2d (0,2) Gauge Theories: Brane Brick and Beyond

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APCTP workshop
"Strings, Branes and Gauge Theories"
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2d (0,2) gauge theories

Smallest
- space-time dimension (for a QFT)
- number of supercharges (with holomorphy)

There seem to be too much freedom.

Are there good ways to organize family of theories?

2d (0,2) theories

Non-linear sigma model 1980's

Gauged linear sigma model [Witten 1993]

…

Heterotic model building Many papers

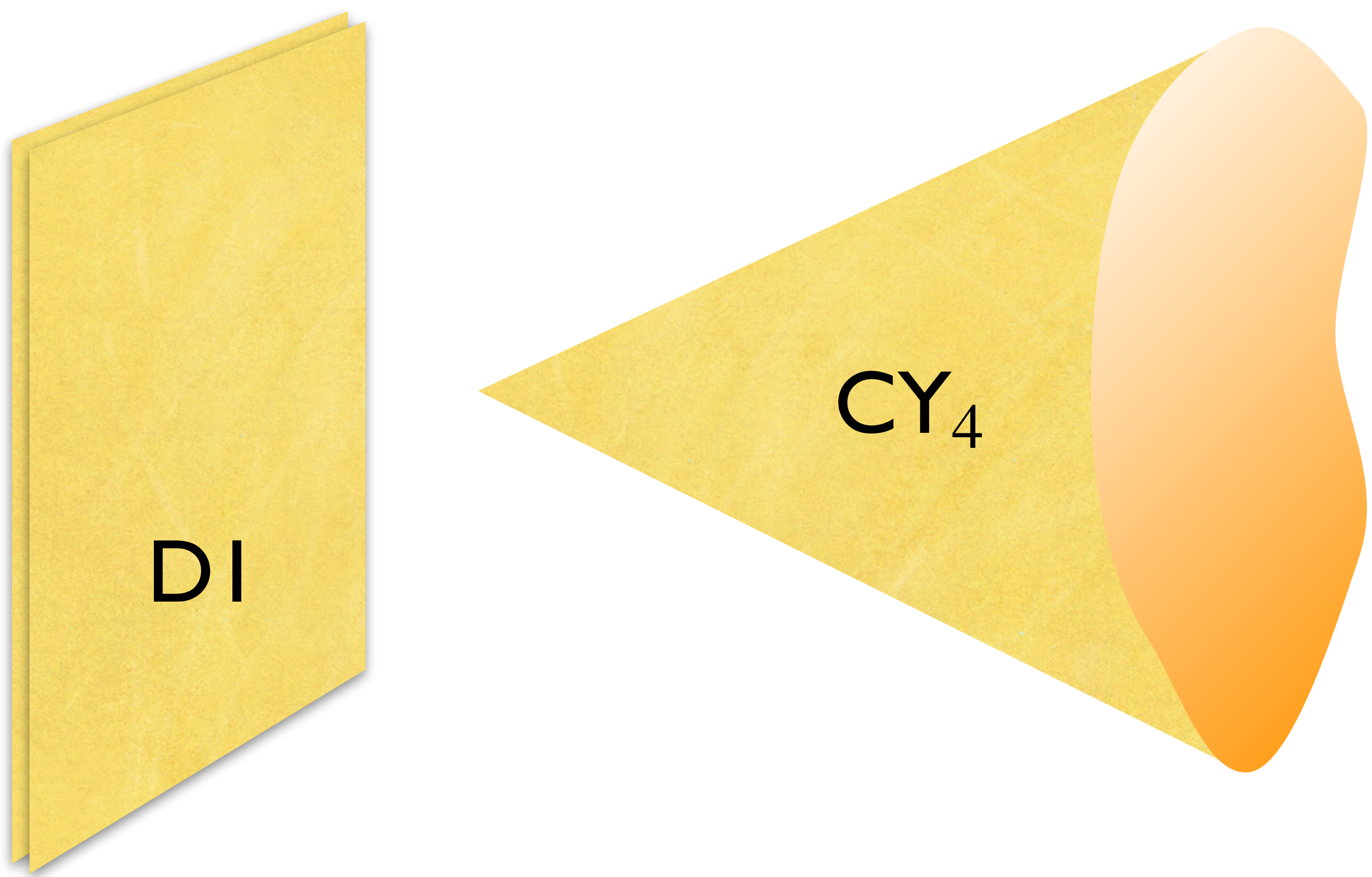
…

Non-abelian / Triality / $T[M_4]$ [Gadde-Gukov-Putrov 2013]

cf) 2d (2,2) theories : [Hori-Vafa 2000], [Hori-Tong 2006]

Brane Brick Models:

2d $\mathcal{N} = (0,2)$ supersymmetric gauge theories
on the world-volume a stack of D1-branes
probing toric Calabi-Yau 4-fold conical singularities.



