Pion and Kaon Structure at the Partonic Level

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The Nature of Hadron Mass and Quark-Gluon Confinement from JLab Experiments in the 12-GeV Era

Outline

1. Pion and Kaon
   - Pion and Kaon Properties in BSE-NJL model
   - Form Factor
   - Parton Distribution Function

2. Medium Effects on FF of the pion and kaon
   - Medium effect on FF

3. Conclusion and Outlook

Outline

Outline

Motivation

- Pion has a special place in QCD
- QCD, as underlying theory of strong interaction, is unable to directly predict structure of hadrons.
- To understand the partonic dynamics in a hadron internal structure, PDF and FF are of fundamental importance and provide complementary information
- Pion structure is simpler than the nucleon, but not so simple. In fact, we do not really understand the structure of the pion. Also the pion is interesting due to the pion is both a dressed quark-antiquark bound state and the Goldstone mode associated with DCSB in Quantum Chromodynamics (QCD).
- From experimental side, next experimental data for the pion and kaon form factors will be coming soon from JLAB (expected 2018), J-PARC as well as COMPASS

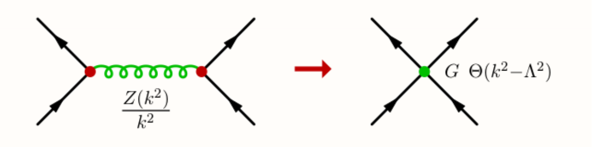
Pion and Kaon in the BSE-NJL Model

The three flavor NJL Lagrangian – containing only four fermion interactions

$$\mathcal{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_\rho \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \quad (1)$$

- $\psi = (u, d, s)$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction

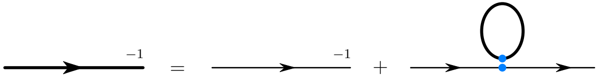


- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole)
- Therefore we regularize using the proper time regularization

$$S(p) = \int_0^\infty d\tau (\not{p} + M_q) e^{-\tau(p^2 - M_q^2)}, \rightarrow e^{(p^2-M_q^2)/\Lambda_{UV}^2} - e^{-(p^2-M^2)/\Lambda_{IR}^2} / [\not{p} + M] \quad (2)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24\ \mathrm{GeV}$ and Λ_{UV} is determined.

- NJL Gap Equation

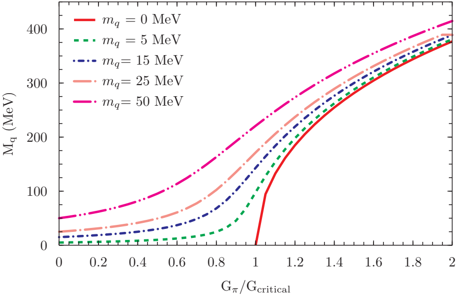


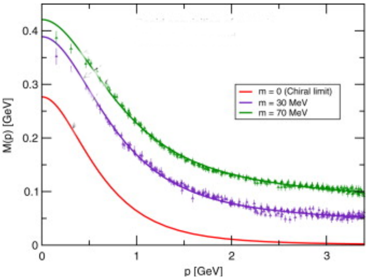
$$
\begin{aligned}
M_q &= m_q + 48i\,G_\pi M_q \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M_q^2 + i\epsilon} \\
&= m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\
&= m_q - 2G_\pi \langle \bar{\psi}\psi \rangle
\end{aligned} \qquad (3)
$$

- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -M_q \frac{3}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

Results for NJL gap equation



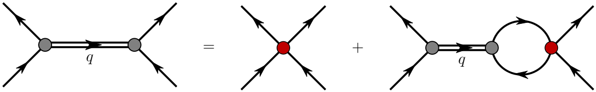


References:

PTPH et al, PRC **94**, 035201 (2016)

C. D.Roberts, PPNP **61**, 50-65 (2008)

Bethe Salpeter Equation for the pion and kaon



- In the NJL model, T-matrix is given by

$$T(q) \;=\; \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K} S(q+k) T(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$T_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab}\, t_\alpha(q) \left[\gamma_t \lambda_\alpha^\dagger\right] \tag{4}$$

