Nuclear Physics School 2018

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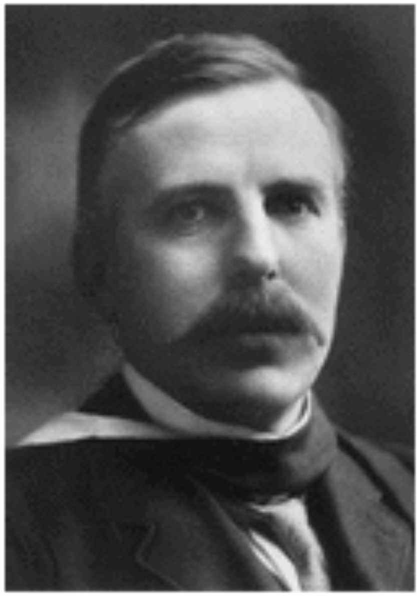
First Lecture

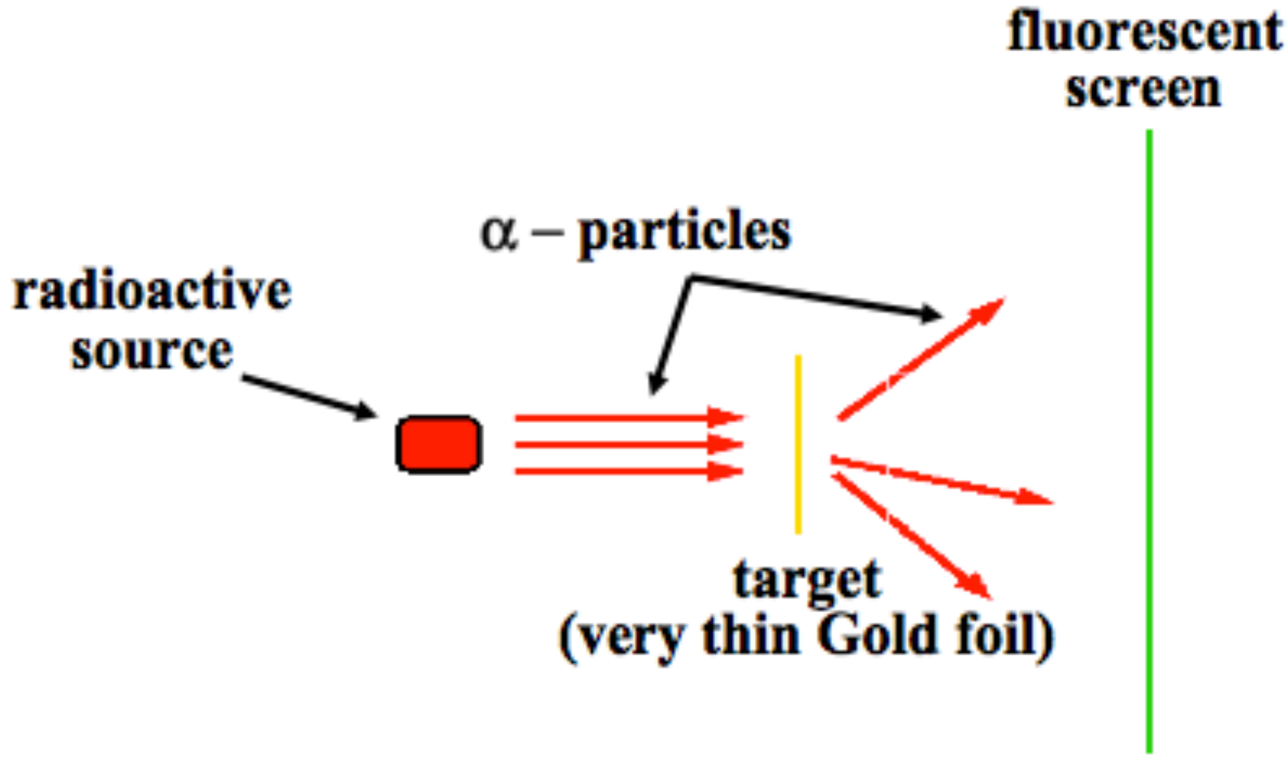
From Atoms to Quarks

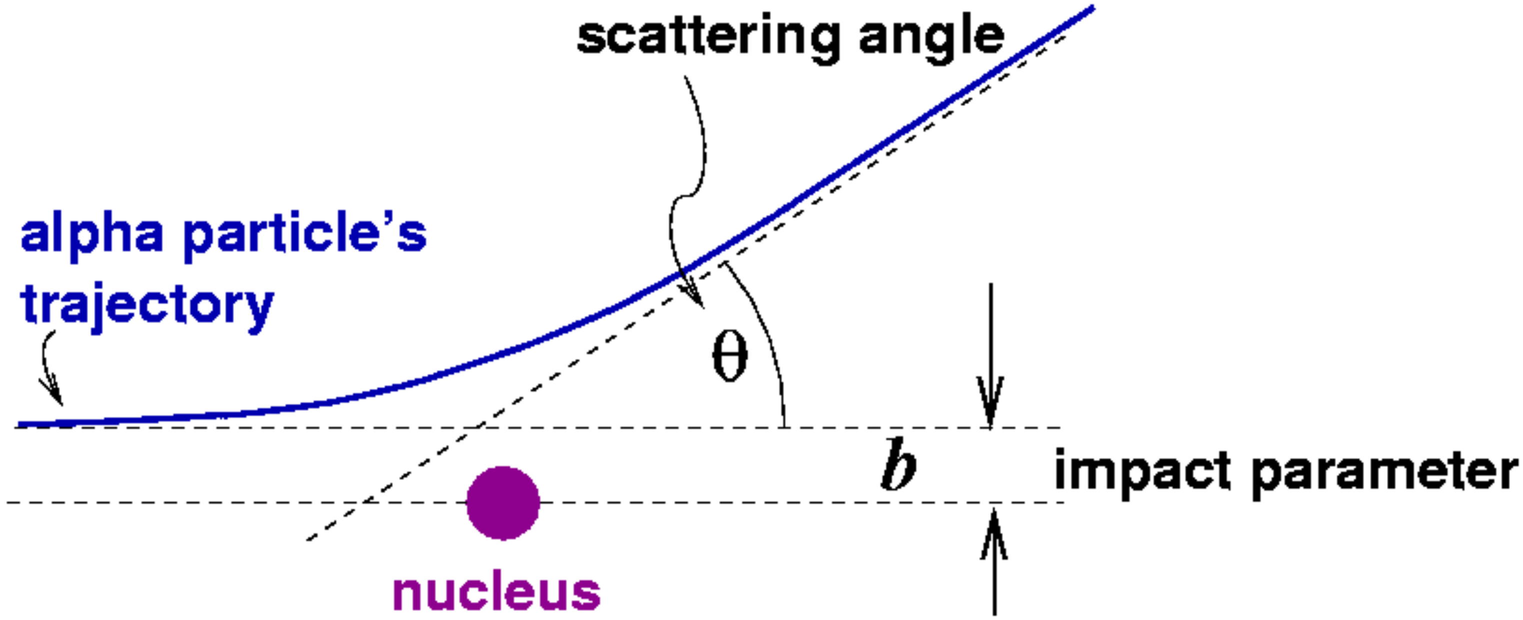
June 28, 2018

1909 – 13: Rutherford's scattering experiments
Discovery of the atomic nucleus

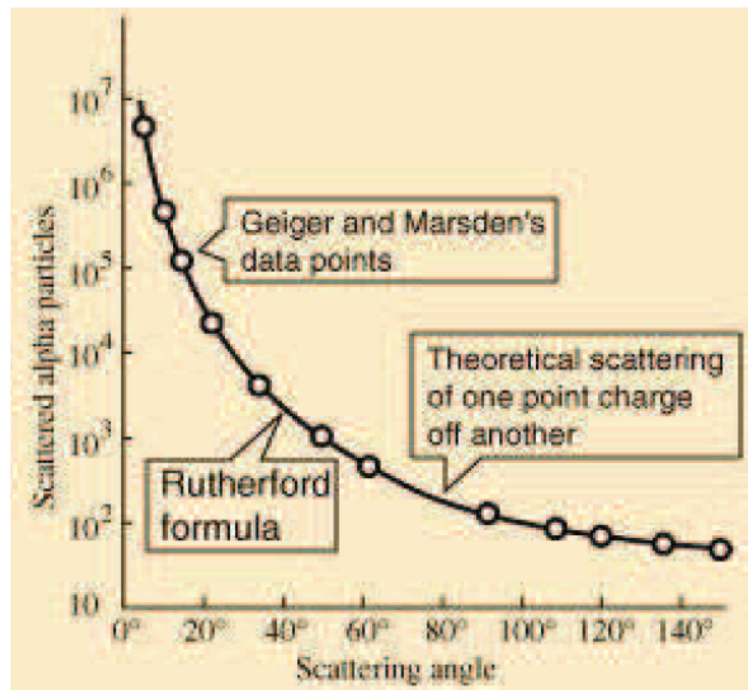


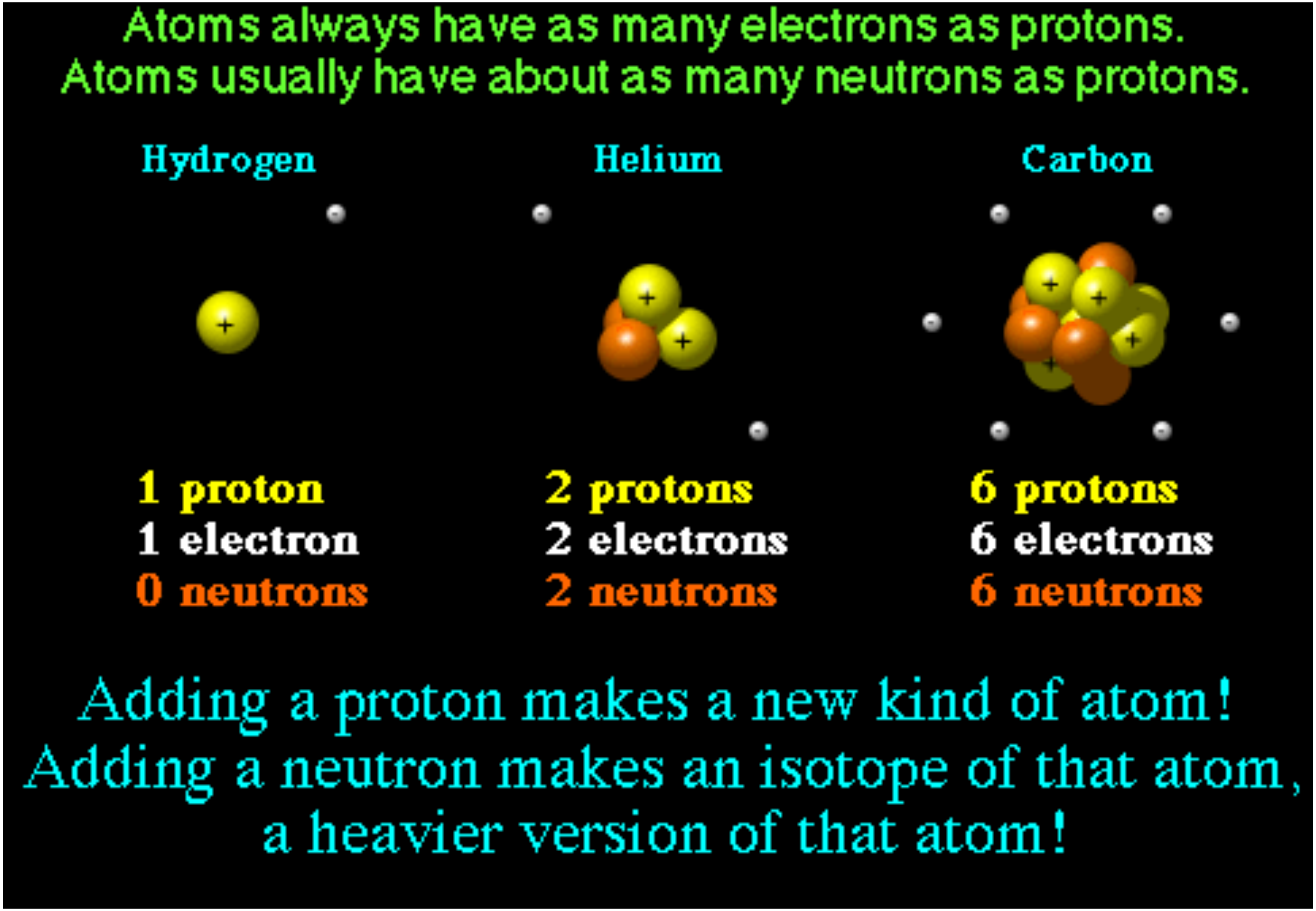
Ernest Rutherford

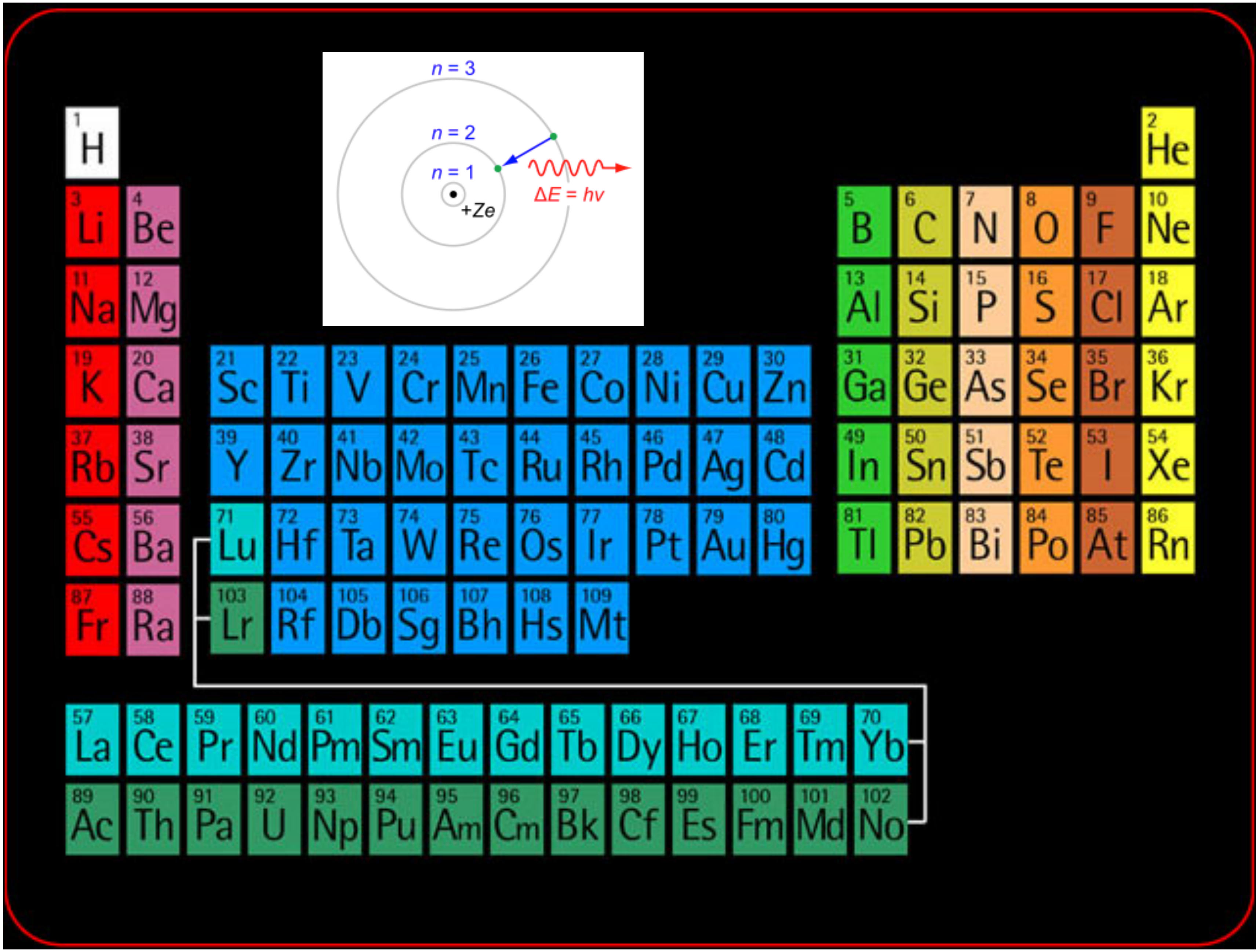




$$\left(\frac{d\sigma}{d\Omega}\right)_{point} \sim \frac{1}{E^2 \sin^4 \frac{\theta}{2}}$$

