On AdS black holes and deconfinement

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This talk is mostly based on:

Sunjin Choi (SNU), Chiung Hwang (KIAS), SK & June Nahmgoong (SNU)
"Deconfining vortices on M2-branes"
work in preparation.

… on M2-brane indices, their free energies & their (remote) relations to AdS black holes.

Also with some general comments on AdS black holes, related to:

Joonho Kim (KIAS), SK & June Nahmgoong (SNU)
work in progress.

So perhaps a better title for today's talk would be….

Deconfining vortices on M2-branes (+ α)

Black holes in AdS

- Schwarzschild black holes in AdS
- Small BH: More or less like BH's in flat space. $M \uparrow T \downarrow$. Negative specific heat.
- Large BH: Plays important roles in AdS thermodynamics

- Hawking-Page transition: transition between two phases, at T of order $1/R_{AdS}$
- Low temperature phase: gas of gravitons in AdS
- High temperature phase: large AdS black holes

- CFT dual: Confinement-deconfinement transition [Witten]
- Confined phase: $F \sim O(N^0)$
- Deconfined phase: $F \sim O(N^2)$ of gluons (~ matrices)

- Studies of the transition were made for weak-coupling 4d Yang-Mills on $S^3 \times R$. Many qualitative aspects of HP transition are already visible at weak coupling. [Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk] :

Various questions: $d = 4$ & $d \neq 4$

- N M2-branes & 3d SCFT: $\sim N^{3/2}$
- N M5-branes & 6d SCFT: $\sim N^3$
- 5d SCFTs: E.g. $AdS_6 \times S^4/Z_2$ of massive IIA. $\sim N^{5/2}$

- Questions:

- Qualitative question:

In various cases, what d.o.f. deconfine in $d \neq 4$…?

Can we address it in any concrete set-up…?

- Quantitative question:

Can we count AdS black holes? Say, SUSY AdS black holes?

Global AdS w/ CFT at the boundary $S^n \times R$

Plan of this talk

1. Study a partition function (~ Witten index) of “vortices” in M2-brane QFT

2. Develop an entropy function of SUSY AdS_4 black holes and discuss its relation to the vortex free energy above (via “superconformal index” on $S^2 \times S^1$)

3. Extend some discussions to SUSY AdS_d black holes with $d = 3,4,5,6,7$

- Entropy functions
- Issues on microscopic counting of these black holes

4. Concluding remarks

M2-brane QFTs

- ABJM: $U(N)_1 \times U(N)_{-1}$ Chern-Simons matter theory.
- Complicated brane engineering. Basically, D2-branes in UV probing D6 & Taub-NUT

  [Aharony, Bergman, Jafferis, Maldacena]

- 3d maximal SYM: $U(N)$ Yang-Mills with $N = 8$ SUSY
- Stack of D2-branes probing flat space R^7
- Somehow, hard to use it to make quantitative studies on M2-branes

- "Mirror dual" of maximal SYM
- N D2-branes probing 1 D6-brane in flat space
- $N = 4$ SUSY in UV. $U(N)$ gauge theory w/ 1 fundamental & 1 adjoint hypers.
- Believed to flow to same maximal SCFT.

- The 3rd QFT: has Higgs branch, vortices, factorization of $Z[S^2 \times S^1]$, $Z[S^3]$, etc.

Higgs branch & vortices

- Higgs vacuum w/ FI $\xi > 0$ (massive deformation ~ chemical potential)

$$qq^\dagger - \tilde{q}^\dagger\tilde{q} + [\phi,\phi^\dagger] + [\tilde{\phi},\tilde{\phi}^\dagger] = \xi\ , \quad q\tilde{q} + [\phi,\tilde{\phi}] = 0$$

$$q^\dagger = (\overset{e^{in_1\varphi}}{\sqrt{N\xi}}, 0, \cdots, 0)$$

- Should set $\tilde{q} = 0$, to have BPS vortices
- For simplicity, also set $\tilde{\phi} = 0$.

$$\phi = \sqrt{\xi}\begin{pmatrix} 0 & \cdots & & & & \\ \overset{e^{in_2\varphi}}{\sqrt{N-1}} & 0 & \cdots & & & \\ 0 & \overset{e^{in_3\varphi}}{\sqrt{N-2}} & 0 & \cdots & & \\ \vdots & & & \ddots & & \\ 0 & \cdots & & \underset{e^{in_{N-1}\varphi}}{\sqrt{2}} & 0 & 0 \\ 0 & \cdots & & 0 & \underset{e^{in_N\varphi}}{1} & 0 \end{pmatrix}$$

