Nuclear Physics School 2018

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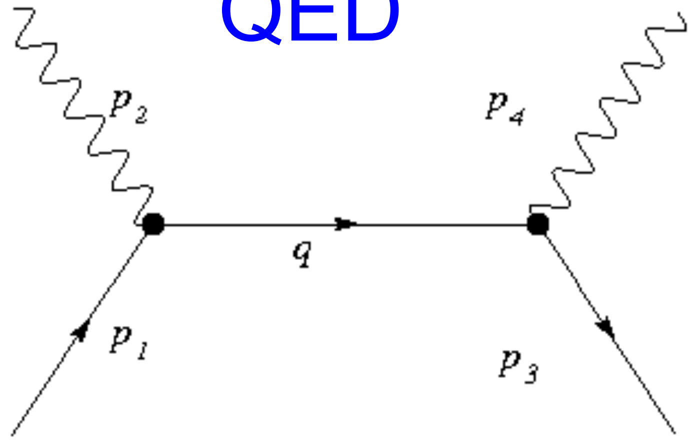
North Carolina State University

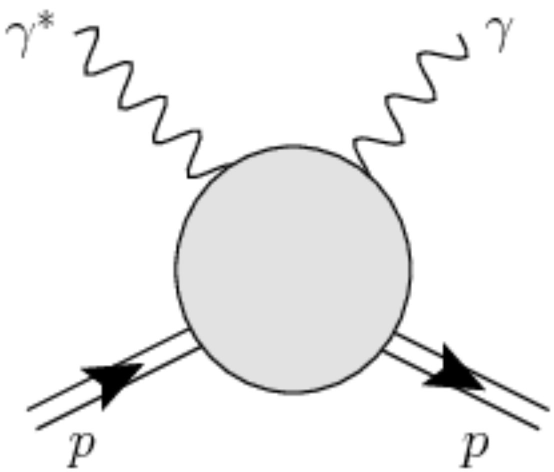
Fourth Lecture

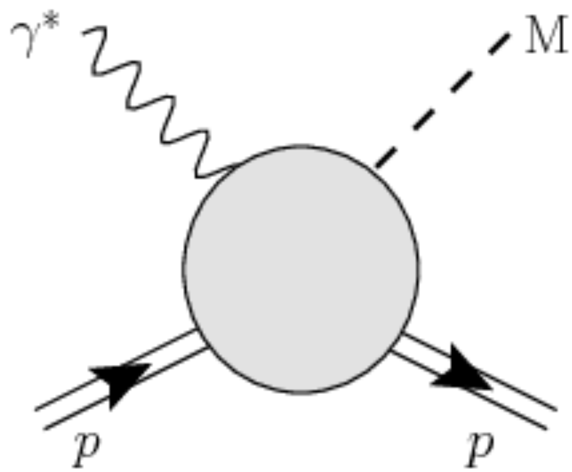
Hadron and Nuclear Structure Study

June 29, 2018

QED

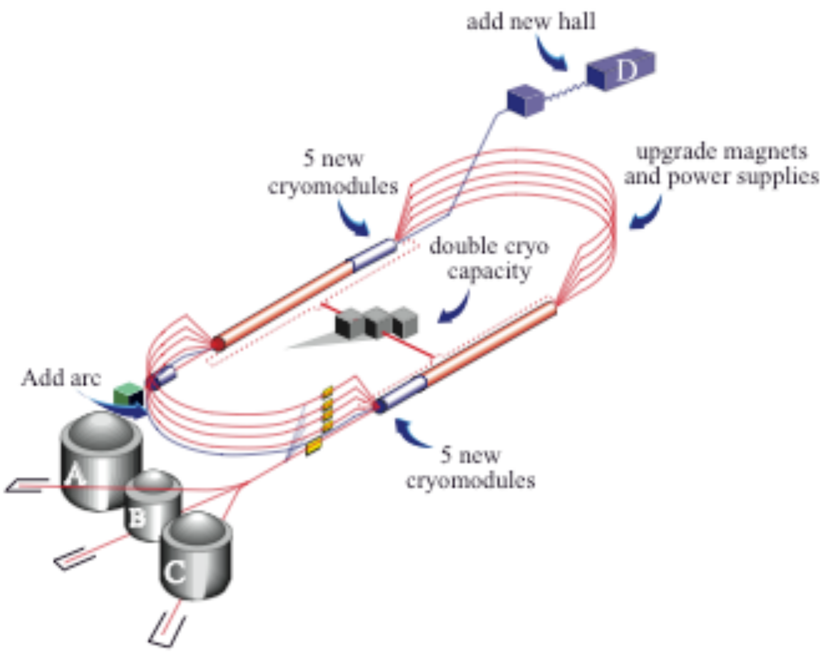


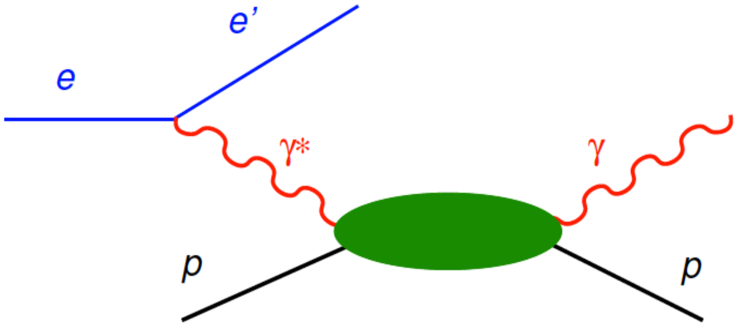


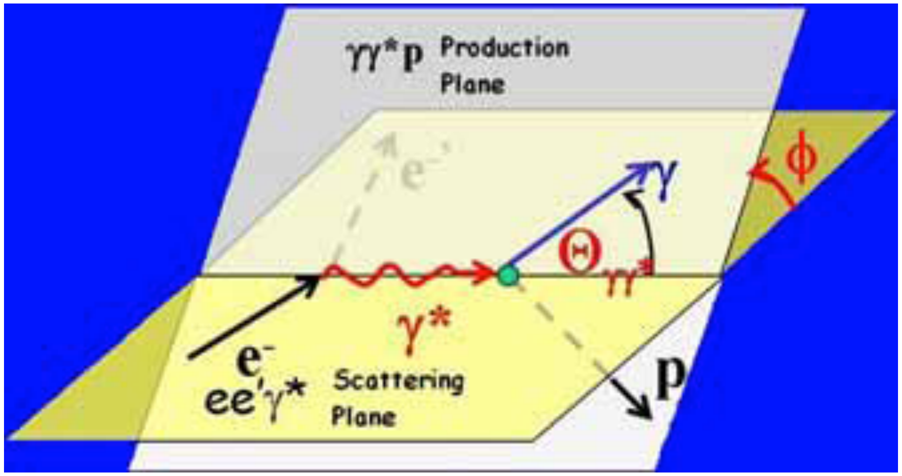


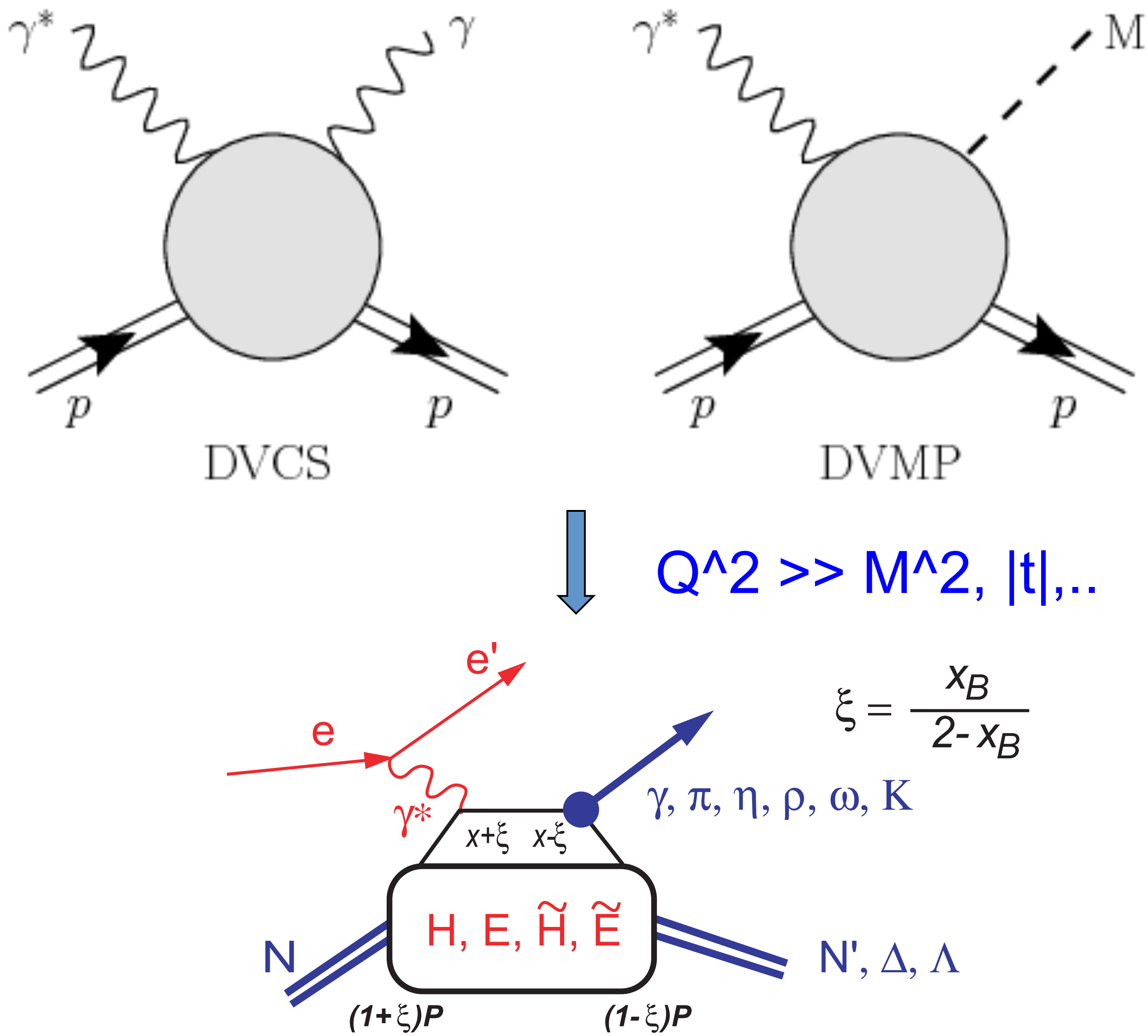
QCD

Hadron Physics at JLab









H, E - unpolarized, H̃, Ẽ - polarized GPD

The GPDs Define Nucleon Structure

Review

International Journal of Modern Physics E
Vol. 22, No. 2 (2013) 1330002 (98 pages)

DOI: 10.1142/S0218301313300026

CONCEPTUAL ISSUES CONCERNING GENERALIZED PARTON DISTRIBUTIONS

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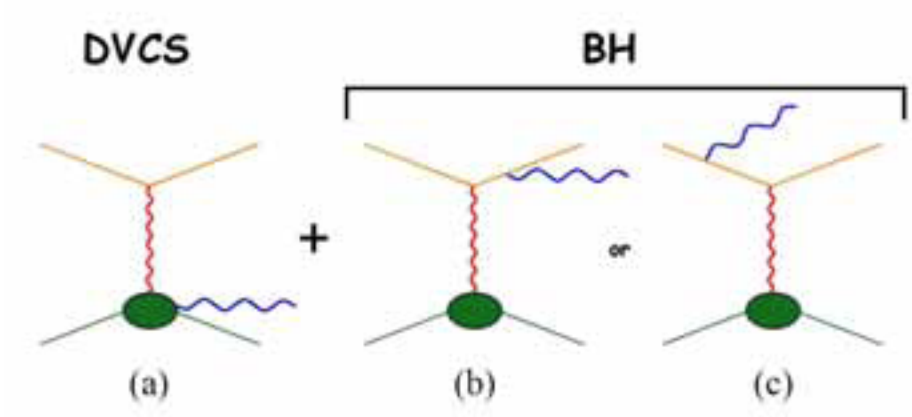
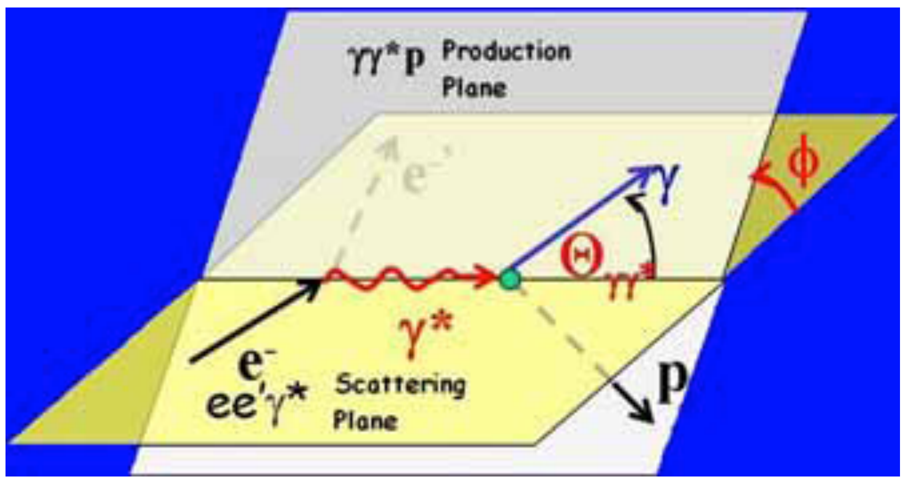
Received 15 July 2012
Revised 30 October 2012
Accepted 6 November 2012
