Off-Shell Pion Electromagnetic Form Factors

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Collaboration with J. de Melo, T. Frederico and C.-R. Ji

Work in progress

Outline

1. Motivation

2. General Structure of off-shell form factors for pion

- Introduce a new form factor $g(Q^2)$ (other than $F_\pi(Q^2)$), which is measurable in the on-mass shell limit

$$g(Q^2 = 0) = \frac{1}{6}\langle r_\pi^2 \rangle$$

3. Manifestly Covariant Model Calculations

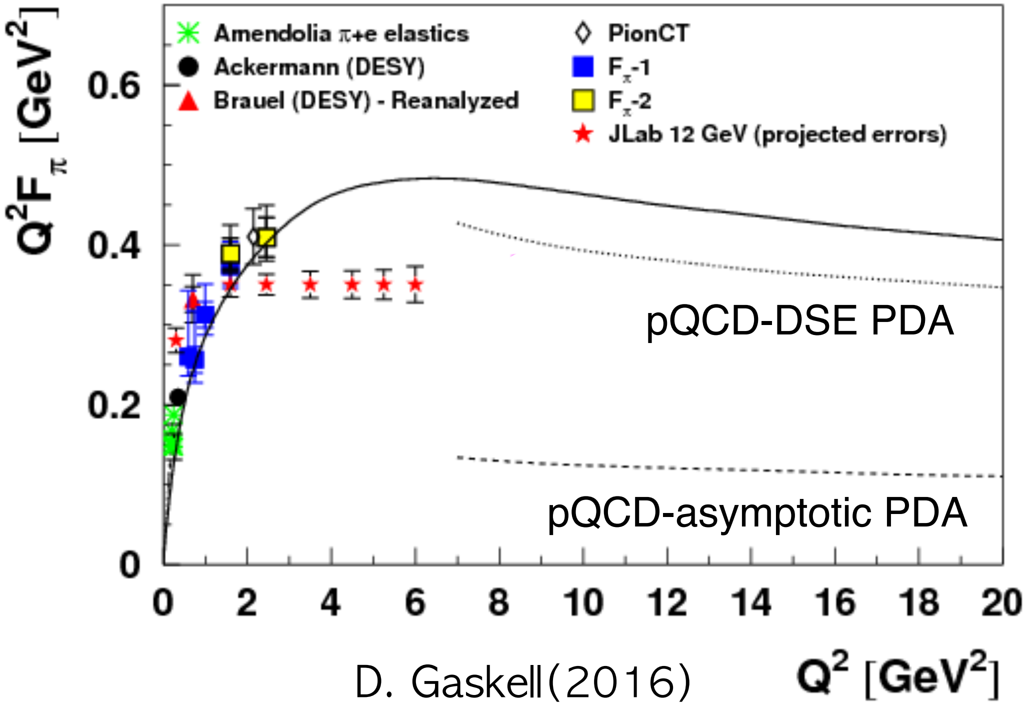
- Discuss a way to compare $g(Q^2)$ with the data extracted from JLAB

4. Summary

1. Motivation

- EM form factors of hadrons provide
- EM information on their bound state properties
- distributions of quarks and gluons inside them

- Pion
- the simplest hadronic system
- parametrized by a single on-shell form factor $F_\pi(Q^2)$



D. Gaskell(2016)

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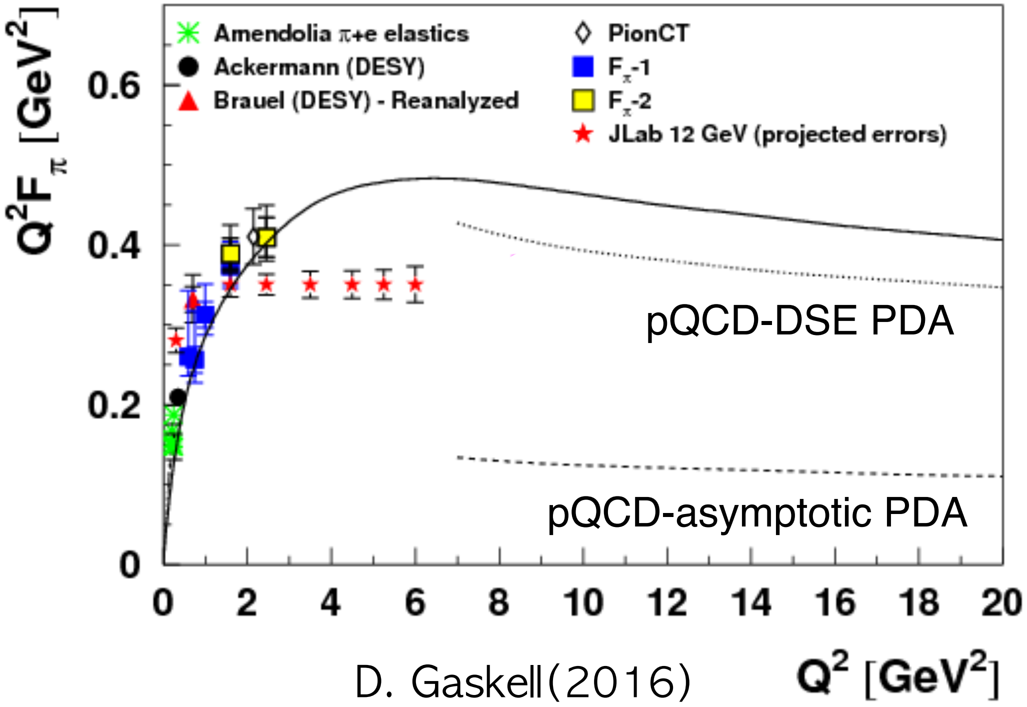
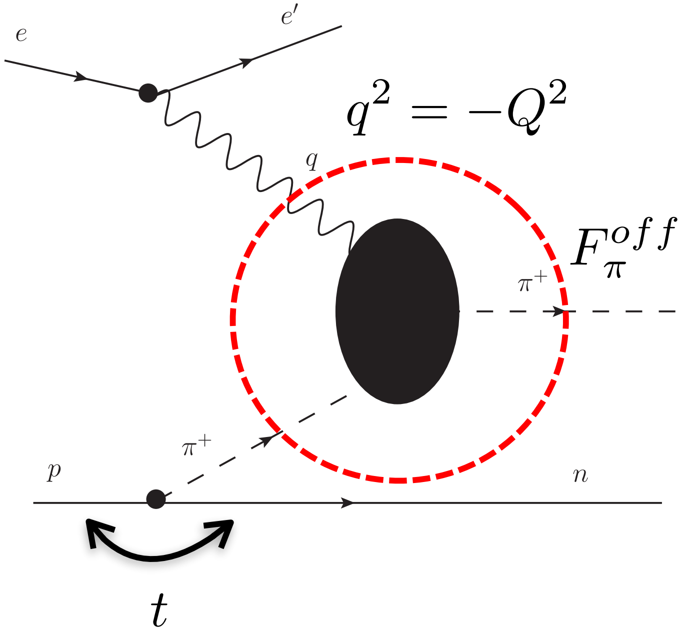


D. Gaskell(2016)

- At low Q^2 (~ 0.25 GeV^2), F_π can be measured directly from the elastic scattering of pions by atomic electrons [Amendolia et al. (1986)]

