

Nuclear Physics School 2018

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North Carolina State University

Third Lecture

Light-Front Quark Model

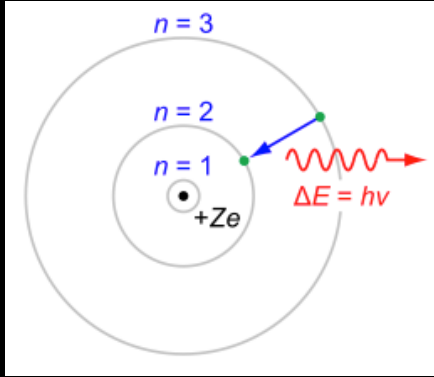
June 29, 2018

1 H	
3 Li	4 Be
11 Na	12 Mg
19 K	20 Ca
37 Rb	38 Sr
55 Cs	56 Ba
87 Fr	88 Ra

21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn
39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd
71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg
103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt			

2 He					
5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn

57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb
89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No



H-atom Spectra

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137.036}$$

$$E_n = -\alpha^2 mc^2 \left(\frac{1}{2n^2} \right) \\ \approx -13.6 eV / n^2.$$

$$\Delta E_{fs} = \Delta E_{rel} + \Delta E_{so} \\ = -\alpha^4 mc^2 \frac{1}{4n^4} \left(\frac{2n}{j + \frac{1}{2}} - \frac{3}{2} \right)$$

Relativistic Correction

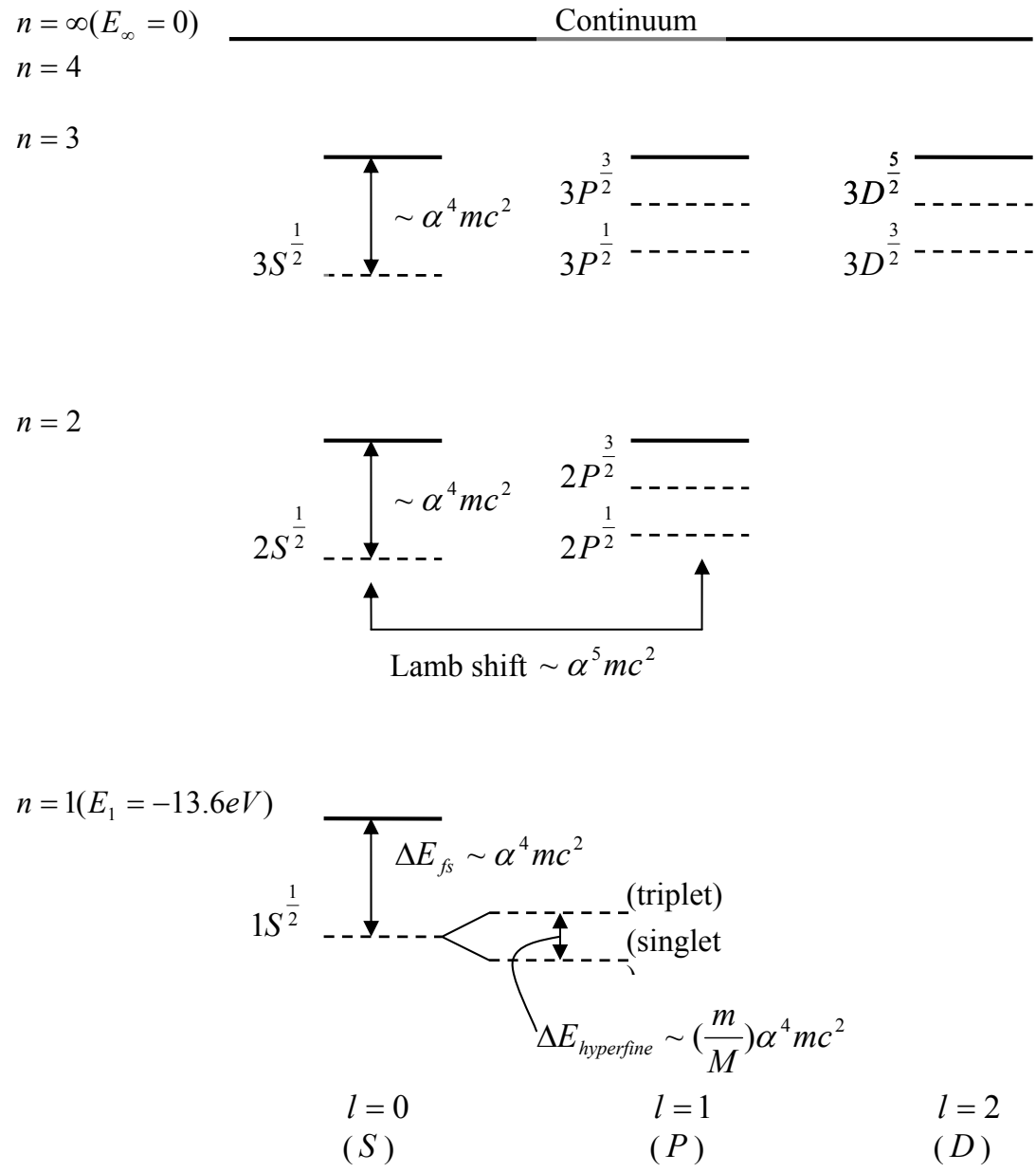
$$T = (\gamma - 1)mc^2 \\ = \frac{1}{2}m\vec{v}^2 + \frac{3}{8}m\frac{\vec{v}^4}{c^4} + \dots \\ = \frac{\vec{p}^2}{2m} - \frac{\vec{p}^4}{8m^3c^2} + \dots,$$

$$\Delta E_{rel} = -\alpha^4 mc^2 \frac{1}{4n^4} \left(\frac{2n}{\ell + \frac{1}{2}} - \frac{3}{2} \right)$$

Spin-Orbit Coupling

$$\Delta H_{so} = \frac{e^2}{2m^2c^2r^3} \vec{L} \cdot \vec{S}$$

$$\langle \Delta E_{so} \rangle = \alpha^4 mc^2 \frac{\left\{ j(j+1) - \ell(\ell+1) - \frac{3}{4} \right\}}{4n^3 \ell \left(\ell + \frac{1}{2} \right) (\ell + 1)}$$



The solid line spectra ($E_n = -\alpha^2 mc^2 (\frac{1}{2n^2})$) are without corrections. The masses of electron and proton are denoted by m and M respectively.

For the positronium, $m/M = 1$:

the reduced mass effect, i.e. $m_{red} = \frac{m}{2}$,

Thus, there is no distinction between the fine structure and the hyperfine structure in the positronium system.

The hyperfine structure is at the same order as the fine structure in the positronium system and the degeneracy of $2S_{1/2}$ and $2P_{1/2}$ states is already broken in the level of fine and hyperfine combined structure.

Therefore, the Lamb shift which breaks the degeneracy of $2S_{1/2}$ and $2P_{1/2}$ states is less interesting in the positronium system.

$$\begin{aligned} E_n^{\text{positronium}} &= -\alpha^2 m_{\text{red}} c^2 \left(\frac{1}{2n^2} \right) \\ &= \frac{1}{2} E_n^{\text{hydrogen}}, \end{aligned}$$

i.e.

$$E_1^{\text{positronium}} = \frac{13.6\text{eV}}{2} = 6.8\text{eV},$$

and

$$\begin{aligned} a^{\text{positronium}} &= \frac{\hbar^2}{m_{\text{red}} e^2} \\ &= 2a^{\text{hydrogen}} \\ &= 1.06 \times 10^{-8} \text{ cm}. \end{aligned}$$

The second one is the electron-positron annihilation effect which modifies the interaction Hamiltonian. In some cases, the positronium decays into two or three photons depending on the positronium state. The energy level correction due to the annihilation effect occurs at the same order as the fine structure, i.e.

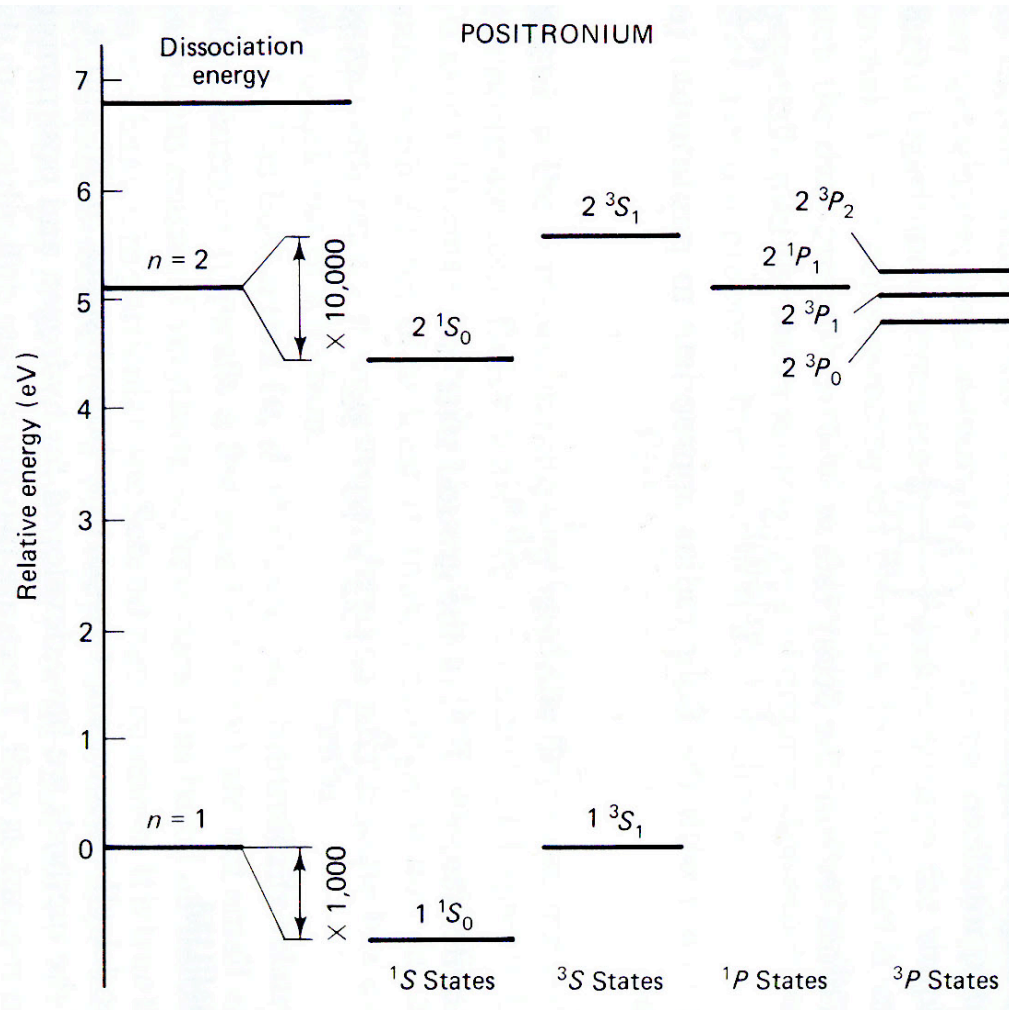
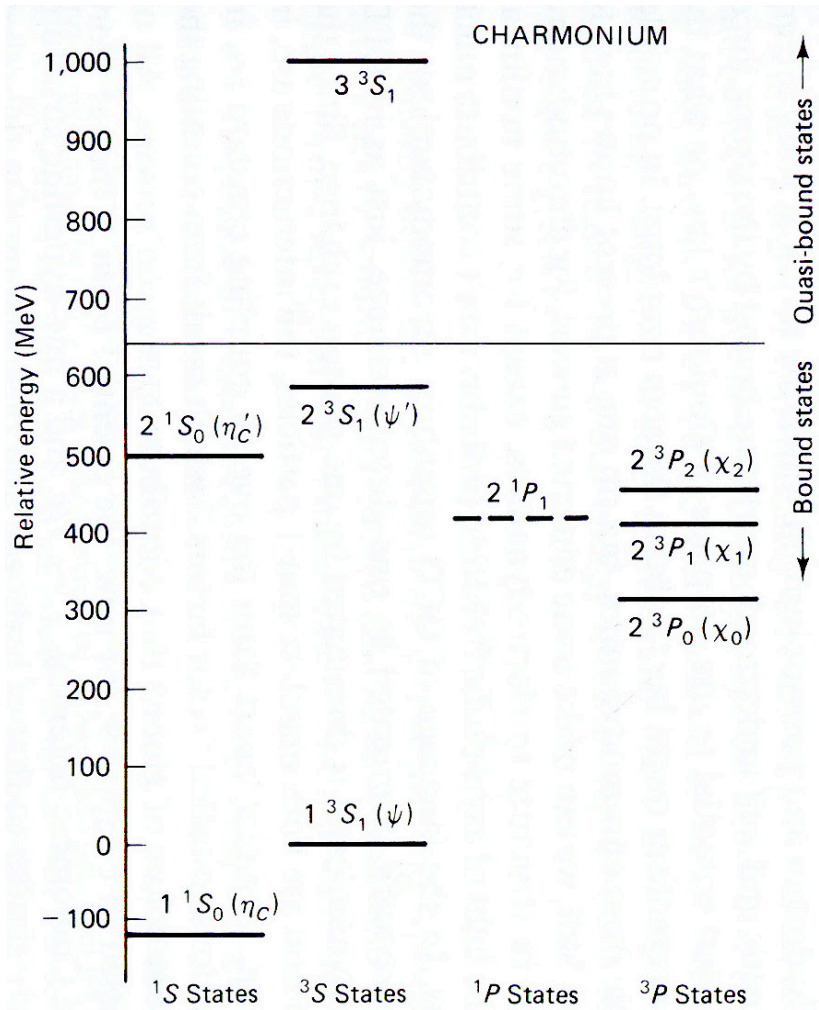
$$\Delta E_{\text{annihilation}} \sim \alpha^4 mc^2.$$

$$\ell = 0 \qquad \mathbf{c} = (-1)^{\ell+s} \qquad \mathbf{c}_{\gamma} = -1$$

$$\tau(^1S_0 \rightarrow 2\gamma) = 1.25 \times 10^{-10} \text{ sec} \qquad \mathbf{s} = 0$$

and

$$\tau(^3S_1 \rightarrow 3\gamma) = 1.45 \times 10^{-7} \text{ sec.} \qquad \mathbf{s} = 1$$



$$V = -\frac{e^2}{r} \longrightarrow (\Delta E)_{\text{positronium}} \sim eV$$

$$V(r) = -\frac{4}{3} \frac{\alpha_s \hbar c}{r} + F_o r \longrightarrow (\Delta E)_{\text{quarkonium}} \sim 100\text{MeV}$$

splitting of the energy level for $n=1$ state, i.e. 1^1S_0 and 1^3S_1

singlet (1^1S_0)

pseudoscalar meson octet

triplet (1^3S_1)

vector meson octet

the mass difference between π and ρ mesons

$$M = m_1 + m_2 + A \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$\begin{aligned} \vec{s}_1 \cdot \vec{s}_2 &= \frac{\vec{s}^2 - \vec{s}_1^2 - \vec{s}_2^2}{2} \\ &= \frac{s(s+1) - \frac{3}{4} - \frac{3}{4}}{2} \hbar^2 \begin{cases} s = 0; & -\frac{3}{4} \hbar^2 \\ s = 1; & \frac{1}{4} \hbar^2. \end{cases} \end{aligned}$$

