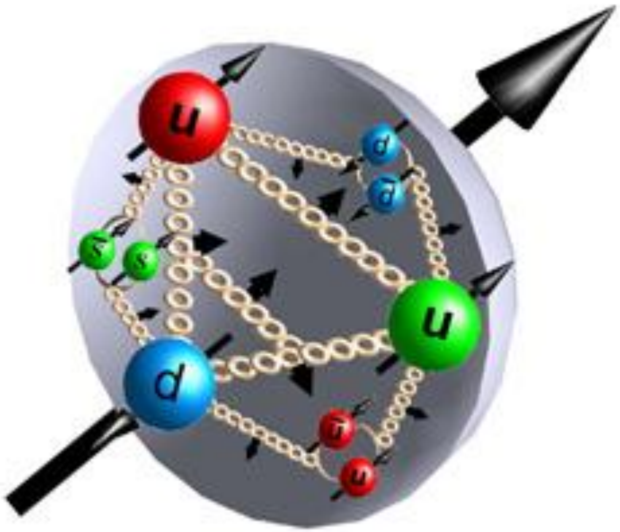
Structure of the nucleon's low-lying excitations

CHEN CHEN

The Institute for Theoretical Physics,
Sao Paulo State University
(IFT- UNESP)



Non-Perturbative QCD:
Confinement & dynamical chiral symmetry breaking (DCSB)

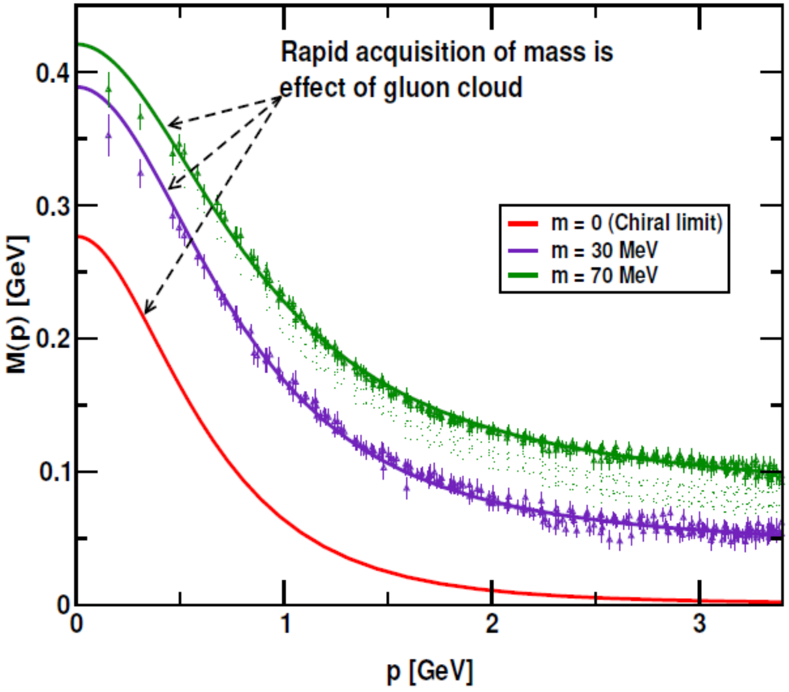
- **Hadrons, as bound states, are dominated by non-perturbative QCD dynamics**
  - Explain how quarks and gluons bind together → **Confinement**
  - Explain the most important mass generating mechanism for visible matter in the Universe → **DCSB**
- **Emergent phenomena**
  - **Confinement**: Colored particles have never been seen isolated
  - **DCSB**: Hadrons do not follow the chiral symmetry pattern
  - Neither of these phenomena is apparent in QCD 's Lagrangian, HOWEVER, They play a dominant role in determining the characteristics of real-world QCD!

Non-Perturbative QCD:
Confinement & dynamical chiral symmetry breaking (DCSB)

- **From a quantum field theoretical point of view**, emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD 's Schwinger functions (propagators and vertices).
- **Dressed-quark propagator in Landau gauge:**

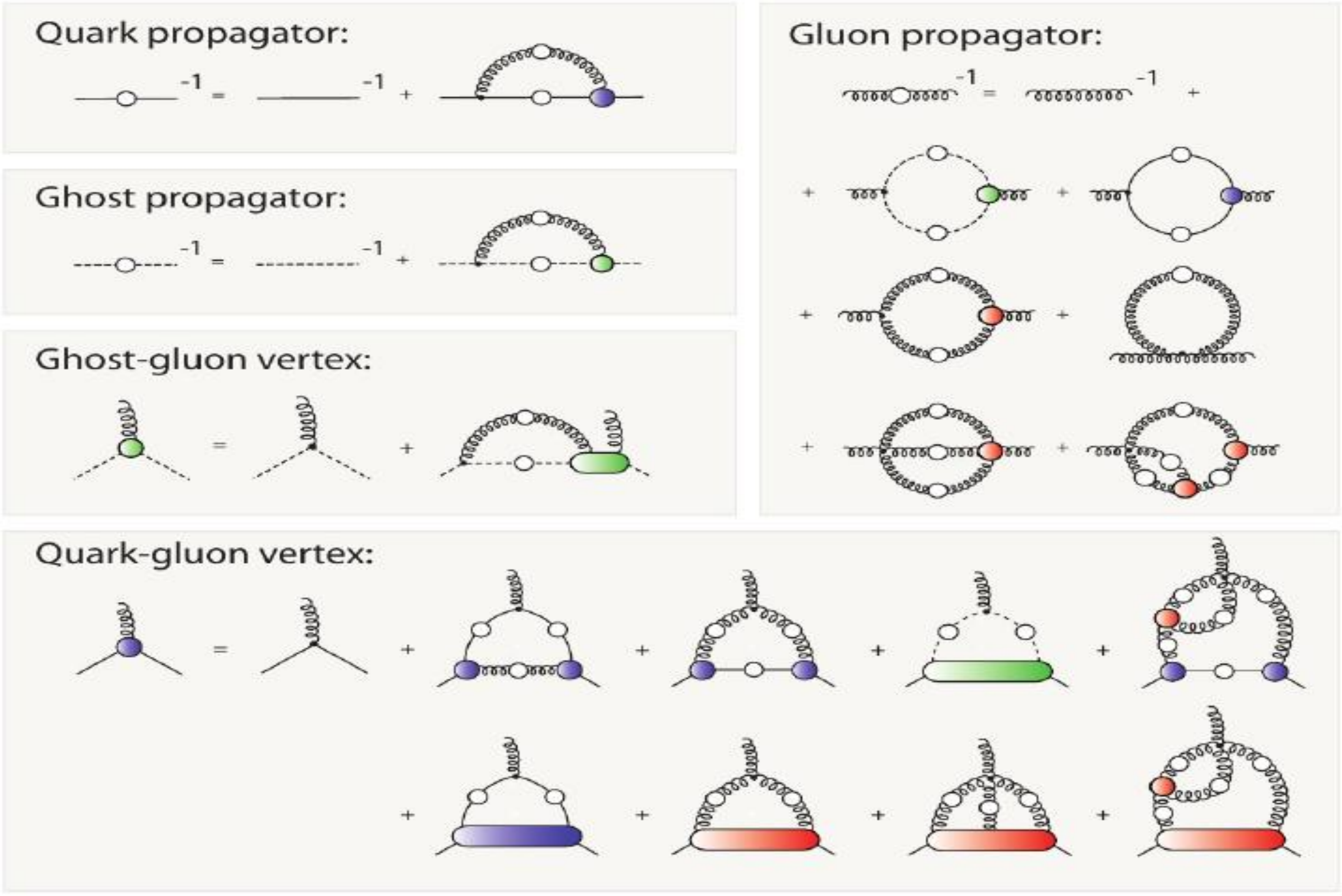
$$S^{-1}(p) = Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) = \left(\frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}\right)^{-1}$$

