

Dualities and 5-Brane Webs for 5d rank 2 SCFTs

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[arXiv:1801.03916] and [arXiv:1806.10569]

APCTP Strings, Branes and Gauge Theories
July 18th, 2018

- A 5d $N=1$ supersymmetric gauge theory can have a superconformal theory as the consistent ultra-violet fixed point.

Seiberg 96

- F_4 superconformal symmetry with $SU(2)_R$
- Web-diagrams of 5-branes in type IIB string theory have provided a powerful tool to study these 5d theories.

Aharony, Hanany 97
Aharony, Hanany, Kol 97

- For example, non-perturbative effects and various dualities of 5d gauge theories could be seen explicitly from their 5-brane webs.

- 5-brane web for $SU(N)_\kappa$ with Chern-Simons level κ for small κ can be obtained from N parallel D5 branes between 2 NS5 branes.
- With the introduction of an orientifold 5 plane, we can realize a $USp(2N)$ or an $SO(N)$ gauge theories.

Brunner, Karch 97,

Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowicz 97

Hanany, Zaffaroni 99

Zafir 15

- 6d (2,0) SCFT of ADE types exist on type IIB ADE singularity C^2/Γ_G [Witten'95, Strominger'95]
- 6d (1,0) SCFTs has been classified.

Heckman, Morrison, Rudelius, Vafa: 1502

- How about the classifying the 5d SCFTs?
 - 5d SCFTs can be obtained from 5-dim compactification of M-theory on local “shrinkable” Calabi-Yau 3-fold.

Jefferson, Katz, H.C. Kim, Vafa, 1801
 - All 5d SCFTs may obtained from a circle compactification with possible twist from 6d theories

- Two progress on the 5d SCFTs
 - milder constraint on the theory in the Coulomb phase with the positivity of monopole tension
Jefferson,HCKim,Vafa,Zafir:1705
 - classification of rank 2 5d SCFTs
Jefferson,Katz,H.C.Kim,Vafa,1801
 - construction of 5d SCFTs with O5-plane, including G_2 gauge theories

- We employ the 5-dim brane webs to two applications for rank 2 cases:
 - Find all brane webs for rank 2 cases
 - employing them for testing duality between different gauge descriptions

1. Introduction
2. 5d gauge theories from 5-brane webs
3. G_2 - $SU(3)_7$ duality
4. G_2 - $SU(3)$ - $Sp(2)$ sequences
5. $SU(3)_{0+1F+1S}$
6. $SU(3)_{3/2+9F}=Sp(2)+1AS+8F$
7. $Sp(2)+ 3 AS = SO(5)_{0+3F}$
8. Conclusion

2. 5d gauge theories from 5-brane webs

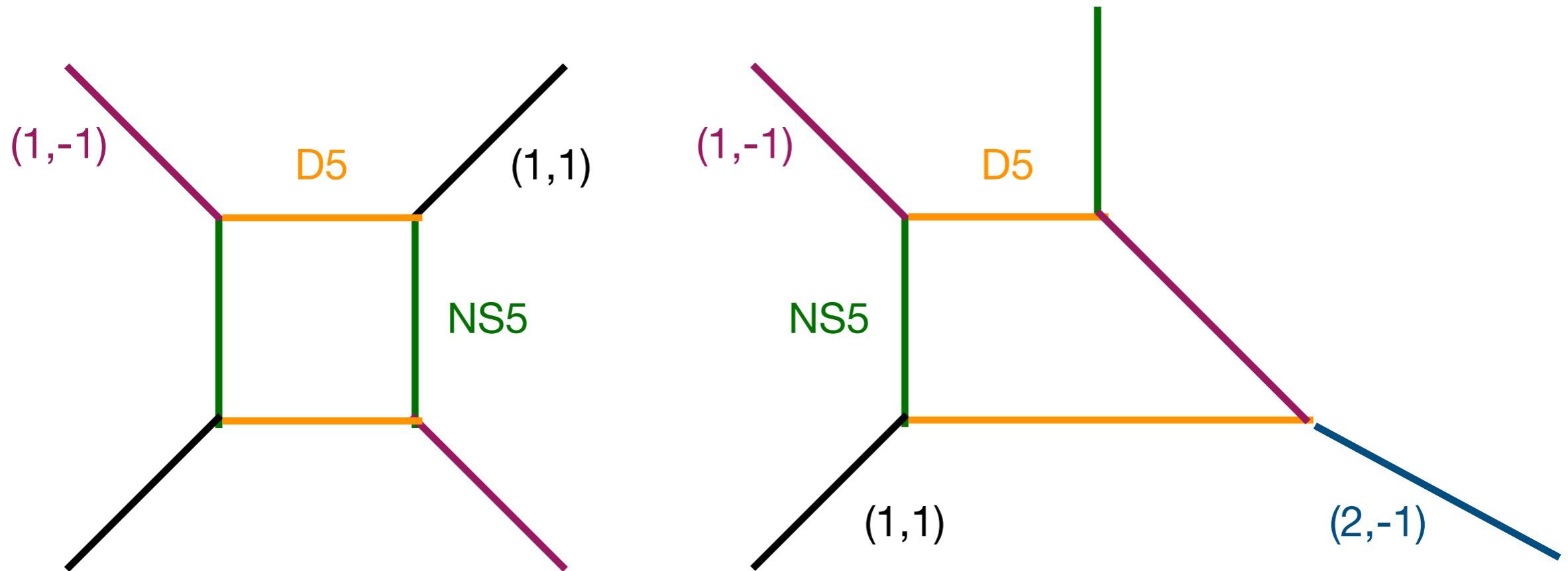
- We can construct a 5d supersymmetric field theory with eight supercharges as the world volume theory on a 5-brane web.
- 5-brane web in type IIB string theory:

	0	1	2	3	4	5	6	7	8	9
D5-brane	×	×	×	×	×	×				
O5-plane	×	×	×	×	×	×				
NS5-brane	×	×	×	×	×		×			
(p, q) 5-brane	×	×	×	×	×	angle				

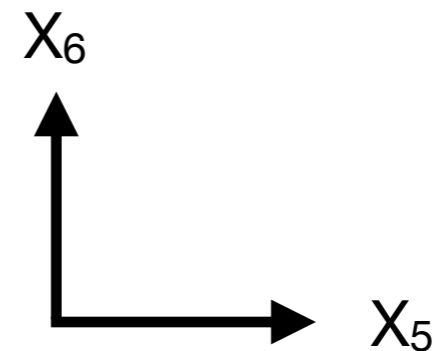
5-brane web

- 5d pure $SU(2)$ gauge theory with $\theta=0,\pi$

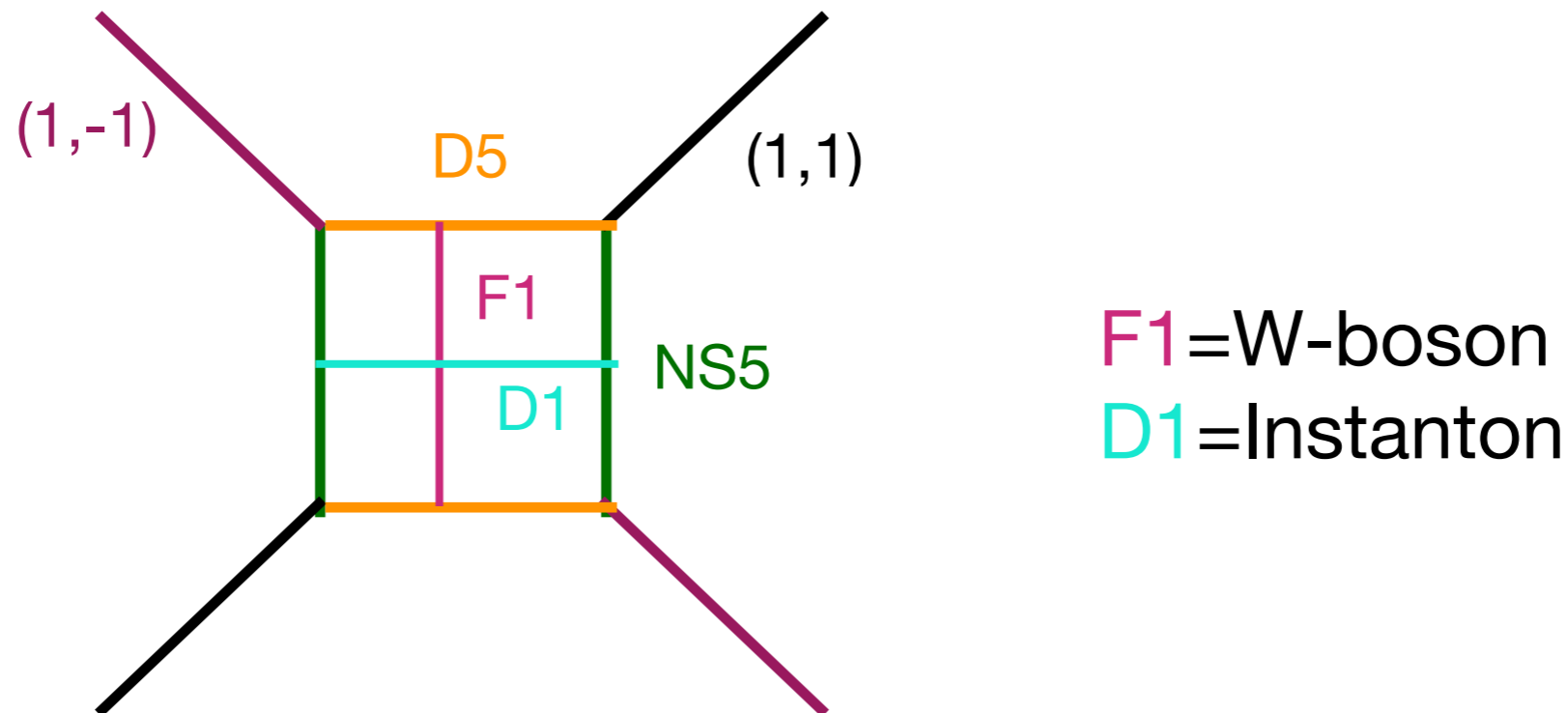
Aharony, Hanany 97
 Aharony, Hanany, Kol 97



- A (p,q) 5-brane (p D5 + q NS5 branes) is a line with slop q/p .

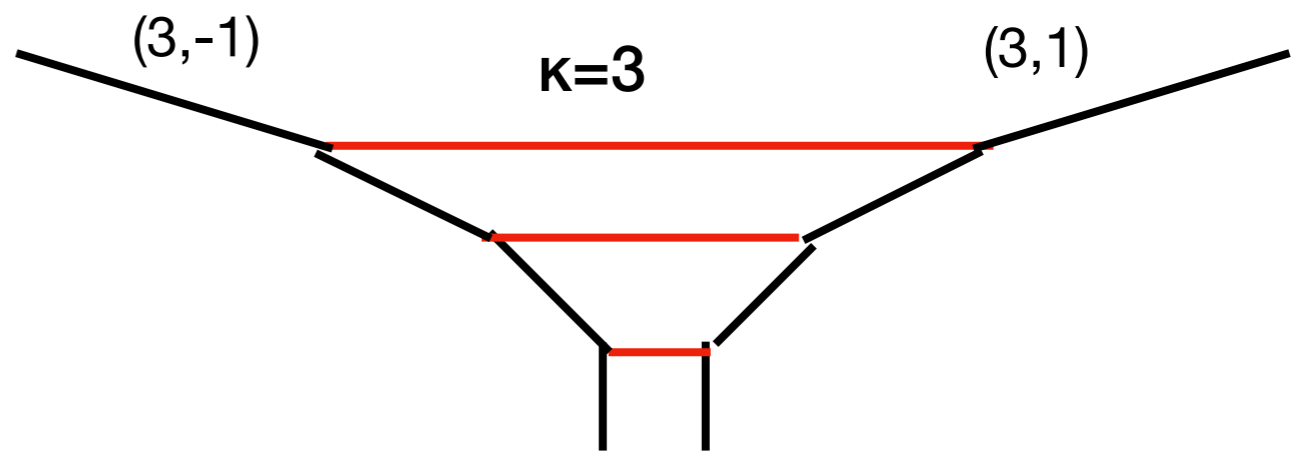
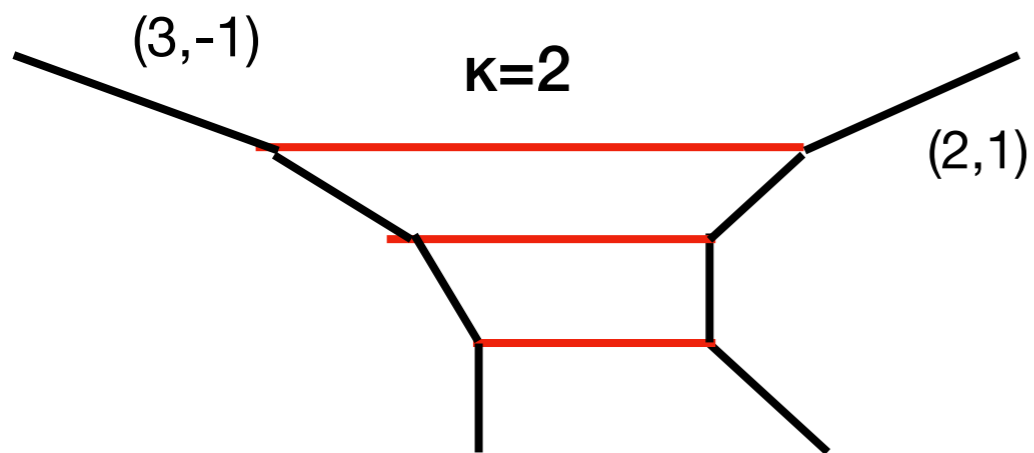
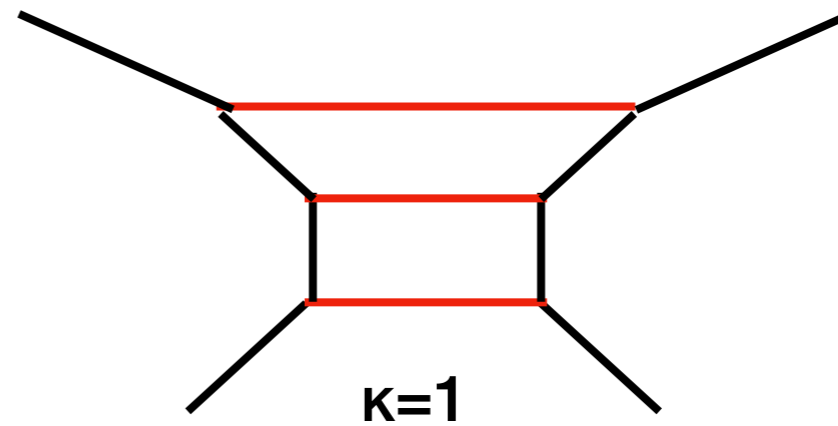
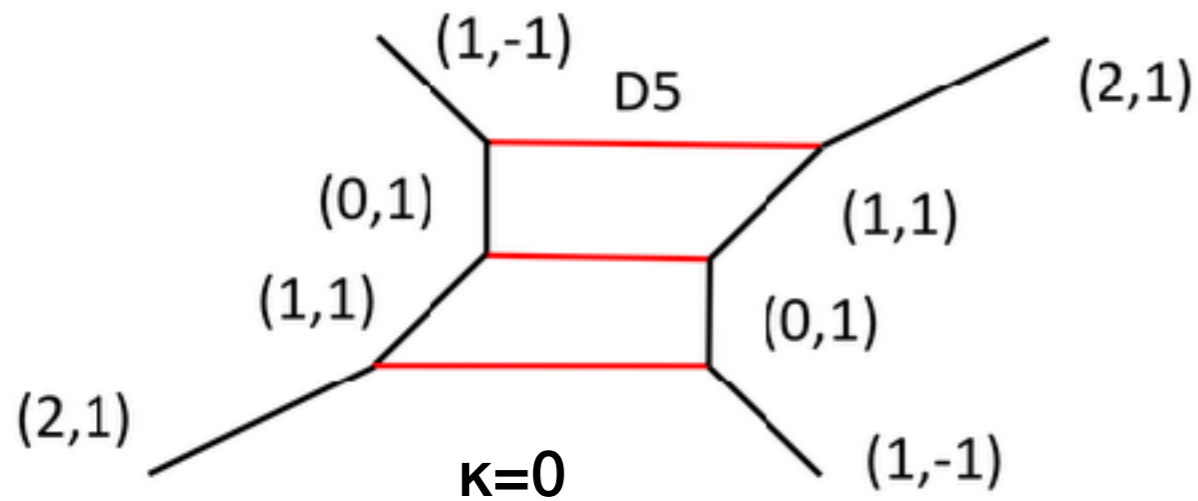