- The reduced t-matrix in this channel take a form

$$t_\alpha(q) \;=\; \frac{-2iG_\pi}{1 + 2G_\pi \Pi_\pi(q^2)}$$

$$t_\beta^{\mu\nu}(q) \;=\; \frac{-2iG_\rho}{1 + 2G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2} \right) \tag{5}$$

- The bubble diagrams appearing read

$$\begin{aligned}\Pi_\pi(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right],\\ \Pi_K(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right],\\ \Pi^{aa}_\nu(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right]\end{aligned} \tag{6}$$

- The kaon and pion masses is given by the pole of the t-matrix

$$\begin{aligned}1+2G_\pi\Pi_\pi(k^2=m_\pi^2) &= 0\\ 1+2G_\pi\Pi_K(k^2=m_K^2) &= 0\end{aligned} \tag{7}$$

Pion and Kaon Masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the kaon and pion masses are given by

$$\begin{aligned} m_\pi^2 &= \left[\frac{m}{M_l}\right] \frac{2}{G_\pi \mathcal{I}_{ll}(m_\pi^2} \\ m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right] \frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2} + (M_s - M_l)^2 \end{aligned} \tag{8}$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \tag{9}$$

The meson-quark-quark coupling constants and Pion and Kaon Decay Constants

The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark -quark coupling constants:

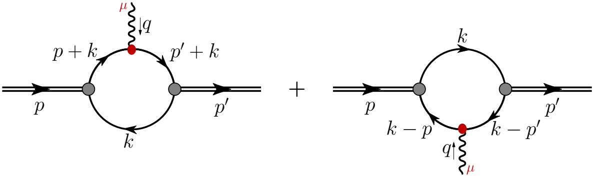
$$\begin{aligned} Z_\pi(q^2) &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\,|_{q^2=m_\pi^2} \\ Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2}\,|_{q^2=m_K^2} \\ Z_\rho(q^2) &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\,|_{q^2=m_\rho^2} \end{aligned} \tag{10}$$

Pion and kaon decay constant in the proper time regularization is given by

$$\begin{aligned} f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\ f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)} \end{aligned} \tag{11}$$

FORM FACTOR IN THE CONFINING NJL MODEL

Diagrammatic representation of the electromagnetic current for the pion and kaon



Feynman diagram for quark [left] and for the anti quark [right]

The matrix element of the electromagnetic current for a pseudoscalar mesons reads

$$J^{\mu}_{\alpha}(p', p) \;=\; \left(p^{'\mu} + p^{\mu}\right) F_{\alpha}(Q^2), \qquad \alpha = \pi, K \tag{12}$$

where p and $p^{'}$ denote the initial and final four momentum of the state, $q^2 = (p^{'} - P)^2 = -Q^2$ and $F_{\alpha}(Q^2)$ is the pion or kaon form factor. The pseudoscalar meson form factor in the NJL model are given by the sum of the two Feynman diagrams, which are respectively given by

$$\begin{aligned} j^{\mu}_{1,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda^{\dagger}_{\alpha} S(p^{'} + k) \hat{Q} \gamma^{\mu} S(p + k) \gamma_5 \lambda_{\alpha} S(k)\right] \\ j^{\mu}_{2,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda_{\alpha} S(k - p) \hat{Q} \gamma^{\mu} S(k - p^{'}) \gamma_5 \lambda^{\dagger}_{\alpha} S(k)\right] \end{aligned} \tag{13}$$

where the Tr is over Dirac, color and flavor indices. The index α labels the state and the λ_{α} are the corresponding flavor matrices

Form Factor in NJL model

We will focus on the quark sector and total form factors for π^+, K^+ and K^0, we find

$$\begin{aligned} F_{\pi^+}^{(bare)}(Q^2) &= (e_u - e_d) f_\pi^{ll}(Q^2) \\ F_{K^+}^{(bare)}(Q^2) &= e_u f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2) \\ F_{K^0}^{(bare)}(Q^2) &= e_d f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2) \end{aligned} \quad (14)$$

