Nuclear Physics School 2018

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Second Lecture

Quantum Chromodynamics

June 28, 2018

Quantum Electrodynamics (QED)

$$\mathcal{L}_{\rm QED} = \overline{\psi}\,(i\,\not{D} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$(i\hbar\gamma^{\mu}\partial_{\mu} - \frac{e}{c}\gamma^{\mu}A_{\mu}(x) - mc)\psi(x) = 0 \qquad J^{\mu} = \overline{\psi}(x)\gamma^{\mu}\psi(x)$$

$$\varepsilon_{\mu\nu\alpha\beta}\partial^{\mu}F^{\alpha\beta} = 0,\ \partial_{\mu}F^{\mu\nu} = \frac{4\pi}{c}J^{\nu} \qquad F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

The invariance under local gauge transformation leads to the current conservation.

$$A_{\mu}(x) \rightarrow A'_{\mu}(x) = A_{\mu}(x) + \partial_{\mu}X(x) \quad \rightarrow \quad \partial_{\mu}J^{\mu} = 0$$

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\rm QCD} = \sum_{i,j=1}^{3}\overline{q_i}\,(i\,\not{D} - m_q)_{ij}\,q_j - \frac{1}{4}\sum_{\alpha=1}^{8}G^{\alpha}_{\mu\nu}G^{\alpha,\mu\nu}$$

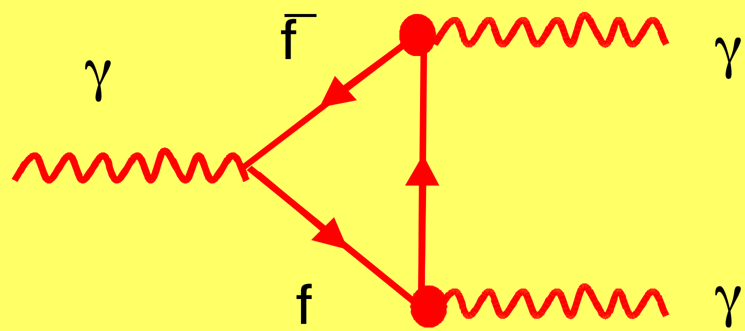
Evidences of $N_C = 3$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$$ indicates fractional charges and N_C=

Δ^{++} (Ω_s) statistic problem: $|\Delta^{++}\rangle = \frac{1}{\sqrt{6}} \varepsilon_{ijk} |u_i \uparrow u_j \uparrow u_k \uparrow\rangle$

Triangle anomaly

Divergent fermion loops



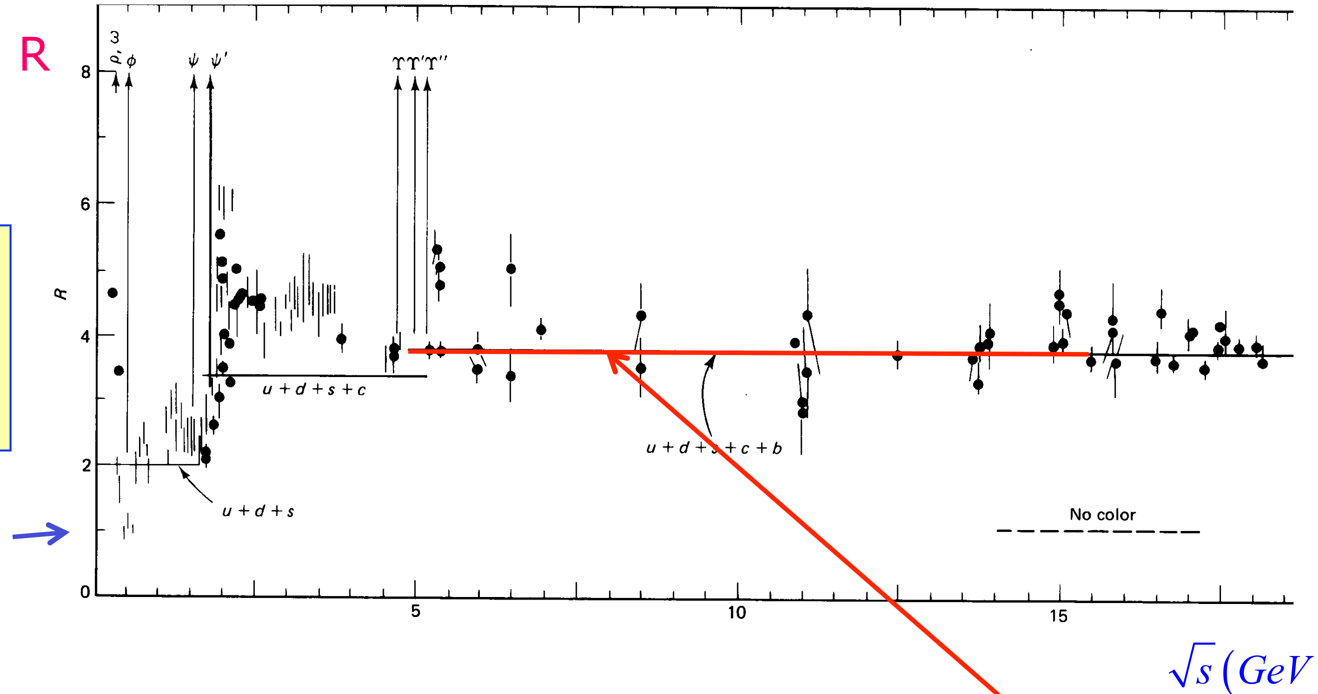
Divergence

$$\sim \sum_f Q_f = \underbrace{(-1)+(-1)+(-1)}_{\text{leptons}} + \underbrace{N_C \cdot \left[\left(\frac{2}{3} - \frac{1}{3}\right) \cdot 3 \right]}_{\text{quarks}}$$

3 generations of u/d-type quark

➡ cancels if $N_C = 3$

R is a function ofi √s. Above the threshold of b production we have:



Without colour: $R = \frac{11}{9}$

With the colour: $R = 3\cdot[(-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2] = \frac{33}{9} = 3.67$

Quantum Chromodynamics – SU(3) Theory

Lagrangian is constructed with quark wave functions $\psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$

Invariance of the Lagrangian under Local SU(3) Gauge Transformation

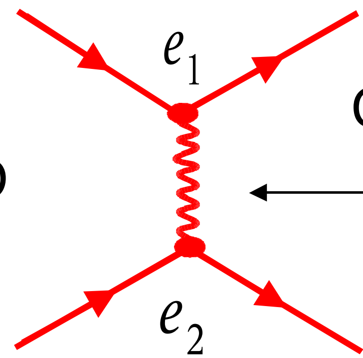
$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x) = e^{i\frac{\alpha_k(x)}{2}\lambda_k}\psi(x)$$

with any unitary (3 x 3) matrix $U(x)$.

$U(x)$ can be given by a linear combination of
8 Gell-Mann matrices $\lambda_1 ... \lambda_8$ [SU(3) group generators]

requires interaction fields – 8 gluons corresponding to these matrices

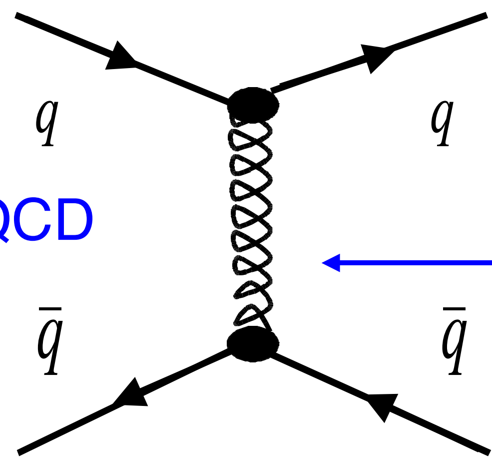
QED



Coupling strength

$e_1 e_2 \, \alpha$

QCD

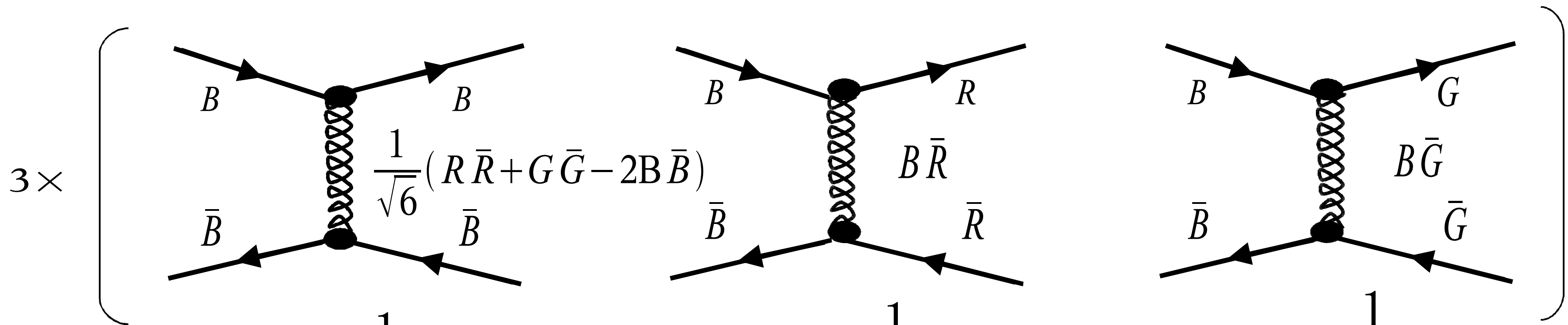


Coupling strength:

$$\underbrace{\frac{1}{2} c_1 c_2}_{C_F} \, \alpha_s$$

Color factor

Color factor for $q\bar{q}$ color singlet state (meson): $\frac{1}{\sqrt{3}}(R\bar{R}+G\bar{G}+B\bar{B})$



$C_F=\frac{1}{3}$ $C_F=\frac{1}{2}$ $C_F=\frac{1}{2}$

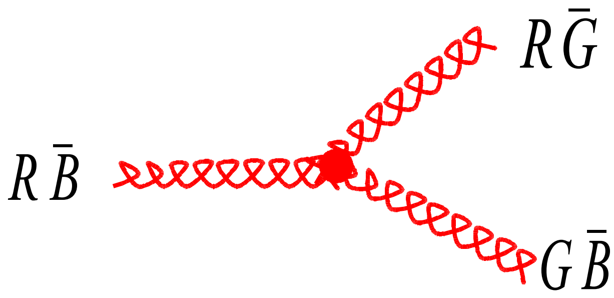
$$C_F=3\cdot\left(\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{2}\right)\right)=\frac{4}{3}$$

Color singlet meson is composed of 3 different possibilities

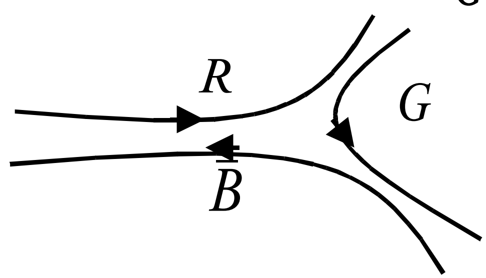
In the case of a color singlet, each initial and final state carries a factor $\frac{1}{\sqrt{3}}$

