Nuclear Physics School 2018

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Third Lecture

Light-Front Quark Model

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3.3 H-atom Spectra $\mathcal{L}_{\mathcal{A}}$ J. **Example 21 and 32 and 33 and 33** correction, i.e. the fine structure. **1. Relativistic Correction** One part of the fine structure comes from the relativistic correction to the kinetic energy. As we Therefore, the actual rate of electron spin change is obtained from Eqs. (3.80), (3.80), (3.80), (3.80) and (3.83) and (3.83), (3.83), (3.80), (3.80), (3.83), (3.83), (3.83), (3.83), (3.83), (3.80), (3.80), (3.83), (3.83) as

$$
\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137.036}
$$
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$$
E_n = -\alpha^2 mc^2 \left(\frac{1}{2n^2}\right)
$$
\n
$$
\approx -13.6eV/n^2.
$$
\n
$$
\Delta E_{fs} = \Delta E_{rel} + \Delta E_{so}
$$
\n
$$
= -\alpha^4 mc^2 \frac{1}{4n^4} \left(\frac{2n}{j + \frac{1}{2}} - \frac{3}{2}\right)
$$
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$$
\vec{\mu}
$$
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$$
= \frac{\vec{\mu}}{2m} \left(\frac{2n}{m^2 c^2} + \cdots\right)
$$
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$$
\Delta E_{so} = \frac{e^2}{2m^2 c^2 r^3} \vec{L} \cdot \vec{s}
$$
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$$
= -\alpha^4 mc^2 \frac{1}{4n^4} \left(\frac{2n}{j + \frac{1}{2}} - \frac{3}{2}\right)
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line spectra($E_n = -\alpha^2 mc^2 \left(\frac{1}{2n^2}\right)$ *n* The solid line spectra($E_n = -\alpha^2 mc^2(\frac{1}{2n^2})$) are without corrections. The m electron and proton are denoted by *m* and *M* respectively. electron and proton are denoted by *m* and *M* respectively. $\left(\frac{1}{2n^2}\right)$ 2 $\frac{2}{2}$ *n* $E_n = -\alpha^2 mc^2 \left(\frac{1}{2m^2}\right)$ are without corrections. The masses of