  - Mass generated from the interaction of quarks with the gluon-medium.
  - Light quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton and the large splitting between parity partners.



Dyson-Schwinger equations (DSEs)

- **The Schwinger functions are solutions of *the quantum equations of motion* (DSEs)**



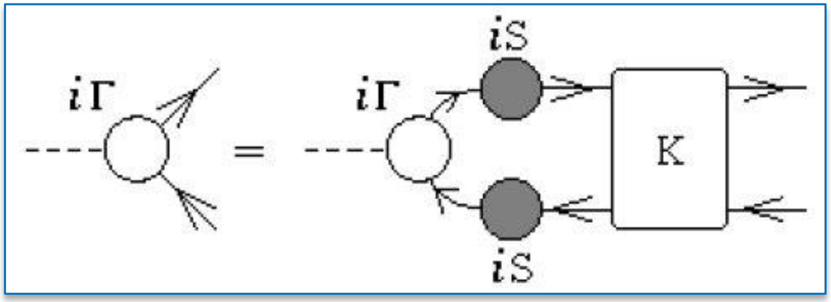
Dyson-Schwinger equations (DSEs)

Dyson-Schwinger equations

- ✓ A Nonperturbative symmetry-preserving tool for the study of Continuum-QCD
- ✓ Well suited to Relativistic Quantum Field Theory
- ✓ A method connects observables with long-range behaviour of the running coupling
- ✓ Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

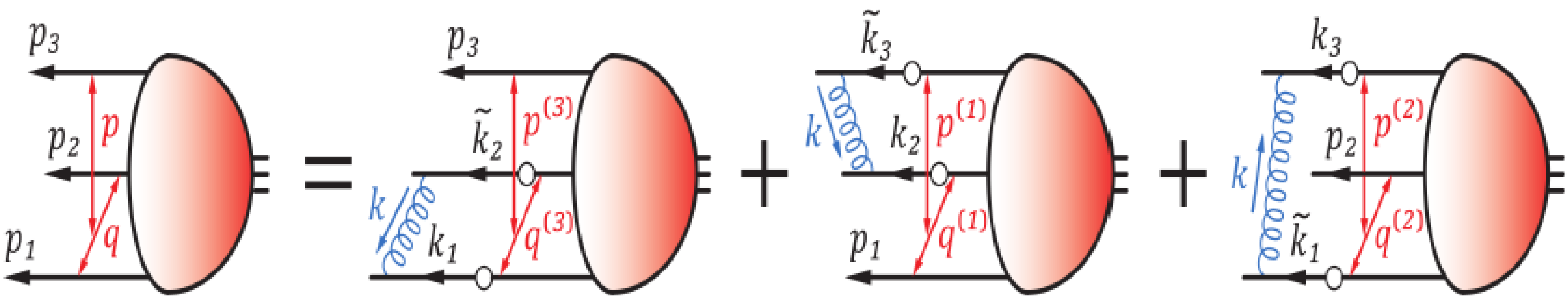
Bound-state equations in QFT

- **Mesons: a 2-body bound state problem in quantum field theory**
  - **Bethe-Salpeter Equation**
  - **K - fully amputated, two-particle irreducible, quark-antiquark scattering kernel**



- **Baryons: a 3-body bound state problem in quantum field theory.**
- **Faddeev equation: sums all possible quantum field theoretical exchanges and interactions that can take place between the three dressed-quarks that define its valence quark content.**

Faddeev equation in rainbow-ladder truncation



2-body correlations: diquarks

- **The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for quark-quark (diquark) correlations within a color-singlet baryon.**

- **Diquark correlations:**
  - A tractable truncation of the Faddeev equation.
  - In our approach: non-pointlike color-antitriplet and fully interacting.
  - Diquark correlations are soft, they possess an electromagnetic size.
  - Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson.

$$\Gamma_{q\bar{q}}(p;P) = - \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{q\bar{q}}(q;P)S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

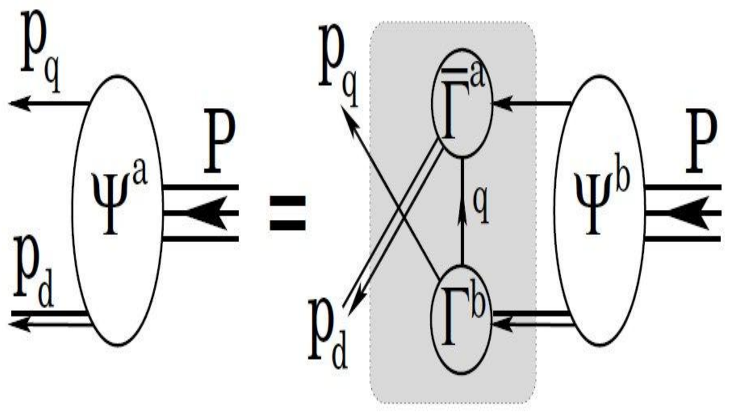
$$\Gamma_{qq}(p;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu\, S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\,\frac{\lambda^a}{2}\,\gamma_\nu$$

2-body correlations: diquarks

- Quantum numbers:
  - (I=0, J^P=0^+): isoscalar-scalar diquark
  - (I=1, J^P=1^+): isovector-pseudovector
  - (I=0, J^P=0^-): isoscalar-pesudoscalar
  - (I=0, J^P=1^-): isoscalar-vector
  - (I=1, J^P=1^-): isovector-vector
  - Tensor diquarks