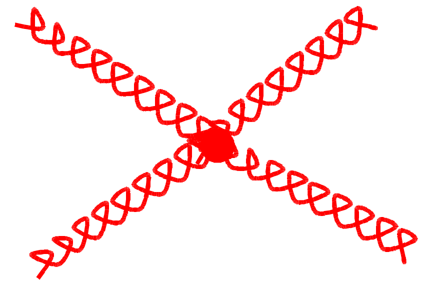
Triple and quadruple gluon Vertex

Gluons carry color charges:
important feature of SU(3)

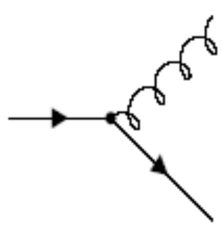


Color flow

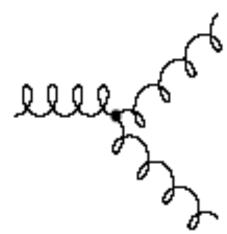




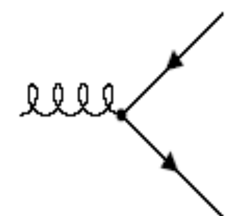
Color factors

$\left| \right.$  $\left. \right|^2 \sim C_F = \frac{4}{3}$

$q \rightarrow qg$

$\left| \right.$  $\left. \right|^2 \sim N_C = 3$

$g \rightarrow gg$

$\left| \right.$  $\left. \right|^2 \sim T_F = \frac{1}{2}$

$g \rightarrow q\bar{q}$

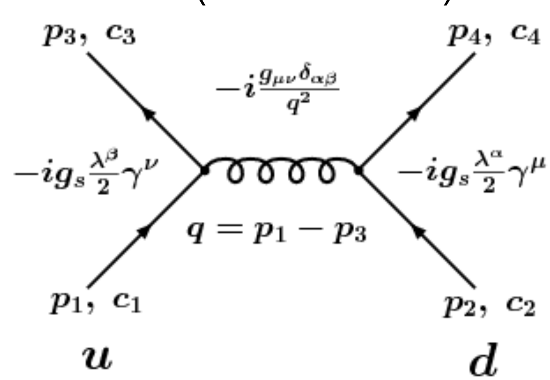
Homework: Compute the color factors between the two quarks and verify that the same colors repel and different colors attract each other.

Hint: Gell-Mann matrices in SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

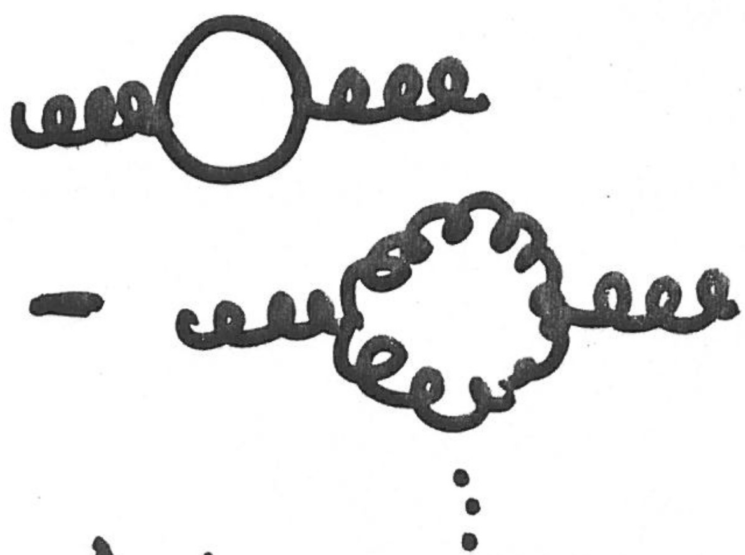


Renormalization of coupling const.

QED

Screening

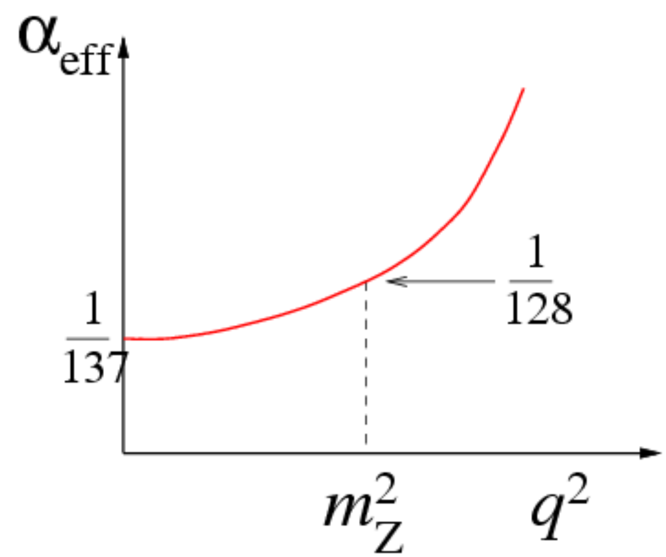
QCD



Anti screening

QED: Running coupling constant

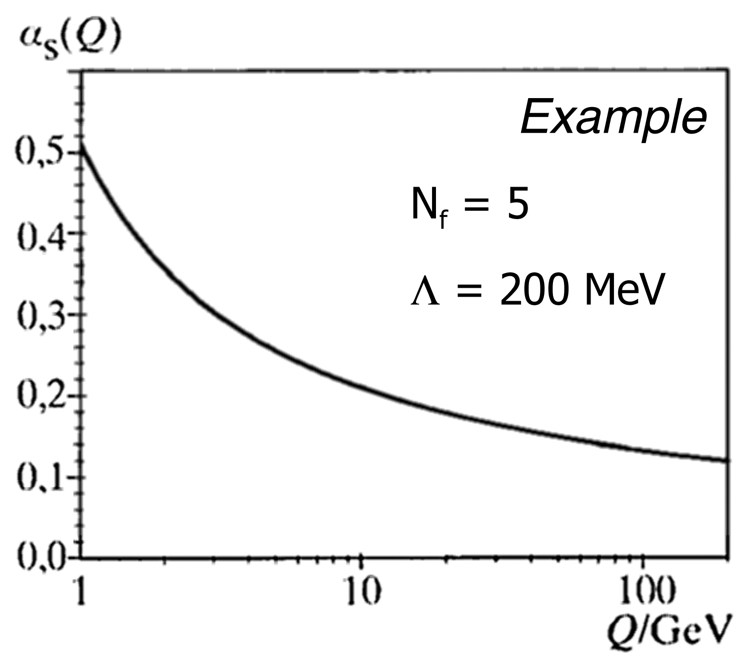
$$\alpha(q^2) = \frac{\alpha}{1 - \underbrace{\frac{\alpha}{3\pi} \cdot \sum_f Q_f^2 \cdot \log \frac{q^2}{m_f^2}}_{-\alpha\, \beta_0^{QED} \log \frac{Q^2}{\mu^2}}}$$



QCD Strong coupling constant α_s

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2) \underbrace{\frac{1}{12\pi}(33 - 2n_f)}_{} \log \frac{Q^2}{\mu^2}}$$

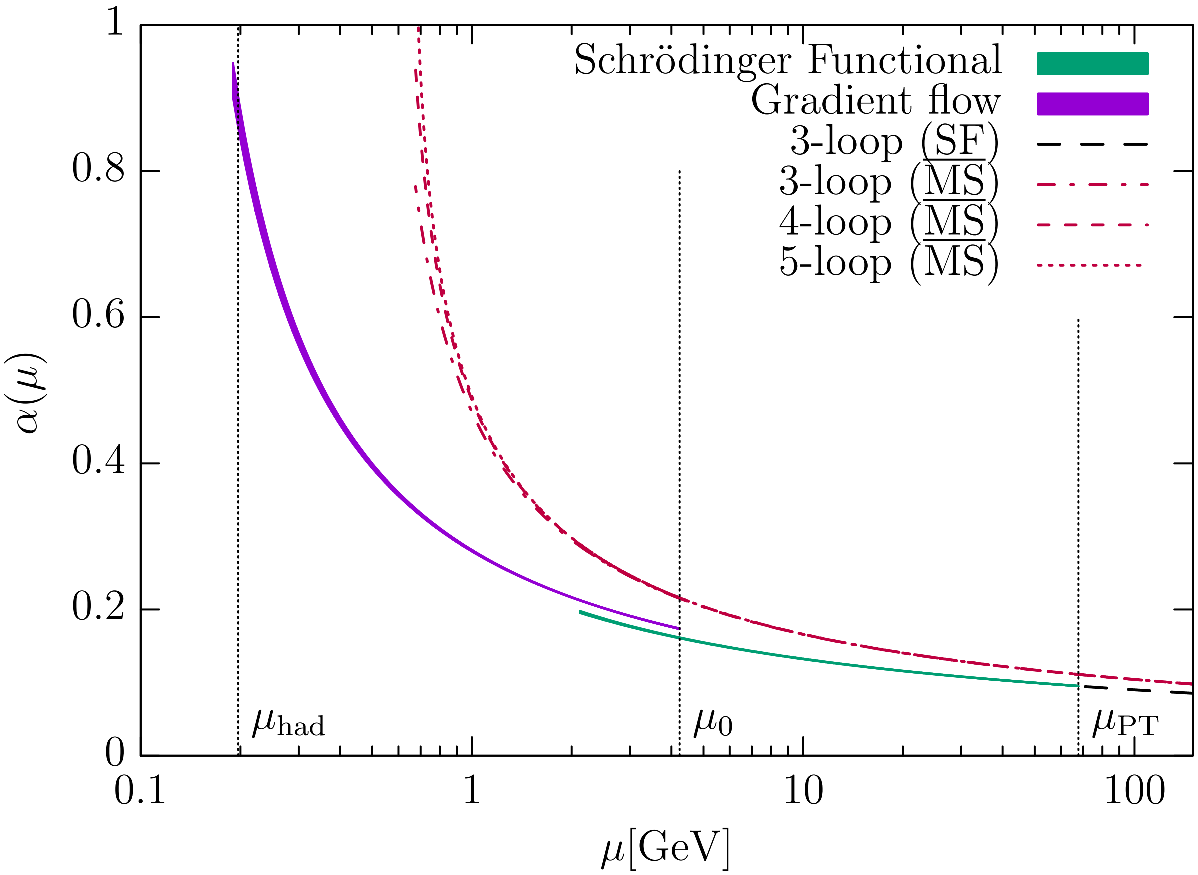
$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$



QCD Coupling from a Nonperturbative Determination of the Three-Flavor Λ Parameter

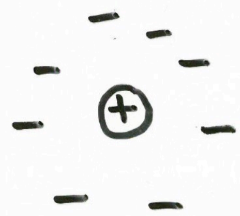
Mattia Bruno,[1] Mattia Dalla Brida,[2] Patrick Fritzsch,[3] Tomasz Korzec,[4] Alberto Ramos,[3] Stefan Schaefer,[5] Hubert Simma,[5] Stefan Sint,[6] and Rainer Sommer[5,7]

(ALPHA Collaboration)



$$\alpha^{(5)}_{\overline{\text{MS}}}(m_Z) = 0.11852(84)$$

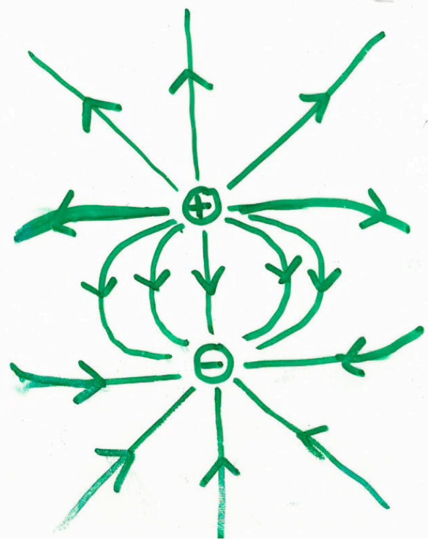
Phenomenological Interpretation of Antiscreening Effect.