Published 6 February 2013

Outline

- JLab Kinematics ($t < -|t_{min}| \neq 0$)
- Original Formulation of DVCS with GPDs (Twist 2)
- Benchmark Barebone Calculation for JLab Kinematics (Exact Result vs. Reduced Result)
- Toward finding the Most General Hadronic Tensor Structure with CFFs (DNA method)
- 5-Fold Differential Cross Section (Notations)
- Hadronic Currents in Meson Production off the Scalar Target (0^{++} vs. 0^{-+})
- Benchmark BSA for π^0 production off ^{4}He
- Conclusion and Outlook

JLab Kinematics $t < -|t_{min}| \neq 0$





$$t = \Delta^2 = -\frac{\varsigma^2 M^2 + \Delta_\perp^2}{1-\varsigma} \quad ; \quad \Delta^+ (\equiv \Delta^0 + \Delta^3) = -\varsigma\, p^+ \quad ; \quad \Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

Coincidence Experiment

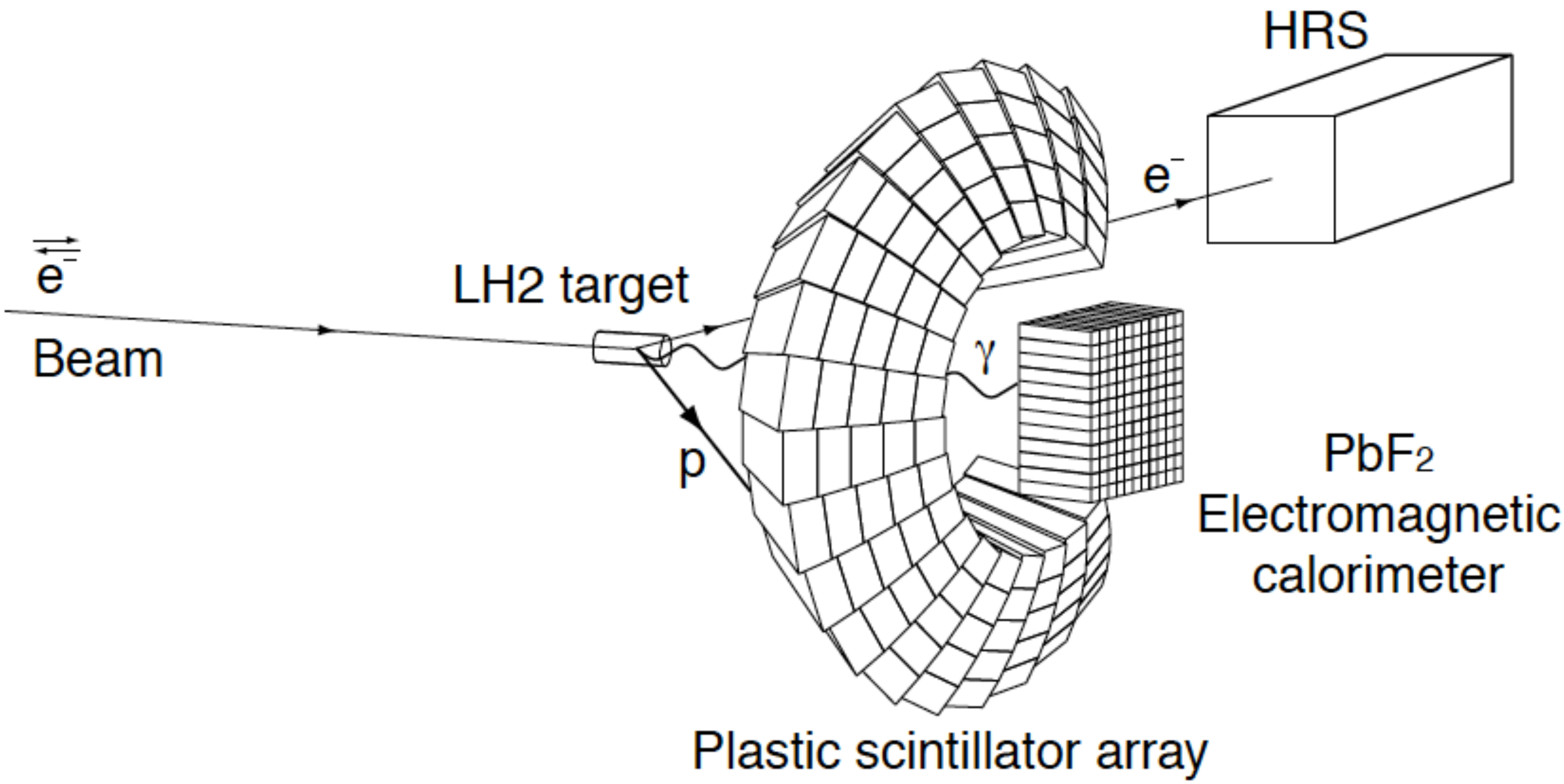


Figure 1.11: E00-110 schematic setup showing the three different detectors used to measured each of the particles in the final state. **Carlos Muñoz Camacho** Thesis('05)