 (QQ) (*ee*), *e e* , $\mathcal M$ For the positronium, m/M = 1: *i* the reduced mass effect, i.e. $m_{red} = \frac{m}{2}$, $m_{red} = \frac{m}{2}$ *m M* the reduced mass effect, i.e. $m_{red} = \frac{m}{2}$, $\overline{\overline{a}}$ and \overline{a} there is no distinction and $2P$ $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{2}{2}$
 $\frac{2}{2}$ and $\frac{2}{2}$ *ffinandnikht
Enydrogen* $\sum_{i=1}^{n}$ 1 $\frac{1}{2}$ 2 2 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ *red positronium* the same order as the upper support. *E positronium*
Le fhe same order as the *n*ine cstrip and \int [⎞] [⎜] χ $\frac{2}{100}$ = $\frac{2}{100}$ = $\frac{2}{100}$ = $\frac{2}{100}$ \overline{z} (since the positronium spectra. The p $13.60V$ T_{α} in $\frac{m}{\sqrt{2\pi}}$ the nontranium m/N = 1. \mathbb{M} is the large difference in the large differe M Thus, there is no distinction between the fine structure. and the hypperflue structure positronium system. ϵ in the hositronium² system and the degeneracy of $2S_{1/2}^{20}$ and $2P_{2/2}$ states is already broken in the level of fine and h \mathcal{D} \mathbb{F} finand \mathcal{D} \mathbb{F} and \mathbb{F} \mathbb{F} Therefore, the Lamb shift which breaks the degeneracy of and 12 1999 states is less interesting in the positronium system.^{2} $\overline{(\overline{\omega_0})(\overline{\omega_0})}$ $\overline{(\overline{\omega_0} + \overline{\omega_0} + \overline{\omega_0} + \overline{\omega_0})(\overline{Q})}$ 2*S*_{1/2} and 2*P*_{1/2} $\overline{\mathbf{B}}$ is less interesting in the positronium system. However, the positronium system in the positronium system in the positronium system. However, the positronium system in the positronium system. However, the posit \mathfrak{m} \Box_{M} the positronium, m/w = 1: $\frac{1}{2}$ the reduced mass eff^{en}t is *m* = $\frac{m}{2}$ L^{-1} **u** *m M* $\frac{1}{\sqrt{16}}$ there is no distinction the fine structure and the hyperfine structure in the positronium system, the divertine structure is at the same order as the nine cstructure and $2P_{1/2}$ s_1 is positromatic system, and the dependency of $\frac{2}{\pi}$ $\frac{1}{2}$ and $\frac{1}{2}$ $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n}$ states is less interesting in the nositronium $\frac{25}{3}$ and $2P_{1/2}$ \overline{O} $\overline{$ $\left(\mathcal{L}\mathcal{L}\right)$ (ee $\mathcal{L}\left(\mathcal{L}\right)$), $\left(2\epsilon\right)$ \boldsymbol{m} *M m* \mathcal{M} **1.** Thus, the fine structure is no distinction between the fine structure \mathcal{M} $t_{1/2} = -t_{1/2}$ and $2P_{1/2}$ and $2P_{1/2}$ and $2P_{1/2}$ and $2P_{2/2}$ $\frac{3.1201}{2}$ and $\frac{2.172}{2}$ $\frac{1}{2}$ is at the hyperfine structure structure is at the $\frac{3}{4}$ $\frac{1}{2}$ $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ states is already broken in the level of fine and hyperfinance of the structure Therefore, the Lamb shift which breaks the degeneracy of anoghing $\frac{2}{2}$ $\frac{2}{2}$ states is less interesting in the positronium system.¹² $\left(\frac{\nu}{\nu}\right)$ is the positron system, <u>F</u> *m M* ϵ ne positronium, m/M = 1: $\sum_{i=1}^{N}$ and reduced indus critery, i.e. m_{red} \rightarrow γ The $\sqrt{4\pi}$ there is no distinglight between the fine structure \mathcal{E} the structure is at the same order as the the cstructure in $2P_{1/2}$ states is already broken in the level of fine and hyperfine mandflates is already broken in the level of fine and hyperfinement. Therefore, the Lamb shift which breaks the degeneracy of and \mathbb{Z} and \mathbb{Z} and \mathbb{Z} \mathbb{Z} and \mathbb{Z} $\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{3}$ $\overline{4}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{$ **Thus, in H-atom, in H-atom, the fine structure in H-atom, the fine structure are clearly are clearly are clear** \mathcal{L} *the reduced mass* $\frac{m}{\sqrt{2}}$ $\frac{m}{\sqrt{2}}$ $\frac{m}{\sqrt{2}}$ $\frac{m}{\sqrt{2}}$ $\Delta S_{1/2}$ and $\Delta P_{1/2}$ $\epsilon_{\rm B}$ and the hyperline equipment $\epsilon_{\rm B}$ in the positronium system. in the positronium² system and 1 ϵ states is already broken in the level of ϵ Therefore, the Lamb shift which be states is less interesting in the position. $\frac{1}{2}$ survey to the much analogous $\frac{1}{2}$ the quarkonium and positronium and positronium are bound-states of q $e^{i(\mathcal{U}\mathcal{U})}$ (eegs) and the function of $e^{i(\mathcal{U}\mathcal{U})}$ $d = 1$ η is replaced by the positronium system, the positron so that the positron so that the factor so that the factor η 61 € *m M* μ , i.e. $m_{red} = \frac{1}{2}$, $2*p*$ the fine structure $\sum_{1/2}^{\infty}$ and $2P_{1/2}$ $same$ order as the *rigge* C_{310} $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ states is already of $2S_{1/2}$ and $\Delta P_{1/2}$ in the combined structure. of fine and hypertinan onlined structure.
This high breaks the 1 2 2 ¹ 2 ² ² 2 ¹ 2 ² ² aks the degeneraty of analyzing μ ₂ and σ σ details to the positronium system $\sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{2} \frac{1}{2}$ and $2P$ σ $\frac{1}{2}$ σ $\frac{1}{2}$ $\sum_{i=1}^{\infty}$ (QQ) and $2P_1(e^{-}e^{+}),$ ²⁰ study of the positronium spectra. The positronium spectra. There are two important aspectra. The positronium spectra. The $\sqrt{2}$ $\sqrt{2}$ T in H-atom, the fine structure structure structure structure structure structure are clearly structure are c M the reduced mass effect, i.e. $m_{i} =$ m m m system, the positron system, the positron so that the factor 2 *M* Thus, there is no distinction between the fine structure. $\overline{\Xi}$ the *d*iverfine structure is at the same order and the rigge csti in the positronium system and the degeneracy of $2S_{1/2}^{20}$ and 2 states is already broken in the level of fine and hope finance makes in the degree of 2 Therefore the Land shift $\mathbb{R}P_1$ breaks the deventor of $\mathbb{R}Q$ \sim less interesting in the positronium system. However, the positronium system \sim $\alpha = \alpha_0$ is very much analogous to the positronium system since $2S_{1/2}$ and $2P$ $\mathbf{q} \in \mathbb{Q}$ uarkonium are bounded-states of \mathbb{Q} uark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark-antiquark- \sqrt{a} \overline{a} $\overline{a$ F athe positronium, m/M = 1: \mathbb{Z} in the large difference in the large differe M the reduced mass effect, i.e. $m_{red} = -$, *m* $\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1}{\sqrt{1-\frac{1}{2}}}\cdot\frac{1$ $\omega_{1/2}$ and $\omega_{1/2}$ and $\omega_{1/2}$ and $\omega_{1/2}$ and ω \bar{z} the divertine structure is at the same order all the rigge estimating in the positronium² system and the degeneracy of $2S_{1/2}^{2D}$ and $2P_{1/2}^{2D}$ states is already broken in the level of fine and hyperfinance in the structure. Therefore, the Lamb shift which breaks the degeneracy of 28 and 2*P*₁ states is less interesting in the positronium system. $2S_{1/2}$ and $2P_{1/2}$ states is less interesting in the positronium system. \overline{QQ} and $\overline{Q}Q$ (*QQ*) and electron-positron (), [−] ⁺ *e e* respectively. This leads to a further (*QQ*) and electron-positron (), [−] ⁺ For the positronium, m/M = 1: \blacksquare \mathbb{Z} of $\frac{1}{2}$ *m* Ω . Thus, there is no distinction between the fine structure structur $2S_{1/2}$ and $2P_{1/2}$ and $2P_{1/2}$ the divertine structure is at the same order and the restricture and in the positronium² system and the degeneracy of $2S_{1/2}^{2D}$ and $2P_{1/2}^{2D}$ states is already broken in the level of fine and h**yperfinannes all the structure**. Therefore, the Lamb shift which $2R$ hreaks the degeneracy of $2R$ \widetilde{R} and $2R$ states is less interesting in the positronium system. However, the positronium system 25.12 and 27.12 e^-e^+ , $\mathcal{Z} \mathcal{Z}$ \mathcal{Z} \mathcal{Z} <u>Tł</u>