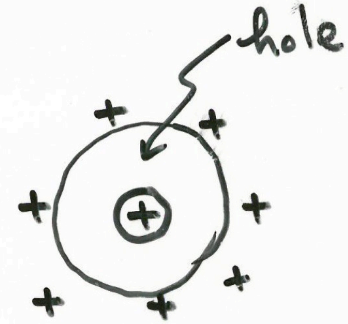


$$\vec{D} = \epsilon \vec{E}$$

$\epsilon > 1$

(Screening)



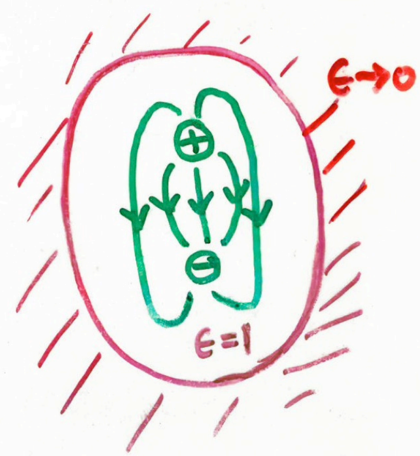


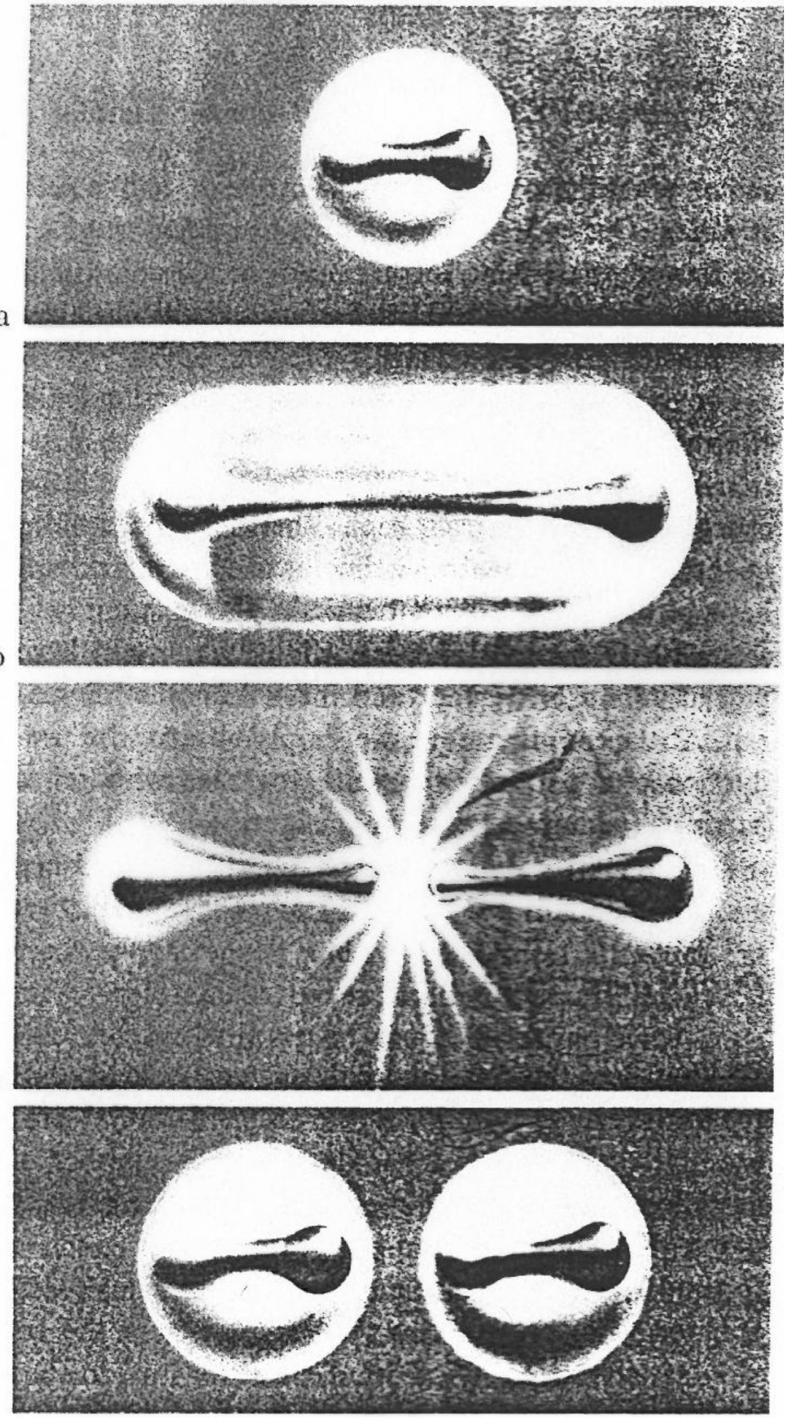
$\epsilon < 1$

(Antiscreening)

$\epsilon \to 0$

size of hole $\to \infty$





a

b

c

d

If vacuum has condensation, vacuum symmetry is broken.

$\Rightarrow \exists$ Goldstone Boson. $(m=0)$

e.g.

Ferromagnetism

magnetization breaks the rotational symmetry

magnon: or spin wave

Superconductivity

Cooper pair condensation breaks the phase sym.

$$\psi \to e^{i\theta}\psi$$

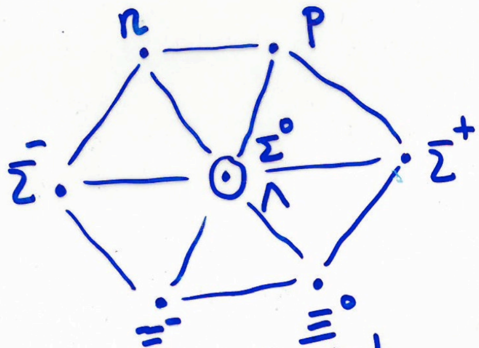
$$v = \langle 0 | \psi_{\downarrow}(x)\psi_{\uparrow}(x) + \psi_{\uparrow}^{\dagger}\psi_{\downarrow}^{\dagger}(x) | 0 \rangle \neq 0$$

Phason: $\chi(x) = \psi_{\downarrow}(x)\psi_{\uparrow}(x) - \psi_{\uparrow}^{\dagger}(x)\psi_{\downarrow}^{\dagger}(x)$

$m_{\chi} = 0$. charge $-2e$.

$\vec{J} = \sigma \vec{E}$ $\quad \sigma \sim \frac{1}{m_{\chi}^2} \to \infty$

In QCD vacuum, chiral sym. is broken.

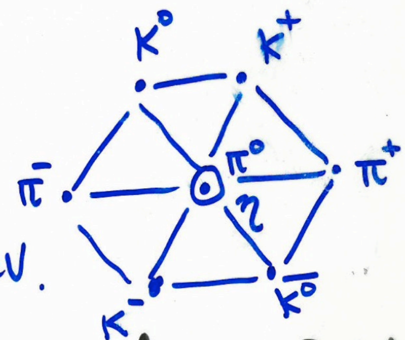


parity $+\frac{1}{2}$

?

parity $-\frac{1}{2}$

Goldstone boson



$m_\pi \simeq 140\,\text{MeV}$.

Low-energy Phenomenology : PCAC.

Phenomenological theory

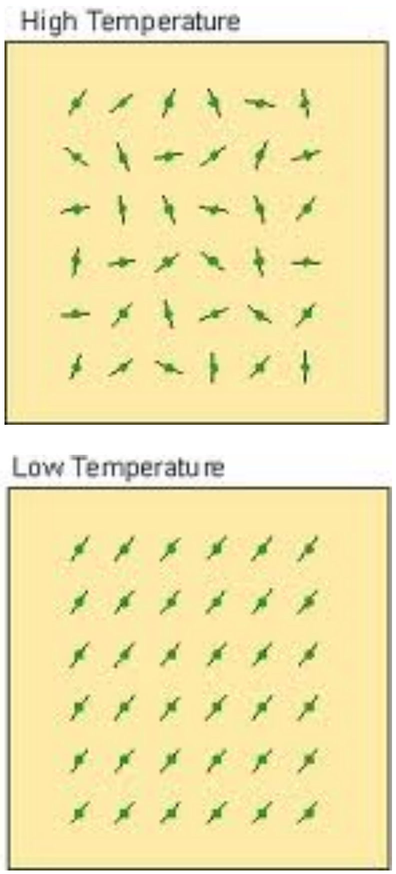
Landau-Ginzburg | Linear σ-model. Nambu-Jona Lassinio model.

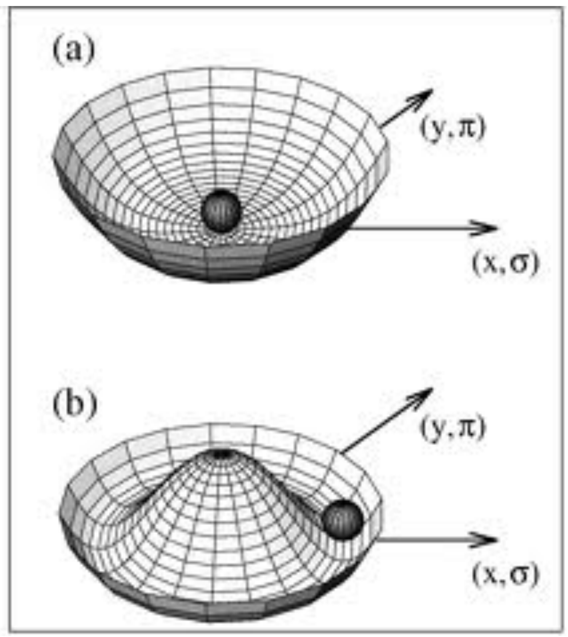
Microscopic Derivation

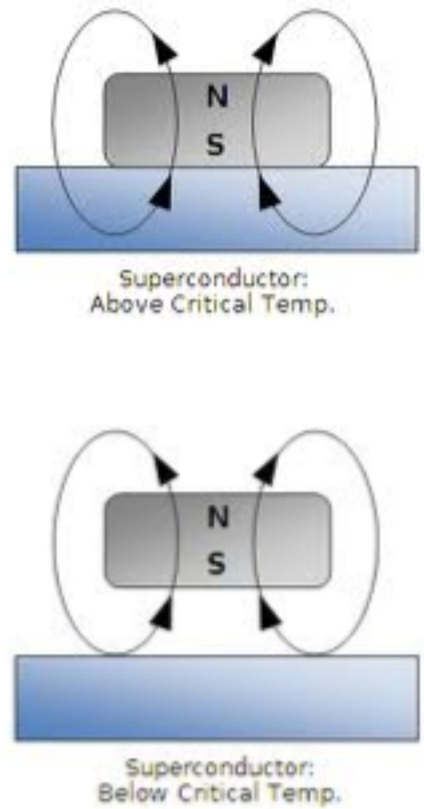
QED (BCS)

QCD (?)