In[277]:=

```
Flatten[{{{"Q^2", "xBj", "k", "k'", "the", "thq", "thq'", "thp'", "q'", "t/Q^2"}},
   Table[Block[{M = 0.938, Q = Sqrt[pars[[i, 1]]], xBj = pars[[i, 2]], Eb = pars[[i, 3]],
      the = pars[[i, 4]] Pi / 180, thetaC = 20 Pi / 180, thq = ArcCos[costhetaqT2]},
     {Q^2, xBj, Eb, PeT1, the 180 / Pi, thq 180 / Pi, ArcCos[costhqf] 180 / Pi, ArcCos[
        costhpf] 180 / Pi, qfT3mu[[1]], MandeltT2 / Q^2}], {i, 1, 12}]}, 1] // MatrixForm
```

Out[277]//MatrixForm=

Q^2	xBj	k	k'	the	thq	thq'	thp'	q'	t/Q^2
1.9	0.36	5.75	2.93669	19.3	18.0503	11.9997	69.6549	2.65582	-0.1555
3.	0.36	6.6	2.15792	26.5	11.6529	6.69292	66.0322	4.23202	-0.131356
4.	0.36	8.8	2.87723	22.9	10.3184	5.96439	64.4845	5.66645	-0.120212
4.55	0.36	11.	4.26285	17.9	10.6859	6.58293	64.1816	6.45567	-0.116055
3.1	0.5	6.6	3.2951	22.5	19.5262	14.4829	60.4087	3.0229	-0.170658
4.8	0.5	8.8	3.68273	22.2	14.4748	10.3331	57.1229	4.76891	-0.13615
6.3	0.5	11.	4.28358	21.1	12.4174	8.76649	55.3539	6.31422	-0.119765
7.2	0.5	11.	3.32409	25.6	10.1755	6.74662	53.737	7.24243	-0.112945
5.1	0.6	8.8	4.26908	21.2	17.7604	13.7928	51.4698	4.06193	-0.172513
6.	0.6	8.8	3.46951	25.6	14.8072	11.1302	49.5692	4.82846	-0.156969
7.7	0.6	11.	4.1592	23.6	13.0416	9.77439	48.0193	6.28128	-0.136317
9.	0.6	11.	3.00426	30.2	10.1946	7.16227	46.0515	7.39496	-0.125229

Table III in E12 - 06 - 114, Julie Roche et al.

Jlab 12 GeV Exclusive Kinematics

Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$p^\mu = \Lambda(\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{1}) \quad ,$$

$$n^\mu = (\overset{ct}{1},\overset{x}{0},\overset{y}{0},\overset{z}{-1})/(2\Lambda) \quad ,$$

$$\overline{P}^\mu = \frac{1}{2}(P+P')^\mu = p^\mu + \frac{M^2-\Delta^2/4}{2} n^\mu ,$$

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} n^\mu \quad , \quad \xi = \frac{Q^2}{2\overline{P}\cdot q} \quad ,$$

$$\Delta^\mu = -\xi\left[p^\mu - \frac{M^2-\Delta^2/4}{2} n^\mu\right] + \Delta_\perp^\mu \ .$$

$$T^{\mu\nu}(p,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int_{-1}^{+1} dx \left(\frac{1}{x-\frac{\xi}{2}+i\varepsilon} + \frac{1}{x+\frac{\xi}{2}-i\varepsilon}\right)$$

$$\times\left[H(x,\Delta^2,\xi)\overline{U}(P')\not{n}U(P) + E(x,\Delta^2,\xi)\overline{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$

$$-\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int_{-1}^{+1} dx \left(\frac{1}{x-\frac{\xi}{2}+i\varepsilon} - \frac{1}{x+\frac{\xi}{2}-i\varepsilon}\right)$$

$$\times\left[\tilde{H}(x,\Delta^2,\xi)\overline{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\xi)\frac{\Delta\cdot n}{2M}\overline{U}(P')\gamma_5 U(P)\right]$$

Just above Eq.(14),

``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>."

Note here that $q'^2 = -\boldsymbol{\Delta}_\perp^2 = 0$.

Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

$$q = q' - \zeta p \quad , \qquad \zeta = \frac{Q^2}{2p \cdot q'} \quad , \qquad r = p - p'$$

$$T^{\mu\nu}(p,q,q') = \frac{1}{2(p \cdot q')} \sum_a e_a^2 \left[\left(-g^{\mu\nu} + \frac{1}{p \cdot q'} (p^\mu q'^\nu + p^\nu q'^\mu) \right) \right.$$
$$\times \left\{ \bar{u}(p') \not{q}' u(p) T_F^a(\zeta) + \frac{1}{2M} \bar{u}(p') (\not{q}' \not{r} - \not{r} \not{q}') u(p) T_K^a(\zeta) \right\}$$
$$\left. + i\varepsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q'_\beta}{p \cdot q'} \left\{ \bar{u}(p') \not{q}' \gamma_5 u(p) T_G^a(\zeta) + \frac{q' \cdot r}{2M} \bar{u}(p') \gamma_5 u(p) T_P^a(\zeta) \right\} \right]$$

At the beginning of Section 2E (Nonforward distributions),
``Writing the momentum of the virtual photon as q=q’-ζp is equivalent to using the Sudakov decomposition in the light-cone `plus’ (p) and `minus’ (q’) components in a situation <u>when there is no transverse momentum</u> .”

Note that here

$$(q - q')^2 = \Delta^2 = t = \zeta^2 M^2 > 0$$

while $t < 0$ in DVCS.

Benchmark Calculation in JLab Kinematics

- To see the effect of taking t<0, we mimic the kinematics at JLab and compute bare bone VCS amplitudes neglecting masses.

$$k^\mu = \left(xp^+, 0, 0, 0\right), \quad k'^\mu = \left(\left(x - \zeta_{eff}\right) p^+, \mathbf{\Delta}_\perp, \frac{\mathbf{\Delta}_\perp{}^2}{2(x - \zeta_{eff}) p^+}\right)$$

$$q^\mu = \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right), \quad q'^\mu = \left(\alpha \frac{\mathbf{\Delta}_\perp{}^2}{Q^2} p^+, -\mathbf{\Delta}_\perp, \frac{Q^2}{2\alpha p^+}\right)$$

Here, for $Q \to \infty$

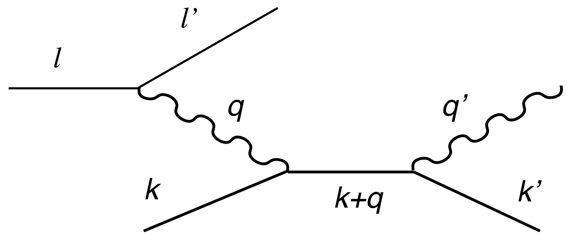
$$\zeta_{\mathrm{eff}} = \zeta + \alpha \frac{\mathbf{\Delta}_\perp^2}{Q^2} \to \zeta$$