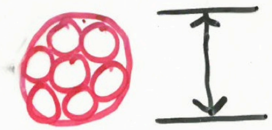



Atoms always have as many electrons as protons.
Atoms usually have about as many neutrons as protons.
Hydrogen
Helium
Carbon
1 proton
1 electron
0 neutrons
2 protons
2 electrons
2 neutrons
6 protons
6 electrons
6 neutrons
Adding a proton makes a new kind of atom!
Adding a neutron makes an isotope of that atom,
a heavier version of that atom!



n = 3
n = 2
n = 1
+Ze
ΔE = hv
1 H
2 He
3 Li
4 Be
5 B
6 C
7 N
8 O
9 F
10 Ne
11 Na
12 Mg
13 Al
14 Si
15 P
16 S
17 Cl
18 Ar
19 K
20 Ca
21 Sc
22 Ti
23 V
24 Cr
25 Mn
26 Fe
27 Co
28 Ni
29 Cu
30 Zn
31 Ga
32 Ge
33 As
34 Se
35 Br
36 Kr
37 Rb
38 Sr
39 Y
40 Zr
41 Nb
42 Mo
43 Tc
44 Ru
45 Rh
46 Pd
47 Ag
48 Cd
49 In
50 Sn
51 Sb
52 Te
53 I
54 Xe
55 Cs
56 Ba
71 Lu
72 Hf
73 Ta
74 W
75 Re
76 Os
77 Ir
78 Pt
79 Au
80 Hg
81 Tl
82 Pb
83 Bi
84 Po
85 At
86 Rn
87 Fr
88 Ra
103 Lr
104 Rf
105 Db
106 Sg
107 Bh
108 Hs
109 Mt
57 La
58 Ce
59 Pr
60 Nd
61 Pm
62 Sm
63 Eu
64 Gd
65 Tb
66 Dy
67 Ho
68 Er
69 Tm
70 Yb
89 Ac
90 Th
91 Pa
92 U
93 Np
94 Pu
95 Am
96 Cm
97 Bk
98 Cf
99 Es
100 Fm
101 Md
102 No

Consider

e → 

$E < \frac{1}{10\,\text{fm}}$ $\qquad \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \sim \frac{1}{E^2 \sin^4\frac{\theta}{2}}$ (Rutherford)

violent large angle scatt.

$E > \frac{1}{10\,\text{fm}}$ $\qquad Ze \to \rho(r)$

spatial extention.

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(Q^2)|^2$$

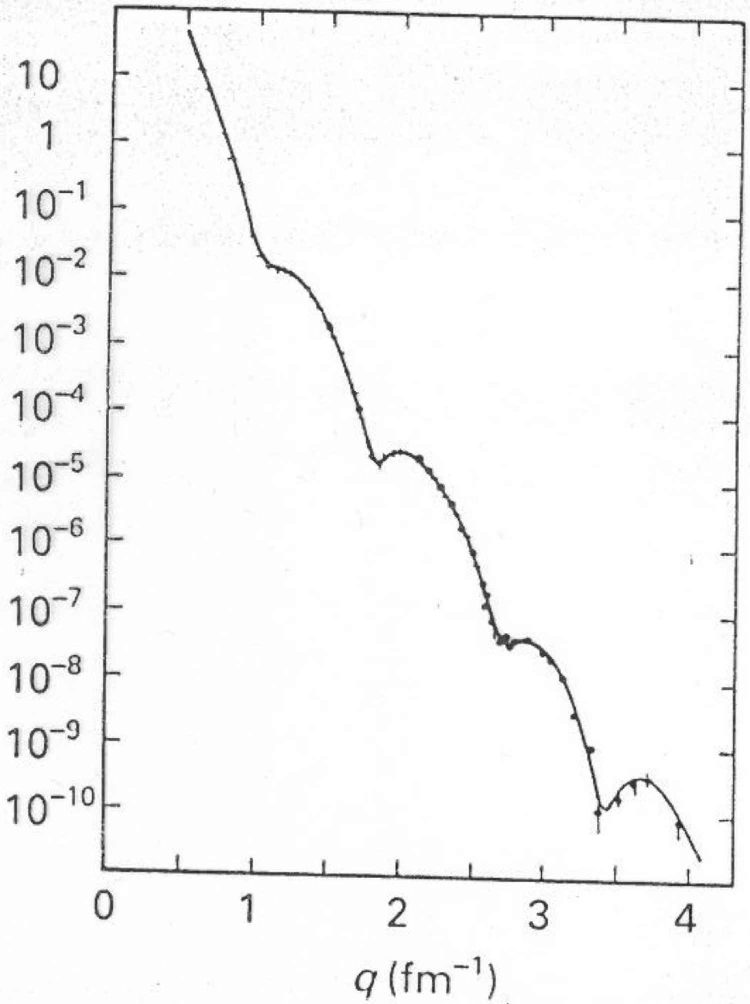
↑ Mom. Transf (Q^2)

Form Factor $F(Q^2) = \int d^3r\, \rho(r)\, e^{i\vec{q}\cdot\vec{r}}$

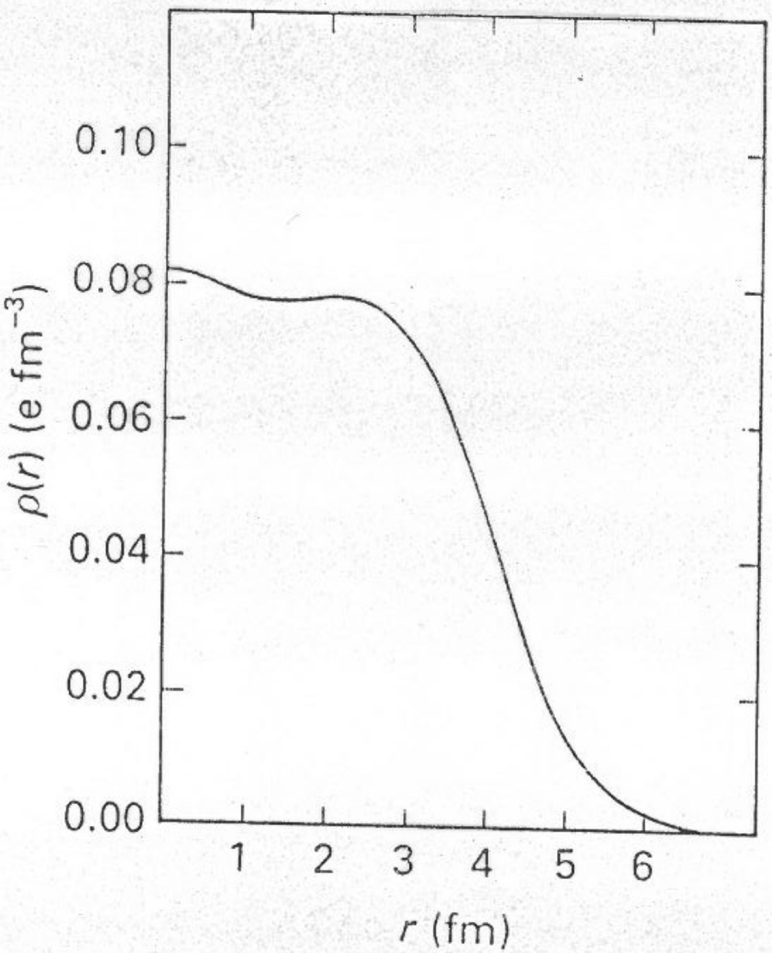
e.g. $\rho(r) \sim e^{-m^2 r^2}$

$F(Q^2) \sim e^{-\frac{Q^2}{4m^2}}$

scattering cross-section $d\sigma/d\Omega$ for the scattering of 450 MeV electrons by $^{58}_{28}Ni$.

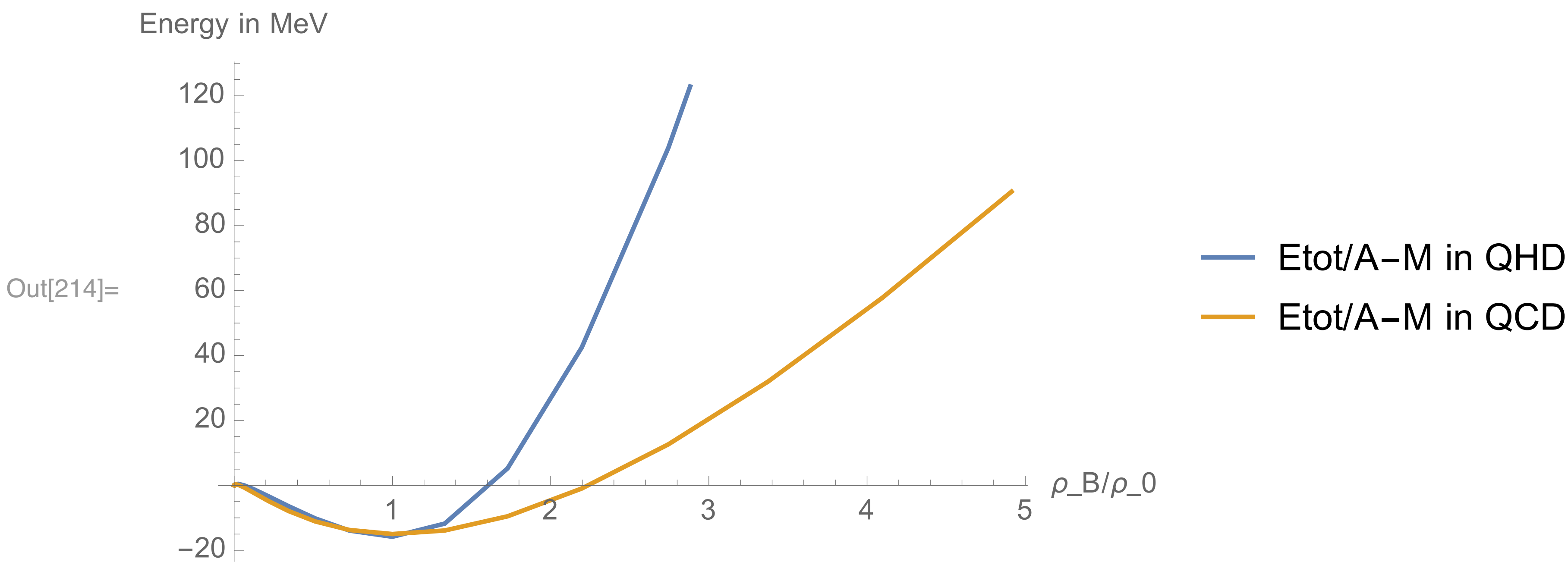


(a)

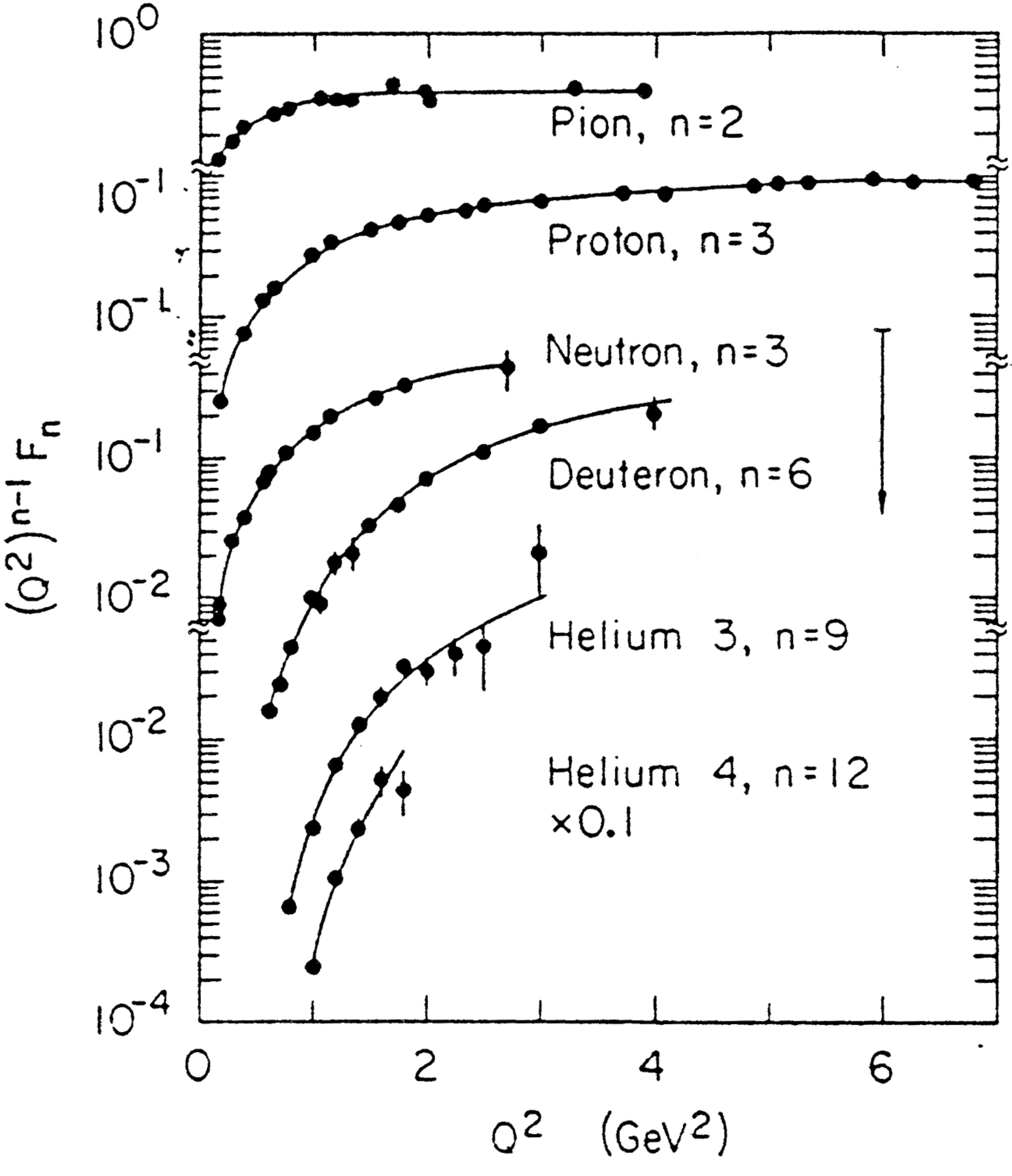


(b)