Some attempt has been discussed


High Temperature
Low Temperature

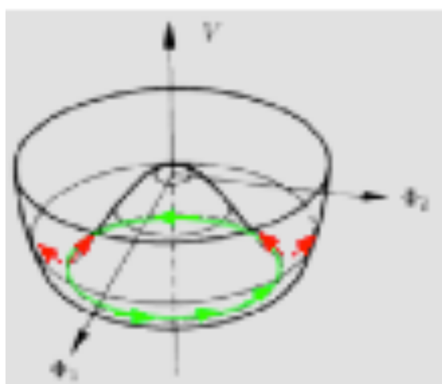

(a)
(y, π)
(x, σ)
(b)
(y, π)
(x, σ)


N
S
Superconductor:
Above Critical Temp.
N
S
Superconductor:
Below Critical Temp.

Meissner Effect of Superconductor

$$m\dot{\vec{x}} = \vec{p} - e\vec{A}$$

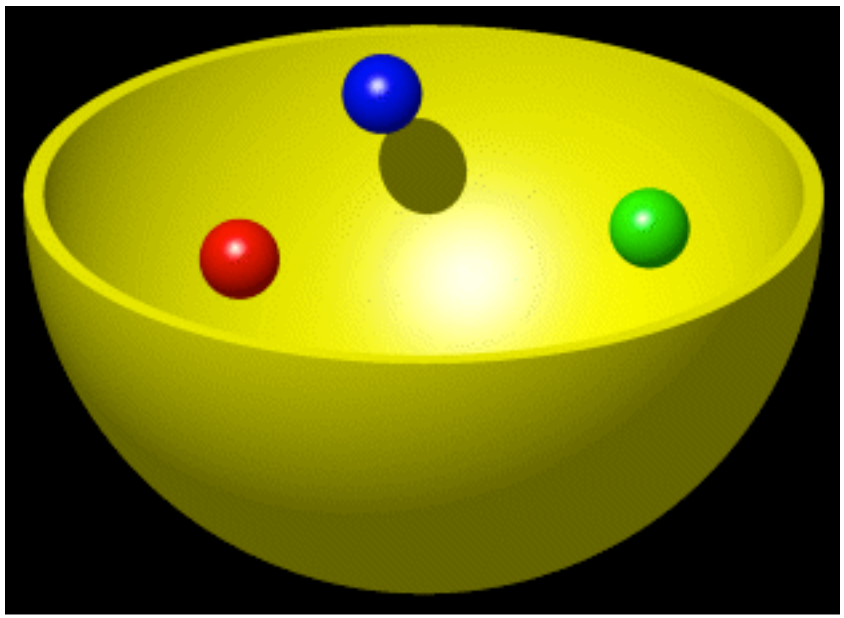
$$D_\mu\phi = \partial_\mu\phi - eA_\mu\phi$$



$$\phi = \begin{pmatrix} 0 \\ V+\eta \end{pmatrix} \Rightarrow m_\eta = \sqrt{\lambda V}\ ,\ m_W = gV\ ,\cdots$$

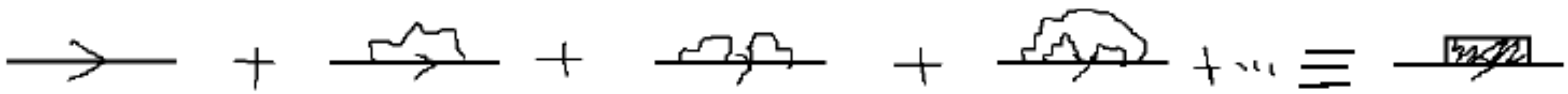
$$M_p = 938.272046 \pm 0.000021\ MeV$$

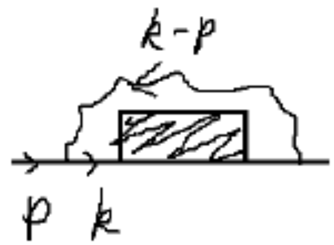
$$M_n = 939.565379 \pm 0.000021\ MeV$$



$$m_u = 2.3^{+0.7}_{-0.5}\ MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\ MeV$$

Dressed quark propagator





$$= i\Sigma(p)$$

We start from the Lagrangian

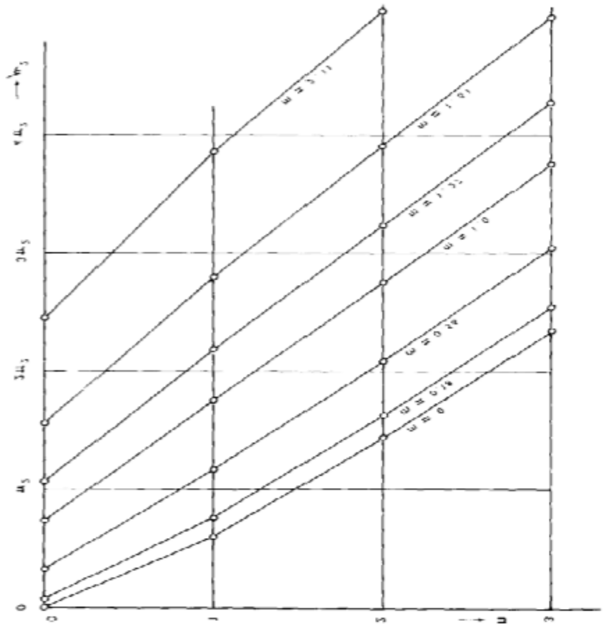
$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}]$ and $D_{\mu} = \partial_{\mu} - igA_{\mu}$

then

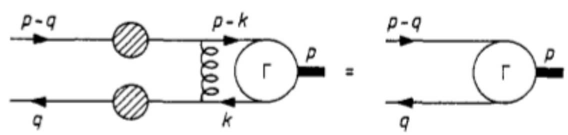
$$i\Sigma(p) = \int \frac{d^2k}{(2\pi)^2} ig\gamma^{\mu} \frac{i}{\not{k} - m - \Sigma(k) + i\epsilon} ig\gamma^{\nu} \frac{-ig_{\mu\nu}}{(p-k)^2}$$

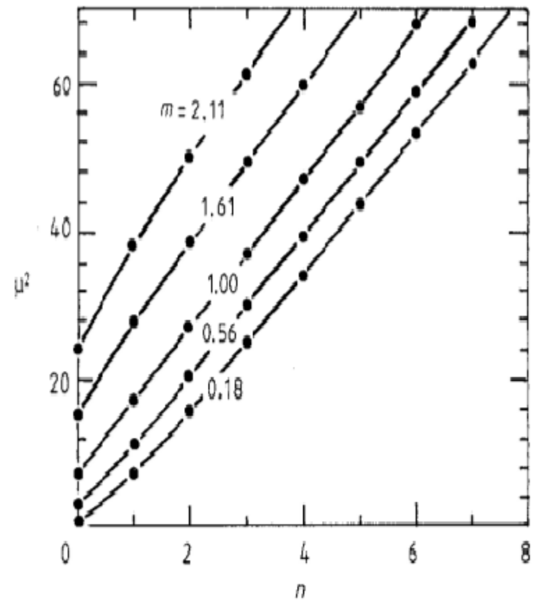
Meson spectroscopy in QCD_{1+1}



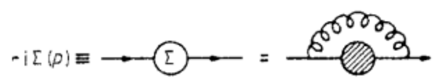
G. 't Hooft, Nucl. Phys. B75, 461 (1974)

LFD ⇦ large N_c ⇨ IFD



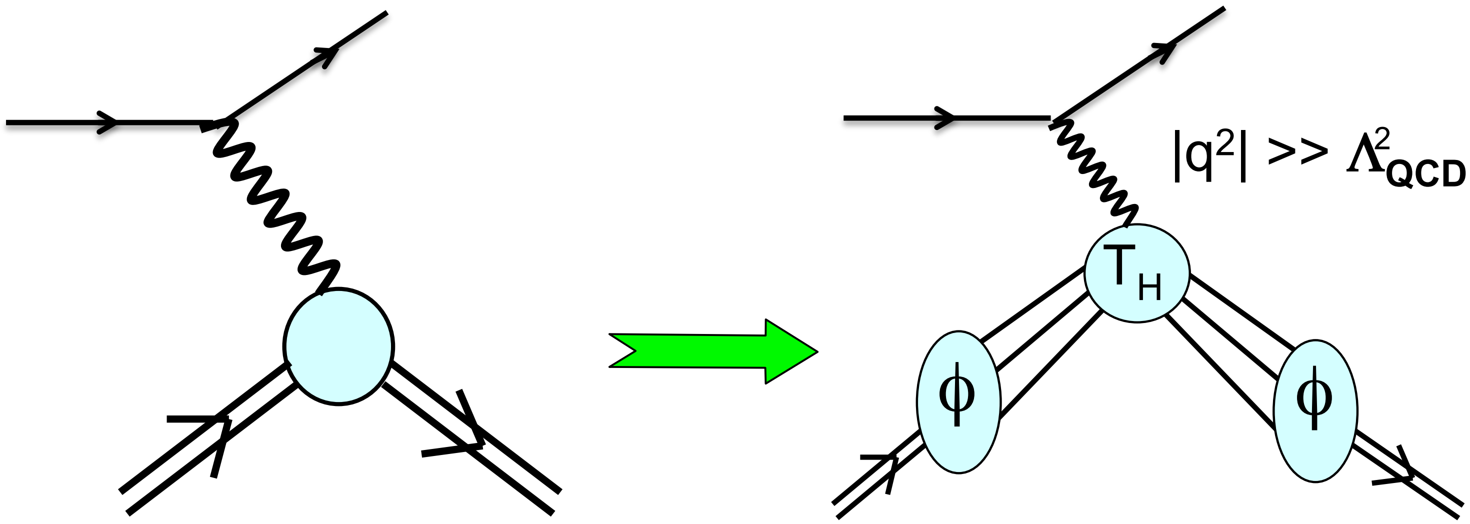


Li, Wilets and Birse, J. Phys. G:Nucl. Phys. 13, 915 (1987)

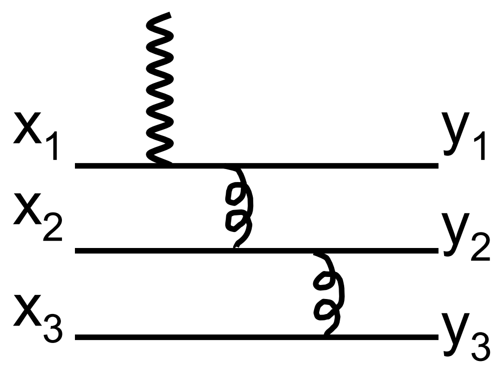


Dynamical quark/gluon mass generation and color confinement in QCD should be understood further.