2d cousin of 4d “brane tiling”

This talk is based on …

1506.03818 - Family of (0,2) theories	SF, DG, SL, RKS, DY
1510.01744 - Brane brick	SF, SL, RKS
1602.01834 - Triality	SF, SL, RKS
1609.01723 - Mirror perspective	SF, SL, RKS
1609.07144 - Orbifold reduction	SF, SL, RKS, CV
1702.02948 - Elliptic genus	SF, DG, SL, RKS

Collaborators: Sebastian Franco, Rak-Kyeong Seong,
Dongwook Ghim, Daisuke Yokoyama, Cumrun Vafa

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On-going collaboration	SF, SG,SL, RKS

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(0,2) multiplets

Multiplet	s-field	off-shell	on-shell
Chiral	Φ	ϕ, ψ_+	ϕ, ψ_+
Fermi	Λ	λ_-, G	λ_-
Gauge	V	A_μ, ρ_-	ρ_-

"(0,2) super-potential"

SUSY : $\sum_a \mathrm{tr}\,[E_a(\Phi) J^a(\Phi)] = 0$

$$\Omega = \sum_a \mathrm{tr}\,[\bar{\Lambda}^a E_a(\Phi) + \Lambda_a J^a(\Phi)]$$

$$\Longrightarrow \quad \mathcal{L}_F = -\sum_a \mathrm{tr}\left(|E_a(\phi)|^2 + |J^a(\phi)|^2\right)$$

Periodic Quiver

The $\mathbb{C}^4$ theory in $(0,2)$ notation

$U(N)$ $(8,8)$ Super-Yang-Mills (dim. red. from 10d)

Decomposition under $SO(8) \supset SU(4) \times U(1)$

$$\begin{aligned} \text{Scalar}: &\quad 8_v \to 4_{+1/2} \oplus \bar{4}_{-1/2} \\ \text{R-Fermi}: &\quad 8_s \to 4_{-1/2} \oplus \bar{4}_{+1/2} \\ \text{L-Fermi}: &\quad 8_c \to 6_0 \oplus 1_{+1} \oplus 1_{-1} \end{aligned}$$

Chiral

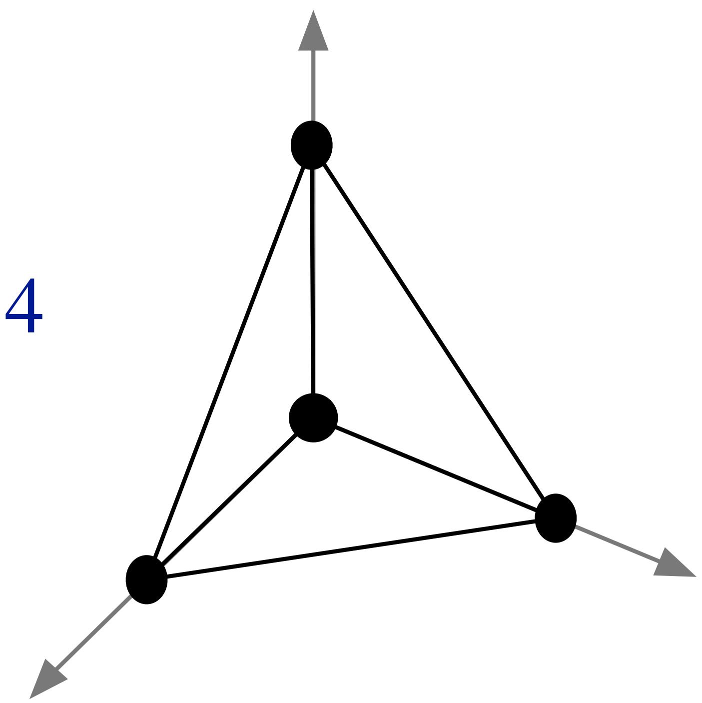
Fermi Gauge

4 chiral + 3 Fermi + 1 gauge … interacting via

$$\begin{array}{lcc} & J & E \\ \Lambda^x: & Y\cdot Z - Z\cdot Y & D\cdot X - X\cdot D \\ \Lambda^y: & Z\cdot X - X\cdot Z & D\cdot Y - Y\cdot D \\ \Lambda^z: & X\cdot Y - Y\cdot X & D\cdot Z - Z\cdot D \end{array}$$

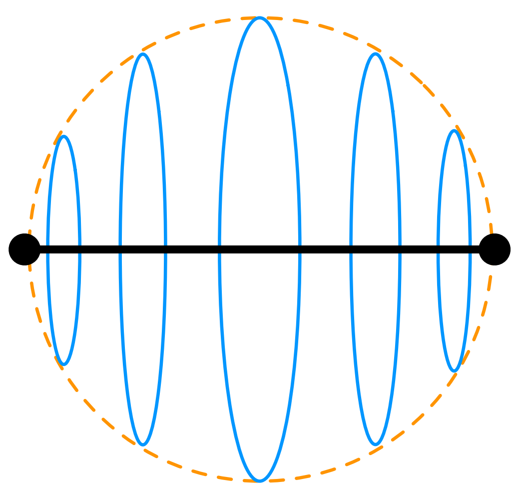
Toric diagram

$\mathbb{C}^4$

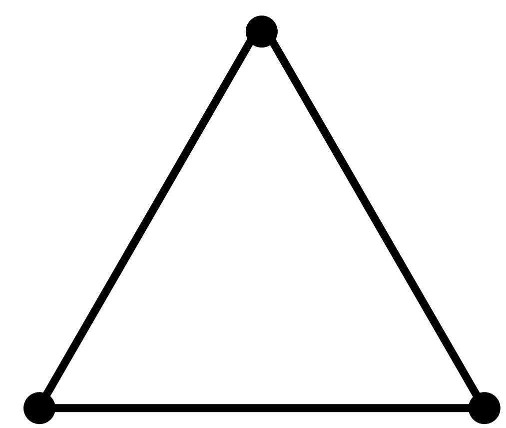


→ Quiver gauge theory ?

