Tetraquark mixing framework for light mesons in the 0^+ channel

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- We explore tetraquark possibility in the light meson system.
- In particular, we reexamine the diquark-antidiquark model by Jaffe and motivate tetraquark mixing framework for the resonances in the 0^+ channel.
- Basically we introduce two types of tetraquark and their strong mixing in order to explain two nonets in PDG.

References;

1) EPJC (2017) 77:173, Hungchong Kim, M.K.Cheoun, K.S.Kim,
2) EPJC (2017) 77:435, K.S. Kim, Hungchong Kim,
3) PRD (2018) 97:094005, Hungchong Kim, K.S.Kim, M.K.Cheoun, M.Oka.

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Jaffe model

A brief review on diquark-antidiquark model

- Well-known model for tetraquark by Jaffe (1977).

- Tetraquarks are constructed by combining diquark (qq) and antidiquark ($\bar{q}\bar{q}$), $qq\bar{q}\bar{q}$, ($q = u, d, s$), while assuming all the quarks are in an S-wave.

- In this construction, the spin-0 diquark with $qq \in J = 0,\ \bar{3}_c, \bar{3}_f,$ is used
  - because this is the most compact object among all possible diquarks.
  - So it can be used as a starting building block for tetraquarks.

⟨qq structure [Jaffe, hep-ph/0001123]⟩

$Spin$	Color	Flavor	$\langle V_{CS}\rangle$	Type
0	$\bar{3}_c$	$\bar{3}_f$	-8	Attractive
1	6_c	$\bar{3}_f$	$-4/3$	Attractive
1	$\bar{3}_c$	6_f	$8/3$	Repulsive
0	6_c	6_f	4	Repulsive

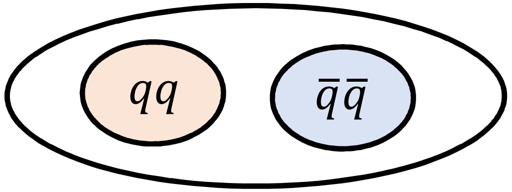
Possible diquarks allowed by Pauli principle.
$\langle V_{CS}\rangle$ is given in a certain unit.

Hyperfine color-spin interaction

$$V_{CS} \propto -\sum_{i\neq j} \lambda_i \cdot \lambda_j \, J_i \cdot J_j$$

λ_i: Gell-Mann matrix for color
J_i: spin,

$qq\bar{q}\bar{q}$ system



Jaffe model

$qq\bar{q}\bar{q}$ from the spin-0 diquark

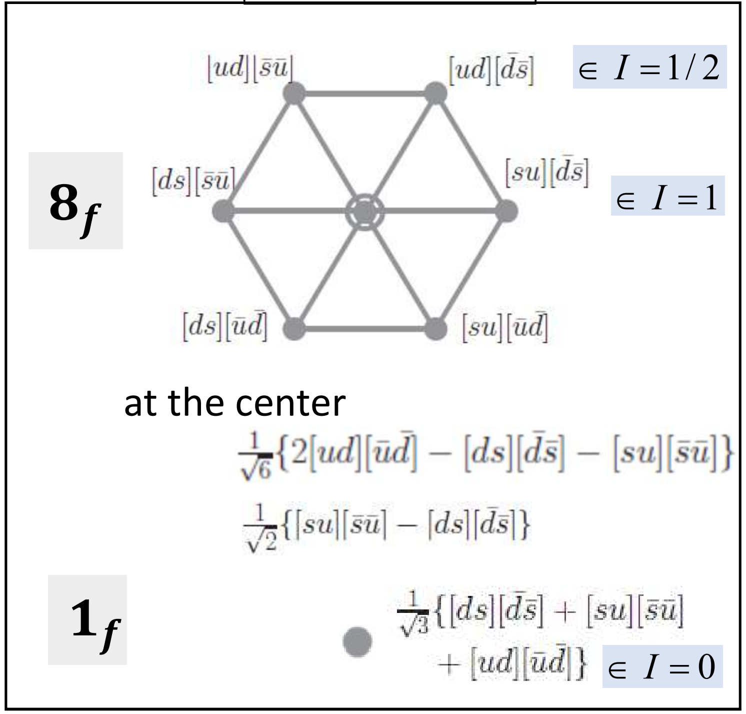
$[qq \in (J=0, \bar{3}_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J=0, 3_c, 3_f)]$

Spin: $[J_{12}=0] \otimes [J_{34}=0] = [J=0] \quad \Longrightarrow \quad |J, J_{12}, J_{34}\rangle = \underline{|000\rangle}$

Color: $\bar{3}_c \otimes 3_c \Rightarrow 1_c$, i.e., $|1_c, \bar{3}_c, 3_c\rangle$, $\quad \frac{1}{\sqrt{12}} \epsilon_{abd} \epsilon^{aef} (q^b q^d)(\bar{q}_e \bar{q}_f)$

Flavor: forming a nonet, $\bar{3}_f \otimes 3_f = 8_f \oplus 1_f \quad \Longrightarrow \quad |8_f, \bar{3}_f, 3_f\rangle \oplus |1_f, \bar{3}_f, 3_f\rangle$

Flavor nonet



Notation: $[ud] = \frac{1}{\sqrt{2}}(ud - du)$, etc.

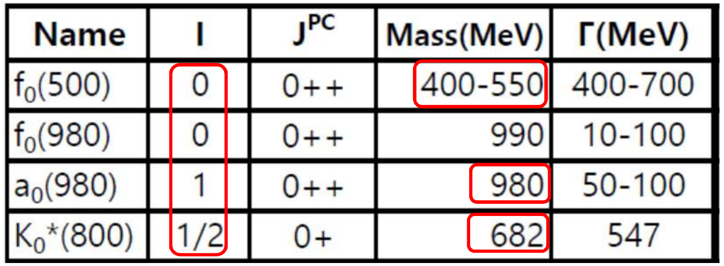
Characteristics of Jaffe's tetraquarks

1. Spin and parity are $J^P = 0^+$.
2. The $I_z = 0$ members have $C = +$.
3. Possible isospins are $I = 0, \frac{1}{2}, 1$.
4. The mass ordering among the octet members, $(I=1) > \left(I = \frac{1}{2}\right) > (I=0)$, ex) $M([su][\bar{d}\bar{s}]) > M([su][\bar{u}\bar{d}])$.

