$\mathrm{AdS}_6/\mathrm{CFT}_5$ in Type IIB

Part I: Warped AdS_6 solutions and 5-brane webs

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UCLA

Strings, Branes and Gauge Theories
APCTP, July 2018

arXiv: 1606.01254, 1611.09411, 1703.08186, 1705.01561, 1706.00433, 1802.07274, 1805.11914, 1806.07898, 1806.08374

with E. D'Hoker, M. Gutperle, A. Karch, C. Marasinou, A. Trivella, O. Varela, O. Bergman, D. Rodríguez-Gómez, M. Fluder

Introduction

Introduction

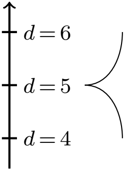
Theories in $d > 4$ crucial part in general understanding of QFT. Often reduce to interesting $d \leq 4$ theories $\rightarrow$ new theories and dualities, geometric realizations of known dualities.

Defining interacting QFTs in $d > 4$ challenging. In particular, Yang-Mills theories non-renormalizable ($\sim \sqrt{-g}R$ in $d = 4$).

Evidence from Strings, Branes and Gauge theories suggests many interacting $d > 4$ QFTs exist, challenging perturbative arguments.

Introduction

Suitable superconformal algebras in $d \leq 6$; maximal ones with $16_Q + 16_S = 32$ supercharges



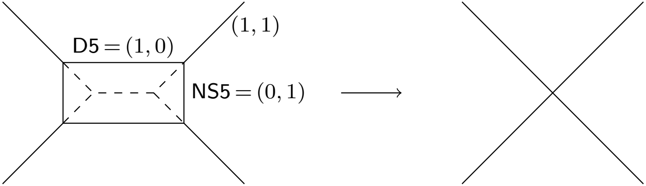
- unique superconf. algebra $F(4)$, 16 supercharges
- strongly-coupled UV fixed points for large classes of gauge theories w/ 8_Q supercharges [Seiberg '96;. . .]
- no standard Lagrangian, existence from Coulomb branch analysis and string theory

Asymptotically safe gauge theories, exceptional global symmetries, dualities, parents to isolated $4d$ theories, relations to $6d$, . . .

5-brane webs in Type IIB

[Aharony,Hanany,Kol '97]

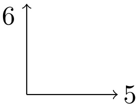
5-brane web: planar arrangement of (p, q) 5-branes at angles fixed by (p, q), junctions w/ conserved charges



Coulomb branch
finite gauge coupling

UV fixed point CFT

	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			



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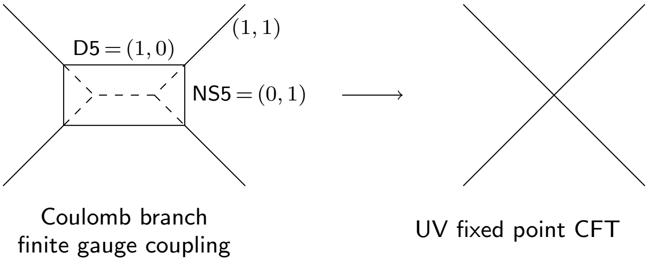

Large classes of 5d SCFTs, with and without gauge theory deformations, quivers, flavors, Chern-Simons terms,...

5-brane webs in Type IIB

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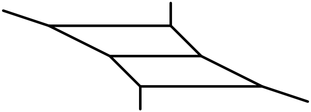
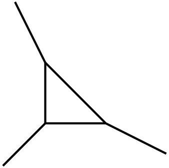
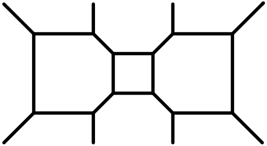


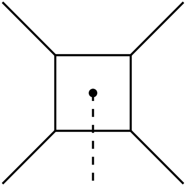
SU(3), CS$= 0$



E_0, no global symmetry



SU(2)$\times$SU(2)$\times$SU(2) quiver

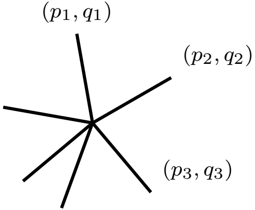


SU(2) $+$ 1 flavor

5-brane webs in Type IIB

[Aharony,Hanany,Kol '97]

General picture: any planar 5-brane junction realizes a 5d SCFT on the intersection point



$p_i, q_i \in \mathbb{Z}$

$$\sum p_i = \sum q_i = 0$$

Characterized entirely by external 5-brane charges. No standard Lagrangian. May or may not have gauge theory deformations.

Supergravity duals?

AdS/CFT to access superconformal fixed points? Harder than in $d \neq 5$, no maximally supersymmetric solutions.