$\mathbb{CP}^1 \subset \mathbb{C}^2$

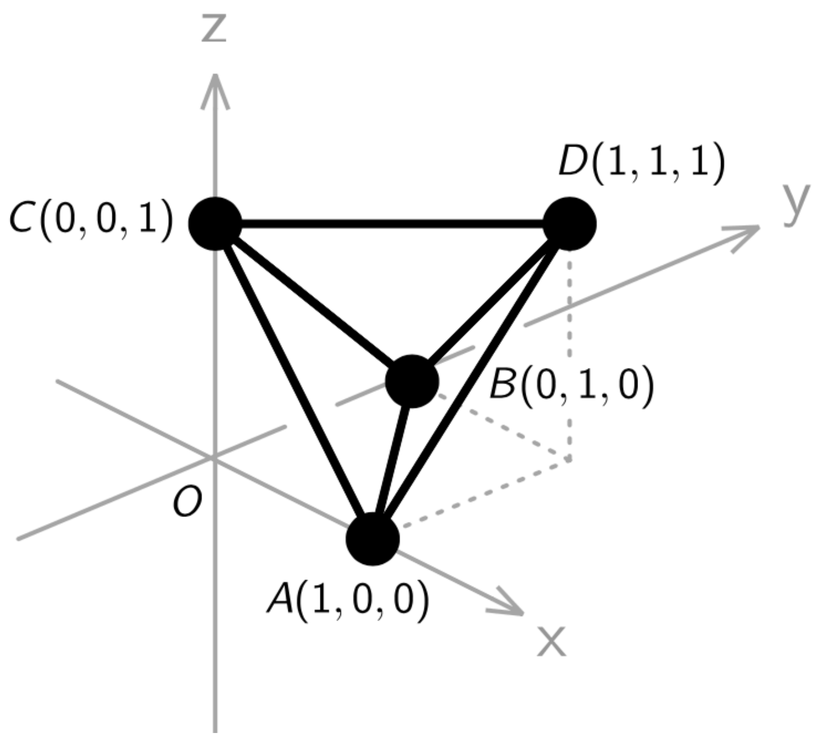


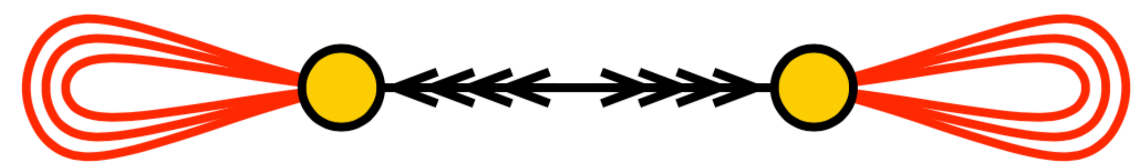
$\mathbb{CP}^2 \subset \mathbb{C}^3$