- The Coulomb phase for the $\theta=0$ case

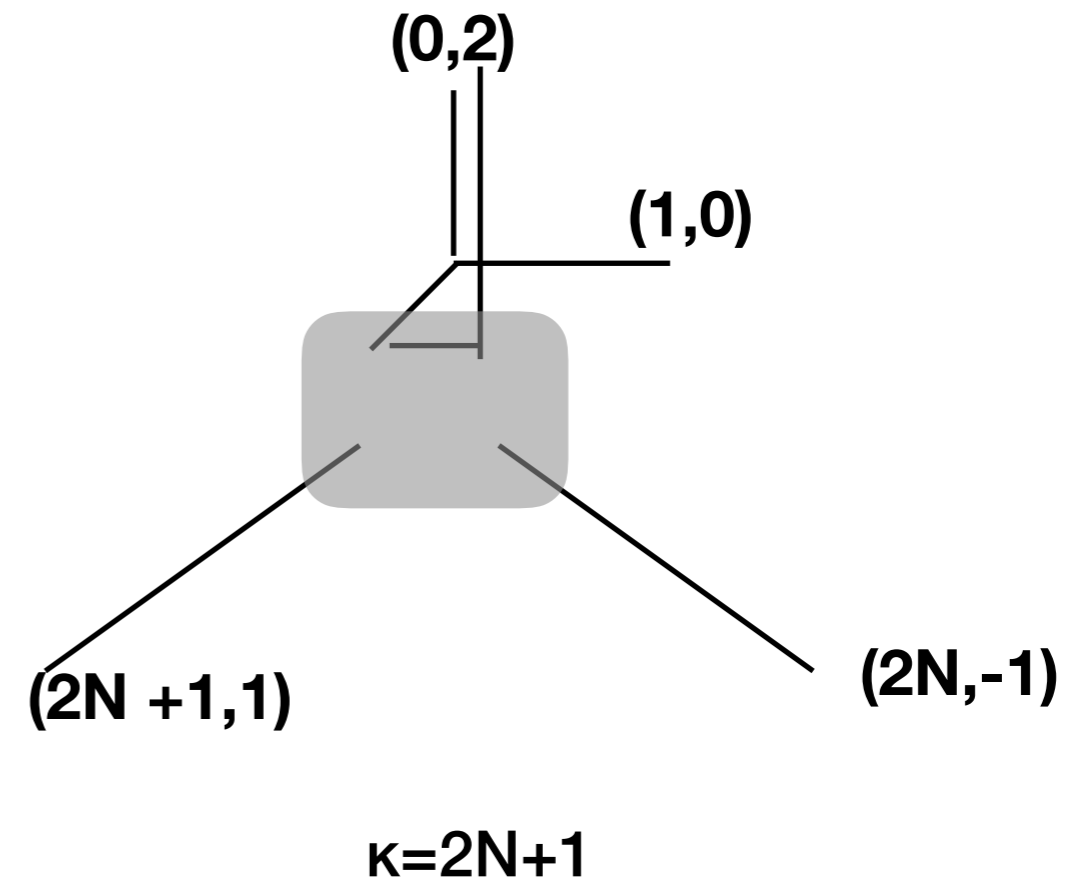
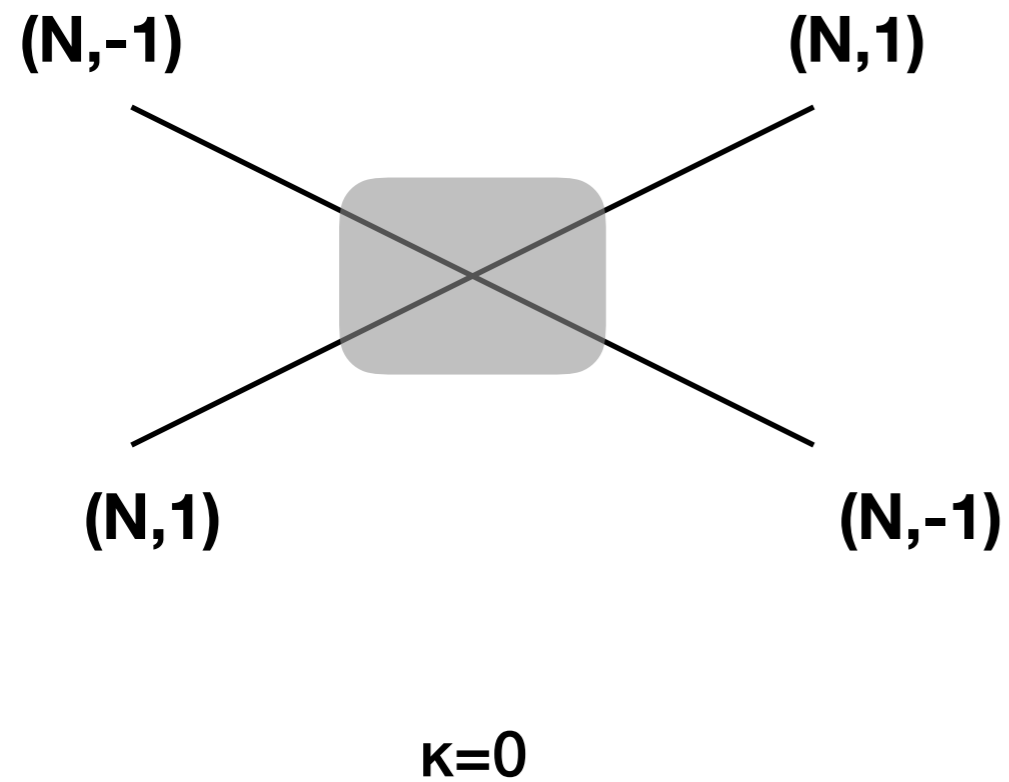


- One massless vector multiplet + two massive vector multiplets mutually nonlocal.
- Cubic prepotential $F(\phi) = F_{\text{tree}} + F_{\text{CS}} + F_{1\text{-loop}}$
 - 1-loop due to W -boson (not instanton)

- 5d pure SU(3) gauge theory with $\kappa=0,1,2,3$



- 5d pure $SU(2N)$ gauge theory with $\kappa=0, \dots, 2N+1$



- Introduce an orientifold 5-plane:
 - N D5-branes on an $O5^+$ or $\widetilde{O5}^+$ -plane: $USp(2N)$
 - N D5-branes on $O5^-$ -plane: $SO(2N)$
 - N D5-branes on $\widetilde{O5}^-$ -plane: $SO(2N+1)$
 - ON-plane + NS5+D5: $SO(2N)$ -type quiver

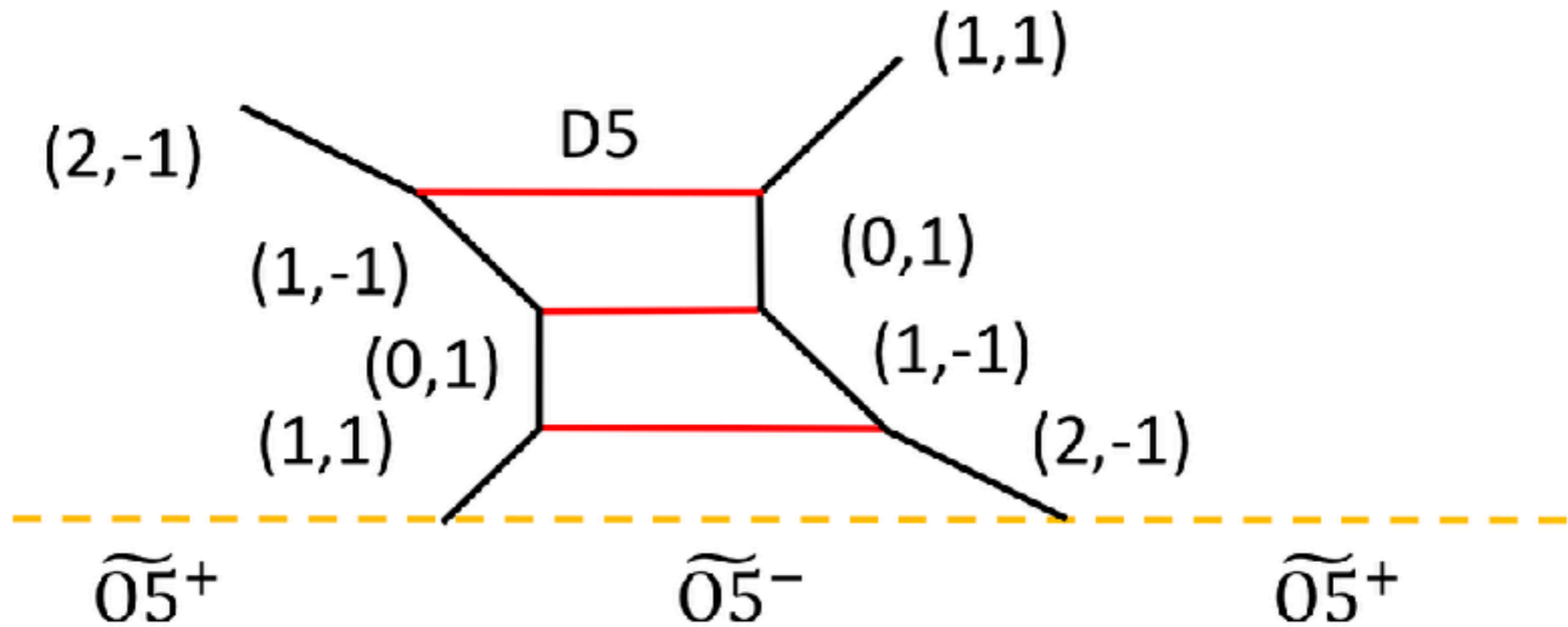
Brunner, Karch 97,

Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowicz 97

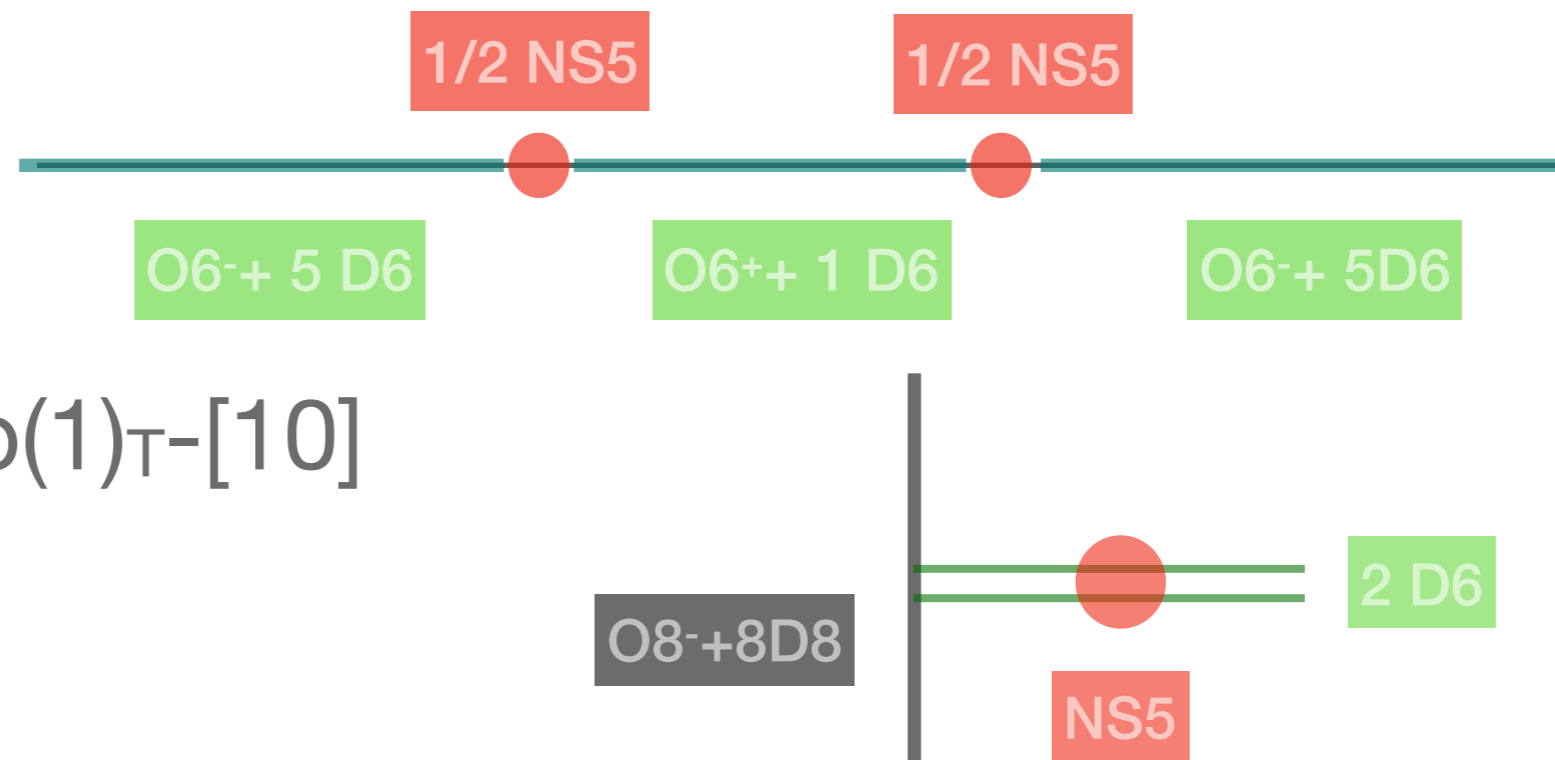
Hanany, Zaffaroni 99

Zafir 15

- the pure $SO(7)$ gauge theory:

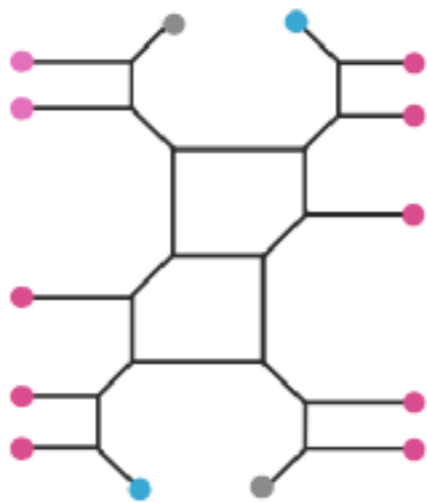


- Start from 6d SCFTs and do the dimensional reduction and flavor decoupling
- 6d SCFT: $Sp(N)_{\tau-[2N+8]}$ with $SO(4N+16)_{\text{flavor}}$
- D-type matter: $Sp(1)_{\tau-[10]}$: 5 D6 brane on $O6^-$ with a single NS5



- E8 wall: $Sp(1)_{\tau-[10]}$

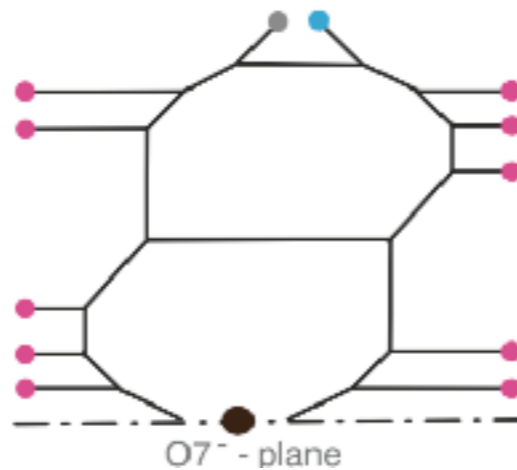
- Circle compactification, T-dual and S-dual
- 3 dual 5d theories: $SU(3)_0$ -[10], $Sp(2)$ -[10], [4]- $Sp(1)$ - $Sp(1)$ -[4]



$SU(3)_0$ -[10]

Hayashi,SSKim,KL,Taki,Yagi:1505

Yonekura:1505

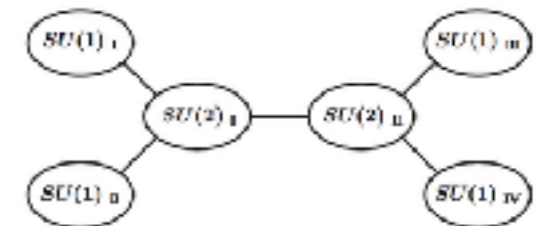


$Sp(2)$ -[10]

Gaiotto,H.C.Kim:1506

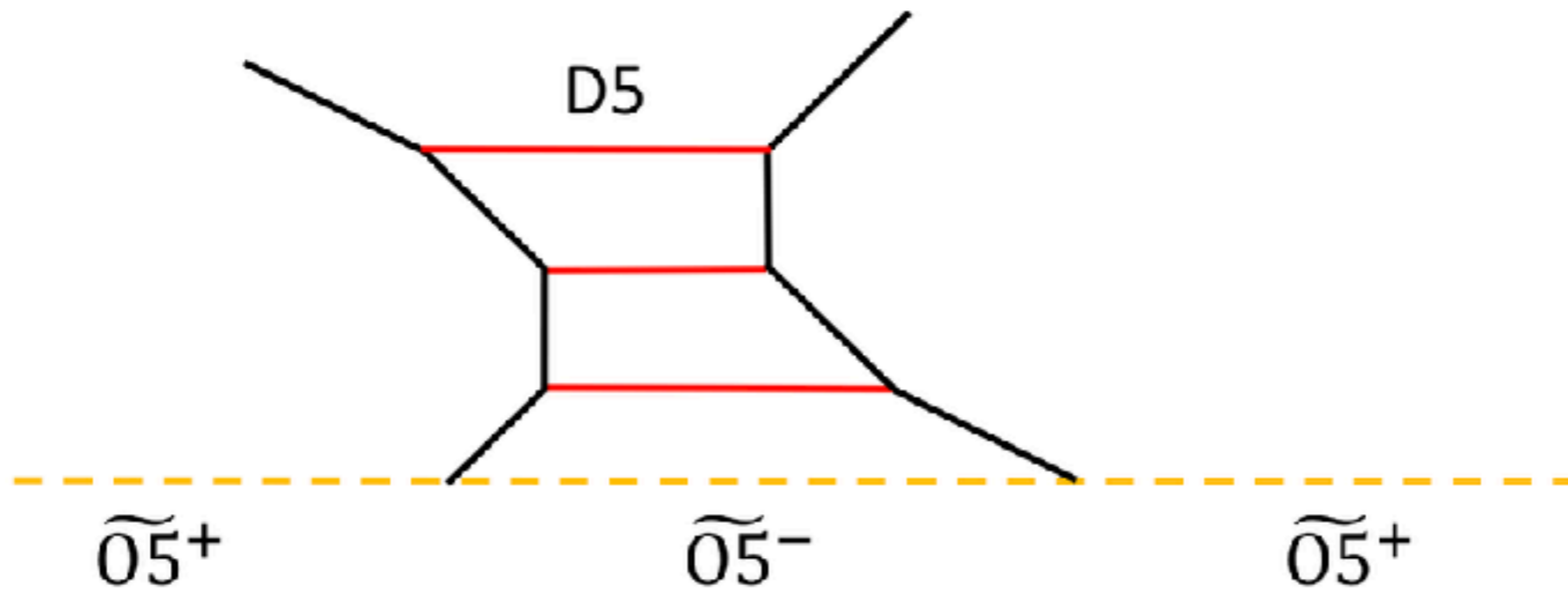


[4]- $Sp(1)$ - $Sp(1)$ -[4]



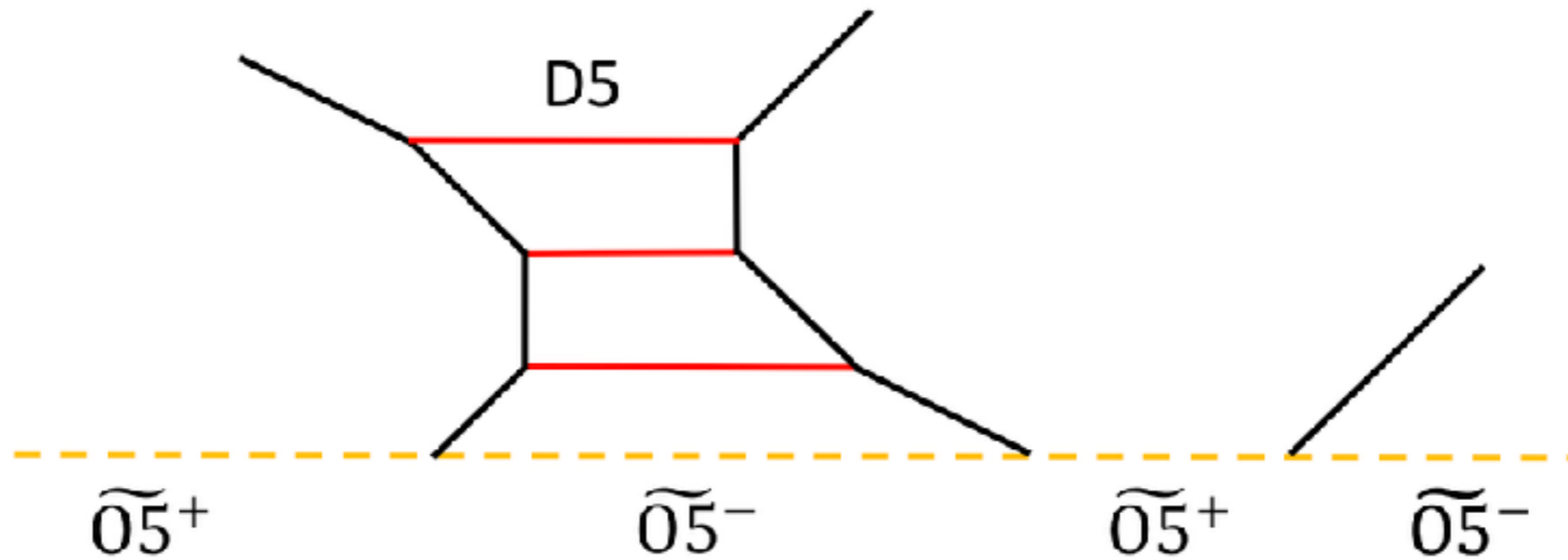
3. G_2 - $SU(3)_7$ Duality

- We start with the web for the 5d pure $SO(7)$ gauge theory with \widetilde{O}_5 plane.