- At larger Q^2, F_π has to be measured indirectly using the "pion cloud" of the proton via ${}^1\mathrm{H}(e, e'\pi^+)n$ reaction [JLAB experiment]

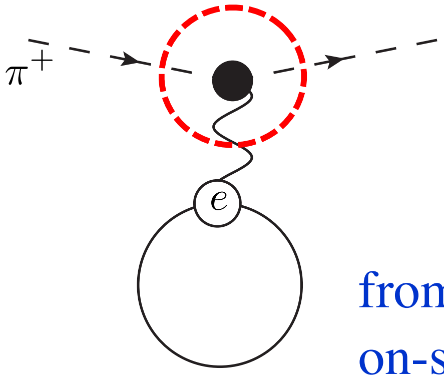
- Determination of F_π from the pion electroproduction on the proton



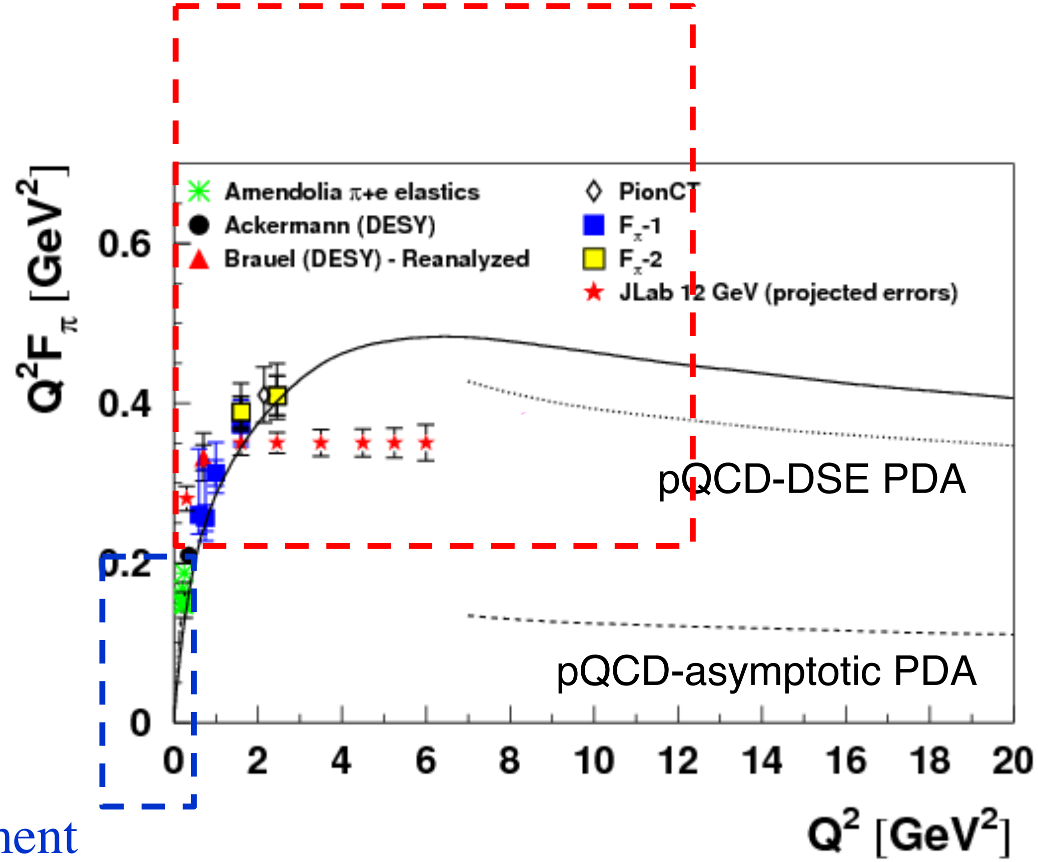
$F_\pi^{off}(Q^2, t)$ Extract → $F_\pi^{on}(Q^2, t = m_\pi^2)$

Need to investigate the off-shell form factors!

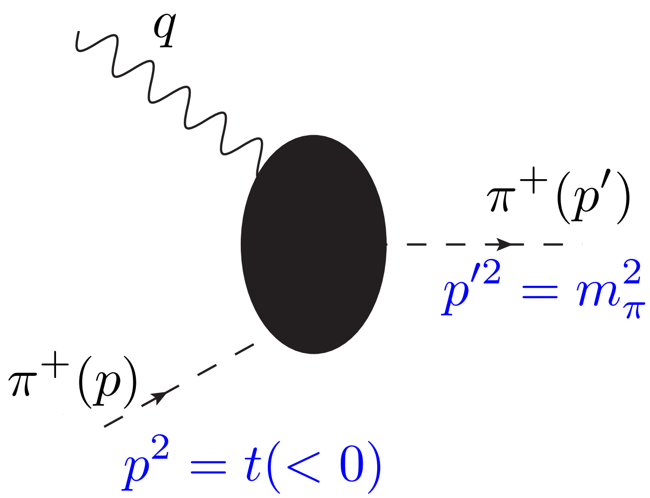
$F_\pi^{on}(Q^2)$



from direct on-shell measurement



2. General Structure of off-shell form factors



- General Structure

$$\Gamma_\mu = e[(p' + p)_\mu G^+ + (p' - p)_\mu G^-]$$

$$G^\pm = G^\pm(p^2 = t, p'^2 = m_\pi^2, q^2 = -Q^2)$$

- Ward-Takahashi Identity (WTI)

$$q^\mu \Gamma_\mu = e\Delta_0^{-1}(p')[\Delta(p) - \Delta(p')]\Delta_0^{-1}(p)$$

where $\Delta_0(p) = \dfrac{1}{p^2 - m_\pi^2 + i\epsilon}$ and $\Delta(p) = \dfrac{1}{p^2 - m_\pi^2 - \Pi(p^2) + i\epsilon}$

constrained by $\Pi(m_\pi^2) = 0$

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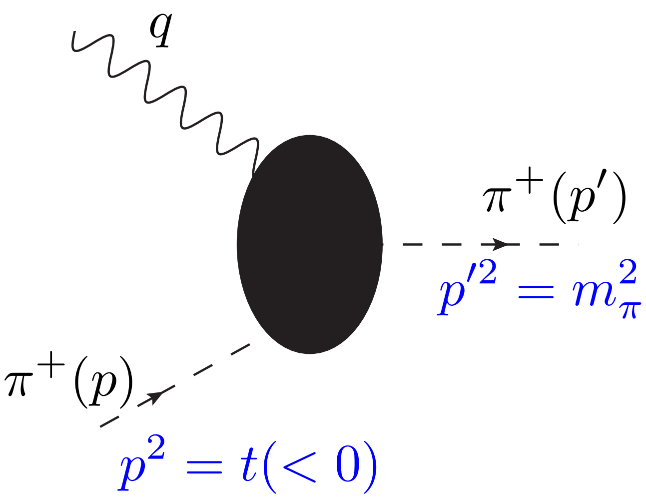