Quark-diquark picture

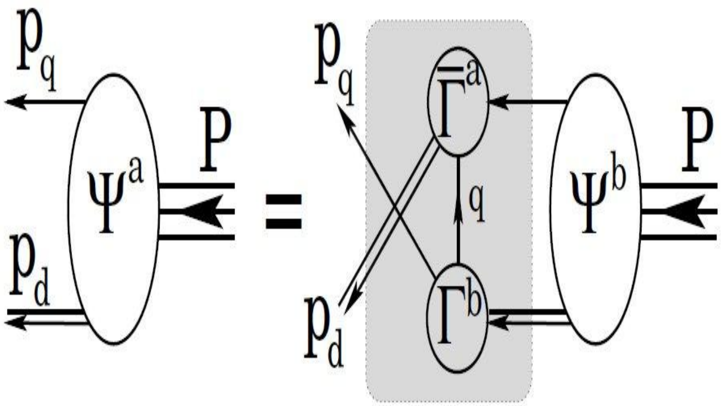
- A baryon can be viewed as a Borromean bound-state, the binding within which has two contributions:
  - ✓ Formation of tight diquark correlations.
  - ✓ Quark exchange depicted in the shaded area.



- The exchange ensures that diquark correlations within the baryon are fully dynamical: no quark holds a special place.
- The rearrangement of the quarks guarantees that the baryon's wave function complies with Pauli statistics.
- Modern diquarks are different from the old static, point-like diquarks which featured in early attempts to explain the so-called missing resonance problem.
- The number of states in the spectrum of baryons obtained is similar to that found in the three-constituent quark model, just as it is in today's LQCD calculations.

A parameter: gDB

- **Some correction of the Faddeev kernels is necessary to overcome an intrinsic weakness of the equation depicted in the figure, whose structure is based on the rainbow-ladder truncation. It is possible that something important is missing.**



- **There is an absence of spin-orbit repulsion owing to an oversimplification of the gluon-quark vertex when formulating the RL bound-state equations. We therefore employ a simple artifice in order to implement the missing interactions.**

  - ✓ **We introduce a single parameter into the Faddeev equation for J^P=1/2^P baryons: gDB, a linear multiplicative factor attached to each opposite-parity (-P) diquark amplitude in the baryon's Faddeev equation kernel.**
  - ✓ **gDB is the single free parameter in our study.**

QCD-kindred model

- **Roper resonance -- solution to the 50 year puzzle**

PRL **115,** 171801 (2015) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2015

Completing the Picture of the Roper Resonance

Jorge Segovia,[1] Bruno El-Bennich,[2,3] Eduardo Rojas,[2,4] Ian C. Cloët,[5] Craig D. Roberts,[5] Shu-Sheng Xu,[6] and Hong-Shi Zong[6]

[1]*Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain*

[2]*Laboratório de Física Teórica e Computacional, Universidade Cruzeiro do Sul, 01506-000 São Paulo, SP, Brazil*

[3]*Instituto de Física Teórica, Universidade Estadual Paulista, 01140-070 São Paulo, SP, Brazil*

[4]*Instituto de Física, Universidad de Antioquia, Calle 70 No. 52-21, Medellín, Colombia*

[5]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[6]*Department of Physics, Nanjing University, Nanjing 210093, China*

(Received 16 April 2015; revised manuscript received 29 July 2015; published 21 October 2015)

We employ a continuum approach to the three valence-quark bound-state problem in relativistic quantum field theory to predict a range of properties of the proton's radial excitation and thereby unify them with those of numerous other hadrons. Our analysis indicates that the nucleon's first radial excitation is the Roper resonance. It consists of a core of three dressed quarks, which expresses its valence-quark content and whose charge radius is 80% larger than the proton analogue. That core is complemented by a meson cloud, which reduces the observed Roper mass by roughly 20%. The meson cloud materially affects long-wavelength characteristics of the Roper electroproduction amplitudes but the quark core is revealed to probes with $Q^2 \gtrsim 3m_N^2$.

DOI: 10.1103/PhysRevLett.115.171801 PACS numbers: 13.40.Gp, 14.20.Dh, 14.20.Gk, 11.15.Tk

Introduction.—The strong-interaction sector of the standard model is thought to be described by quantum chromodynamics (QCD), a relativistic quantum field broken, express the intrinsic mass scale(s) and features associated with confinement and DCSB, and employ realistic kernels in baryon bound-state equations, which

SOLUTIONS & THEIR PROPERTIES

- The four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets: nucleon, roper, N(1535), N(1650)
- **Masses**
- **Rest-frame orbital angular momentum**
- **Diquark content**
- **Pointwise structure**

SOLUTIONS & THEIR PROPERTIES:
Masses

- We choose gDB=0.43 so as to produce a mass splitting of 0.1 GeV between the lowest-mass P=- state and the first excited P=+ state, viz. the empirical value.
- Our computed values for the masses of the four lightest 1/2^{+-} baryon doublets are listed here, in GeV:

g_{DB}	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
0.43	1.19	1.73	1.83	1.91
1.0	1.19	1.73	1.43	1.61

- Pseudoscalar and vector diquarks have no impact on the mass of the two positive-parity baryons, whereas scalar and pseudovector diquarks are important to the negative parity systems.
- Although 1/2^- solutions exist even if one eliminates pseudoscalar and vector diquarks, 1/2^+ solutions do not exist in the absence of scalar and pseudovector diquarks.
- It indicates that, with our Faddeev kernel, the so-called P-wave (negative-parity) baryons can readily be built from positive-parity diquarks. This indicates that the energy-cost associated with introducing quark-diquark orbital angular momentum is not very high.

SOLUTIONS & THEIR PROPERTIES:
Masses

- The quark-diquark kernel omits all those resonant contributions which may be associated with meson-baryon final-state interactions that are resummed in dynamical coupled channels models in order to transform a bare baryon into the observed state.
- The Faddeev equations analyzed to produce the results should therefore be understood as producing the dressed-quark core of the bound state, not the completely dressed and hence observable object.
- In consequence, a comparison between the empirical values of the resonance pole positions and the computed masses is not pertinent. Instead, one should compare the masses of the quark core with values determined for the meson-undressed bare excitations, e.g.,