Proton Charge Form Factor

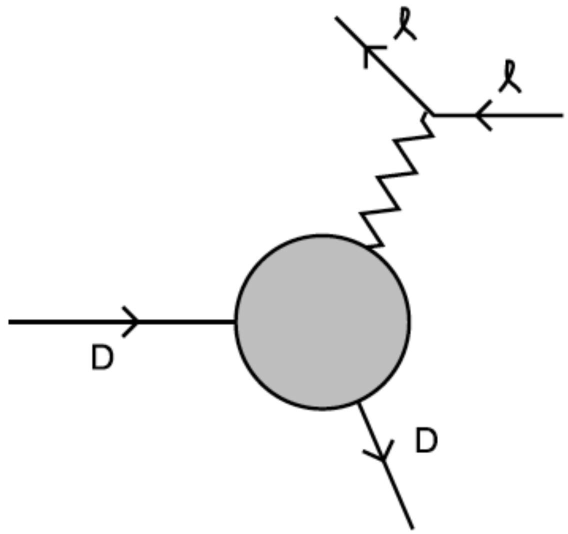


$$T_H = \sum \Big[$$



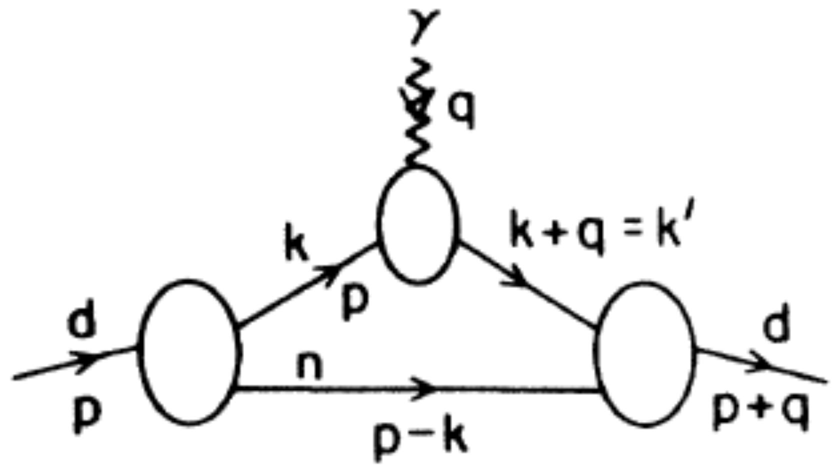
$$+ \quad . \quad . \quad . \Big]$$

$$= \frac{\alpha_s^{\,2}}{Q^4} f(x_i, y_i)$$



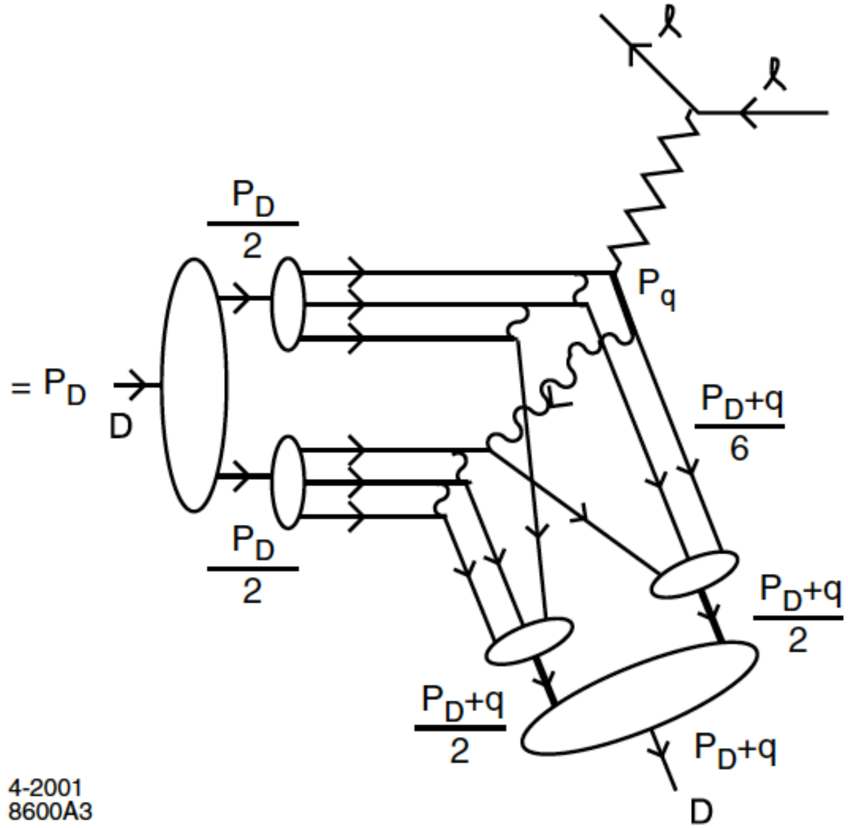
Impulse Approximation is valid only for

$$Q^2 < 2M_d\epsilon_d \qquad Q \lesssim 100 \text{ MeV}$$



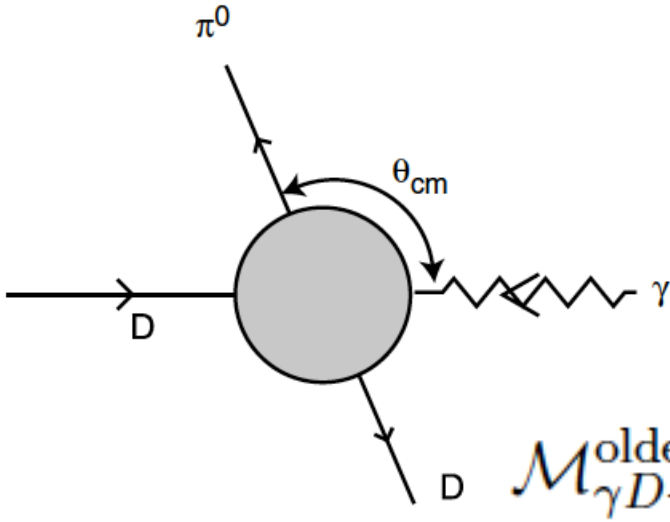
$$F_d(Q^2) = F_d^{\text{body}}(Q^2) F_N(Q^2)$$

S.J.Brodsky & C.Ji, PRD 33, 2653 (1986)



Reduced Form Factor

$$F_D(Q^2) \rightarrow f_d(Q^2) F_N^2\left(\frac{Q^2}{4}\right)$$

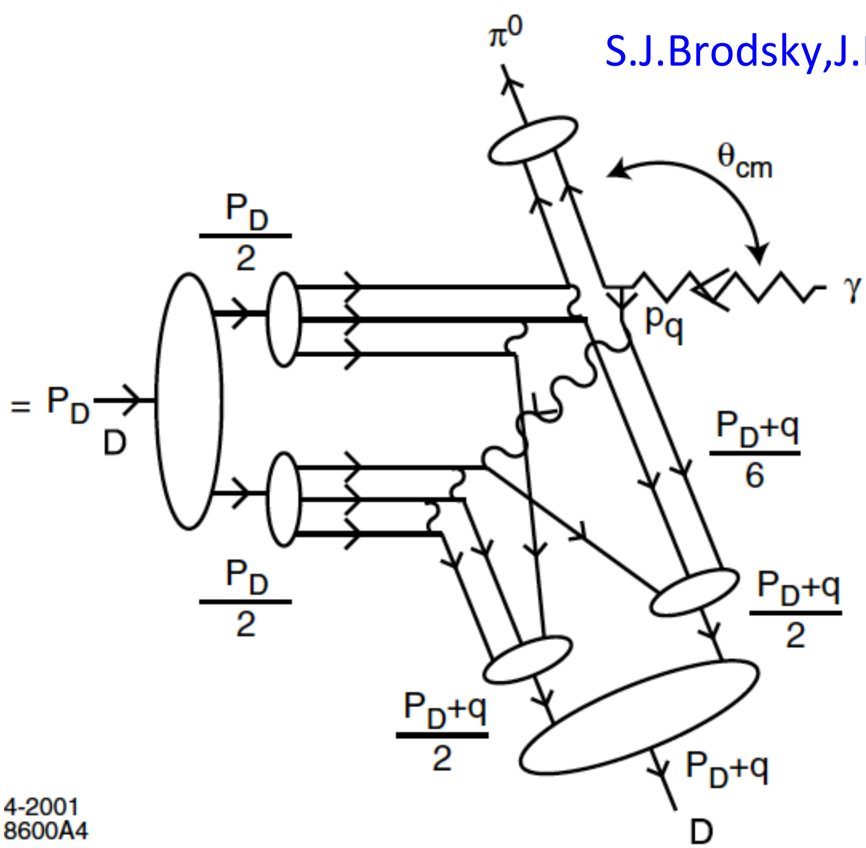


$$[x_1(p_d + q) + q/2]^2 \sim (1 + 2x_1)q^2/4$$
$$\sim q^2/3 \text{ (using } x_1 \sim 1/6)$$

S.J.Brodsky,J.R.Hiller, PRC 28, 475 (1983)

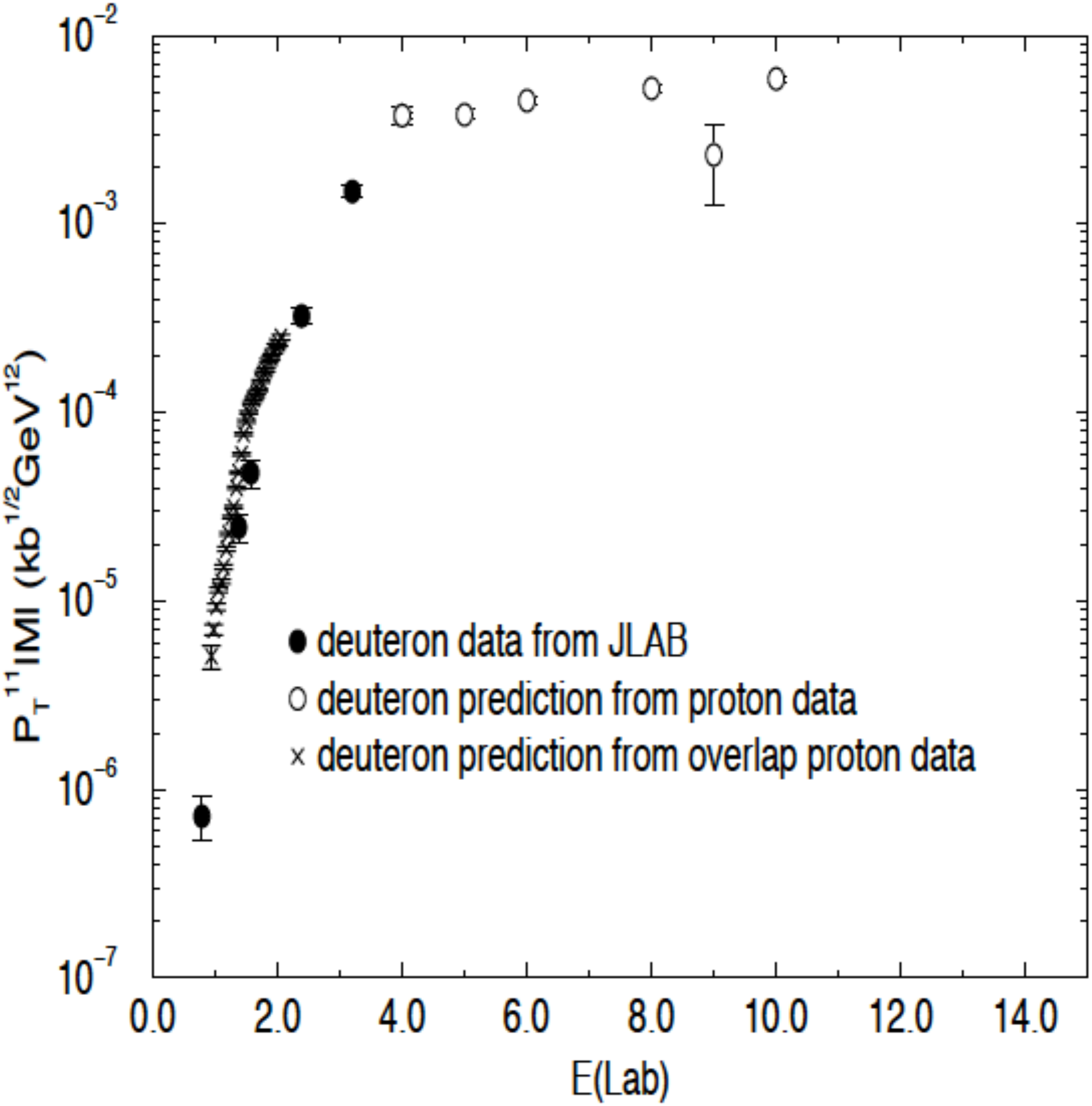
$$\mathcal{M}^{\text{older}}_{\gamma D \to \pi^0 D}(u,t) \simeq m_{\gamma d \to \pi^0 d}(u,t) F_N^2(t/4)$$