(*QQ*) and electron-positron (), [−] ⁺

 α and is very much analogous to the positronium system sy

 $e^{-}e^{+}$ $\qquad \qquad$ \qquad \qquad

positronium spectra distinguished from the hydrogen spectra \mathcal{L}

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studies the positronium spectra. The positronium spectra. The positronium spectra. The positronium spectra are two inportant aspectra. The positronium spectra are two inportant aspects of $\rho = \rho \sqrt{2\pi r^2}$

(*QQ*) and electron-positron (), [−] ⁺

$$
E_n^{\text{positronium}} = -\alpha^2 m_{\text{red}} c^2 \left(\frac{1}{2n^2}\right)
$$

$$
= \frac{1}{2} E_n^{\text{hydrogen}},
$$

 \overline{z}

i.e.

$$
E_1^{positronium} = \frac{13.6eV}{2} = 6.8eV,
$$

and

$$
a^{positronium} = \frac{\hbar^2}{m_{red}e^2}
$$

$$
= 2a^{hydrogen}
$$

$$
= 1.06x10^{-8}cm.
$$

The second one is the electron-positron annihilation effect which modifies the interaction Hamiltonian. In some cases, the positronfum decays into two
or three photons depending on the positron state. The energy level the interaction Hamiltonian. In some cases, the positronium decays into two
or three photons depending on the positromium state. The energy level correction due to the annihilation effect occurs at the same order as the \pm ine structure, i.e. $\Delta E \frac{\Delta E}{annihilation}$ annihilation⁴ \widetilde{m} C₂. MC $\,$. $\mathcal{A} = \mathbf{0}$ or $\mathbf{0} = (1)^{l+s}$ into $\mathbf{0} = 1$ \sim α γ it the state is ϵ = 0. ℓ = 0. ϵ = 0. 3γ and 2γ and 2γ $3\gamma \ 3\gamma \ \tau(^1S_0 \rightarrow 2\gamma) = 1.25x10^{-10}$ see +s S=0 and $c = (\sigma \psi^{\ell+s}_{(n)})^{\ell+s}$ 0 and $\rho = 0$ γ_{ν} $\frac{3\gamma}{\gamma}$ is not different to the decay to the de $\mathcal{L}\mathcal{W}^{3}S_{1} \rightarrow 3\gamma$) = 1.45x10⁻⁷ sec. S= $2\gamma_2$ $3\cancel{2}\gamma$ 3γ τ ⁽ $S_{a,1}^{\sim}$ $\stackrel{a}{\rightarrow}$ 2 γ) $\bar{\bar{\tau}}$ A^{\dagger} S^{\dagger} S^{\dagger} $\stackrel{a}{\rightarrow}$ $\bar{\gamma}$ S^{\dagger} S^{\dagger} _{1 \equiv} 1($\bar{\psi}$ S^{\dagger} χ 10⁻¹⁰ sec ($\bar{\psi}$ rrection due to ΔE _{annihilation} $4\gamma_{MC}^2$ α^4 mc^2 . \mathbf{A} and \mathbf{A} into \mathbf{A} into \mathbf{A} into \mathbf{A} into \mathbf{A} into \mathbf{A} into photons, the orbital into photons, the orbital into photons, the orbital intervals of \mathbf{A} into \mathbf{A} into \mathbf{A} i $\alpha = \alpha$ and $\alpha = (-1)$ its spin state is $\ell = 0$. $\ell = 0$. 3γ and 2γ and 2γ $3\gamma \ 3\gamma \ \tau({}^1S_0 \rightarrow 2\gamma) = 1.25x_0^10^{-10}$ sec_{ts} $S=0$ $\ell=0.$ Since $\ell=0.$ Since $\ell=0$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{3}{8}$ $\frac{3}{1}$ $\frac{3}{1}$ $\frac{145 \times 10^{-7}}{9}$ sec s⁻¹ p_{μ} p_{μ} Δy (p_{μ} Δy) = 1.45.10 sec. $p-1$ f^2 2γ f^2 3γ decays are given by $\tau^a($ $S_a \rightarrow \frac{Z}{2}$ γ $\frac{Z}{2}$ $\frac{Z}{25}$ $\frac{Z}{20}$ $\frac{Z}{20}$ $\frac{Z}{20}$ $\frac{Z}{20}$ $\frac{Z}{20}$ rection due to the annihilation effect occurs at the same order as the find. $c = (-1)$ ^{applygation} $c = (-1)$ ^{applygation} 2γ $\Omega_{\rm eff}=\Delta E \frac{\Delta E}{m}$ annihilation $\Omega_{\rm eff}^{\rm 2}$ or $\Omega_{\rm eff}^{\rm 2}$ involves one more fine- \mathcal{S} is not different to the decay $p \hspace{.2cm} \hat{\hspace{.2cm}} \hspace{.2cm} \hat{\hspace{.2cm}} \hspace{1cm} \hat{\hspace{$ \sim *OL*_Y ℓ HC S_I c = (-1 $\tau({}^{1}S_{\theta} \rightarrow 2\gamma) = 1.25x10^{-10}$ *sec_{+s} s*=0 and $\gamma^3 S_1 \rightarrow 3\gamma$) = 1.45 x10⁻⁷ sec. S=1 $\tilde{\tau}$ $(\frac{1}{5}S \rightarrow 2\gamma) = 1.25 \times 10^{-10}$ sec τ ^{($S_a \stackrel{\alpha}{\rightarrow} 2\gamma$) $\bar{\tau}$ ^{[(25x10⁻¹⁰) \sum $\bar{\zeta}$ (25x10⁻¹⁰) sec (2)}} \overline{c} **b** \overline{c} (\overline{c}) and the reason \overline{c} \overline{c}). correction due to the annihilation effect occurs at the same order as the fine. su ucune, i.e. $c = (-1)^{\frac{amihiflation}{4}} \frac{2}{\sqrt{2}}$ $3\gamma \frac{\Delta E \frac{\Delta E}{annihilation}$ annihilation \cdots $c = (-1)^{\ell+s}$ $e = 0$ $\ell = 1$ 3γ 2γ 2γ $3\gamma \hspace{0.1cm} 3\gamma \hspace{0.1cm} \tau(\hspace{0.1cm} S_{\theta} \rightarrow 2\gamma) = 1.22 \chi \underline{\text{IV}} \underline{\text{U}} \left(-\frac{8}{3} \right)^{e+s} \hspace{1cm} \text{S} = 0$ γ 3γ and $\ell = 0$. $\frac{\partial y}{\partial x}(\frac{1}{2}S_{\alpha} \rightarrow 2y) = 1.25 \times 10^{-10} \text{ sec}$ a_{N} and $\alpha = \left(-1 \right)^{\ell+s}$ with $\ell = 0$. $\ell = 0$. $3\gamma = 2\gamma = 2\gamma$
 $3\gamma = 3\gamma = 1.25x10^{-10}$ see +s S= and $c = (c \pm \int_{0}^{c+s} 1)^{c+s}$ $= 2\gamma_{2y}$ $= 3\frac{2\gamma^{3}S_{1} - 3\gamma}{3\gamma} = 1.45x10^{-7}$ sec. $S =$ $\tau(S_a \rightarrow 2\gamma) = 1.25 \times 10^{-10} \text{ se}$
 $\tau(S_a \rightarrow 2\gamma) = 1.25 \times 10^{-10} \text{ se}$ $\mathbf{v} = (\mathbf{v}_1 - \mathbf{v}_2 - \mathbf{v}_3)$ is not decay to the decay to the decay to the decay to the decay of the decay of $\mathbf{v}_i = \mathbf{v}_i$ correction due to the annihilation effect of the same order at the same order at the same order as the same of the fine same action Hamiltonian. In some cases, the positron $\lim_{n \to \infty} Q_n$ into two $c=-1$ $\ell = 0.$ \mathbf{v} . $s=0$ 2ν 3 $s=1$ \overline{a} 10^{-10} sec \overline{Q}) The second one is the electron-positron annihilation effect which modifies
the interaction Hamiltonian. In some cases, the positron^fum decays into two or three photons depending on the positromation state. The energy level correction due to the annihilation effect occurs at the same order as μ e fing.
structure, i.e. ΔE_{ampl $_{\text{rational}} \sim \alpha_{\text{r}}^{\text{A}}$ $\alpha = (-1)^{\ell+s}$ $_{\ell=0,\ell=0}$ $_{\ell=0}$. $C = (\sigma \psi^{\ell+s})^{\ell+s}$ into $C = (-1)^{\ell+s}$ 2γ and 3γ its spin state is $\ell = 0$. 2γ $\frac{3}{2}$ behavior decays into $2\gamma(3S_1 \rightarrow 3\gamma) = 1.45x10^{-7}$ sec. s=1, then its spin state is spin state into $s=1$ $d^2\gamma$ is $\frac{3\gamma}{\tau}({}^1S \to 2\gamma) = 1.25x10^{-10}$ sec $c = (-1)^{\ell+s} \frac{f(s)g_{\ell}(s)}{\tau(s)g_{\ell}(s)} \frac{f'(s)g_{\ell}(s)}{\tau(s)} \frac{f'(s)g_{\ell}(s)}{\tau(s)} \frac{f'(s)g_{\ell}(s)}{\tau(s)} \frac{f'(s)g_{\ell}(s)}{\tau(s)} \text{ sec} (Q)$ which modifies lecays into two
he energy level $c_{\overline{y}}=1$. $\epsilon = 0$ \mathbf{s} - \mathbf{v} $f_{\ell} = \begin{bmatrix} 1 \\ \ell \end{bmatrix} = 0.$ 2γ ³ $\Delta E_{\text{application}} \sim \alpha_{\text{2}}^{3}mc^{2}.$ $B_V \propto L \overline{L}$ annihilation $\omega^T m \overline{c}^2$. a number of the positronium must be $\ell = 0$. $\mathcal{C}=\left(-1\right)^{1/2}$ is $\ell=\mathbb{Q}$ and $\ell=\mathbb{Q}$ into $2q-1$ 3γ 2γ 2γ $3\gamma \, 3\gamma \, \tau(^1S_0 \to 2\gamma) = 1.25x10^{-10} \, \text{kg}^{\alpha+s} \quad \text{S=0}$ P_1^{\prime} $Q_{\mathcal{H}}^3 S_r \to 3\nu$ = 1.45x10⁻⁷ sec s=1 $3\cancel{2}\cancel{\gamma}$ $3\cancel{y}$ $3\cancel{y}$ $3\cancel{y}$ $3\cancel{z}$ $\overline{\mathcal{F}}_{\mathcal{V}}^{1!} \mathfrak{F}_{\mathcal{A}}^{5} \mathfrak{X}_{10}^{10^{-10}} \mathfrak{F}_{\mathcal{S}}^{eq}$ $^{10}_{1} \mathfrak{F}_{\mathcal{S}}^{10} \mathfrak{F}_{\mathcal{S}}^{10} \mathfrak{F}_{\mathcal{S}}^{10} \mathfrak{F}_{\mathcal{S}}^{10} \mathfrak{X}_{10}^{10} \mathfrak{F}_{\mathcal{S}}^{10} \mathfrak{F}_{\mathcal{S}}^{10}$