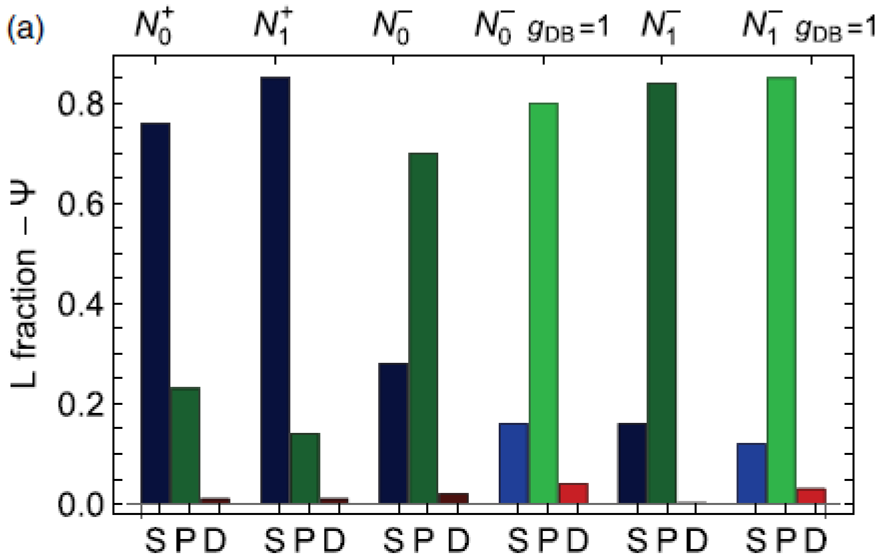
	m_N	$m_{N(1440)}^{1/2^+}$	$m_{N(1535)}^{1/2^-}$	$m_{N(1650)}^{1/2^-}$
herein	1.19	1.73	1.83	1.91
M_B^0 [53]		1.76	1.80	1.88

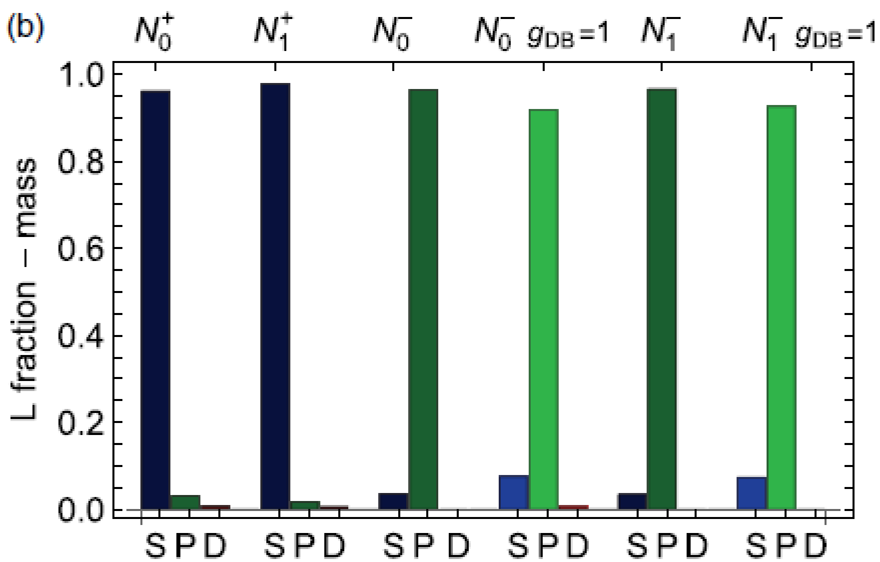
where M^0_B is the relevant bare mass inferred in the associated dynamical coupled-channels analysis.

- Notably, the relative difference between our predicted quark core masses and the relevant bare mass inferred in the associated dynamical coupled-channels analysis is just 1.7%, even though no attempt was made to secure agreement. We consider this to be a success of our formulation of the bound-state problem for a baryon's dressed-quark core.

SOLUTIONS & THEIR PROPERTIES:

Rest-frame orbital angular momentum

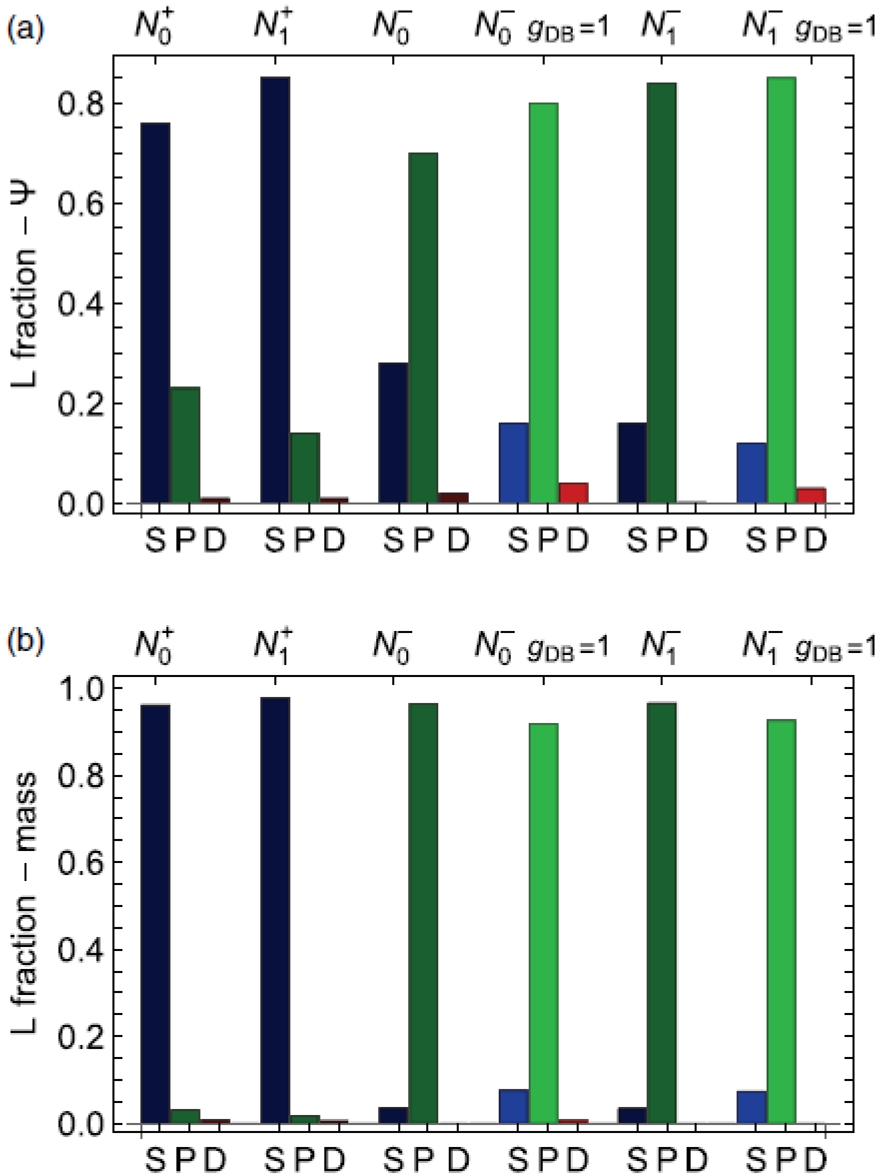




- (a) Computed from the wave functions directly.