S.J.Brodsky,J.R.Hiller,C.Ji,G.A.Miller, PRC 64, 055204 (2001)

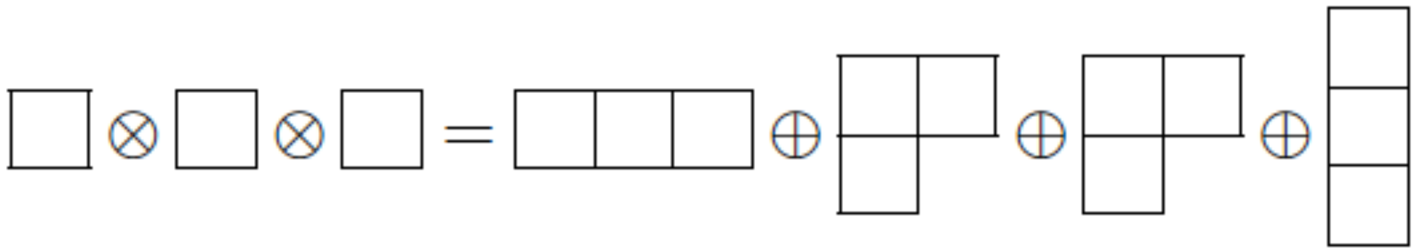


New Improved Reduced Nuclear Amplitude

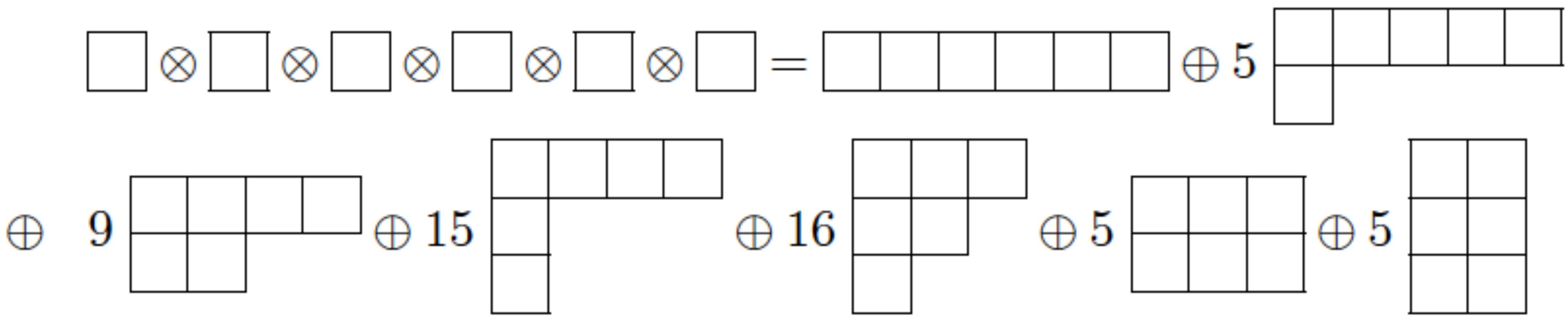
$$\frac{\mathcal{M}_{\gamma D \to \pi^0 D}}{\mathcal{M}_{eD \to eD}} = C' \frac{\mathcal{M}_{\gamma p \to \pi^0 p}}{\mathcal{M}_{ep \to ep}}$$



10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
P_T^{11} IMI (kb$^{1/2}$GeV^{12})
deuteron data from JLAB
deuteron prediction from proton data
deuteron prediction from overlap proton data
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
E(Lab)



$$\{3\} \otimes \{3\} \otimes \{3\} = \{10\} \oplus \{8\}_{\mathrm{S}} \oplus \{8\}_{\mathrm{A}} \oplus \{1\}$$

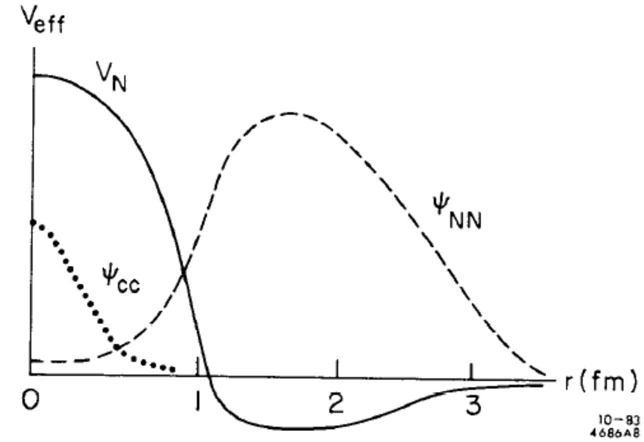


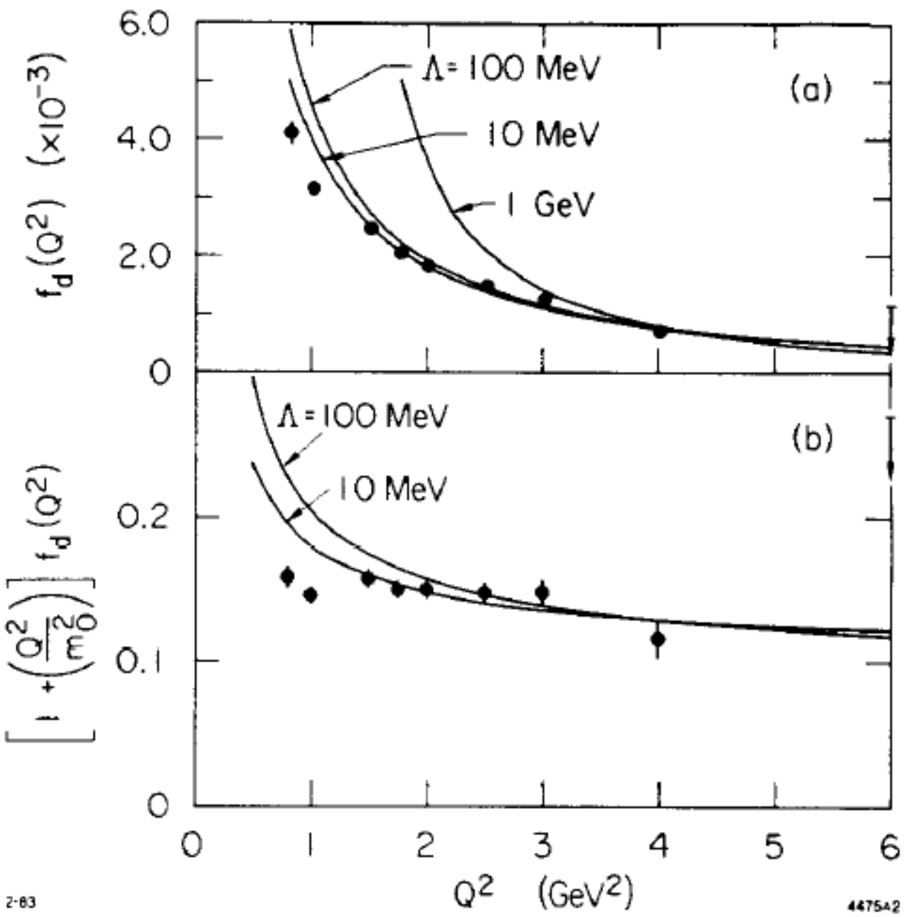
$$\{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} = 729$$
$$= \{28\} \oplus 5\{35\} \oplus 9\{27\} \oplus 15\{10\} \oplus 16\{8\} \oplus 5\{10^*\} \oplus 5\{1\}$$

$$\phi_{NN}(x_i,Q) = 0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.64\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots$$

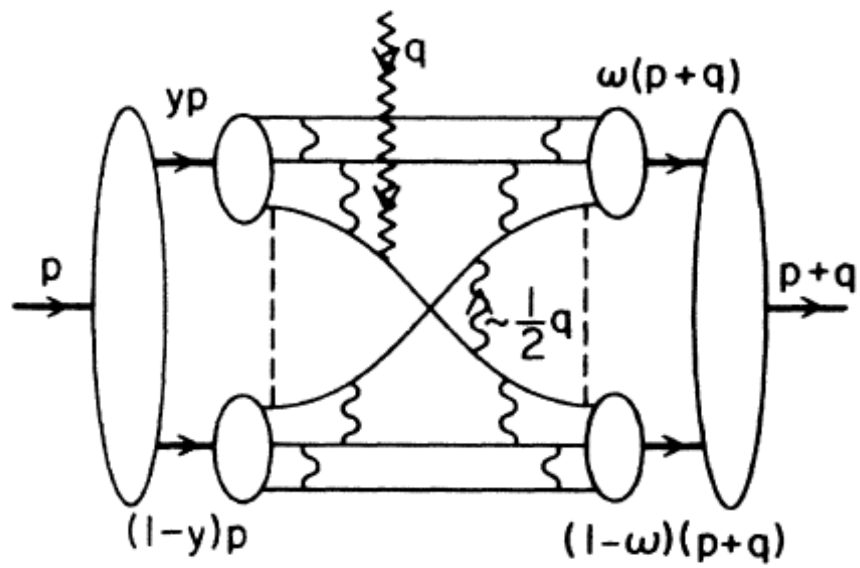
$$\phi_{\Delta\Delta}(x_i,Q) = -0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.59\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots$$