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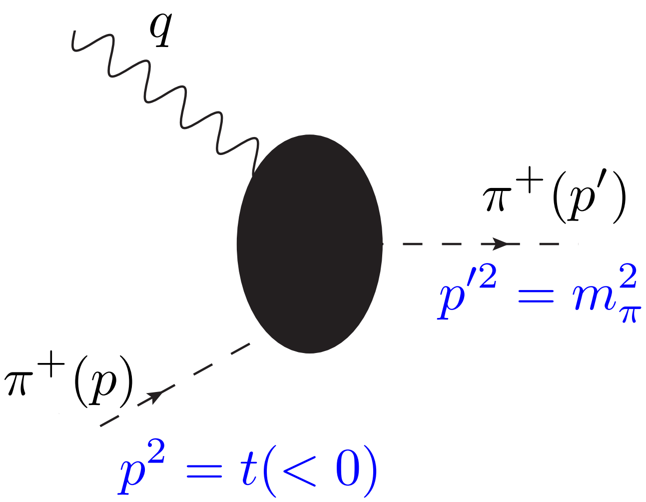
In the half on-shell limit,

$$\Delta_0^{-1}(p')\Delta(p') \to 1$$

$$\Delta_0^{-1}(p')\Delta(p) \to 0$$

$\Rightarrow$

$$(m_\pi^2 - t)G^+ + q^2 G^- = m_\pi^2 - t$$



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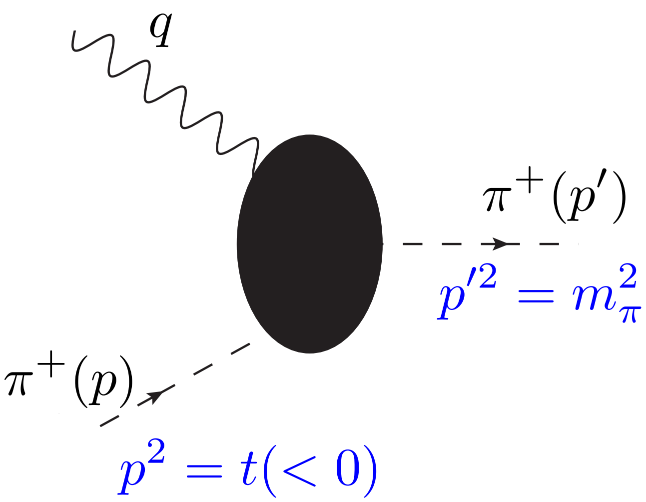
- Time-reversal invariance: $p \leftrightarrow p'$

$$G^+(Q^2, p^2, p'^2) = G^+(Q^2, p'^2, p^2) \text{ and } G^-(Q^2, p^2, p'^2) = -G^-(Q^2, p'^2, p^2)$$



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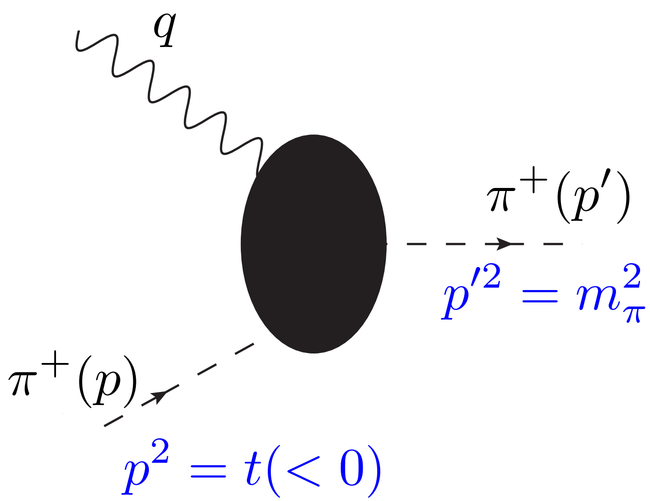

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0 for $p^2 = p'^2 = m_\pi^2$

$$G^+(Q^2, p^2, p'^2) = G^+(Q^2, p'^2, p^2) \text{ and } G^-(Q^2, p^2, p'^2) = -G^-(Q^2, p'^2, p^2)$$

Defining $\quad F_1(Q^2, t) \equiv G^+(q^2, t, m_\pi^2),\ F_2(Q^2, t) \equiv G^-(q^2, t, m_\pi^2)$

We have

$$F_2(Q^2, t) = \frac{t - m_\pi^2}{Q^2}[1 - F_1(Q^2, t)] \Rightarrow$$

$$F_1(Q^2 = 0, t = m_\pi^2) = 1$$

$$F_2(Q^2, t = m_\pi^2) = 0$$

- New method to make $F_2(Q^2,t)$ measurable in the on-shell limit.

$$F_1(Q^2,t) + Q^2 \frac{F_2(Q^2,t)}{t-m_\pi^2} = 1$$

$$\boxed{\frac{F_2(Q^2,t)}{t-m_\pi^2} \equiv g(Q^2,t)}$$

$$\Downarrow$$

$$F_1(Q^2,t) + Q^2 g(Q^2,t) = 1$$

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$$F_1(Q^2,t) + Q^2 g(Q^2,t) = 1$$

$$\frac{\partial}{\partial Q^2}$$

$$g(Q^2,t) + Q^2 \frac{\partial g(Q^2,t)}{\partial Q^2} = -\frac{\partial}{\partial Q^2} F_1(Q^2,t)$$

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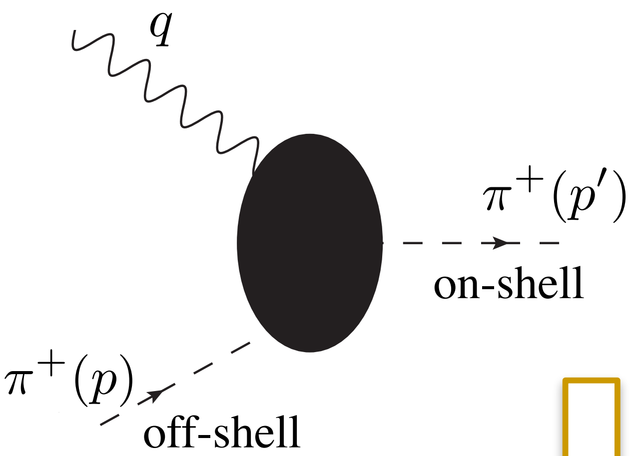
On-shell limit $t = m_\pi^2$

$$g(Q^2=0,m_\pi^2) = -\frac{\partial}{\partial Q^2} F_1(Q^2=0,m_\pi^2) = \frac{1}{6}\langle r_\pi^2\rangle$$