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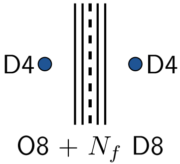
One well known AdS_6 solution from type I' construction:



N D4 probing O8+N_f D8: USp(N) with antisymmetric and N_f flavors

$\rightarrow$ warped $\text{AdS}_6 \times \text{S}^4$ in massive IIA [Brandhuber, Oz]

Locally unique [Passias], orbifolds dual to quiver gauge theories [Rodriguez-Gomez, Bergman], T-duals in IIB [Cvetic et al.; Lozano et al.]

Holographic duals for 5d SCFTs

Holographic duals for SCFTs realized by 5-brane webs in Type IIB?
Not a standard near-horizon limit – fully localized intersections.

Type IIB BPS equations studied by [Apruzzi,Fazzi,Passias,Rosa,Tomasiello; H.Kim,N.Kim,Suh;H.Kim,N.Kim] . No explicit solutions.

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Part I: Warped AdS_6 solutions in Type IIB (today)

Part II: AdS_6/CFT_5 – Tests and applications (next week)

Holographic duals for 5d SCFTs

Warped AdS_6 solutions in Type IIB

- Ansatz and general local $AdS_6 \times S^2 \times \Sigma$ solution
- Global solutions on the disc and 5-brane webs
- Solutions with more general Σ?
- Solutions with $SL(2,\mathbb{R})$ monodromy $\rightarrow$ 7-branes

AdS_6 solutions in Type IIB
– ansatz and local solution –

Symmetries and ansatz

AdS_6 + 16 susies $\rightarrow$ F(4) $\supset$ bosonic SO(2,5) $\oplus$ SO(3)

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$$\mathrm{AdS}_6 + 16 \text{ susies} \;\rightarrow\; \mathrm{F}(4) \;\supset\; \text{bosonic } \underbrace{\mathrm{SO}(2,5)}_{\nearrow\ \mathrm{AdS}_6} \oplus \underbrace{\mathrm{SO}(3)}_{\nwarrow\ \mathrm{S}^2}$$

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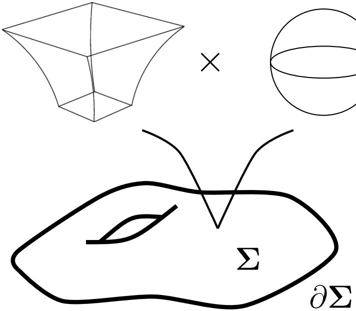
$\nearrow$ AdS_6 $\quad$ $\nwarrow$ S^2



General ansatz: AdS_6 and S^2 warped over Riemann surface Σ

$$\mathcal{M} = (\text{AdS}_6 \times \text{S}^2) \times_{\text{w}} \Sigma$$

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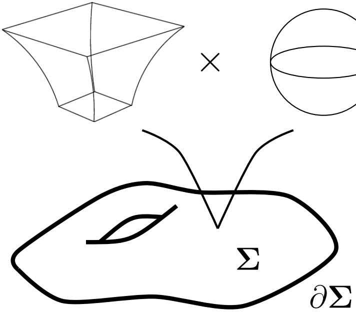
AdS$_6$ $\nearrow$ (SO(2,5))   S^2 $\nwarrow$ (SO(3))



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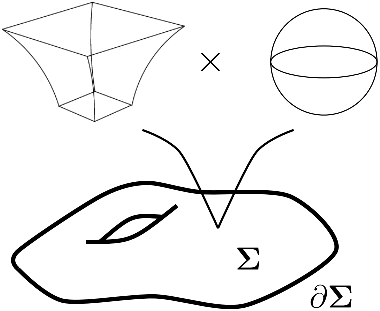
↗ AdS$_6$ ↖ S^2



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Remaining bosonic fields: $C_{(4)} = 0 \quad C_{(2)} \propto \mathrm{vol}_{\mathrm{S}^2} \quad \tau = \chi + ie^{-2\phi}$

General local solution

With complex coordinate w on Σ

$$ds^2 = f_6(w,\bar{w})^2 ds^2_{\mathrm{AdS}_6} + f_2(w,\bar{w})^2 ds^2_{\mathrm{S}^2} + 4\rho(w,\bar{w})^2 |dw|^2$$

$$C_{(2)} = \mathcal{C}(w,\bar{w})\,\mathrm{vol}_{\mathrm{S}^2} \qquad B(w,\bar{w}) = \frac{1+i\tau(w,\bar{w})}{1-i\tau(w,\bar{w})}$$

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Preserve 16 supersymmetries $\rightarrow$ BPS eq.

$$\delta\psi_M = D_M\epsilon - \frac{1}{96}\left(\Gamma_M(\Gamma\cdot G) + 2(\Gamma\cdot G)\Gamma_M\right)\mathcal{B}^{-1}\epsilon^* \stackrel{!}{=} 0$$
$$\delta\lambda = i(\Gamma\cdot P)\mathcal{B}^{-1}\epsilon^* - \frac{i}{24}(\Gamma\cdot G)\epsilon \stackrel{!}{=} 0$$