The results are denoted as "*bare*" because the quark-photon vertex is elementary result, that is, $\Lambda_{\gamma q}^{\mu(bare)} = \hat{Q}\gamma^\mu$. The quark-sector form factors for a hadron α are defined by

$$F_\alpha(Q^2) = e_u F_\alpha^u(Q^2) + ed F_\alpha^d(Q^2) + e_s F_\alpha^s(Q^2) + \cdots \quad (15)$$

therefore the "*bare*" pseudoscalar meson quark-sector form factors are easily read from the total form factor equation above

The first superscript on the body form factors, $f_\alpha^{ab}(Q^2)$, indicates the struck quark and the second the spectator, where

$$
\begin{aligned}
f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(M_a^2+x(1-x)Q^2)} \\
&+ \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int_0^{1-x} dz \int d\tau \\
&\times e^{-\tau((x+z)(x+z-1)m_\alpha^2+(x+z)M_a^2+(1-x-z)M_b^2+xzQ^2)} \\
&\times \left[(x+z)m_\alpha^2 + (M_a - M_b)^2(X+Z) + 2M_b(M_a - M_b)\right]
\end{aligned}
\tag{16}
$$

Importantly, these expression satisfy charge conservation.

In general the quark-photon vertex is not elementary ($\hat{Q}\gamma^\mu$) but instead dressed, with this dressing given by the inhomogeneous BSE. The general solution for the dressed quark-photon vertex for a quark of flavor q has the form

$$\Lambda^{\mu}_{\gamma Q}(p', p) = e_q \gamma^\mu + \left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2}\right) F_Q(Q^2) \to \gamma^\mu F_{1Q}(Q^2) \quad (17)$$

where the final result is used because the $\frac{q^\mu \not{q}}{q^2}$ term cannot contribute to a hadron electromagnetic current because of current conservation

For the dressed u, d and s quarks we find

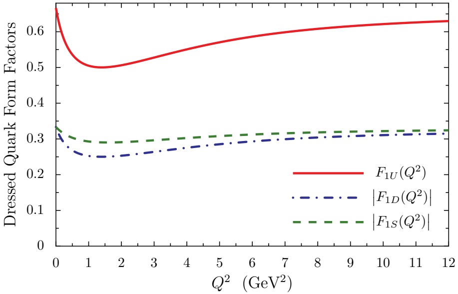
$$F_{1U/D}(Q^2) = e_{u/d}\frac{1}{1+2G_\rho\Pi_v^{ll}(Q^2)}$$
$$F_{1S}(Q^2) = e_s\frac{1}{1+2G_\rho\Pi_v^{ss}(Q^2)} \quad (18)$$

where the explicit form of the bubble diagram is

$$\Pi_v^{qq}(Q^2) = \frac{3Q^2}{\pi^2}\int_0^1 dx\int\frac{d\tau}{\tau}x(1-x)e^{-\tau\left[M_q^2+x(1-x)Q^2\right]} \quad (19)$$

Form Factor in NJL model

The dressed quark form factors obtained as solutions to the inhomogeneous BSE:



In the limit $Q^2 \to \infty$ these form factors reduce to the elementary quark charges, as expected because of asymptotic freedom in QCD. For small Q^2 these results are similar to expectations from vector meson dominance, where the dressed u and d quarks are dressed by ρ and ω mesons and the dressed s quark by ϕ meson.

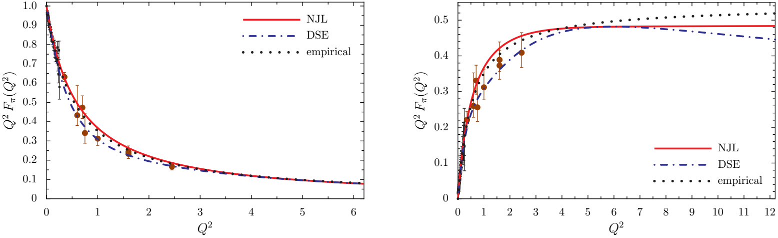
Form Factor in NJL model

The complete results for the pseudoscalar meson form factor – with a dressed quark-photon vertex – read

$$
\begin{aligned}
F_{\pi^+}(Q^2) &= \left[F_{1U}(Q^2)F_{1D}(Q^2)\right] f_{\pi}^{ll}(Q^2) \\
F_{K^+}(Q^2) &= F_{1U}(Q^2)f_K^{ls}(Q^2) - F_{1S}(Q^2)f_K^{sl}(Q^2) \\
F_{K^0}(Q^2) &= F_{1D}(Q^2)f_K^{ls}(Q^2) - F_{1S}(Q^2)f_K^{sl}(Q^2)
\end{aligned} \tag{20}
$$