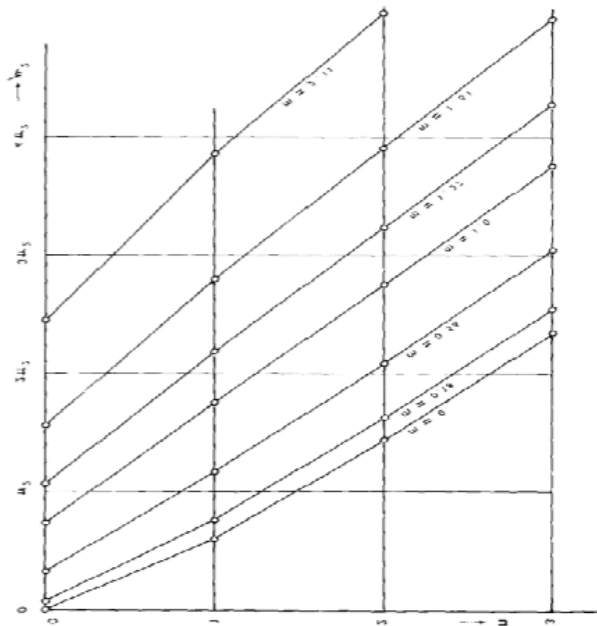
$$M = m_1 + m_2 + A \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

	Eq.(3.101)	Experiment
π	140	138
K	484	496
η	559	549
ρ	780	776
ω	780	783
K*	896	892
ϕ	1032	1020

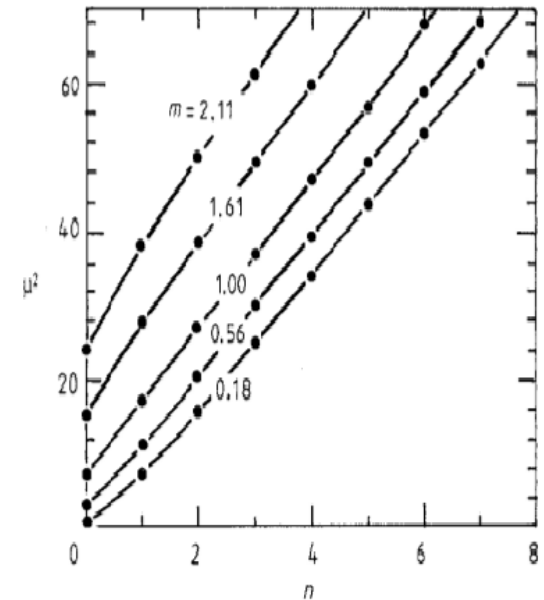
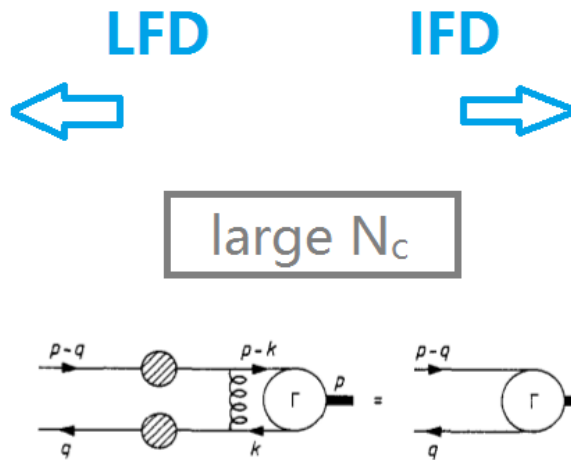
$$m_u = m_d = 310 \text{MeV} / c^2, \quad m_s = 483 \text{MeV} / c^2$$

$$A = \left(\frac{2m_u}{\hbar} \right)^2 160 \text{MeV} / c^2$$

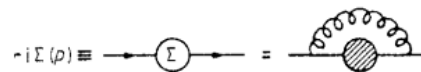
Meson spectroscopy in QCD_{1+1}



G. 't Hooft, Nucl. Phys. B75, 461 (1974)



Li, Willets and Birse, J. Phys. G:Nucl. Phys. 13, 915 (1987)



Dynamical quark/gluon mass generation and color confinement in QCD should be understood further.

Effective Constituent Quark Model for Low Q^2

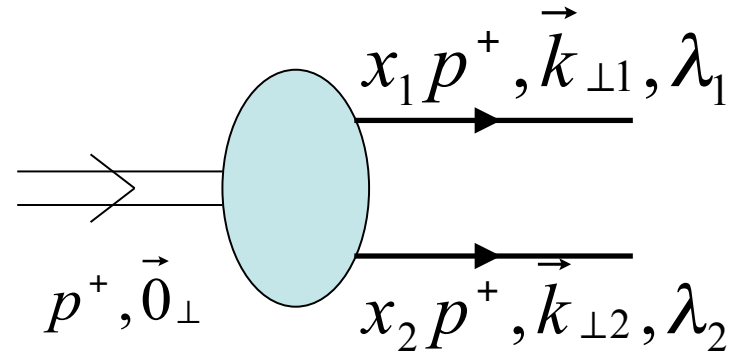
$$|Meson\rangle = \psi_{q\bar{q}} |q\bar{q}\rangle + \psi_{q\bar{q}g} |q\bar{q}g\rangle + \dots$$

$$\approx \Psi_{Q\bar{Q}} |Q\bar{Q}\rangle,$$

where

$$|Q\rangle = \psi_q^Q |q\rangle + \psi_{qg}^Q |qg\rangle + \dots$$

$$|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}} |\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}} |\bar{q}g\rangle + \dots$$



$$\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i}) \chi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Radial

(Dependent on the model potential)

$$H = T + V$$

V includes Coulomb, Confinement,
Spin-Spin, Spin-Orbit interactions.

Spin-Orbit

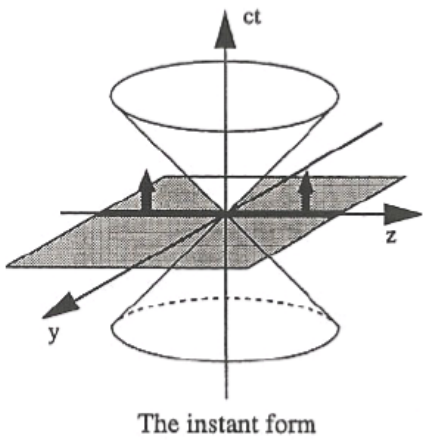
(Interaction independent Melosh transformation)

$$J^{PC} = 0^{++} (f_0, a_0, \dots)$$

$$0^{-+} (\pi, K, \eta, \eta', \dots)$$

$$1^{-} (\rho, K^*, \omega, \phi, \dots)$$

...



Equal t

$$p^0 \Leftrightarrow$$

$$(p^1, p^2) \Leftrightarrow$$

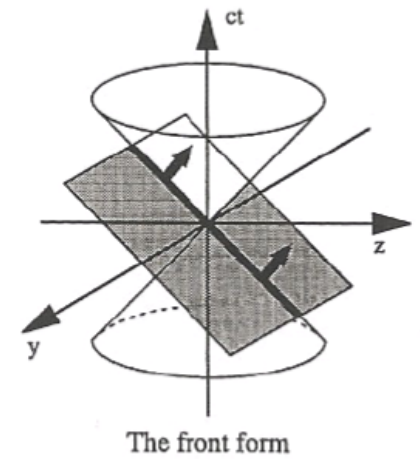
$$p^3 \Leftrightarrow$$

Equal τ

$$p^- = p^0 - p^3$$

$$\vec{p}_\perp$$

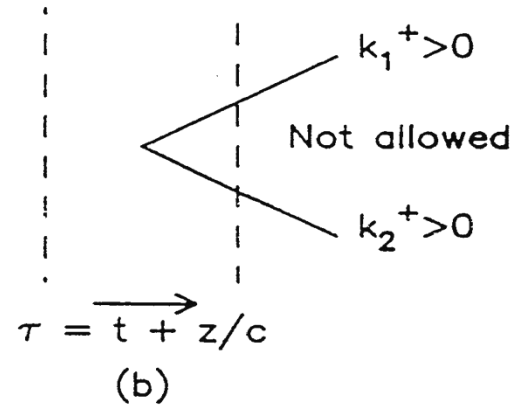
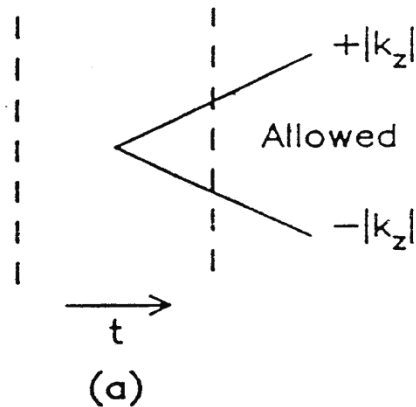
$$p^+ = p^0 + p^3$$



Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^2 + m^2}$$

$$p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$$



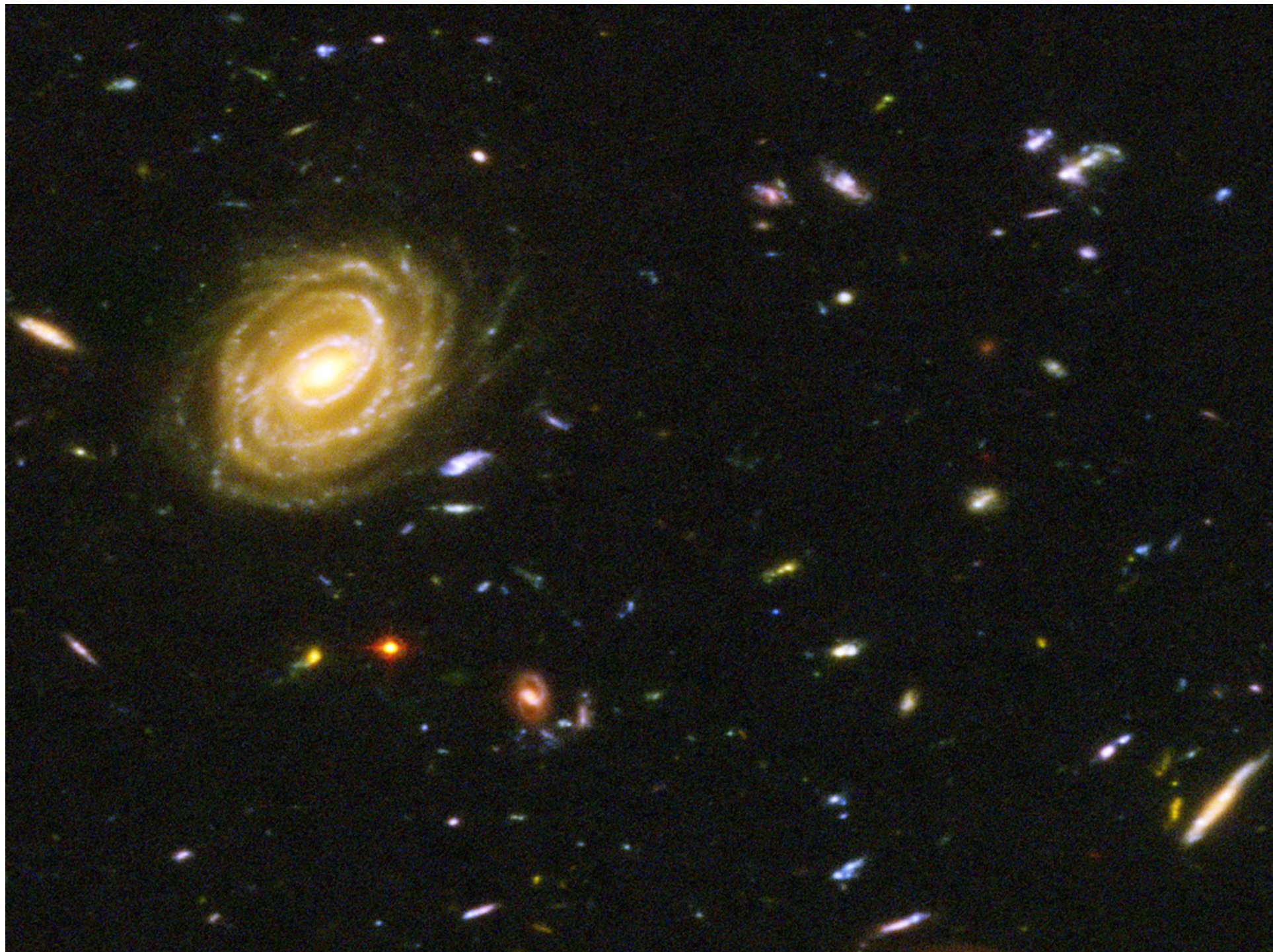
Zero-modes
 $k_1^+ = 0, k_2^+ = 0$

Stability Group

6

7

(maximum)



PHYSICAL REVIEW C **92**, 055203 (2015)

Variational analysis of mass spectra and decay constants ...