- BPS vortex: space-dependent VEV, windings at infinity.
- Vortex charges: winding numbers $n_i \geq 0$ vs. $U(1)^N$ fluxes k_i

$$n_1 = k_1\ , \quad n_2 = k_2 - k_1\ , \quad \cdots\ , \quad n_N = k_N - k_{N-1}$$

$$M = \xi\sum_{i=1}^{N} k_i\ , \quad k_1 \leq k_2 \leq \cdots \leq k_N$$

- Note that an order is required to k_i's:
- In large N limit, one can only occupy finitely many $U(1)^N$ fluxes at low E
- Large N, low E spectrum doesn't see N: makes it possible to have a "gravity dual"

Vortex partition function

- $Z[R^2_\beta \times S^1]$ with suitable Higgs branch VEV at infinity

$$Z(q,t,z,\tilde{Q}) = \mathrm{Tr}\left[(-1)^F q^{R+r+2j} t^{R-r} z^{2L} \tilde{Q}^T\right]$$

- R, r: $SO(4)$ R-charges ($q = e^{-\beta}$: Ω-deformation)
- z: flavor symmetry for adjoint hyper. $\tilde{Q}$: vortex charge, topological $U(1)_T$

- Closely related to $Z[D_2 \times S^1]$, with suitable boundary conditions at $\partial D_2 = S^1$

$$D_n q = 0\ ,\quad \tilde{q} = 0\ ,\quad D_n \phi = 0\ ,\quad \tilde{\phi} = 0$$

(similar D/N b.c. given to fields in vector multiplet)

- Contour integral expression [Yoshida, Sugiyama]:

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N}\left[\frac{ds_a}{2\pi i s_a} s_a^{-2\pi r\zeta}\right] \prod_{a=1}^{N} \frac{(s_a t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)_\infty}{(s_a t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)_\infty} \cdot \frac{\prod_{a\neq b}(s_a s_b^{-1}; q^2)_\infty}{\prod_{a,b=1}^{N}(s_a s_b^{-1} t^{-1} q; q^2)_\infty} \cdot \prod_{a,b=1}^{N} \frac{(s_a s_b^{-1} z t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)_\infty}{(s_a s_b^{-1} z t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)_\infty}$$

$$\tilde{Q} \equiv q^{4\pi r\zeta}. \qquad (a;q)_\infty \equiv \prod_{n=0}^{\infty}(1 - a q^n)$$

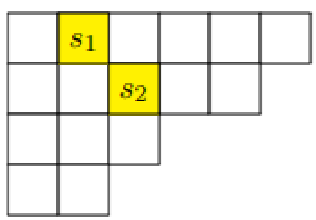
“Confining” spectrum

- For $\tilde{Q} \ll 1$, can express it as a residue sum. $Z = Z_{prefactor}\, Z_{pert}\, Z_{vortex}$

$$Z_{\text{prefactor}} = \frac{\left(u^N v^{\frac{N(N-1)}{2}}\right)^{2\pi r\zeta}}{(q^2;q^2)_\infty^N} \qquad Z_{\text{pert}} = \prod_{a=1}^{N} \frac{(u^{-2}v^a q^2;q^2)_\infty}{(v^a;q^2)_\infty} \qquad Z_{\text{vortex}} = \sum_{0\le k_1\le\cdots\le k_N} \tilde{Q}^{k_1+\cdots+k_N} Z_{k_1,\cdots,k_N}$$

- $Z_{k_1,\cdots,k_N}$ admits and expression in terms of Young diagram

$$\prod_{s\in Y} \frac{(1-u^{-2}q^{-2a(s)}v^{-l(s)})(1-u^{-2}vq^2q^{2a(s)}v^{l(s)})(1-v^Nq^{2x(s)}v^{-y(s)})}{(1-q^{-2}q^{-2a(s)}v^{-l(s)})(1-vq^{2a(s)}v^{l(s)})(1-u^{-2}q^2v^Nq^{2x(s)}v^{-y(s)})} \qquad \begin{array}{l} u = t^{1/2}q^{1/2} \\ v = zt^{1/2}q^{1/2} \end{array}$$



$$\begin{cases} a(s): & \text{arm (horizontal) length} = \text{number of boxes to the right of } s \\ l(s): & \text{leg (vertical) length} = \text{number of the boxes below } s \\ x(s): & \text{horizontal position} = \text{number of boxes to the left of } s \\ y(s): & \text{vertical position} = \text{number of the boxes above } s \end{cases}$$

$$a(s_1) = 4,\ l(s_1) = 3,\ x(s_1) = 1,\ y(s_1) = 0,$$
$$a(s_2) = 2,\ l(s_2) = 1,\ x(s_1) = 2,\ y(s_1) = 1.$$