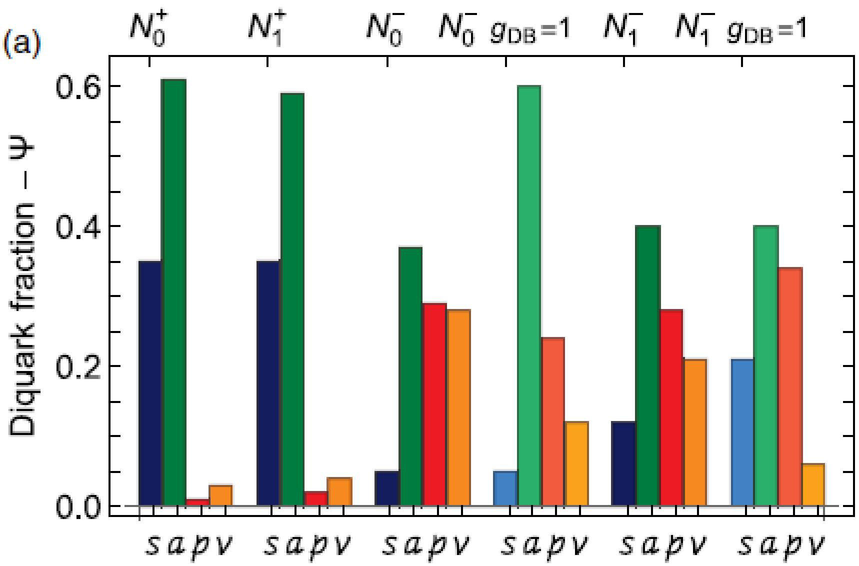
  (b) Computed from the relative contributions to the masses.
- (b) delivers the same qualitative picture of each baryon's internal structure as that presented in (a). Therefore, there is **little** mixing between partial waves in the computation of a baryon's mass.

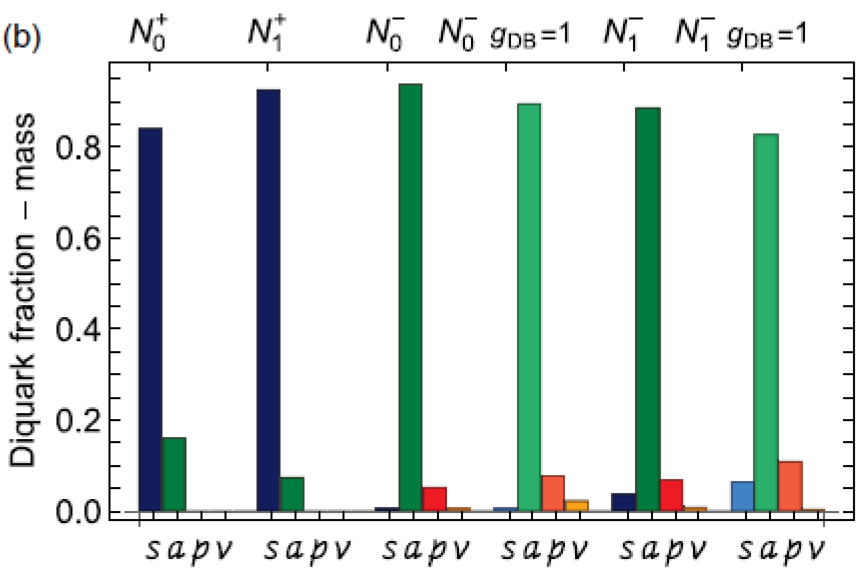
SOLUTIONS & THEIR PROPERTIES:
Rest-frame orbital angular momentum



- The nucleon and Roper are primarily **S-wave** in nature, since they are not supported by the Faddeev equation unless **S-wave** components are contained in the wave function. On the other hand, the N(1535)1/2^- ,N(1650)1/2^- are essentially **P-wave** in character.
- These observations provide support in quantum field theory for the constituent-quark model classifications of these systems.

SOLUTIONS & THEIR PROPERTIES:
Diquark content





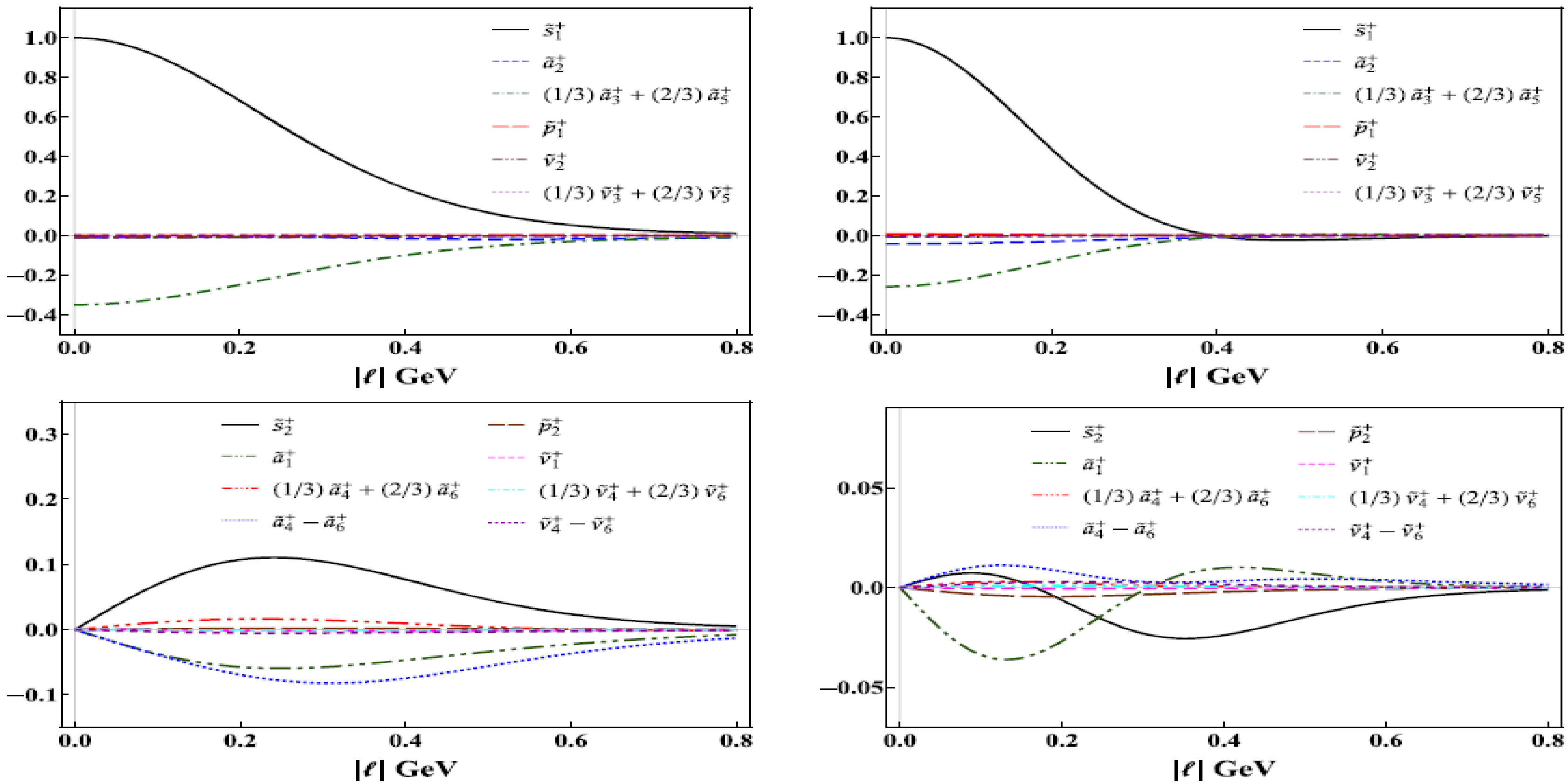
- (a) Computed from the amplitudes directly.