Stability of Nuclear Matter

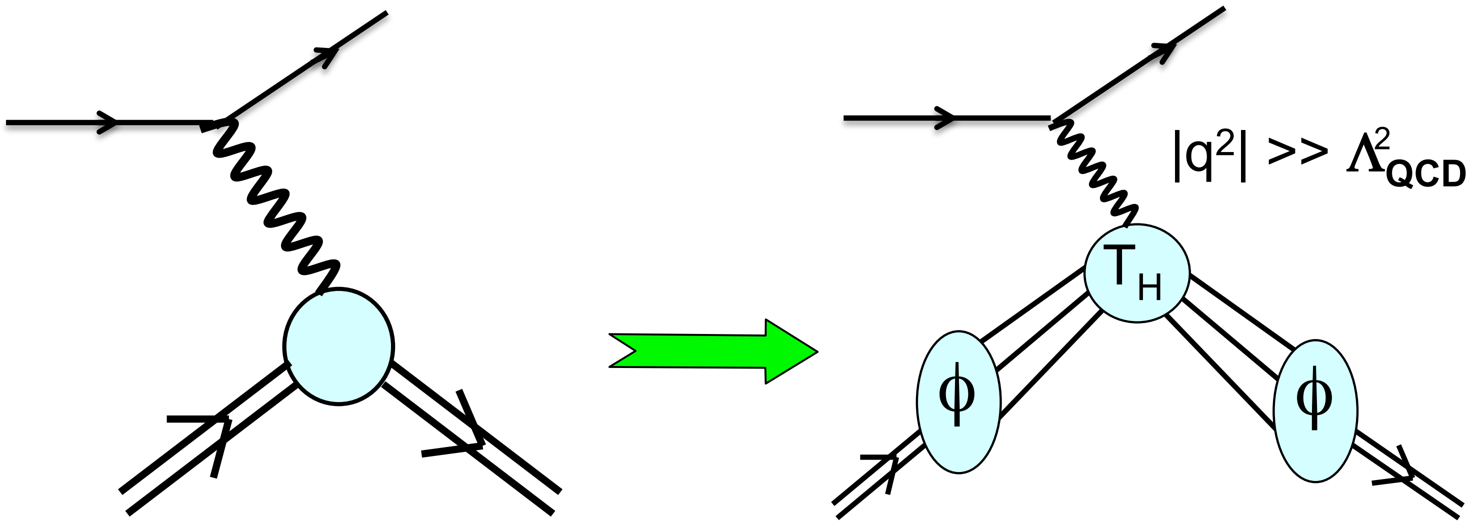


- Relativity is crucial for the stabilization of nuclear matter.
- Proton has quarks and gluons inside.
- Quantum Chromodynamics (QCD) governs them.

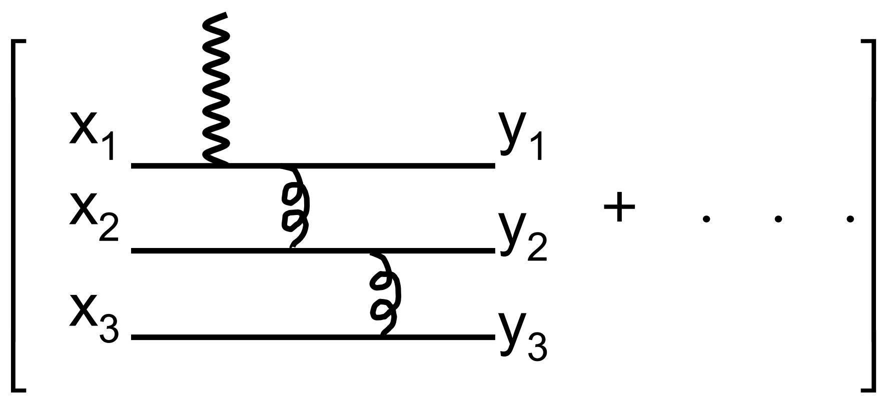


10^0
10^-1
10^-1
10^-1
10^-2
10^-2
10^-3
10^-4
Pion, n=2
Proton, n=3
Neutron, n=3
Deuteron, n=6
Helium 3, n=9
Helium 4, n=12
×0.1
(Q^2)^n-1 F_n
0
2
4
6
Q^2 (GeV^2)

Proton Charge Form Factor

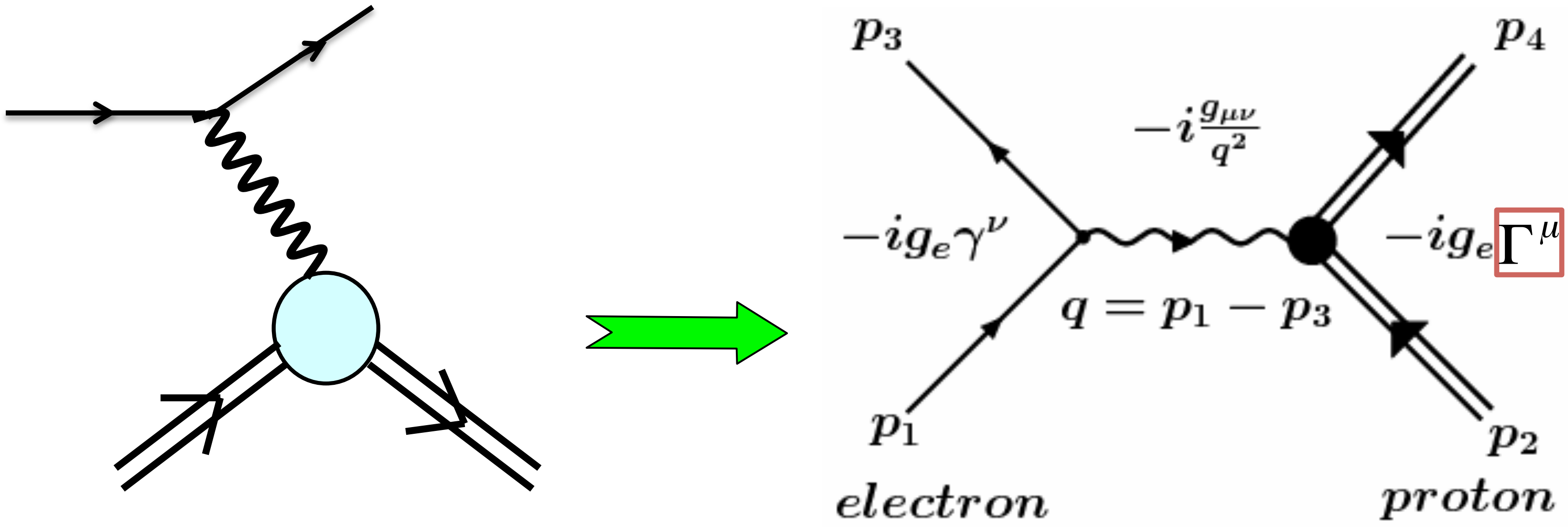


$$T_H = \sum$$



$$= \frac{\alpha_s^{\,2}}{Q^4} f(x_i, y_i)$$

Feynman Diagram of Electron-Proton Scattering



$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2Mc}$$

$F_1(0) = 1$ (Charge)

$F_2(0) = 1.7928$ (Anomalous magnetic moment)

If the target is a point-like object such as the electron, then

$$F_1(q^2) = 1$$

$$F_2(q^2) = 0,$$

for any q^2 .

Differential Cross Section of Elastic Scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4ME\sin^2\frac{\theta}{2}} \right)^2 \frac{E'}{E} \left[2K_1 \sin^2\frac{\theta}{2} + K_2 \cos^2\frac{\theta}{2} \right],$$

$$E' = \frac{E}{1 + \left(\frac{2E}{Mc^2} \right) \sin^2\frac{\theta}{2}}.$$

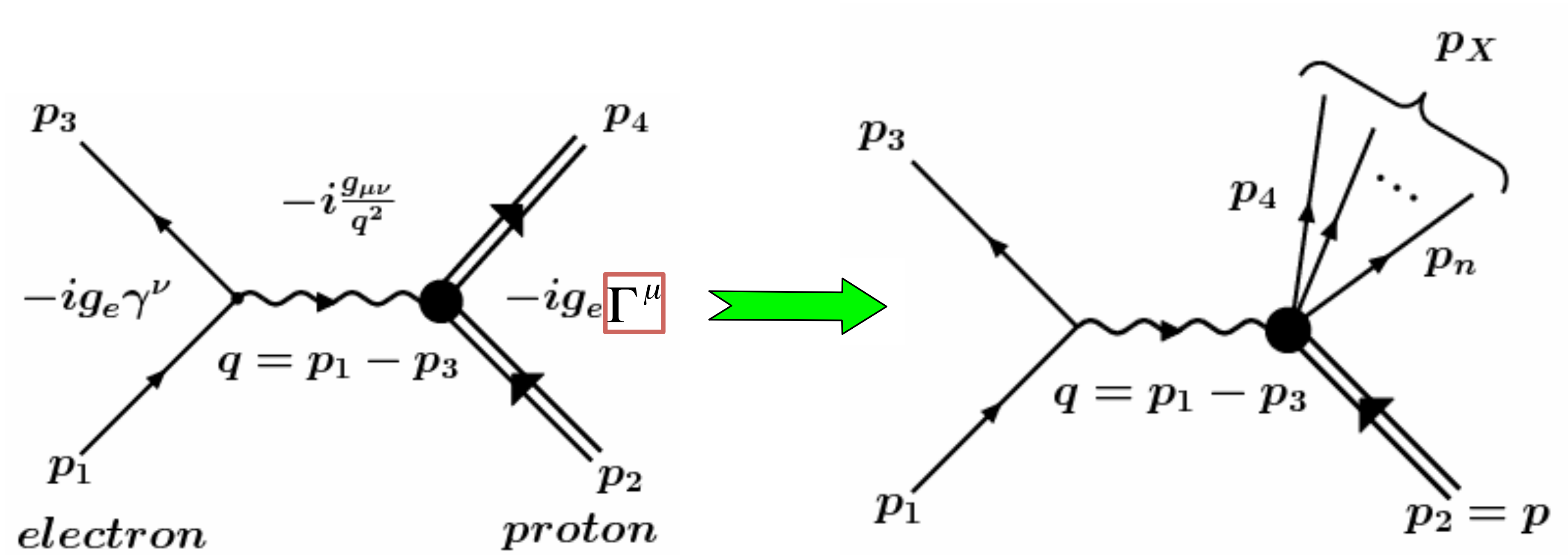
$$K_1 = -q^2 (F_1 - 2F_2)^2,$$

$$K_2 = -(2Mc)^2 (F_1^2 - \frac{q^2}{(Mc)^2} F_2^2).$$

If the proton was the point-like object,

$$K_1 = -q^2, \qquad K_2 = (2Mc)^2.$$

Deep Inelastic Scattering (DIS)



$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4ME\sin^2\frac{\theta}{2}}\right)^2 \frac{E'}{E}\left[2K_1\sin^2\frac{\theta}{2} + K_2\cos^2\frac{\theta}{2}\right] \Rightarrow \frac{d\sigma}{dE'd\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2\frac{\theta}{2}}\right)^2\left(2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right),$$

where the structure functions W_1 and W_2 are the functions of q^2 and $x = -\frac{q^2}{2p\cdot q}$.

In the elastic scattering case, E' is fixed and

$$q^2 = -2p \cdot q$$

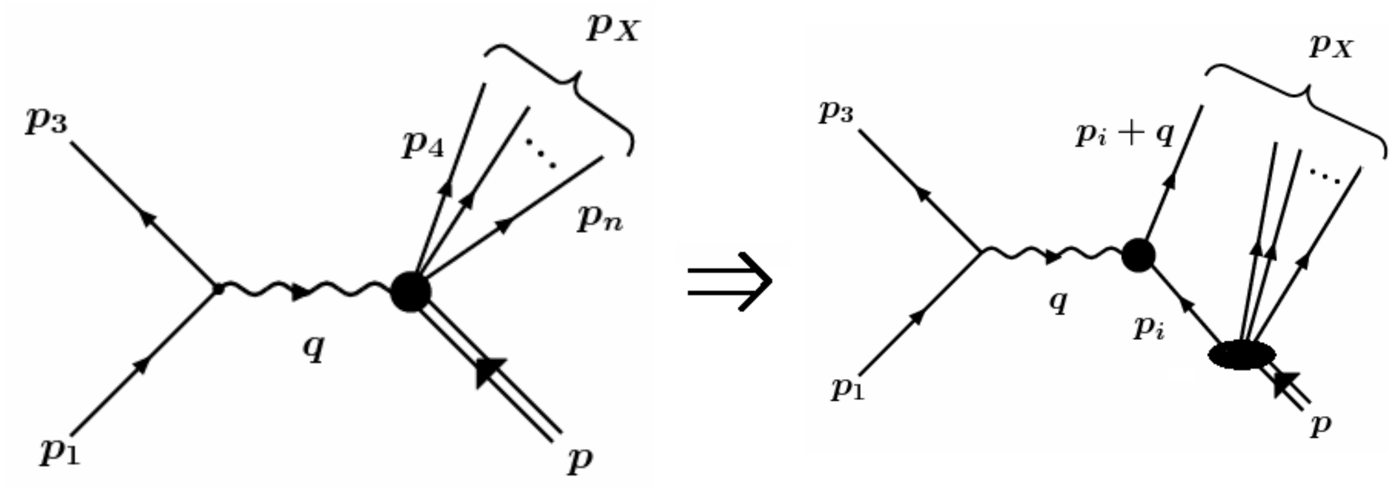
so that

$$x = 1.$$

Thus W_1 and W_2 are related to K_1 and K_2 as follows,

$$W_{1,2}(q^2, x) = -\frac{K_{1,2}(q^2)}{2Mq^2}\delta(x-1).$$

Parton Model



$$W_{1,2}(q^2, x) = \sum_i f_i(z_i) w^i_{1,2}(q^2, z_i), \text{ where } p_i = z_i p.$$