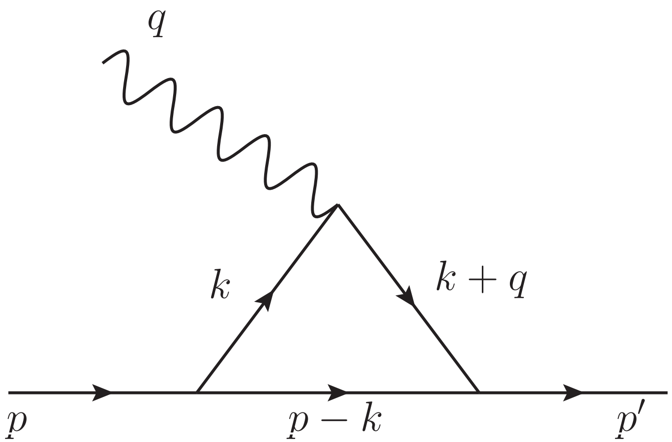
$$\boxed{g(Q^2,t=m_\pi^2) = \frac{1}{6}\langle r^2\rangle + \alpha Q^2 + \cdots}$$

vs. $F_2(Q^2,m_\pi^2) = 0$

3. Exactly Solvable Model Calculation



$$\Gamma^{\mu}_{\mathrm{off-shell}} = (p' + p)^{\mu} F_1(Q^2, t) + (p' - p)^{\mu} F_2(Q^2, t)$$



$$\Gamma^{\mu} = i\mathcal{N} \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{S}^{\mu}}{N_k N_{k+q} N_{p-k}} H_{\mathrm{cov}} H'_{\mathrm{cov}}$$

$$\mathrm{S}^{\mu} = \mathrm{Tr}[\gamma_5(\not{k} + \not{q} + m_q)\gamma^{\mu}(\not{k} + m_q)\gamma_5(\not{k} - \not{p} + m_q)]$$

$$N_p = p^2 - m_q^2 + i\epsilon \text{ etc.}$$

Some essential procedures for the calculations:

$$\Gamma^\mu = i\mathcal{N}\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{S}^\mu}{N_k N_{k+q} N_{p-k}} H_{\rm cov} H'_{\rm cov}$$

$$\frac{1}{N_1 N_2 N_3} = \int_0^1 dx \int_0^x dy \frac{2!}{[N_1 + x(N_2 - N_1) + y(N_3 - N_2)]^3},$$

Wick rotation and Regularization in $d = 4 - 2\epsilon$ dimension:

$$\int \frac{d^d k_E}{(2\pi)^d}\frac{1}{(k_E^2 + C)^n} = \frac{1}{(4\pi)^2}\frac{1}{2C}$$

$$\int \frac{d^d k_E}{(2\pi)^d}\frac{k_E^2}{(k_E^2 + C)^n} = \frac{1}{(4\pi)^2}(1 - \frac{\epsilon}{2})C^{-\epsilon}\Gamma(\epsilon) = \frac{1}{(4\pi)^2}\left[\frac{1}{\epsilon} - \gamma - \frac{1}{2} - \mathrm{Log}C\right],$$

$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + \frac{1}{12}(6\gamma^2 + \pi^2) + \mathcal{O}(\epsilon^2)$$

$$\Gamma^\mu = F_1(Q^2,t)(p'+p)^\mu + F_2(Q^2,t)(p'-p)^\mu$$

where

$$F_1(Q^2,t) = N_c \frac{g^2}{8\pi^2}\int_0^1 dx \int_0^x dy \left[(1+3y)\left(\gamma + \frac{1}{2} + \text{Log}C\right) + \frac{\alpha}{C}\right],$$

$$F_2(Q^2,t) = N_c \frac{g^2}{8\pi^2}\int_0^1 dx \int_0^x dy \left[3(1-2x+y)\left(\gamma + \frac{1}{2} + \text{Log}C\right) + \frac{2\beta-\alpha}{C}\right],$$