Elastic Form Factor Results

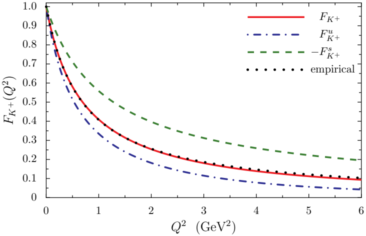
Results for the pion form factor and $Q^2 F_\pi(Q^2)$ – including effects from the dressed quark-photon vertex

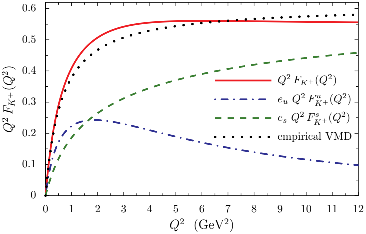


$\rightarrow$ We find that excellent agreement with existing data and the modest difference with the DSE result for $Q^2 \leq 6\ \mathrm{GeV}^2$
$\rightarrow$ Our result for $Q^2 F_\pi(Q^2)$ is very similar to the empirical monopole result but begins to plateau for $Q^2 \geq 6\ \mathrm{GeV}^2$, where $Q^2 F_\pi(Q^2) \sim 0.49$. This maximum is almost identical to that obtaining using the DSEs, which is not surprising because in both approaches it is driven by DCSB.

Elastic Kaon Form Factor Results

Results for the K^+ form factor and the quark sector components and the $Q^2 F_{K^+}(Q^2)$ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



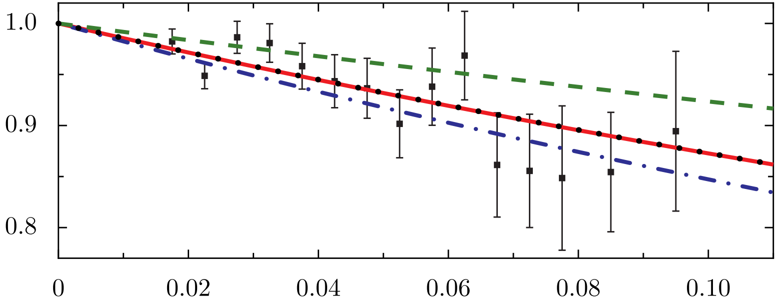


→ For the quark sector form factors, We observe very large difference in their Q^2 evolution with s-quark component much harder than u-quark form factor.

→ The s quark component begins to dominate the K^+ form factor for $Q^2 \geq 1.6\mathrm{GeV}^2$

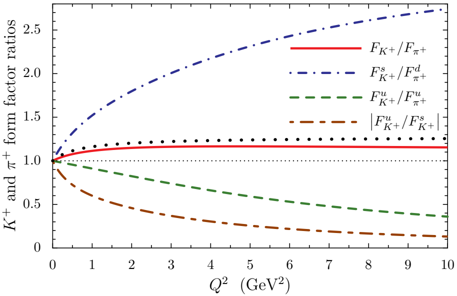
Elastic Kaon Form Factor Results

Results for the K^+ form factor and the quark sector components comparing to the existed data – each including effects from the dressed quark-photon vertex



$\rightarrow$ All existing data for the kaon form factor lies in domain $0 < Q^2 < 0.1\ \mathrm{GeV}^2$. Therefore, we eagerly wait any new data at Q^2 similar to the pion.

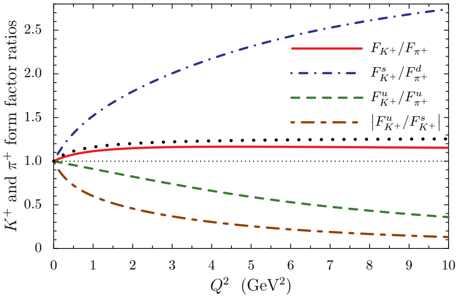
Results for the $Q^2 F_{K^+}(Q^2)$ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



$\rightarrow$ The ratio $F_K^+(Q^2)/F_{\pi^+}(Q^2)$ is always greater than unity and becomes almost constant for $Q^2 \geq 3$ GeV^2.