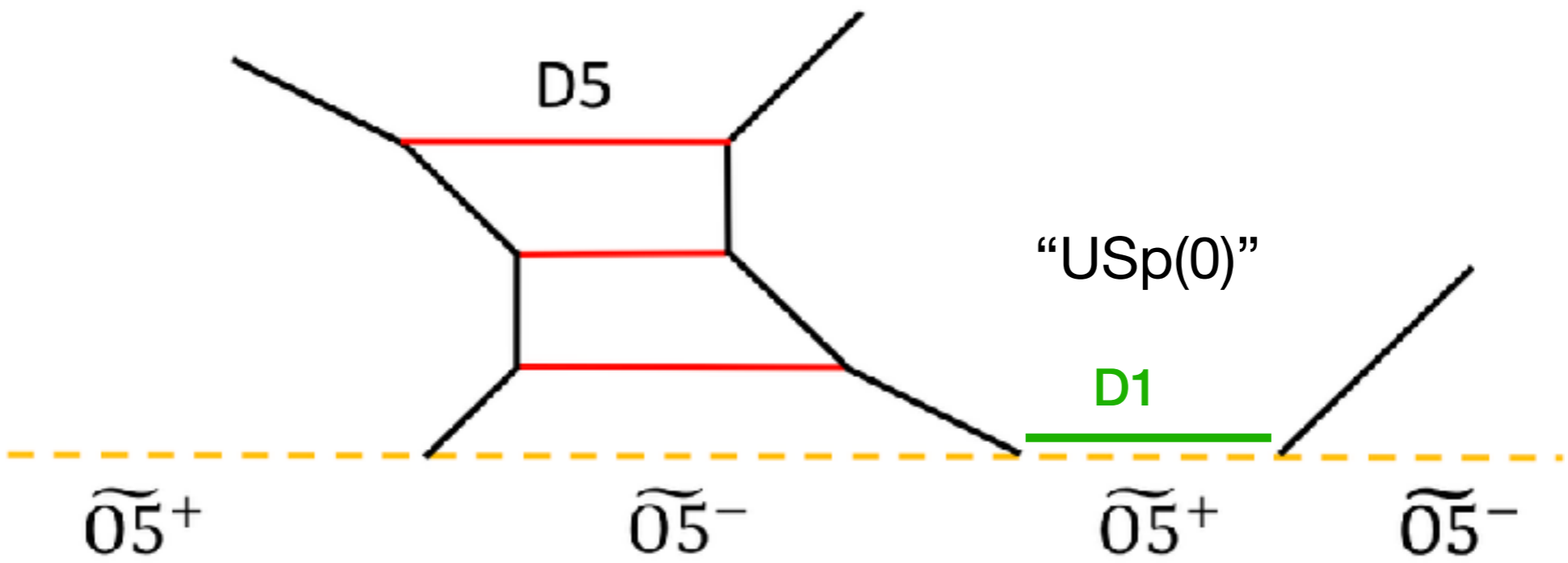
- with spinor matter for the $SO(7)$ gauge theory:

Zafir 15



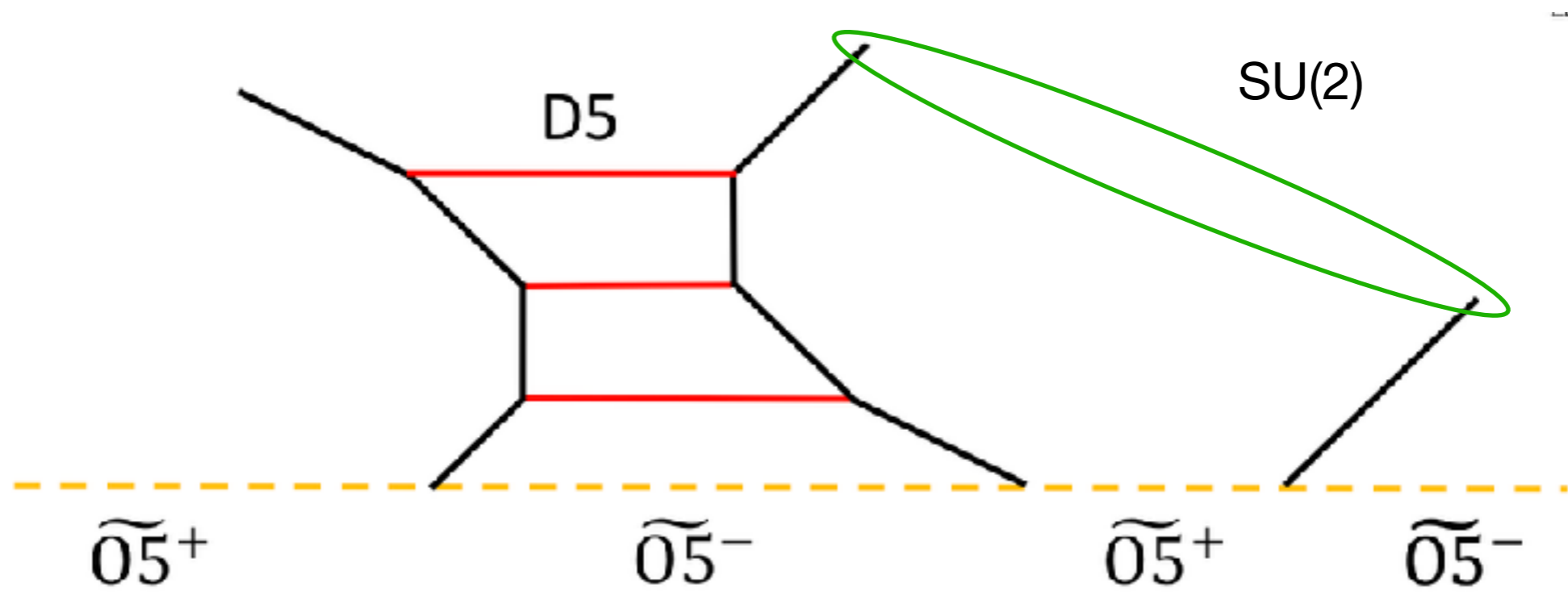
- with spinor matter for the $SO(7)$ gauge theory:

Zafir 15

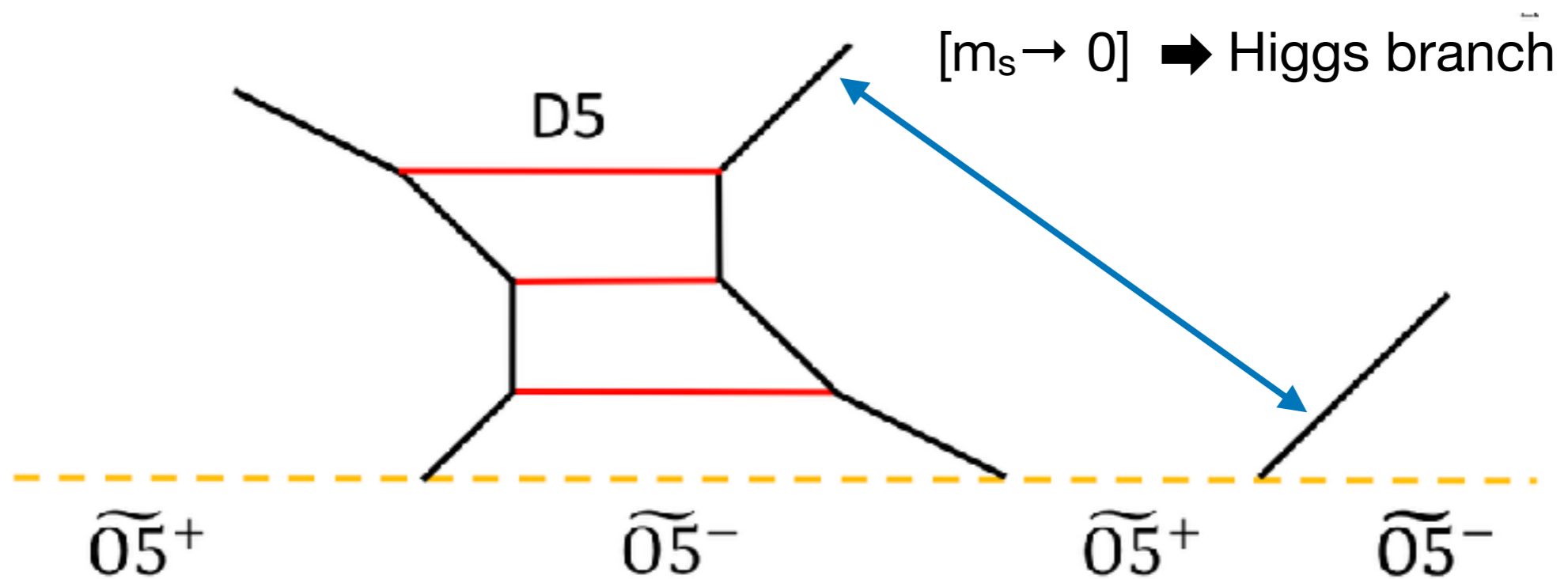


D1-brane = "USp(0) instantons" = the spinor matter

- with spinor matter for the $SO(7)$ gauge theory:

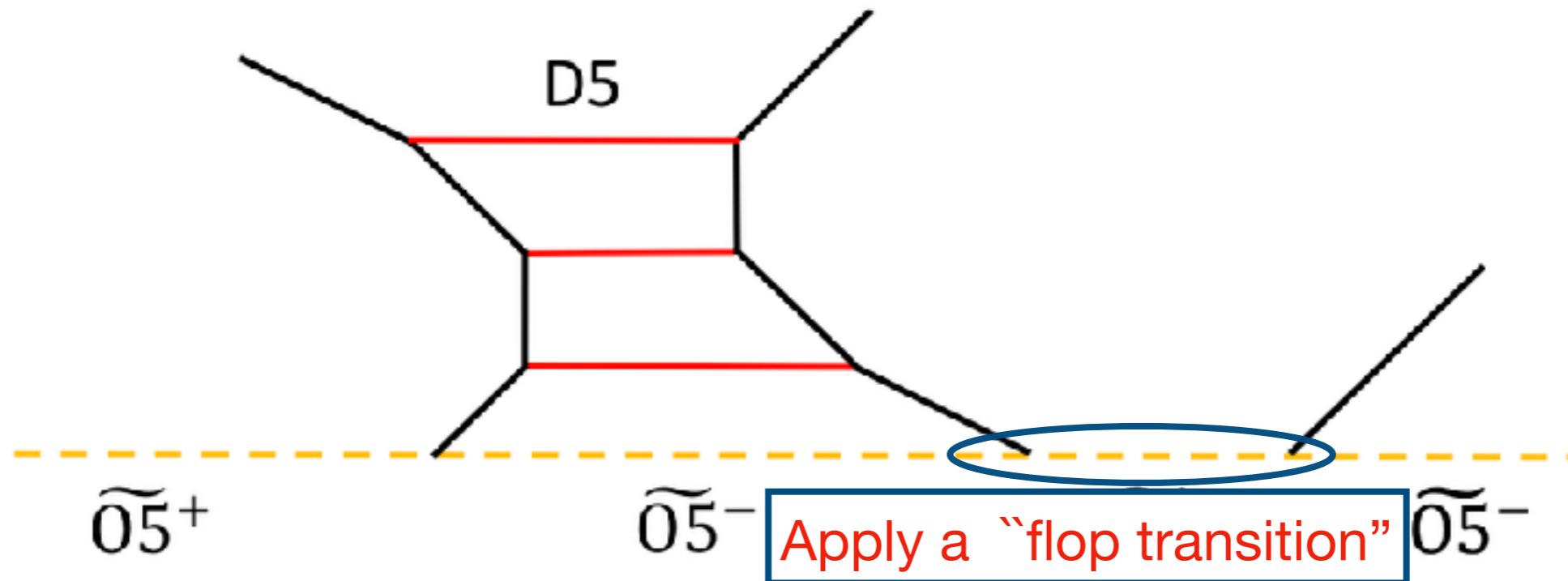


- with spinor matter for the $SO(7)$ gauge theory:

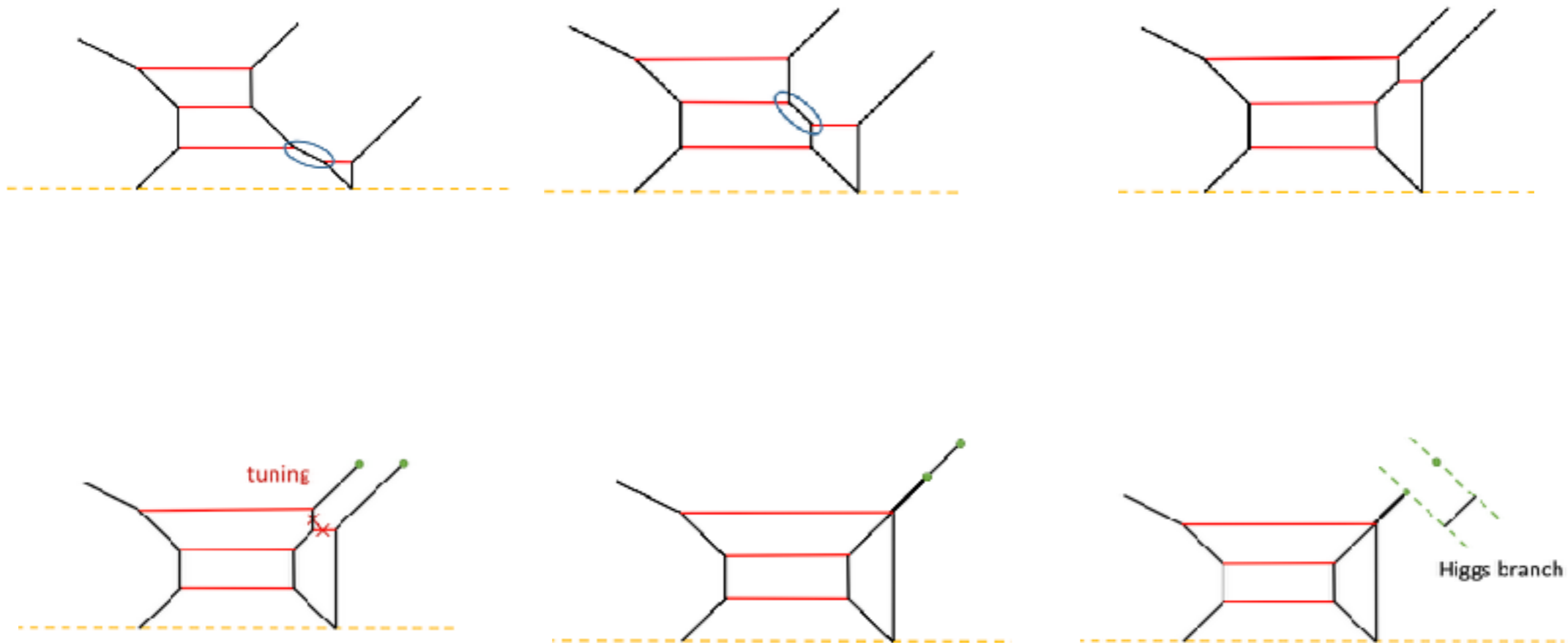


A Higgsing associated to the global $SU(2)$ yields G_2 .