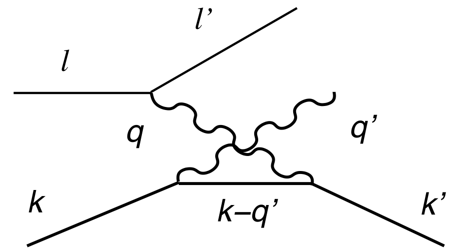
$$\alpha = \frac{(x - \zeta) Q^2}{2\mathbf{\Delta}_\perp^2} \left(1 - \sqrt{1 - \frac{4\zeta}{x - \zeta} \frac{\mathbf{\Delta}_\perp^2}{Q^2}}\right) \to \zeta$$

$$q'^- \to q^- = \frac{Q^2}{2\zeta p^+}$$

"Bare Bone" VCS Operators & Amplitudes

$S=(k+q)^2$





$U=(k-q')^2$

$$\mathcal{O}_s = \frac{\not\epsilon^*(q';h')(\not k+\not q)\not\epsilon(q;h)}{(k+q)^2} \qquad \mathcal{O}_u = \frac{\not\epsilon(q;h)(\not k-\not q')\not\epsilon^*(q';h')}{(k-q')^2}$$

$$\mathcal{O}_s|_{Q\to\infty} = \frac{\not\epsilon^*(q';h')\gamma^+\, q^-\not\epsilon(q;h)}{2\,(x-\zeta)p^+\, q^-} \qquad \mathcal{O}_u|_{Q\to\infty} = \frac{\not\epsilon(q;h)\gamma^+\,(-q'^-)\not\epsilon^*(q';h')}{2\,xp^+\,(-q'^-)}$$

⇓ ⇓

$$\mathcal{O}_s|_{\mathrm{GPDRed}} = \frac{\not\epsilon^*(q';h')\gamma^+\not\epsilon(q;h)}{2p^+}\,\frac{1}{x-\zeta} \qquad \mathcal{O}_u|_{\mathrm{GPDRed}} = \frac{\not\epsilon(q;h)\gamma^+\not\epsilon^*(q';h')}{2p^+}\,\frac{1}{x}$$

$$\mathcal{H}(\{s',s\}\{h',h\}) = \bar{u}(k';s')(\mathcal{O}_s+\mathcal{O}_u)u(k;s)$$

$$\mathcal{L}(\{\lambda',\lambda\}h) = \bar{u}(\ell';\lambda')\not\epsilon^*(q;h)u(\ell;\lambda)$$

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda',\lambda\}h)\frac{1}{q^2}\mathcal{H}(\{s',s\}\{h',h\})$$

Using the identity $\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu} = g^{\mu\alpha}\gamma^{\nu} + g^{\alpha\nu}\gamma^{\mu} - g^{\mu\nu}\gamma^{\alpha} + i\varepsilon^{\mu\alpha\nu\beta}\gamma_{\beta}\gamma_5$

and Sudakov vectors $n(+)^{\mu} = (1,0,0,0),\ n(-)^{\mu} = (0,0,0,1)$

we compare the exact amplitude

$$
\begin{aligned}
T_s{}^{\mu\nu} \; = \; & \frac{1}{s}[(\{(k^{+}+q^{+})n^{\mu}(+) + q^{-}n^{\mu}(-) + q_{\perp}{}^{\mu}\}n^{\nu}(+) \\
& + \{(k^{+}+q^{+})n^{\nu}(+) + q^{-}n^{\nu}(-) + q_{\perp}{}^{\nu}\}n^{\mu}(+) - g^{\mu\nu}q^{-}) \\
& \times \bar{u}(k';s')\not{n}(-)u(k;s) \\
& - i\epsilon^{\mu\nu\alpha\beta}\{(k^{+}+q^{+})n_{\alpha}(+) + q^{-}n_{\alpha}(-) + q_{\perp\alpha}\}n_{\beta}(+) \\
& \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)]
\end{aligned}
$$

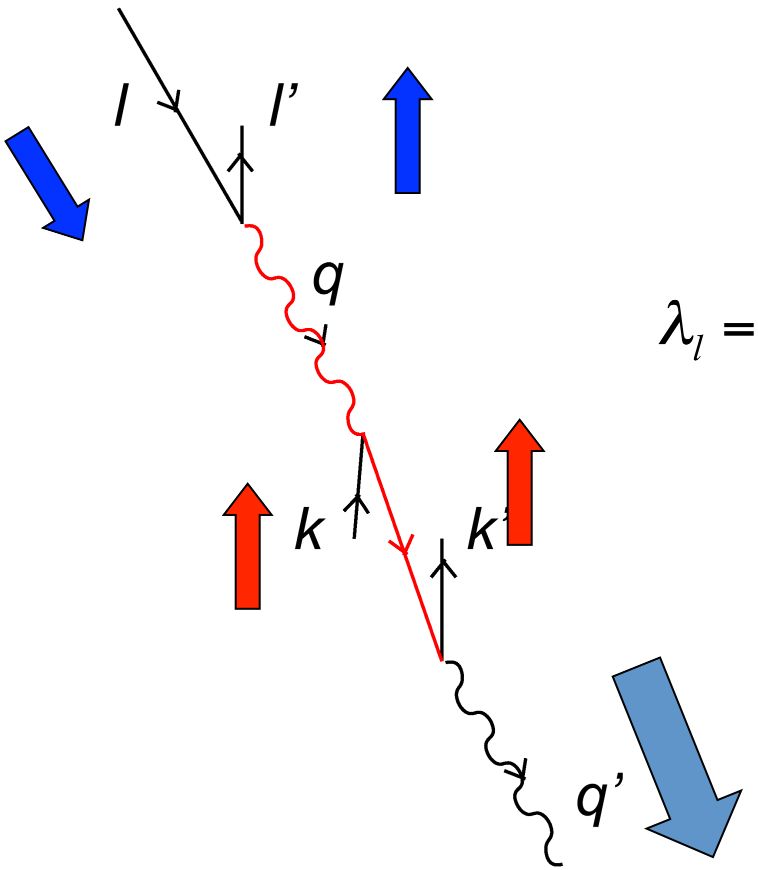
and the reduced amplitude that agrees in the DVCS limit

$$
\begin{aligned}
T_s{}^{\mu\nu} \; = \; & \frac{q^{-}}{s}[\{n^{\mu}(-)n^{\nu}(+) + n^{\nu}(-)n^{\mu}(+) - g^{\mu\nu}\} \\
& \times \bar{u}(k';s')\not{n}(-)u(k;s) \\
& - i\epsilon^{\mu\nu\alpha\beta}n_{\alpha}(-)n_{\beta}(+) \times \bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)]
\end{aligned}
$$

The tensor structure of the reduced amplitude is identical to the ones given by X. Ji and A.V. Radyushkin.

Sanity Checks of Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.

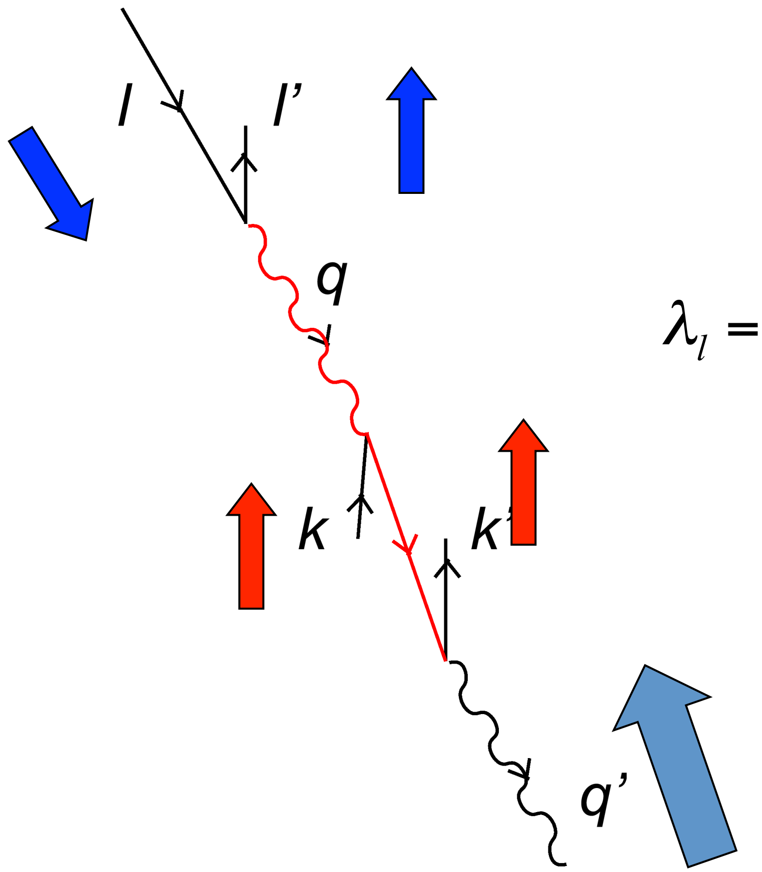


$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\, s_k = s_{k'} = +\frac{1}{2}\,,\, h_{q'} = +1\,;$$

Allowed !

Checking Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



$$\lambda_l = \lambda_{l'} = +\frac{1}{2}\,,\ s_k = s_{k'} = +\frac{1}{2}\,,\ h_{q'} = -1\,;$$

Prohibited !