※ Note, two-quark system ($q\bar{q}$) has the opposite ordering.

Possible candidates must be sought from the resonances with $J^{P(C)} = 0^{+(+)}$

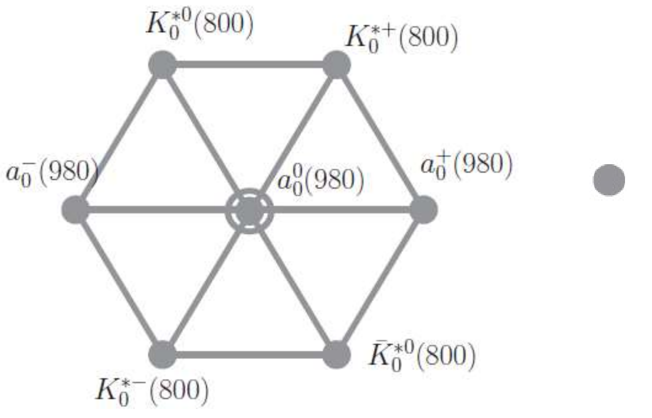
Light nonet (Jaffe’s selection)



Name	I	J^PC	Mass(MeV)	Γ(MeV)
$f_0(500)$	0	0++	400-550	400-700
$f_0(980)$	0	0++	990	10-100
$a_0(980)$	1	0++	980	50-100
$K_0^*(800)$	1/2	0+	682	547

The lowest-lying resonances in $J^{P(C)} = 0^{+(+)}$

- In PDG, the lowest-lying states in $J^P = 0^+$, $f_0(500), f_0(980), K_0^*(800)$, $a_0(980)$, seem to form a nonet ($8_f \oplus 1_f$)
  - A clue for the octet ? Gell-Mann−Okubo mass relation works within $\sim 14\%$, $M^2[a_0(980)] + 3M^2[f_0(500)] \approx 4M^2[K_0^*(800)]$.
- They satisfy the tetraquark characteristics above,
  - the anticipated isospins, $I = 0, \frac{1}{2}, 1$, and
  - the mass ordering, $M[a_0(980)] > M[K_0^*(800)] > M[f_0(500)]$.



Light nonet is the strong candidate for the tetraquark although their masses are rather small to be four-quark states.

Two states in $I = 0$
may be a mixture of: $f_0(500)$, $f_0(980)$

our claim

Another tetraquark in 0^+ can be constructed by the spin-1 diquark

because this spin-1 diquark also forms a bound state even though it is less compact than the spin-0 diquark.

⟨qq structure⟩

Spin	Color	Flavor	$\langle V_{CS} \rangle$
0	$\bar{3}_c$	$\bar{3}_f$	-8
1	6_c	$\bar{3}_f$	$-4/3$
1	$\bar{3}_c$	6_f	$8/3$
0	6_c	6_f	4

$qq\bar{q}\bar{q}$ from the spin-1 diquark in $J^P = 0^+$ channel

Spin: $[J_{12}=1]\otimes[J_{34}=1] \Rightarrow [J=0] \quad \Longrightarrow |J, J_{12}, J_{34}\rangle = \underline{|011\rangle}$

Color: $6_c \otimes \bar{6}_c \Rightarrow 1_c$, i.e., $|1_c, 6_c, \bar{6}_c\rangle$, $\frac{1}{\sqrt{96}}(q^a q^b + q^b q^a)(\bar{q}_a \bar{q}_b + \bar{q}_b \bar{q}_a)$

Flavor: $\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$ also form a nonet in flavor !

⇒ This 2nd tetraquark also satisfies the tetraquark characteristics above.

- In fact, this 2nd tetraquark is more compact than the one from the spin-0 diquark.
  ⇒ The spin-1 diquark configuration is also important as well,
  ⇒ and cannot be ignored in the construction of tetraquark.
- But this 2nd type tetraquark requires another nonet to be found in PDG
  ⇒ do we have the candidates ?

Yes ! PDG seems to have another nonet that satisfies the same characteristics.

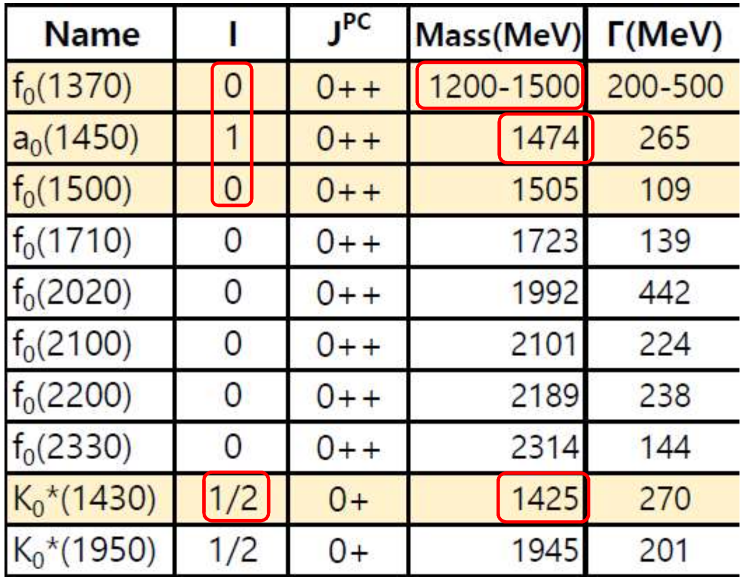
our observation

Heavy nonet (our selection)