$$\alpha = E^2 - q\cdot E - m_q^2 + y[E^2 + 2p\cdot E - q\cdot p - m_q^2],$$

$$\beta = E^2 + p\cdot E - m_q^2 - (x-y)[E^2 + 2p\cdot E - q\cdot p - m_q^2].$$

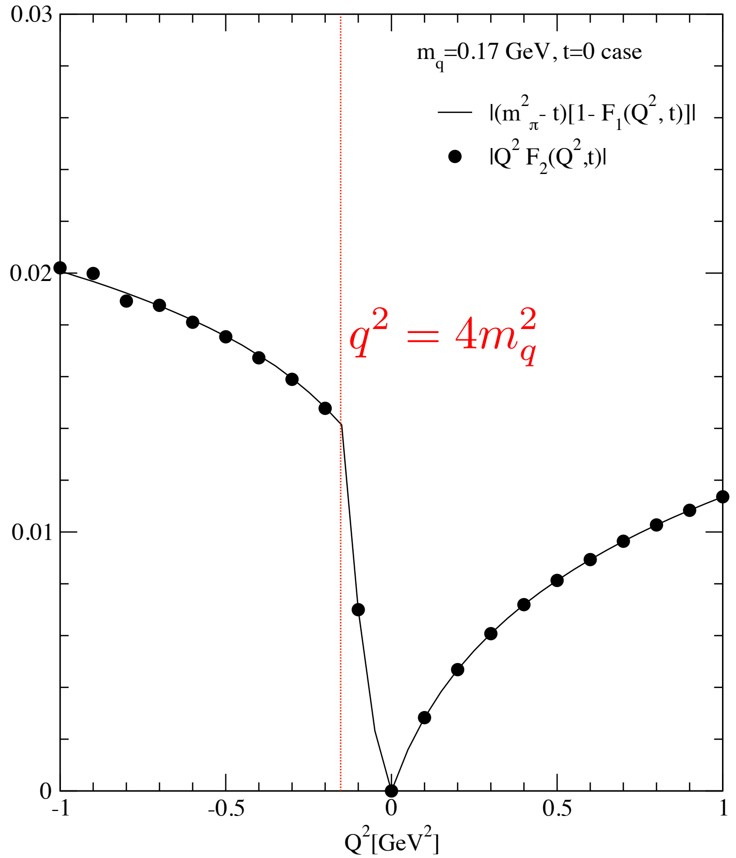
$$E = (x-y)q - yp,$$

$$C = (x-y)(x-y-1)q^2 - y(1-y)t - 2y(x-y)q\cdot p + m_q^2,$$

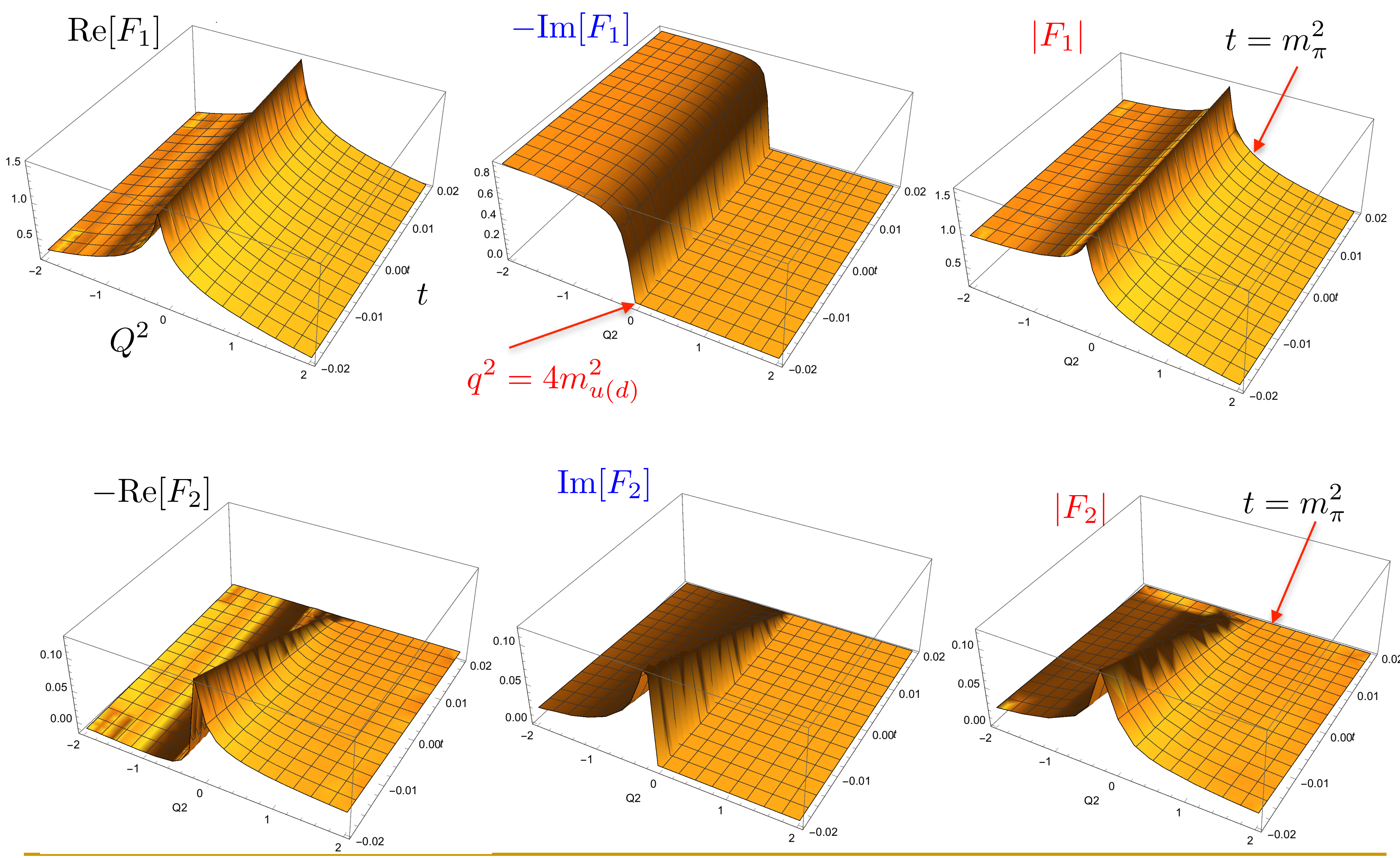
- Numerical Results: $m_u = m_d = 0.17$ GeV, $m_\pi = 0.14$ GeV