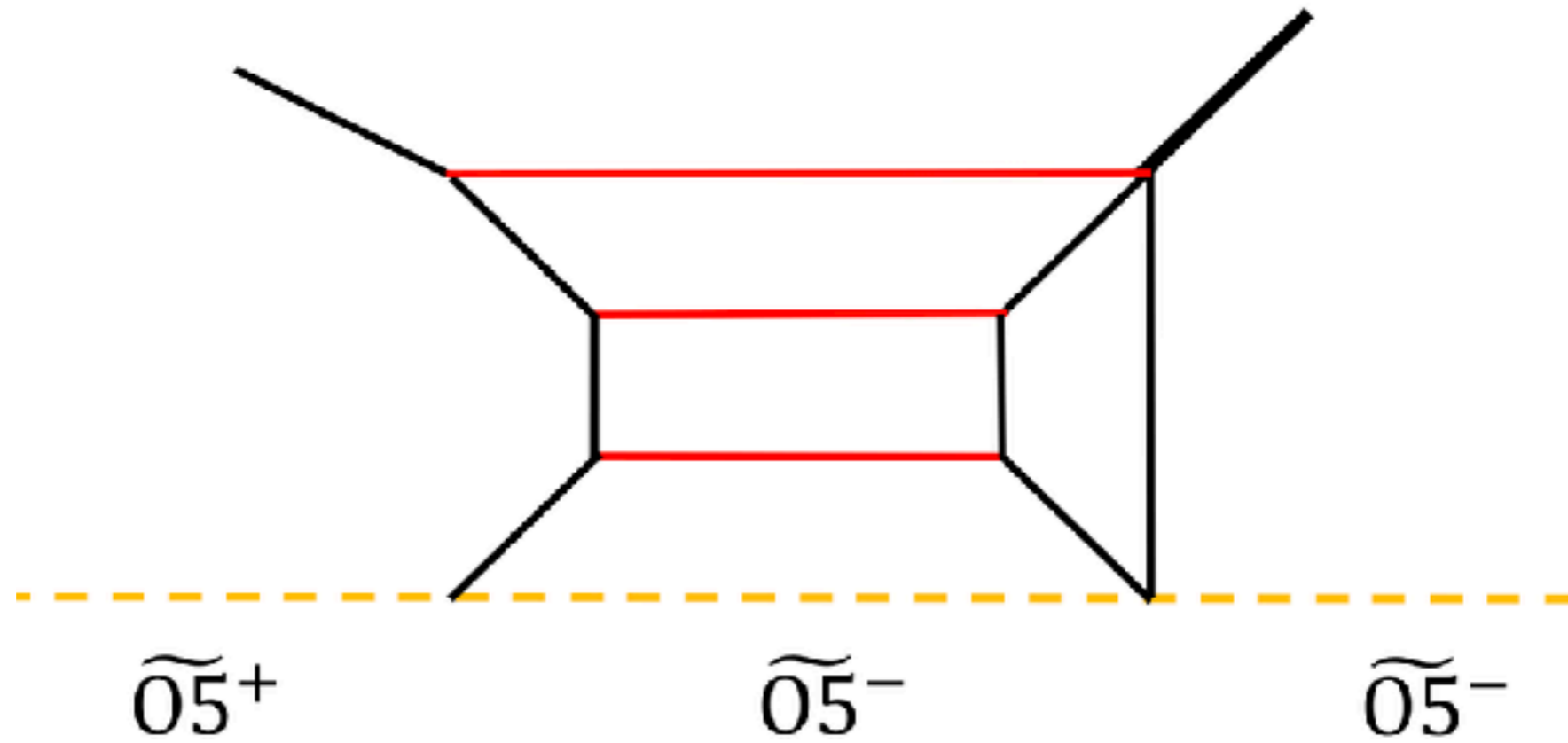
- Higgsing 5d SO(7) gauge theory with a spinor:
- Apply flop transitions:



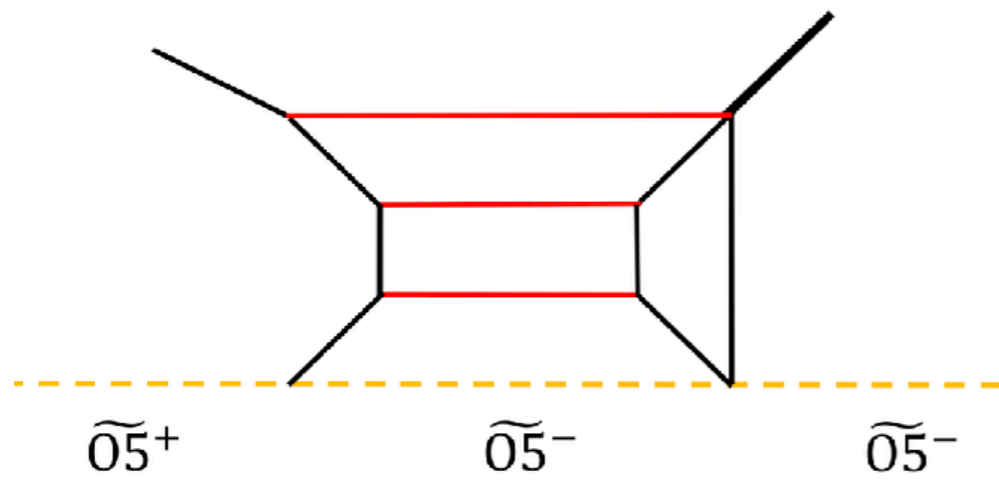
- Higgsing 5d $SO(7)$ gauge theory with a spinor:
 - flops, tuning mass parameters, Higgsing



- A 5-brane web configuration for the 5d pure G_2

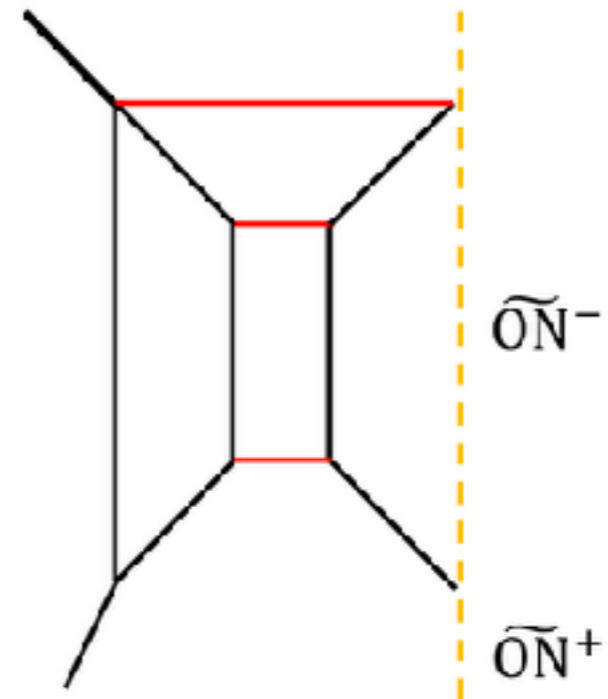


- pure $G_2 \leftrightarrow SU(3)_7$ duality



pure G_2

S-dual



$SU(3)_7$

- Confirm the duality between G_2 and $SU(3)_7$ by comparing the 1-loop corrected pre-potential.
- identical Coulomb parameters ϕ_i but mutually non-local W -bosons.
- monopole string tensions from the prepotential and the web diagram.
- monopole string tension is given by the area of the face of the diagram

Hanany, Witten 96

Aharony, Hanany 97

Aharony, Hanany, Kol 97

SU(3)₇ gauge theory

- SU(N)_κ + N_F Fundamental hyper

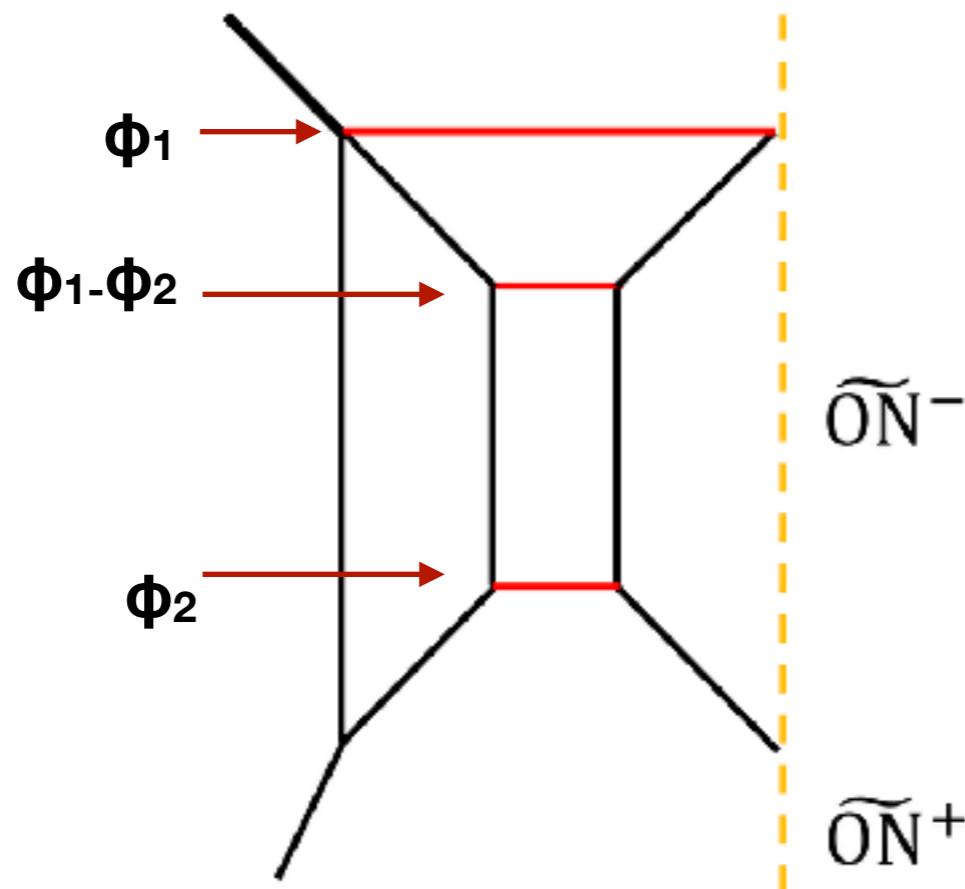
- Prepotential $a_1 \geq a_2 \geq \dots \geq a_N, \sum_{i=1}^N a_i = 0$

$$\mathcal{F}_{SU(N)} = \sum_{i=1}^N \left(\frac{m}{2} a_i^2 + \frac{\kappa}{6} a_i^3 \right) + \frac{1}{6} \sum_{i < j}^N |a_i - a_j|^3 - \frac{1}{12} \sum_F \sum_i^N |a_i - m_F|^3$$

$$\mathcal{F}_{SU(3)_7} = m_0(\phi_1^2 - \phi_1\phi_2 + \phi_2^2) + \frac{4}{3}\phi_1^3 + 3\phi_1^2\phi_2 - 4\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

- Monopole tension $a_1 = \phi_1, a_2 = \phi_2 - \phi_1, a_3 = \phi_3 - \phi_4, \dots, a_N = -\phi_{N-1}$

$$T_i = \partial \mathcal{F} / \partial \phi_i, \quad i = 1, 2, \dots, N-1$$



- A 5-brane web configuration for the 5d pure G_2

- $G_2 + N_F$ Fundamental hyper

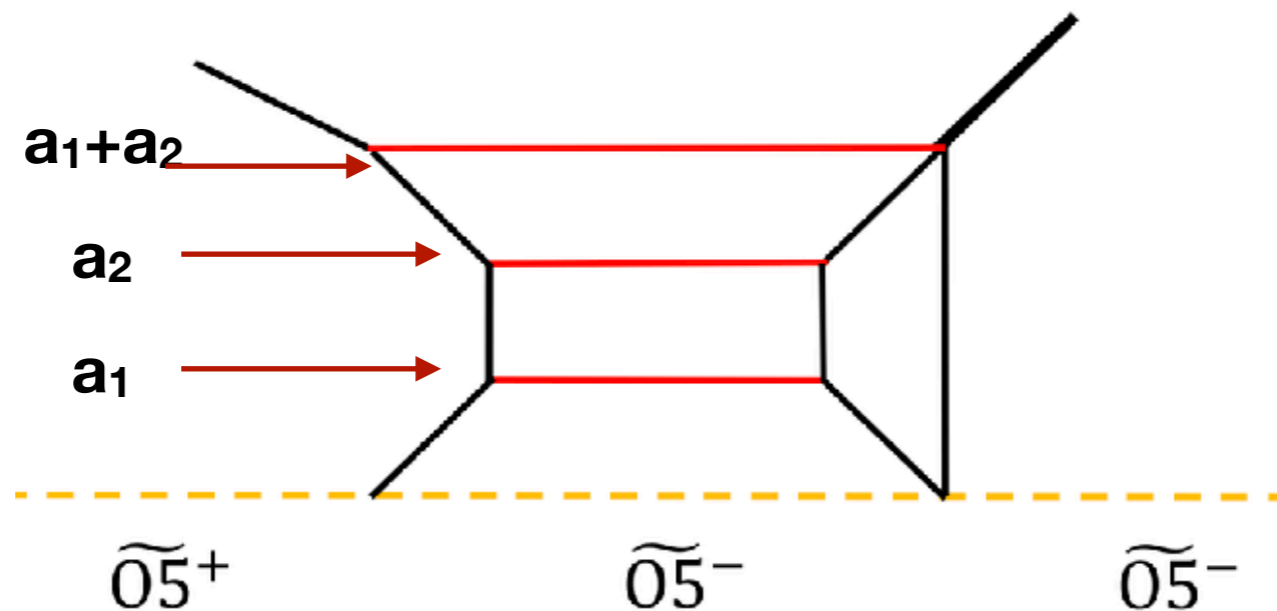
- Prepotential
$$\mathcal{F} = \frac{m}{2} h_{ij} \phi_i \phi_j + \frac{1}{12} \left(\sum_{\mathbf{R}} |\mathbf{R} \cdot \phi|^3 - \sum_f \sum_{\mathbf{w} \in \mathbf{W}_f} |\mathbf{w} \cdot \phi|^3 \right)$$

$$2\phi_1 - 3\phi_2 = a_2 - a_1, \quad -\phi_1 + 2\phi_2 = a_1$$

$$\mathcal{F}_{G_2} = m_0(\phi_1^2 - 3\phi_1\phi_2 + 3\phi_2^2) + \frac{4}{3}\phi_1^3 - 4\phi_1^2\phi_2 + 3\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

- Monopole tension

$$T_1 = \partial \mathcal{F} / \partial \phi_1, \quad T_2 = \partial \mathcal{F} / \partial \phi_2$$



- Duality between G_2 & $SU(3)_7$

Parameter Map

$$m_0^{SU(3)} = -\frac{m_0^{G_2}}{3},$$

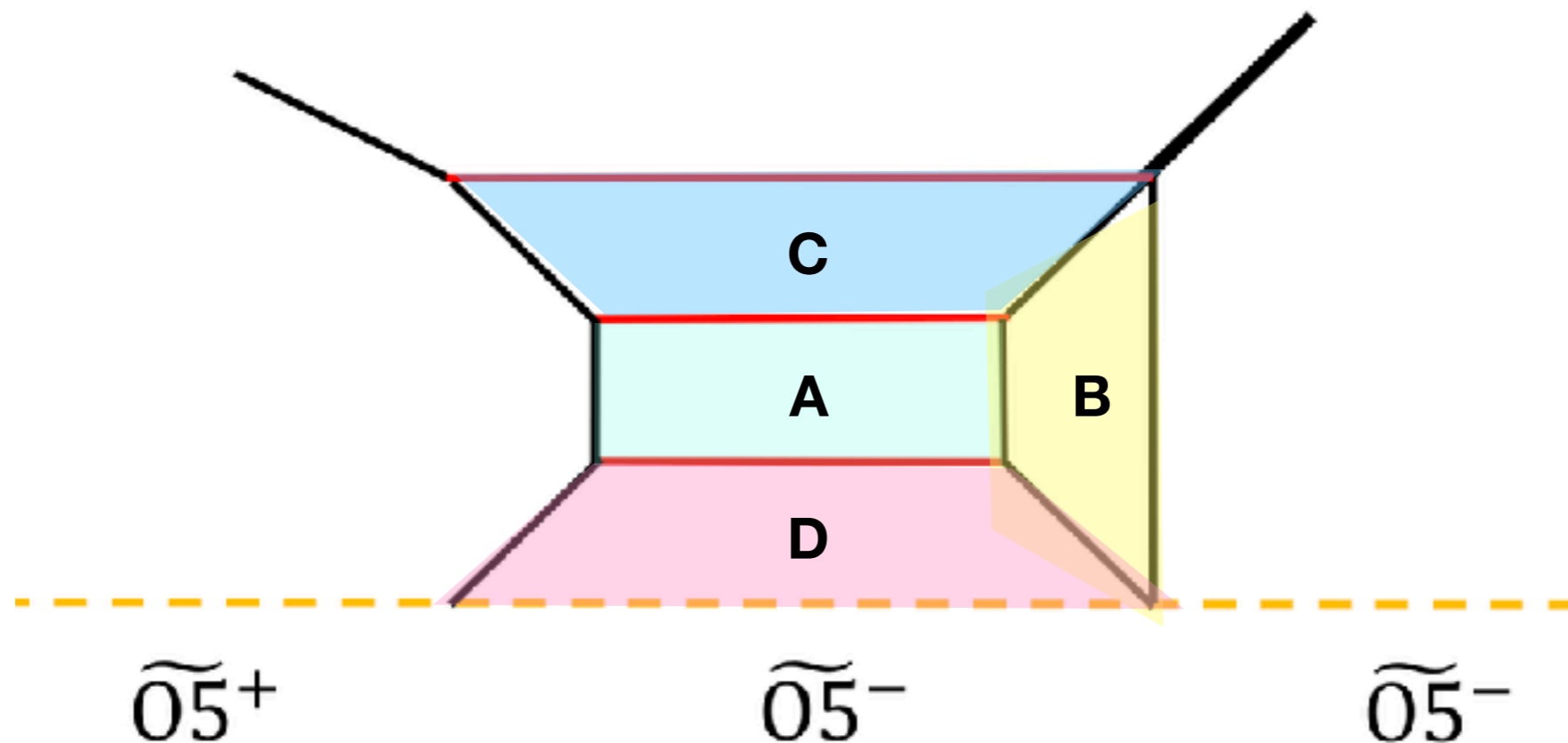
$$\phi_1^{SU(3)} = \phi_2^{G_2} + \frac{1}{3}m_0^{G_2},$$