- A similar nonet can be selected from higher resonances in $J^P = 0^+$, $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$
  - GMO relation within $\sim 4\%$, $M^2[a_0(1450)] + 3M^2[f_0(1370)] \approx 4M^2[K_0^*(1425)]$
- They have the anticipated isospins, $I = 0, \frac{1}{2}, 1$, and
- their mass ordering, though marginal, still holds here,

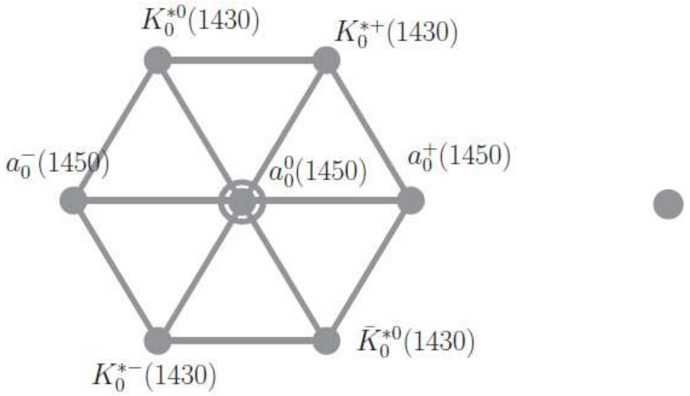
$M[a_0(1450)] > M[K_0^*(1430)]$ with $\Delta M \sim 50$ MeV, $M[K_0^*(1430)] \gtrsim M[f_0(1370)]$.

The 'marginal' ordering can be explained partially by our hyperfine masses (more later!).



Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$f_0(1370)$	0	0++	1200-1500	200-500
$a_0(1450)$	1	0++	1474	265
$f_0(1500)$	0	0++	1505	109
$f_0(1710)$	0	0++	1723	139
$f_0(2020)$	0	0++	1992	442
$f_0(2100)$	0	0++	2101	224
$f_0(2200)$	0	0++	2189	238
$f_0(2330)$	0	0++	2314	144
$K_0^*(1430)$	1/2	0+	1425	270
$K_0^*(1950)$	1/2	0+	1945	201

$J^{P(C)} = 0^{+(+)}$ with higher masses



Two states in $I = 0$ may be a mixture of $f_0(1370), f_0(1500)$

Heavy nonet could be the 2nd candidate for the tetraquark !

our model setup

We have two tetraquark types in $J^P = 0^+$,

- differed by the spin and color configuration which we denote by

  $|000\rangle_{\bar{3}_c,3_c} \Longrightarrow |000\rangle \qquad |011\rangle_{6_c,\bar{6}_c} \Longrightarrow |011\rangle.$
- Both form a nonet in flavor separately ($8_f \oplus 1_f$).

PDG also has two nonets in $J^P = 0^+$ with the tetraquark characteristics.

Light nonet (Jaffe's selection)

The lowest-lying in 0^+,
$f_0(500), f_0(980),\ K_0^*(800),\ a_0(980)$

Heavy nonet (additional selection by us)

From higher resonances in 0^+,
$f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

The huge mass gap between the two $\gtrsim 500$ MeV

What kind of correspondence one can make between the two sets ?

Two tetraquark types $\Longleftrightarrow$ Two nonets in PDG

A crucial observation is that

- the two tetraquarks, $|000\rangle$, $|011\rangle$, mix through the hyperfine color-spin interaction !

$$V_{CS} \propto \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

λ_i: Gell-Mann matrix for color,
J_i: spin,
m_i: constituent quark mass

- The mixing terms are nonzero, $\langle 011|V_{CS}|000\rangle \neq 0$.
- $\langle V_{CS}\rangle$ forms a 2x2 matrix in the bases, $|000\rangle, |011\rangle$,
  $\Longrightarrow$ constituting the hyperfine mass matrix.

The upshot is that

- physical resonances, the two nonets in PDG, can be identified by the eigenstates that diagonalize the 2x2 matrix,
  i.e., the two nonets in PDG must be superposition of $|000\rangle, |011\rangle$.
- In fact, the mixing is found to be strong so it can explain the large mass gap between the two nonets.

This is our tetraquark mixing framework for the two nonets in $J^P = 0^+$.

☞ We look for its phenomenological signatures from experimental observables such as masses or decay properties !

digression

One question

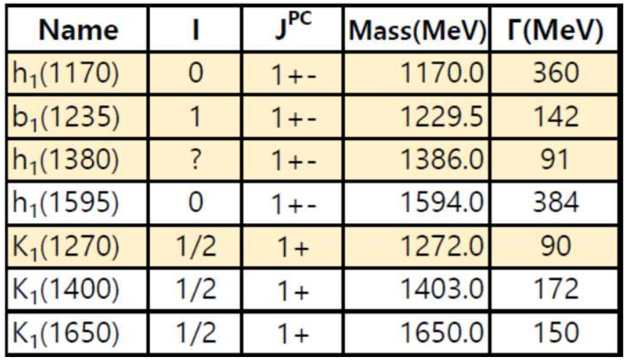
- The spin-1 diquark scenario requires additional nonets to be found in $J^P = 1^{+-}, 2^{++}$ corresponding to the configurations

$$|111\rangle_{6_c,\bar{6}_c} \qquad |211\rangle_{6_c,\bar{6}_c}$$

※ One can prove that C-parity is negative for $J = 1$, positive for $J = 2$.

Are there such nonets in PDG ? My answer is 'Maybe'.