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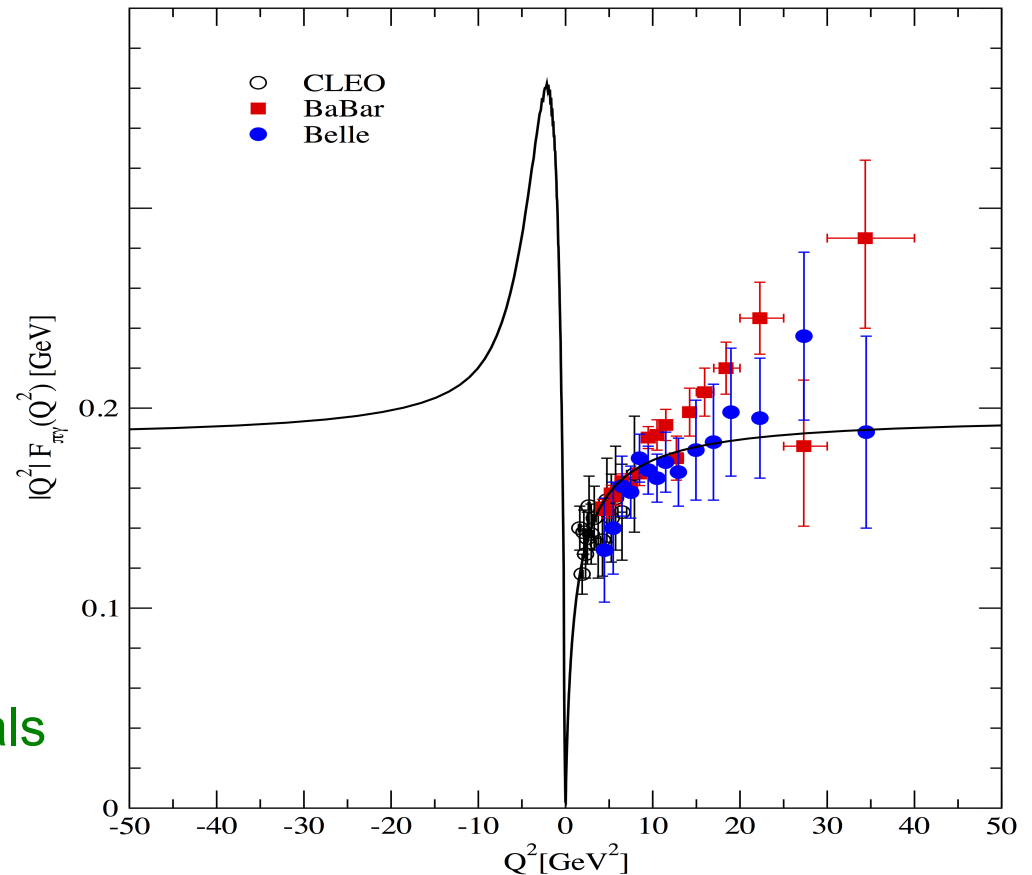
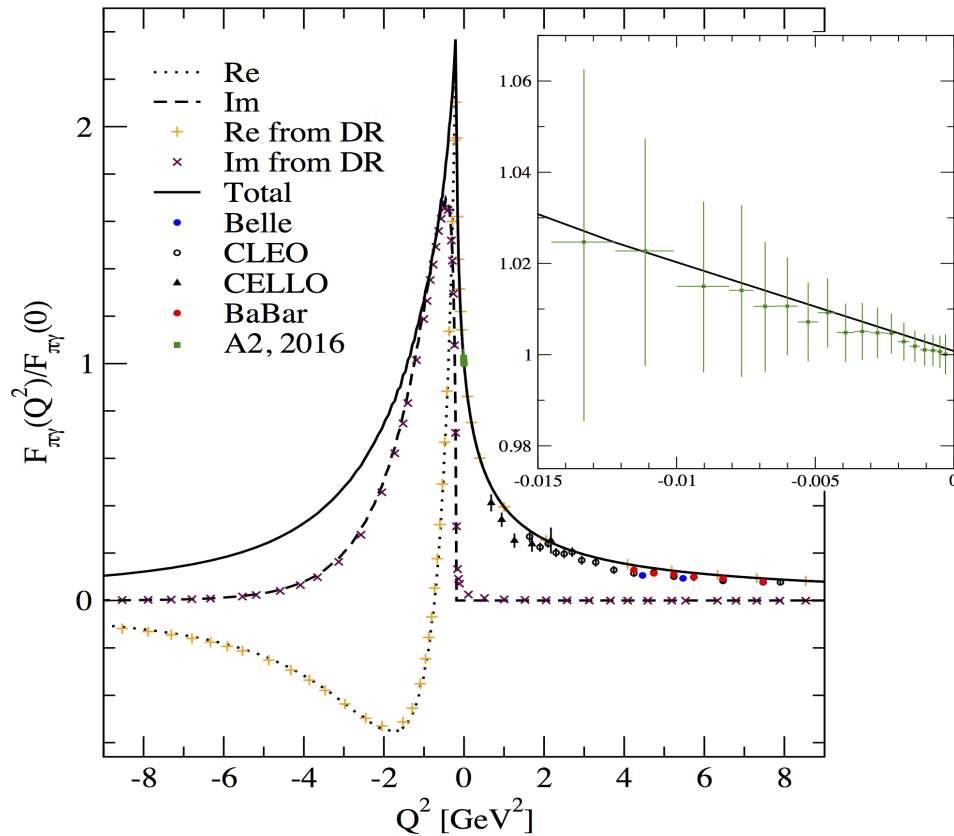
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²*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA*

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$(\overline{9657}) \eta_b(\overline{9389}) \underline{9407}_{+19}^{-18}$	$(\overline{9691}) \underline{Y(9460)} \underline{9434}_{-6}^{+6}$
$(\overline{6459}) B_c(\overline{6277}) \underline{6301}_{+14}^{-12}$	$(\overline{6494}) \underline{B_c^*(?)}$ $\underline{6330}_{-5}^{+3}$
$(\overline{5375}) B_s(\overline{5366}) \underline{(5314)}$ $(\overline{5235}) B(\overline{5279}) \underline{(5233)}$	$(\overline{5424}) B_s^*(\overline{5415}) \underline{(5333)}$ $(\overline{5315}) B(\overline{5325}) \underline{(5268)}$
$(\overline{3171}) \eta_c(\overline{2980}) \underline{3055}_{+25}^{-18}$	$(\overline{3225}) J/\psi(\overline{3097}) \underline{3102}_{-8}^{+4}$
$(\overline{2011}) D_s(\overline{1968}) \underline{(1981)}$ $(\overline{1836}) D(\overline{1870}) \underline{(1875)}$	$(\overline{2109}) D_s^*(\overline{2112}) \underline{(2031)}$ $(\overline{1998}) D(\overline{2010}) \underline{(1962)}$
$(\overline{958}) \eta'(\overline{958}) \underline{(958)}$ $(\overline{548}) \eta(\overline{548}) \underline{(548)}$ $(\overline{478}) K(\overline{494}) \underline{(510)}$ $(\overline{140}) \pi(\overline{140}) \underline{(140)}$	$(\overline{850}) \underline{(1020)} \phi(\overline{1020}) \underline{(1020)} \underline{(835)}$ $(\overline{782}) \underline{(770)} / \rho(\overline{775}) \underline{(780)} \underline{(782)}$ $K^*(892) \quad \omega(782)$
CJ Model Exp. This work	CJ Model Exp. This work

Both spacelike and timelike form factors can be computed in LFQM.

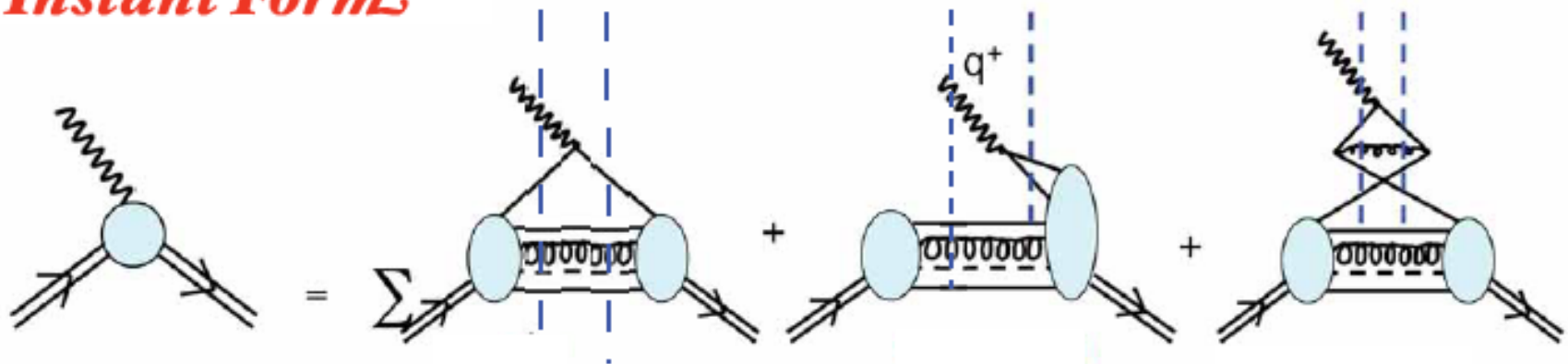


H.-M. Choi, H.-Y. Ryu, C.-R. Ji,
PRD96, 056008 (2017)

LFQM applications since 1997
~ 28 papers in peer reviewed journals

Calculation of Form Factors in Equal-Time Theory

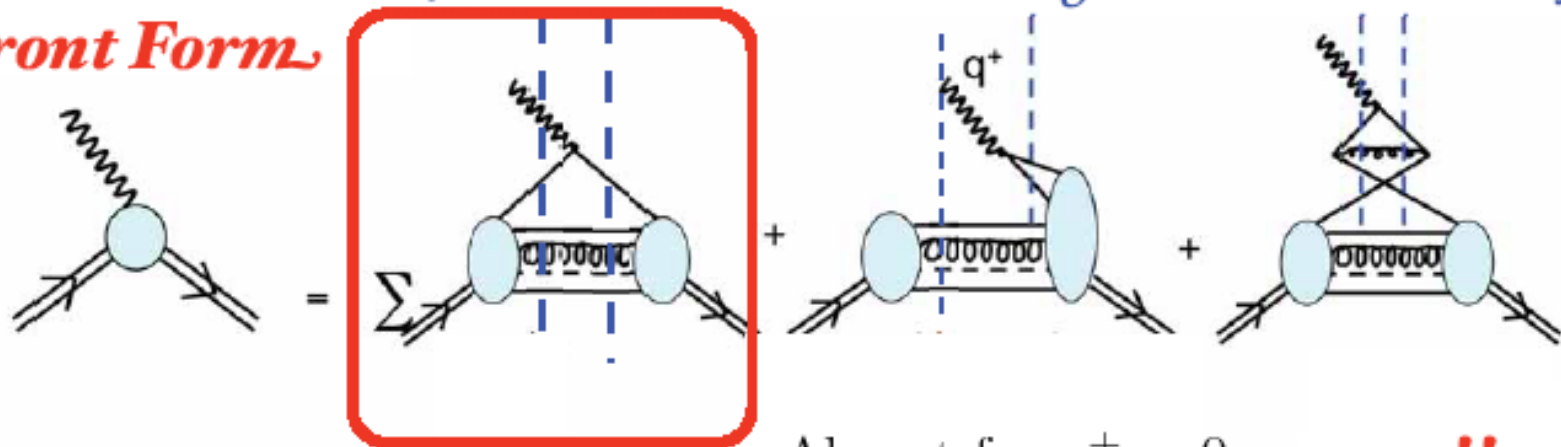
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

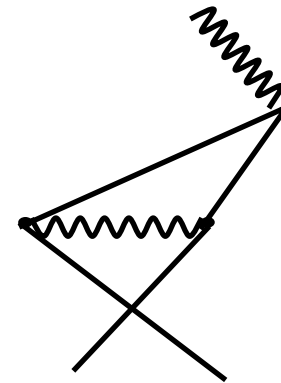
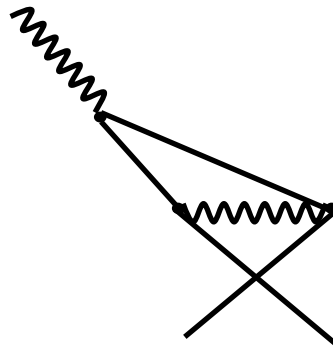
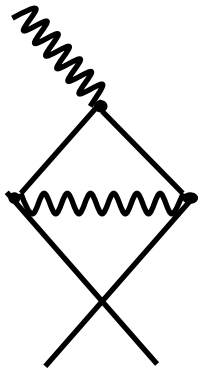
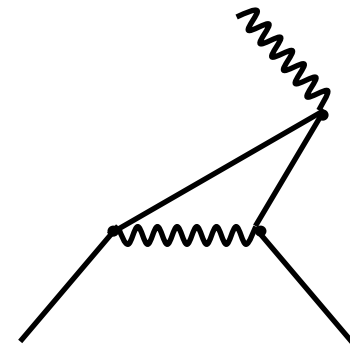
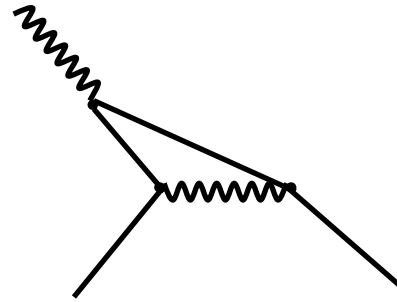
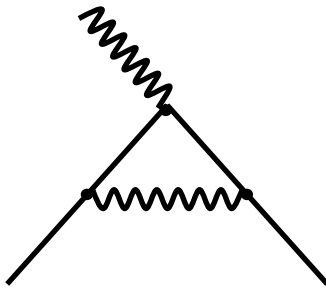
Front Form



