The simplest 3d N=4 SCFT

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Based on Arxiv 1806.07714 with Masahito Yamazaki (Kavli-IPMU)

Introduction : Minimal (S)CFTs

Def : Theory with Smallest Stress-energy tensor central charge C_T

$$\langle T(x)T(0)\rangle = \frac{C_T}{|x|^{2d}}(tensor)$$

Examples : 2d Ising model, 3d N=0,1,2 Ising model
4d N=2 Argyres-Douglas theory,
6d N=(2,0) theory of su(2) type

Why?

- **Starting point of classification**
- **Minimal ~ More Stable in RG~ More Universal**
- **Barometer testing level of our understanding**

OUTLINE OF TALK

- SUSY enhancement of $\mathbf{U(1)_{-3/2} + \Phi}$: N=2 → N=4
  - Three evidences
  - Properties on IR 3d N=4 SCFT (e.g, No Vacuum moduli)
  - Speculation on minimality of the IR SCFT
- 3d N=2 Minimal model with U(1) flavor symmetry
- Some other examples of enhancement, N=3 → N=5

$U(1)_{-3/2} + \Phi$: Symmetry and operator Spectrum

The UV Lagrangian

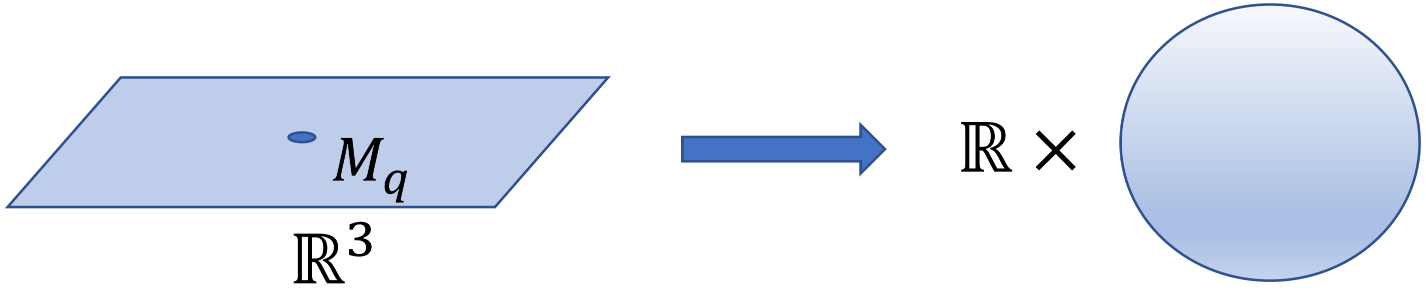
$$\frac{k}{4\pi}\mathrm{AdA} + (D_A\phi)^*(D_A\phi) + i\bar{\psi}\gamma^\mu(D_A)_\mu\psi - \frac{4\pi^2}{k^2}(\phi^*\phi)^3 - \frac{6\pi}{k}(\phi^*\phi)\bar{\psi}\psi \qquad \left(k = -\frac{3}{2}\right)$$

A : U(1) gauge field, ϕ : Complex scalar, ψ : Dirac fermion

Manifest symmetries

Symmetry	Acting on	Noether Current	N=2 multiplet
N=2 SUSY	$\phi \longleftrightarrow \psi$	$\phi^*\partial\psi$	Stress-energy tensor $Q^{n=0,1,2}\cdot(\bar{\psi}\psi)$
U(1) R-symmetry	ϕ : +1/2, ψ : -1/2	$J_R = \phi^*\partial\psi - \bar{\psi}\gamma\psi$	
U(1) Top'l symmetry	M_q : ½ BPS Monopole operator	$J_{\text{top}} = \epsilon^{\mu\nu\rho}(\partial_\nu A_\rho - \partial_\nu A_\rho)$	Conserved current $Q^{n=0,1,2}\cdot(\phi^*\phi)$

Radial quantization :



$|M_q\rangle \Leftrightarrow$ Semi-classical ½ BPS configuration with $\int_{S^2} F = 2\pi q$

$U(1)_{-3/2} + \Phi$: Symmetry and operator Spectrum

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Local operator spectrum (Semi-classical analysis)

Creation operators	U(1) gauge	U(1) R	U(1) Top'l
$a_j^+(\phi)$: $j \in \frac{\lvert q \rvert}{2} + \mathbb{Z}_{\geq 0}$	+1	1/3	0
$a_j^+(\phi^*)$: $j \in \frac{\lvert q \rvert}{2} + \mathbb{Z}_{\geq 0}$	-1	-1/3	0
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Determined By F-maximization