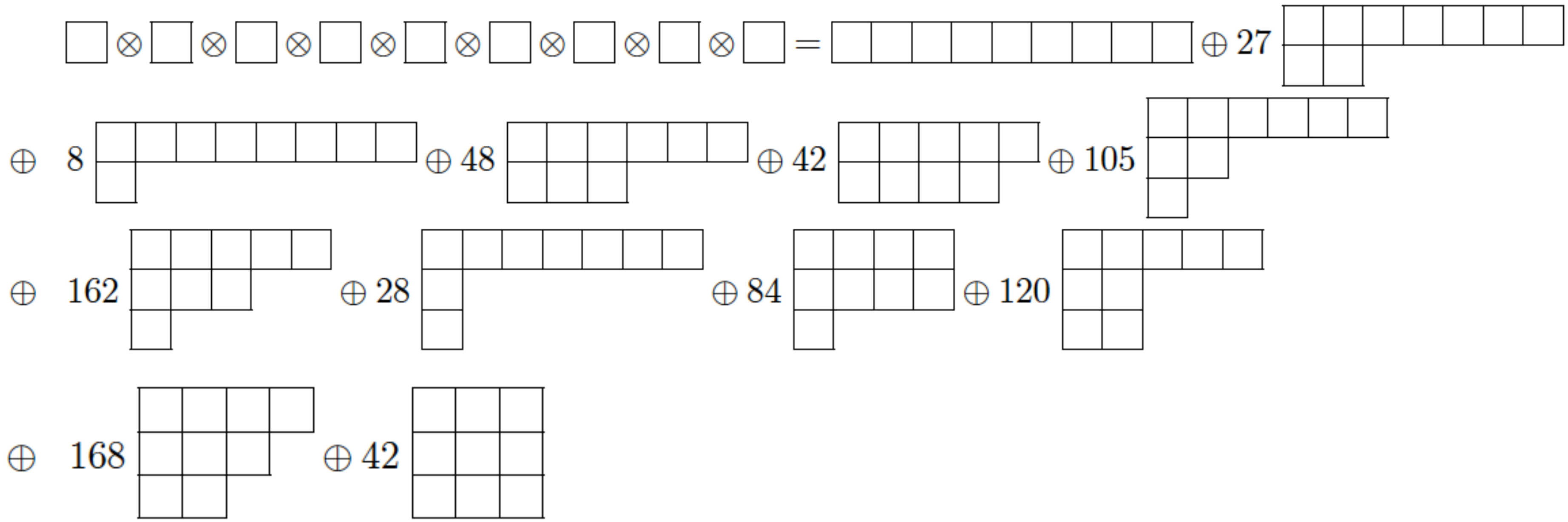
$$\phi_{CC}(x_i,Q) = -0.70\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.35\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots$$





C.Ji & S.J.Brodsky, PRD 34, 1460 (1986);
S.J.Brodsky, C.Ji, G.P.Lepage, PRL 51, 83 (1983);
S.J.Brodsky & C.Ji, PRD 33, 1406 (1986)





$$\{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} \otimes \{3\} = 19683$$

$$= \{55\} \oplus 27\{81\} \oplus 8\{80\} \oplus 48\{64\} \oplus 42\{35^*\} \oplus 105\{35\} \oplus 162\{27\} \oplus 28\{28\} \oplus 84\{10^*\}$$

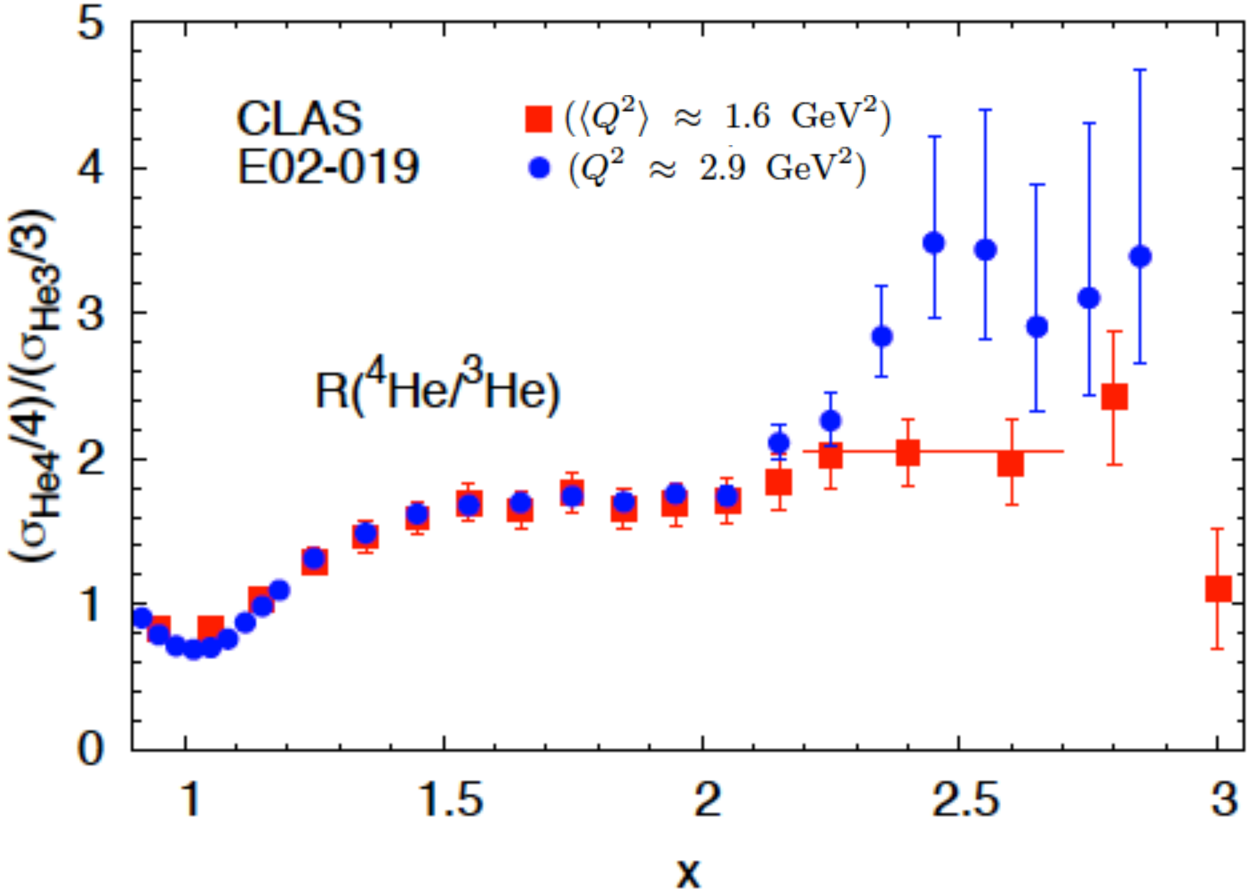
$$\oplus\ 120\{10\} \oplus 168\{8\} \oplus 42\{1\}.$$

(42-1)/(5-1) = 41/4 >10

More than an order of magnitude increase!

B.L.G.Bakker & C.Ji, Prog. in Part. and Nucl.Phys. 74, 1 (2014)

Three-Nucleon Short Range Correlation



New measurements of high-momentum nucleons and short-range structures in nuclei.

E02-019 Hall C Expt, PRL 108, 092502 (2012)

Mesons $Q\bar{Q}$

Baryons QQQ

Glueball States $GG \quad GGG$

"tetraquark" states $Q\bar{Q}\,Q\bar{Q}$

$Z_c\,(c\bar{c}\,u\bar{d})$

"pentaquark" states $QQQ\,Q\bar{Q}$

"hexaquark" states $QQQ\,QQQ$

F.J.Dyson & N. Xuong, PRL 13, 815 (1964)

"H dibaryon" UUD DSS R.L.Jaffe, PRL 38, 195 (1977)

Possible mechanisms underlying confinement multiply as the number of quarks and gluon constituents increase.

Do the constituents always cluster as color-singlet subsystems?

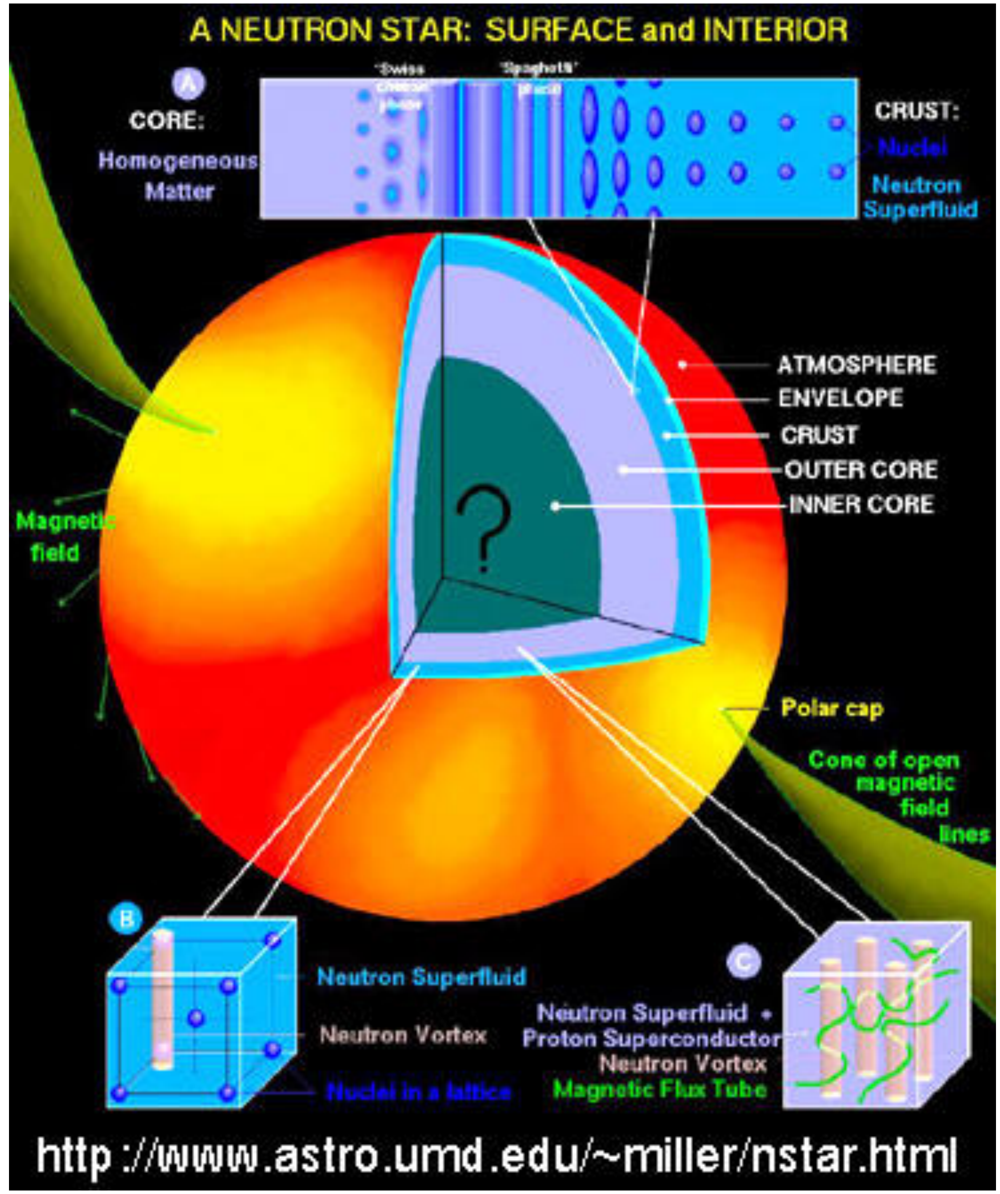
Predominantly "yes" for ordinary nuclei, but there are also rare configurations in which other multiquark color configurations "hidden color" can enter.