 $\overline{}$ $\overline{\$ **greater by a factor of 100 million for charmonium. In positronium the various** D^0 *D*⁰

$$
\langle \Delta H_{rel} \rangle = -\frac{1}{2mc^2} \Big(E_n^2 - 2E_n \langle V \rangle + \langle V^2 \rangle \Big),
$$

\n
$$
\in D^0 \qquad V \overline{\mathcal{D}}^0 \frac{e^2}{r} \qquad \frac{\mathcal{V}}{\mathcal{V}} \qquad \qquad (\Delta E)_{positronium} \sim eV
$$

 $t_{\rm 57}$ Pedestrian Approach to Particle Physics 57

$Schrödinger$

Pedestrian Approach to Particle Physics 67

s₁ and s₂
\nsplitting of the energy level for
$$
\frac{n=1}{2}
$$
 state, i.e. 1^1S_0 and $\binom{3}{5}\binom{3}{5}$
\n $\binom{3}{1}\binom{3$

 $\begin{array}{ccc} 2 & & \text{ s = 1;} \end{array}$

2

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=

 $s(s+1) - \frac{1}{4} - \frac{1}{4} s = 0$

 $s(s+1) - \frac{3}{4} - \frac{3}{4}$ $s = 0$
 $s = 2$

=

1; $\frac{1}{4}$

68

 $\overline{}$

s

 $\overline{}$

2

 $+1) - \frac{3}{4} - \frac{3}{4}$ $s = 0$ ₆₈

 \hbar

4

⎨

 $\overline{\mathsf{I}}$

1 \cdots \cdots

 $\frac{\overline{1}}{\hbar^2}$ and $\frac{1}{\sqrt{2}}$

2

 \hbar

 m_1 *m*

 $\frac{1-\hbar^2}{4n}$:

 \hbar

2

4

68

68

Table 3.2: Meson Spectra (in the unit of MeV/c² , 1 2 1 2 1 2 *m m ^s ^s ^M ^m ^m ^A* ! ! [⋅] ⁼ ⁺ ⁺ (3.101) 1 2 1 2 1 2 *m m ^s ^s ^M ^m ^m ^A* ! ! [⋅] ⁼ ⁺ ⁺ (3.101)

$$
m_{u} = m_{\phi} = \frac{310MeV_2}{m_d} = 3(6MeV)e^{2}W_u = \frac{2m_g}{s} = 3.488RMeV_3e^{2} = 483MeV^2e^{2}
$$

$$
A = \left(\frac{2m_{u}}{\hbar}\right)^2 160MeV/c^2 \left(\frac{2m_{u}}{\hbar}S_1\right)^2 \left(\frac{36MeV}{s^2} + \frac{3}{2}\right)^2
$$

$$
B = \frac{5^2 - 5^2}{s^2} = \frac{5^2 - 5^2}{s^2}
$$

[−] [−] [⋅] ⁼

Meson spectroscopy in QCD_{1+1}

Dynamical quark/gluon mass generation and color confinement in QCD should be understood further.