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Decomposing Killing spinors, reducing BPS eq. on AdS_6 and S^2
$\rightarrow$ coupled PDEs on Σ for supergravity fields & Killing spinors

FF ▶▶

– AdS$_6\times$S^2 gravitino eq. $\rightarrow$ radii in terms of Killing spinors

– dilatino eq. $\rightarrow$ 3-form in terms of B, Killing spinors

– Killing spinors in terms of ρ^2, B, holomorphic $\partial_w\mathcal{A}_\pm$

– decouple and integrate remaining equations for ρ^2, B

General local solution

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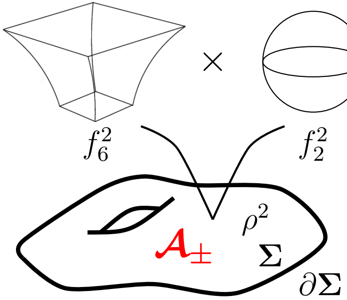

Arbitrary locally holomorphic

$$\mathcal{A}_\pm : \Sigma \rightarrow \mathbb{C}$$

yield metric functions f_6^2, f_2^2, ρ^2, axion-dilaton B, two-form field $\mathcal{C}$ and Killing spinors solving BPS eq.

SU$(1,1) \otimes \mathbb{C}$ transf. of $\mathcal{A}_\pm$ induce SL(2,$\mathbb{R}$) on supergravity fields

General local solution

Explicit solution for supergravity fields:

$$f_6^2 = \sqrt{6\mathcal{G}}\left[\frac{1+R}{1-R}\right]^{1/2} \qquad f_2^2 = \frac{1}{9}\sqrt{6\mathcal{G}}\left[\frac{1-R}{1+R}\right]^{3/2}$$

$$\rho^2 = \frac{\kappa^2}{\sqrt{6\mathcal{G}}}\left[\frac{1+R}{1-R}\right]^{1/2} \qquad B = \frac{\partial_w \mathcal{A}_+ \, \partial_{\bar{w}}\mathcal{G} - R\, \partial_{\bar{w}}\bar{\mathcal{A}}_- \partial_w \mathcal{G}}{R\, \partial_{\bar{w}}\bar{\mathcal{A}}_+ \partial_w \mathcal{G} - \partial_w \mathcal{A}_- \partial_{\bar{w}}\mathcal{G}}$$

$$\mathcal{C} = \frac{4i}{9}\left[\frac{(1+R^2)\partial_w \mathcal{G}\, \partial_{\bar{w}}\bar{\mathcal{A}}_- - 2R\partial_{\bar{w}}\mathcal{G}\, \partial_w \mathcal{A}_+}{(1+R)^2\, \kappa^2} - \bar{\mathcal{A}}_- - 2\mathcal{A}_+\right]$$

with composite quantities

$$\kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2 \qquad \partial_w \mathcal{B} = \mathcal{A}_+ \partial_w \mathcal{A}_- - \mathcal{A}_- \partial_w \mathcal{A}_+$$

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \qquad R + \frac{1}{R} = 2 + 6\,\frac{\kappa^2\,\mathcal{G}}{|\partial_w \mathcal{G}|^2}$$

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General local solution

General type IIB supergravity solution with 16 supersymmetries on $\mathrm{AdS}_6 \times \mathrm{S}^2$ warped over Σ, in terms of locally holomorphic $\mathcal{A}_\pm$ on Σ.

Solves IIB supergravity equations of motion for arbitrary $\mathcal{A}_\pm$ ✓

[arXiv:1712.04463 Corbino, D'Hoker, CFU]

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$\rightarrow$ narrow down to globally regular solutions

AdS_6 solutions in Type IIB
– global solutions –

Regularity conditions

Imposing physical regularity conditions imposes constraints. Real geometry with consistent spacetime signature, $\mathrm{Im}(\tau) > 0$:

$$\kappa^2\big|_{\mathrm{int}(\Sigma)} > 0 \qquad\qquad \mathcal{G}\big|_{\mathrm{int}(\Sigma)} > 0$$

$\rightarrow \Sigma$ must have a boundary ($\partial_w \partial_{\bar{w}} \mathcal{G} = -\kappa^2$ by construction)

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For 10d geometry w/o boundary, collapse S^2 on $\partial\Sigma$ (AdS_6 finite):

$$\kappa^2\big|_{\partial\Sigma} = 0 \qquad\qquad \mathcal{G}\big|_{\partial\Sigma} = 0$$

Not all independent, $\mathcal{G}|_{\mathrm{int}(\Sigma)} > 0$ implied by the other conditions.

Solving the regularity conditions

Fix topology of Σ, 1) construct locally holomorphic $\mathcal{A}_{\pm}$ producing regular κ^2, 2) implement additional constraints for regular $\mathcal{G}$.