$U(1)_{-3/2} + \Phi$: Symmetry and operator Spectrum

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*. Bare monopole $\lvert M_q \rangle$ has

Gauge charge : $-\frac{\lvert q \rvert}{2} - \frac{3q}{2}$

U(1) R-charge : $\frac{\lvert q \rvert}{3}$

U(1) Top'l-charge : q

|q| fermionic zero modes

$\rightarrow \delta Q = -\frac{\lvert q \rvert}{2} Q(a_j^\dagger)$

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*. No gauge-invariant chiral primary

$[a_j^+(\phi)]^{2q} \, |M_q\rangle$: Dyonic operator having non-zero spin, ¼ BPS

$U(1)_{-3/2} + \Phi$: Symmetry enhancement

Conjecture :

UV theory : $U(1)_{-3/2} + \Phi$	→	IR fixed point : a 3d N=4 SCFT T_{IR}
N=2 Supersymmetry		N=4 Superconformal
$U(1)_R \times U(1)_{top}$.		$SO(4)_R$.

Evidences :

1) $\langle J_R^{\mu}(x) J_R^{\delta}(0)\rangle = \langle J_{top}^{\mu}(x) J_{top}^{\delta}(0)\rangle = \frac{C_J}{|x|^4}\left(\delta^{\mu\delta} - 2\frac{x^\mu x^\delta}{|x|^2}\right),\qquad C_J = \frac{2}{25}\left(8 - \frac{5\sqrt{5+2\sqrt{5}}}{\pi}\right)$

$\mathbb{Z}_2 \in weyl(SO(4)) : U(1)_R \Leftrightarrow U(1)_{top}$

$C_J(U(1)_R) = -\frac{2}{\pi^2}\partial_b\partial_b F(b,\zeta=0)\Big|_{b=1}$, $C_J(U(1)_{top}) = 8\partial_\zeta\partial_\zeta F(b=1,\zeta)\Big|_{\zeta=0}$

b : Squashing paramter, ζ : FI parameter

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2) Two extra SUSY multiplets

Superconformal index $I(x) := Tr(-1)^R x^{\frac{R}{2}+j_3} = 1 - x - 2x^{\frac{3}{2}} + \cdots$ ...

$*.\ \frac{1}{4}\, BPS\ operators\ with\ \Delta = R + j_3$

$*.\ -x^{\frac{3}{2}}$ term comes only from either

(a descendent of extra susy multiplet) or ~~(Chiral primary of R=3)~~

$[R = 1, \Delta = 2, j_3 = 1] \in Q^{n=0,1,2} \cdot \left[R = 0, \Delta = \frac{3}{2}, j = \frac{1}{2}\right]$

Semiclassical analysis

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2) Two extra SUSY multiplets

3) An IR Duality

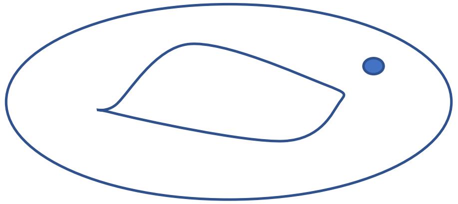
$(U(1)_{-3/2} + \Phi) \otimes (U(1)_{-3/2} + \Phi) \cong$ **Gauging diagonal SU(2) of T[SU(2)] with CS level +3**

T[SU(2)] : U(1) coupled to two hypers of charge +1
SU(2)xU(1) → SU(2)x SU(2)

(The non-Lagrangian gauging preserves N=4 SUSY)

Duality from 3d/3d correspondence

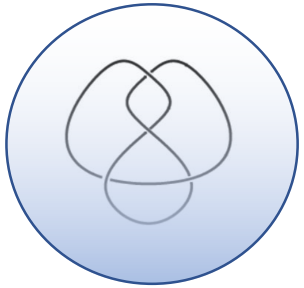
4d/2d correspondence :



Punctured Riemann surface Σ

$\rightarrow$ $4d\ N = 2\ \ T[\Sigma]$

3d/3d correspondence :

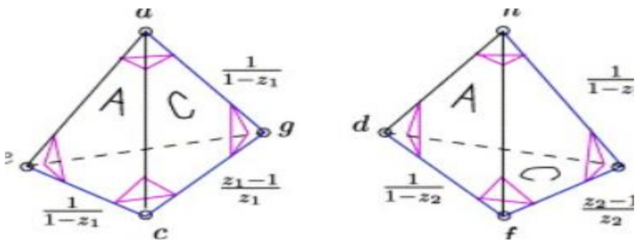


3-manifold N=S^3-(figure-eight)