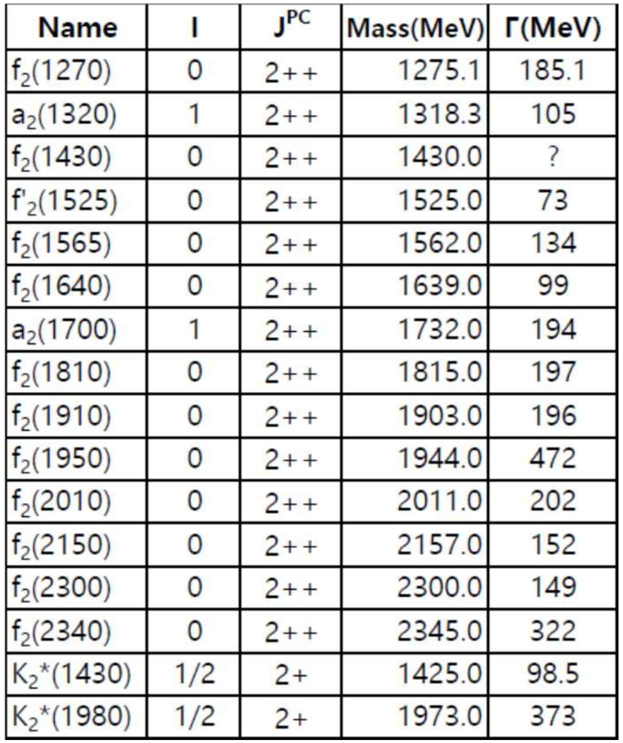
- There are lots of resonances to choose but the candidate selection is not definite.



Name	I	J^PC	Mass(MeV)	Γ(MeV)
h_1(1170)	0	1+-	1170.0	360
b_1(1235)	1	1+-	1229.5	142
h_1(1380)	?	1+-	1386.0	91
h_1(1595)	0	1+-	1594.0	384
K_1(1270)	1/2	1+	1272.0	90
K_1(1400)	1/2	1+	1403.0	172
K_1(1650)	1/2	1+	1650.0	150

$J^{P(C)} = 1^{+(-)}$ resonances

- Highlighted members can be selected but with some ambiguity,
  - unknown isospin of $h_1(1380)$,
  - the mass ordering, slightly violated, $M[b_1(1235)] < M[K_1(1270)]$



Name	I	J^PC	Mass(MeV)	Γ(MeV)
f_2(1270)	0	2++	1275.1	185.1
a_2(1320)	1	2++	1318.3	105
f_2(1430)	0	2++	1430.0	?
f'_2(1525)	0	2++	1525.0	73
f_2(1565)	0	2++	1562.0	134
f_2(1640)	0	2++	1639.0	99
a_2(1700)	1	2++	1732.0	194
f_2(1810)	0	2++	1815.0	197
f_2(1910)	0	2++	1903.0	196
f_2(1950)	0	2++	1944.0	472
f_2(2010)	0	2++	2011.0	202
f_2(2150)	0	2++	2157.0	152
f_2(2300)	0	2++	2300.0	149
f_2(2340)	0	2++	2345.0	322
K_2^*(1430)	1/2	2+	1425.0	98.5
K_2^*(1980)	1/2	2+	1973.0	373

$J^{P(C)} = 2^{+(+)}$ resonances

digression

The selection is ambiguous

- maybe due to further mixings with additional tetraquarks constructed by other diquarks, and possible contamination from two-quark component with $\ell = 1$.
- This ambiguity does not mean that $|111\rangle, |211\rangle$ do not exist.
  ⇒ It simply says that the candidates do not stand out in a well-separated entity.
  ⇒ It does not rule out our mixing framework in the 0^+ channel.

testing ground

Testing ground of our tetraquark framework is the two nonets.

Isospin	Light nonet	Heavy nonet	
$I = 1$	$a_0(980)$	$a_0(1450)$	
$I = 1/2$	$K_0^*(800)$	$K_0^*(1430)$	
$I = 0$	$f_0(500)$	$f_0(1370)$	$\Leftarrow$ close to the 8_f member
	$f_0(980)$	$f_0(1500)$	$\Leftarrow$ close to the 1_f member

Flavor mixing on isoscalars

- The $I = 0$ members are subject to additional flavor mixing between $|\mathbf{8}_f\rangle_{I=0}, |\mathbf{1}_f\rangle_{I=0}$, known as the OZI rule.
- Depending on how the flavor mixing is implemented, we consider three cases

(Hungchong Kim et.al., PRD2018),

– $SU(3)_f$ Symmetric Case, SSC (no flavor mixing)
– Ideal Mixing Case, IMC
– Realistic Case with Fitting, RCF

According to our mixing scheme

- first we need to calculate the hyperfine masses, $\langle V_{CS}\rangle$, w.r.t. $|000\rangle, |011\rangle$ in each isospin channel.

Color-spin interaction for four-quark system

hyperfine masses

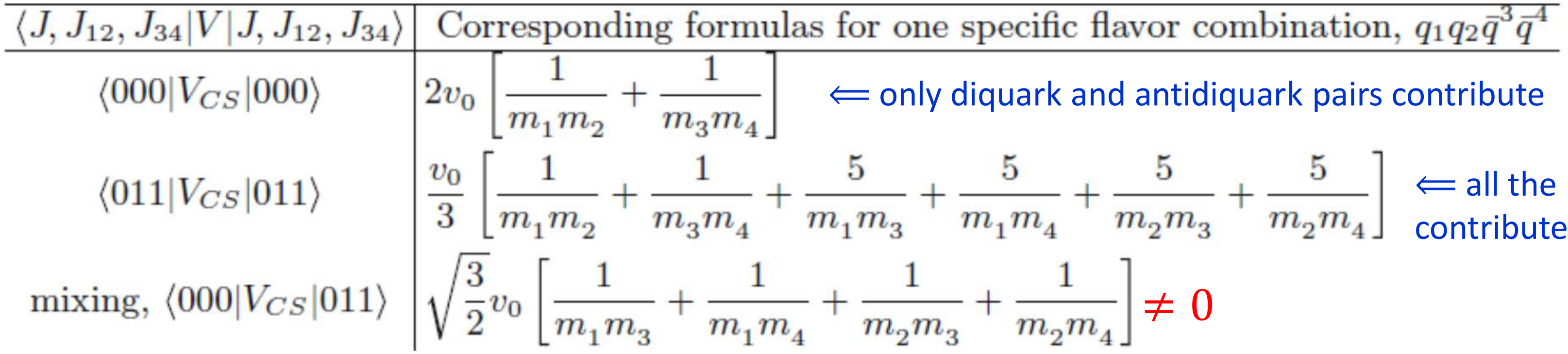
$$V_{CS} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