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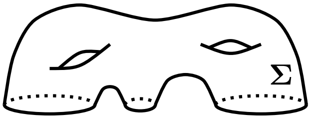

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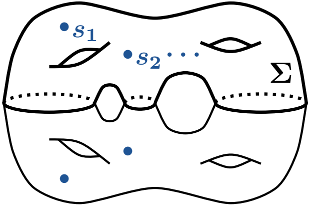

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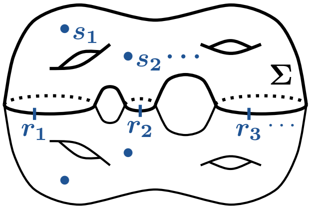

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1b) From Φ to $(\partial_w)\mathcal{A}_\pm$: **poles** r_ℓ on $\partial\Sigma$, integ. constants $\mathcal{A}^0_\pm$

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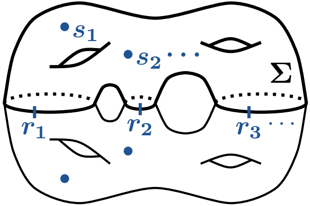

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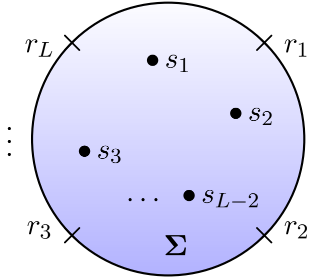
2) $\mathcal{G}|_{\partial\Sigma} = 0$: constraints on $\{s_n, r_\ell, \mathcal{A}^0_\pm\}$

Regular solutions on the disc

$\Sigma =$disc/upper half plane: L poles $\sim L-2$ "charges" $\Longrightarrow L \geq 3$



$$\mathcal{A}_\pm = \mathcal{A}^0_\pm + \sum_{\ell=1}^{L} Z^\ell_\pm \ln(w - r_\ell)$$

$$Z^\ell_+ = \sigma \prod_{n=1}^{L-2} (r_\ell - s_n) \prod_{k \neq \ell}^{L} \frac{1}{r_\ell - r_k}$$

$$\mathcal{A}_-(w) = -\overline{\mathcal{A}_+(\bar{w})} \qquad \sum_\ell Z^\ell_+ = 0$$

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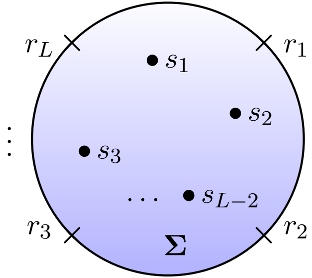

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$\mathcal{G}|_{\partial\Sigma} = 0 \sim$ one local condition per pole $\rightarrow 2L-2$ free parameters

$$\mathcal{A}^0_+ Z^k_- - \mathcal{A}^0_- Z^k_+ + \sum_{\ell\neq k} Z^{[\ell,k]} \ln|p_\ell - p_k| = 0$$

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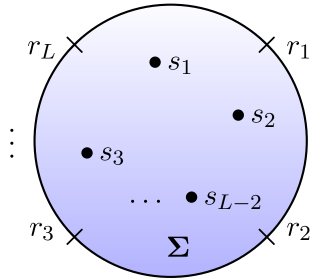

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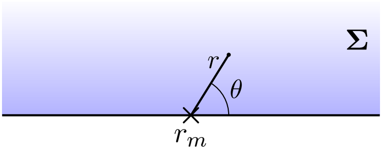
Solutions regular everywhere, except for possibly the poles. . .

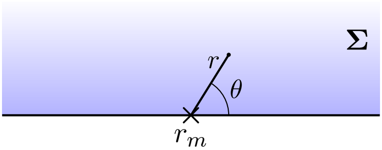
Regular solutions on the disc – behavior near poles



string frame metric, $r \ll 1$:

$$\widetilde{ds}^2 \approx \frac{2}{3} \left| Z_+^m - Z_-^m \right| \left[3\,|\ln r| ds^2_{\mathrm{AdS}_6} + \overbrace{\frac{dr^2}{r^2} + d\theta^2}^{\Sigma} + \sin^2\theta \, ds^2_{\mathrm{S}^2} \right]$$

Regular solutions on the disc – behavior near poles



string frame metric, $r \ll 1$:

$$\widetilde{ds}^2 \approx \frac{2}{3}\left|Z_+^m - Z_-^m\right| \Big[\underbrace{3\,|\ln r| ds^2_{\mathrm{AdS}_6}}_{\to\,\mathbb{R}^{1,5}} + \overbrace{\frac{dr^2}{r^2} + d\theta^2}^{\Sigma} \underbrace{ + \sin^2\theta\, ds^2_{\mathrm{S}^2}}_{\mathsf{S}^3}\Big]$$