$\rightarrow$ $3d\ N = 2\ \ T[\mathrm{N,A}]$

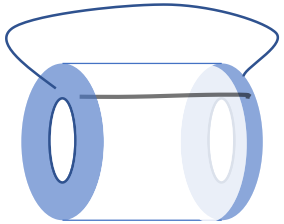
$(\ A \in H_1(\partial N, \mathbb{Z}))$

S^3-(figure-eight)



$\rightarrow$ **$(U\,(1)_{-3/2} + \Phi) \otimes (U\,(1)_{-3/2} + \Phi)$**

$\cong$



Gluing with
a twist $ST^3 \in PSL(2,Z)$

$\rightarrow$ **Gauging diagonal SU(2) of T[SU(2)]**
with CS level +3

Basic properties of IR fixed N=4 SCFT

***. No chiral primary operator, vacua**

Cf) 4d, 5d SCFT with 8 supercharges

***. No UV Lagrangian with manifest N=4**

Cf) 4d case [Maruyoshi-Song :'16]

***. Simpler than a free hyper theory**

$$C_T = 4C_J = \frac{8}{25}\left(8 - \frac{5\sqrt{5+2\sqrt{5}}}{\pi}\right) = 0.9925.. < C_T\ (a\ free\ hyper) = 2$$

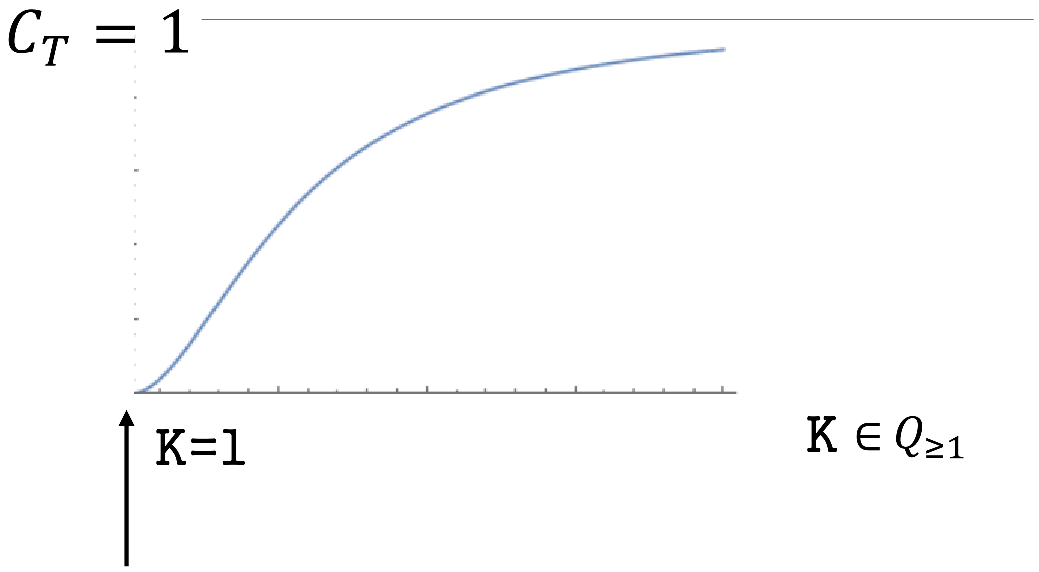
$$F = 1/2\ \mathrm{Log}(1/2(5+\sqrt{5})) = 0.6429 < F(a\ free\ hyper) = \log 2 = 0.693147$$

3d N=2 Minimal model with U(1) flavor symmetry

*. Simplest 3d N=2 with U(1) flavor symmetry (?)

$(U(1)_{-1/2-K} + \Phi) := (U(1)_{pq-1/2\, q^2} + \Phi(\text{charge } +q))$, $K := p/q$

S-duality : $(U(1)_{-1/2-K} + \Phi) \sim (U(1)_{-1/2-1/K} + \Phi)$



a 3d N=4 SCFT T_{IR} = Simplest 3d N=2 with U(1) flavor symmetry (?)

Some other examples of N -> N+2 enhancement

ABJM k=1,2 : N=6 → N=8

U(1) + a hyper with CS level +1 : N=3 → N=5

U(1) + a chiral with CS level -3/2 : N=2 → N=4

U(1) + a complex Scalar with CS level +3 : N=0 → N=2 (?)

Summary OF TALK

- SUSY enhancement of $\mathbf{U(1)_{-3/2} + \Phi}$: N=2 → N=4
    - Three evidences
    - Properties on IR 3d N=4 SCFT (e.g, No Vacuum moduli)
    - Speculation on minimality of the IR SCFT
- 3d N=2 Minimal model with U(1) flavor symmetry
- Some other examples of enhancement, N=3 → N=5