for all the pairs among 4 quarks

$$= v_0\left[\lambda_1 \cdot \lambda_2 \frac{J_1 \cdot J_2}{m_1 m_2} + \lambda_3 \cdot \lambda_4 \frac{J_3 \cdot J_4}{m_3 m_4} + \lambda_1 \cdot \lambda_3 \frac{J_1 \cdot J_3}{m_1 m_3} + \lambda_1 \cdot \lambda_4 \frac{J_1 \cdot J_4}{m_1 m_4} + \lambda_2 \cdot \lambda_3 \frac{J_2 \cdot J_3}{m_2 m_3} + \lambda_2 \cdot \lambda_4 \frac{J_2 \cdot J_4}{m_2 m_4}\right]$$

Master formulas for $\langle V_{CS} \rangle$

$v_0 = (-192.9\ \mathrm{MeV})^3$ from the mass splitting, $D_2^*(2463) - D_0^*(2318)$



$\langle J, J_{12}, J_{34} \vert V \vert J, J_{12}, J_{34} \rangle$	Corresponding formulas for one specific flavor combination, $q_1 q_2 \bar{q}^3 \bar{q}^4$
$\langle 000 \vert V_{CS} \vert 000 \rangle$	$2v_0\left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4}\right]$
$\langle 011 \vert V_{CS} \vert 011 \rangle$	$\frac{v_0}{3}\left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} + \frac{5}{m_1 m_3} + \frac{5}{m_1 m_4} + \frac{5}{m_2 m_3} + \frac{5}{m_2 m_4}\right]$
mixing, $\langle 000 \vert V_{CS} \vert 011 \rangle$	$\sqrt{\frac{3}{2}} v_0\left[\frac{1}{m_1 m_3} + \frac{1}{m_1 m_4} + \frac{1}{m_2 m_3} + \frac{1}{m_2 m_4}\right]$ $\neq 0$

⟸ only diquark and antidiquark pairs contribute

⟸ all the pairs contribute

Ex) For the $I = 1$ members,

Since their flavor is, $[su][\bar{d}\bar{s}] = \frac{1}{2}(su - us)\,(\bar{d}\bar{s} - \bar{s}\bar{d})$,

we sum over all flavor combinations,

$$\langle V_{CS} \rangle = \frac{1}{4}[\langle V_{CS} \rangle_{su\bar{d}\bar{s}} + \langle V_{CS} \rangle_{su\bar{s}\bar{d}} + \langle V_{CS} \rangle_{us\bar{d}\bar{s}} + \langle V_{CS} \rangle_{us\bar{s}\bar{d}}\,].$$

hyperfine masses

Hyperfine mass matrix in the $I = 1$ channel,

- Diagonalization leads to the physical hyperfine masses

$$\begin{array}{c|cc} \langle V_{CS} \rangle & |000\rangle & |011\rangle \\ \hline |000\rangle & \underline{-173.9} & \underline{-222.3} \\ |011\rangle & -222.3 & \underline{-331.5} \end{array} \quad \rightarrow \quad \begin{array}{c|cc} \langle V_{CS} \rangle & |0_A^{a_0}\rangle & |0_B^{a_0}\rangle \\ \hline |0_A^{a_0}\rangle & -16.8 & 0.0 \\ |0_B^{a_0}\rangle & 0.0 & -488.5 \end{array}$$

and eigenstates corresponding to $a_0(980), a_0(1450)$

$$|0_A^{a_0}\rangle = -0.817|000\rangle + 0.577|011\rangle \quad \Rightarrow |a_0(1450)\rangle$$

$$|0_B^{a_0}\rangle = 0.577|000\rangle + \underline{0.817|011\rangle} \quad \Rightarrow |a_0(980)\rangle$$

This identification follows from $\langle 0_A^{a_0}|V_{CS}|0_A^{a_0}\rangle > \langle 0_B^{a_0}|V_{CS}|0_B^{a_0}\rangle$

As advertised,

- $|011\rangle$ is found to be more compact, $\langle 000|V_{CS}|000\rangle > \langle 011|V_{CS}|011\rangle$.
  ☞$a_0(980)$ has more probability to stay in $|011\rangle$ than in $|000\rangle$!!
  ※ The similar result was reported also by Black et.al [PRD59,074026 (1999)]. There, this mixing is used to explain why the light nonet is 'so light' without identifying the heavy nonet.
  ☞ We emphasize that $|011\rangle$ must be considered in tetraquark studies.
- The **strong mixing** causes **large separation** in hyperfine masses.
  ☞ This can explain the large mass gap (500 MeV or so)
  ☞ in addition to the lightness of the light nonet.

Similar consequences can be seen in the other isospin channels.

Including all the members,

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle \ = \ \beta|000\rangle + \alpha|011\rangle$$

(diagonal) hyperfine masses

Isospin	α	β	Light	$\langle V_{CS}\rangle$	Heavy	$\langle V_{CS}\rangle$
$I=1$	0.867	0.577	$a_0(980)$	-488.5	$a_0(1450)$	-16.8
$I=1/2$	0.813	0.582	$K_0^*(800)$	-592.7	$K_0^*(1430)$	-26.9
$I=0$ (RCF)	0.814	0.581	$f_0(500)$	-667.5	$f_0(1370)$	-29.2
$I=0$ (RCF)	0.816	0.578	$f_0(980)$	$\underline{-535.1}$	$f_0(1500)$	$\underline{-20.1}$

~100

~10

close to the 8_f member

※ Approximately, $\alpha \approx \sqrt{2/3},\ \beta \approx \sqrt{1/3}$.