Comparison

Complete DVCS amplitudes, $\sum_h \mathcal{L}(\{\lambda', \lambda\}, h)\frac{1}{q^2}\mathcal{H}(\{h', h\}\{s', s\})$ in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless, $\lambda' = \lambda$ and $s' = s$.

λ h' s	this work	AVR	XJ
$\frac{1}{2}$ 1 $\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$ 1 $-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$	0
$\frac{1}{2}$ -1 $\frac{1}{2}$	$-\frac{4}{Q^3}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$
$\frac{1}{2}$ -1 $-\frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\mathbf{\Delta}_\perp^2}{Q^2}\right)$

AVR=XJ, taking into account the real photon helicity swap for the exact collinear kinematics vs. the nonlinear kinematicsin LFD:

$$q'^{\mu} = \left(\alpha\frac{\mathbf{\Delta}_\perp^{\,2}}{Q^2}p^+, -\mathbf{\Delta}_\perp, \frac{Q^2}{2\alpha p^+}\right) \leftrightarrow \left(0, 0_\perp, \frac{Q^2}{2\zeta p^+}\right)$$

$$+h' \leftrightarrow -h'$$

C.Carlson and C.Ji, Phys.Rev.D67,116002 (2003);
B.Bakker and C.Ji, Phys.Rev.D83,091502(R) (2011).

For any orders in Q

			Exact	Reduced
λ	h'	s	$\mathcal{A} = \Sigma \mathcal{L} \frac{1}{q^2} \mathcal{H}$	$\mathcal{A}_{\rm red} = \Sigma \mathcal{L} \frac{1}{q^2} \mathcal{H}_{\rm red}$
$\frac{1}{2}$	1	$\frac{1}{2}$	$4 \sqrt{\frac{x}{(x-\zeta)D_+}} \frac{Q^3}{Q^4-4(\zeta p^+)^4}$	$-4(\zeta p^+)^2 \sqrt{\frac{x-\zeta}{xD_+}} \frac{4Q\Delta(\zeta p^+)^2 - D_- Q^4}{\Delta(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$2 \frac{2Q\{Q^3(x-\zeta)-4\Delta\zeta(\zeta p^+)^2\} - D_-\{Q^4(x-\zeta)-4\zeta(\zeta p^+)^4\}}{\sqrt{x(x-\zeta)D_+} Q(Q^4-4(\zeta p^+)^4)}$	$-8 \sqrt{\frac{xD_+}{x-\zeta}} \frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$
$\frac{1}{2}$	-1	$\frac{1}{2}$	$2 \frac{4(\zeta p^+)^2\{2Q\Delta\zeta-(\zeta p^+)^2(x-\zeta)D_+\} - D_- Q^4 \zeta}{\sqrt{x(x-\zeta)D_+} Q(Q^4-4(\zeta p^+)^4)}$	$2 \sqrt{\frac{xD_+}{x-\zeta}} \frac{Q^3}{Q^4-4(\zeta p^+)^4}$
$\frac{1}{2}$	-1	$-\frac{1}{2}$	$-16 \sqrt{\frac{x}{(x-\zeta)D_+}} \frac{(\zeta p^+)^4}{Q(Q^4-4(\zeta p^+)^4)}$	$4 \sqrt{\frac{x-\zeta}{xD_+}} \frac{Q^3\Delta - (\zeta p^+)^2 D_- Q^2}{\Delta(Q^4-4(\zeta p^+)^4)}$

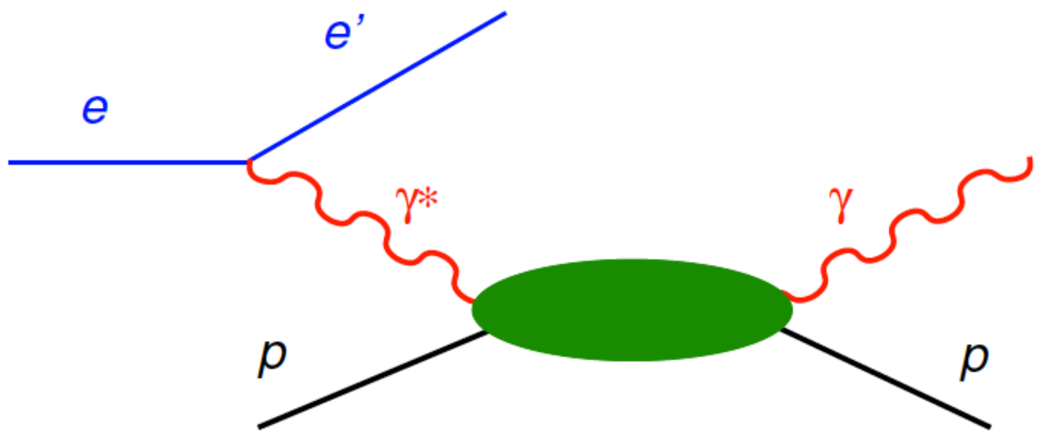
$$D = \frac{4\zeta\Delta^2}{(x-\zeta)Q^2}, \quad D_\pm = 1 \pm \sqrt{1-D}$$

Here, $\Delta = |\boldsymbol{\Delta}_\perp|$

CFFs should be generalized for $t = \Delta^2 \neq 0$ beyond the leading twist definition given by

$$\mathrm{H}(\Delta^2,\xi) = \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} + \frac{1}{x + \frac{\xi}{2} - i\varepsilon} \right) H(x,\Delta^2,\xi)$$

Number of Independent Amplitudes in VCS



Nucleon Target

$$3 \times 2 \times 2 \times 2/2 = 12$$

12 independent tensor structures

M.Perrottet, Lett. Nuovo Cim. 7, 915 (1973);
R.Tarrach, Nuovo Cim. 28A, 409 (1975);
D.Drechsel et al.,PRC57,941(1998);
A.V.Belitsky, D.Mueller and A.Kirchner, NPB629, 323(2002);
A.V.Belitsky and D.Mueller, PRD82, 074010(2010)

DNA Method

$$d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\beta} g^{\nu\alpha}$$

$$G^{\mu\nu}(q'q) = q'_\alpha d^{\mu\nu\alpha\beta} q_\beta = q' \cdot q \, g^{\mu\nu} - q^\mu q'^\nu,$$
$$G^{\mu\nu}(qq) = q_\alpha d^{\mu\nu\alpha\beta} q_\beta = q^2 \, g^{\mu\nu} - q^\mu q^\nu,$$
$$G^{\mu\nu}(q'q') = q'_\alpha d^{\mu\nu\alpha\beta} q'_\beta = q'^2 \, g^{\mu\nu} - q'^\mu q'^\nu,$$
$$G^{\mu\nu}(\overline{P}q) = \overline{P}_\alpha d^{\mu\nu\alpha\beta} q_\beta = \overline{P} \cdot q \, g^{\mu\nu} - q^\mu \overline{P}^\nu,$$
$$G^{\mu\nu}(q'\overline{P}) = q'_\alpha d^{\mu\nu\alpha\beta} \overline{P}_\beta = \overline{P} \cdot q' \, g^{\mu\nu} - \overline{P}^\mu q'^\nu.$$

$$\begin{aligned} \tilde{T}^{\mu\nu}_{\rm DNA} := \sum_{i=1}^{5} \mathcal{S}_i \, \tilde{T}^{(i)\,\mu\nu}_{\rm DNA} &= \mathcal{S}_1 \, G^{\mu\nu}(q'q) \\ &+ \mathcal{S}_2 \, G^{\mu\lambda}(q'q') \, G_\lambda{}^\nu(qq) \\ &+ \mathcal{S}_3 \, G^{\mu\lambda}(q'\overline{P}) \, G_\lambda{}^\nu(\overline{P}q) \\ &+ \mathcal{S}_4 \, [G^{\mu\lambda}(q'\overline{P}) \, G_\lambda{}^\nu(qq) + G^{\mu\lambda}(q'q') \, G_\lambda{}^\nu(\overline{P}q)] \\ &+ \mathcal{S}_5 \, G^{\mu\lambda}(q'q') \overline{P}_\lambda \overline{P}_{\lambda'} G^{\lambda\nu}(qq). \end{aligned}$$

Compton Form Factors (CFFs) : $\mathcal{S}_i, \; i = 1, 2, ..., 5$

C.Ji & B.Bakker, PoS QCDEV2017,038(2017);
B.Bakker & C.Ji, Few Body Syst. 58,no.1,8(2017)

Most General Hadronic Tensor for Scalar Target

$$T^{\mu\nu} = G_{qq'}^{\mu\nu} S_1 + G_q^{\mu\lambda} G_{q'\lambda}{}^{\nu} S_2 + G_{q\bar{P}}^{\mu\lambda} G_{\bar{P}q'\lambda}{}^{\nu} S_3$$

$$+ (G_{q\bar{P}}^{\mu\lambda} G_{q'\lambda}{}^{\nu} + G_q^{\mu\lambda} G_{\bar{P}q'\lambda}{}^{\nu}) S_4 + G_q^{\mu\lambda} \bar{P}_\lambda \bar{P}_{\lambda'} G_{q'}^{\lambda'\nu} S_5$$

$$G_{qq'}^{\mu\nu} = g^{\mu\nu} q \cdot q' - q'^\mu q^\nu$$

$$G_q^{\mu\nu} = g^{\mu\nu} q^2 - q^\mu q^\nu$$

$$G_{q'}^{\mu\nu} = g^{\mu\nu} q'^2 - q'^\mu q'^\nu$$

$$G_{q\bar{P}}^{\mu\nu} = g^{\mu\nu} q \cdot \bar{P} - \bar{P}^\mu q^\nu$$

$$G_{\bar{P}q'}^{\mu\nu} = g^{\mu\nu} q' \cdot \bar{P} - q'^\mu \bar{P}^\nu$$

For $q'^2 = 0$, only S_1, S_2 and S_4 contribute.