point-like structure functions $w^i_{1,2}(q^2, z_i)$

$$w^i_1(q^2, z_i) = \frac{Q_i^2}{2m_i}\delta(x_i - 1)$$

$$w^i_2(q^2, z_i) = -\frac{2m_i c^2 Q_i^2}{q^2}\delta(x_i - 1),$$

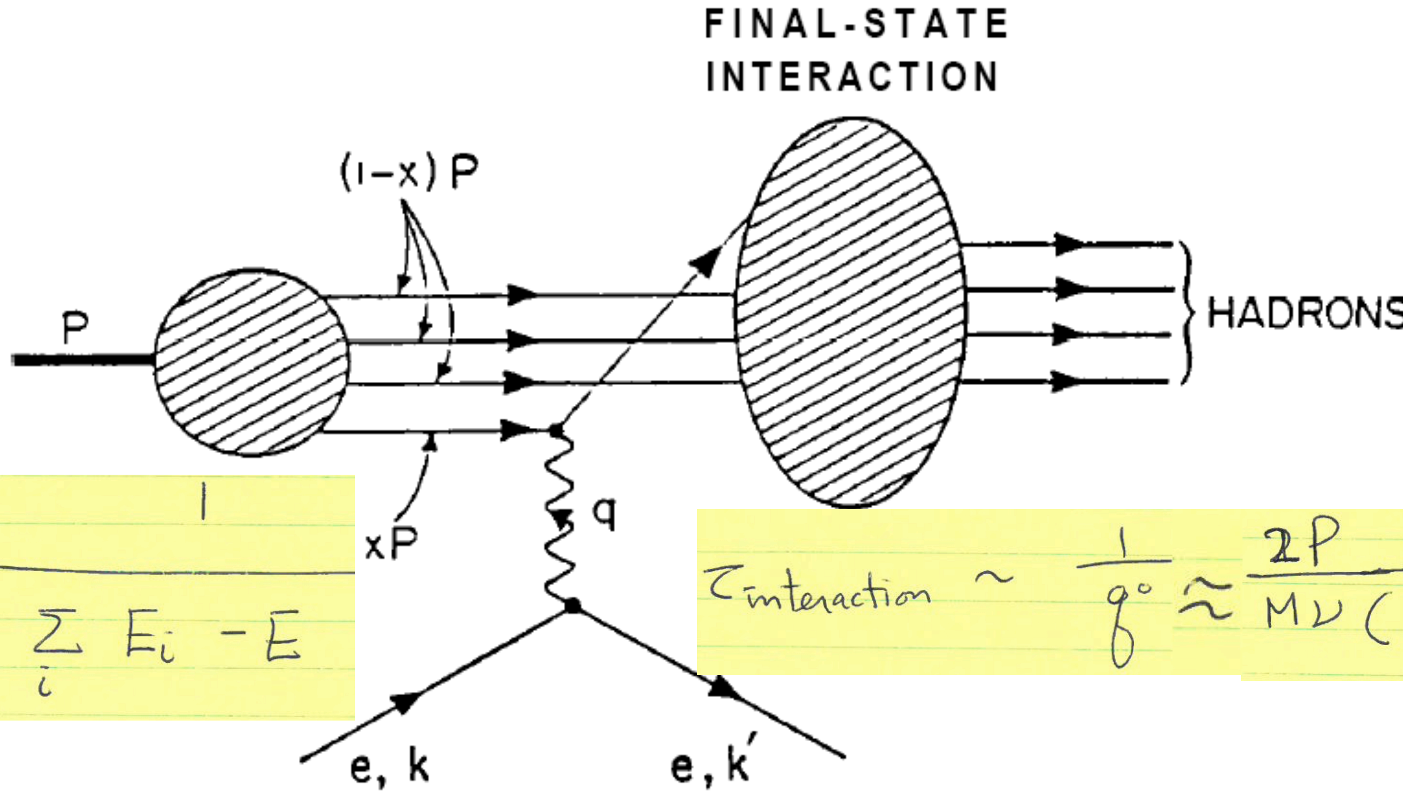
where Q_i is the charge of *i'th* parton,

$$m_i = z_i M$$

and

$$x_i = -\frac{q^2}{2p_i \cdot q} = \frac{x}{z_i}.$$

Validity of Parton Model



$\tau_{\text{lifetime of free partons}} \sim \dfrac{1}{\sum_i E_i - E}$

$\tau_{\text{interaction}} \sim \dfrac{1}{q^0} \simeq \dfrac{2P}{M\nu\left(1+\frac{q^2}{2M\nu}\right)}$

If $2M\nu, -q^2 \gg M^2$ and $\dfrac{-q^2}{2M\nu} \neq 1$ (end point singularity)

$\tau_{life} \gg \tau_{int}$

Structure Functions in Parton Model

$$w_1^i(q^2, z_i) = \frac{Q_i^2}{2M}\delta(x - z_i)$$

$$w_2^i(q^2, z_i) = -\frac{2xMc^2}{q^2} Q_i^2 \delta(x - z_i)$$

$$W_1 = \sum_i \int_0^1 \frac{Q_i^2}{2M}\delta(x - z_i) f_i(z_i) dz_i$$

$$= \frac{1}{2M}\sum_i Q_i^2 f_i(x)$$

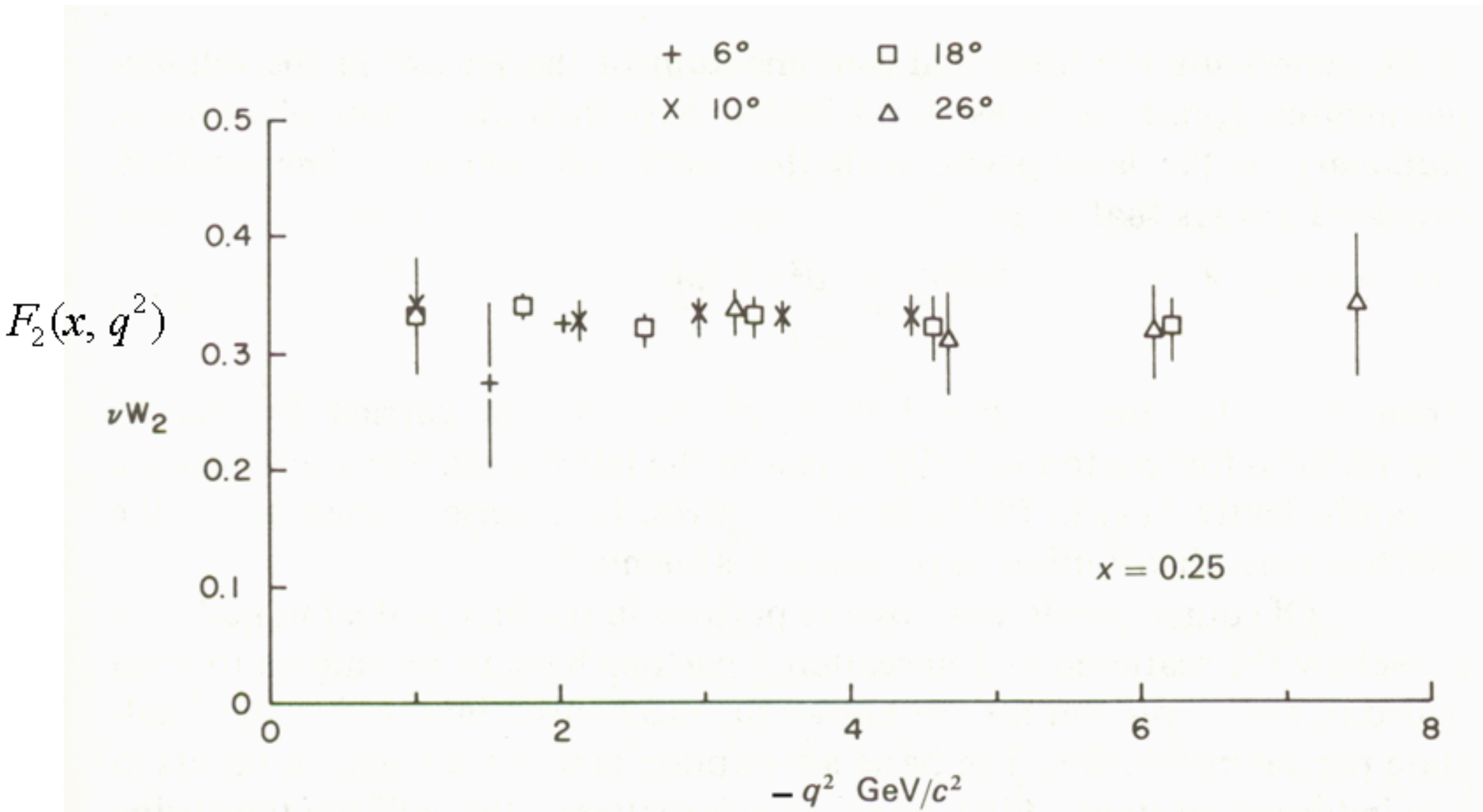
$$W_2 = \sum_i \int_0^1 \left(-\frac{2x^2Mc^2}{q^2}\right) Q_i^2 \delta(x - z_i) f_i(z_i) dz_i$$

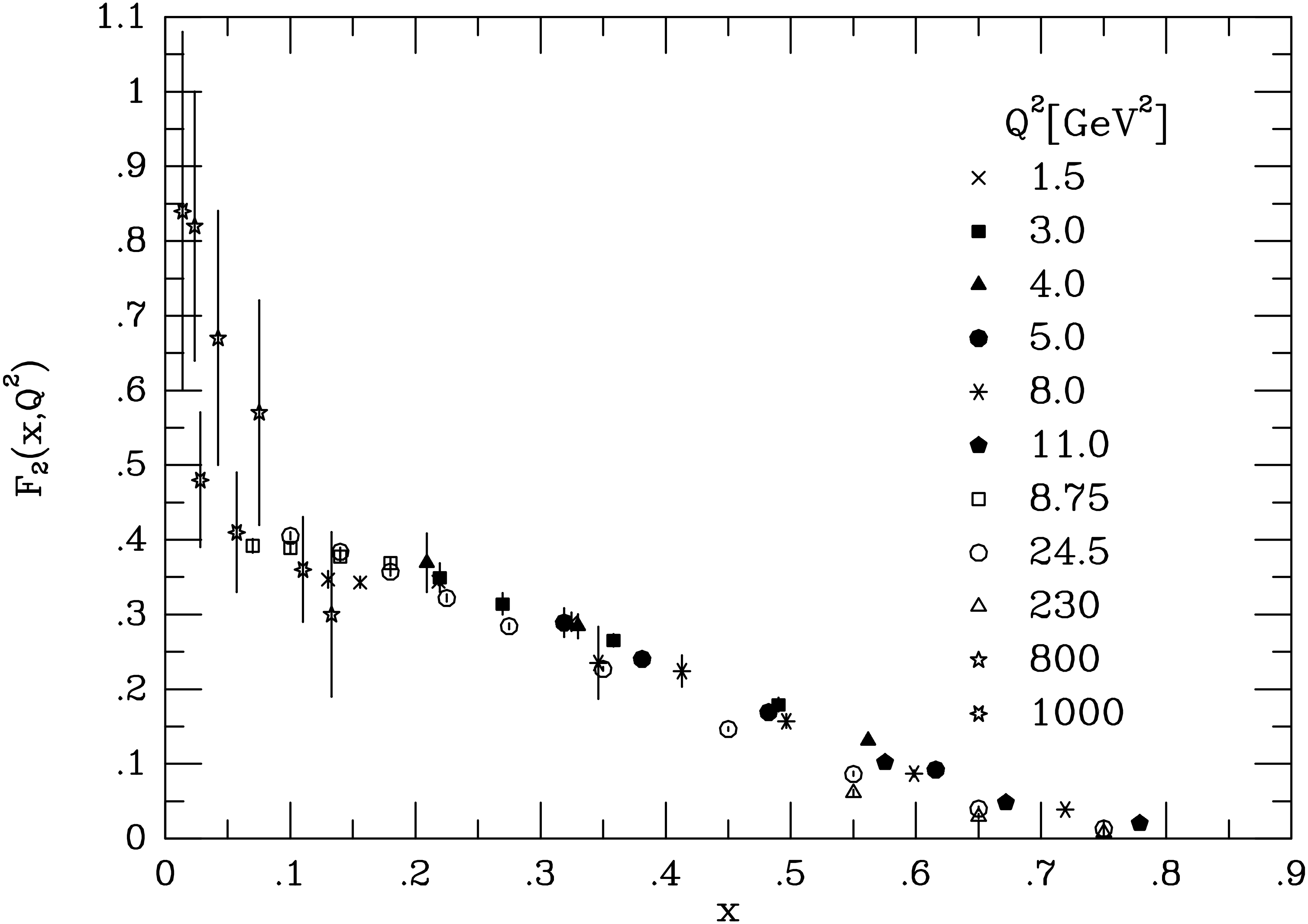
$$= -\frac{2Mc^2}{q^2} x^2 \sum_i Q_i^2 f_i(x).$$

Scaling of Structure Functions in Parton Model

$$F_1(x,q^2) = MW_1 = \frac{1}{2}\sum_i Q_i^2 f_i(x)$$

$$F_2(x,q^2) = -\frac{q^2}{2Mc^2x}W_2 = x\sum_i Q_i^2 f_i(x)$$





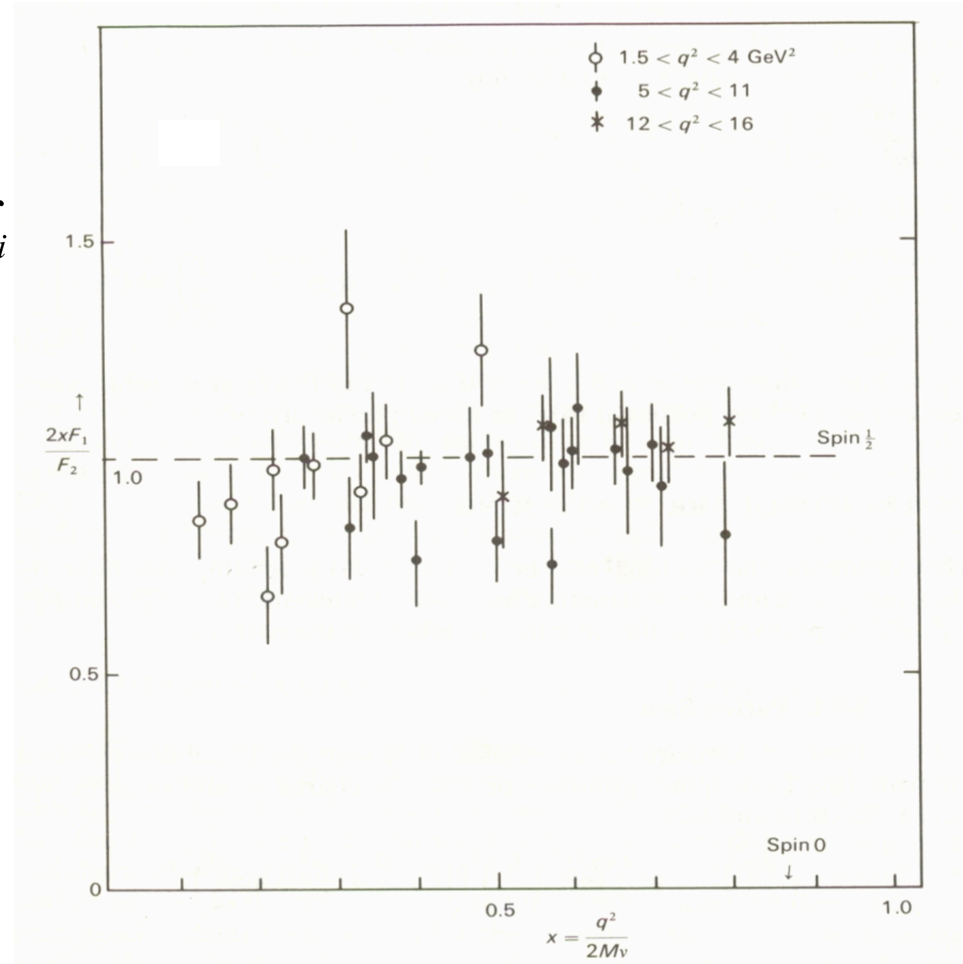
1.1
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
$F_2(x,Q^2)$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
x
Q^2[GeV^2]
1.5
3.0
4.0
5.0
8.0
11.0
8.75
24.5
230
800
1000

The scaling behavior of
the structure functions indicate
that the proton is made of
the more fundamental point-like
object, parton or quark.