$\rightarrow$ For very large Q^2 this ratio plateaus to value 1.10 in agreement with the QCD result in conformal limit, $F_K^+(Q^2)/F_{\pi^+}(Q^2) \rightarrow f_K^2/f_\pi^2$.

Elastic Kaon Form Factor Results



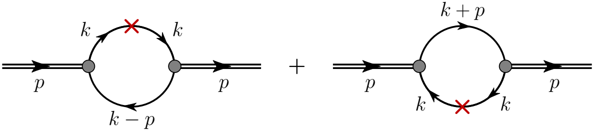
$\rightarrow$ The large constant ratio $F_K^+(Q^2)/F_{\pi^+}(Q^2)$ conceals dramatic flavor breaking effects in the quark sector form factors that grow with increasing Q^2.

$\rightarrow$ At $Q^2 = 10$ GeV^2, we find $F^u_{K^+}/F^u_{\pi^+} \equiv 0.36$ and $F^s_{K^+}/F^d_{\pi^+} \equiv 2.74$. Therefore we find very large flavor breaking at large Q^2.

VALENCE QUARK DISTRIBUTION IN THE CONFINING NJL MODEL

Valence quark distribution

The valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams



The operator insertion $\gamma^+ \delta(k^+ - xp^+) \hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$\hat{P}_{u/d} = \frac{1}{2}\left(\frac{2}{3}\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$$
$$\hat{P}_s = \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8 \qquad (21)$$

The valence quark and anti-quark distributions in the pion or kaon are given by

$$
\begin{aligned}
q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ - xp^+\right) \\
&\times \; Tr\left[\gamma_5 \lambda_\alpha^\dagger S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha S(k-p)\right] \\
\bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ + xp^+\right) \\
&\times \; Tr\left[\gamma_5 \lambda_\alpha S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha^\dagger S(k+p)\right] \qquad (22)
\end{aligned}
$$

To evaluate these expression we first take the moments

$$
\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \qquad (23)
$$

where $n = 1, 2, 3, \cdots$ is an integer.

Using the Ward-like identity $S(k)\gamma^+ S(k) = \frac{-\partial S(k)}{\partial k_+}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^+ we find:

$$\begin{aligned} q_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_s^2 + (1-x)M_l^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \\ \bar{q}_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_l^2 + (1-x)M_s^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \qquad (24) \end{aligned}$$

Results for the π^+ are obtained by $M_s \to M_l$ and $Z_K \to Z_\pi$, giving the result $u_\pi(x) = \bar{d}_\pi(x)$

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx\,[u_{K^+}(x) - \bar{u}_{K^+}(x)] \;=\; \int_0^1 [\bar{s}_{K^+}(x) - s_{k^+}(x)] = 1 \qquad (25)$$

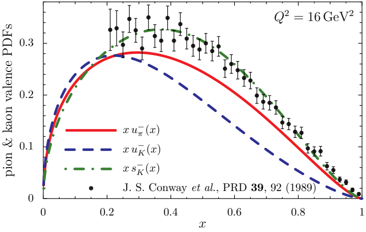
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx\,[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)] \;=\; 1 \qquad (26)$$

Analogous results holds for the remaining kaons and the pions.

Kaon VQDIS Results

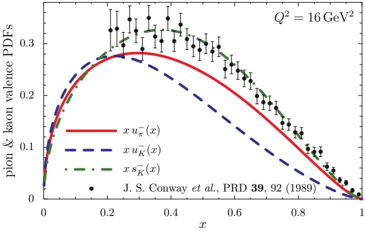
Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.



$\rightarrow$ At the model scale, the momentum fraction carried by the u and s quarks in the K^+, $< xu >= 0.42$ and $< xs >= 0.58$.
$\rightarrow$ The flavor breaking effects of $[< xs > - < xu >]/[< xs > + < xu >] \sim 16\%$ which is similar to that seen in the masses, $[M_s - M_u]/[M_s + M_u] \sim 21\%$.