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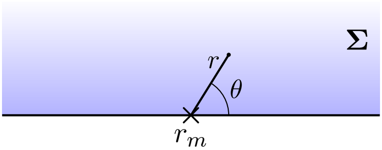

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$$dC_{(2)} \approx \frac{8}{3} Z_+^m \mathrm{vol}_{\mathrm{S}^3} \qquad e^{-2\phi} \approx \frac{\sqrt{3}\,\kappa_m^2}{4\,\mathrm{Re}(Z_+^m)^2}\,\frac{r}{\sqrt{|\ln r|}} \qquad \chi \approx \frac{\mathrm{Im}(Z_+^m)}{\mathrm{Re}(Z_+^m)}$$

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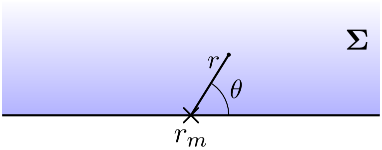

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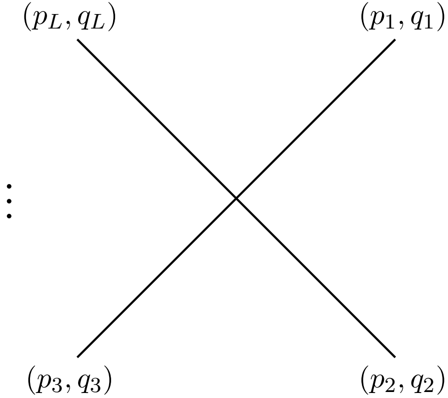
Entire near-pole solution matches (p,q) 5-branes of [Lu,Roy '98]

$$q + ip \quad \longleftrightarrow \quad Z_+^m$$

Solutions regular with isolated poles corresponding to 5-branes ✓

AdS$_6$ solutions in Type IIB
– connection to 5-brane webs –

5-brane web picture



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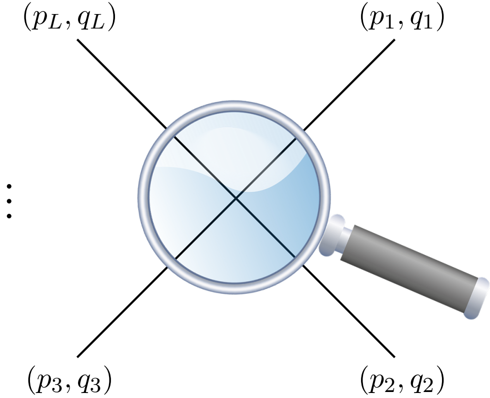

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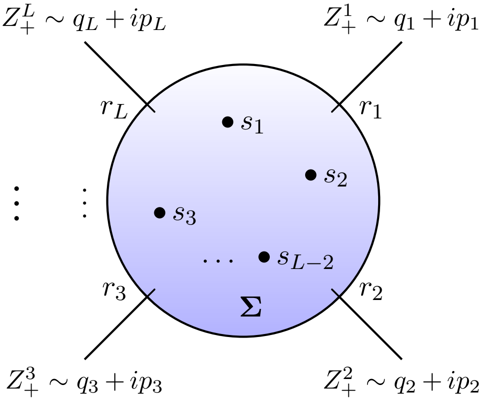

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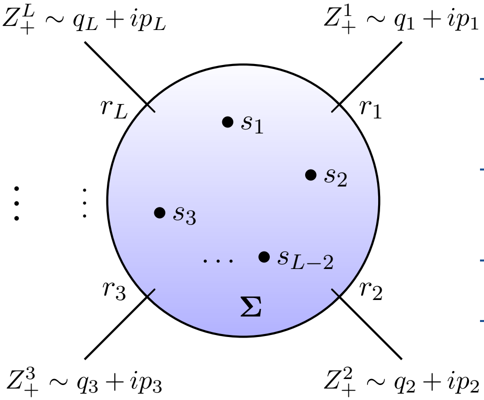

- external 5-branes explicitly (p,q) charge conserved
- parametrized by choice of residues mod charge cons.
- $\mathrm{AdS}_6 + 16$ susies $= F(4)$
- need $L \geq 3$, p and q charge

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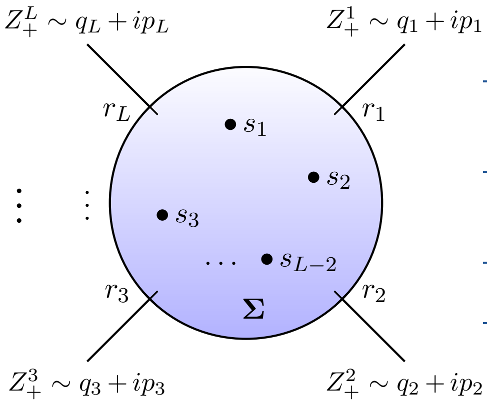

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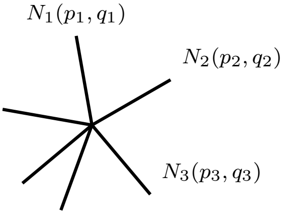
Supergravity solutions for fully localized 5-brane intersections ✓

“Large N”

Classical supergravity: $Z_+^\ell \sim q_\ell + ip_\ell \in \mathbb{C}$. Limit of string theory with large 5-brane charges $\rightarrow$ effectively continuous.