Callen-Gross Relation

$$F_1(x,q^2) = MW_1 = \frac{1}{2}\sum_i Q_i^2 f_i(x)$$

$$F_2(x,q^2) = -\frac{q^2}{2Mc^2x}W_2 = x\sum_i Q_i^2 f_i$$



$$F_2(x) = 2xF_1(x)$$

If the partons were spin 0 particles, we would have

$$W_i^{\mu\nu} \propto (2x_F P^\mu + q^\mu)(2x_F P^\nu + q^\nu)$$

and it is easy to check that this leads to $F_1 = 0$ ($\sigma_{\text{transverse}} = 0$)

$$F_2(x,q^2) = -\frac{q^2}{2Mc^2x}W_2 = x\sum_i Q_i^2 f_i(x)$$

$$F_2^p(x) = x\left\{\left(\frac{2}{3}\right)^2 u(x) + \left(-\frac{1}{3}\right)^2 d(x)\right\},$$

where *u(x)* and *d(x)* are the probability functions of *u* and *d* quarks.

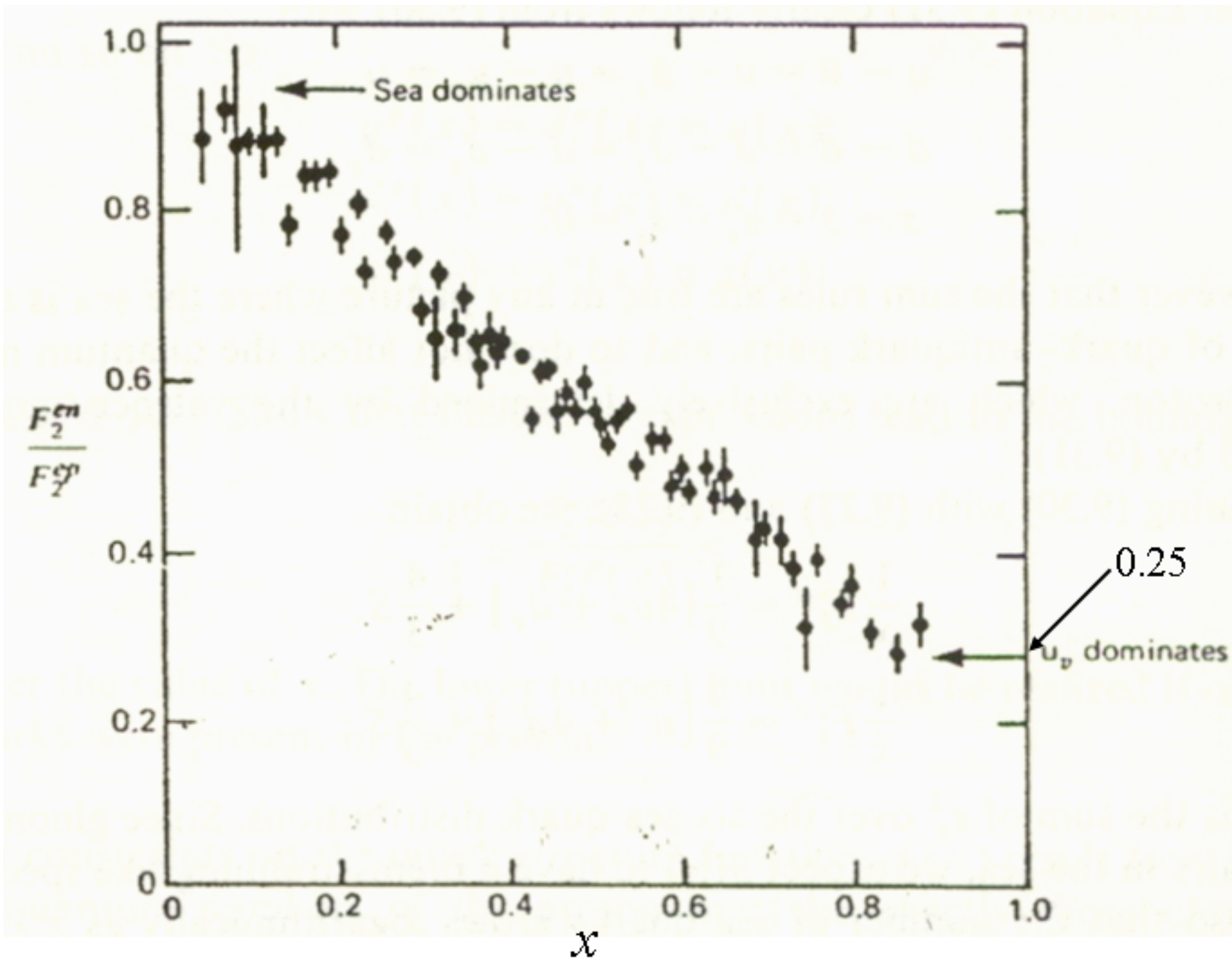
As the proton has twice more u quarks than d quark, one may predict that u(x) dominates over d(x) as x → 1.

$$F_2^p(x) \xrightarrow[x\to 1]{} \left(\frac{2}{3}\right)^2 u(x)$$

Due to the isospin symmetry, i.e. $u \Leftrightarrow d$ when $p \Leftrightarrow n$,

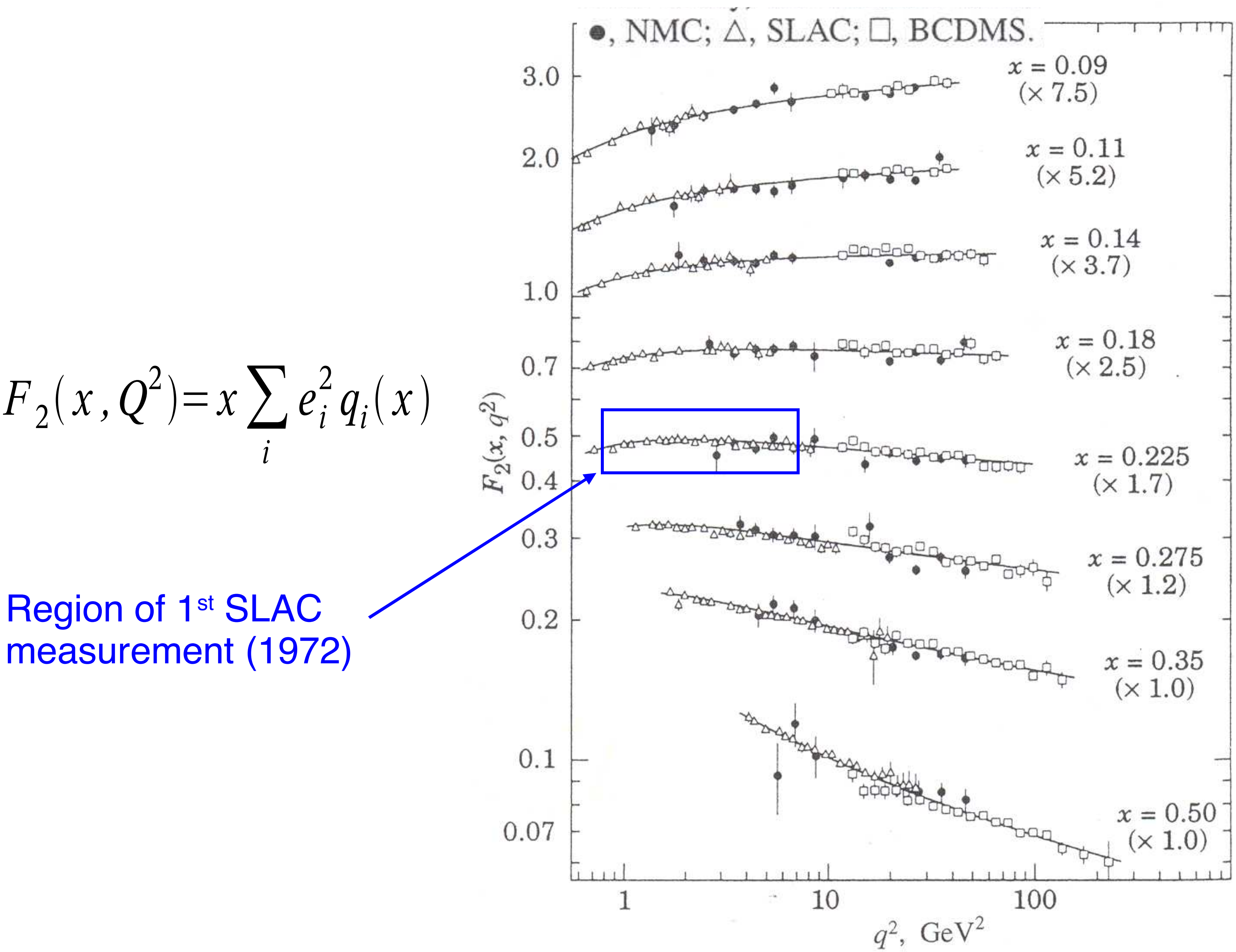
$$F_2^n(x) \xrightarrow[x\to 1]{} \left(-\frac{1}{3}\right)^2 u(x)$$

$$\frac{F_2^n(x)}{F_2^p(x)} \xrightarrow[x\to 1]{} \frac{\left(-\frac{1}{3}\right)^2}{\left(\frac{2}{3}\right)^2} = \frac{1}{4}$$