Effective Constituent Quark Model for Low Q^2

$$
|Meson\rangle = \psi_{q\overline{q}} |q\overline{q}\rangle + \psi_{q\overline{q}g} |q\overline{q}g\rangle + ...
$$

$$
\approx \Psi_{Q\overline{Q}} |Q\overline{Q}\rangle,
$$

where

$$
|Q\rangle = \psi_q^Q |q\rangle + \psi_{qg}^Q |qg\rangle + ...
$$

$$
|\overline{Q}\rangle = \psi_q^{\overline{Q}} |\overline{q}\rangle + \psi_{qg}^{\overline{Q}} |\overline{q}g\rangle + ...
$$

$$
\overrightarrow{p^*,0_{\perp}}\sqrt{\frac{x_1p^*,\vec{k}_{\perp 1},\lambda_1}{x_2p^*,\vec{k}_{\perp 2},\lambda_2}}
$$

$$
\Psi_{Q\overline{Q}}(x_i,\vec{k}_{\perp i},\lambda_i) = \Phi(x_i,\vec{k}_{\perp i})\chi(x_i,\vec{k}_{\perp i},\lambda_i)
$$

 $H = T + V$

V includes Coulomb, Confinement, Spin-Spin,Spin-Orbit interactions.

 $0^{-+}(\pi, K, \eta, \eta', ...)$ $J^{PC} = 0^{++}(f_0, a_0,...)$ Radial **Spin-Orbit** Spin-Orbit (Dependent on the model potential) (Interaction independent Melosh transformation)

$$
1-(\rho,K^*,\omega,\phi,...)
$$

...

Energy-Momentum Dispersion Relations

PHYSICAL REVIEW C 92, 055203 (2015) Variational analysis of mass spectra and decay constants \cdots

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vacuum fluctuations into the zero modes in the limit of*p*⁺ → 0 [11–13]. This simplification is a remarkable advantage in LFD and facilitates the partonic interpretation of the amplitudes. Based on the advantages of the LFD, the LFQM has been developed [14] and subsequently applied for various meson phenomenologies such as the mass spectra of both heavy and light mesons [15], the decay constants, distribution amplitudes,

has been quite useful in providing a good physical picture useful in providing a good physical picture and picture

Calculation of Form Factors in Equal-Time Theory **Instant Form.**

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

• Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

• Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

Dirac's Proposition

Traditional approach I raditional approach
evolved from NR dynamics and the relativistic dynamics

for relativistic dynamics

Close contact with

T-dept QFT, LQCD, IMF, etc.

Euclidean space

Euclidean space

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

Interpolation between Instant and Front Forms

Feynman Diagram: Invariant under all Poincaré generators

 (a)

 (b)

Individual Time-Ordered Diagrams: Invariant under stability group Kinematic vs. Dynamic Generators

S.Weinberg, PR158,1638(1967) "Dynamics at Infinite Momentum"

Note however this is still in the instant form.

 (a)

 (b)

 $\left(a\right)$

 $\Sigma(a)+\Sigma(b)=1/(s-m^2)$; s=2 GeV², m=1GeV

 (b)

 $P_z = -\sqrt{\frac{s(1-C)}{2C}}$ **J-shape peak & valley** : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$ As $C \rightarrow 0$, $P^+ = P^0 + P_z \rightarrow 0$ leads to LF Zero-modes.

Nontrivial Vacuum State

$$
|0\rangle \rightarrow |0\rangle
$$

Translation in scalar field: $\quad \phi \longrightarrow \phi' = \phi + \nu$

$$
|\Omega\rangle = \exp\left(i\int_{-\ell}^{+\ell} dx^2 v \,\pi(x^2)\right)|0\rangle
$$

$$
\pi(x^{\hat{+}} = 0, x^{\hat{-}}) = -i \sum_{n=-\infty}^{\infty} \left(\frac{\pi}{\ell}\right) \sqrt{\frac{\omega_n}{4\pi}} \left[a_n e^{-i\left(\frac{n\pi}{\ell}\right)x^{\hat{-}}} - a_n^{\hat{+}} e^{i\left(\frac{n\pi}{\ell}\right)x^{\hat{-}}} \right]
$$

$$
1 \Omega = \exp \left[- (C^{1/2} m \ell) \frac{v^2}{2} \right] \exp \left[- (C^{1/2} m \ell)^{1/2} v a_0^{\dagger} \right] |0>
$$

condensation of Zero-Modes

Vacuum Energy $P_{\hat{+}}$ | Ω > = E_{Ω} | Ω >

$$
a e^{\alpha a^+} 10 \ge a e^{\alpha a^+} 10 \ge
$$

$$
P_{\hat{+}} | \Omega \rangle \rightarrow \left[\frac{m v}{C^{1/2}} a_0^{\dagger} a_0 + \frac{(m^3 \ell)^{1/2} v}{C^{1/4}} (a_0 + a_0^{\dagger}) \right] \exp \left[-(C^{1/2} m \ell)^{1/2} v a_0^{\dagger} \right] | 0 \rangle
$$

= $(-m^2 v^2 \ell) \exp \left[-(C^{1/2} m \ell)^{1/2} v a_0^{\dagger} \right] | 0 \rangle$

$$
E_{\Omega} = -m^2 v^2 \ell = \int_{-\ell}^{+\ell} (-\frac{1}{2} m^2 v^2) dx^{\hat{+}}
$$

Independent of interpolation angle!