Proof of WTI : $$F_1(Q^2, t) + Q^2 \frac{F_2(Q^2, t)}{t - m_\pi^2} = 1$$

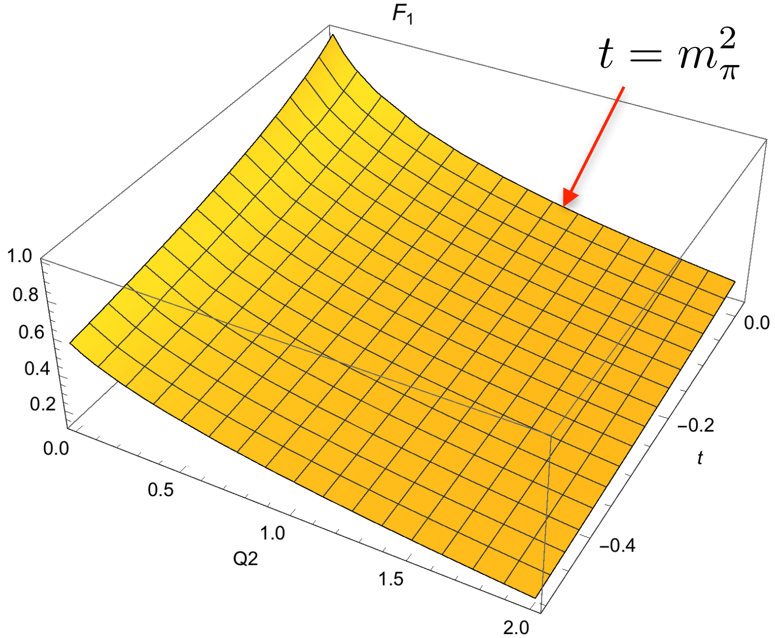
or $$q^2 F_2(Q^2, t) = (m_\pi^2 - t)[F_1(Q^2 = 0, t) - F_1(Q^2, t)]$$

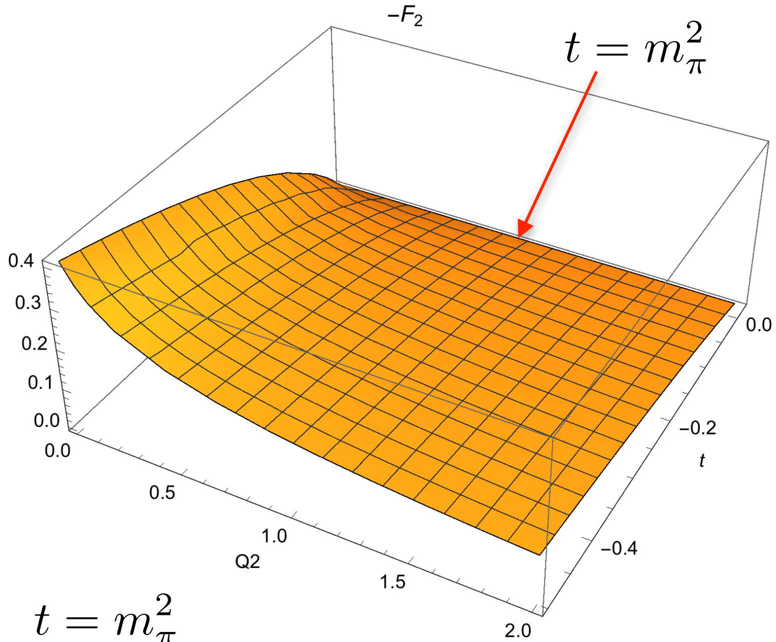


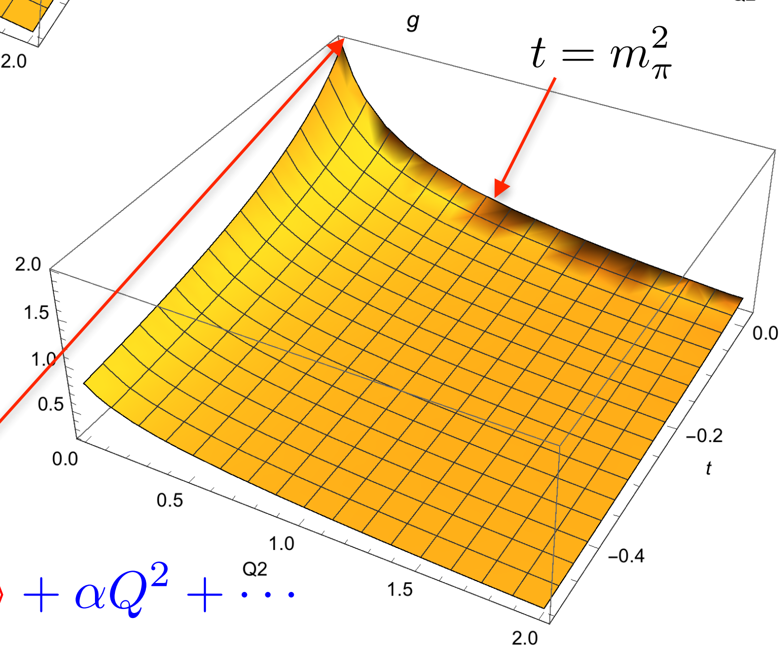
3D Imaging of $F_i(Q^2,t)$ $\quad -2 \leq Q^2 \leq 1\ \mathrm{GeV}^2, \quad -m_\pi^2 \leq t \leq m_\pi^2$