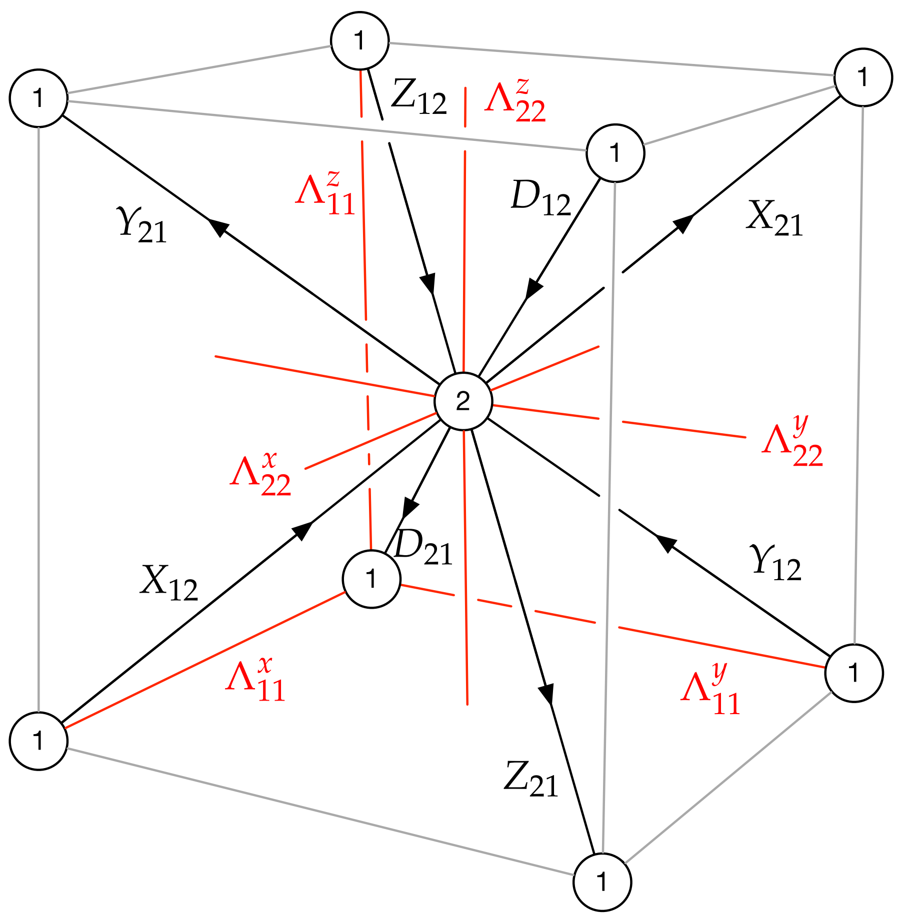


Periodic quiver : geometrization of $U(1)^3 \subset SU(4)$

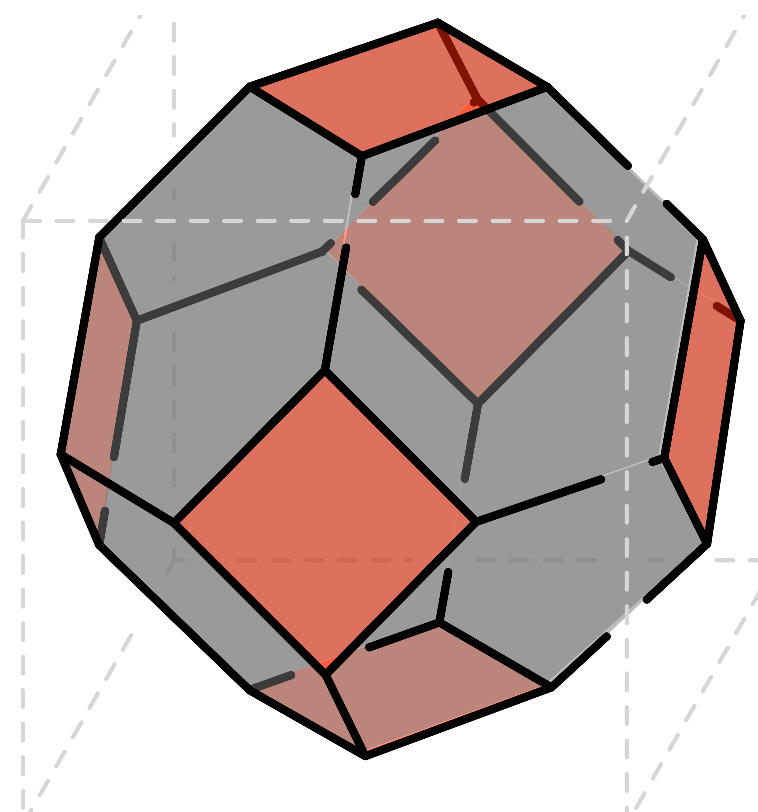
$\mathbb{C}^4/\mathbb{Z}_2$







