# 6d strings and exceptional instantons

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Based on 1801.03579 [HC.Kim,J.Kim,S.Kim,KHL,J.Park]

# Instantons in 4d $\mathcal{N} = 2$

- Instantons : 0d object satisfying $F = \star F$ on $\mathbb{R}^4$
- Prepotential in Coulomb branch gets instanton corrections [Seiberg,Witten 1994]
- Corrections computed by instanton partition function [Nekrasov 2002]
- ADHM : matrix model describing instanton moduli space [Atiyah et.al.1978]

# Instantons in higher dimensions

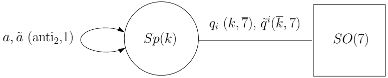
- 5d/6d w/ 8 supercharges
- At 5d/6d, instantons are $\frac{1}{2}$ BPS particles/strings
- ADHM : 1d/2d gauge theories of instanton dynamics
- ADHM is realized by Dp-D(p-4) in string theory
- Instantons not realized by D-branes & F1's : "exceptional"
- "Exceptional" theories appear in 6d SCFTs, dualities
- I will deal with $SO(7)$ w/ matters in **8** rep and $G_2$ w/ matters in **7** rep

# Strategy for exceptional instantons

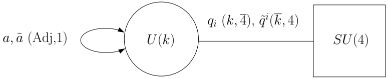
- Consider only Coulomb/Tensor branch on Ω-background
- Suffices to study massive fluctuations near isolated points of moduli space
- Consider classical $H \subset G$ whose instanton captures part of inst moduli
- Describe subset of moduli space by ADHM of $H$
- Add more 1d/2d matters to describe the $G$ instanton moduli
- Buttom up approach as model building, relying on the guess works
- Test by instanton partition functions

# $SO(7)$ ADHM

- Traditional ADHM construction: $Sp(k)_G \times SO(7)_F$, $N = (0, 4)$



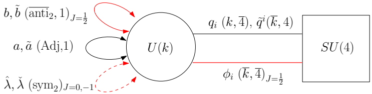

- Alternative construction: consider $SU(4) \subset SO(7)$: $\mathbf{21} \to \mathbf{15} \oplus \mathbf{6}$
- $SU(4)$ instanton ADHM




- It captures part of $SO(7)$ instanton moduli
- Add more ADHM matters as zero modes from vector in $\mathbf{6}$

# Alternative $SO(7)$ instanton ADHM

- Alternative ADHM of $SO(7)$ instanton




- Symmetries: $U(k)$, $SU(4)$, $\mathcal{N} = (0,1)$ SUSY
- Witten index of alternative ADHM

$$
\begin{aligned}
Z_k = & \sum_{\vec{Y};|\vec{Y}|=k} \prod_{i=1}^{4} \prod_{s\in Y_i} \frac{2\sinh(\phi(s))\cdot 2\sinh(\phi(s)-\epsilon_+)}{\prod_{j=1}^{4} 2\sinh\frac{E_{ij}(s)}{2}\cdot 2\sinh\frac{E_{ij}(s)-2\epsilon_+}{2}\cdot 2\sinh\frac{\epsilon_+-\phi(s)-v_j}{2}} \\
& \times \prod_{i\leq j}^{4} \prod_{s_{i,j}\in Y_{i,j};s_i<s_j} \frac{2\sinh\frac{\phi(s_i)+\phi(s_j)}{2}\cdot 2\sinh\frac{\phi(s_i)+\phi(s_j)-2\epsilon_+}{2}}{2\sinh\frac{\epsilon_{1,2}-\phi(s_i)-\phi(s_j)}{2}}
\end{aligned}
$$

$$
\begin{aligned}
\phi(s) &= v_i - \epsilon_+ - (n-1)\epsilon_1 - (m-1)\epsilon_2 \\
E_{ij}(s) &= v_i - v_j - \epsilon_1 h_i(s) + \epsilon_2(v_j(s)+1) \\
s &= (m,n)\in Y_i \quad (i=1,\cdots,4)
\end{aligned}
$$

- It shows the enhanced Weyl symmetry of $SO(7)$
- Matches with index of traditional ADHM of $SO(7)$ instantons

$$
\left(2\sinh\frac{\epsilon_{1,2}}{2}\right) Z_1^{\text{standard}} = \frac{1}{2}\sum_{a=1}^{3}\sum_{s=\pm} \frac{2\cosh\frac{su_a}{2}\cdot 2\cosh\frac{2\epsilon_+-su_a}{2}\cdot 2\sinh(\pm(su_a-\epsilon_+))}{2\sinh(su_a)\cdot 2\sinh(\epsilon_+-su_a)\prod_{b(\neq a)} 2\sinh\frac{su_a\pm u_b}{2}\cdot 2\sinh\frac{2\epsilon_+-su_a\pm u_b}{2}}
$$

$$
v_1 = \frac{u_1+u_2+u_3}{2}, \quad v_2 = \frac{u_1-u_2-u_3}{2}, \quad v_3 = \frac{-u_1+u_2-u_3}{2}, \quad v_4 = \frac{-u_1-u_2+u_3}{2}
$$