$$\rho_{NS} = (5 \sim 10)\rho_0$$

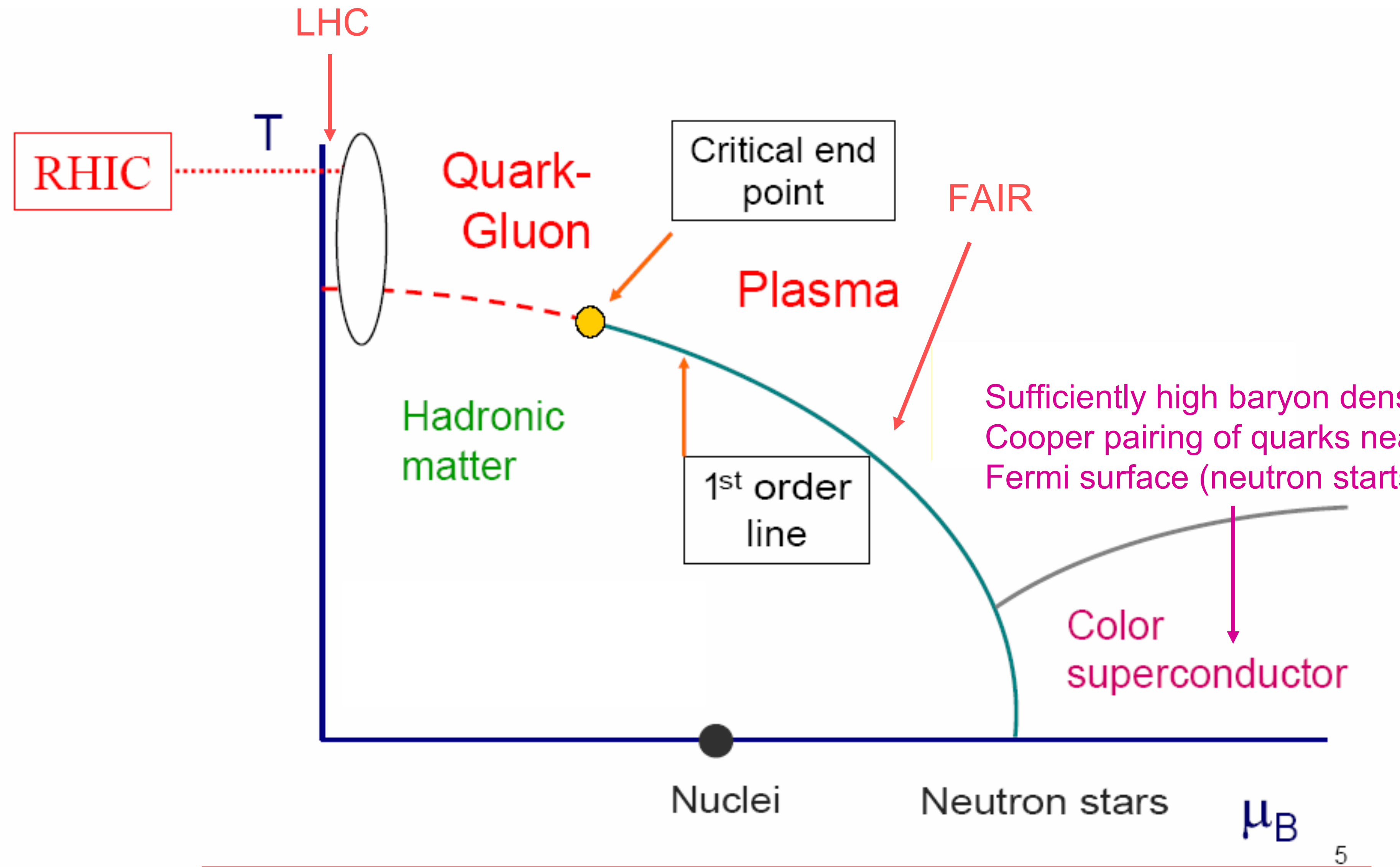
$$\rho_0 = 2.65 \times 10^{14} g/cm^3$$

$$\rho_\Theta = 1.4 g/cm^3$$

Large Enhancement of Hidden Color Effect is expected.



Phase diagram as of today



Reminder: Chemical Potential

= Change of energy when adding additional baryon

= net baryon density

$$\mu_i = \left(\frac{\partial U}{\partial N_i}\right)_{S,V,N_{j\neq i}}$$

Phase Transition & Critical Temperature

Energy density of hadron gas

(ideal gas)

$$\epsilon = \int \frac{d^3p}{(2\pi)^3} \sum_i \frac{E_i}{e^{\beta E_i} \pm 1}$$

at large T
(ignoring masses)

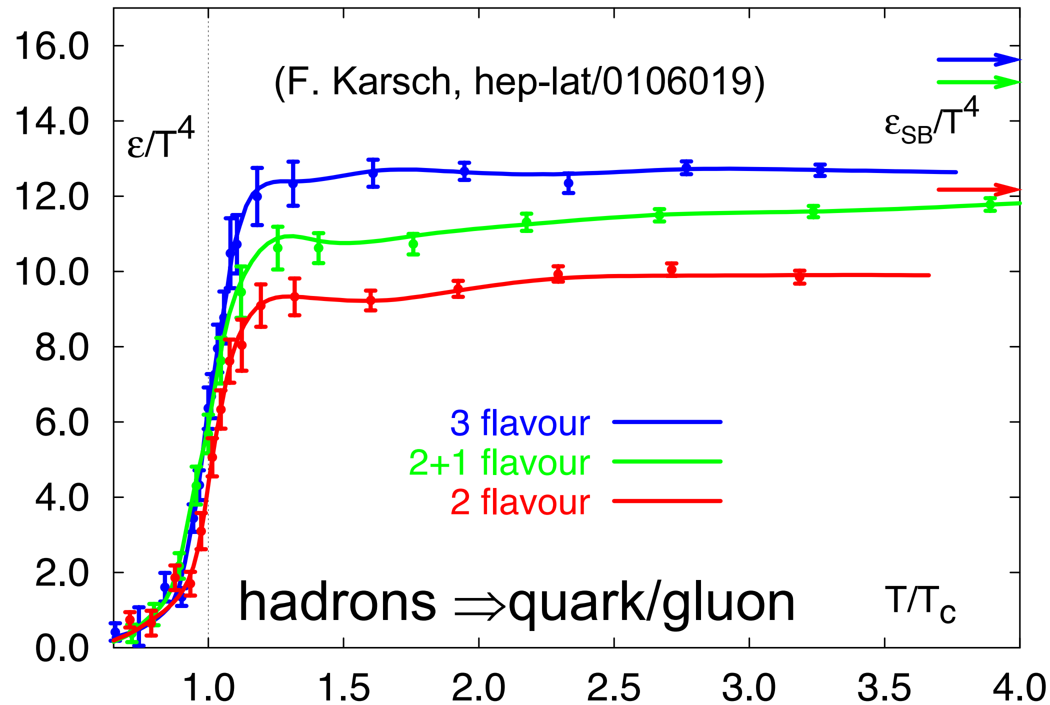
$$\epsilon \sim \frac{\pi^2}{30} N T^4$$

Degrees of freedom

QGP: $N = 2 \times 8 + \frac{7}{4}(2 \times 3 \times N_f)$

Hadronic gas phase (only pions): N = 3

Lattice QCD predicts phase transition



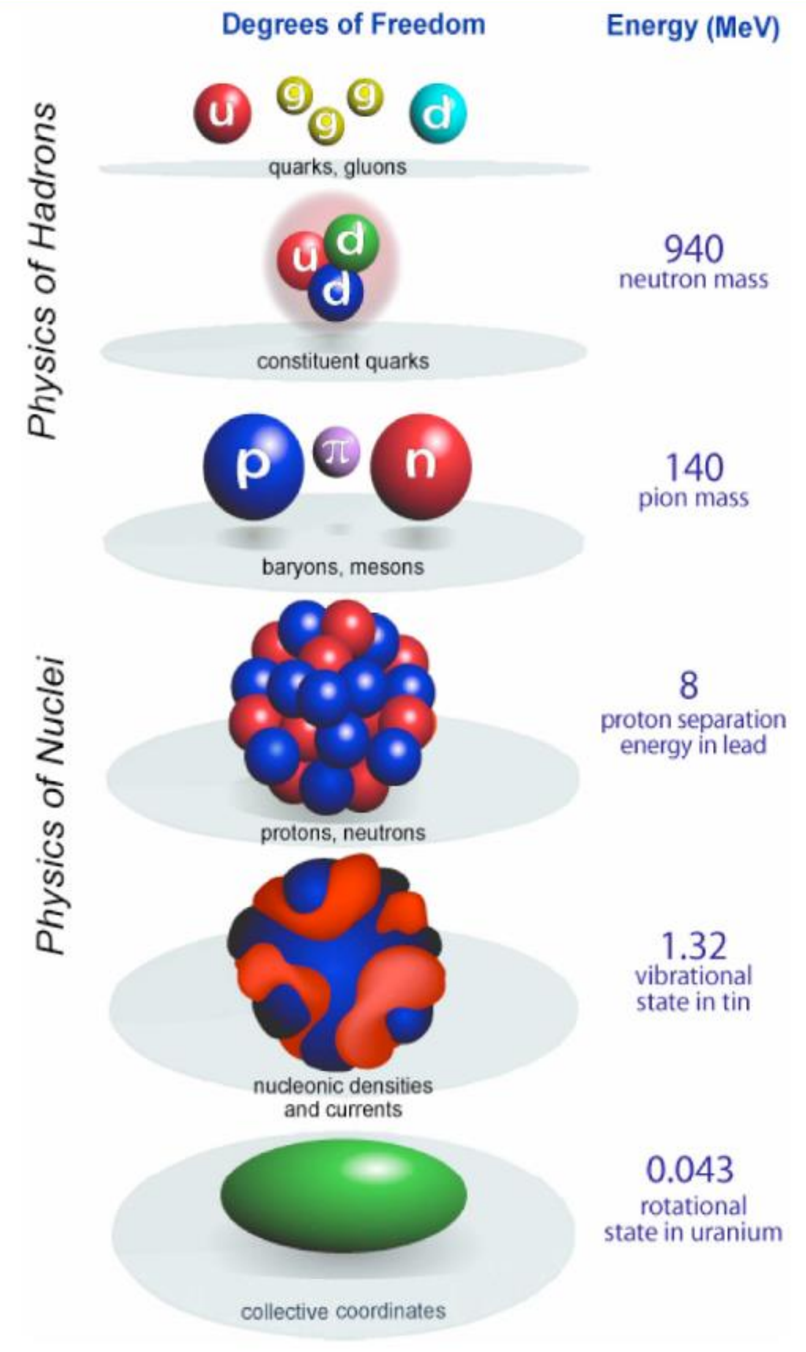
Critical temperature: T_C = 173 MeV
(zero net-baryon density)

Critical density: $\rho_c \sim 0.7$ GeV fm^{-3}

nuclear density: $\rho = 0.15$ GeV fm^{-3}

Inside nucleon: $\rho = 0.5$ GeV fm^{-3}

Relevant Degrees of Freedom in Strongly Interacting Systems



Summary and Outlook

- Proliferation of hidden color degrees of freedom is dramatic as the number of quarks increase.

  → 3N SRC may be enhanced as Q gets large.

  → Q dependence of deuteron b_1 structure function may be important to check the effect of hidden color degrees of freedom.

  → Recent observation of d^* resonance raises the possibility of producing other novel color-singlet six-quark dibaryon configurations allowed by QCD.

- The link between the traditional nuclear physics and the quark-gluon picture may be provided by the reduced nuclear amplitudes.