$$\phi_2^{SU(3)} = \phi_1^{G_2} + \frac{2}{3}m_0^{G_2},$$

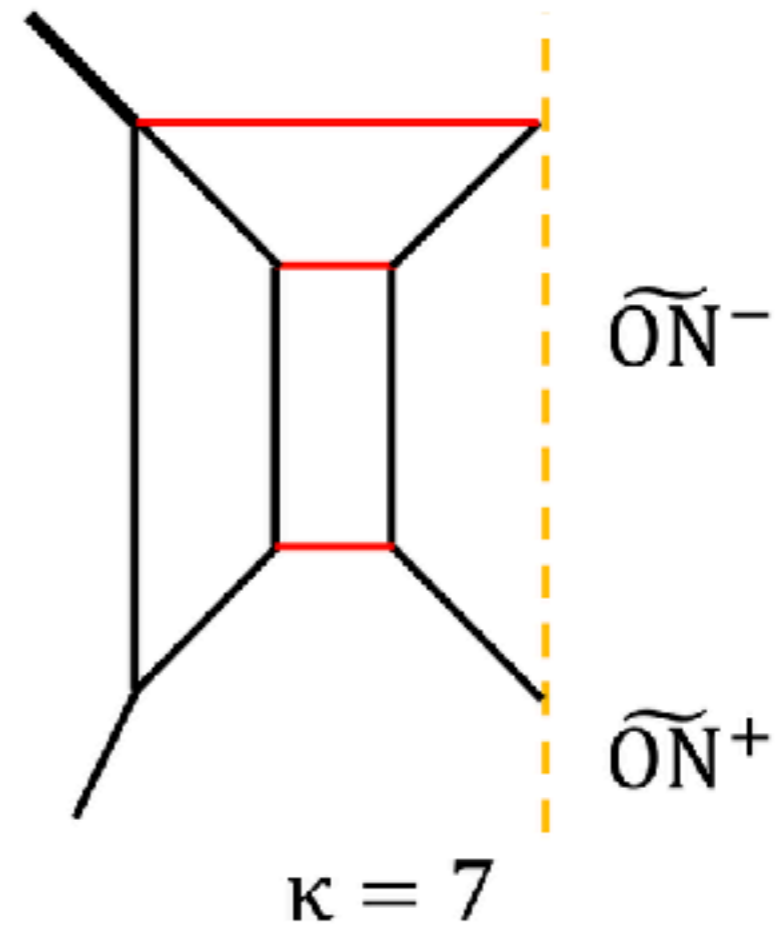
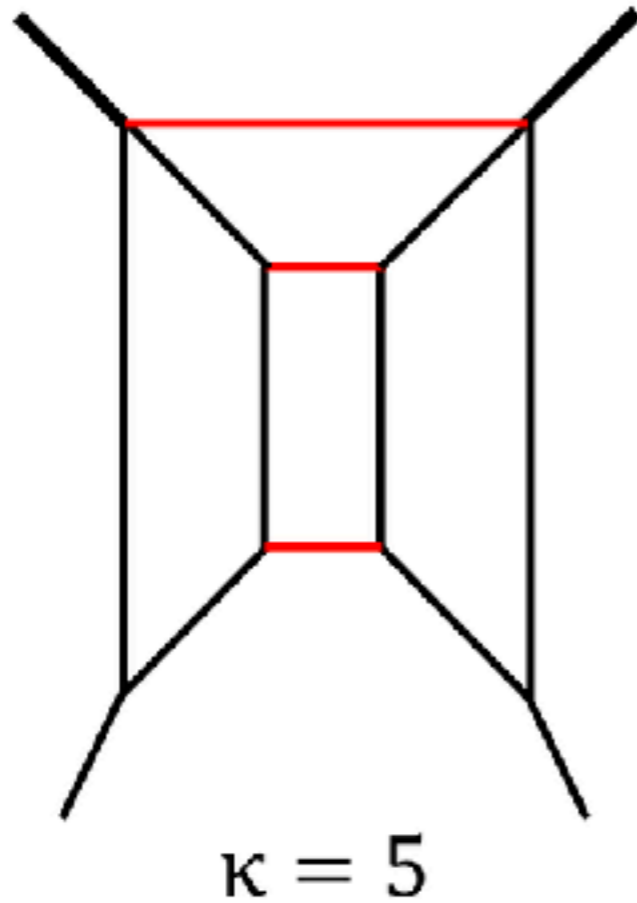
Monopole tension

$$T_1^{SU(3)_7} = T_2^{G_2} = \mathbf{B + C + 2D}$$

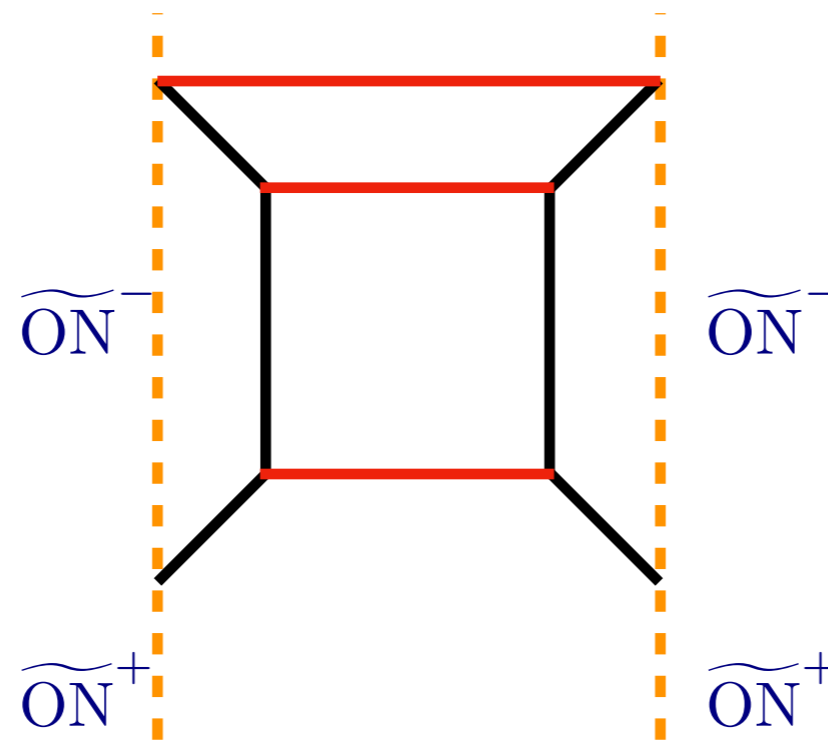
$$T_2^{SU(3)_7} = T_1^{G_2} = \mathbf{A}$$



- In fact, we can further increase the Chern-Simons levels.



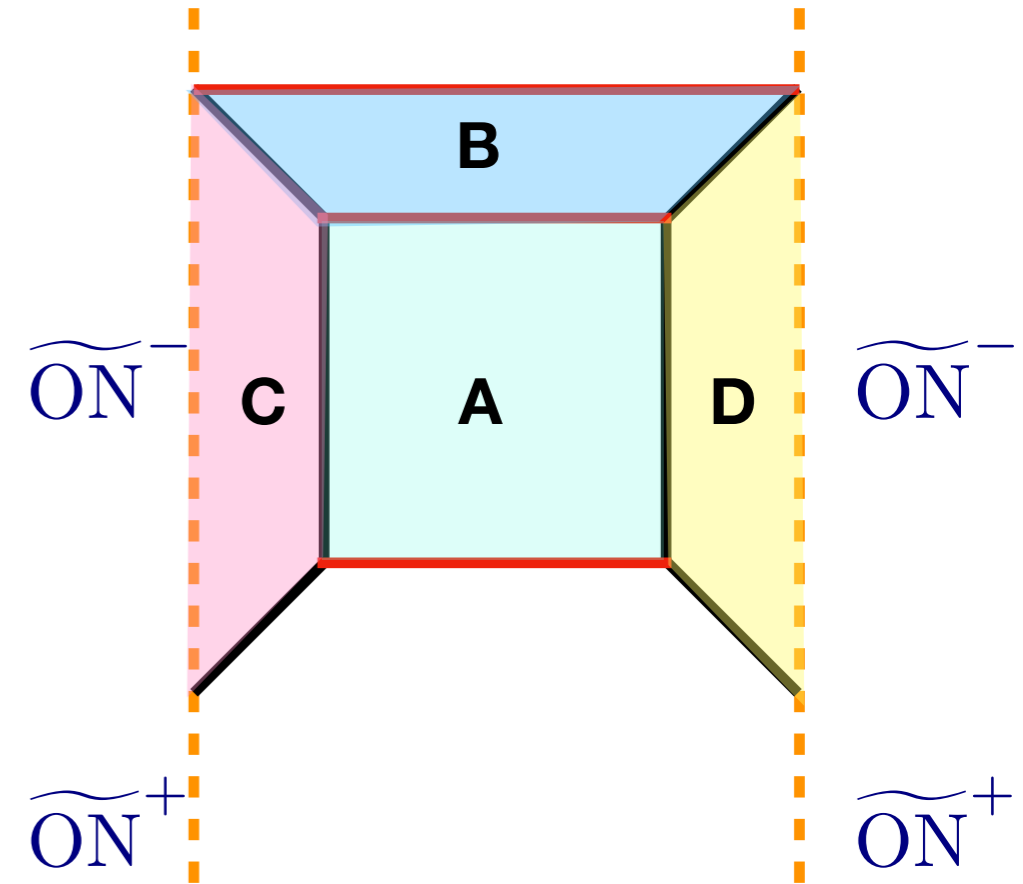
- A 5-brane web diagram for $SU(3)$ with Chern-Simons level $\kappa=9$



- The $SU(3)_9$ theory has been found and constructed from geometry.

Jefferson, Kim, Vafa, Zafrir 17
 Jefferson, Katz, Kim, Vafa 18

- Check: compare monopole tension from the prepotential and the 5-brane webs.
- Two independent D3 brane faces
- $A, B + 2C + 2D$

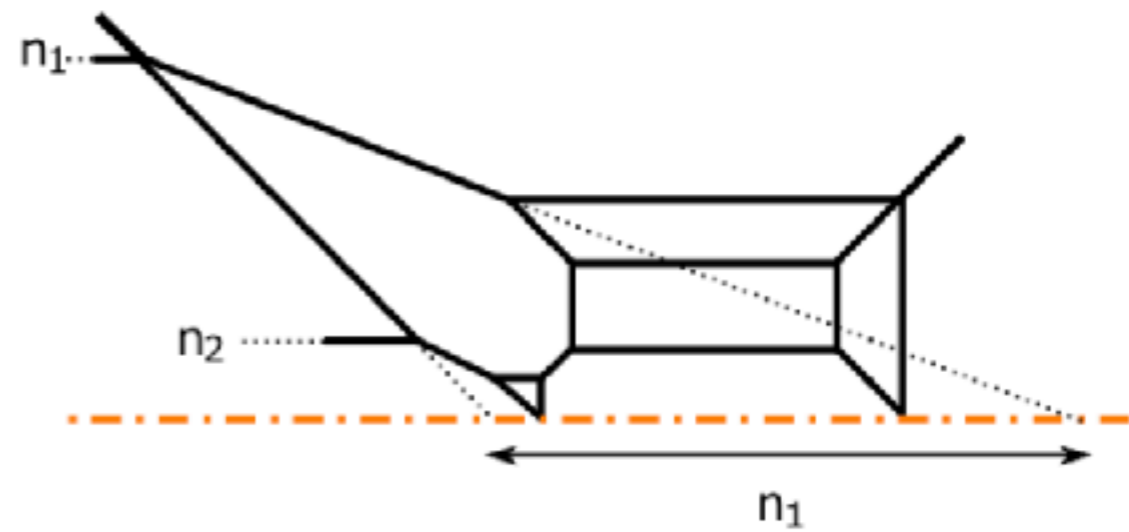
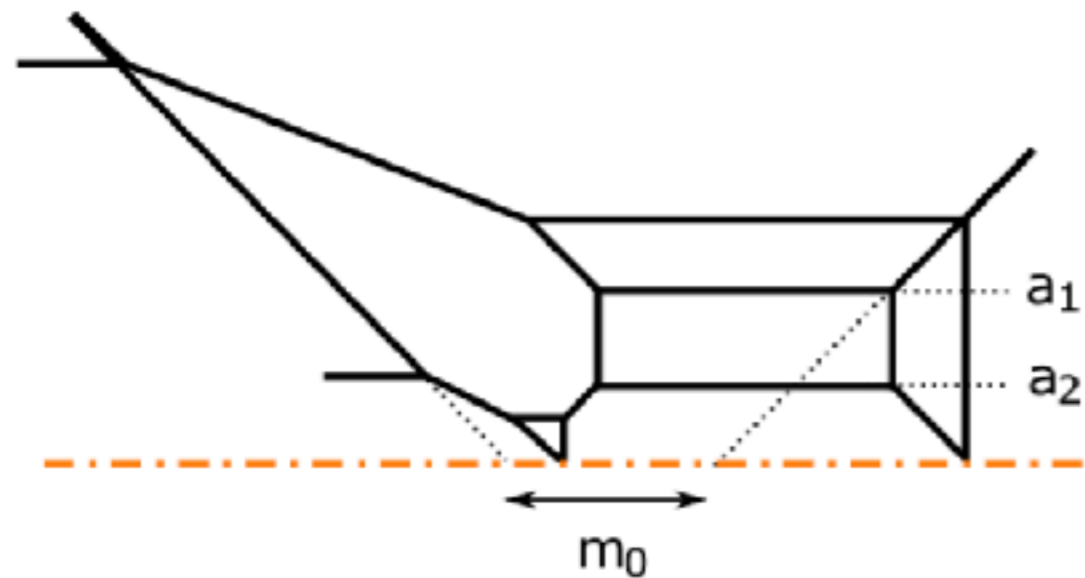


$$\begin{aligned}
 \mathcal{F}_{SU(3)_9} &= \frac{m_0}{2}(a_1^2 + a_2^2 + a_3^2) + \frac{1}{6}((a_1 - a_2)^3 + (a_1 - a_3)^3 + (a_2 - a_3)^3) + \frac{9}{6}(a_1^3 + a_2^3 + a_3^3) \\
 &= m_0(\phi_1^2 - \phi_1\phi_2 + \phi_2^2) + \frac{4}{3}\phi_1^3 + 4\phi_1^2\phi_2 - 5\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3, \quad (5.5)
 \end{aligned}$$

4. G_2 -SU(3)-Sp(2) sequence

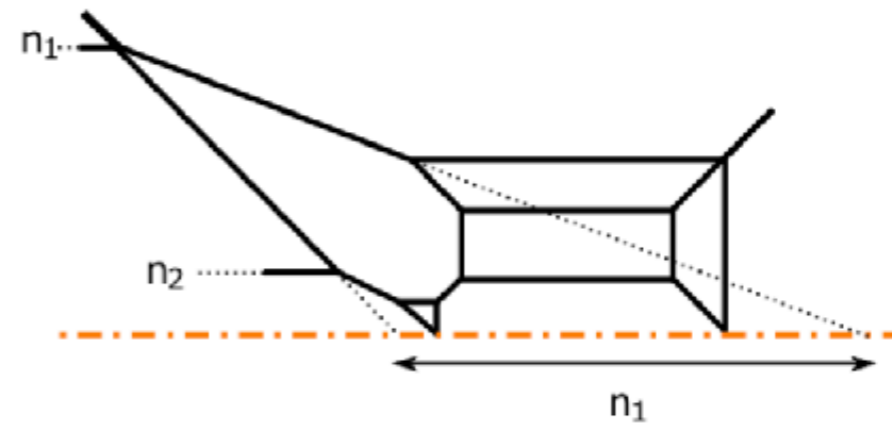
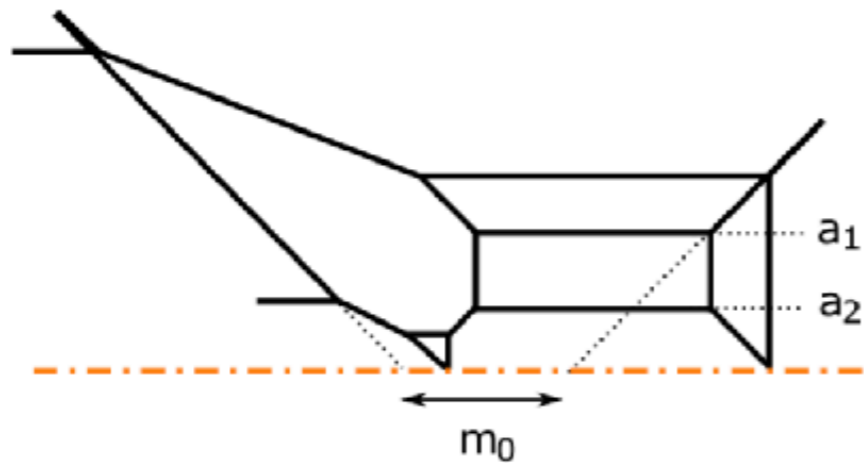
- $G_2 + 2F$

- $SO(7) + 3S \rightarrow G_2 + 2F + 2 \text{ singlets (after Higgsing with a spinor)}$



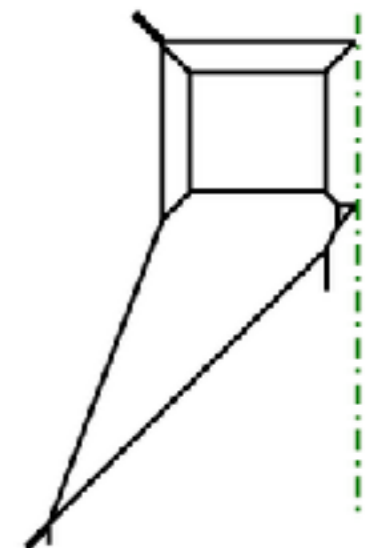
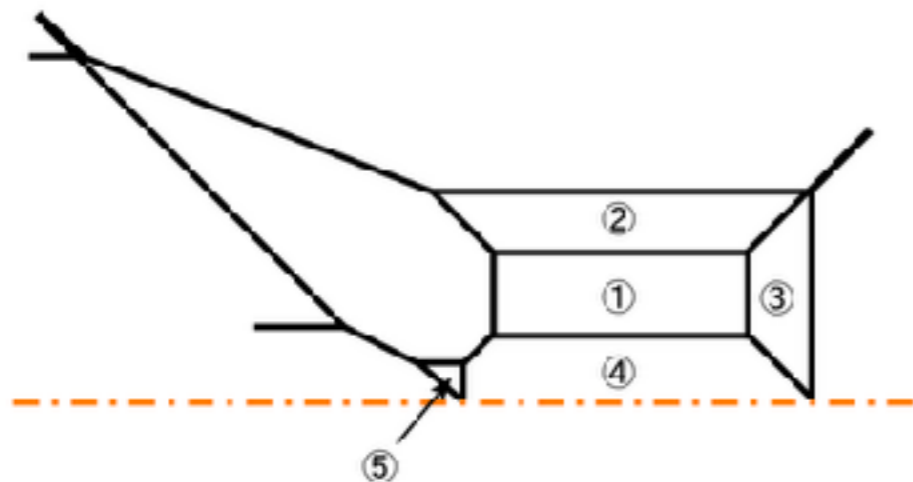
- $G_2 + 2F$

- $SO(7) + 3S \rightarrow G_2 + 2F + 2 \text{ singlets (after Higgsing with a spinor)}$



- Monopole $\textcircled{1}$, $\textcircled{1} + \textcircled{2} + \textcircled{3} + 2\textcircled{4} + \textcircled{5}$