Recovery of Trivial Vacuum in LFD?

$$
|\Omega\rangle = \exp\left[-(C^{1/2}m\ell)\frac{v^2}{2}\right] \exp\left[-(C^{1/2}m\ell)^{1/2}va_0^+\right]|0\rangle
$$

$$
|\Omega \rangle \rightarrow |0 \rangle \quad \text{as} \quad C \rightarrow 0
$$

However, E_{Ω} and $< \Omega$ | $\phi(x)$ | $\Omega >= -v$

are still independent of interpolation angle!

What is going on?

 $< \Omega$ | $\phi(x)$ | Ω > $=< 0 \text{ } | \exp \left[(C^{1/2}m\ell)^{1/2} v (a_0^* - a_0) \right]$ $a_0 + a_0^+$ $2{\left(\displaystyle C^{1/2}m\ell\right)^{1/2}}$ $\sqrt{}$ \setminus $\left(\frac{a_0 + a_0^+}{2\sqrt{C^{1/2} \cos \theta_0}}\right)$ \int $\left[- (C^{1/2}m\ell)^{1/2} v(a_0^* - a_0) \right]$ | 0 > $=-V$

Complication is transferred from vacuum to operator.

Coulomb Gauge

Light-front Gauge

$$
\sum_{\lambda=\pm} \epsilon_{\mu}^{*}(\lambda)\epsilon_{\nu}(\lambda) = -g_{\mu\nu} + \frac{(q \cdot n)(q_{\mu}n_{\nu} + q_{\nu}n_{\mu})}{q_{\perp}^{2}C + q_{\perp}^{2}} - \frac{Cq_{\mu}q_{\nu}}{q_{\perp}^{2}C + q_{\perp}^{2}} - \frac{q^{2}n_{\mu}n_{\nu}}{q_{\perp}^{2}C + q_{\perp}^{2}}
$$
\n
$$
-\eta_{\mu\nu} + \frac{(q \cdot n)(q_{\mu}n_{\nu} + q_{\nu}n_{\mu})}{(q \cdot n)^{2}} - \frac{q^{2}n_{\mu}n_{\nu}}{(q \cdot n)^{2}} - \eta_{\mu\nu} + \frac{(q \cdot n)(q_{\mu}n_{\nu} + q_{\nu}n_{\mu})}{(q \cdot n)^{2} - q^{2}} - \frac{q_{\mu}q_{\nu}}{(q \cdot n)^{2} - q^{2}} - \frac{q^{2}n_{\mu}n_{\nu}}{(q \cdot n)^{2} - q^{2}}
$$
\n
$$
-\eta_{\mu\nu} + \frac{(q \cdot n)(q_{\mu}n_{\nu} + q_{\nu}n_{\mu})}{(q \cdot n)^{2} - q^{2}} - \frac{q_{\mu}q_{\nu}}{(q \cdot n)^{2} - q^{2}} - \frac{q^{2}n_{\mu}n_{\nu}}{(q \cdot n)^{2} - q^{2}}
$$

where $t_m \equiv t - m^2$, $u_m \equiv u - m^2$, and $s_m \equiv s - 4m^2$.

$$
|\mathcal{M}|^2 = 2e^4 \left(\frac{u}{t} + \frac{t}{u}\right)
$$

Scattering Angle Dependence of the Annihilation Amplitudes: Chirality

When $m_e=0$, chirality is conserved.

Scattering Angle Dependence of the Annihilation Amplitudes: Chirality

Outlook

- "A method is more important than a discovery, since the right method will lead to new and even more important discoveries." - Lev Landau
- LFQM saves a lot of dynamical effort in the hadron spectroscopy and structure study.
- Correspondence between IFD and LFD may shed more light on bridging the LFQM and the QCD.