Absent for $q^+ = 0$ **zero !!**

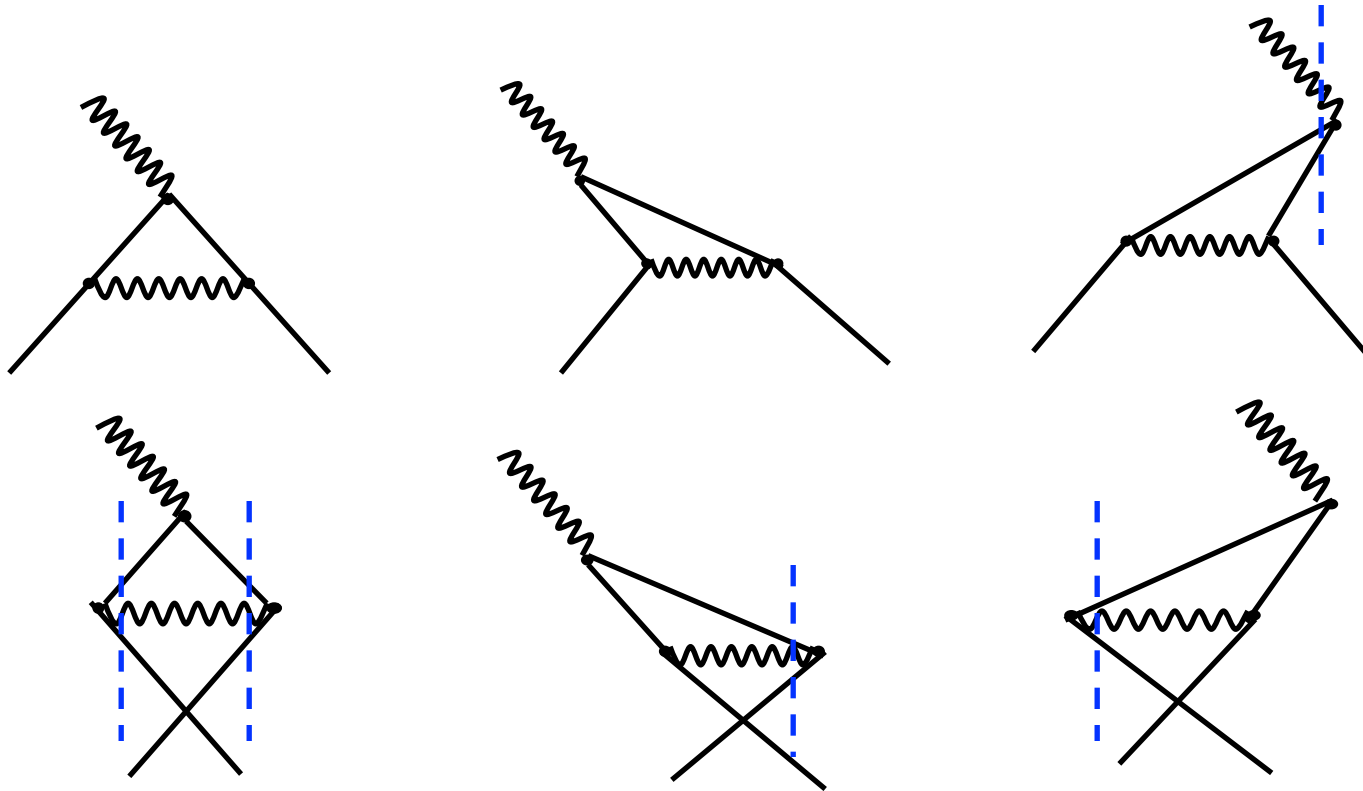
g-2 calculation

t →

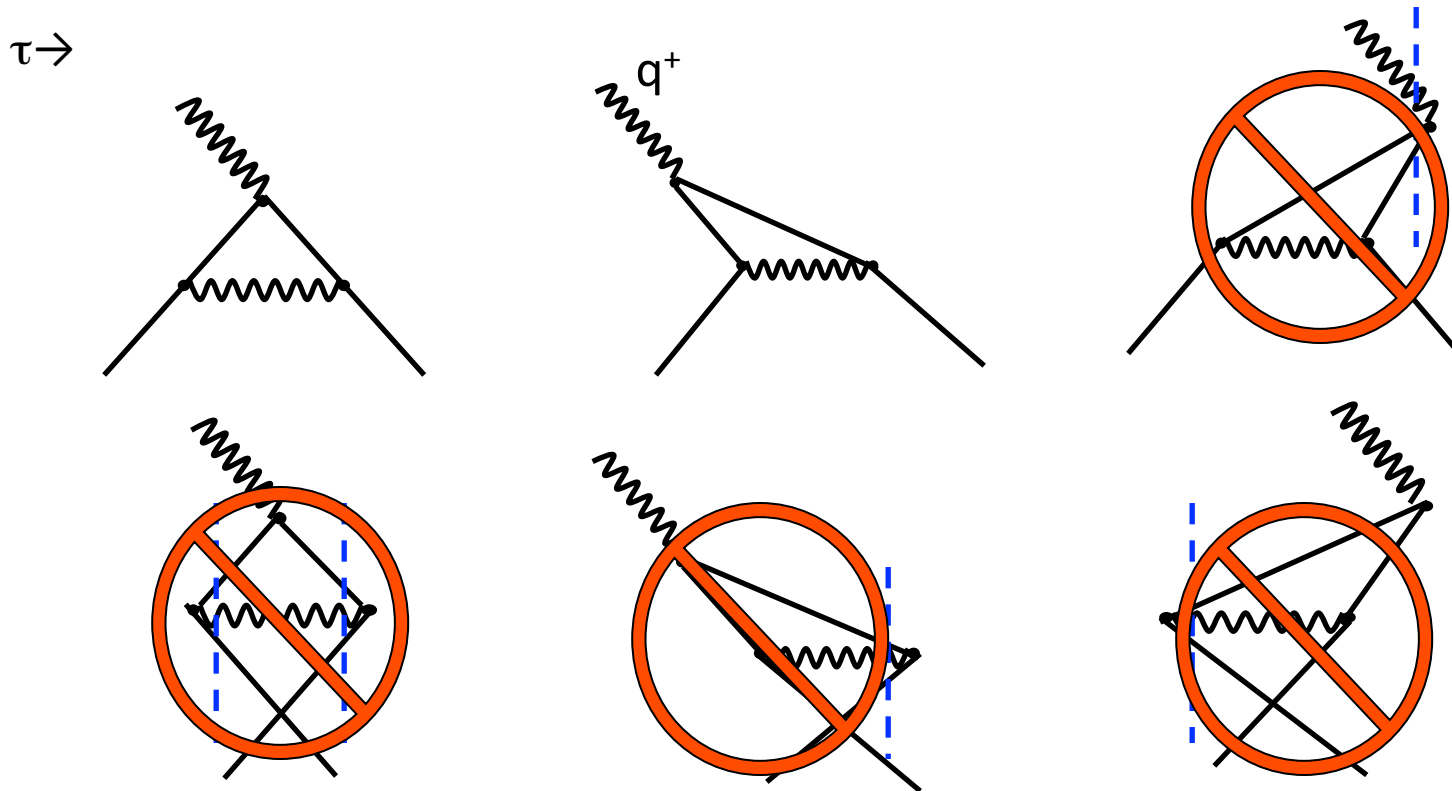


g-2 calculation

t →

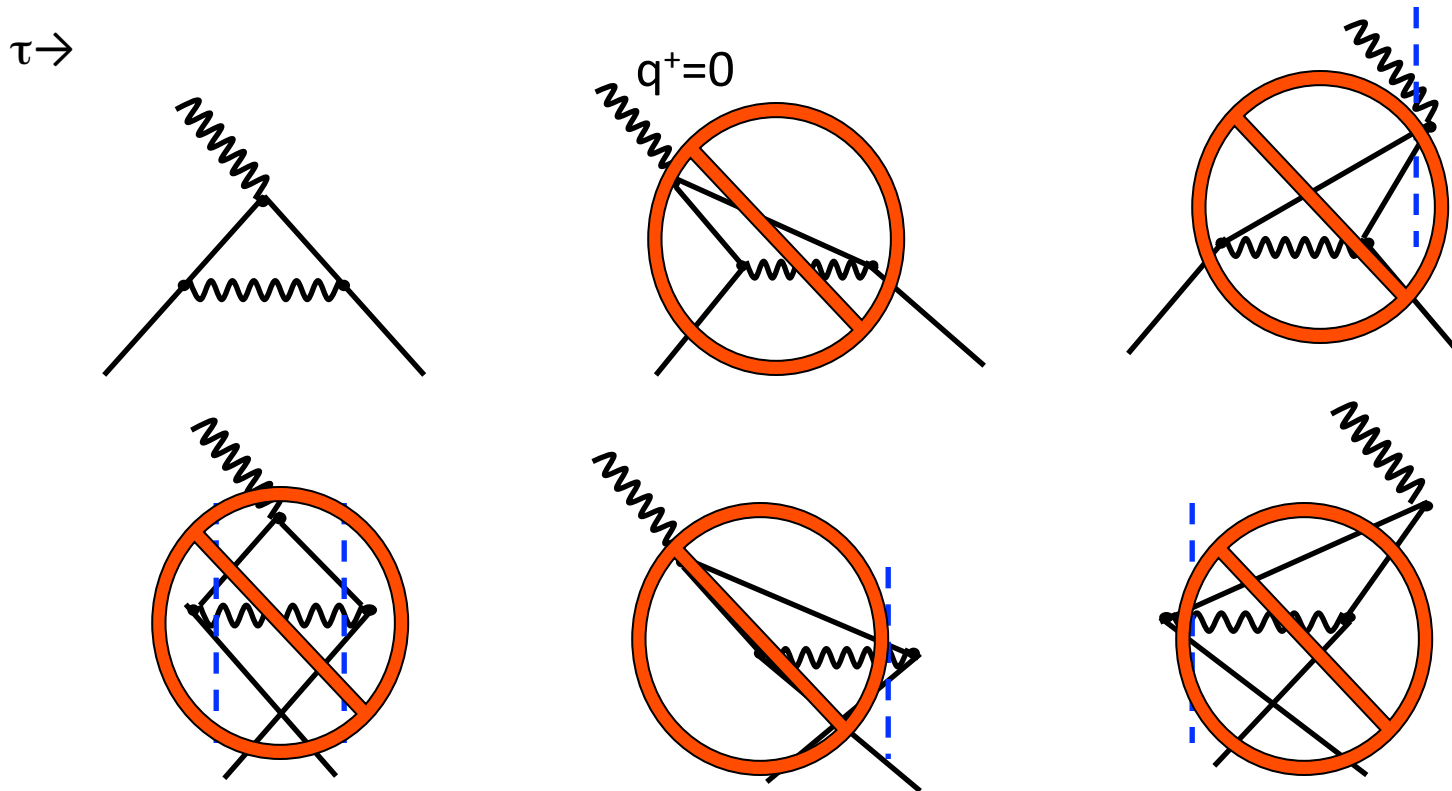


g-2 calculation



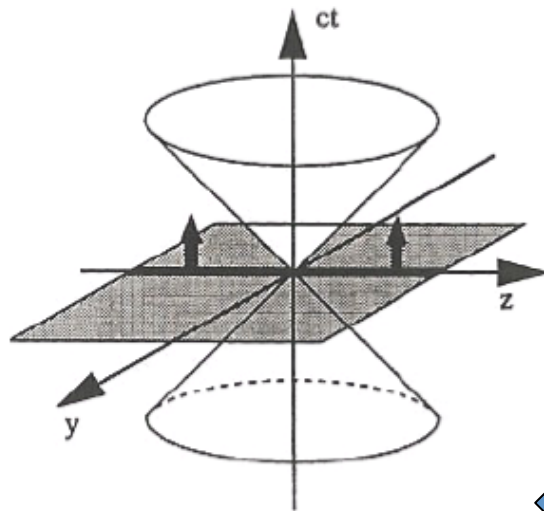
- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

g-2 calculation



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

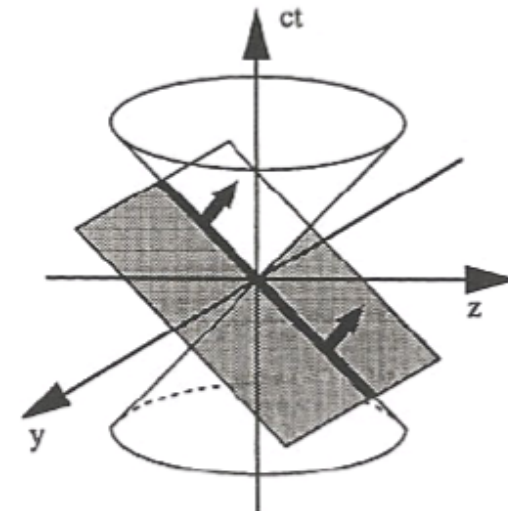
Dirac's Proposition



The instant form



1949



The front form



Can they be linked?

Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

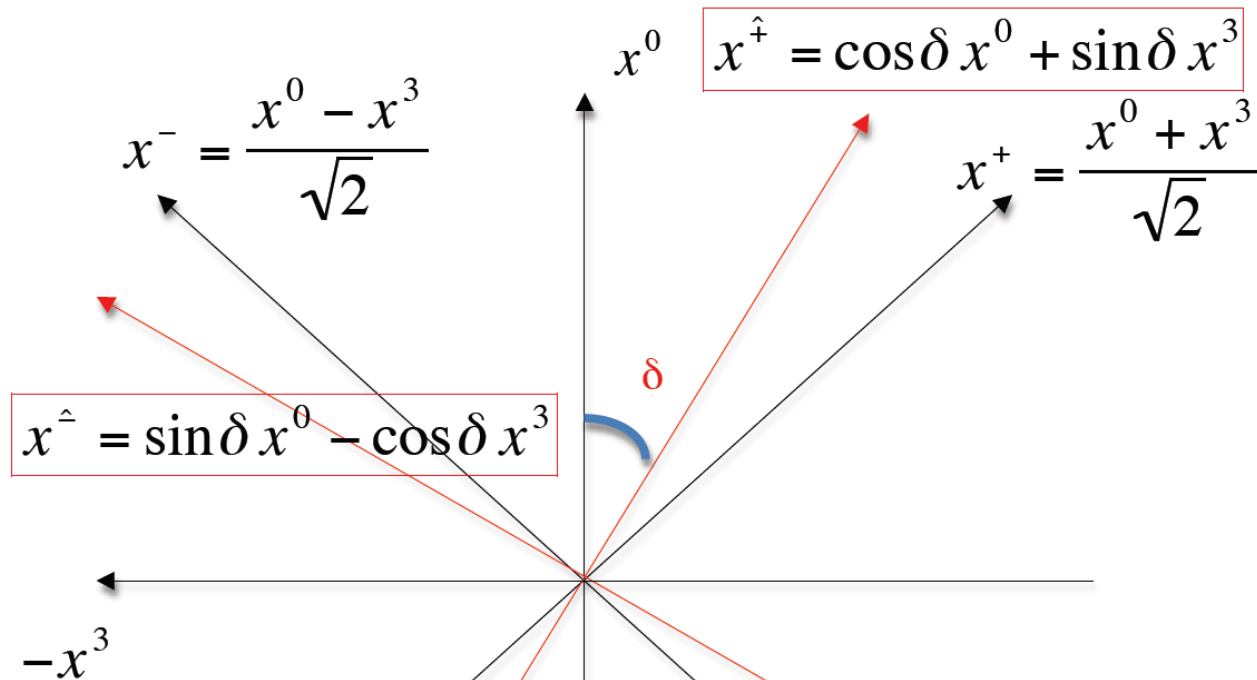
T-dept QFT, LQCD, IMF, etc.

Innovative approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

Interpolation between Instant and Front Forms



K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

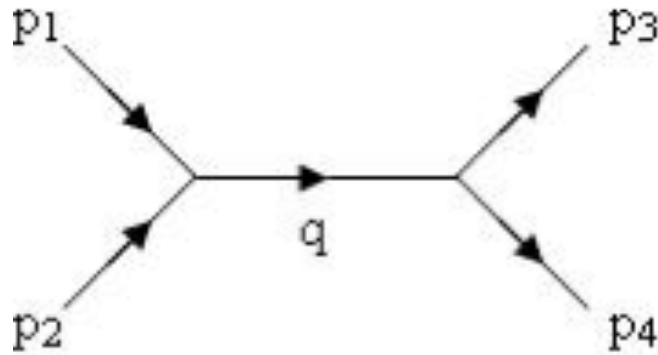
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

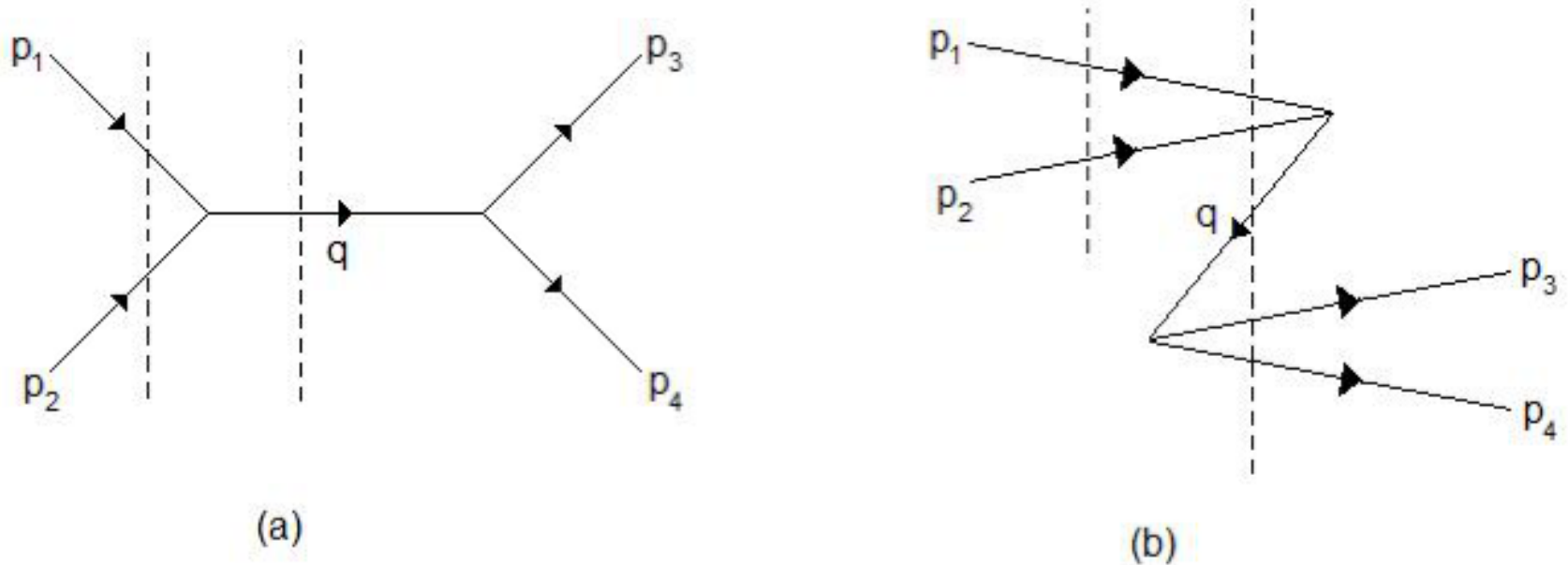
C.Ji, Z.Li, B.Ma and A.Suzuki, arXiv:1805.06599 – QED

$e^+e^- \rightarrow \mu^+\mu^-$

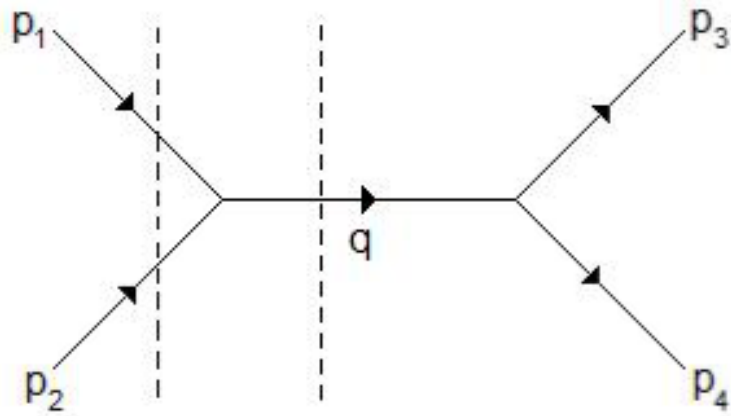


$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2}$$

Feynman Diagram: Invariant under all Poincaré generators

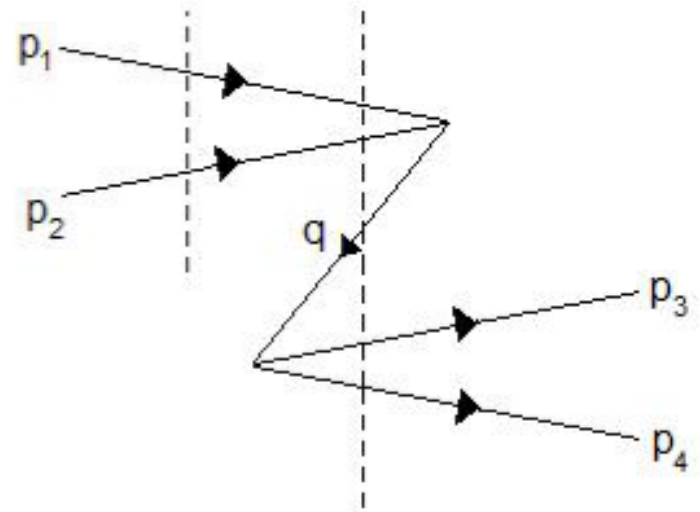


Individual Time-Ordered Diagrams: Invariant under stability group
Kinematic vs. Dynamic Generators



(a)

$$\frac{1}{E_1 + E_2 - Eq}$$

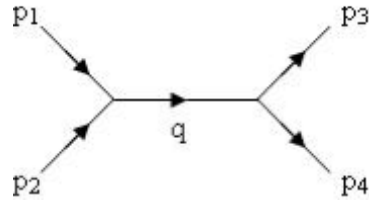


(b)

$$\begin{aligned} & - \frac{1}{Eq + E_3 + E_4} \\ & = - \frac{1}{Eq + E_1 + E_2} \\ & \rightarrow 0 \end{aligned}$$

S.Weinberg, PR158,1638(1967)
 “Dynamics at Infinite Momentum”

Note however this is still in the instant form.



$$\delta = 0$$

$$p_0 = p^0$$

$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$

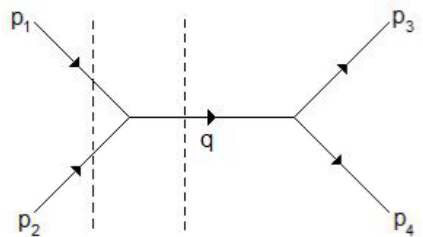
$$p_{\hat{+}} = p^0 \cos \delta - p^3 \sin \delta$$

$$p_{\hat{-}} = p^0 \sin \delta + p^3 \cos \delta$$

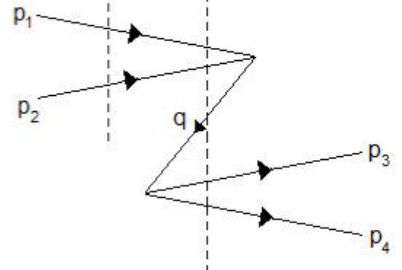
$$\delta = \pi/4$$

$$p_+ = p^-$$

$$p_- = p^+$$



(a)



(b)

$$\frac{1}{2q^0} \left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0} \right)$$

$$\frac{1}{2\omega_q} \left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}} \right)$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}(\bar{\mathbf{q}}_{\perp}^2 + m^2)}$$

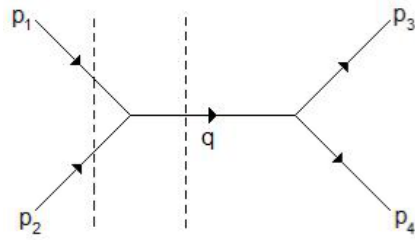
$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

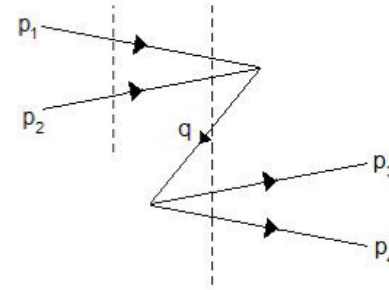
$$\frac{1}{P^+} \left\{ P^- - \frac{(\bar{\mathbf{P}}_{\perp}^2 + m^2)}{2P^+} \right\}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \rightarrow \frac{2}{\mathbb{C}} - \frac{\bar{\mathbf{q}}_{\perp}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$

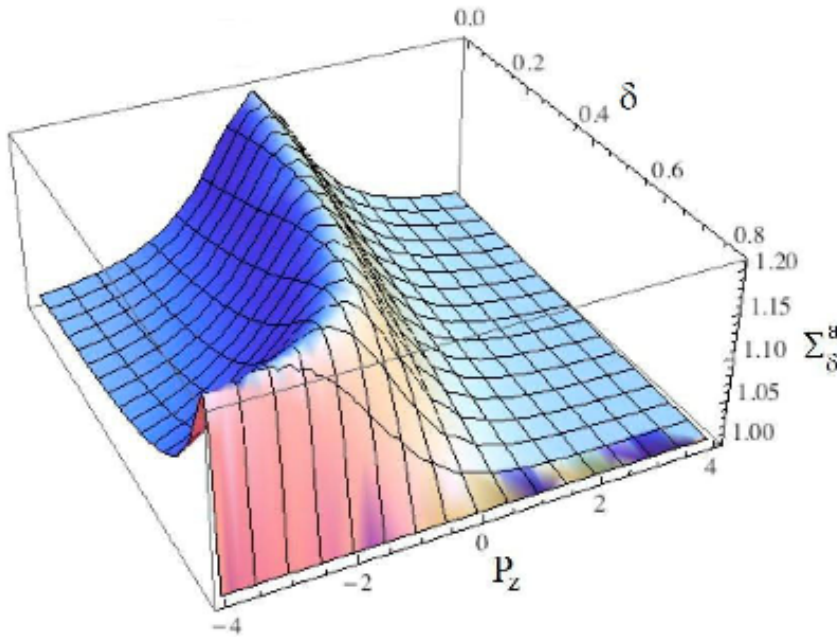
$$\rightarrow \infty \text{ as } \mathbb{C} \rightarrow 0$$



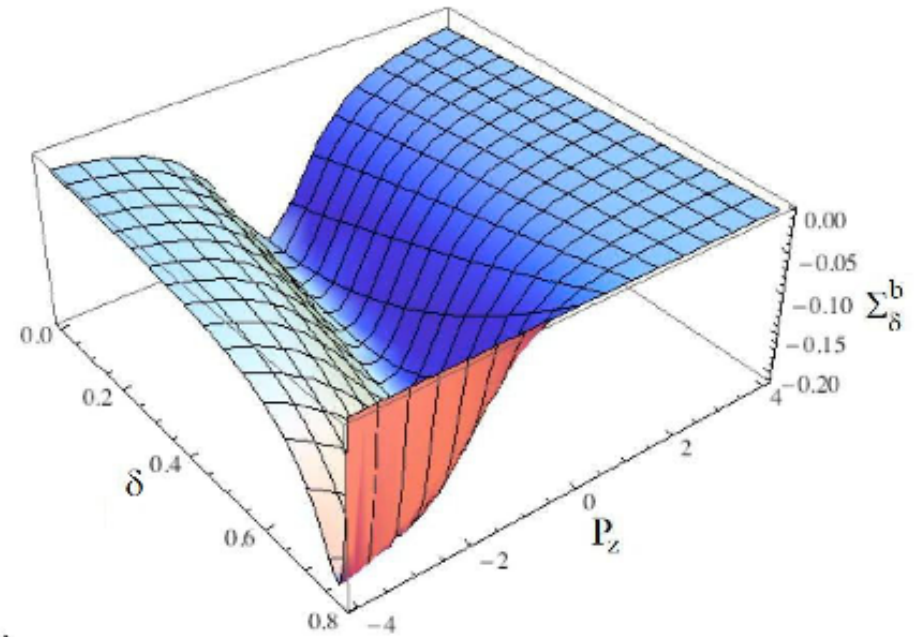
(a)