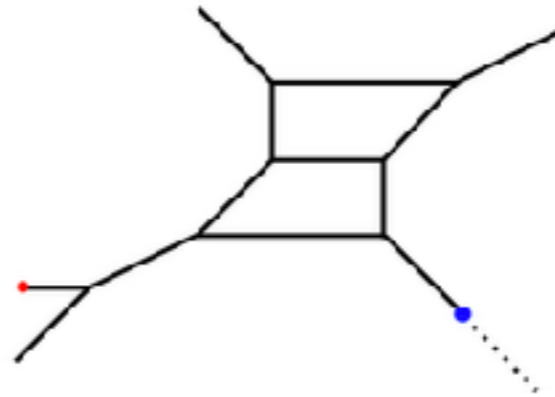
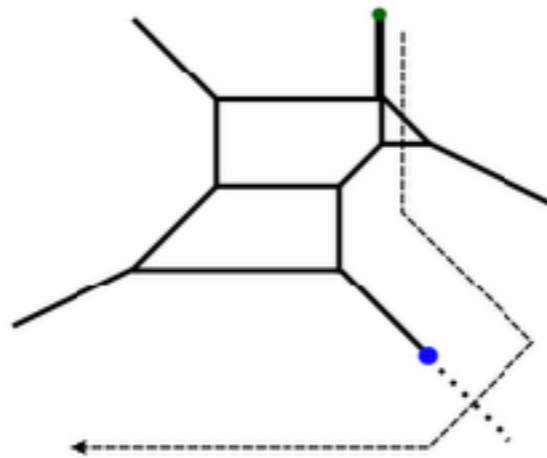
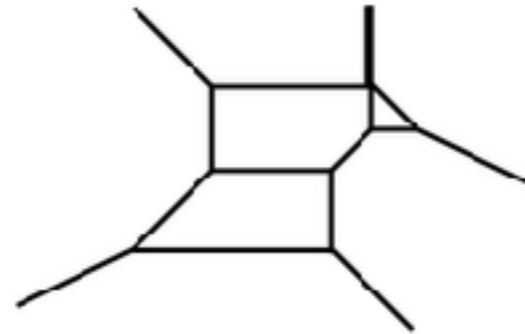
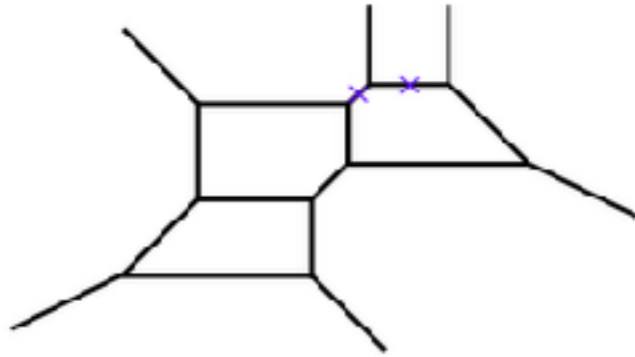
- S-dual



ON

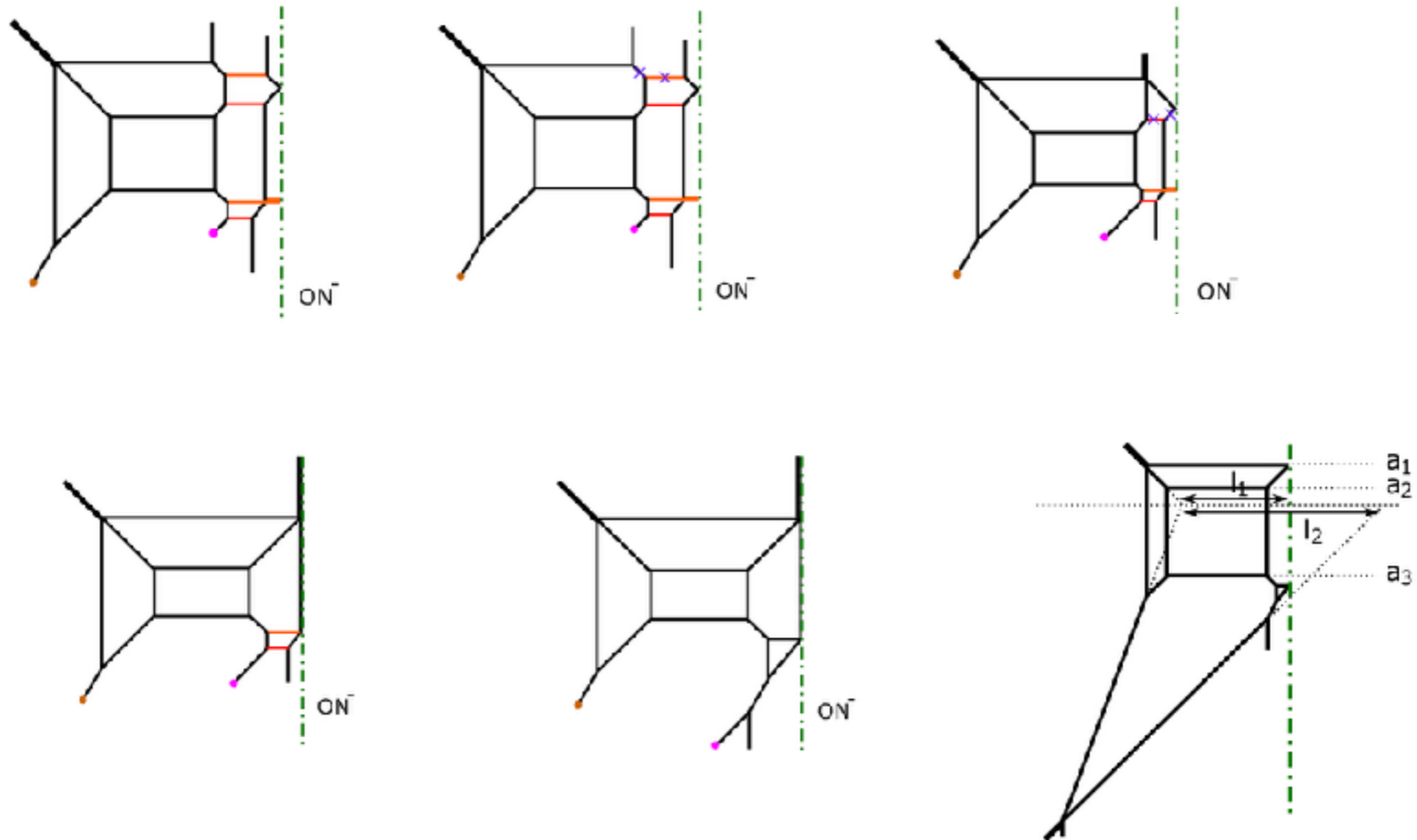
- $SU(3)_{6+} 2F$

- $SU(3)_{-1} \times SU(2)$ Higgsed to $SU(3)_{1/2+} 1AS = SU(3)_{1/2} + 1F$



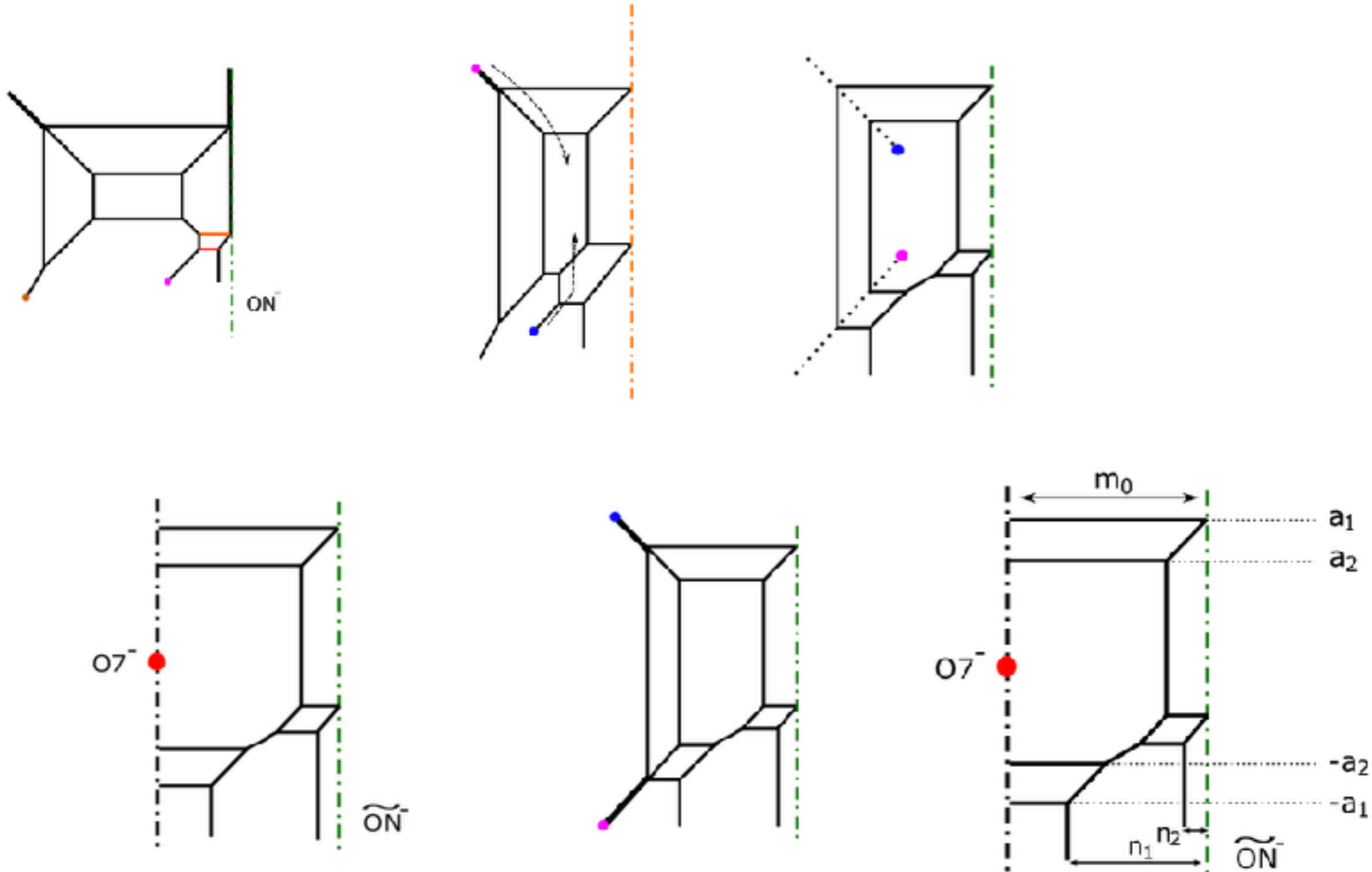
- $SU(3)_6 + 2F$

- $SU(2) \times SU(3)_3 \times SU(2)$ Higgsed to $SU(3)_6 + 2AS$

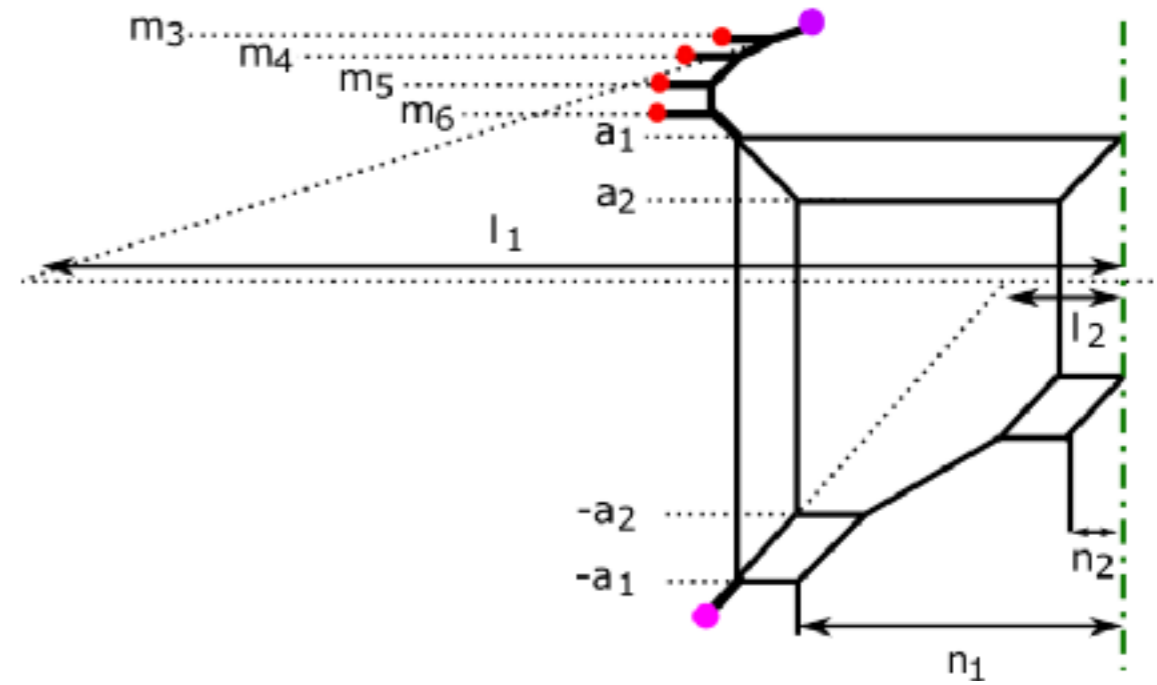
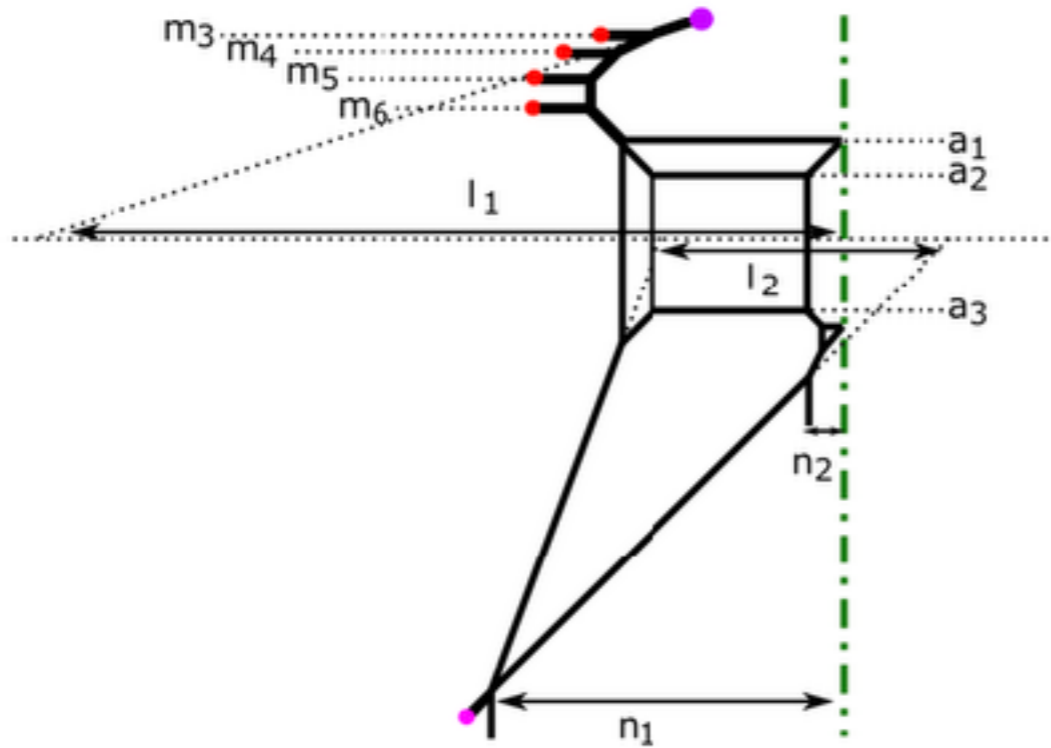
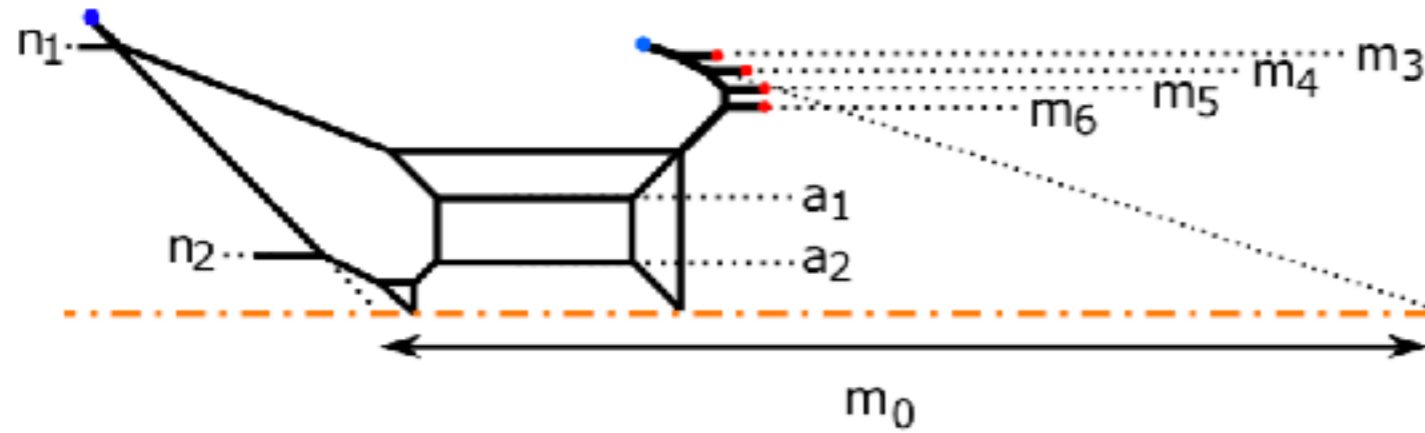