Large numbers of like-charged coincident 5-branes at each pole



$p_i, q_i \in \mathbb{Z}$, relatively prime

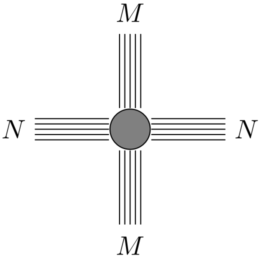
$N_i \gg 1 \ \forall i$

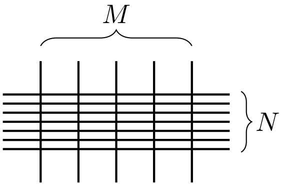
$Z_+^\ell \sim N_\ell q_\ell + ip_\ell$

In particular: generally large D5 and NS5 brane charges.

SCFT/gauge theory connections

NS5/D5 intersection [Aharony,Hanany,Kol '97]





$$N - \underbrace{SU(N) \times \cdots \times SU(N)}_{SU(N)^{M-1}} - N$$

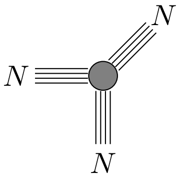
$\leftrightarrow$ 4-pole solution with $Z_+^1 = -Z_+^3 \sim iN \ , \ Z_+^2 = -Z_+^4 \sim M$

large D5 and large NS5 charge $\leftrightarrow$ large number of nodes in quiver deformation and (at least some) large-rank gauge groups

SCFT/gauge theory connections

5d T_N theories: junction of N D5, N NS5 and N $(1,1)$ 5-branes

[Benini,Benvenuti,Tachikawa '09]



– Reduce on S^1 to 4d $T[A_{N-1}]$

– IR gauge theory [Bergman,Zafrir '14]

$$N - SU(N-1) - \cdots - SU(2) - 2$$

$\leftrightarrow$ 3-pole solution with $Z^1_+ \sim iN$, $Z^2_+ \sim N$, $Z^3_+ \sim (1+i)N$

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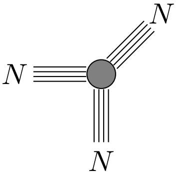

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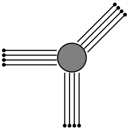
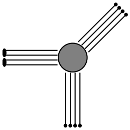
Potential subtlety: s-rule constraints from 5-branes on 7-branes



vs.



$SU(4)^3$

vs.

$SU(4)^2 \times SU(2) \to E_7$

Supergravity sol. $\sim$ unconstrained junctions (details next week).

Global solutions on the disc

Physically regular solutions for $\Sigma =$disc, $L \geq 3$ poles on $\partial\Sigma$, $2L - 2$ real parameters = residues at poles with zero sum.

Poles required by regularity, correspond to (p, q) 5-branes with $q + ip \sim Z_+^\ell$. Natural identification with 5-brane junctions.

Recipe for Σ w/ arbitrary numbers of handles and boundaries.

AdS_6 solutions in Type IIB
– More general Riemann surfaces? –

More general Riemann surfaces?

Construction of regular κ^2 works for general Riemann surfaces.
Boundary condition $\mathcal{G}_{\partial\Sigma} = 0$ not automatic.

Two natural options: (i) add handles (ii) add further boundaries

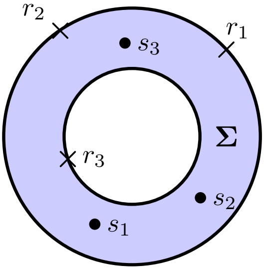
Technically more challenging. Charge distribution on doubled surface + Green's function $\to \kappa^2$, Green's functions not as simple.

More general Riemann surfaces?

Next-to-simplest option: annulus, no handles, two boundaries

Doubled surface=torus $\rightarrow$ quasi-periodic Jacobi ϑ-functions in Green's function

$L \geq 2$ poles, N "charges"



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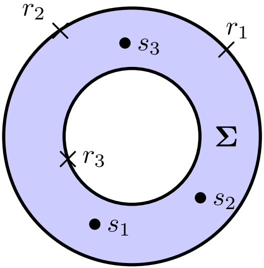

$$\mathcal{A}_+(w|\tau) = \mathcal{A}_+^0 + \alpha_+ w + \sum_{\ell=1}^{L} Z_+^\ell \ln \vartheta_1(w - r_\ell|\tau)$$

$$Z_+^\ell = \sigma \, \frac{\prod_{n=1}^{N} \vartheta_1(r_\ell - s_n|\tau)}{\prod_{n\neq\ell} \vartheta_1(r_\ell - p_n|\tau)} \exp\left\{ -\frac{2\pi i}{\tau} \, r_\ell \, \Lambda_+ - \frac{\pi i}{\tau} \sum_{n=1}^{N} (r_n^2 + r_n) \right\}$$

More general Riemann surfaces?