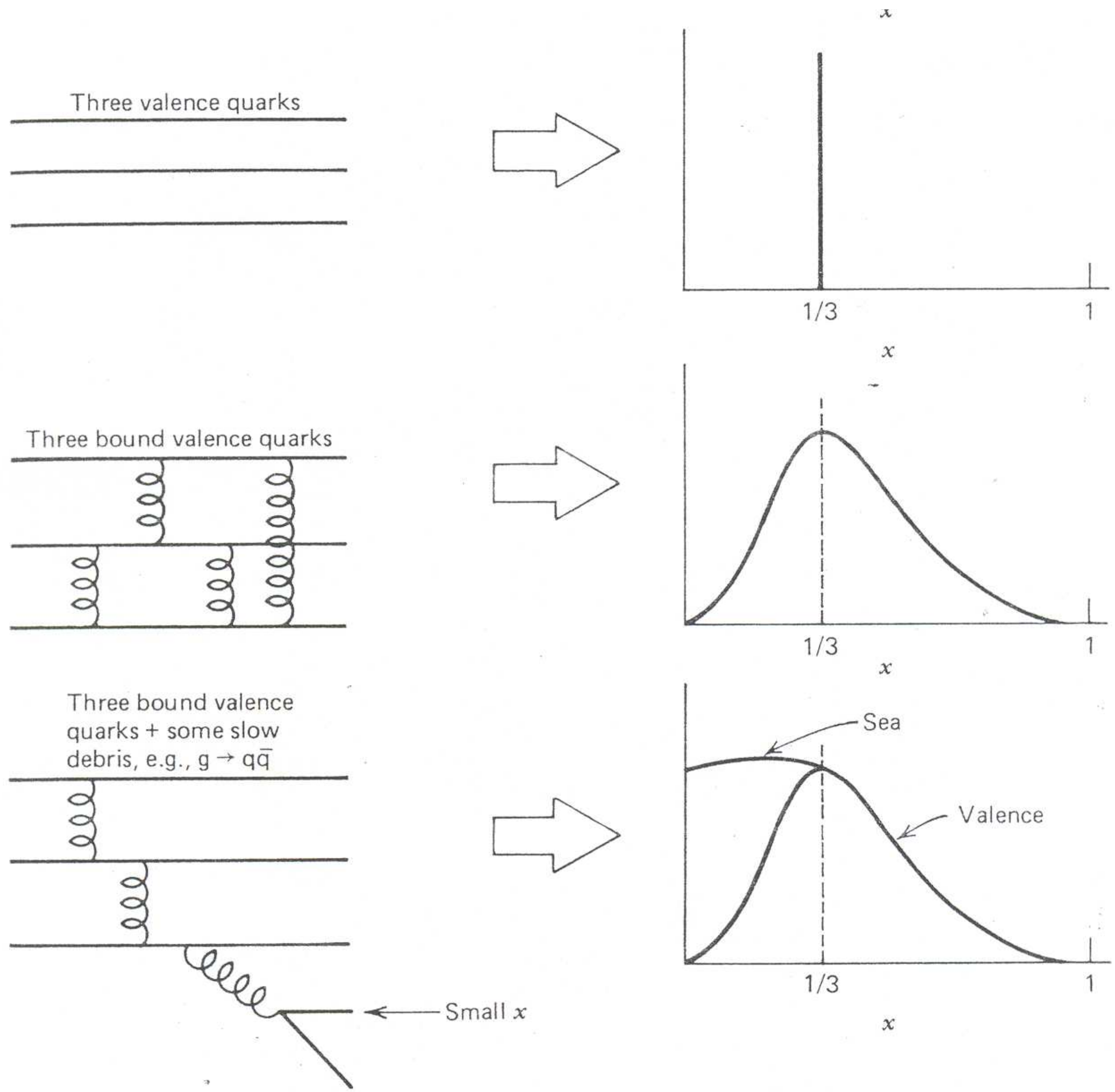
Scaling violation

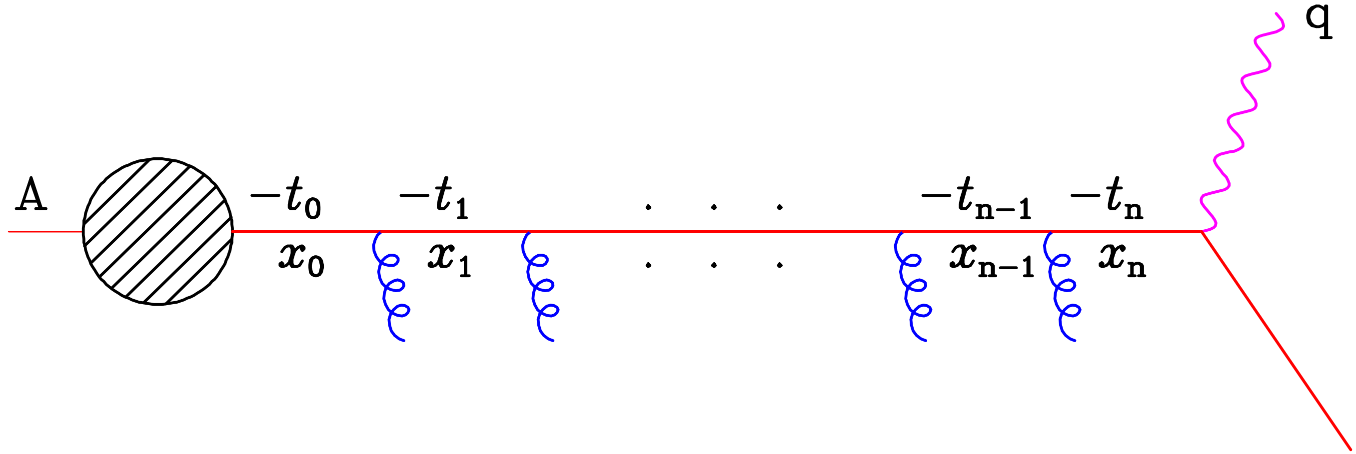
$$F_2 = F_2(x, Q^2) = x \sum_i e_i^2 q_i(x)$$



Proton model

$$F_2(x) = x \sum_i e_i^2 q_i(x)$$





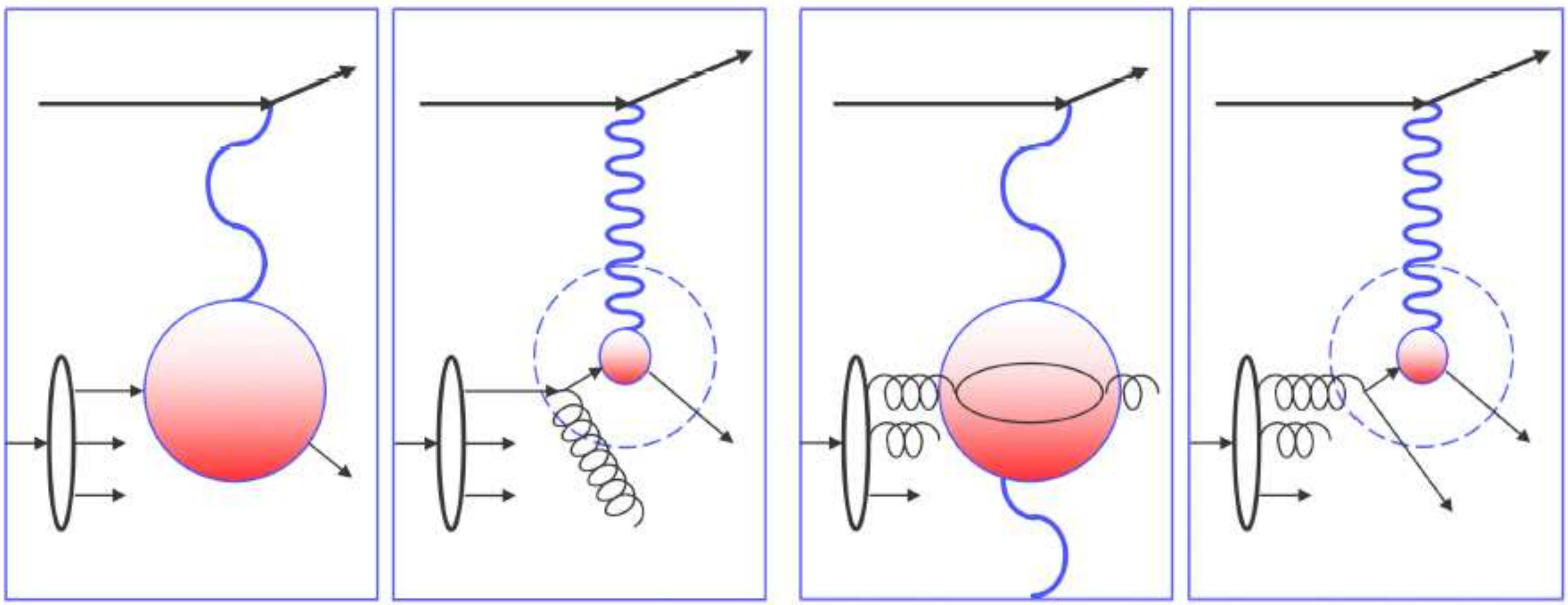
Incoming quark from target hadron, initially with low virtual mass-squared $-t_0$ and carrying a fraction x_0 of hadron's momentum, moves to more virtual masses and lower momentum fractions by successive small-angle emissions, and is finally struck by photon of virtual mass-squared $q^2 = -Q^2$.

Cross section will depend on Q^2 and on momentum fraction distribution of partons seen by virtual photon at this scale

QCD explains observed scaling violation

Large x: valence quarks

Small x: Gluons, sea quarks

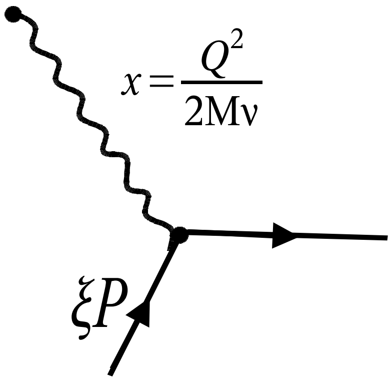


$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x

$Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed x

Quantitative description of scaling violation

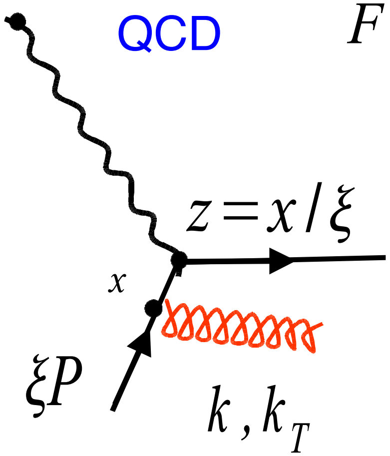
Quark Parton Model

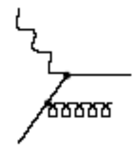


$$F_2(x) = x \sum_i e_i^2 \int_0^1 q_i(\xi) \cdot \delta(x-\xi)\, d\xi = x \sum_i e_i^2 q_i(x)$$

The photon "catches" a quark with the "right" x

QCD





$$F_2(x,Q^2) = x \sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} q_i(\xi) \cdot \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\mu_0^2} \right]$$

$$\hat{\sigma}(\gamma^* q \to qg) \sim \frac{\alpha_s}{2\pi} P_{qq}(z) \int_{\mu_0^2}^{Q^2} \frac{dk_T^2}{k_T^2}$$

$$\sim \frac{\alpha_s}{2\pi} P_{qq}(z) \log\left(\frac{Q^2}{\mu_0^2}\right)$$

P_{qq} - probability of a quark to emit a gluon and thus to become a quark with momentum reduced by fraction z.

μ^0 cutoff parameter

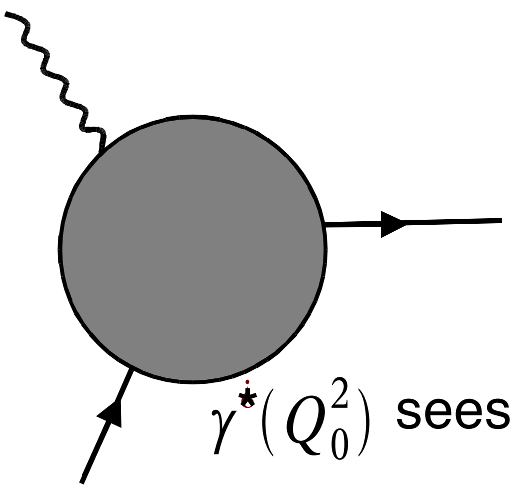
Changing to the quark densities:

$$q_i(x,Q^2)=q_i(x)+\underbrace{\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_0^2}\int_0^1\frac{d\xi}{\xi}q_i(\xi)\,P_{qq}\left(\frac{x}{\xi}\right)}_{\Delta q(x,Q^2)}$$

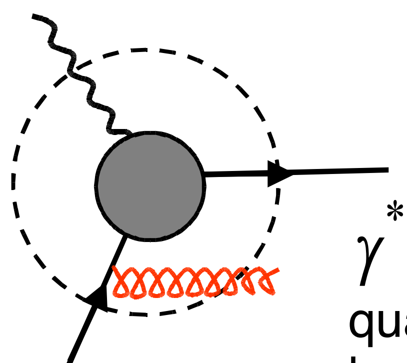
Integro-differential equation for $q(x,Q^2)$:

$$\frac{d}{d\log Q^2}q(x,Q^2)=\frac{\alpha_s}{2\pi}\int_0^1\frac{d\xi}{\xi}q(\xi,Q^2)\,P_{qq}\left(\frac{x}{\xi}\right)$$

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **A**ltarelli, **P**arisi, 1972 – 1977)



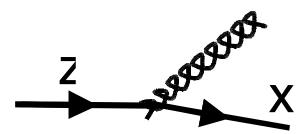
$\gamma^*(Q_0^2)$ sees q(x)

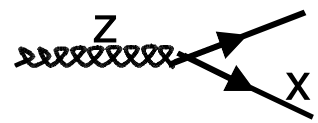


$\gamma^*(Q^2>Q_0^2)$ sees that quarks q(x) are surrounded by softer quarks

Evolution of parton densities (quarks and gluons)

evolution of quark density with $\ln Q^2$

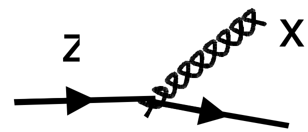


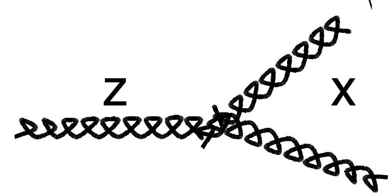


$$\underbrace{\frac{\partial q(x,Q^2)}{\partial \ln Q^2}}= \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)\,P_{qq}\left(\frac{x}{z}\right)+g(z,Q^2)\,P_{qg}\left(\frac{x}{z}\right)\right]$$

$$\underbrace{\frac{\partial g(x,Q^2)}{\partial \ln Q^2}}= \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)\,P_{gq}\left(\frac{x}{z}\right)+g(z,Q^2)\,P_{gg}\left(\frac{x}{z}\right)\right]$$

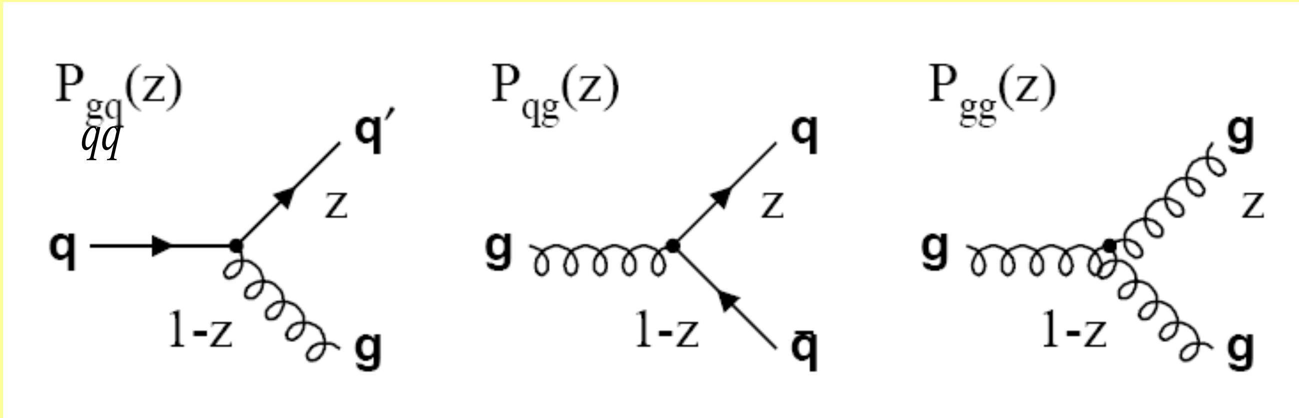
evolution of gluon density with $\ln Q^2$





Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction $\varepsilon = x/z$ of the parent parton.