- $Sp(2)_{\pi} + 2AS$

- $SU(2) \times SU(3)_3 \times SU(2)$ Higgsed to $SU(3)_6 + 2AS$



- Marginal Theories: $G_2+6F = SU(3)_4+6F = Sp(2)+2AS+4F$



- Marginal Theories: $G_2+6F = SU(3)_4+6F=Sp(2)+2AS+4F$

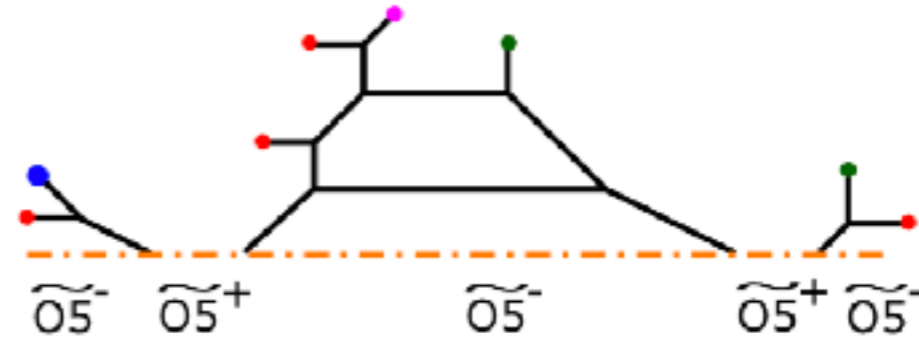
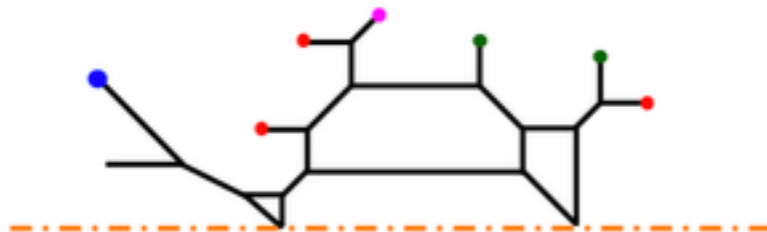
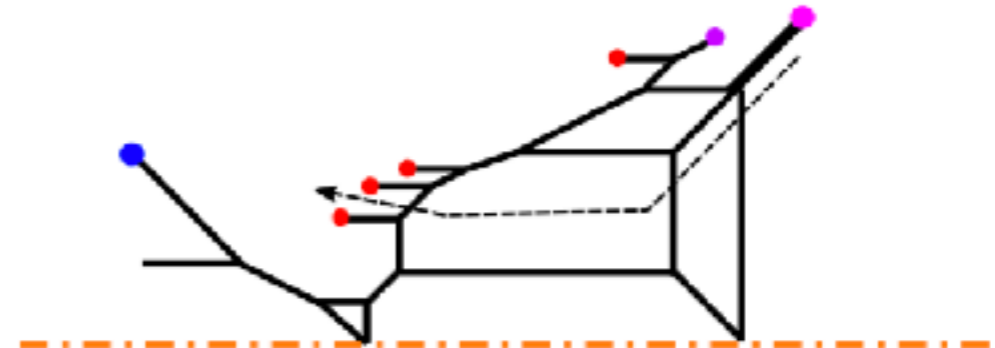
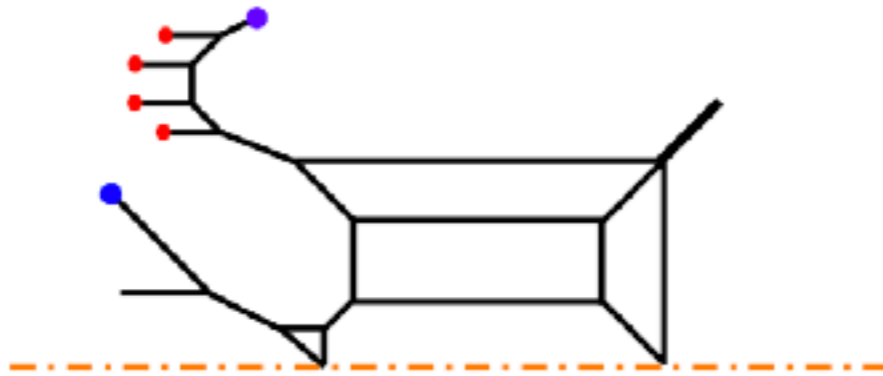
$$\begin{aligned}
 m_0^{Sp(2)} &= \frac{m_0^{G_2}}{2}, \\
 m_{\mathbf{AS},1}^{Sp(2)} &= m_{\mathbf{F},1}^{G_2}, \\
 m_{\mathbf{AS},2}^{Sp(2)} &= m_{\mathbf{F},2}^{G_2}, \\
 m_{\mathbf{F},i}^{Sp(2)} &= m_{\mathbf{F},i}^{G_2} - \lambda_2, \quad (i = 3, \dots, 6), \\
 \phi_1^{Sp(2)} &= \phi_2^{G_2} - \lambda_2, \\
 \phi_2^{Sp(2)} &= \phi_1^{G_2} - 2\lambda_2,
 \end{aligned}$$

$$\lambda_2 = -\frac{1}{2}m_0^{G_2} + \frac{1}{2}\sum_{i=3}^6 m_{\mathbf{F},i}^{G_2}.$$

$$\begin{aligned}
 m_0^{Sp(2)} &= m_0^{SU(3)} + \frac{1}{2}\sum_{i=1}^2 m_{\mathbf{AS},i}^{SU(3)} - \lambda_3, \\
 m_{\mathbf{AS},i}^{Sp(2)} &= m_{\mathbf{AS},i}^{SU(3)} - 2\lambda_3, \quad (i = 1, 2) \\
 m_{\mathbf{F},j}^{Sp(2)} &= m_{\mathbf{F},j}^{SU(3)} - \lambda_3, \quad (j = 3, \dots, 6), \\
 \phi_1^{Sp(2)} &= \phi_1^{SU(3)} - \lambda_3, \\
 \phi_2^{Sp(2)} &= \phi_2^{SU(3)} - 2\lambda_3,
 \end{aligned}$$

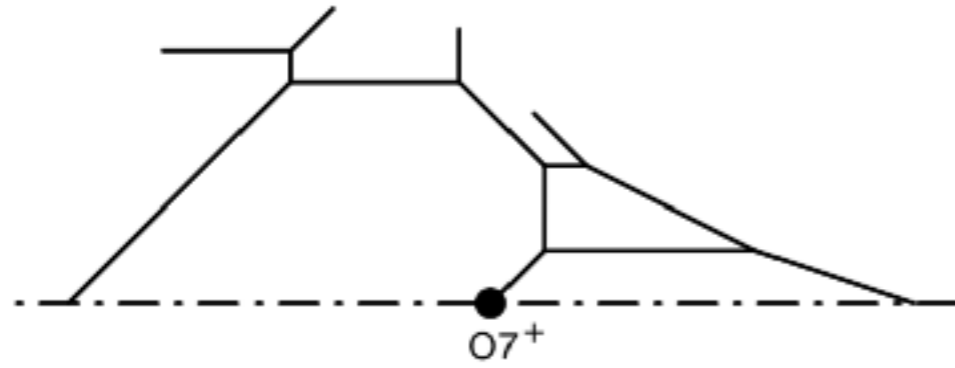
$$\lambda_3 = -\frac{1}{2}m_0^{SU(3)} + \frac{1}{4}\sum_{i=1}^2 m_{\mathbf{AS},i}^{SU(3)} + \frac{1}{4}\sum_{j=3}^6 m_{\mathbf{F},j}^{SU(3)}.$$

- Marginal Theories: $G_2+6F = SU(3)_4+6F =$
 $Sp(2)+2AS+4F = SO(5)+ 2V+ 4S$

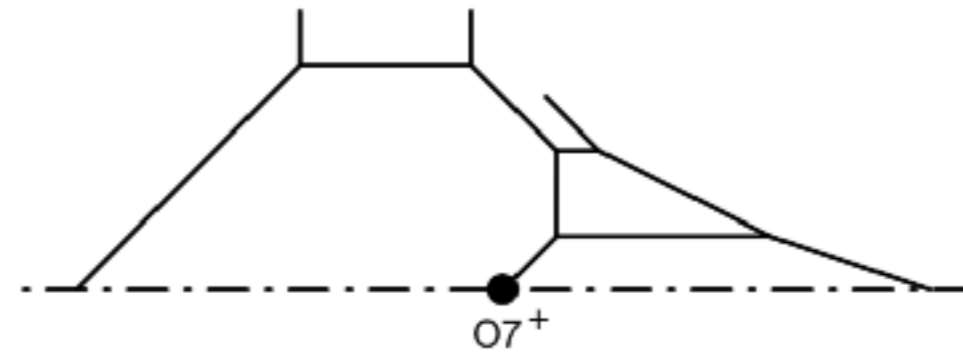


5. $SU(3)_0+1F+1S$

- Marginal Theory: $SU(3)_0 + 1F + 1Sym$ $SU(N)_0 + 1AS + 1Sym$



(a) $SU(3)_0 + 1F + 1Sym$



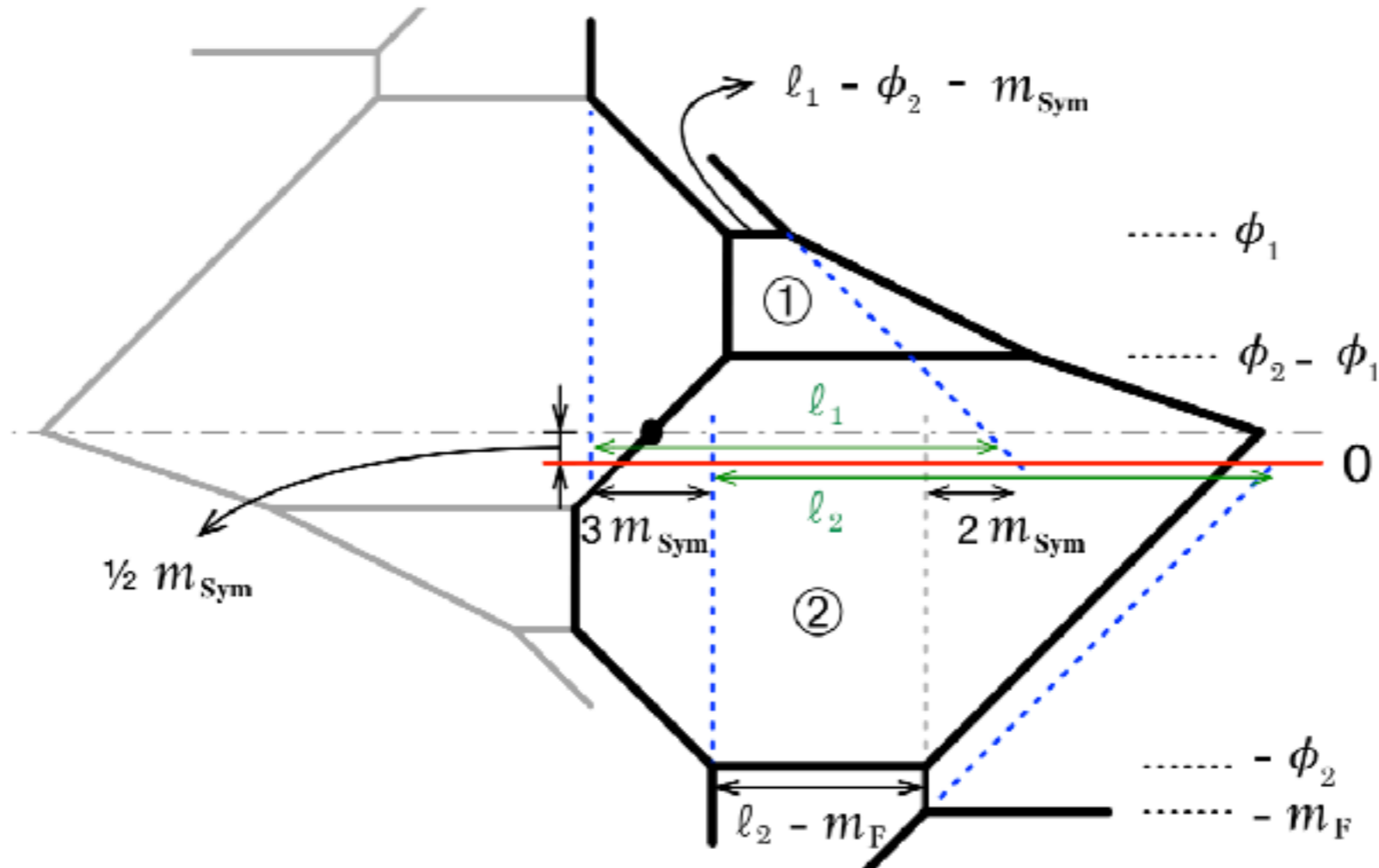
(b) $SU(3)_{-\frac{1}{2}} + 1Sym$



$SU(3)_{-\frac{7}{2}} + 1F$

• Marginal Theory: $SU(3)_0+1F+1Sym$

$SU(N)_0+1AS+1Sym$

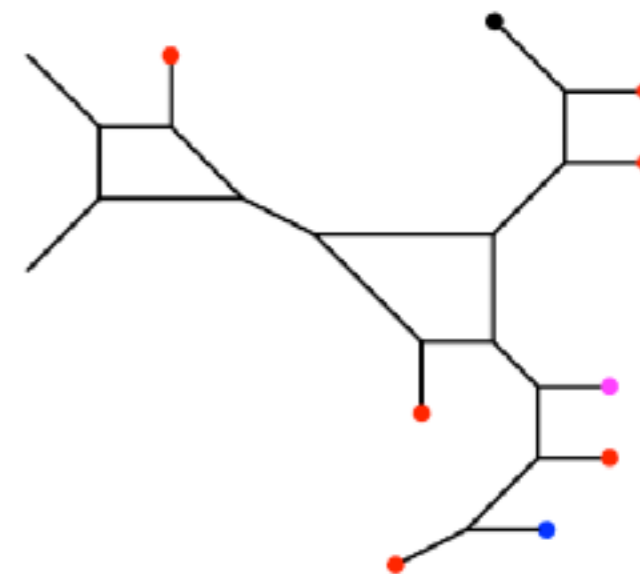
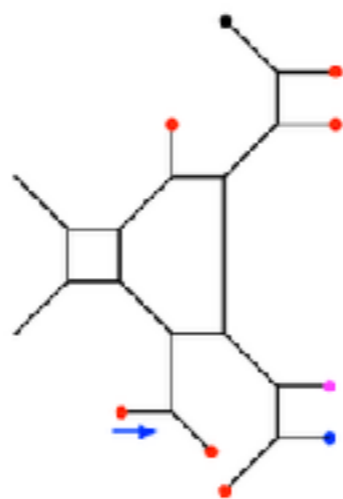
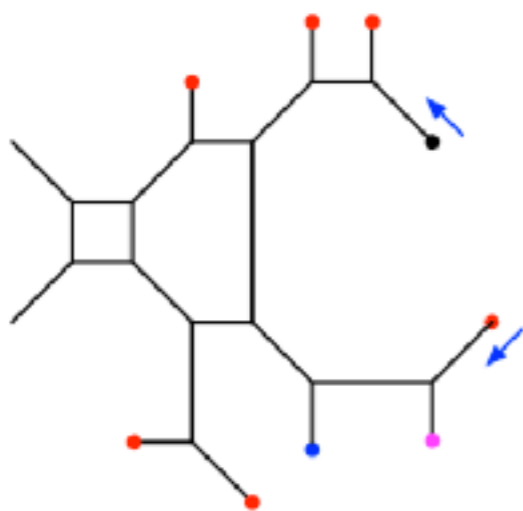
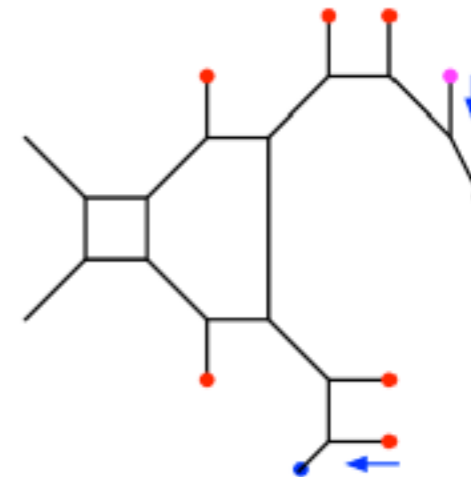
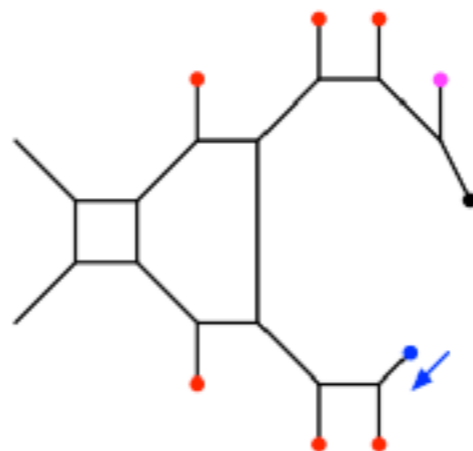
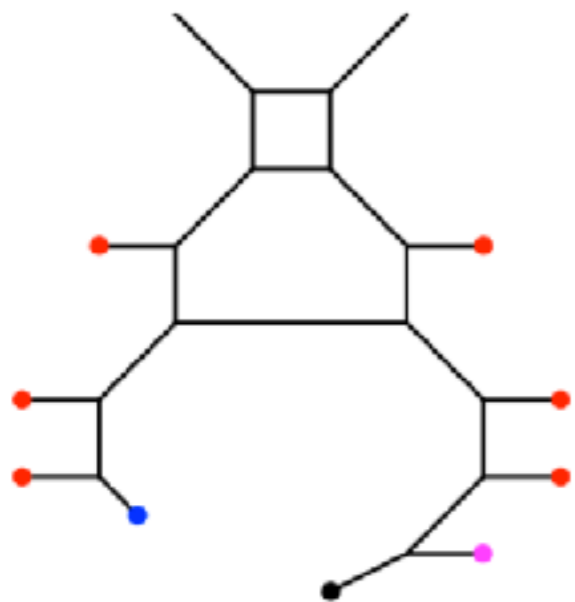


$$m_F \geq \phi_2 \geq \phi_1 \geq \frac{1}{2} \phi_2 \geq m_{Sym} \geq 0.$$

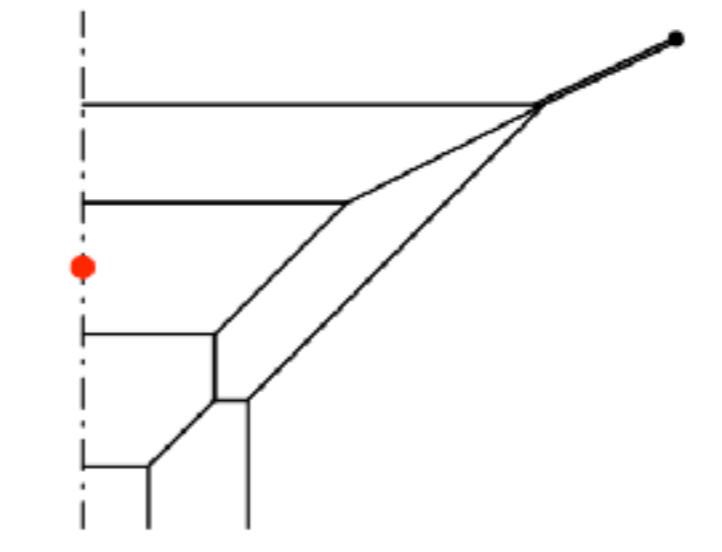
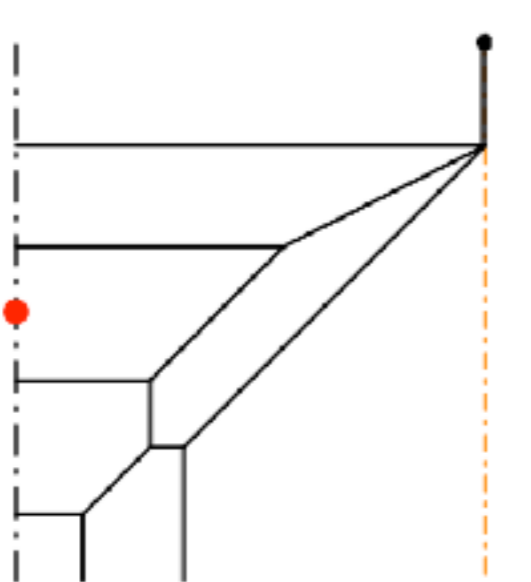
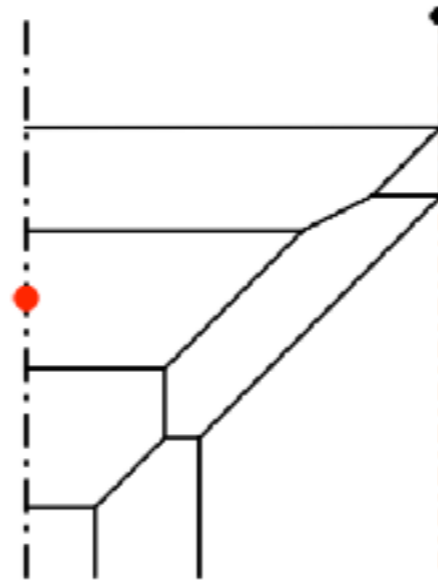
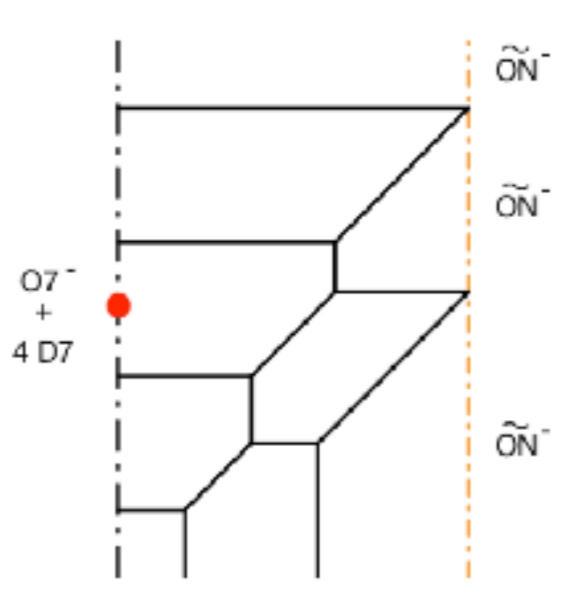
$$\mathcal{F}_{SU(3)_0+1F+1Sym}(\phi_1, \phi_2, m_0, m_F, m_{Sym})$$