- For the octet members, our hyperfine masses are ordered,
  $\langle V_{CS}\rangle_{I=1} > \langle V_{CS}\rangle_{I=\ /2} > \langle V_{CS}\rangle_{I=0}$, the same as the masses, $M[a_0] > M[K_0^*] > M[f_0]$.
  ☞ $\langle V_{CS}\rangle$ is partially responsible for the mass ordering.

- But $\langle V_{CS}\rangle$ splitting is much narrower for heavy nonet,
  ~ 100 MeV for light nonet,
  ~ 10 MeV or less for heavy nonet.

Our hyperfine masses explain partially the marginal mass ordering seen in the heavy nonet !

Mass splitting formula

- Our first task is to test our framework in generating masses through the mass splitting formula,

  $\Delta M_H \approx \Delta\langle V_{CS}\rangle$ $\quad V_{CS} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$

- It says that the mass difference between hadrons with the same flavor content and color configuration can be approximated by their hyperfine mass splitting (we understand why).

  ※ For example, this seems to work well for the lowest-lying baryons and mesons, $\Delta - N, \Sigma - \Lambda, \Xi^* - \Xi,\ K^* - K, D^* - D, etc.$

  PLB(1986)171:293, Lipkin, EPJA (2016) 52:184, PRD(2015)91:014021, H.Kim et.al.

- This splitting formula minimizes the parameter dependence so its prediction could be reliable.

- Our tetraquarks, $|000\rangle$, $|011\rangle$, have different color configurations. But the color-electric terms, $V_{CE} = v_1 \sum_{i<j} \frac{\lambda_i \cdot \lambda_j}{m_i\, m_j}$, almost cancel in the difference, $\Delta\langle V_{CE}\rangle \approx 0$ (backup slides).

Results on mass splitting between the two nonets

$\Delta M_H \approx \Delta\langle V_{CS}\rangle$

For $I = 1$

Heavy nonet	Light nonet	ΔM_{exp} (MeV)	$\Delta\langle V_{CS}\rangle$ (MeV)		
			SSC	IMC	RCF
$a_0(1450)$	$a_0(980)$	**494**	**471.7**	-	-

☞Our mixing scheme works very well !

equal when
$m_u = m_d$

For $I = 0, 1/2$

$f_0(1500)$	$f_0(980)$	**515**	**541.7**	471.7	515
$f_0(1370)$	$f_0(500)$	875	611.7	681.7	638.3
$K_0^*(1430)$	$K_0^*(800)$	743	565.8	-	-

M_{exp} is broad or not fixed well

Note that the $I = 0$ results do not depend much on how the flavor mixing is implemented. For the last two lines, precise agreement is not anticipated as the participating resonances are either very broad or their masses are poorly known.

At least, we can say from all these that the strong mixing qualitatively generates the huge gap between the two nonets.

Our second task is to test

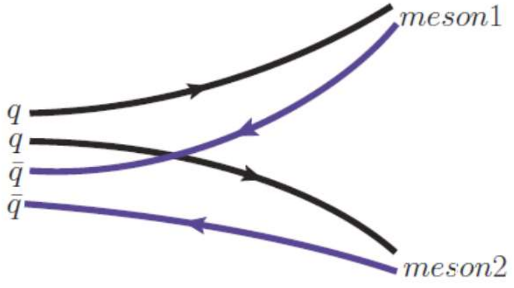
Tetraquark signatures from the $a_0(980)$, $a_0(1450)$ decays

※ We do not discuss the $I = 0, 1/2$ cases due to lack of expt. data for comparison.

Tetraquark decays dominantly through the fall-apart mechanism.

- In this mechanism, the quark-antiquark pairs simply fall apart into two mesons.
- This decay is possible because our tetraquark have two-meson open channel.

  ☞ Namely, rearranging $q_1 q_2 \bar{q}^3 \bar{q}^4$ into quark-antiquark pairs, $(q_1 \bar{q}^3)(q_2 \bar{q}^4)$, we see the nonzero component with two color singlet pairs,



$qq\bar{q}\bar{q}$ fall-apart decay

$$\overset{(24)}{\underset{(13)}{q_1 q_2 \bar{q}^3 \bar{q}^4}} \Longrightarrow [(8_c)_1 \otimes (8_c)_{24}]_{1_c} \oplus \boxed{[(1_c)_{13} \otimes (1_c)_{24}]_{1_c}}$$

two-meson modes

fall-apart strength

Fall-apart strength of $a_0(980)$, $a_0(1450)$

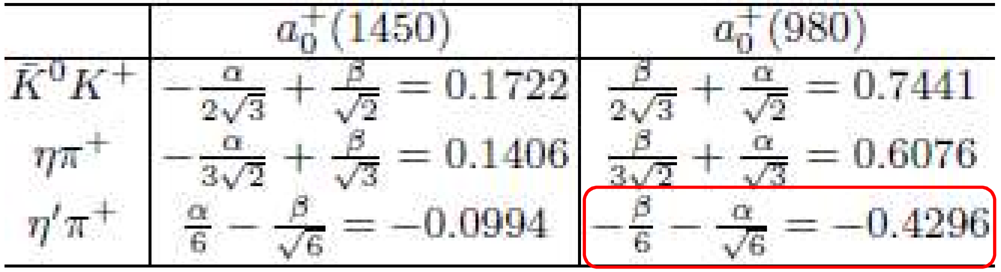
$$|a_0(1450)\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|a_0(980)\rangle = \beta|000\rangle + \alpha|011\rangle$$

$\alpha = 0.817, \beta = 0.577$

- $|000\rangle, |011\rangle$ fall apart into two mesons, each forming a color singlet, spin-0 state.
- The relative sign difference leads to the coupling strengths suppressed for $a_0(1450)$ but enhanced for $a_0(980)$. (K.S.Kim, Hungchong Kim, EPJC2017).