- One can take smooth large N limit, if vortex is heavier than critical value

$$Z_{\text{vortex}}^{N\to\infty} = PE\left[\frac{(1-u^{-2})(1-u^{-2}vq^2)}{(1-q^{-2})(1-v)}\frac{\tilde{Q}}{1-q^2\tilde{Q}u^{-2}}\right]$$

- Agrees with ‘suitably projected’ graviton index on $AdS_4 \times S^7$.

“Deconfining” phenomenon?

- Index does not see deconfinement properly, but we can probe a hint of it.
- Study various parameter regimes & see $\log Z \sim N^{3/2}$.
- Deconfining vortices. Elementary d.o.f. of QFT shouldn't. $\log Z \sim N^2$ is wrong.

- Strategy: large N calculus of $Z[D_2 \times S^1]$ contour integral

- For technical reasons, we only do so in $\beta \to 0$ limit, $q = e^{-\beta}$. (“large volume”, “high T”)

$$Z \sim \frac{1}{N!} \int \prod_{a=1}^{N} \frac{ds_a}{2\pi i s_a} \exp\left[-\frac{W(s,\cdots)}{2\beta}\right]$$

$$\begin{aligned} W \;=\; & N\left[\mathrm{Li}_2(zt^{-\frac{1}{2}}) - \mathrm{Li}_2(zt^{\frac{1}{2}}) - \mathrm{Li}_2(t^{-1})\right] + \sum_{a=1}^{N}\left[\xi \log s_a + \mathrm{Li}_2(s_a t^{-\frac{1}{2}}) - \mathrm{Li}_2(s_a t^{\frac{1}{2}})\right] \\ & + \sum_{a\neq b}\left[\mathrm{Li}_2(s_a s_b^{-1}) - \mathrm{Li}_2(s_a s_b^{-1} t^{-1}) + \mathrm{Li}_2(s_a s_b^{-1} z t^{-\frac{1}{2}}) - \mathrm{Li}_2(s_a s_b^{-1} z t^{\frac{1}{2}})\right] \end{aligned}$$

- Real fugacities. Eigenvalue distribution on real axis: $s_a = s_0^- \exp\left[N^\alpha x_a\right] \quad (x_1 < x_a < x_2)$

- Like $Z[S^3]$ or $Z_{top}(S^2 \times S^1)$, arrange 2-body interactions into short-ranged ones.

$$W \approx W_0 + N^{1+\alpha}(\xi + T)\int_0^{x_2} dx\rho(x)x + N^{1+\alpha}\xi\int_{x_1}^{0} dx\rho(x)x + \frac{N^{2-\alpha}}{8}T(4f^2 - T^2)\int_{x_1}^{x_2} dx\rho(x)^2$$

$$W_0 \equiv \tfrac{N^2}{4}T(2f+T) \qquad\qquad \tilde{Q} = e^{-\xi},\ t = e^{T},\ z = e^{f}$$

- Formal 0-point energy: a remnant of our weak-coupled UV QFT. Ignore it.

Result

- The large N free energy (saddle point exists only in certain parameter regimes, especially when the quantity inside square-root is positive)

$$\log Z \sim \mathrm{sgn}(\xi)\frac{\sqrt{2}N^{\frac{3}{2}}}{6}\frac{\sqrt{-\xi(\xi+T)(4f^2-T^2)}}{\beta}$$

$$Q \equiv q^{\frac{1}{2}}t^{-\frac{1}{2}}\tilde{Q} = e^{-\xi-\frac{T}{2}-\frac{\beta}{2}} \equiv e^{-\xi_{\text{ren}}} \text{ is the canonical parameter for the } SO(8)$$

$$(T_1,T_2,T_3,T_4) = \left(\frac{T}{2}+f, \frac{T}{2}-f, -\frac{T}{2}+\xi_{\text{ren}}, -\frac{T}{2}-\xi_{\text{ren}}\right)$$

$$\log Z \sim \mathrm{sgn}(\xi_{\text{ren}}-T/2)\frac{\sqrt{2}N^{\frac{3}{2}}}{12\beta}\sqrt{-(4\xi_{\text{ren}}^2-T^2)(4f^2-T^2)} \equiv \mathrm{sgn}(\xi_{\text{ren}}-T/2)\frac{\sqrt{2}N^{\frac{3}{2}}}{3\beta}\sqrt{-T_1T_2T_3T_4}$$