graph dual



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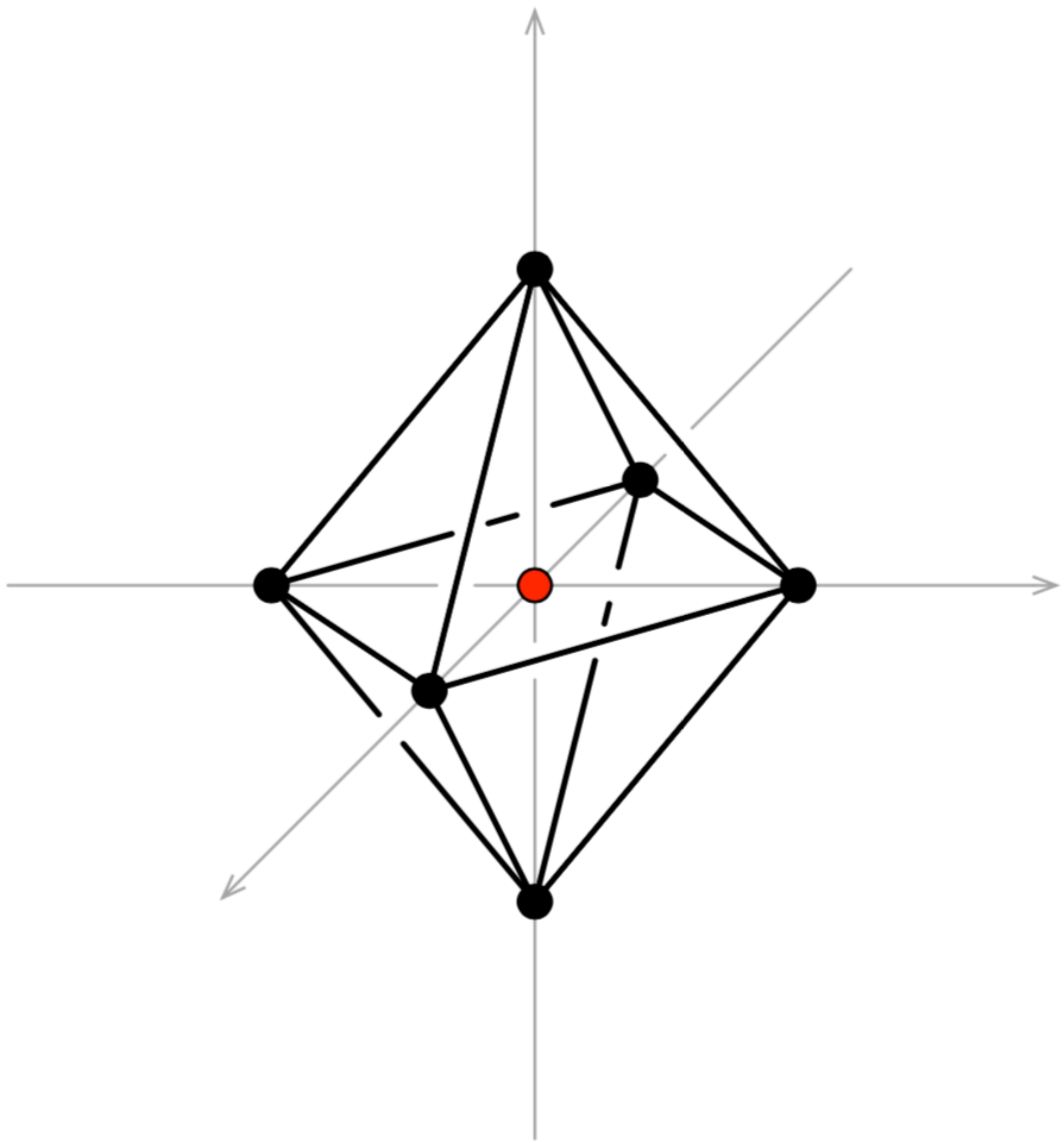
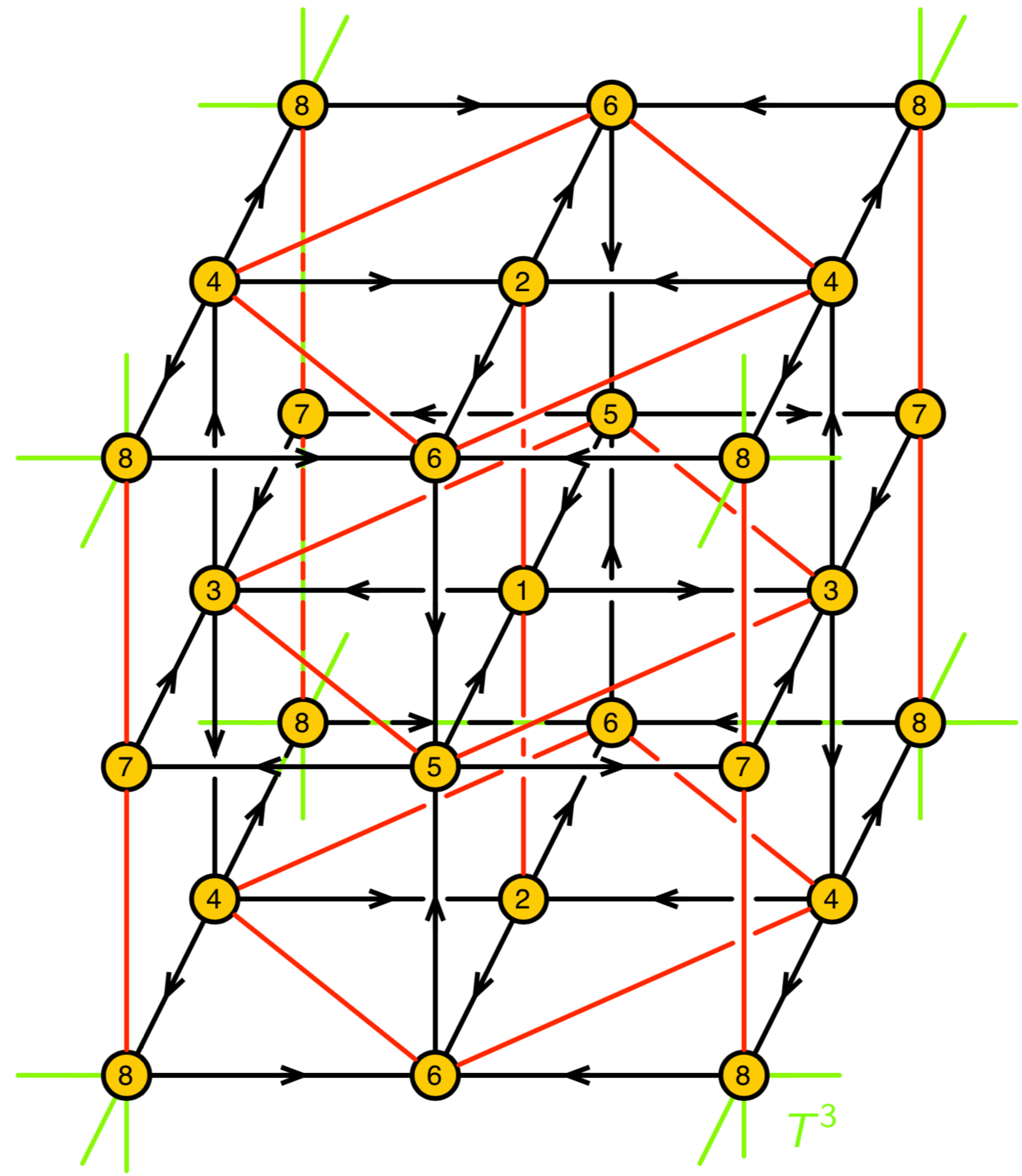
$Q^{1,1,1}/\mathbb{Z}_2$