(b)



(a)

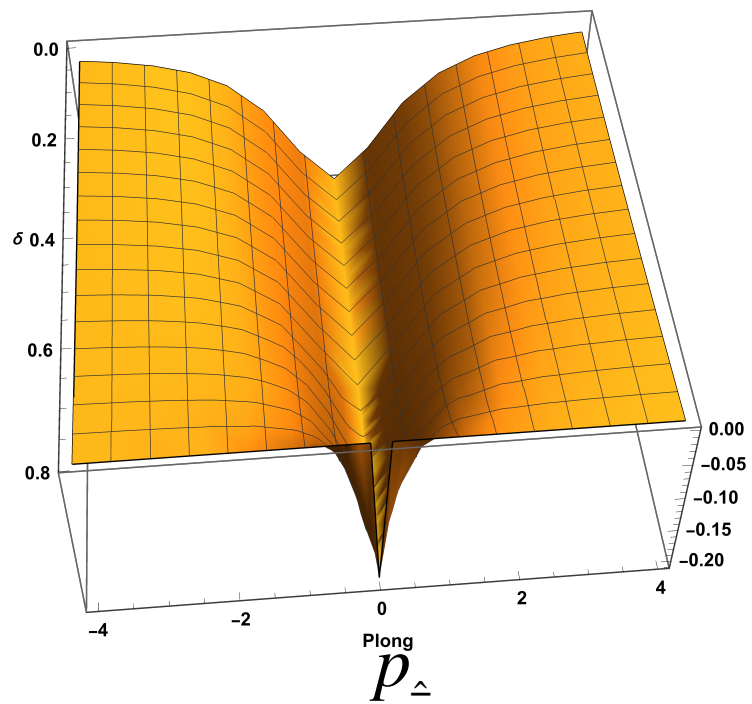
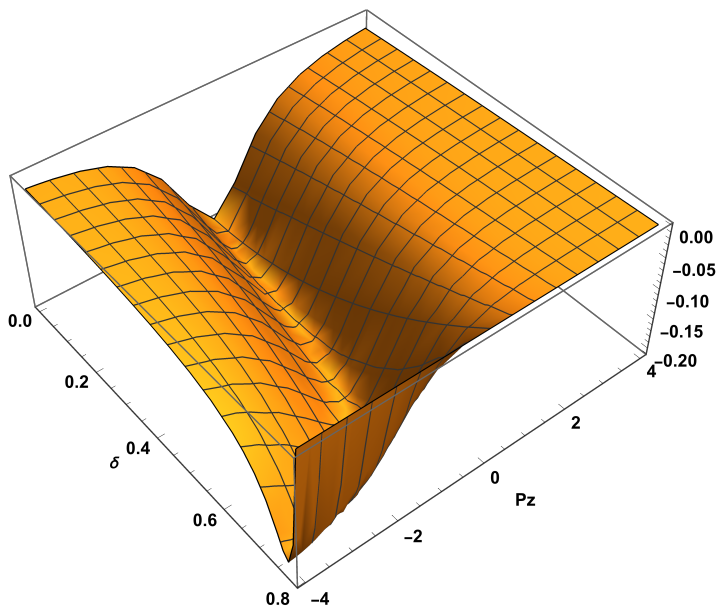
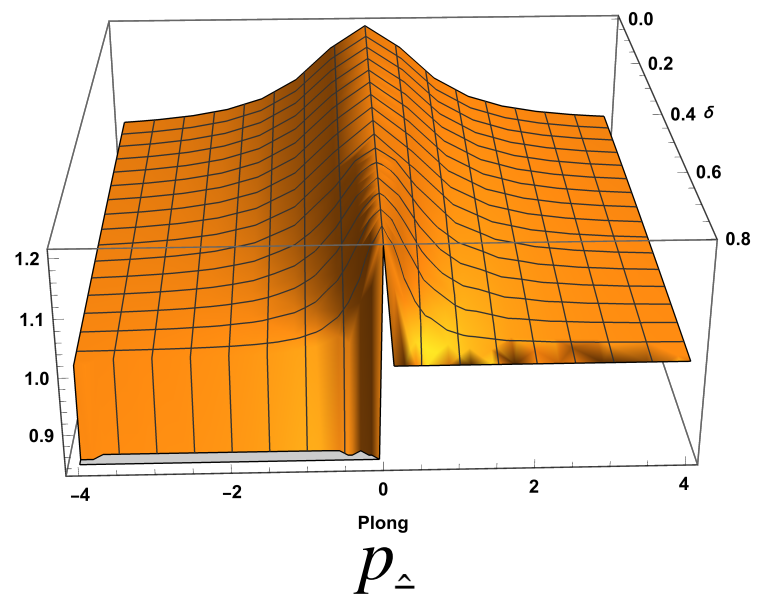
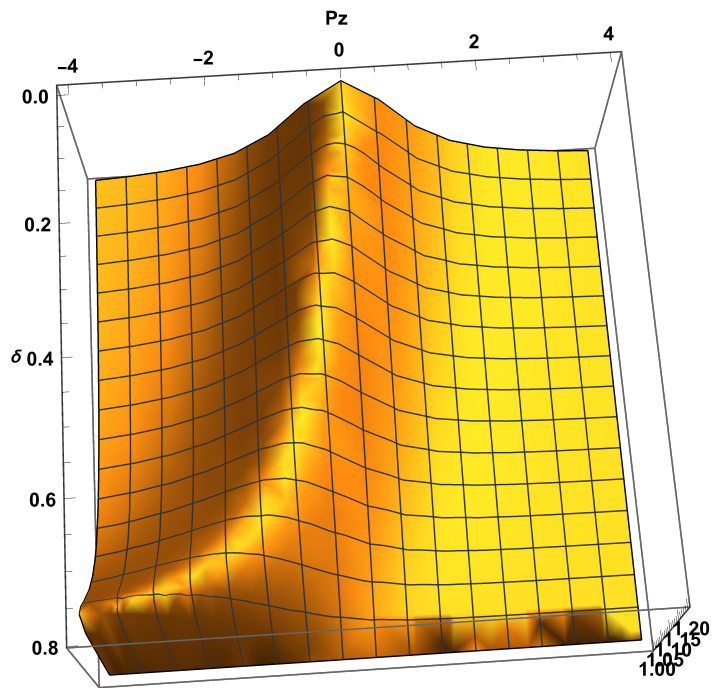


(b)

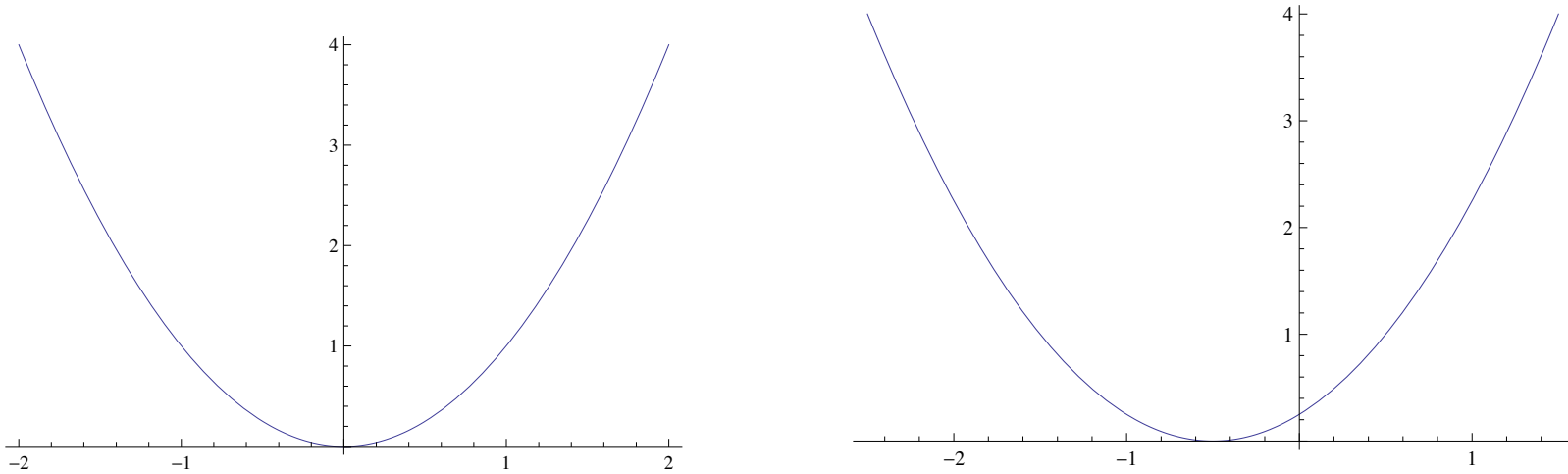
$$\Sigma(a)+\Sigma(b)=1/(s-m^2) ; s=2 \text{ GeV}^2, m=1\text{GeV}$$

$$\text{J-shape peak \& valley : } P_z = -\sqrt{\frac{s(1-C)}{2C}} ; C = \cos(2\delta)$$

As $C \rightarrow 0$, $P^+ = P^0 + P_z \rightarrow 0$ leads to LF Zero-modes.



Symmetry Breaking



$$\phi \rightarrow \phi' = \phi + v$$

$$L \rightarrow L' = L - m^2 v \phi - \frac{1}{2} m^2 v^2$$

$$P_{\hat{\dagger}} \rightarrow P'_{\hat{\dagger}} = P_{\hat{\dagger}} + \frac{(m^3 \ell)^{1/2} v}{C^{1/4}} (a_0 + a_0^+), \text{ where } C = \cos 2\delta$$

Nontrivial Vacuum State

$$|0\rangle \rightarrow |\Omega\rangle$$

Translation in scalar field: $\phi \rightarrow \phi' = \phi + v$

$$|\Omega\rangle = \exp\left(i \int_{-l}^{+l} dx^{\hat{}} v \pi(x^{\hat{}})\right) |0\rangle$$

$$\pi(x^{\hat{}} = 0, x^{\hat{}}) = -i \sum_{n=-\infty}^{\infty} \left(\frac{\pi}{l}\right) \sqrt{\frac{\omega_n}{4\pi}} \left[a_n e^{-i\left(\frac{n\pi}{l}\right)x^{\hat{}}} - a_n^+ e^{i\left(\frac{n\pi}{l}\right)x^{\hat{}}} \right]$$

$$|\Omega\rangle = \exp\left[-(C^{1/2} m l) \frac{v^2}{2}\right] \exp\left[-(C^{1/2} m l)^{1/2} v a_0^+\right] |0\rangle$$

Condensation of Zero-Modes

Vacuum Energy

$$P_{\hat{\dagger}} |\Omega\rangle = E_{\Omega} |\Omega\rangle$$

$$a e^{\alpha a^+} |0\rangle = \alpha e^{\alpha a^+} |0\rangle$$

$$P_{\hat{\dagger}} |\Omega\rangle \rightarrow \left[\frac{m\nu}{C^{1/2}} a_0^+ a_0 + \frac{(m^3 \ell)^{1/2} \nu}{C^{1/4}} (a_0 + a_0^+) \right] \exp\left[-(C^{1/2} m \ell)^{1/2} \nu a_0^+\right] |0\rangle$$

$$= (-m^2 \nu^2 \ell) \exp\left[-(C^{1/2} m \ell)^{1/2} \nu a_0^+\right] |0\rangle$$

$$E_{\Omega} = -m^2 \nu^2 \ell = \int_{-\ell}^{+\ell} \left(-\frac{1}{2} m^2 \nu^2\right) dx^{\hat{\dagger}}$$

Independent of interpolation angle!

Recovery of Trivial Vacuum in LFD?

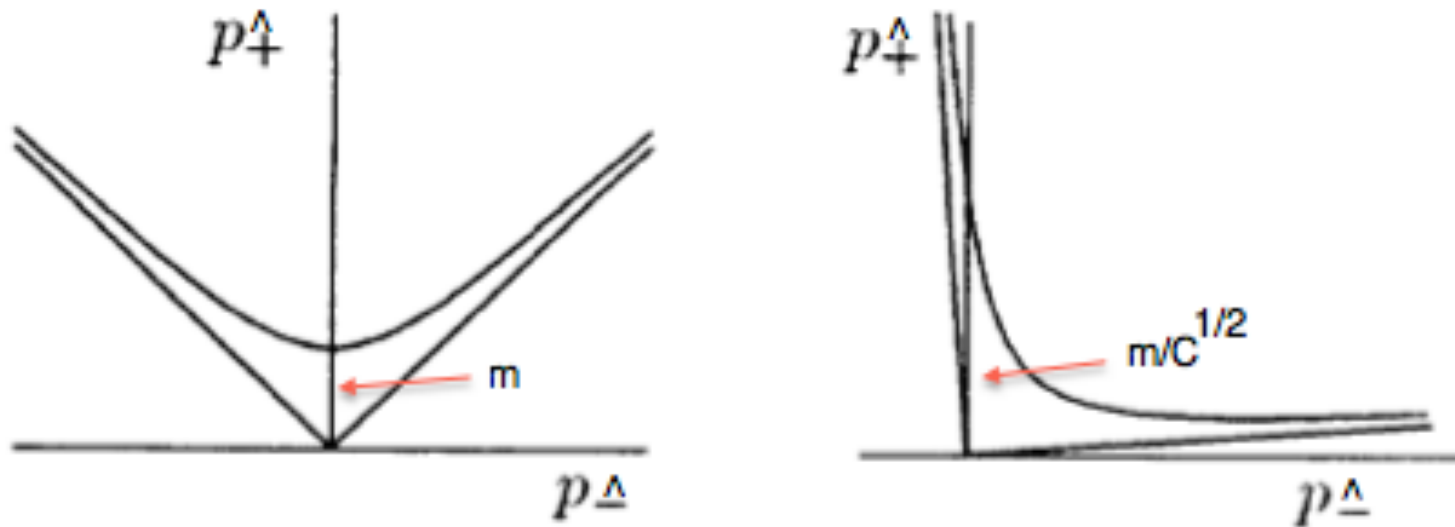
$$|\Omega\rangle = \exp\left[-(C^{1/2}m\ell)\frac{v^2}{2}\right]\exp\left[-(C^{1/2}m\ell)^{1/2}va_0^+\right]|0\rangle$$

$$|\Omega\rangle \rightarrow |0\rangle \quad \text{as} \quad C \rightarrow 0$$

However, E_Ω and $\langle \Omega | \phi(x) | \Omega \rangle = -v$

are still independent of interpolation angle!