Main challenge: $\mathcal{G} = 0$ on two boundaries $\Rightarrow$ non-local condition.

Necessary for $\mathcal{G}|_{\partial\Sigma} = 0$: $\mathcal{G}$ constant along regular boundary segments, no monodromy around poles $\sim$ local conditions.

This is sufficient for a single boundary component:

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} \qquad \partial\mathcal{B} = \mathcal{A}_+\partial\mathcal{A}_- - \mathcal{A}_-\partial\mathcal{A}_+$$

$\mathcal{B}$ defined up to integration constant $\rightarrow$ use it to set $\mathcal{G} = 0$.

Annulus needs extra constraint, connecting both boundaries.
Found <u>no solutions</u> for $L = 2, 3, 4, 5$ ✗

AdS$_6$ solutions in type IIB
– solutions with monodromy –

5-brane webs with 7-branes

[DeWolfe,Hanany,Iqbal,Katz '99]

Supergravity fields single-valued in solutions so far. Could allow for $\mathsf{SL}(2,\mathbb{R})$ monodromy, induced by simple action on $\mathcal{A}_\pm \Rightarrow$ 7-branes

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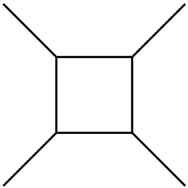
	0	1	2	3	4	5	6	7	8	9
D5	×	×	×	×	×	×				
NS5	×	×	×	×	×		×			
D7	×	×	×	×	×			×	×	×



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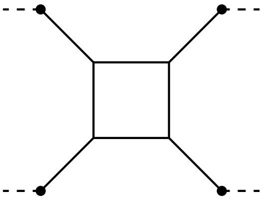

(i) terminate external (p,q) 5-branes on $[p,q]$ 7-branes

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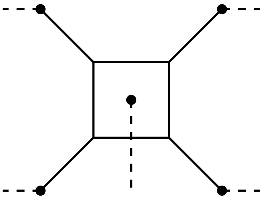

(i) terminate external (p,q) 5-branes on $[p,q]$ 7-branes

(ii) add 7-branes into faces of web

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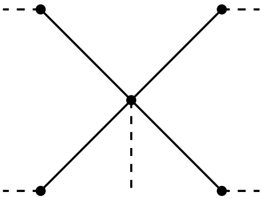

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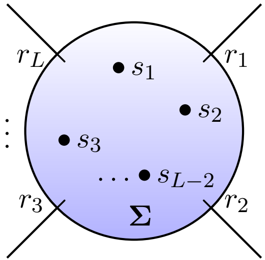
In conformal and "near-horizon" limit, (ii) should be represented in corresponding supergravity solution.

Solutions with monodromy

Construction from disc solutions without monodromy:



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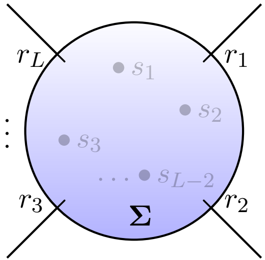

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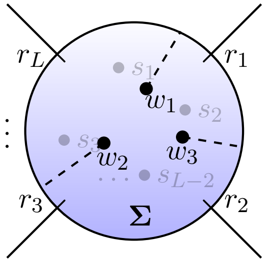

– pick parabolic $SL(2,\mathbb{R})$ matrix

$$M_{[p,q]} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix}$$

– add punctures w_i, "charge" n_i, orientation of branch cut γ_i

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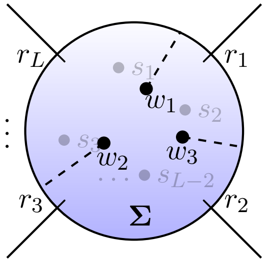

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Encode branch cut structure in $f = \sum_{i=1}^{I} \frac{n_i^2}{4\pi} \ln\left(\gamma_i \, \frac{w-w_i}{w-\bar{w}_i}\right)$.

Differentials $\partial\mathcal{A}_\pm = \partial\mathcal{A}_\pm^s + \eta_\pm f\left(\eta_-\partial\mathcal{A}_+^s - \eta_+\partial\mathcal{A}_-^s\right)$, $\eta_\pm = p \mp iq$.

Solutions with monodromy

Differentials $\partial\mathcal{A}_\pm$ realize commuting $SL(2,\mathbb{R})$ monodromies $M_{[n_i p, n_i q]}$ around w_i; sum of residues not necessarily zero.