Coupling strength of the fall-apart modes into two PS mesons

up to an overall constant



	$a_0^+(1450)$	$a_0^+(980)$
$\bar{K}^0K^+$	$-\frac{\alpha}{2\sqrt{3}}+\frac{\beta}{\sqrt{2}}=0.1722$	$\frac{\beta}{2\sqrt{3}}+\frac{\alpha}{\sqrt{2}}=0.7441$
$\eta\pi^+$	$-\frac{\alpha}{3\sqrt{2}}+\frac{\beta}{\sqrt{3}}=0.1406$	$\frac{\beta}{3\sqrt{2}}+\frac{\alpha}{\sqrt{3}}=0.6076$
$\eta'\pi^+$	$\frac{\alpha}{6}-\frac{\beta}{\sqrt{6}}=-0.0994$	$-\frac{\beta}{6}-\frac{\alpha}{\sqrt{6}}=-0.4296$

kinematically not allowed

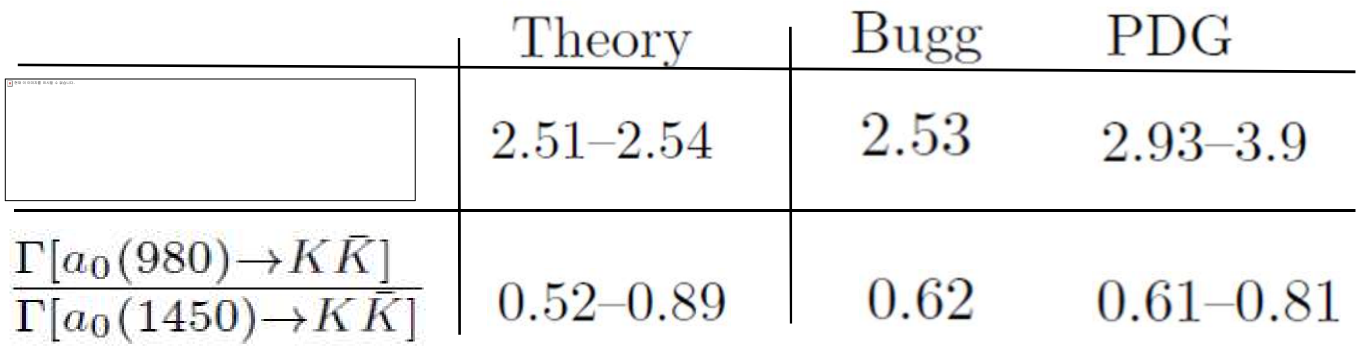
- The relative enhancement factor is about ‘four’ !
- Similar enhancement can be seen for the other channels (Hungchong Kim et.al., PRD2018).

Could be a clear signature for the tetraquark mixing framework.

partial width ratios

This signature can be tested most effectively from the following ratios !

Based on expt. analysis



	Theory	Bugg	PDG
	2.51–2.54	2.53	2.93–3.9
$\dfrac{\Gamma[a_0(980)\to K\bar{K}]}{\Gamma[a_0(1450)\to K\bar{K}]}$	0.52–0.89	0.62	0.61–0.81

(backup slide)

Bugg: PRD78,074023(2008)

※ The ratios eliminate the dependence on the overall constant.

The agreement is quite good !

- Only disagreement is in the 1st ratio in comparison with the PDG ratio but both results still point toward the enhancement and suppression of the couplings.
- Our tetraquark mixing framework seems to work for the decays.

my view

Some comments on a two-quark picture

1. Is it possible to explain the two nonets (0^+) in a two-quark picture $(q\bar{q})$ with $\ell = 1$? My answer is ‘No’.

$$q\bar{q}: (S = 0,1) \otimes (\ell = 1) \Longrightarrow J = 0,1,2$$

Total J	Configuration	# of confs.
$J = 0$	$(S = 1, \ell = 1)$	one
$J = 1$	$(S = 0, \ell = 1), (S = 1, \ell = 1)$	two
$J = 2$	$(S = 1, \ell = 1)$	one

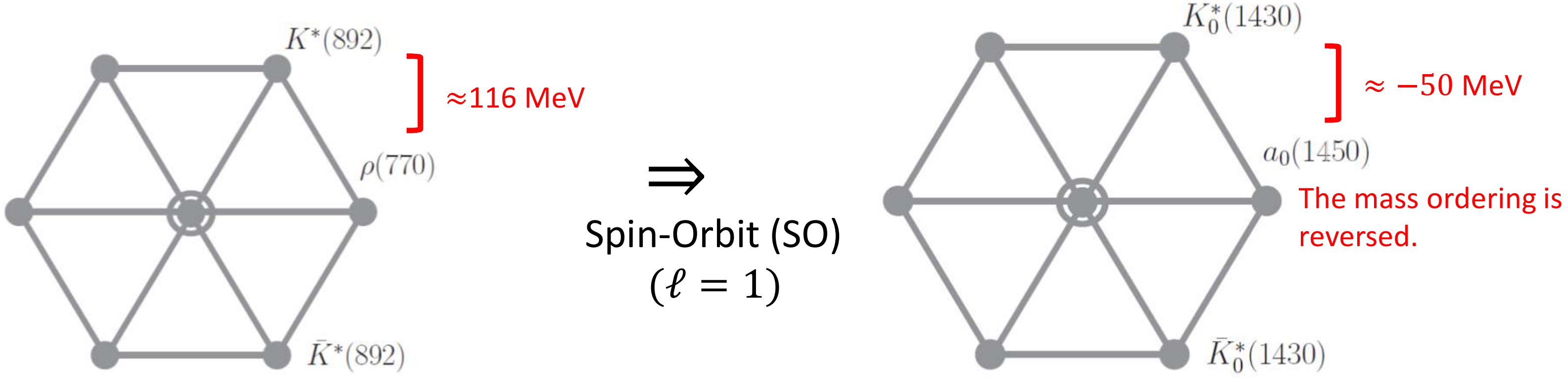
☞This picture yields only one configuration in $J^P = 0^+$.