  (b) Computed from the relative contributions to the masses.
- From (a): although gDB < 1 has little impact on the nucleon and Roper, it has a significant effect on the structure of the negative parity baryons, serving to enhance the net negative-parity diquark content. The amplitudes associated with these negative-parity states contain roughly equal fractions of even and odd parity diquarks.
- From (b): In each case depicted in the lower panel, there is a single dominant diquark component. There are **significant interferences** between different diquarks.

SOLUTIONS & THEIR PROPERTIES:

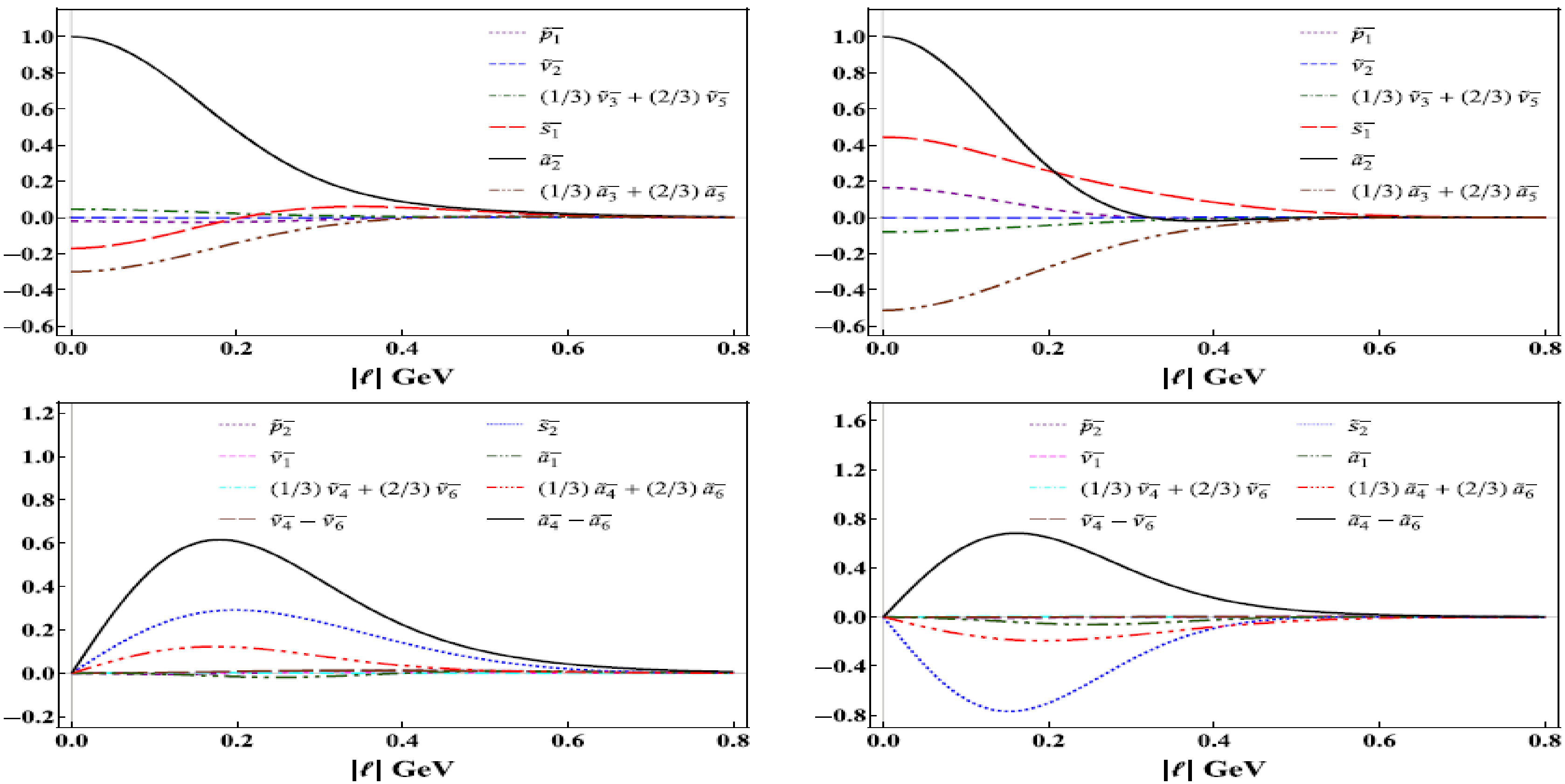
Pointwise structure

- We consider the zeroth Chebyshev moment of all S- and P-wave components in a given baryon's Faddeev wave function.
- Nucleon's first positive-parity excitation: all S-wave components exhibit a single zero; and four of the P-wave projections also possess a zero. This pattern of behavior for the first excited state indicates that it may be interpreted as a radial excitation.



SOLUTIONS & THEIR PROPERTIES:
Pointwise structure

- For N(1535)1/2^-,N(1650)1/2^- : the contrast with the positive-parity states is stark.
  In particular, there is no simple pattern of zeros, with all panels containing at least one function that possesses a zero.
- In their rest frames, these systems are predominantly P-wave in nature, but possess material S-wave components; and the first excited state in this negative parity channel—*N(1650)1/2^–*—has little of the appearance of a radial excitation, since most of the functions depicted in the right panels of the figure do not possess a zero.