Kaon VQDIS Results

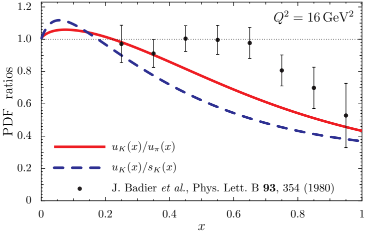
Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.



$\rightarrow$ $SU(3)$ flavor breaking at the model scale $u_K(x)$ peaks at $x_u = 0.237$ and $\bar{s}_K$ peaks at the $x_s = 1 - x_u = 0.763$. This implies flavor breaking effects of around $[x_s - x_u]/[x_s + x_u] \sim 53\%$. For the pion the peak at $x = 0.5$ when $m_u = m_d$.

Kaon VQDIS Results

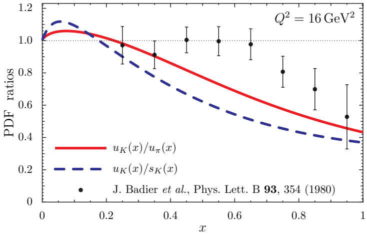
The ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO evolution to $Q^2 = 16$ GeV^2



$\rightarrow$ The ratio of $u_K/u_\pi \rightarrow 0.434 \sim M_u/M_s$ as $x \rightarrow 1$, which is in a good agreement with existing data.

$\rightarrow$ The x-dependence differs from much of data in the valence region. This may lie with the absence of the momentum dependence in the NJL Bethe Salpeter vertices, or with data itself.

Kaon VQDIS Results



→ The ratio $u_K(x)/s_K(x)$ approaches 0.37 as $x \to 1$. It is evident that the flavor breaking effects have a sizeable x dependence, being maximal at large x and becoming negligible at small x where the perturbative effects from DGLAP evolution dominate.

→ The Drell-Yan-West (DYW) relation, $F(Q^2) \sim \frac{1}{Q^{2n}} <--->$ $q(x) \sim (1-x)^{2n-1}$. For the pion, $F_\pi \sim 1/Q^2$ and the DYW relation implies $q_\pi(x) \sim (1-x)$. Kaon PDF do behave as the pion. → As refelection of the expectations of may be expected by DTW like relations, $u_K/s_K < 1$ as $x \to 1$ and $\mid F_K^u/F_K^s \mid < 1$ for $Q^2 >> \Lambda_{QCD}$.

MEDIUM EFFECTS ON PION AND KAON FF

In collaboration with:

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The in-medium properties of the kaon is calculated in the NJL model combined with the QMC model (see Kazuo's presentation on July 4 Session 6)

The in-medium dressed quark propagator takes a form

$$S_q^*(k) = \frac{\not{k} + V^0 + M_q^*}{(k + V^0)^2 - M_q^* + i\epsilon}, \qquad (27)$$

where the medium modification enter as the shift of the quark momentum through $k^\mu \to k^\mu + V^\mu$ where vector potential, $V^\mu = (\delta_0^\mu V^0, \vec{0})$ [Miller, PRL 103, 082301 (2009)]. The asterik denotes the in-medium quantity.

Results for the in-medium space-like electromagnetic form factors of the pion (in preparation (2018))

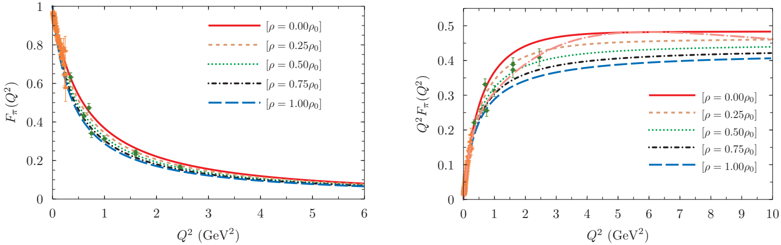


Figure: Results for the π form factors (red solid line) for various nuclear densities. The form factors are calculated using the inputs from the QMC model for each $\rho_B/\rho_0 = [0.00, 0.25 , 0.50, 0.75, 1.00]$ which are represented by respectively [solid, dashed, dotted, dashed-dotted, long dashed] lines.

Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii

Table: Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii. This is calculated using the inputs from the QMC model. All charge radii in the medium along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_π	r_u	r_d	r^{expt}
0.00	0.629	0.629	0.629	0.672 ± 0.008
0.25	0.667	0.667	0.667	
0.50	0.705	0.704	0.705	
0.75	0.740	0.739	0.740	
1.00	0.771	0.771	0.771	

Medium Effects on kaon FF Results

Results for the in-medium space-like electromagnetic form factors of the pion (in preparation (2018))

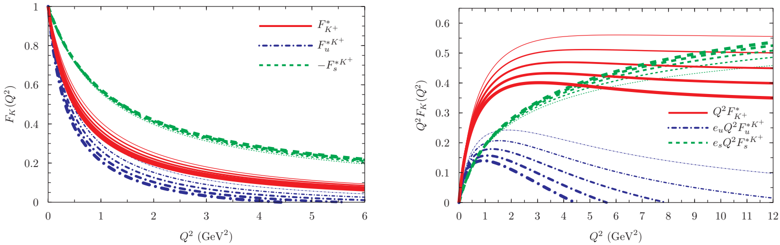


FIGURE: Results for the in-medium $Q^2F^*_{K^+}(Q^2)$ (solid line) together with the charge-weighted quark-sector contributions. The in-medium form factors are calculated using the inputs from the calculated NJL model combined with the QMC model for $\rho_B/\rho_0 = [0.00, 0.25 , 0.50, 0.75, 1.00]$. The difference densities are represented by thinner to thicker lines.

Medium Effects on kaon FF Results

Results for the charge radii of the charged kaon in the nuclear medium and its quark sector charge radii

Table: Results for the in-medium charge radii of the charged kaon and its in-medium quark sector charge radii. This is calculated in the NJL model using the inputs from the standard QMC model. All in-medium charge radii along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_K	r_u	r_s	r^{expt}
0.00	0.59	0.65	0.44	0.56 ± 0.03
0.25	0.62	0.69	0.44	
0.50	0.65	0.73	0.44	
0.75	0.68	0.77	0.44	
1.00	0.71	0.81	0.44	

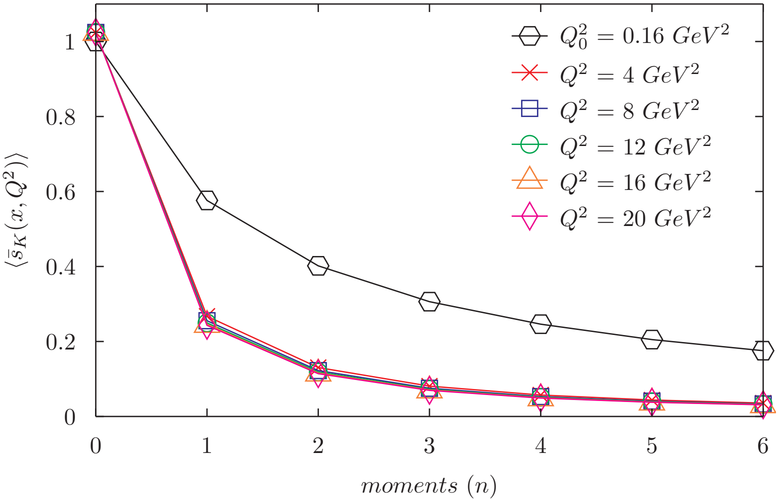
Conclusion and outlook

- We have studied a kaon and pion structure via form factor and VQDIS in vacuum
- Our prediction in form factor and VDIS are good agreement with experimental data
- We have extend our study on medium FF and PDF of pion and kaon in order to understand the feature of FF and PDF of the kaon and pion in the medium. The result looks very interesting and promising
- It would be interesting to extend calculation to PDF of the pion, kaon, and ρ meson in medium
- It would be interesting to extend the calculation to GPD, TMD of the pion and kaon.

THANK YOU VERY MUCH FOR ATTENTION !!

EXTRA SLIDES

Moments PDF of the s quark in the Kaon



Moment PDF of the u quark in the kaon