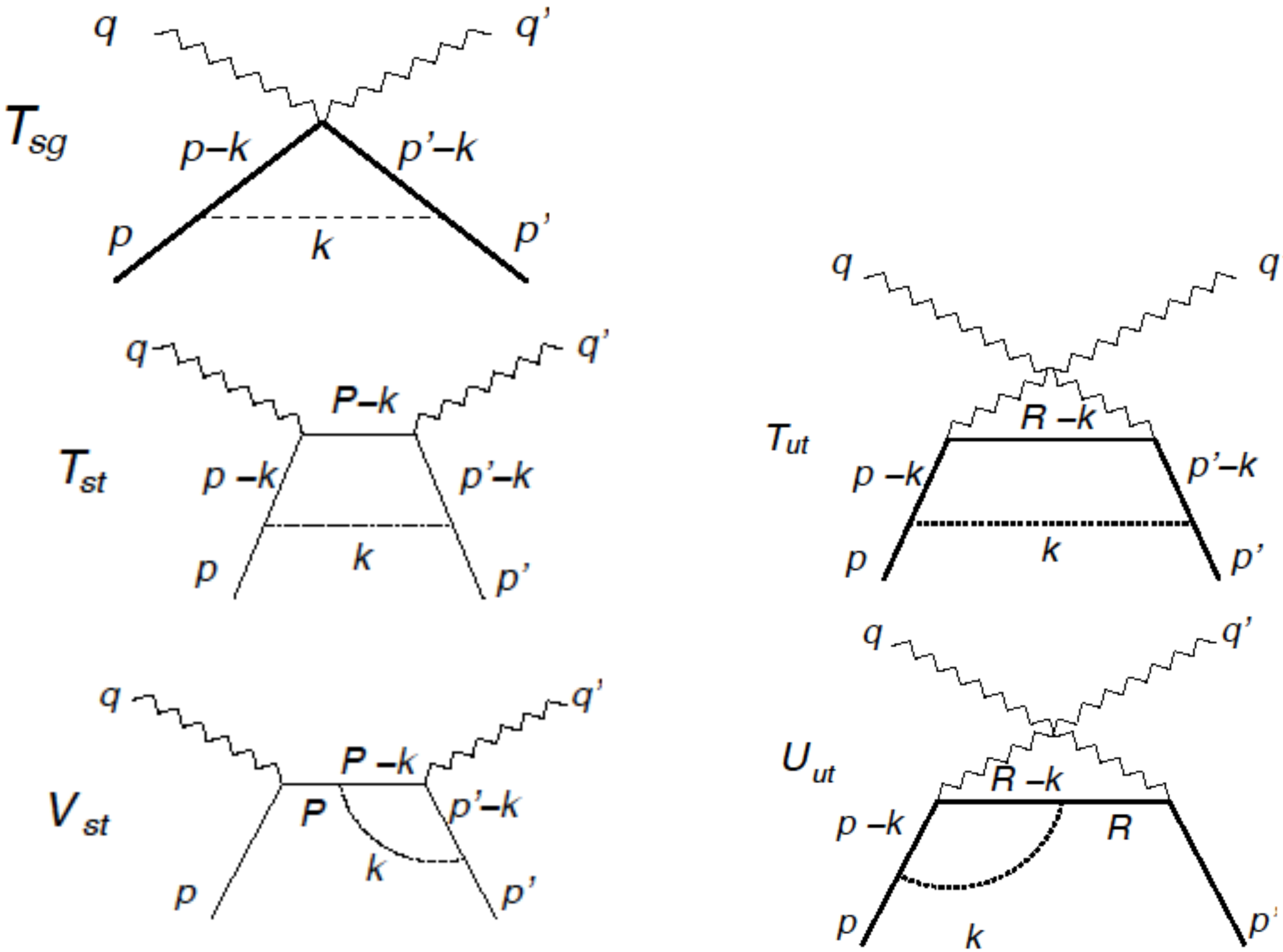
Metz's approach $S_1 = -B_1, S_2 = B_3, S_3 = -B_2, S_4 = B_4, S_5 = B_{19}$

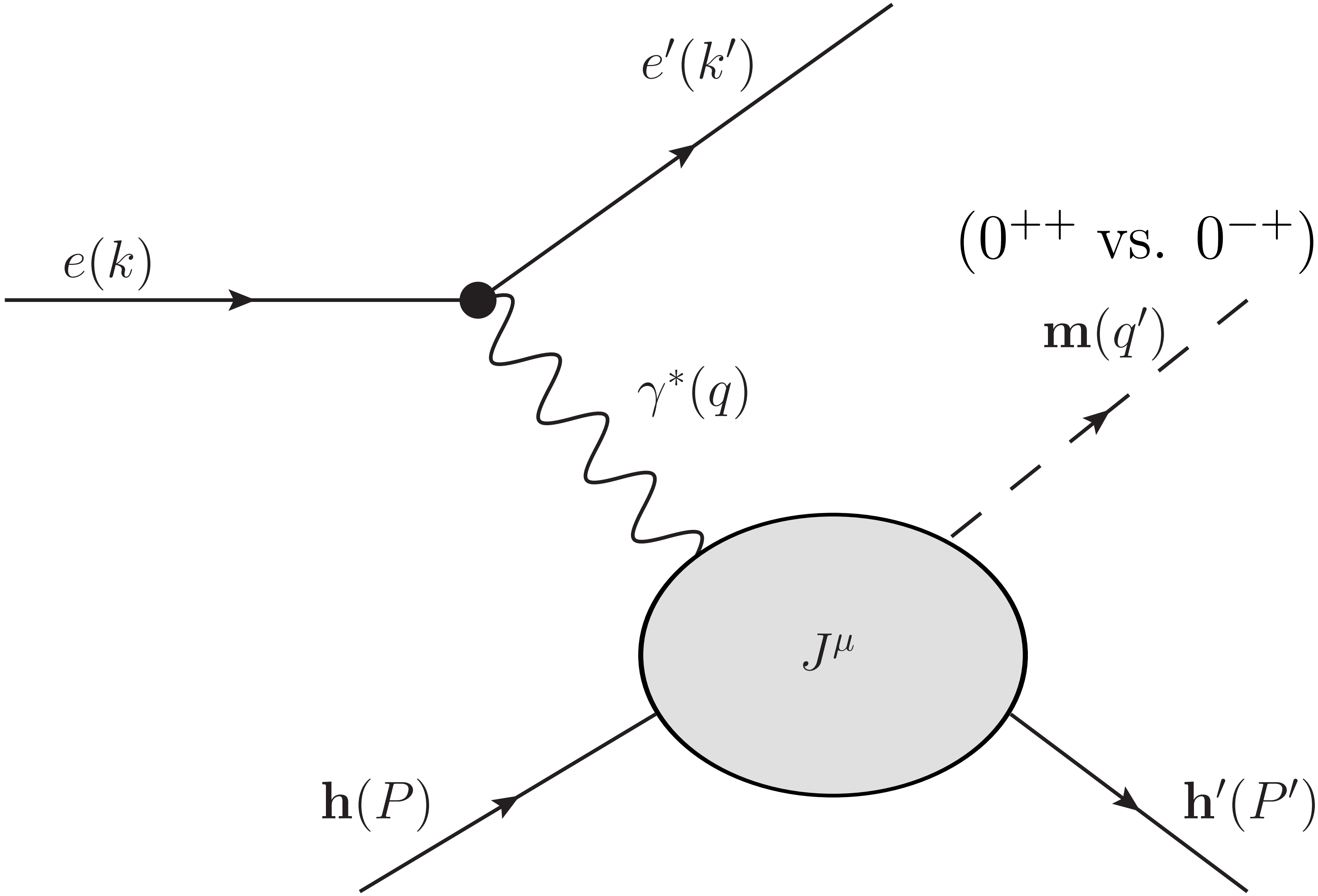
The method using the projectors introduces a kinematical singularity at $q' \cdot q = 0$. In Tarrach's paper a method is described to remove these kinematic poles. Here we give the final result of that algorithm as obtained in the thesis of Metz[3]. His CFFs are denoted as B_1, B_2, B_3, B_4, and B_{19}. They are implicitly given in terms of the elementary tensor by the following equations:

$$
\begin{aligned}
M^{\mu\nu} &= B_1 M_1^{\mu\nu} + B_2 M_2^{\mu\nu} + B_3 M_3^{\mu\nu} + B_4 M_4^{\mu\nu} + B_{19} M_{19}^{\mu\nu}, \\
M_1^{\mu\nu} &= -q' \cdot q \, g^{\mu\nu} + q^\mu q'^\nu, \\
M_2^{\mu\nu} &= -(\bar{P} \cdot q)^2 g^{\mu\nu} - q' \cdot q \, \bar{P}^\mu \bar{P}^\nu + \bar{P} \cdot q \, (\bar{P}^\mu q'^\nu + q^\mu \bar{P}^\nu), \\
M_3^{\mu\nu} &= q'^2 q^2 g^{\mu\nu} + q' \cdot q \, q'^\mu q^\nu - q^2 q'^\mu q'^\nu - q'^2 q^\mu q^\nu, \\
M_4^{\mu\nu} &= \bar{P} \cdot q \, (q'^2 + q^2) \, g^{\mu\nu} - \bar{P} \cdot q \, (q'^\mu q'^\nu + q^\mu q^\nu) \\
&\quad - q^2 \bar{P}^\mu q'^\nu - q'^2 q^\mu \bar{P}^\nu + q' \cdot q \, (\bar{P}^\mu q^\nu + q'^\mu \bar{P}^\nu), \\
M_{19}^{\mu\nu} &= (\bar{P} \cdot q)^2 q'^\mu q^\nu + q'^2 q^2 \bar{P}^\mu \bar{P}^\nu - \bar{P} \cdot q \, q^2 q'^\mu \bar{P}^\nu - \bar{P} \cdot q \, q'^2 \bar{P}^\mu q^\nu .
\end{aligned}
$$

[3] A. Metz, *Virtuelle Comptonstreuung un die Polarisierbarkeiten de Nukleons* (in German), PhD thesis, Universität mainz, 1997.

Gauge invariance requires more than handbag amplitudes





C.Ji, H.-M.Choi, A.Lundeen, B.Bakker
arXiv:1806.01379 [nucl-th]

5-Fold Differential Cross Section

for unpolarized target and without recoil polarization

The following notation of the coincidence cross section will be used in our calculations. Further details can be found in D. Drechsel and L. Tiator, J. Phys. G 18 (1992) 449-497. (scanned version) (click here for a larger image)

$$\frac{d\sigma}{d\Omega_f dE_f d\Omega} = \Gamma \frac{d\sigma_v}{d\Omega}, \quad \Gamma = \frac{\alpha}{2\pi^2} \frac{E_f}{E_i} \frac{k_\gamma}{Q^2} \frac{1}{1-\varepsilon}$$

$$\frac{d\sigma_v}{d\Omega} = \frac{d\sigma_T}{d\Omega} + \varepsilon \frac{d\sigma_L}{d\Omega} + \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{LT}}{d\Omega} \cos\phi$$

$$+ \varepsilon \frac{d\sigma_{TT}}{d\Omega} \cos 2\phi + h\sqrt{2\varepsilon(1-\varepsilon)} \frac{d\sigma_{LT'}}{d\Omega} \sin\phi$$

https://maid.kph.uni-mainz.de/maid2007/cross.html

$$d\sigma \equiv \frac{d^5\sigma}{dy\,dx\,dt\,d\phi_{k'}\,d\phi_{q'}} = \kappa\langle|\mathcal{M}|^2\rangle,$$

$$\kappa \equiv \frac{1}{(2\pi)^5}\frac{yx}{32Q^2\sqrt{1+(\frac{2Mx}{Q})^2}}.$$

$$y = P\cdot q/P\cdot k$$

$$t = (P-P')^2$$

$$x = Q^2/(2P\cdot q)$$

$$\epsilon = -\frac{2M^2x^2y^2+2Q^2(y-1)}{2M^2x^2y^2+Q^2(y^2-2y+2)}$$

$$\epsilon_L = \frac{Q^2}{\nu^2}\epsilon$$

$$\langle|\mathcal{M}|^2\rangle = \left(\frac{e^2}{q^2}\right)^2\mathcal{L}^{\mu\nu}\mathcal{H}_{\mu\nu} = \left(\frac{e^2}{q^2}\right)^2\left[\frac{2q^2}{\epsilon-1}\langle|\tau_{fi}|\rangle^2 + 2i\lambda\epsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta J^\dagger_\mu J_\nu\right]$$

$d\sigma_T$ $d\sigma_{TT}$ $d\sigma_L$ $d\sigma_{LT}$ $d\sigma_{LT'}$

$$\langle|\tau_{fi}|\rangle^2 = \frac{1}{2}(|H_x|^2+|H_y|^2) + \frac{\epsilon}{2}(|H_x|^2-|H_y|^2) + \epsilon_L|H_z|^2 - \sqrt{\frac{1}{2}\epsilon_L(1+\epsilon)}(H_x^*H_z + H_z^*H_x)$$

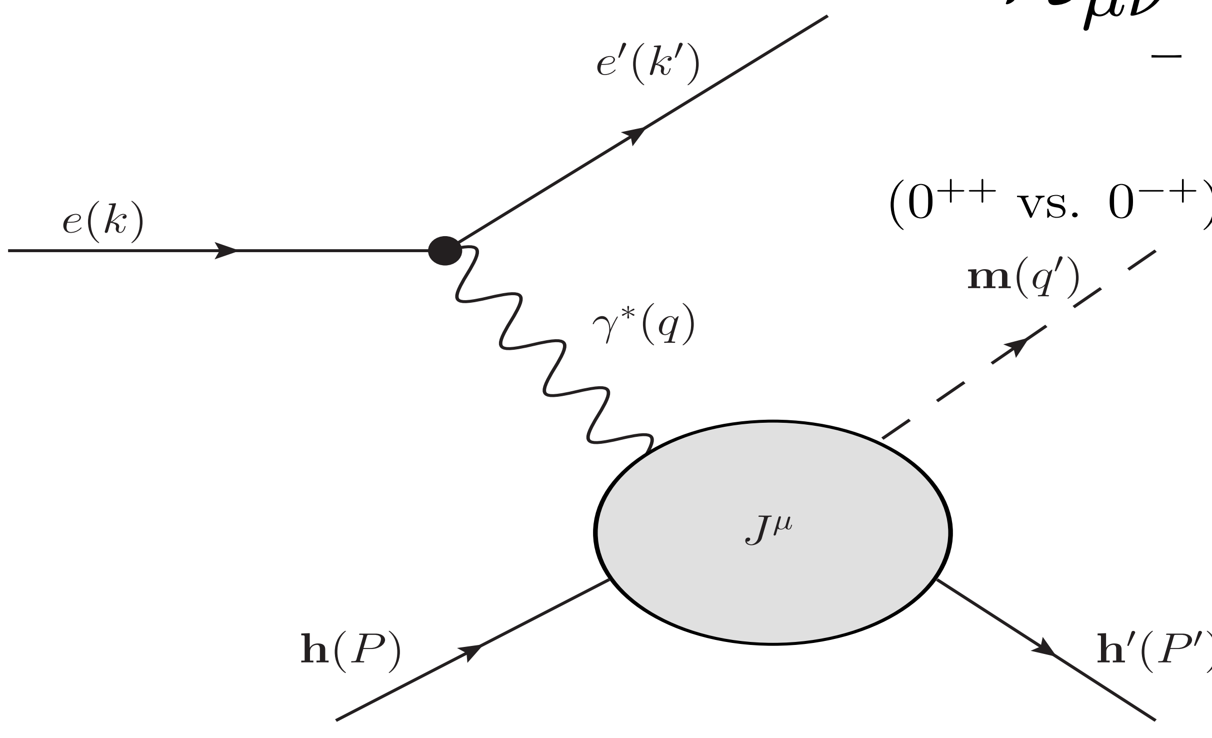
R. Williams, C.Ji, S. Cotanch (= WJC), PRC46, 1617 (1992)

$$\langle |\mathcal{M}|^2 \rangle = \left(\frac{e^2}{q^2} \right)^2 \mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu}$$

$$\mathcal{L}^{\mu\nu} = q^2 \Big[g^{\mu\nu} + \tfrac{2}{q^2} (k^\mu k'^\nu + k'^\mu k^\nu) \Big] + 2i\lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta$$