3D Imaging of $F_1(Q^2,t)$, $F_2(Q^2,t)$, and $g(Q^2,t)$ in spacelike region



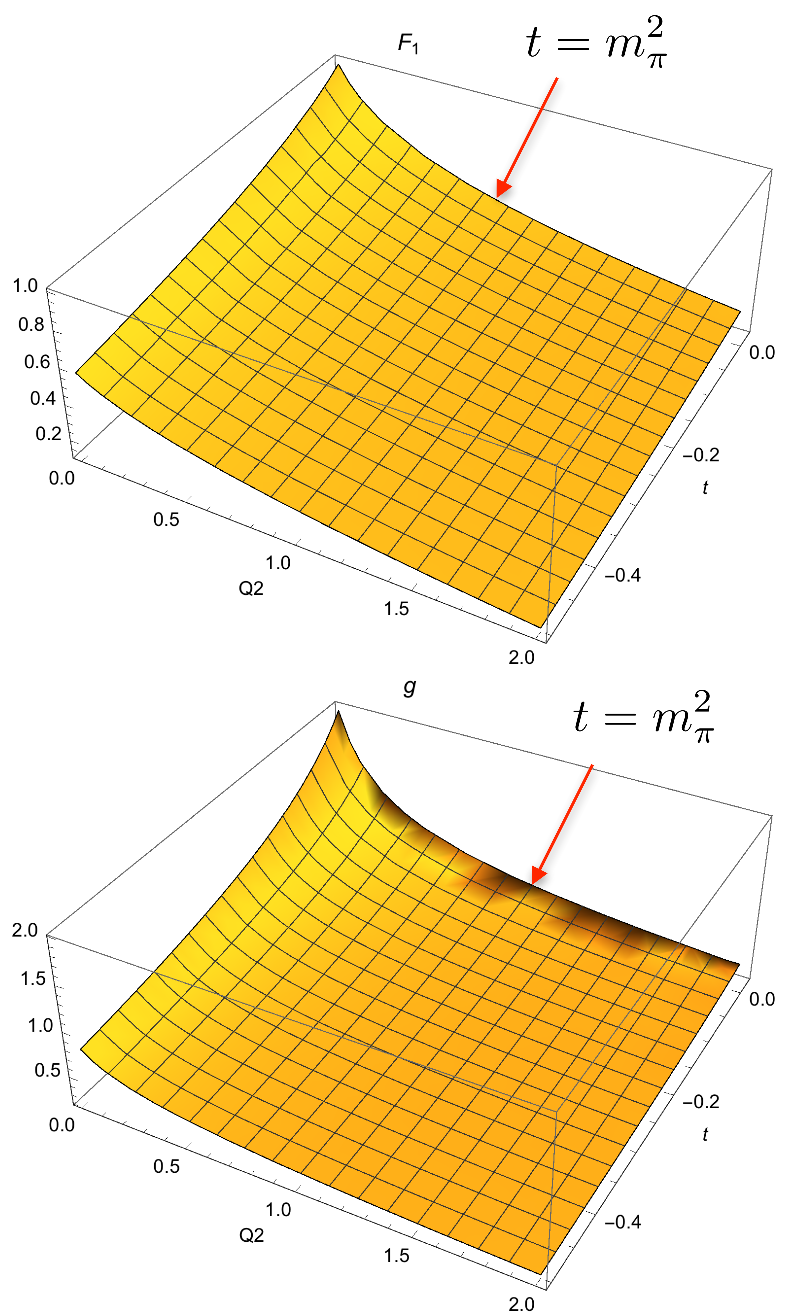


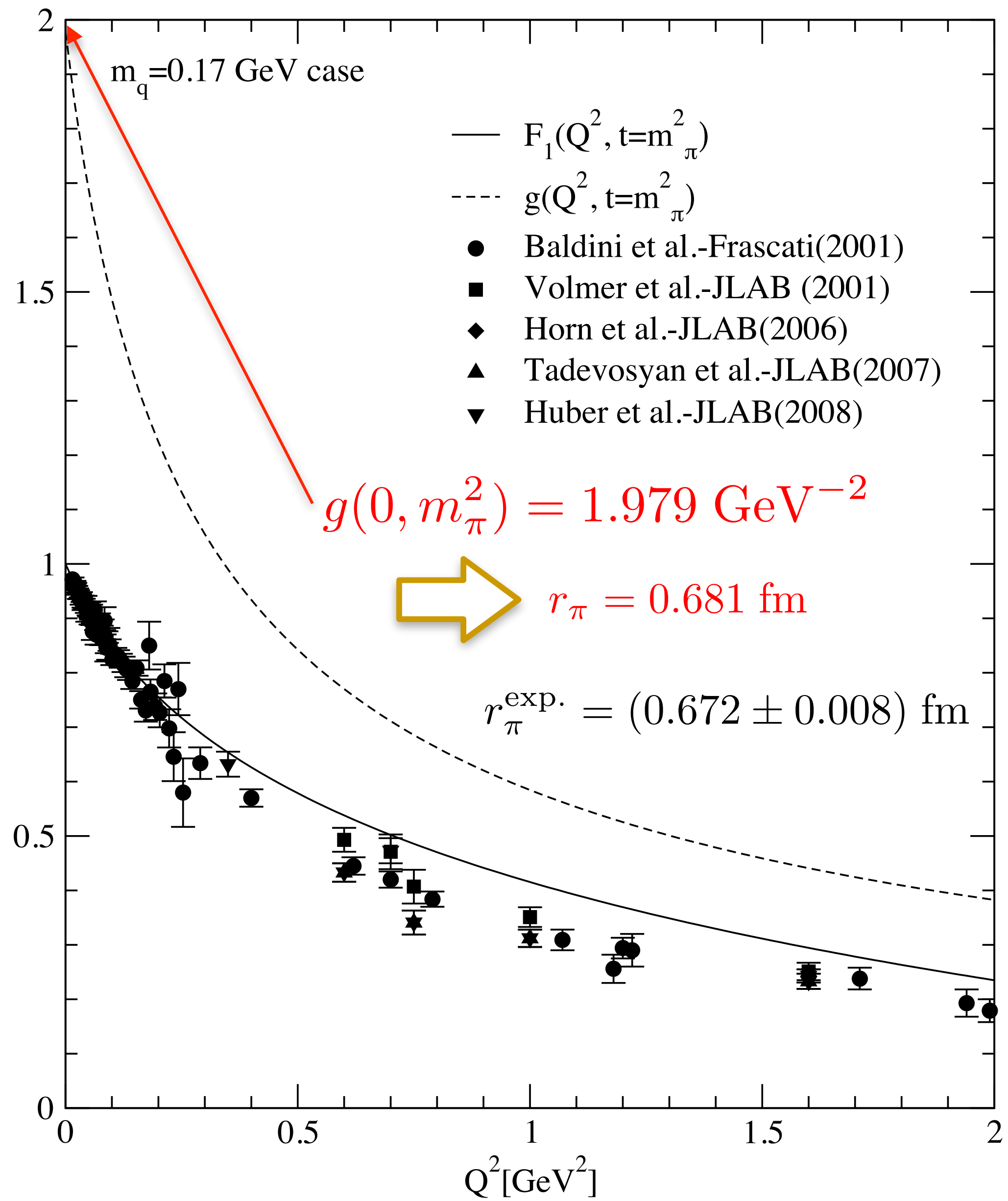


$$\frac{F_2(Q^2,t)}{t-m_\pi^2} \equiv g(Q^2,t)$$

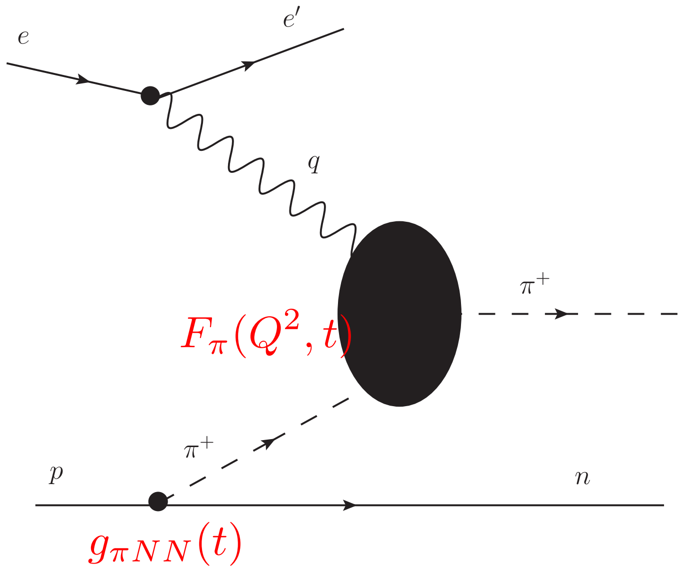
$$g(Q^2, t=m_\pi^2) = \frac{1}{6}\langle r^2\rangle + \alpha Q^2 + \cdots$$

On-shell predictions





4. Data Extracted from JLAB



Cross section for $^1\mathrm{H}(e,e'\pi^+)n$

$$\rightarrow \left(\frac{d\sigma_L}{dt}, \frac{d\sigma_T}{dt}, \frac{d\sigma_{LT}}{dt}, \frac{d\sigma_{TT}}{dt}\right)$$

- At small -t, the pion pole process dominates σ_L

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)$$

where

$$g_{\pi NN}(t) = g_{\pi NN}(m_\pi^2)\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right)$$

Inputs:

$g_{\pi NN}(m_\pi^2) = 13.4,\ \Lambda_\pi = 0.80$ GeV

σ_L^{exp}

[Ref.] H.Blok et al. PRC 78, 045202(2008)