Splitting functions are calculated as power series in α_s up to a given order:



$$P_{ij}(z,\alpha_s)=P_{ij}^0(z)+\frac{\alpha_s}{2\pi}P_{ij}^1(z)+\ldots$$

In leading order: $P_{ij}(z,\alpha_s)\equiv P_{ij}^0(z)$

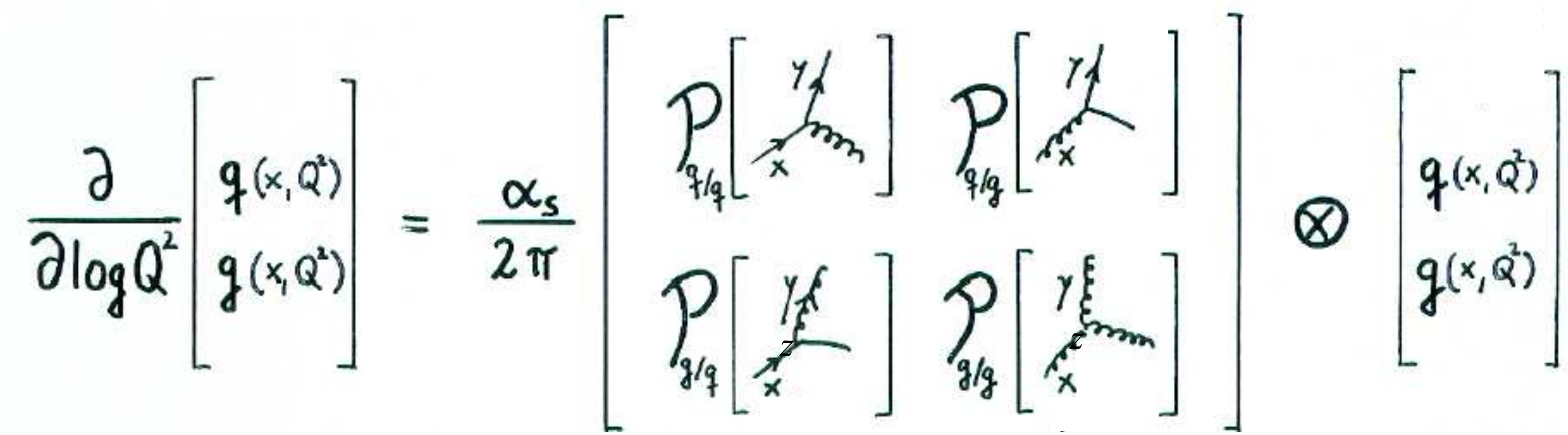
$$P_{qq}(z)=\frac{4}{3}\frac{1+z^2}{1-z}$$

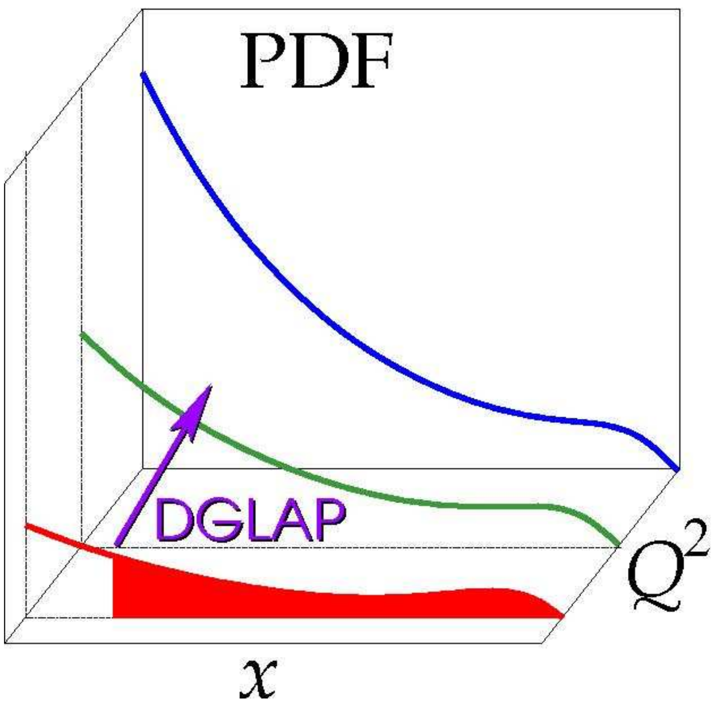
$$P_{gq}(z)=\frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{qg}(z)=\frac{z^2+(1-z)^2}{2}$$

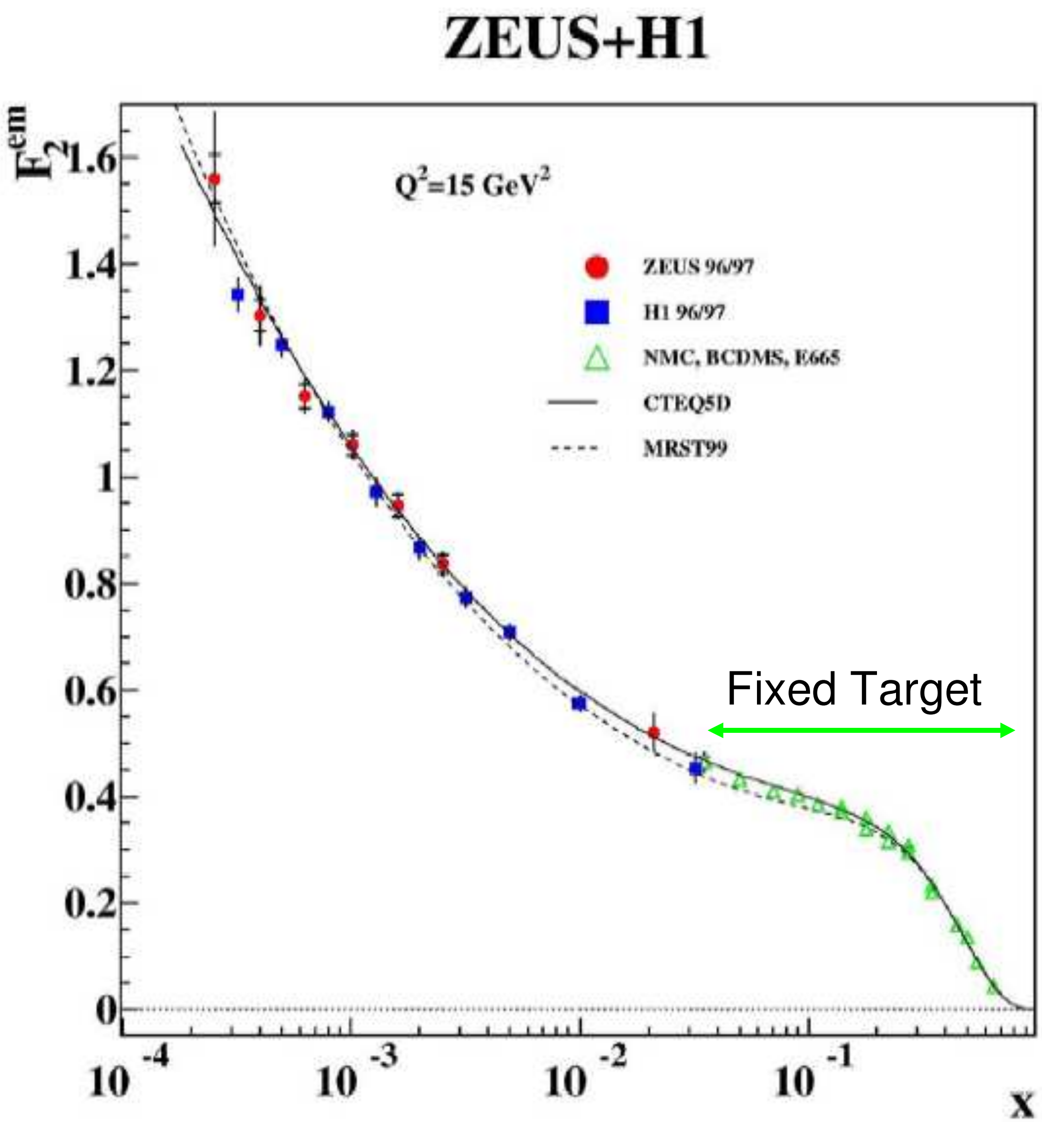
$$P_{gg}(z)=6\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)$$

DGLAP Evolution ("symbolic"):



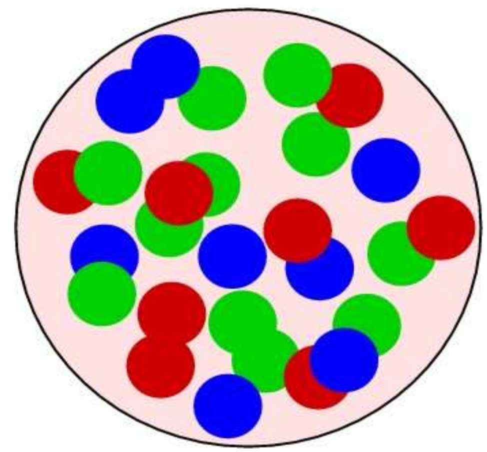


$$P \otimes f(x, Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z, Q^2)$$



Infinite rise will violate unitarity limit

At low x gluons should start to “overlap”

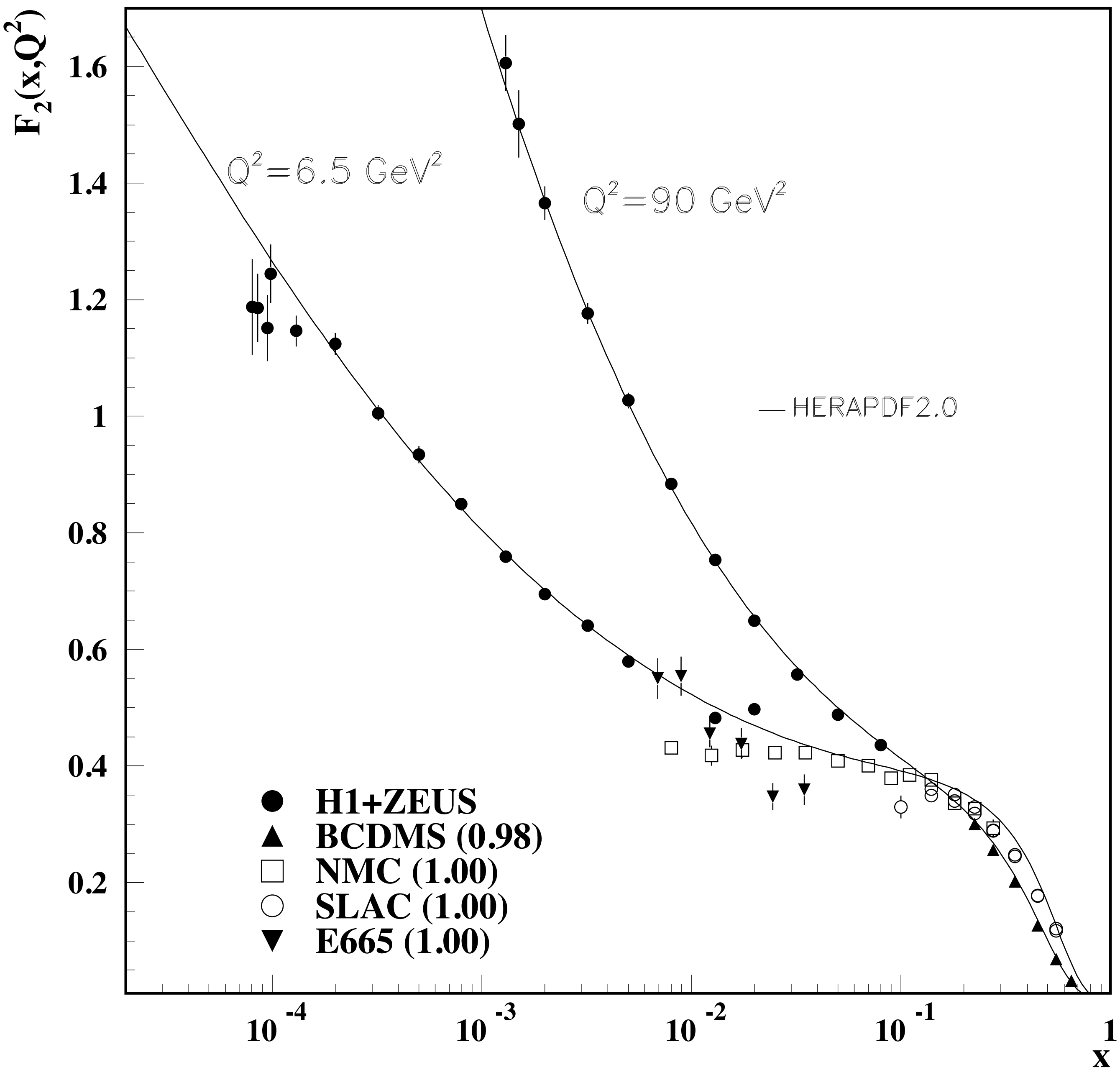


Expect to see slowing rise – deviation from $x^{-\lambda}$ dependence

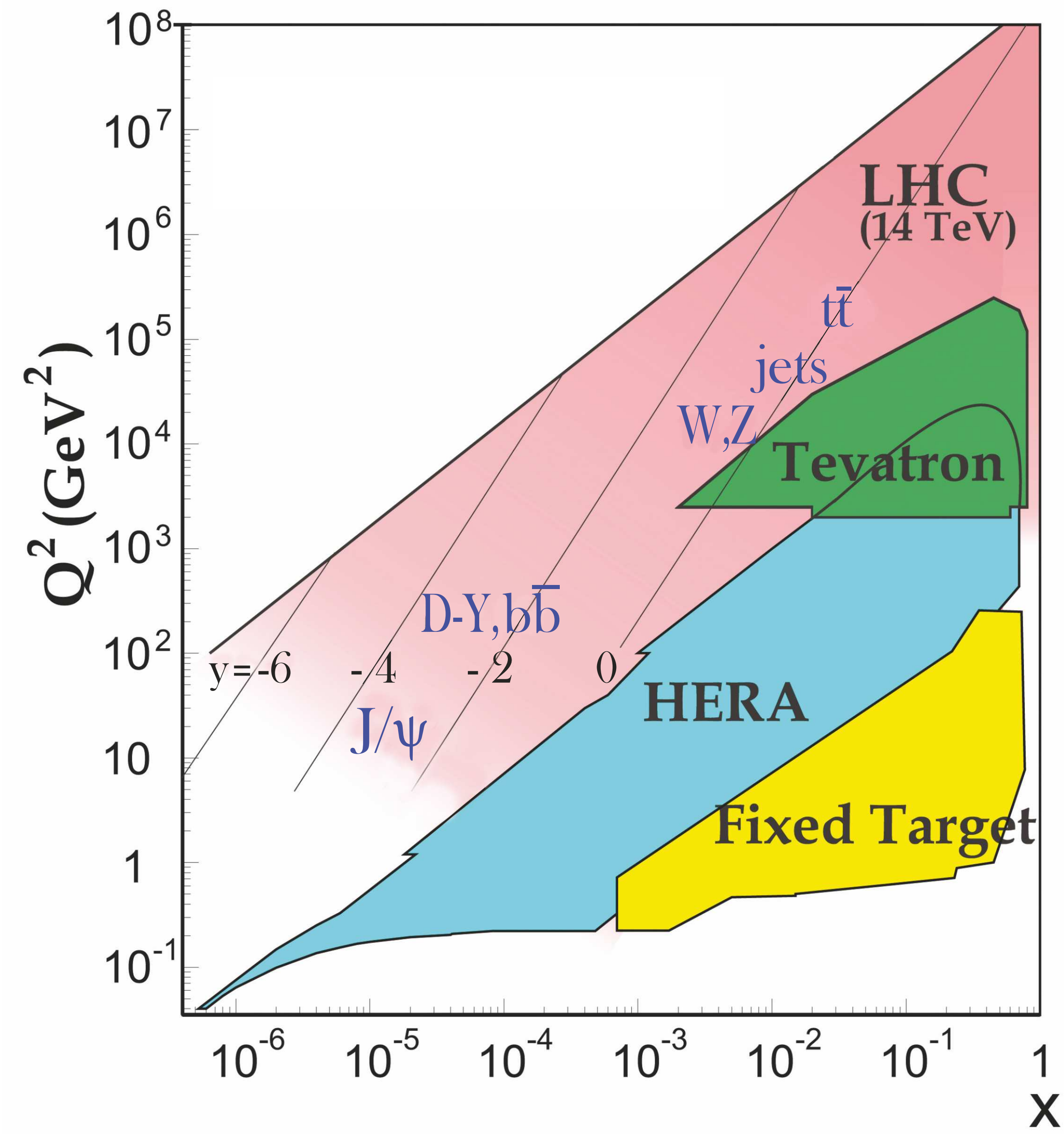
Look at very low x

No deviations observed so far

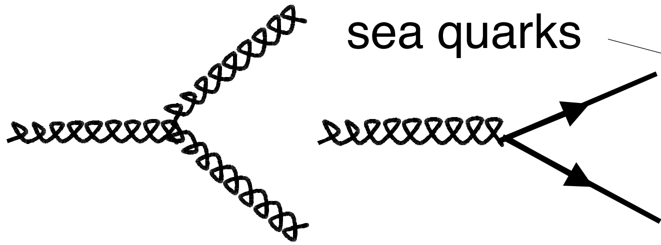
Most Updated $F_2(x,Q^2)$ of Proton from PDG