Locally holomorphic functions $\mathcal{A}_\pm$ involve "polylogarithm integrals" Regular κ^2 by construction, $\mathcal{G}|_{\partial\Sigma} = 0$ still one condition per pole.

Demanding monodromies to lift to $SL(2,\mathbb{R})$ transformations on $\mathcal{A}_\pm$ yields further constraints, forcing punctures onto curves in Σ.

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$\rightarrow$ regular solutions for disc with punctures and commuting parabolic $SL(2,\mathbb{R})$ monodromies

Solutions with monodromy

At punctures, solution as expected for 7-brane wrapping $\mathrm{AdS}_6 \times \mathrm{S}^2$.
Infinitesimal monodromy: recover probe D7 (DBI, κ symmetry) ✓

Additional parameters due to punctures:
charge ✓ branch cut orientation ✓ position in Σ **??**

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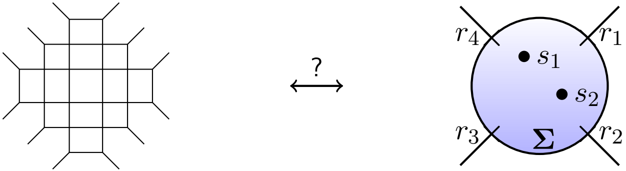
Internal structure of brane web encoded in Σ?



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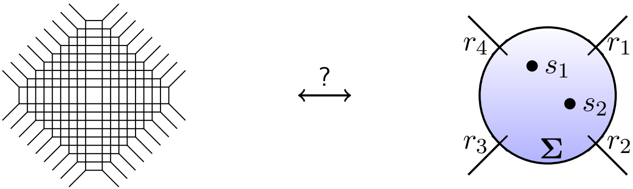
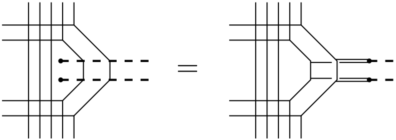


Dense grid at large N. Position in $\Sigma \leftrightarrow$ choice of face.
Continuous parameter, remains meaningful in conformal limit.

Solutions with monodromy

Hanany/Witten: pull 7-branes out of web $\rightarrow$ 5-brane creation



s-rule: max one D5 between NS5 and D7
[Benini,Benvenuti,Tachikawa]

In general not the same as (unconstrained) 5-brane junction.

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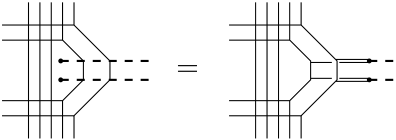

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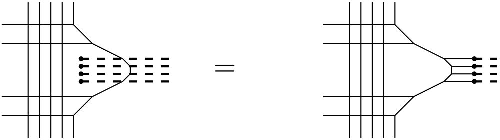
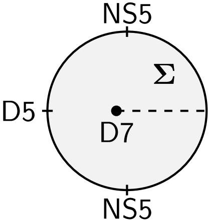
Exception: 7-branes "trapped" by only one NS5 brane



Same as unconstrained 5-brane junction with additional D5s

Solutions with monodromy

Supergravity picure: family of 3-pole solutions



- two NS5 poles one D5 pole with *fixed* residues
- one D7 puncture, on "equator" charge depends on location

Solutions with monodromy

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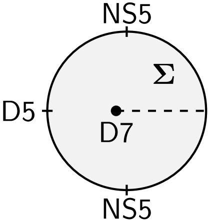

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- one D7 puncture, on "equator" charge depends on location

In general, on-shell action (=free energy) different from 4-pole solution w/o monodromy.

As $\mathrm{D7} \to \partial\Sigma$ w/ fixed 5-brane charges, puncture turns into pole.

Consistent with 5-brane webs just shown. Confirmation: next week.

Summary & Outlook

Summary

Supergravity solutions for fully localized 5-brane junctions in type IIB. Holographic duals for the corresponding 5d SCFTs.

Solutions regular everywhere except for isolated physically meaningful singularities.

Extension to solutions with parabolic SL(2,$\mathbb{R}$) monodromy, for 5-brane junctions with (mutually local) 7-branes.

Outlook

Establish $\text{AdS}_6/\text{CFT}_5$ dualities rigorously: confront holography with QFT results ("stringy" operators, $F(\mathrm{S}^5)$ $\rightarrow$ next week).

AdS/CFT with warped products: spectra, correlation functions, defects, Wilson lines, truncation to 6d $F(4)$ gauged sugra...

Solutions with non-commuting SL(2,$\mathbb{R}$) monodromies for mutually non-local 7-branes, more general Riemann surfaces, ...

$AdS_2 \times S^6$: local solution in [arXiv:1712.04463] , global solutions?

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Thank you!