$$\mathcal{H}_{\mu\nu} = J^\dagger_\mu J_\nu$$

$$\mathcal{H}_{\mu\nu} \neq \mathcal{H}_{\nu\mu}$$



Pseudoscalar(0^{-+}) Meson vs. Scalar(0^{++}) Meson

$$\epsilon^{\mu\nu\alpha\beta} \quad \text{vs.} \quad d^{\mu\nu\alpha\beta} = g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}$$

$$J_S^\mu = (S_q q_\alpha + S_{\bar{P}}\bar{P}_\alpha)d^{\mu\nu\alpha\beta}q_\beta\Delta_\nu$$

$$F_{PS}(Q^2, t, x)$$

$$J_{PS}^\mu = F_{PS}\epsilon^{\mu\nu\alpha\beta}q_\nu\bar{P}_\alpha\Delta_\beta$$

$$F_1 = S_q - S_{\bar{P}}$$

$$F_2 = S_{\bar{P}}$$

$$F_1(Q^2, t, x) \qquad F_2(Q^2, t, x)$$

$$J_S^\mu = F_1(q^2\Delta^\mu - q^\mu q\cdot\Delta) + F_2[(\bar{P}\cdot q + q^2)\Delta^\mu - (\bar{P}^\mu + q^\mu)q\cdot\Delta]$$

$$q \;;\; \bar{P} = P + P' \;;\; \Delta = P - P' = q' - q$$

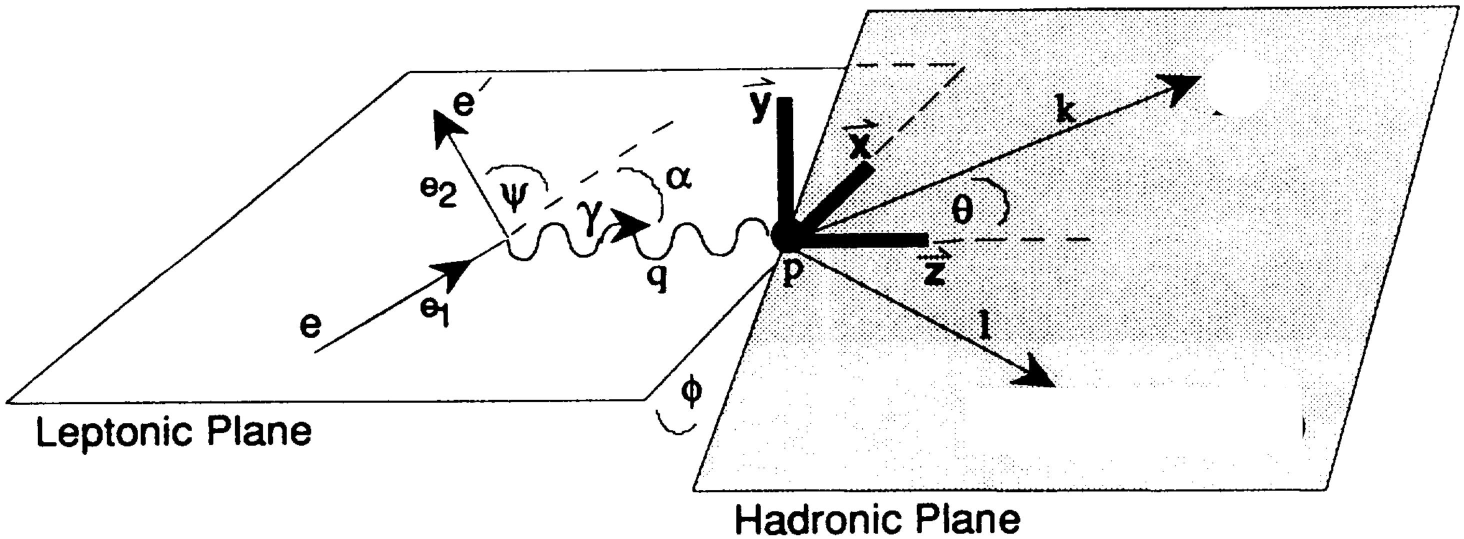
$$\boxed{J^{\mu}_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta} q_{\nu}\bar{P}_{\alpha}\Delta_{\beta}}$$

$$\begin{aligned}\mathcal{H}_{\mu\nu} &= J^{\dagger}_{\mu} J_{\nu} \\ &= |F_{PS}|^2 \epsilon_{\mu\alpha\beta\gamma}\epsilon_{\nu\alpha'\beta'\gamma'} q^{\alpha}\bar{P}^{\beta}\Delta^{\gamma} q^{\alpha'}\bar{P}^{\beta'}\Delta^{\gamma'} \\ &= \mathcal{H}_{\nu\mu}\end{aligned}$$

$$\boxed{\epsilon^{\mu\nu\alpha\beta} k_{\alpha} k'_{\beta}\mathcal{H}_{\mu\nu} = 0}$$

$$\frac{d\sigma^{PS}_{\lambda=+1} - d\sigma^{PS}_{\lambda=-1}}{d\sigma^{PS}_{\lambda=+1} + d\sigma^{PS}_{\lambda=-1}} = 0$$

$$J^{\mu}_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta$$



$$H_z = 0$$

$$d\sigma^{PS} = d\sigma^{PS}_{T} + d\sigma^{PS}_{TT}\epsilon\cos 2\phi$$
$$= d\sigma^{PS}_{T}(1 - \epsilon\cos 2\phi)$$

$$d\sigma^{PS}_{T} = \kappa\frac{e^4|F_{PS}(Q^2,t,x)|^2\sin^2\theta}{4M^2x^4(1-\epsilon)}\left(4M^2x^2+Q^2\right)\left[x^2\left(t^2-4m^2M^2\right)+Q^4+2Q^2tx\right]$$

Scalar Meson Production

$$J_S^\mu = F_1(q^2\Delta^\mu - q^\mu q\cdot\Delta) + F_2[(\bar{P}\cdot q + q^2)\Delta^\mu - (\bar{P}^\mu + q^\mu)q\cdot\Delta]$$

$$d\sigma_\lambda^S = d\sigma_T^S(1+\epsilon\cos(2\phi)) + d\sigma_L^S\epsilon_L + d\sigma_{LT}^S\cos\phi\sqrt{\epsilon_L(1+\epsilon)} + \lambda\, d\sigma_{\mathrm{BSA}}^S$$

$$\begin{bmatrix} d\sigma_T^S \\ d\sigma_L^S \\ d\sigma_{LT}^S \\ d\sigma_{\mathrm{BSA}}^S \end{bmatrix} = \begin{bmatrix} T_1 & T_2 & T_3 & 0 \\ L_1 & L_2 & L_3 & 0 \\ I_1 & I_2 & I_3 & 0 \\ 0 & 0 & 0 & S_A \end{bmatrix} \begin{bmatrix} |F_1|^2 \\ |F_2|^2 \\ F_{12}^+ \\ F_{12}^- \end{bmatrix}$$

$$F_{12}^\pm = F_1F_2^* \pm F_2F_1^*$$

$$\frac{d\sigma_{\lambda=+1}^S - d\sigma_{\lambda=-1}^S}{d\sigma_{\lambda=+1}^S + d\sigma_{\lambda=-1}^S} = \frac{d\sigma_{\mathrm{BSA}}^S}{d\sigma_T^S(1+\epsilon\cos(2\phi)) + d\sigma_L^S\epsilon_L + d\sigma_{LT}^S\cos\phi\sqrt{\epsilon_L(1+\epsilon)}}$$

$$\sim\ F_1F_2^* - F_2F_1^*$$

$$F_1 F_2^* - F_2 F_1^* \neq 0$$

as far as at least one of F_1 and F_2 develops an imaginary part.

However, if $q = q' - \zeta P$ due to A.V.Radyushkin,PRD56,5524(1996) is imposed, then two form factors merge together: $J_S^\mu = \zeta(F_1 + F_2)(q^2 P^\mu - q^\mu q \cdot P)$

No BSA in that case as $\mathcal{H}_{\mu\nu} = \mathcal{H}_{\nu\mu}$

Thus, BSA measurement of scalar meson production off ^{4}He would be important.

Conclusion and Outlook

- Although the existing formulation meant already good progress, the realistic experimental setup requires the extension of the formalism to cover the broader kinematic regions of the DVCS experiments.
- The determination of most general hadron tensor structure is important not only for CFFs also for the discussion of GPDs.
- The DNA of the most general hadronic tensor structure for scalar target is found and applicable to DVCS and DVMP off ^4H_e .
- BSA from exclusive π^0 production off ^{4}He is predicted to be absent from the symmetry of general hadronic current structure consideration, which may provide a benchmark for BSA analysis.