What is going on?

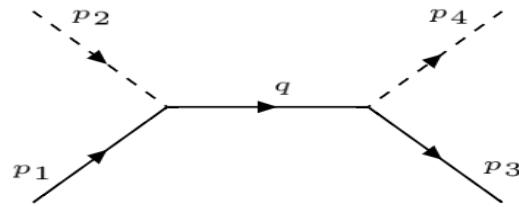


$$\langle \Omega | \phi(x) | \Omega \rangle$$

$$= \langle 0 | \exp\left[(C^{1/2} m \ell)^{1/2} \nu (a_0^+ - a_0)\right] \left(\frac{a_0 + a_0^+}{2(C^{1/2} m \ell)^{1/2}}\right) \exp\left[-(C^{1/2} m \ell)^{1/2} \nu (a_0^+ - a_0)\right] | 0 \rangle$$

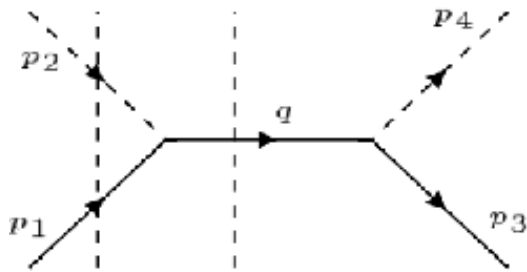
$$= -\nu$$

Complication is transferred from vacuum to operator.

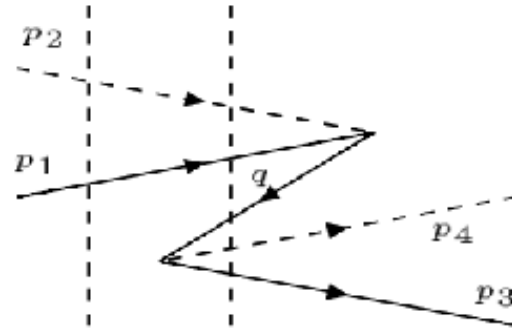


Constrained Degrees of Freedom of Fermion in Light-Front Dynamics

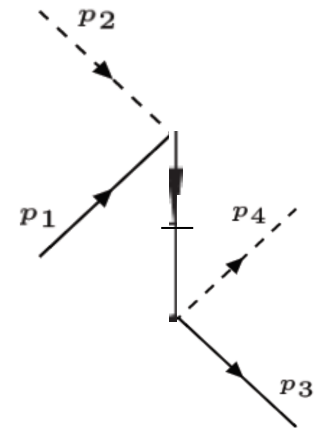
Fermion Propagator



(a)



(b)



$$\begin{aligned}
 \Sigma_a^{\text{IFD}} + \Sigma_b^{\text{IFD}} &= \frac{1}{2q_{on}^0} \left(\frac{\not{q} + m}{q^0 - q_{on}^0} - \frac{\not{q} + m}{q^0 + q_{on}^0} \right) \\
 &= \frac{1}{2q_{on}^0} \frac{2q_{on}^0 (\not{q} + m)}{(q^0)^2 - (q_{on}^0)^2} \\
 &= \frac{\not{q} + m}{q^2 - m^2}
 \end{aligned}$$

$$\Sigma_{a, \delta \rightarrow \frac{\pi}{4}} = \frac{\not{q}_{on} + m}{q^2 - m^2}$$

$$\Sigma_{b, \delta \rightarrow \frac{\pi}{4}} = \frac{\gamma^+}{2q^+}$$

*S.-J. Chang and T.-M. Yan,
PRD7, 1147(1973)*

$$\frac{1}{\not{q} - m} = \sum_s \frac{u(q, s) \bar{u}(q, s)}{q^2 - m^2} + \frac{\gamma^+}{2q^+}$$

$$A^{\hat{+}} = 0, \quad \partial_{\hat{-}} A_{\hat{-}} + \partial_{\perp} \mathbf{A}_{\perp} \mathbb{C} = 0 \quad (\mathbb{C} = \cos 2\delta)$$

$$\delta \rightarrow 0 \\ (\mathbb{C} \rightarrow 1)$$

$$\delta \rightarrow \pi/4 \\ (\mathbb{C} \rightarrow 0)$$

$$A^0 = 0, \quad \nabla \cdot \mathbf{A} = 0$$

Coulomb Gauge

$$A^+ = 0$$

Light-front Gauge

C.Ji, Z. Li, and A. T. Suzuki, PRD91, 065020(2015)

$$\sum_{\lambda=\pm} \epsilon_{\hat{\mu}}^*(\lambda) \epsilon_{\hat{\nu}}(\lambda) = -g_{\hat{\mu}\hat{\nu}} + \frac{(q \cdot n)(q_{\hat{\mu}} n_{\hat{\nu}} + q_{\hat{\nu}} n_{\hat{\mu}})}{q_1^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{\mathbb{C} q_{\hat{\mu}} q_{\hat{\nu}}}{q_1^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{q^2 n_{\hat{\mu}} n_{\hat{\nu}}}{q_1^2 \mathbb{C} + q_{\hat{-}}^2}$$

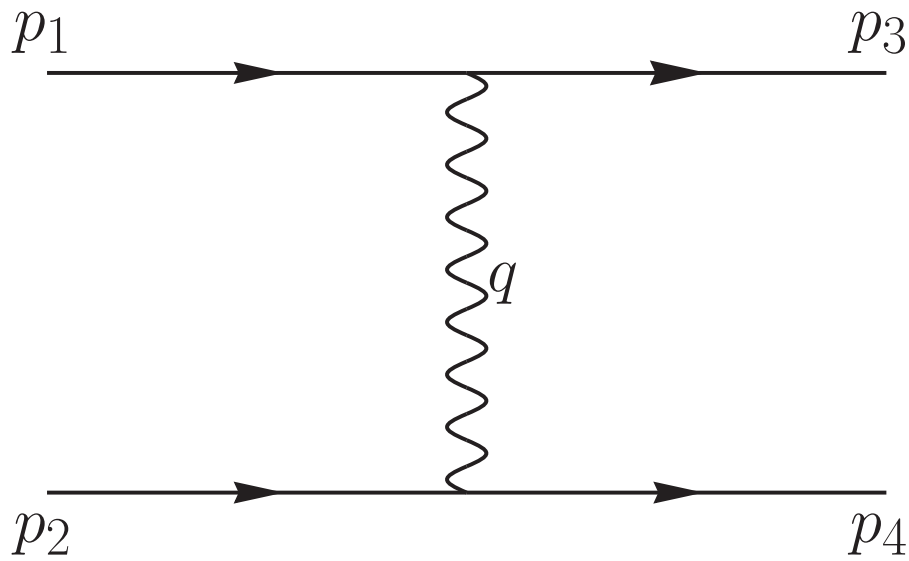
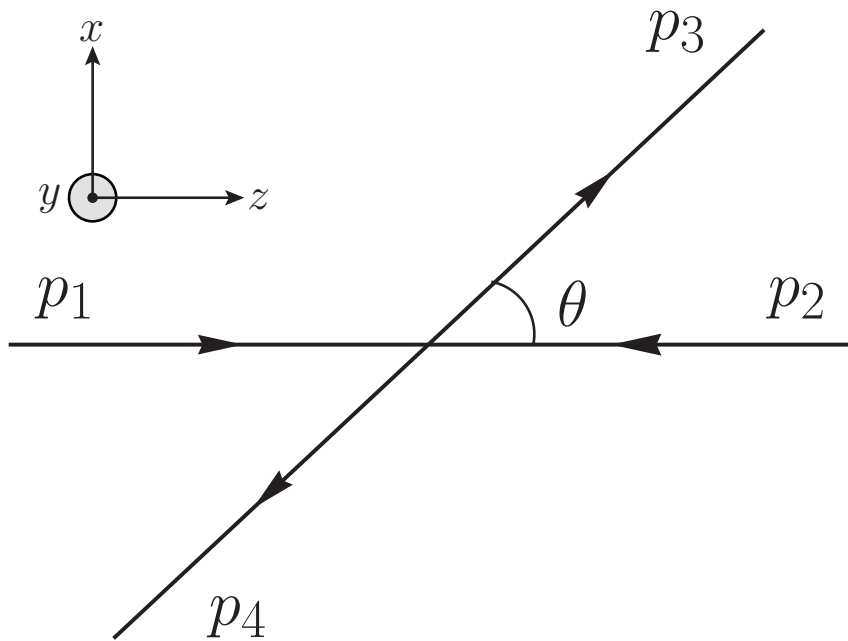
IFD

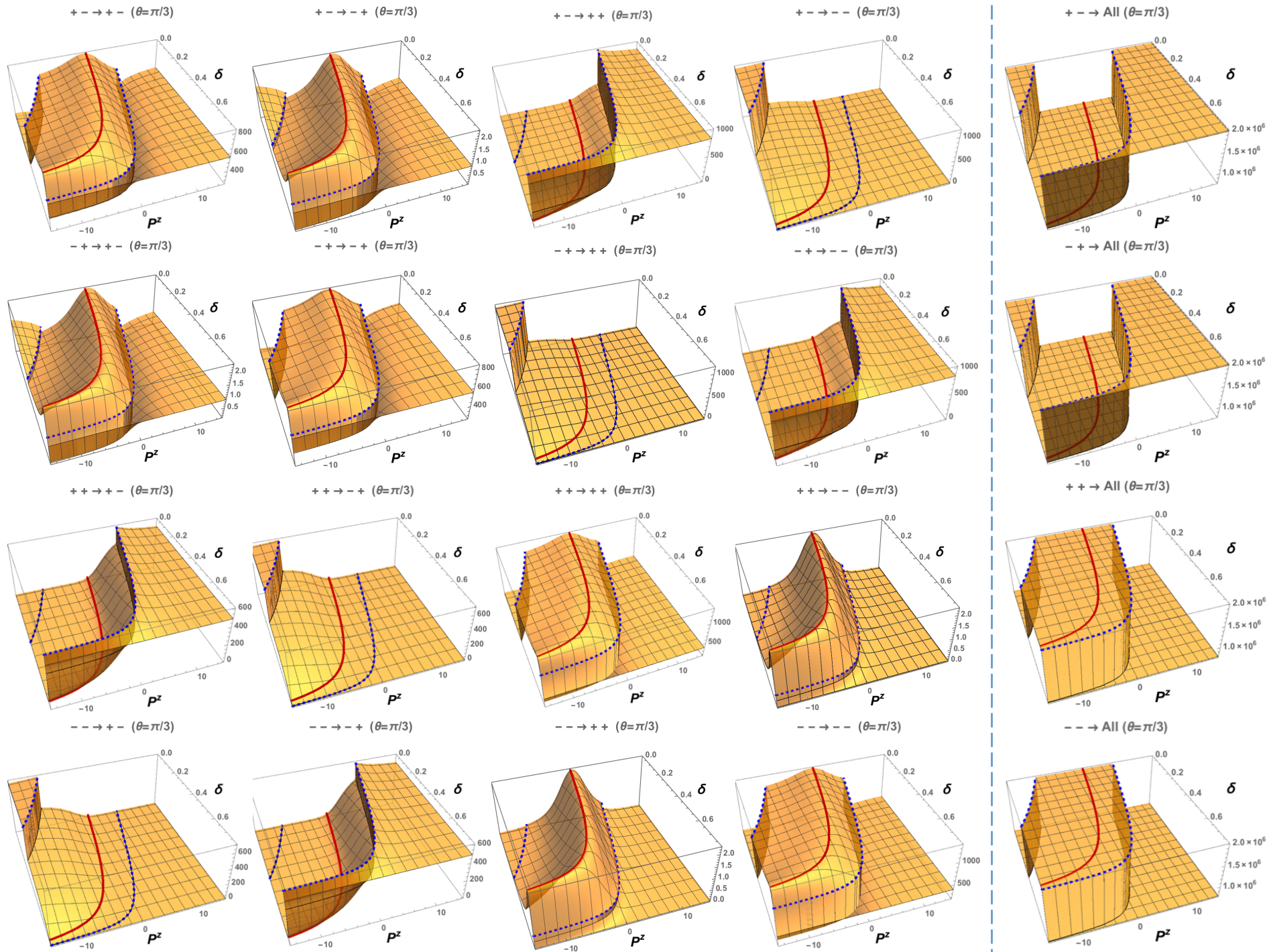
LFD

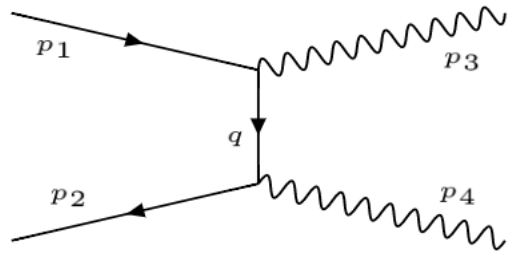
$$-g_{\mu\nu} + \frac{(q \cdot n)(q_{\mu} n_{\nu} + q_{\nu} n_{\mu})}{(q \cdot n)^2} - \frac{q^2 n_{\mu} n_{\nu}}{(q \cdot n)^2}$$

