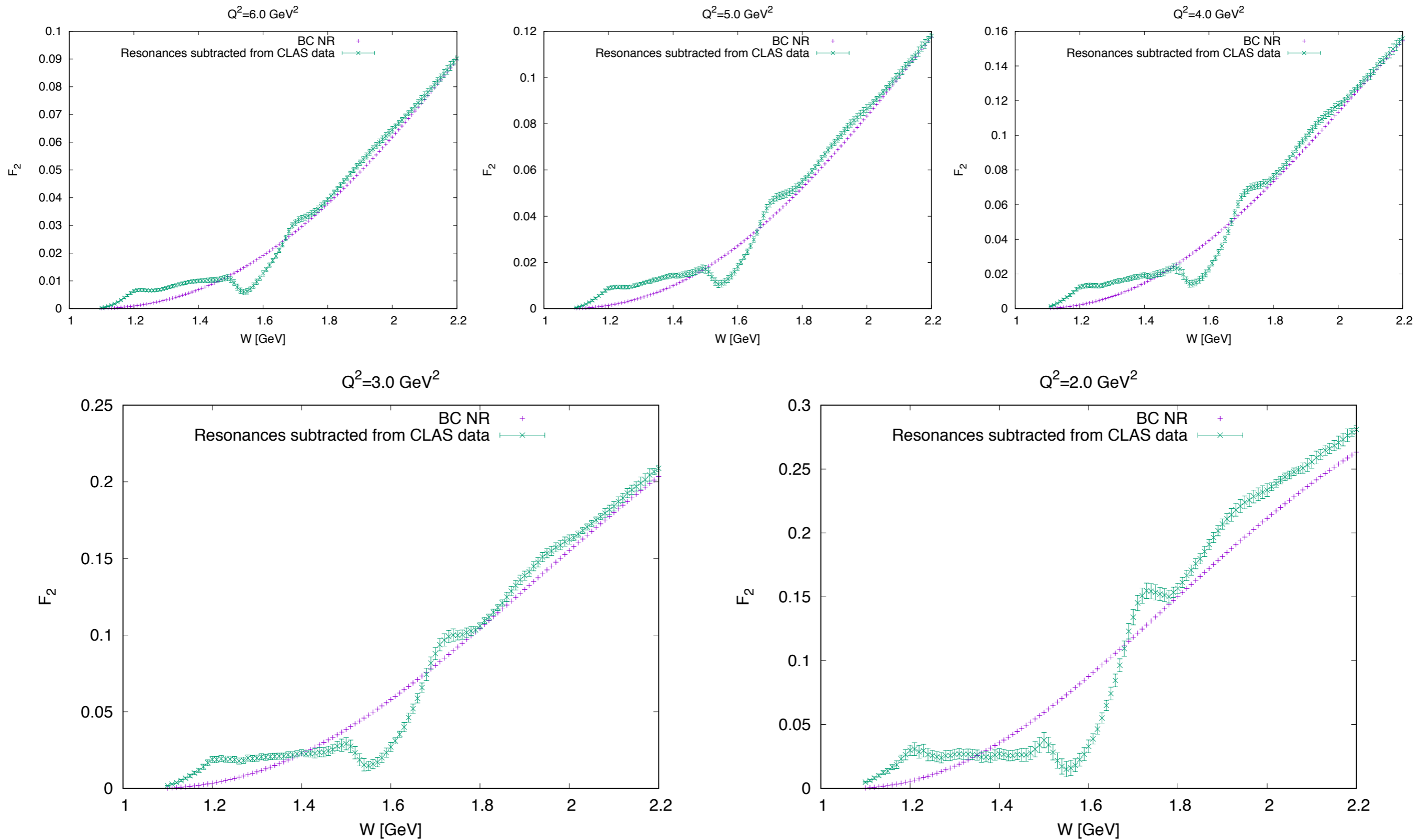
From CLAS data and fits

Δ	M_r [MeV]	Γ_r [MeV]	J_r	L_r	$BF_{\pi N}$
1232	1232	117	$\frac{3}{2}$	1	0.99
1620	1630	140	$\frac{1}{2}$	0	0.25
1700	1700	300	$\frac{3}{2}$	2	0.15
1905	1880	330	$\frac{5}{2}$	3	0.12
1910	1890	280	$\frac{1}{2}$	1	0.23
1950	1930	285	$\frac{7}{2}$	3	0.40
2420	2420	400	$\frac{11}{2}$	5	0.10