Experimental Kinematic Domain in x and Q^2



sea quarks

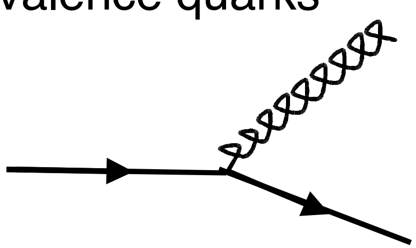


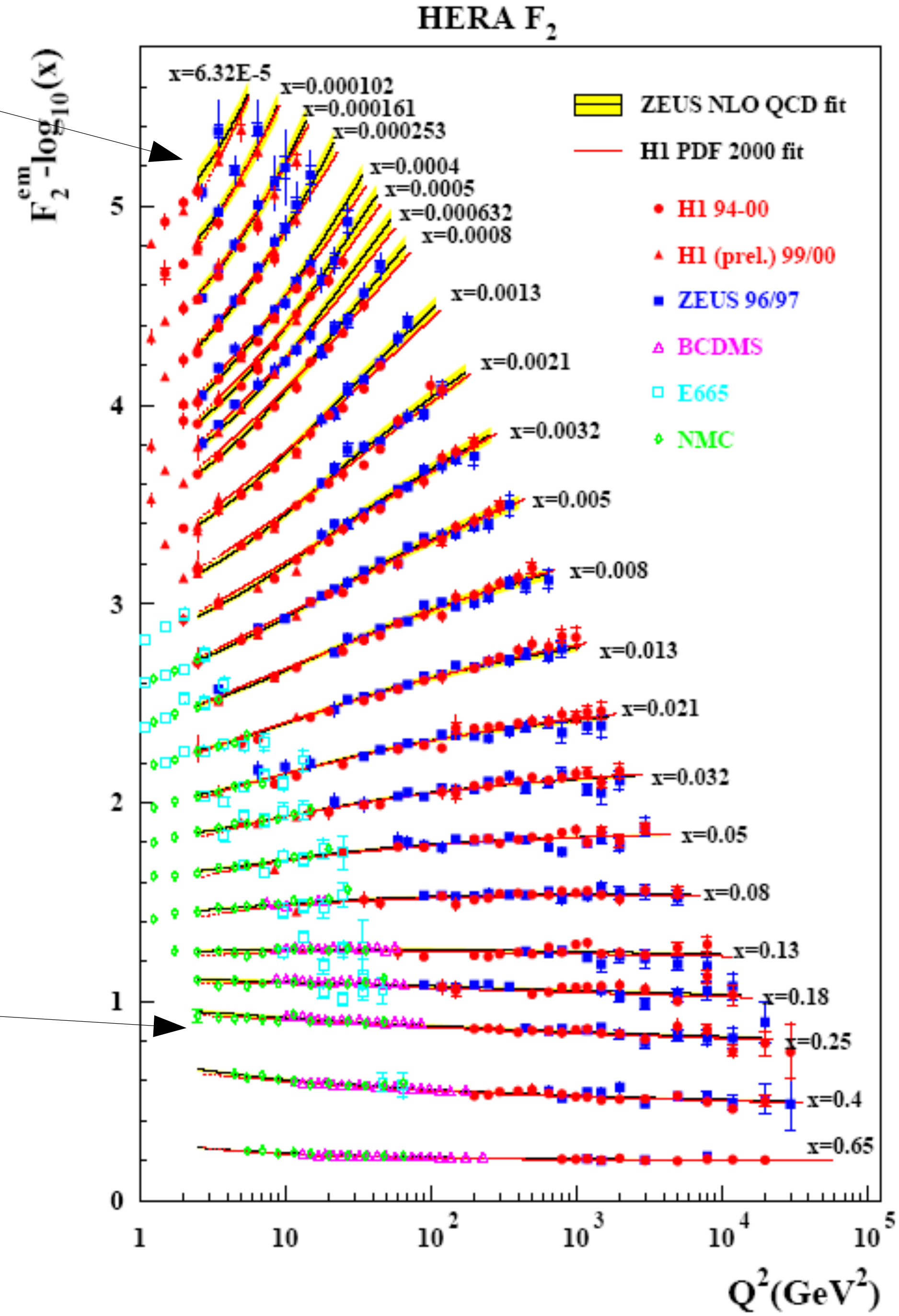
5 orders of magnitude
in x and Q²

$$F_2(x, Q^2)$$

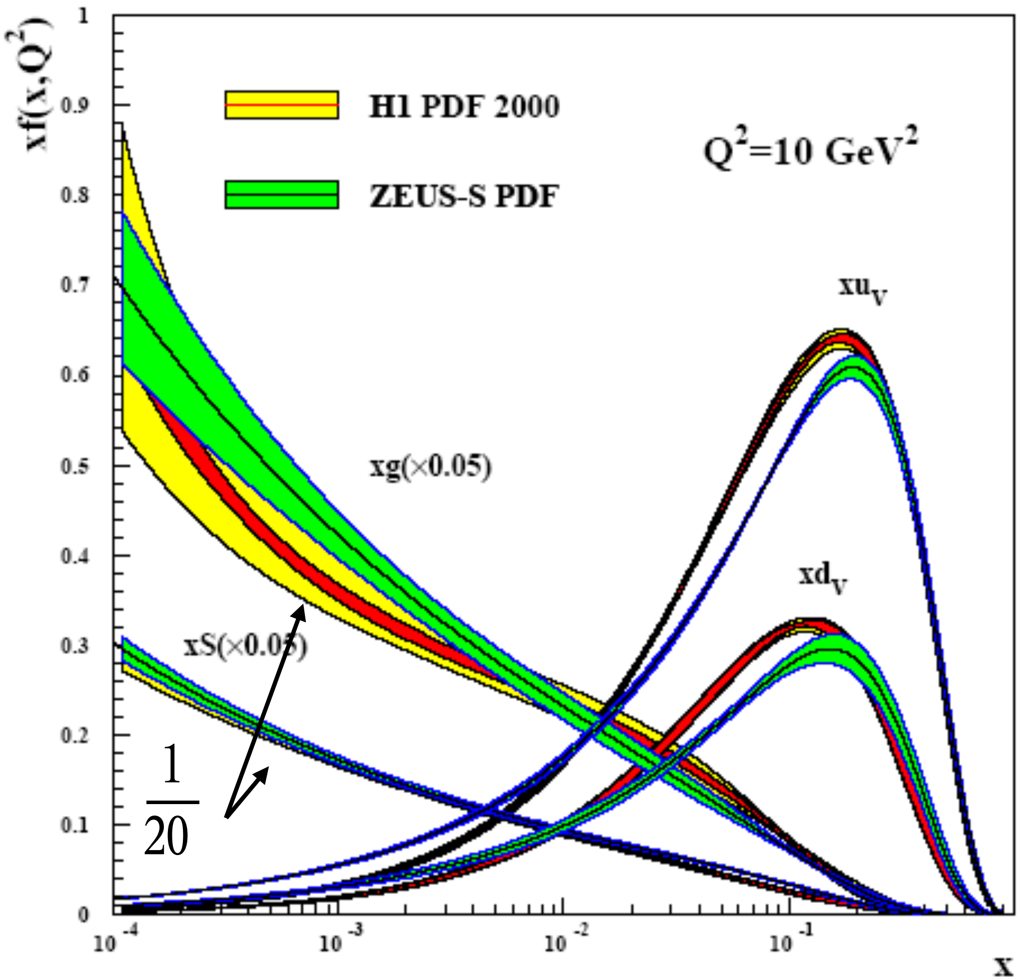
described by QCD evolution

valence quarks



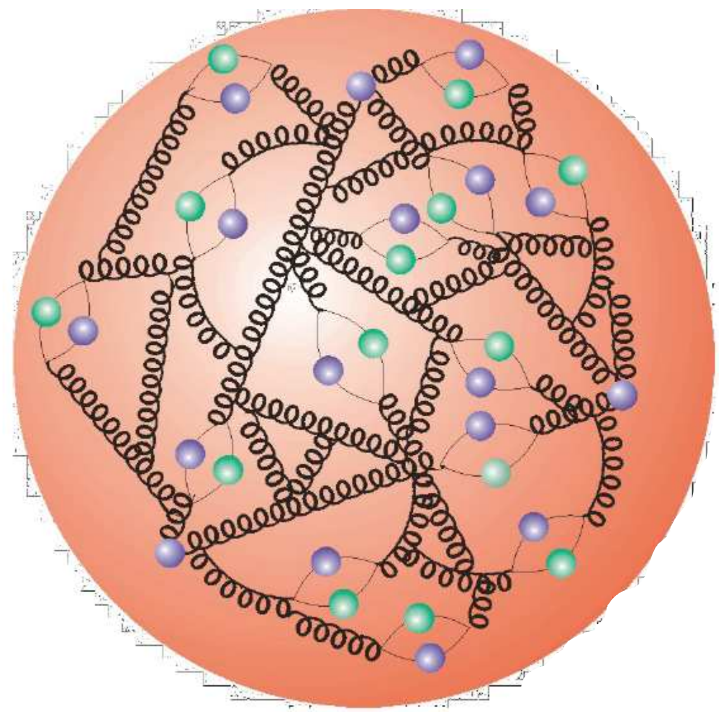


Structure of the proton as seen by HERA



$$\#\ \text{Valenzquarks} = \int u_v(x) + d_v(x)\, dx = 3$$

$$\#\ \text{Gluonen} = \int g(x)\, dx > 30$$



PDF fits

Many options - uncertainties:

- Which datasets? [HERA only? Also some fixed target? Also pp data?]
- Which order of perturbation theory [LO, NLO, NNLO]?
- Form of parameterization q(x), g(x) [How many parameters?]

$$x\,p(x,Q^2) = A_p x^{a_p} (1-x)^{b_p} P(x, c_p, \ldots)$$

characterizes at x -> 0
sea: a < 0, valence a>0

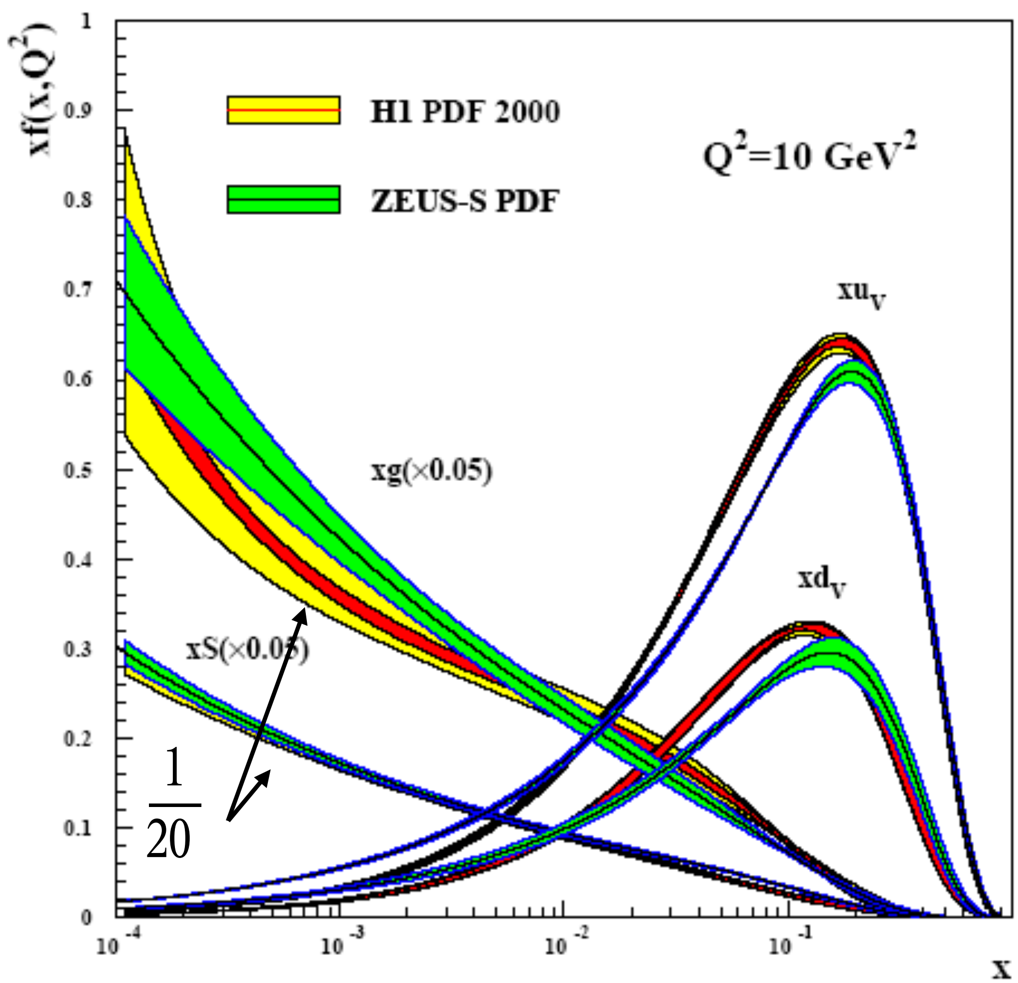
characterizes at x -> 1
always b > 0

“fine tuning”
weakly x-dependent function

- Which PDFs? For each flavour? Some combination?
- Pure DGLAP or some extention/alternative?
- Start-up scale Q^2
- Sum rules
- Heavy quark treatment [What to do with c(x), b(x) at low Q^2?]

H1 and ZEUS do their own fits based mostly on their own data.
Theor. groups (e.g. CTEQ, MRST/MSTW,...) do combined fits of many datasets

Current knowledge of PDFs



Uncertainties:

- *u*-density: ~3%
- *d*-density: ~10%
- *g*-density: 10-20% and more

u is better known than *d*
due to el. charge (squared):
$F_2 = x(8/9\ u + 1/9\ d + ...)$

gluon is known worse,
as it is determined from
scaling violations (derivatives)

Most Updated PDF from PDG