# $SO(7)$ with matters in $\mathbf{8}$ rep

- $\mathbf{8} \to \mathbf{4} \oplus \overline{\mathbf{4}}$ : add two Fermi multiplets $\Psi_a(k,1), \tilde{\Psi}_a(\overline{k},1)$
- $n_{\mathbf{8}} \leq 4$ in 5d, $n_{\mathbf{8}} = 2$ in 6d (anomaly cancelation)
- 5d instanton index

$$\begin{aligned}
\hat{Z}_1^{n_{\mathbf{8}}=1} &= \prod_{i<j} \frac{t^2}{(1-t^2 b_i^{\pm} b_j^{\pm})} \left[\chi_{\mathbf{2}}^{Sp(1)} f_0(v) + f_1(v)\right] \\
\hat{Z}_1^{n_{\mathbf{8}}=2} &= \prod_{i<j} \frac{t^2}{(1-t^2 b_i^{\pm} b_j^{\pm})} \left[\chi_{\mathbf{5}}^{Sp(1)} f_0(v) + \chi_{\mathbf{4}}^{Sp(2)} f_1(v) + f_2(v)\right] \\
\hat{Z}_1^{n_{\mathbf{8}}=3} &= \prod_{i<j} \frac{t^2}{(1-t^2 b_i^{\pm} b_j^{\pm})} \left[\chi_{\mathbf{14'}}^{Sp(3)} f_0(v) + \chi_{\mathbf{14}}^{Sp(3)} f_1(v) + \chi_{\mathbf{6}}^{Sp(3)} f_2(v) + f_3(v)\right]
\end{aligned}$$

$$\begin{aligned}
f_0(v) &= \chi_{\mathbf{9}}^{SU(2)} + \chi_{\mathbf{7}}^{SU(2)}(\chi_{\mathbf{7}}^{SO(7)}+1) + \chi_{\mathbf{5}}^{SU(2)}(-\chi_{\mathbf{35}}^{SO(7)} + \chi_{\mathbf{7}}^{SO(7)} + 1) \\
&\quad + \chi_{\mathbf{3}}^{SU(2)}(-\chi_{\mathbf{35}}^{SO(7)} + \chi_{\mathbf{27}}^{SO(7)} + 1) + \chi_{\mathbf{105}}^{SO(7)} - \chi_{\mathbf{21}}^{SO(7)} + \chi_{\mathbf{7}}^{SO(7)} \\
f_1(v) &= -\chi_{\mathbf{8}}^{SU(2)}\chi_{\mathbf{8}}^{SO(7)} - \chi_{\mathbf{6}}^{SU(2)}\chi_{\mathbf{8}}^{SO(7)} + \chi_{\mathbf{4}}^{SU(2)}\chi_{\mathbf{112}}^{SO(7)} - \chi_{\mathbf{2}}^{SU(2)}\chi_{\mathbf{168}}^{SO(7)} \\
f_2(v) &= \chi_{\mathbf{7}}^{SU(2)}\chi_{\mathbf{35}}^{SO(7)} - \chi_{\mathbf{5}}^{SU(2)}(\chi_{\mathbf{7}}^{SO(7)} - \chi_{\mathbf{35}}^{SO(7)} + \chi_{\mathbf{105}}^{SO(7)}) - \chi_{\mathbf{3}}^{SU(2)}(\chi_{\mathbf{21}}^{SO(7)} + \chi_{\mathbf{27}}^{SO(7)} - \chi_{\mathbf{35}}^{SO(7)} \\
&\quad - \chi_{\mathbf{77}}^{SO(7)} + \chi_{\mathbf{168'}}^{SO(7)} + 1) - \chi_{\mathbf{7}}^{SO(7)} + \chi_{\mathbf{21}}^{SO(7)} + \chi_{\mathbf{27}}^{SO(7)} - \chi_{\mathbf{105}}^{SO(7)} + \chi_{\mathbf{189}}^{SO(7)} + \chi_{\mathbf{330}}^{SO(7)} \\
f_3(v) &= -\chi_{\mathbf{112'}}^{SO(7)}\chi_{\mathbf{6}}^{SU(2)} + (\chi_{\mathbf{48}}^{SO(7)} - \chi_{\mathbf{112'}}^{SO(7)} + \chi_{\mathbf{512}}^{SO(7)})\chi_{\mathbf{4}}^{SU(2)} - (\chi_{\mathbf{112'}}^{SO(7)} + \chi_{\mathbf{448}}^{SO(7)})\chi_{\mathbf{2}}^{SU(2)} .
\end{aligned}$$

- Elliptic genus of 6d $SO(7)$ instanton strings

$$Z_1(\tau, \epsilon_{1,2}, v_i, m_a) = \frac{1}{\theta(\epsilon_{1,2})} \sum_{i=1}^{4} \frac{\theta(4\epsilon_+ - 2v_i)\prod_{a=1}^{2}\theta(m_a \pm (v_i - \epsilon_+))}{\prod_{j(\neq i)}\theta(v_{ij})\theta(2\epsilon_+ - v_{ij})\theta(2\epsilon_+ - v_i - v_j)}$$