Extract off-shell $F_1(Q^2,t) = F_\pi(Q^2,t)$

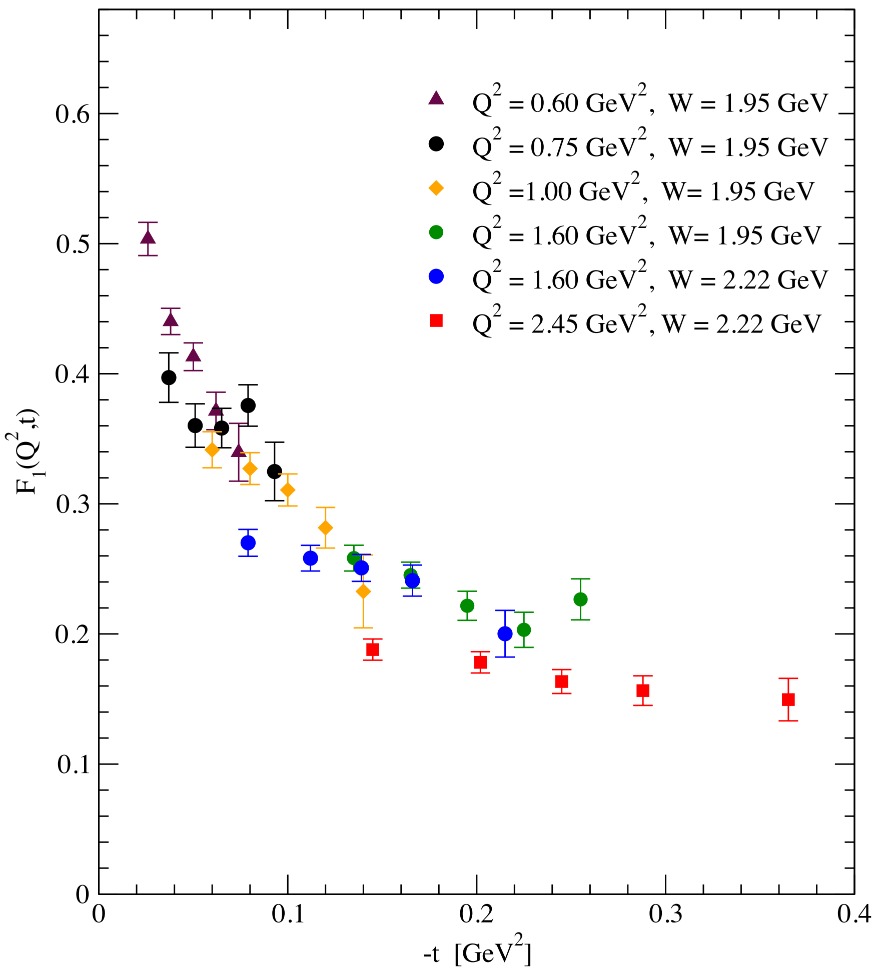
Extract

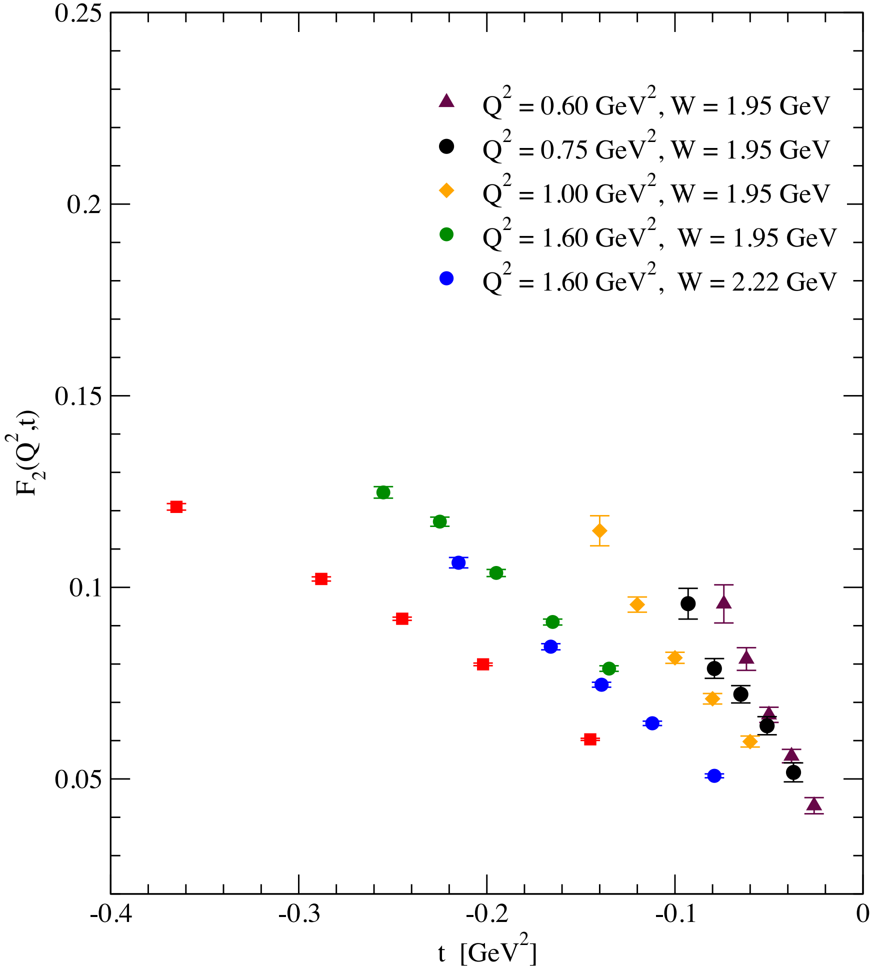
$$F_1(Q^2,t) + Q^2 \frac{F_2(Q^2,t)}{t-m_\pi^2} = 1$$

- Off-shell $F_1(Q^2, t), F_2(Q^2, t)$ extracted from JLAB data

[Ref.] H.Blok et al. PRC 78, 045202(2008)

PRELIMINARY





On-shell form factors:

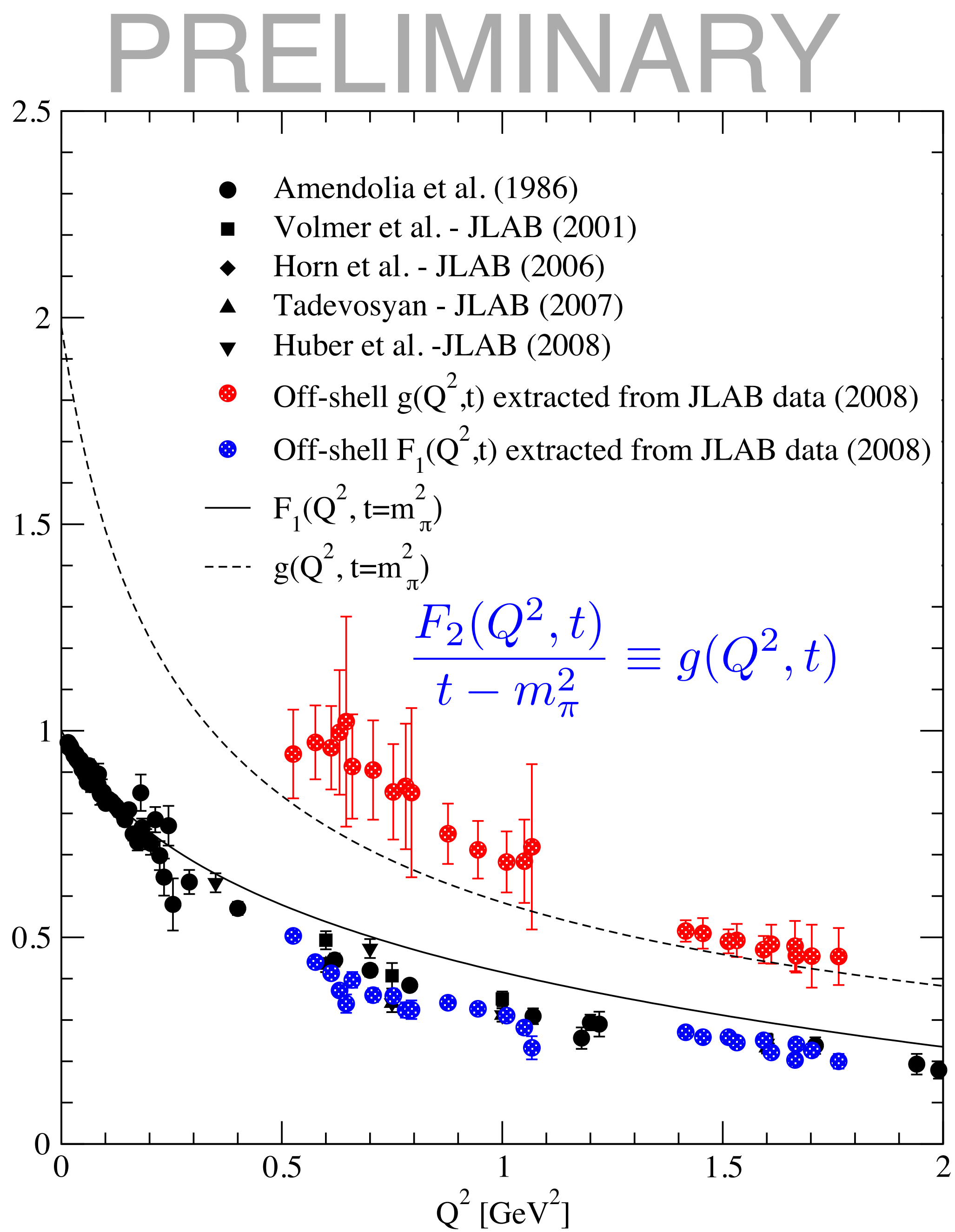
$$F_1(Q^2, t = m_\pi^2), \quad g(Q^2, t = m_\pi^2)$$

compared with JLAB data.

$g(Q^2, t = m_\pi^2)$

can give new constraint

in extracting on-shell $F_\pi(Q^2)$



5. Summary

- Obtain the general off-shell pion form factors: $F_1(Q^2, t)$ and $F_2(Q^2, t)$

$$F_1(Q^2, t) + Q^2 \frac{F_2(Q^2, t)}{t - m_\pi^2} = 1 \quad \text{from WTI}$$

$$\Rightarrow \quad F_1(Q^2 = 0, t = m_\pi^2) = 1 \qquad F_2(Q^2, t = m_\pi^2) = 0$$

- Find the new observable in the on-shell limit: $g(Q^2, t) = \dfrac{F_2(Q^2, t)}{t - m_\pi^2}$

$$g(Q^2, t = m_\pi^2) = \frac{1}{6}\langle r^2 \rangle + \alpha Q^2 + \cdots$$

$\Rightarrow$ can be extracted from JLAB experiment and would give the new constraint to extract the on-shell charge form factor!

$F_1(Q^2, t = -m_\pi^2)$ vs. $F_2(Q^2, t = -m_\pi^2)$

$$-5 \le Q^2 \le 2 \text{ GeV}^2$$