6. $SU(3)_{3/2} + 9F$, $Sp(2)+1A+8F$

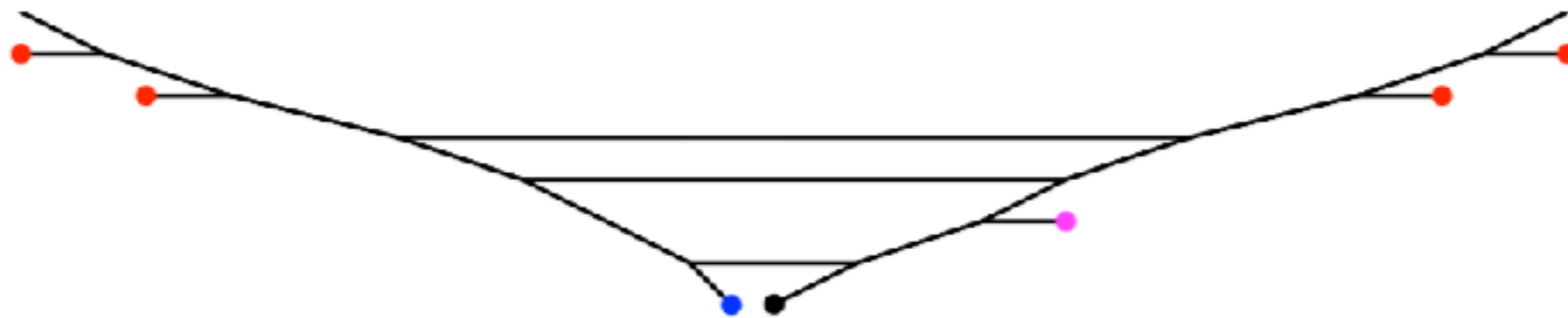
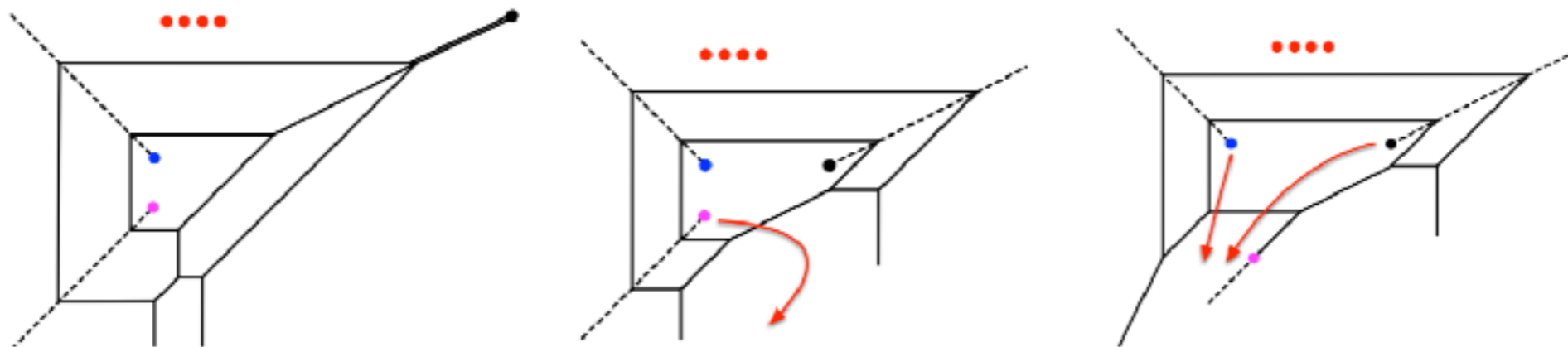
- $SU(3)_{5/2+} + 7F = SU(2)_{\pi} \times [SU(2)_{+5F}]$



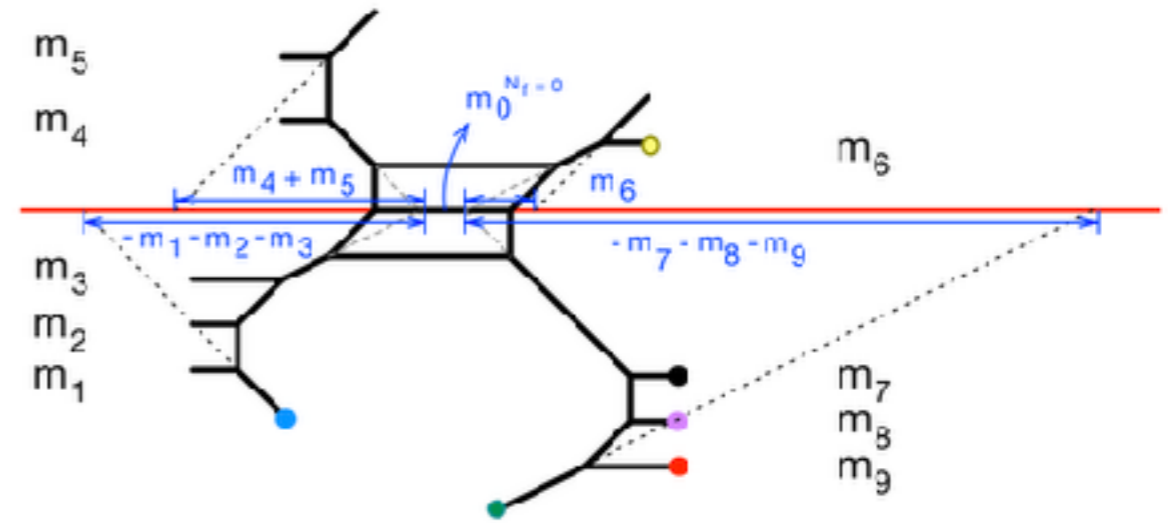
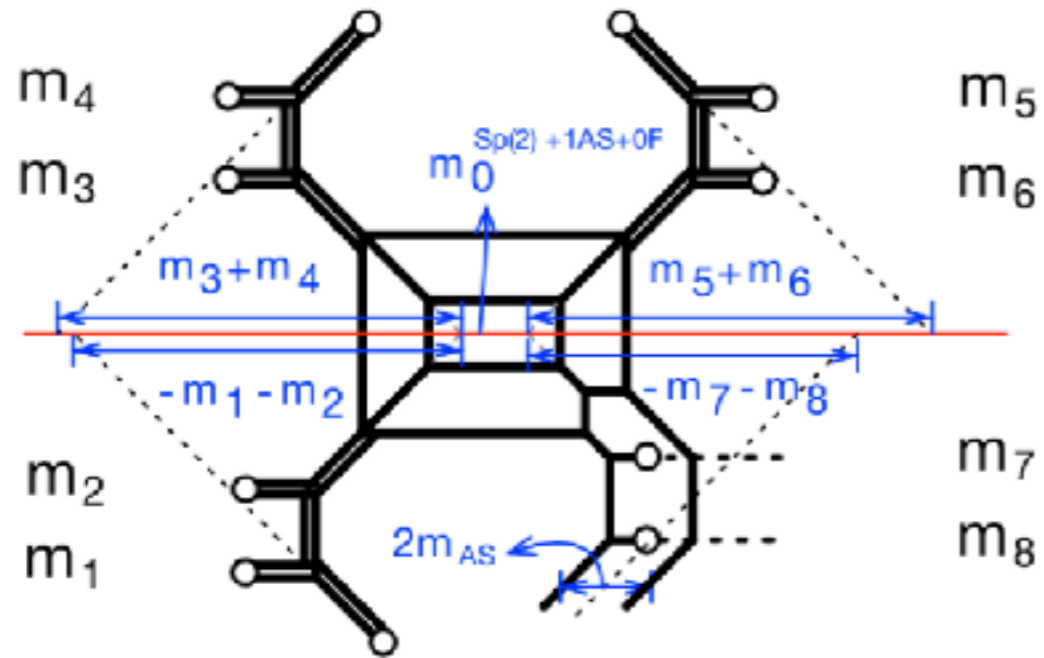
- $Sp(2)+ 2AS+ 4F$ & decoupling to $Sp(2)+1AS+ 4F$



- $Sp(2)+1AS+4F = SU(3)_{3/2}+5F$

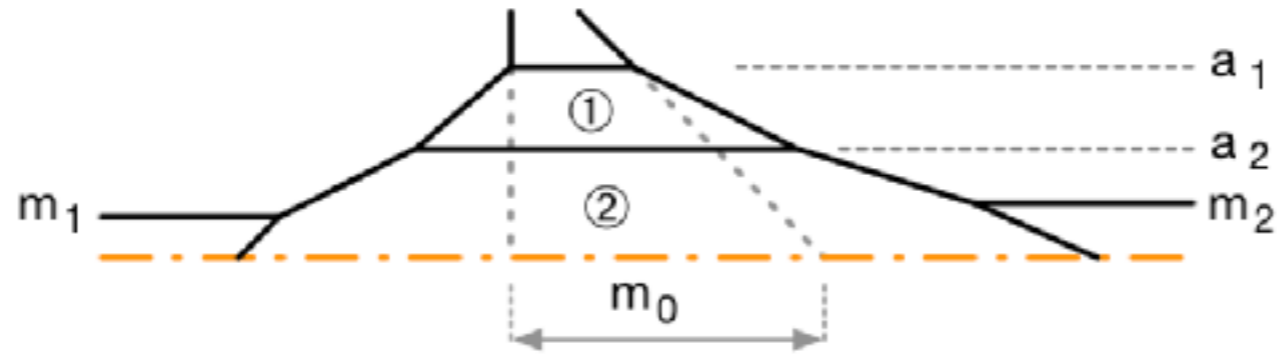


- Generalize to $Sp(2)+1AS+ 8F = SU(3)_{3/2}+9F$

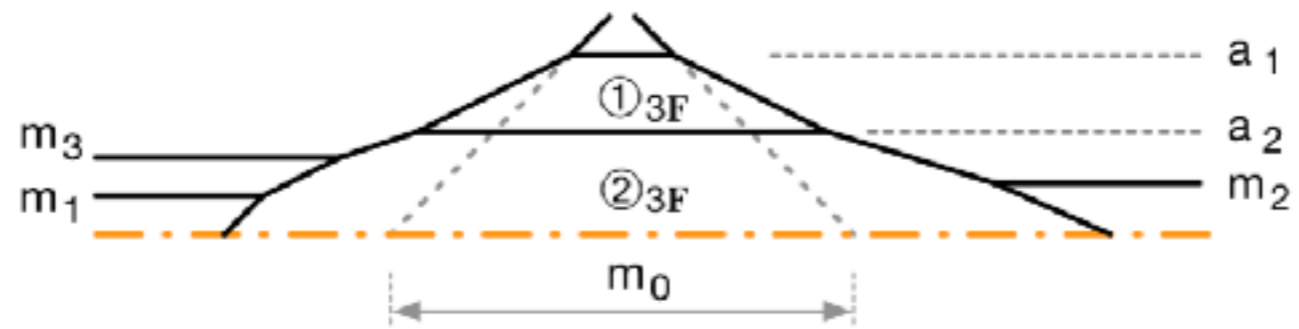


$$7. \text{Sp}(2) + 3 \text{AS} = \text{SO}(5)_0 + 3\text{F}$$

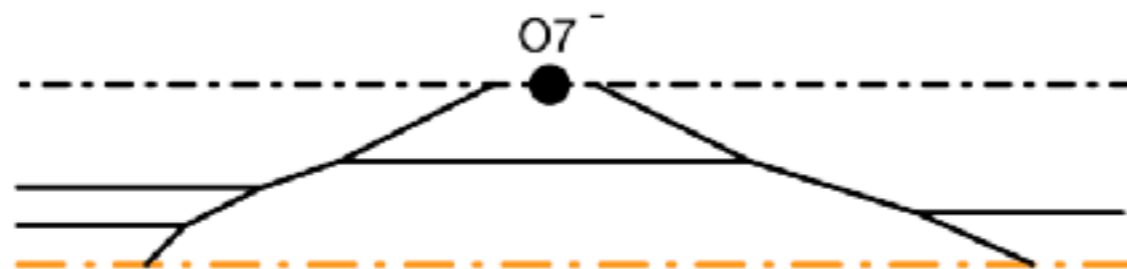
- $SO(5)_0 + 2F$



- $SO(5)_0 + 3F$

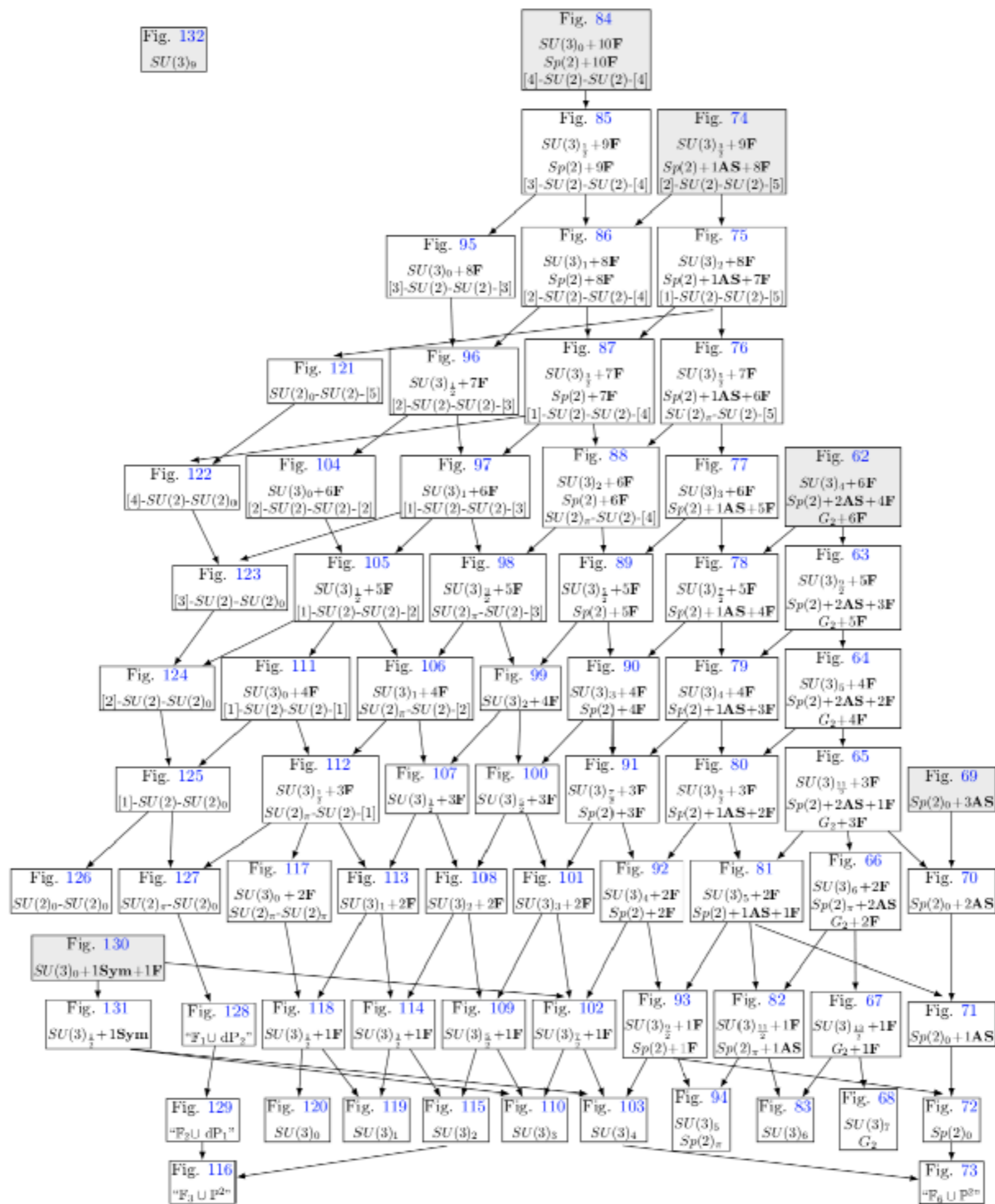


(a)



8. Conclusion

Fig. 132
 $SU(3)_0$



- We recovered all the (p,q) 5-brane web for the 5d rank 2 SCFTs
- We showed the dualities between different theories in the Coulomb phase.
- Some progress in exceptional case in higher rank theories.

- More challenges remain.
 - defects
 - geometry
 - classification & duality
 - large rank and d.o.f. ($N^{5/2}$)
 - relation to 6 and 4 dim theories