# $G_2$ instantons from Higgsing

- Give non-zero VEV to one of matter fields $\Phi$ in **8** representation
- $SO(7)\,\text{w/ single}\,\Phi \xrightarrow{\langle\Phi\rangle} G_2$
- In our $SU(4)$ formalism, $SU(4) \xrightarrow{\langle\Phi\rangle} SU(3)$
- $SU(3)(\subset G_2)$ ADHM formalism :

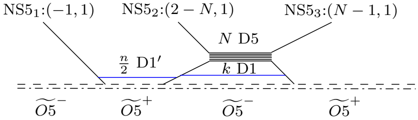
  $q(k,\overline{4}), \tilde{q}(\overline{k},4) \to q(k,\overline{3}), \tilde{q}(\overline{k},3),\ \phi_i(\overline{k},\overline{4}) \to \phi_i(\overline{k},\overline{3}) \oplus \phi(\overline{k},1)$
  from $SU(4)$ formalism

- $\Psi_a(k,1),\ \tilde{\Psi}_a(\bar{k},1)$ for $n_{\mathbf{7}}$ matters in **7** representation
- $\langle\Phi\rangle \neq 0$ : $Z_{SO(7)} \xrightarrow{m=\epsilon_+,\ v_4=0} Z_{G_2}$
- Index at $n_{\mathbf{7}} = 0$ matches with [Cremonesi,Ferlito,Hanany,Mekareeya 2014]
  e.g. k=1

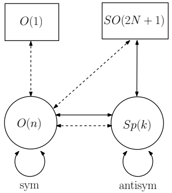
$$\left(2\sinh\frac{\epsilon_{1,2}}{2}\right) Z_1 = \frac{t^3(1+t^2)(1+t^2\chi_{\mathbf{7}}^{G_2}(v)+t^4)}{\prod_{i<j}(1-t^2e^{v_{ij}})(1-t^2e^{-v_{ij}})} = t^3\sum_{n=0}^{\infty}\chi_{(0,n)}^{G_2}(v)t^{2n}$$

# Test by brane construction

- Brane construction of 5d $SO(7)$ w/ matter in **8** rep. [Zafrir 2015]




- matter in **8** rep is realized non-perturbatively by D1$'$




- Witten index computes canonical partition function at fixed $n$
- The full index(grand canonical ptn ftn) matches with ours

# More applications in 6d

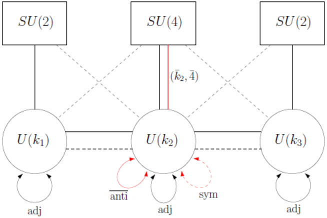
- 6d SCFTs classified geometrically from F-theory [Morrison,Taylor 2012][Heckman,Morrison,Rudelius,Vafa 2015]
- Among 'atoms', 3 SCFTs contain $G_2$ or $SO(7)$ w/ **8** matters

| base | 3, 2 | 3, 2, 2 | 2, 3, 2 |
|---|---|---|---|
| gauge symmetry | $G_2 \times SU(2)$ | $G_2 \times SU(2) \times \{\ \}$ | $SU(2) \times SO(7) \times SU(2)$ |
| matter | $\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$ | $\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$ | $\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$ |

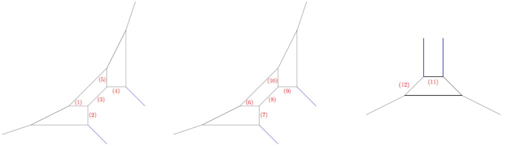
- Constructed 2d quiver gauge theories of instanton strings
- I will explain with the last case which can be tested

# $SU(2) \times SO(7) \times SU(2)$ instanton strings

- 2d quiver gauge theory of $SU(2) \times SO(7) \times SU(2)$ instanton strings




- 5d description on $S^1$ [Del Zotto,Vafa,Xie 2015] [Hayashi,Ohmori 2017]



$$(1) : \frac{q}{y_1 y_2 y_3^2}, \quad (2) : e^{-v}, \quad (3) : e^{-u}\frac{y_1 y_2 y_3^2}{\sqrt{q}}, \quad (4) : e^{-\tilde{v}}, \quad (5) : \frac{q}{y_1 y_2 y_3^2}, \qquad (11) : e^{-2u-v-\tilde{v}}\frac{y_1^2 y_2^2 y_3^4}{q}, \quad (12) : y_1 y_2$$

$$(6) : \frac{y_2}{y_1}, \quad (7) : \frac{w^2 e^{-v}}{y_2 y_3}, \quad (8) : e^{-u}\frac{y_1 y_2^2 y_3^3}{w\tilde{w}\sqrt{q}}, \quad (9) : \frac{\tilde{w}^2 e^{-\tilde{v}}}{y_2 y_3}, \quad (10) : \frac{y_2}{y_1}$$

- Tests elliptic genus of our formalism by topological vertex

## Conclusion and remarks

- We proposed ADHM-like formalism of $SO(7)$ w/ matters in **8**, $G_2$ w/ **7**
- We tested them by their Witten index/elliptic genus
- We applied our method to 6d quiver SCFTs

- Not yet generalized to other exceptional instantons
- Test 5d dualities on instanton spectrum