- A small test: highly squashed $S^3_{b\ll 1}$, regarded as another IR regulator of $R^2 \times S^1$

$$ds^2 = b^2|dz_1|^2 + b^{-2}|dz_2|^2 = \left(b^2\sin^2\theta + b^{-2}\cos^2\theta\right)d\theta^2 + b^2\cos^2\theta d\phi_1^2 + b^{-2}\sin^2\theta d\phi_2^2$$

$$\approx b^{-2}(d\rho^2+\rho^2 d\phi_2^2) + b^2 d\phi_1^2 \qquad\qquad 0 \le \rho \equiv \sin\theta \le 1$$

- After suitable analytic continuation (although a priori unjustified), and freezing to the values of T_I's on S^3, we reproduce $Z[S^3]$ at large N. (computed from ABJM)

Superconformal index

- Factorization of superconformal index into two vortex partition functions
  - Many Higgs vacuum points contribute $qq^\dagger - \tilde{q}^\dagger\tilde{q} + [\phi,\phi^\dagger] + [\tilde{\phi},\tilde{\phi}^\dagger] = \xi \ , \quad q\tilde{q} + [\phi,\tilde{\phi}] = 0$
  - The factorization formula: $$Z_{S^2\times S^1}(Q,t,z,q) = \sum_{p\in\text{Higgs}} Z^{(p)}_{\text{pert}}(t,z,q) Z^{(p)}_{\text{vortex}}(Q,t,z,q) Z^{(p)}_{\text{vortex}}(Q^{-1},t^{-1},z^{-1},q^{-1})$$
  - We have no good control at large N, except one Higgs vacuum we discussed so far.
  - Anyway, we can "assume" that the known term dominates large N, and write down its asymptotic free energy: $$\log Z_{S^2\times S^1} \sim \pm i\frac{2\sqrt{2}N^{\frac{3}{2}}}{3\beta}\sqrt{T_1T_2T_3T_4} \qquad T_1+T_2+T_3+T_4\approx 0$$
  - The formula has a curious implication to the "black hole entropy function."

- Does this index deconfine? Not properly. We just view it as an indirect signal of deconfined d.o.f. I will present an analogy in 4d N=4 SYM index.
- 'Standard lore' is that the index does not deconfine in 4d N=4 SYM.
  - No large N phase transition seen at finite T. [Kinney, Maldacena, Minwalla, Raju]
  - Take $\beta \to 0$ first. Still, $\log Z \sim (a-c)\beta^{-1} = 0$ for N=4 SYM. [Di Pietro, Komargodski]

Analogy: index of 4d N=4 SYM

- The index for 4d N=4 SYM:

$$Z[S^3 \times S^1] = \frac{1}{N!} \int \prod_{i=1}^{N} \frac{d\alpha_i}{2\pi} \prod_{i<j} \left(2\sin\frac{\alpha_i - \alpha_j}{2}\right)^2 PE\left[\left(1 - \frac{\prod_{i=1}^{3}(1-t^2 v_i)}{(1-t^3 y)(1-t^3/y)}\right) \sum_{i,j=1}^{N} e^{i(\alpha_i - \alpha_j)}\right]$$

v_i satisfying $v_1 v_2 v_3 = 1$ are the fugacities for $SU(3) \subset SO(6)$ part of R-symmetry.

- An expression valid for making large N approx. (ignore Cartans)

$$Z[S^3 \times S^1] \sim \frac{1}{N!} \int \prod_{i=1}^{N} \frac{d\alpha_i}{2\pi} \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \frac{\prod_{i=1}^{3} 2\sinh\frac{n\Delta_i}{2}}{2\sinh\frac{n\omega_1}{2} \cdot 2\sinh\frac{n\omega_2}{2}} \sum_{i \neq j} e^{in(\alpha_i - \alpha_j)}\right]$$

$$\Delta_1 + \Delta_2 + \Delta_3 = \omega_1 + \omega_2 \qquad (e^{-\omega_1}, e^{-\omega_2}, e^{-\Delta_i}) = (t^3 y, t^3/y, t^2 v_i)$$