8 gauge nodes

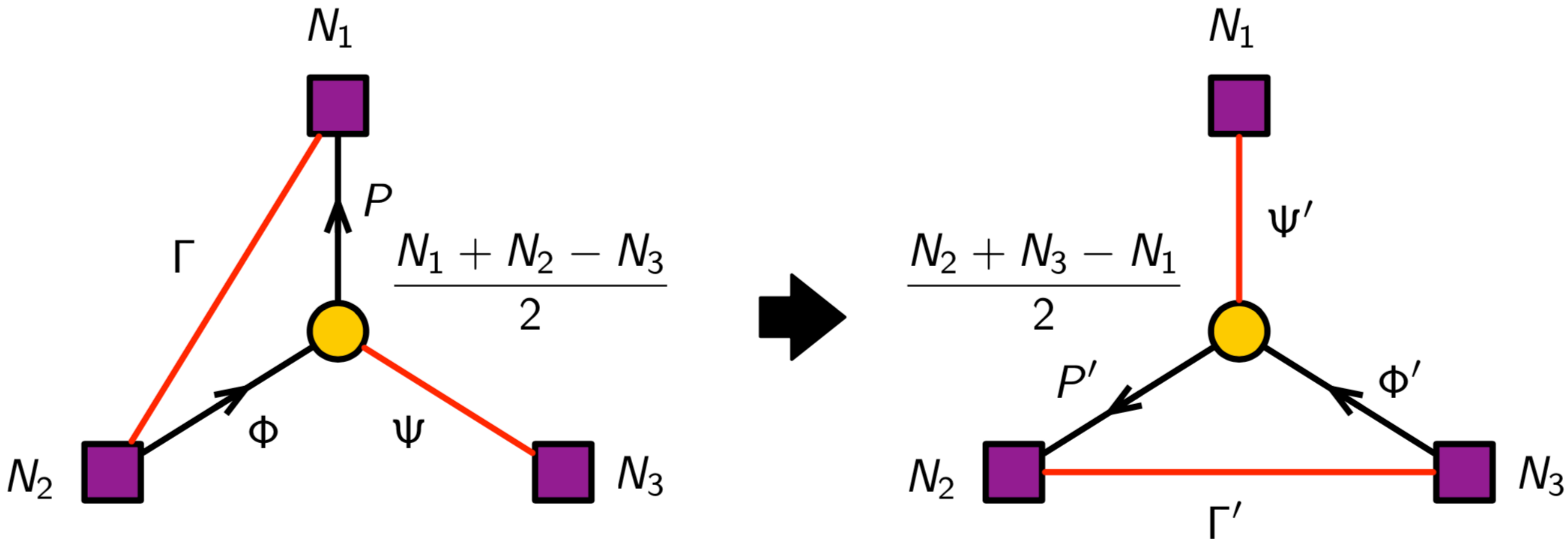


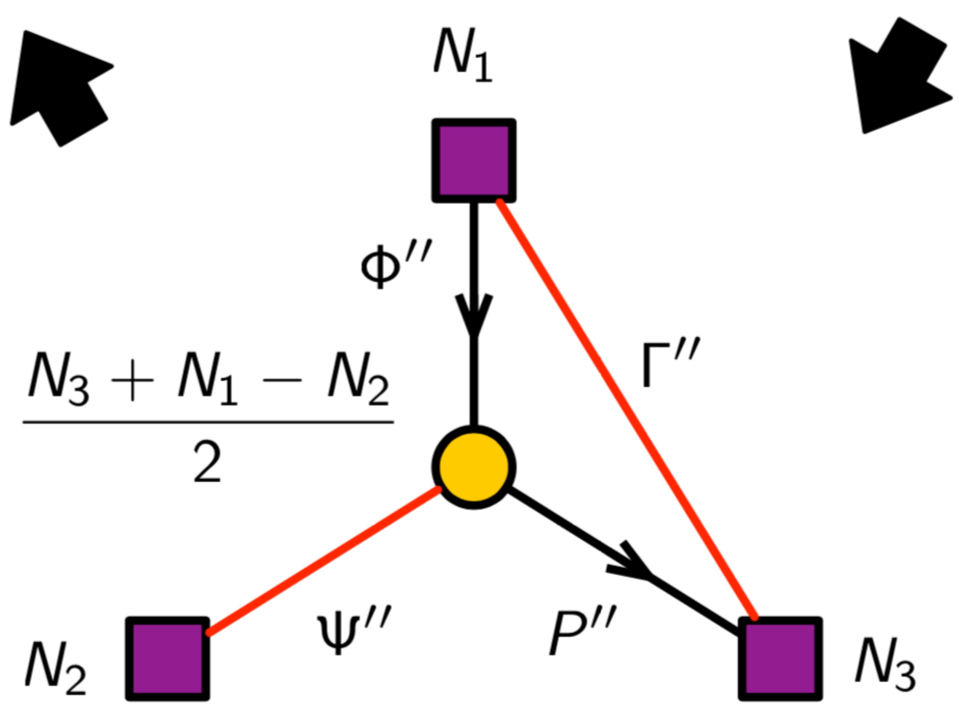
Triality

Basic triality

[Gadde,Gukov,Putrov 2013]

All three flow to the same IR fixed point!