P.Srivastava and S. Brodsky, PRD64,045006(2001)

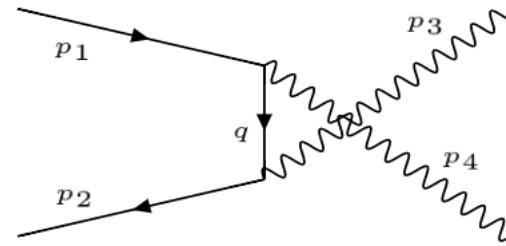
$$-\eta_{\mu\nu} + \frac{(q \cdot n)(q_{\mu} n_{\nu} + q_{\nu} n_{\mu})}{(q \cdot n)^2 - q^2} - \frac{q_{\mu} q_{\nu}}{(q \cdot n)^2 - q^2} - \frac{q^2 n_{\mu} n_{\nu}}{(q \cdot n)^2 - q^2}$$





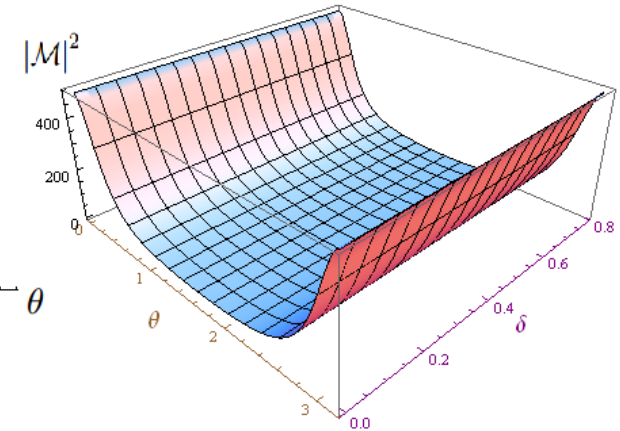
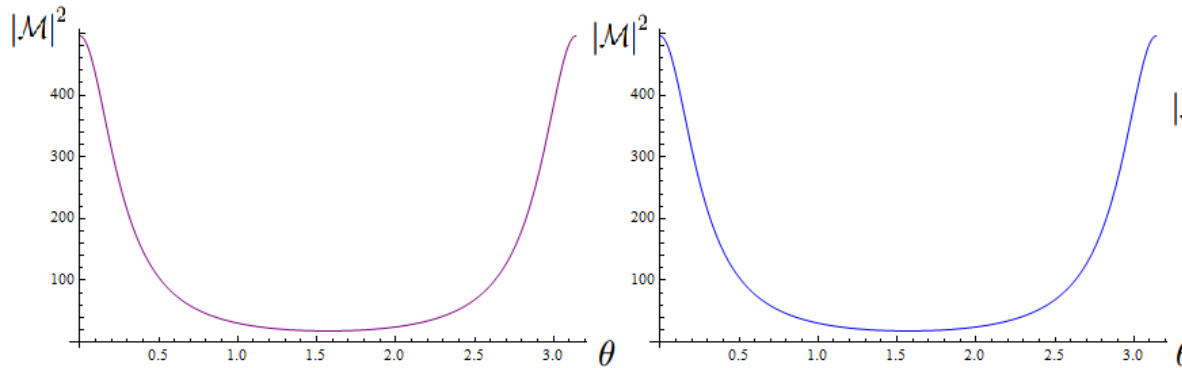


Direct Diagram



Exchanged Diagram

Including electron mass



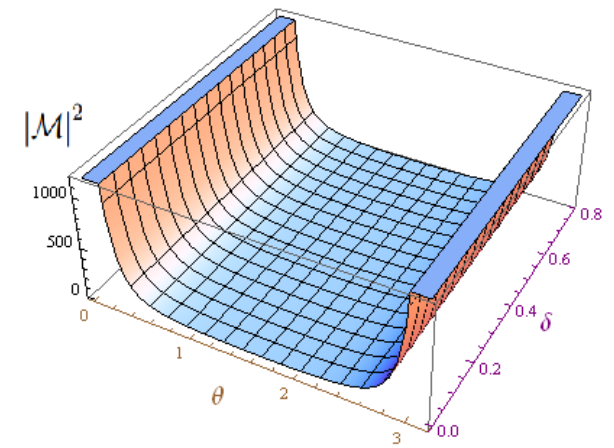
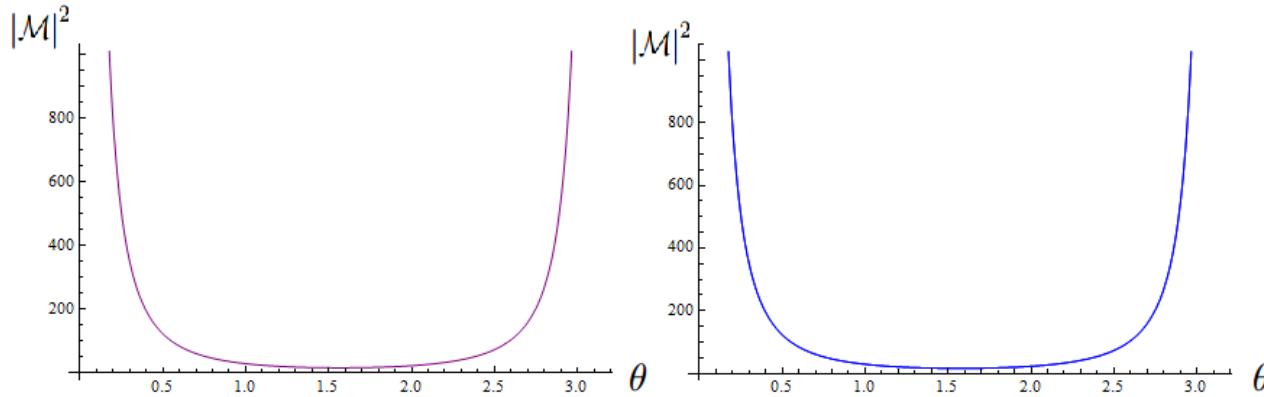
— Textbook calculation

— Our interpolation method

$$|\mathcal{M}|^2 = 2e^4 \left[\frac{u_m}{t_m} + \frac{t_m}{u_m} + 2m^2 \left(\frac{s_m}{t_m u_m} - \frac{1}{t_m} - \frac{1}{u_m} \right) - 4m^4 \left(\frac{1}{t_m^2} + \frac{1}{u_m^2} \right) \right]$$

where $t_m \equiv t - m^2$, $u_m \equiv u - m^2$, and $s_m \equiv s - 4m^2$.

Taking electron mass zero

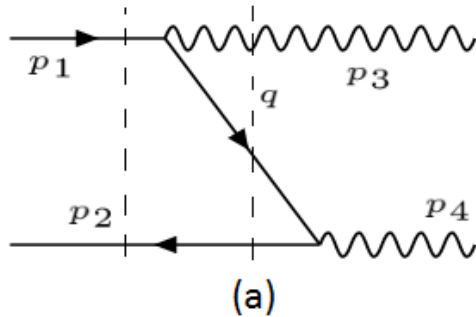


— Textbook calculation

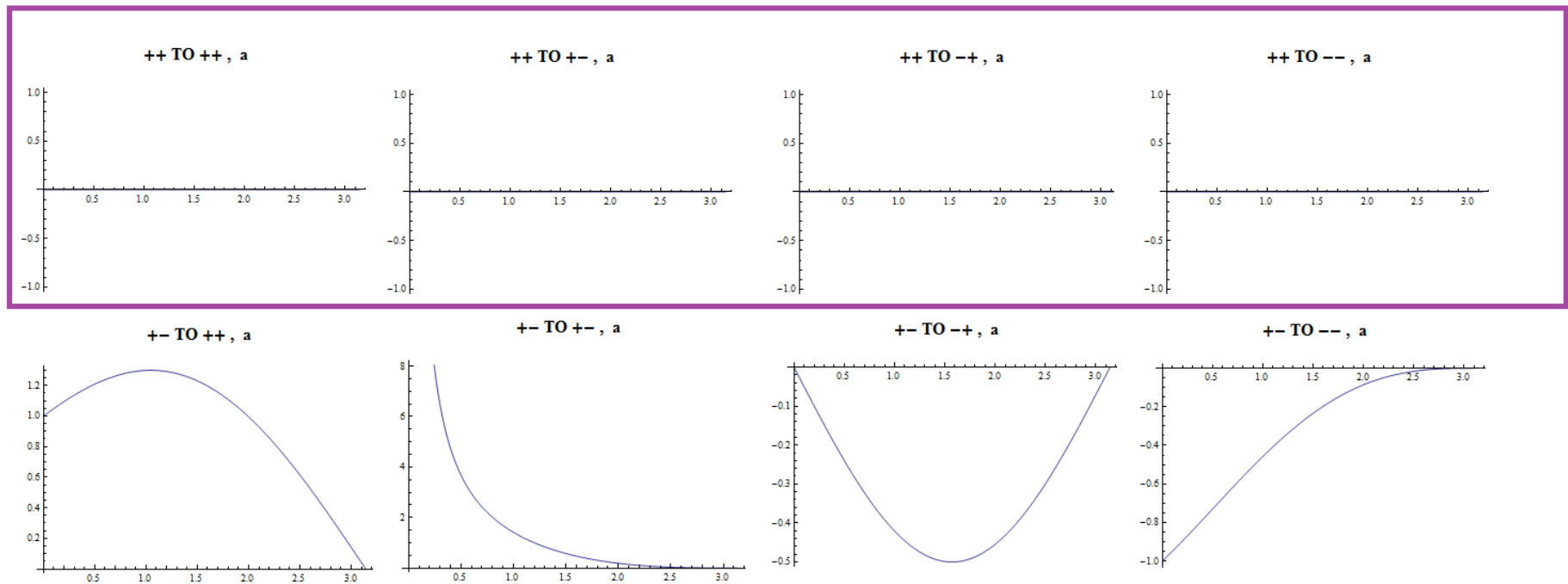
— Our interpolation method

$$|\mathcal{M}|^2 = 2e^4 \left(\frac{u}{t} + \frac{t}{u} \right)$$

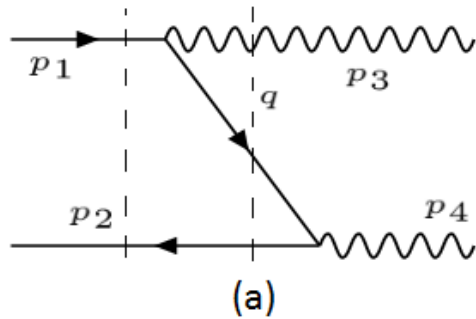
Scattering Angle Dependence of the Annihilation Amplitudes: Chirality



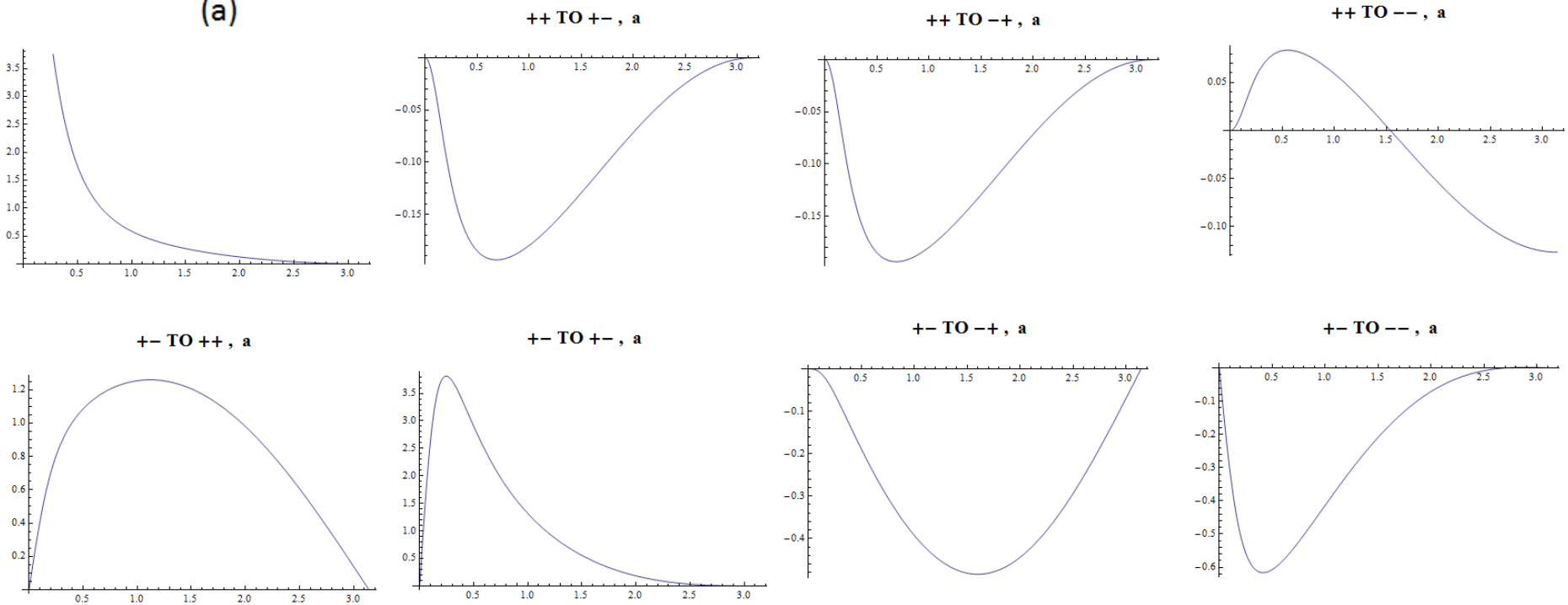
When $m_e=0$, chirality is conserved.



Scattering Angle Dependence of the Annihilation Amplitudes: Chirality



When $m_e \neq 0$, no such property.



Outlook

- “A method is more important than a discovery, since the right method will lead to new and even more important discoveries.” - Lev Landau
- LFQM saves a lot of dynamical effort in the hadron spectroscopy and structure study.
- Correspondence between IFD and LFD may shed more light on bridging the LFQM and the QCD.