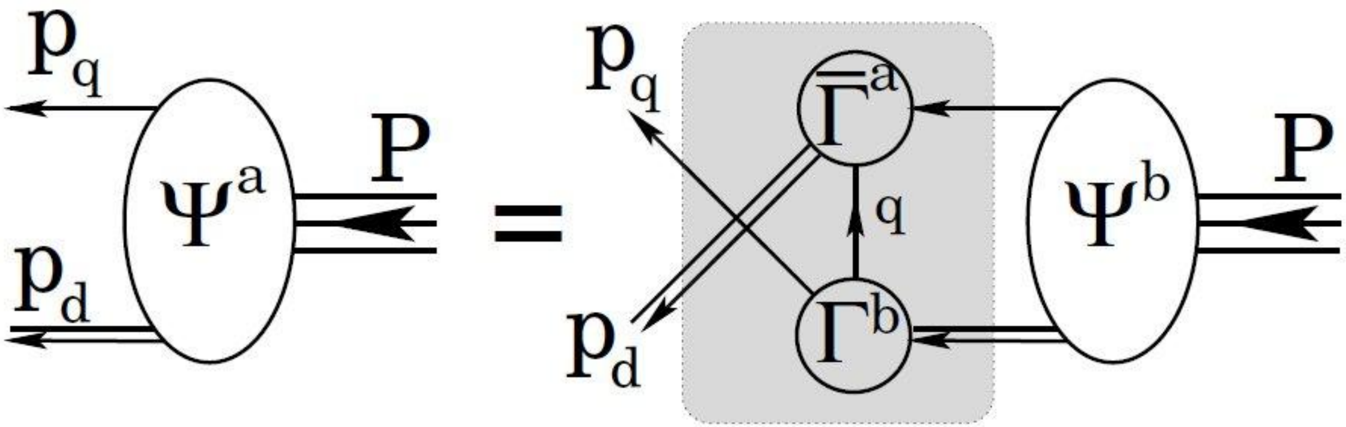
Summary & Outlook

- By including all kinds of diquarks, we performed a comparative study of the four lightest baryon (I=1/2, J^P=1/2^{+-}) isospin doublets in order to both elucidate their structural similarities and differences.
- The first **ON-SHELL** DSE treatment of these systems.
- The two lightest (I=1/2, J^P=1/2^+) doublets are dominated by scalar and pseudovector diquarks; the associated rest-frame Faddeev wave functions are primarily S-wave in nature; and the first excited state in this 1/2^+ channel has very much the appearance of a radial excitation of the ground state.
- In the two lightest (I=1/2, J^P=1/2^-) systems, **TOO**, scalar and pseudovector diquarks play a material role. In their rest frames, the Faddeev amplitudes describing the dressed-quark cores of these negative-parity states contain roughly equal fractions of even and odd parity diquarks; the associated wave functions of these negative-parity systems are predominantly P-wave in nature, but possess measurable S-wave components; and, the first excited state in this negative parity channel has little of the appearance of a radial excitation.
- NEXT: N(1720)3/2^+, N(1520)3/2^-, baryons with strangeness, form factors, PDAs, PDFs, …, GPDs, TMDs.

Thank you!

QCD-kindred model

- The dressed-quark propagator
- Diquark amplitudes
- Diquark propagators
- Faddeev amplitudes



QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)$$

- **When calculations require the evaluation of numerous multidimensional integrals → algebraic form:**

$$\bar{\sigma}_S(x) = 2\bar{m}\mathcal{F}(2(x+\bar{m}^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)[b_0 + b_2\mathcal{F}(\epsilon x)], \quad \text{(A3a)}$$

$$\bar{\sigma}_V(x) = \frac{1}{x+\bar{m}^2}[1-\mathcal{F}(2(x+\bar{m}^2))], \quad \text{(A3b)}$$

with $x = p^2/\lambda^2$, $\bar{m} = m/\lambda$,

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}, \quad \text{(A4)}$$

$\bar{\sigma}_S(x) = \lambda\sigma_S(p^2)$ and $\bar{\sigma}_V(x) = \lambda^2\sigma_V(p^2)$. The mass scale, $\lambda = 0.566$ GeV, and parameter values,

$\bar{m}$	b_0	b_1	b_2	b_3
0.00897	0.131	2.90	0.603	0.185

(A5)

associated with Eq. (A3) were fixed in a least-squares fit to light-meson observables [79,80]. [$\epsilon = 10^{-4}$ in Eq. (A3a) acts only to decouple the large- and intermediate-p^2 domains.]

QCD-kindred model

- **The dressed-quark propagator**

$$S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$$

  - Based on solutions to the gap equation that were obtained with a dressed gluon-quark vertex.
  - Mass function has a real-world value at p^2 = 0, NOT the highly inflated value typical of RL truncation.
  - Propagators are entire functions, consistent with sufficient condition for confinement and completely unlike known results from RL truncation.
  - Parameters in quark propagators were fitted to a diverse array of meson observables. **ZERO** parameters changed in study of baryons.
  - Compare with that computed using the DCSB-improved gap equation kernel. Evidently, although simple and introduced long beforehand, the parametrization is a sound representation of contemporary numerical results.

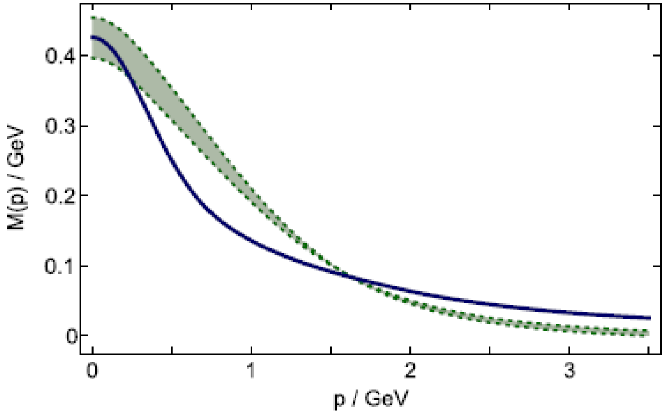


FIG. 6. Solid curve (blue)—quark mass function generated by the parametrization of the dressed-quark propagator specified by Eqs. (A3) and (A4) (A5); and band (green)—exemplary range of numerical results obtained by solving the gap equation with the modern DCSB-improved kernels described and used in Refs. [16,81–83].