- Appearance of the two nonets in 0^+ cannot be explained by this two-quark picture.
- This gives another motivation for constructing the tetraquark framework.

my view

2. Alternatively, one may view the heavy nonet in a $q\bar{q}$ picture while maintaining the $qq\bar{q}\bar{q}$ picture for the light nonet. We think this is not realistic.

- The heavy nonet, if viewed as $q\bar{q}$ with $\ell = 1$, must have the configuration $(S = 1$, vector nonet$) \otimes (\ell = 1) \Longrightarrow J = 0$
  $\Rightarrow$ orbital excitations of the vector mesons, ρ, ω, K^*, ϕ.



The mass ordering is reversed.

- In this picture, SO makes the heavy nonet ‘heavier’ than the vector nonet.
- To reproduce the expt. gap (≈ -50 MeV), SO must have strong dependence on isospin channels, strong enough to flip the mass ordering normally established by the quark masses.
  ☞ This picture seems not realistic !

my view

3. One may view the two nonets as a mixture of a two-quark ($q\bar{q}$), and four-quark ($qq\bar{q}\bar{q}$) ?

   - But $q\bar{q}$, $qq\bar{q}\bar{q}$ do not mix under the color-spin interaction !

     $\langle q\bar{q}|qq\bar{q}\bar{q}\rangle = 0$, $\langle q\bar{q}|V_{CS}|qq\bar{q}\bar{q}\rangle = 0.$

   - Normally this scenario requires ad hoc mixing.

my view

Some comments on hadronic molecules

- One may view the heavy nonet as meson-meson bound states.

- Since mesons are colorless, this picture provides shallow bound states
  ⇒ Expected to be less probable to be formed in collision processes.

- Since the lowest-lying mesons form a nonet in flavor, the flavor structure of the meson-meson states would be much diverse including 27-plet
  ⇒ PDG does not support this picture. (ex. no 0^+ resonances with $I = 2$.)

Summary

- We propose a tetraquark mixing framework for light mesons in the 0^+ channel.
  - Two types of tetraquark $|000\rangle, |011\rangle$, have been introduced, one from the spin-0 diquark and the other from the spin-1 diquark.
  - We emphasize that $|011\rangle$ is important in the tetraquark studies.
  - The two tetraquarks are found to mix strongly through the color-spin interaction.
  - We report that their mixture, which diagonalize the hyperfine mass, can generate the two nonets in PDG, the light and heavy nonets.

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

with $\alpha \approx \sqrt{2/3}, \beta \approx \sqrt{1/3}$

- Our mixing framework has been tested relatively well phenomenologically.
  - It reproduces the mass splitting between the two nonets.
  - Its another consequence in the decay couplings, namely coupling enhancement for the light nonet and suppression for the heavy nonet, has been tested relatively well for the decays, $a_0(980), a_0(1450) \Longrightarrow K\bar{K}, \eta\pi$.

Our work may provide a new view on tetraquarks, especially how they are realized in the actual spectrum, i.e., through ``mixing framework’’.

Back up slides

backup slide 1

Explanation for $\Delta\langle V_{CE}\rangle \approx 0$

$$\begin{pmatrix} \langle 000|V_{CE}|000\rangle & 0 \\ 0 & \langle 011|V_{CE}|011\rangle \end{pmatrix}$$ $\Rightarrow$ a diagonal matrix in $J = 0$ channel

$\langle 000|V_{CE}|011\rangle = 0$ because V_{CE} is blind on spin

$$\langle 000|V_{CE}|000\rangle \approx \langle 011|V_{CE}|011\rangle$$

$$\begin{pmatrix} -23.8 & 0 \\ 0 & -24.57 \end{pmatrix} \approx -24.57 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- It is almost a multiple of the identity matrix in $|000\rangle, |011\rangle$ basis, unchanged under diagonalization.
- $\Delta\langle V_{CE}\rangle \approx 0 \Longrightarrow$ does not contribute to the mass splitting.

backup slide 2

Theoretical partial widths of $a_0 \Longrightarrow K\bar{K}, \eta\pi$

- calculated by constructing effective Lagrangians but with the coupling strengths fixed from our fall-apart decays.
- The width is averaged over the mass distribution $f(M)$ determined by the total decay width and its central mass.

$$\langle\Gamma(M_c,\Gamma_{exp})\rangle = \frac{\int_{m_1+m_2}^{\infty}\Gamma(M)f(M)dM}{\int_{m_1+m_2}^{\infty}f(M)dM}$$

Meson	M_c(MeV)	Γ_{exp}(MeV)
$a_0(980)$	980	50-100
$a_0(1450)$	1474	265

Expt. partial widths of $a_0 \Longrightarrow K\bar{K}, \eta\pi$

For $a_0(980)$, its partial widths can be estimated relatively well from PDG,

$$\Gamma[a_0(980) \to \pi\eta] \approx 60 \text{ MeV} ,$$
$$\Gamma[a_0(980) \to K\bar{K}] \approx 10.98 \text{ MeV}$$

For $a_0(1450)$, two sets are available from experimental analysis.

Partial width	Bugg(MeV)	PDG(MeV)
$\Gamma[a_0(1450) \to \pi\eta]$	23.7	15.38–20.49
$\Gamma[a_0(1450) \to K\bar{K}]$	17.7	13.53–18.03

Bugg, PRD78,074023(2008)