- Take the limit $\omega_{1,2} \to 0$, keeping Δ_I imaginary (& analytically continue later)

$$\log Z_{S^3 \times S^1} \sim \frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1 \omega_2} + \cdots$$

- Analogous to the procedures we took in 3d. $\log Z_{S^2 \times S^1} \sim \pm i \frac{2\sqrt{2} N^{\frac{3}{2}}}{3\beta} \sqrt{T_1 T_2 T_3 T_4}$

- 4d: N^2 itself is clearly visible as elementary fields, so this may not look that novel.
- In 3d, probably more interesting since we don't see $N^{3/2}$ explicitly.

Entropy function of SUSY AdS black holes

- Now I slightly change the gear, to discuss BPS black holes in AdS.
- BPS black holes in $AdS_4 \times S^7$ [Kostelecky, Perry] [Cvetic, et.al.] :

- Can have 5 charges: $Q_1, Q_2, Q_3, Q_4 \in SO(8)$ electric charge, $J \in SO(3)$ spin.
- Only 4 independent parameters. BPS AdS black holes come with a “charge relation”

- ‘Entropy function’ for these BH’s (inspired by [Hosseini, Hristov, Zafaroni] on AdS5, AdS7)

$$S = -F_{BPS}(T_i, \omega) + \omega J + \sum_{i=1}^{4} T_i Q_i = \pm i \frac{4\sqrt{2}N^{\frac{3}{2}}}{3} \frac{\sqrt{T_1 T_2 T_3 T_4}}{\omega} + \omega J + \sum_{i=1}^{4} T_i Q_i$$

- Naively extremizing in T_i, ω, one obtains $S_* = 0$. (homogeneous degree 1.)
- Realizes the fact that BH’s don’t exist w/ independent charges
- Constrained extremization gives right entropy & charge relation [Choi,Hwang,SK,Nahmgoong]

$$T_1 + T_2 + T_3 + T_4 - \omega = 2\pi i$$

- It is tempting to interpret F_{BPS} as the ‘off-shell’ large N free energy.

- Limiting chemical potentials to those of the index, and taking asymptotic limit $\omega = 2\beta \to 0$, one indeed finds

$$\log Z_{S^2 \times S^1} \sim \pm i \frac{2\sqrt{2}N^{\frac{3}{2}}}{3\beta} \sqrt{T_1 T_2 T_3 T_4} \qquad T_1 + T_2 + T_3 + T_4 \approx 0$$

Other entropy functions

- Our asymptotic free energy of index supports the "off-shell" interpretation of the entropy function.
- "Deriving" this entropy function, together with the constraint mechanism, is an important issue. (work in progress [Joonho Kim, SK, Nahmgoong] for AdS5)

- Similar entropy functions exist for other BPS AdS_d black holes.

- $AdS_5 \times S^5$: [Hosseini, Hristov, Zaffaroni] (2017)

$$\log Z \sim \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2}$$

- $AdS_7 \times S^4$: [Hosseini, Hristov, Zaffaroni] (2018)

$$\log Z \sim \frac{N^3}{12} \frac{(\Delta_1 \Delta_2)^2}{\omega_1 \omega_2 \omega_3}$$

- $AdS_6 \times S^4/Z_2$: [Choi, Hwang, SK, Nahmgoong] (to appear)

$$\log Z \sim \pm \# N^{\frac{5}{2}} i \frac{\Delta^3}{\omega_1 \omega_2}$$

- Even works with some AdS_3/CFT_2 models & some 2d QFT toy models…

Concluding remarks

- Studied vortices on M2. Studied their indices on $D_2 \times S^1$, $S^2 \times S^1$.
- Asymptotic free energies at large temperature-like parameter.
- $\log Z \sim N^{3/2}$ at large N.
- Indirect signal that vortices (or monopole operators) deconfine in 3d.

- Curious relation to the entropy function of SUSY AdS_4 black holes.
- Asymptotic index on $S^2 \times S^1$ agrees w/ “off-shell” free energy of BPS AdS_4 black holes.
- But only on a subspace of fugacity space, on which BPS black holes do not exist.

- This is closely related to the fact that BPS black holes satisfy a charge relation.

- We are trying to derive the entropy functions, at least in some asymptotic limits, without using SUSY index. We aim to really count BPS AdS black holes.
- Work in progress for AdS5 and AdS7 [Joonho Kim, SK, Nahmgoong]