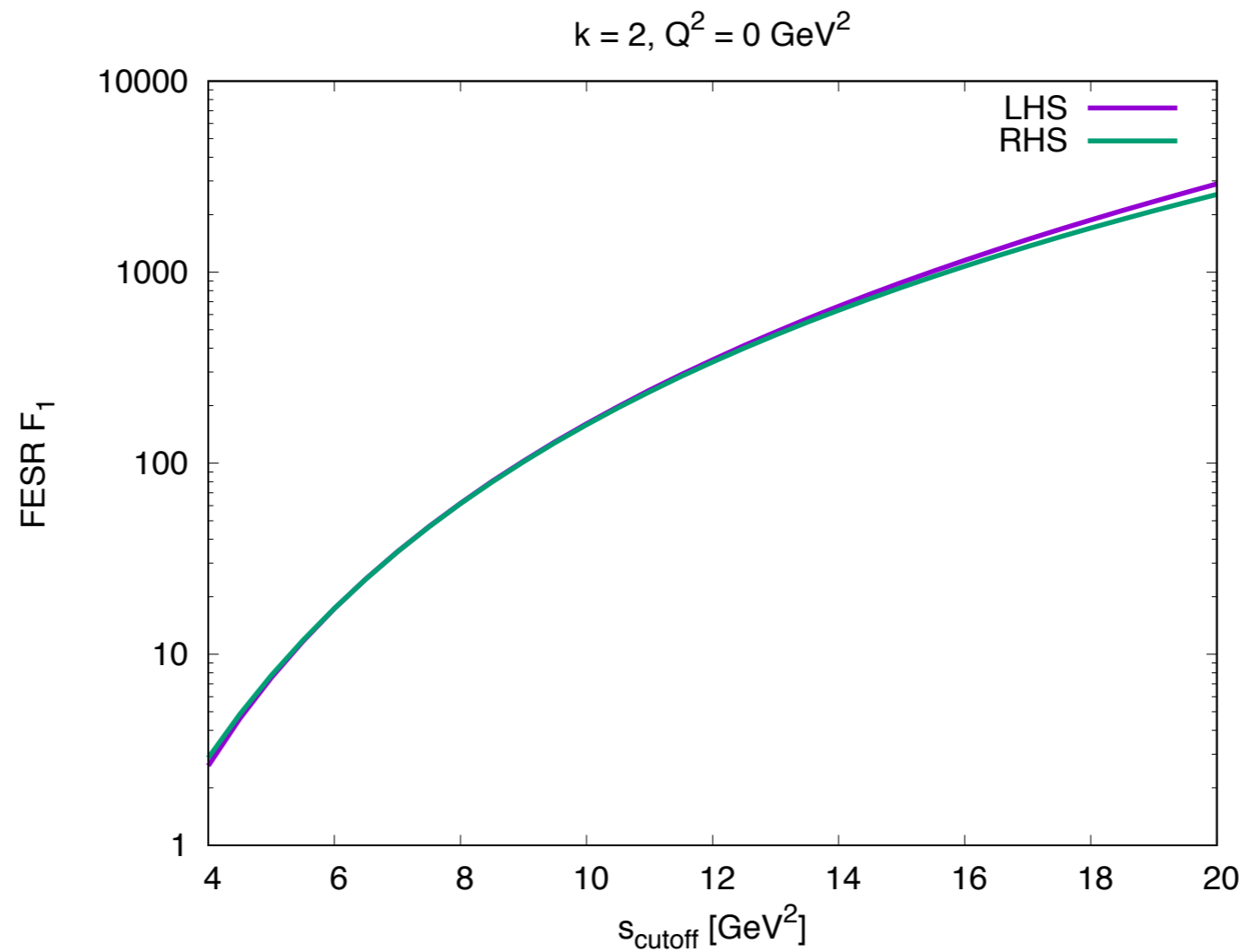
From Bonn model

Ronniger and Metsch, EPJA 49 (2013) 8

Duality in CLAS F_2 data



FESR cutoff dependence



FESR variation with k