QCD-kindred model

- **Diquark amplitudes**: five types of correlation are possible in a ***J=1/2*** bound state: isoscalar scalar(I=0,J^P=0^+), isovector pseudovector, isoscalar pseudoscalar, isoscalar vector, and isovector vector.
- The leading structures in the correlation amplitudes for each case are, respectively (Dirac-flavor-color),

$$\Gamma^{0^+}(k;K) = g_{0^+}\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\vec{\Gamma}_\mu^{1^+}(k;K) = i g_{1^+}\gamma_\mu C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{1^+}^2),$$

$$\Gamma^{0^-}(k;K) = i g_{0^-} C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{0^-}^2),$$

$$\Gamma_\mu^{1^-}(k;K) = g_{1^-}\gamma_\mu\gamma_5 C\tau^2 \vec{H}\mathcal{F}(k^2/\omega_{1^-}^2),$$

$$\vec{\Gamma}_\mu^{\bar{1}^-}(k;K) = i g_{\bar{1}^-}[\gamma_\mu, \gamma\cdot K]\gamma_5 C\vec{\tau}\vec{H}\,\mathcal{F}(k^2/\omega_{\bar{1}^-}^2),$$

- Simple form. Just one parameter, which connects mass with width.
- Match expectations based on solutions of meson and diquark Bethe-Salpeter amplitudes.

QCD-kindred model

- **Diquark masses (in GeV):**

$$m_{0^+} = 0.8, \quad m_{1^+} = 0.9, \quad m_{0^-} = 1.2, \quad m_{1^-} = 1.3,$$

- **The first two values provide for a good description of numerous dynamical properties of the nucleon, Δ-baryon and Roper resonance.**
- **Masses of the odd-parity correlations are based on those computed from a contact interaction.**
- **Such values are typical; and in truncations of the two-body scattering problem that are most widely used, isoscalar- and isovector-vector correlations are degenerate.**
- **Normalization condition → couplings:**

$$g_{0^+} = 14.8, \qquad g_{1^+} = 12.7,$$
$$g_{0^-} = 12.8, \qquad g_{1^-} = 5.4, \qquad g_{\bar{1}^-} = 2.5.$$

- **Faddeev kernels: 22 × 22 matrices are reduced to 16 × 16 !**

QCD-kindred model

- **The diquark propagators**

$$\Delta^{0^\pm}(K) = \frac{1}{m_{0^\pm}^2}\mathcal{F}(k^2/\omega_{0^\pm}^2),$$

$$\Delta_{\mu\nu}^{1^\pm}(K) = \left[\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^\pm}^2}\right]\frac{1}{m_{1^\pm}^2}\mathcal{F}(k^2/\omega_{1^\pm}^2).$$

- **The *$\mathcal{F}$-functions*: Simplest possible form that is consistent with infrared and ultraviolet constraints of confinement (IR) and 1/q^2 evolution (UV) of meson propagators.**

- **Notably, our diquarks are confined.**
  - **free-particle-like at spacelike momenta**
  - **pole-free on the timelike axis**
  - **This is NOT true of RL studies. It enables us to reach arbitrarily high values of momentum transfer.**

QCD-kindred model

- **The Faddeev amplitudes:**

$$\begin{aligned}\psi^{\pm}(p_i,\alpha_i,\sigma_i) = {} & [\Gamma^{0^+}(k;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\Delta^{0^+}(K)[\varphi^{\pm}_{0^+}(\ell;P)u(P)]^{\alpha_3}_{\sigma_3} \\ & + [\Gamma^{1^+ j}_{\mu}]\Delta^{1^+}_{\mu\nu}[\varphi^{j\pm}_{1^+\nu}(\ell;P)u(P)] \\ & + [\Gamma^{0^-}]\Delta^{0^-}[\varphi^{\pm}_{0^-}(\ell;P)u(P)] \\ & + [\Gamma^{1^-}_{\mu}]\Delta^{1^-}_{\mu\nu}[\varphi^{\pm}_{1^-\nu}(\ell;P)u(P)], \qquad (9)\end{aligned}$$

- **Quark-diquark vertices:**

$$\varphi^{\pm}_{0^+}(\ell;P) = \sum_{i=1}^{2} s^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{j\pm}_{1^+\nu}(\ell;P) = \sum_{i=1}^{6} a^{j\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\pm},$$

$$\varphi^{\pm}_{0^-}(\ell;P) = \sum_{i=1}^{2} p^{\pm}_i(\ell^2,\ell\cdot P)\mathcal{S}^i(\ell;P)\mathcal{G}^{\mp},$$

$$\varphi^{\pm}_{1^-\nu}(\ell;P) = \sum_{i=1}^{6} v^{\pm}_i(\ell^2,\ell\cdot P)\gamma_5\mathcal{A}^i_\nu(\ell;P)\mathcal{G}^{\mp},$$

where $\mathcal{G}^{+(-)} = \mathbf{I}_{\rm D}(\gamma_5)$ and

$$\mathcal{S}^1 = \mathbf{I}_{\rm D}, \qquad \mathcal{S}^2 = i\gamma\cdot\hat{\ell} - \hat{\ell}\cdot\hat{P}\mathbf{I}_{\rm D}$$

$$\mathcal{A}^1_\nu = \gamma\cdot\ell^{\perp}\hat{P}_\nu, \qquad \mathcal{A}^2_\nu = -i\hat{P}_\nu\mathbf{I}_{\rm D}, \qquad \mathcal{A}^3_\nu = \gamma\cdot\hat{\ell}^{\perp}\hat{\ell}^{\perp}_\nu$$

$$\mathcal{A}^4_\nu = i\hat{\ell}^{\perp}_\nu\mathbf{I}_{\rm D}, \quad \mathcal{A}^5_\nu = \gamma^{\perp}_\nu - \mathcal{A}^3_\nu, \quad \mathcal{A}^6_\nu = i\gamma^{\perp}_\nu\gamma\cdot\hat{\ell}^{\perp} - \mathcal{A}^4_\nu,$$

QCD-kindred model

- Both the Faddeev amplitude and wave function are Poincare covariant, i.e. they are qualitatively identical in all reference frames.
- Each of the scalar functions that appears is frame independent, but the frame chosen determines just how the elements should be combined.
- In consequence, the manner by which the dressed quarks' spin, ***S***, and orbital angular momentum, ***L***, add to form the total momentum ***J***, is frame dependent: ***L***, ***S*** are not independently Poincare invariant.
- The set of baryon rest-frame quark-diquark angular momentum identifications:

$$^2S:\ \mathcal{S}^1, \mathcal{A}_\nu^2, (\mathcal{A}_\nu^3 + \mathcal{A}_\nu^5),$$

$$^2P:\ \mathcal{S}^2, \mathcal{A}_\nu^1, (\mathcal{A}_\nu^4 + \mathcal{A}_\nu^6),$$

$$^4P:\ (2\mathcal{A}_\nu^4 - \mathcal{A}_\nu^6)/3,$$

$$^4D:\ (2\mathcal{A}_\nu^3 - \mathcal{A}_\nu^5)/3,$$

- The scalar functions associated with these combinations of Dirac matrices in a Faddeev wave function possess the identified angular momentum correlation between the quark and diquark.