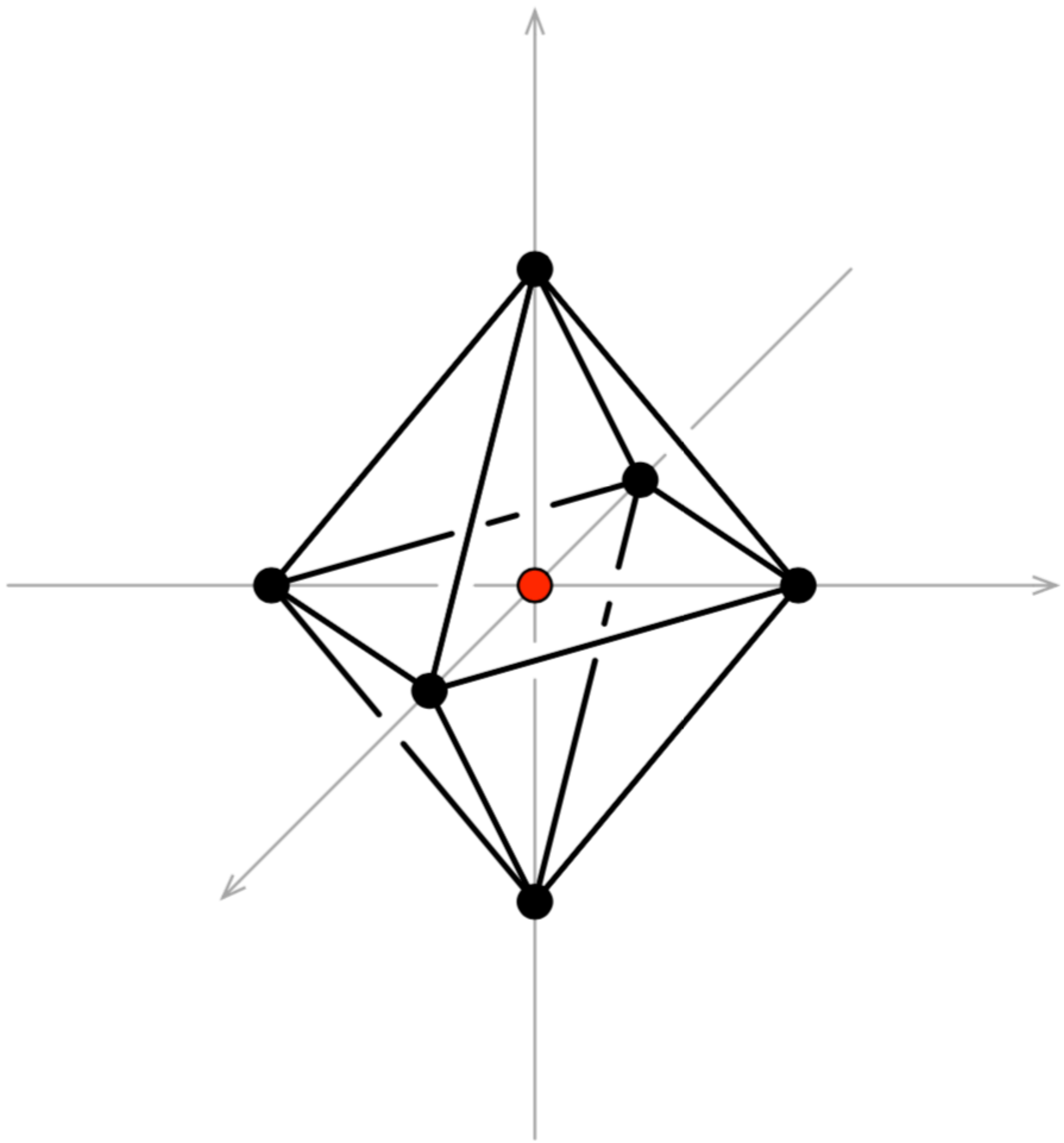


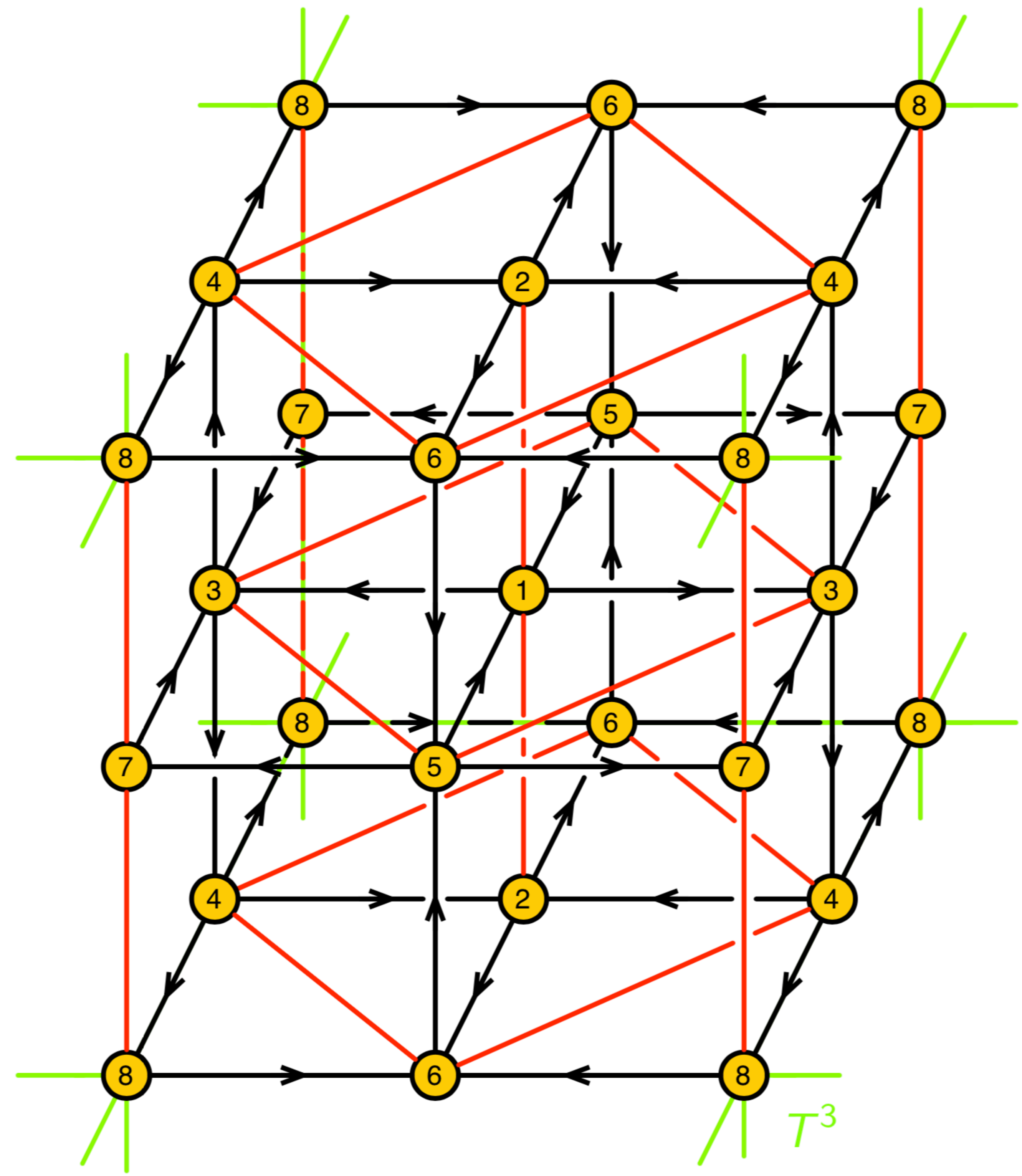
A simple choice:

$$N_1 = N_2 = N_3 = 2N$$

flavor gauge

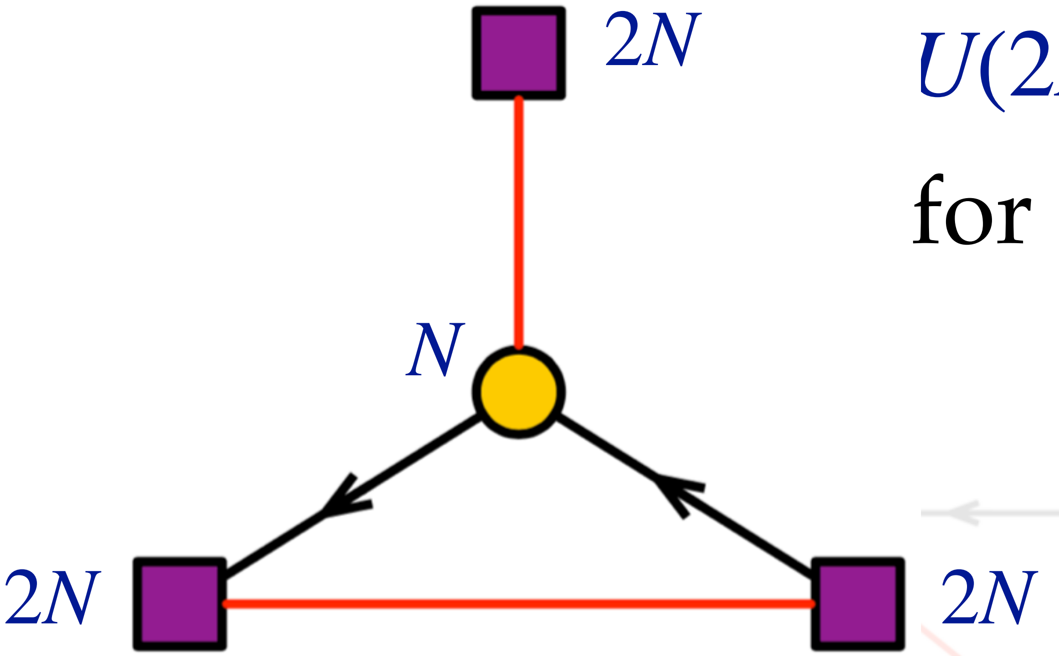
$Q^{1,1,1}/\mathbb{Z}_2$

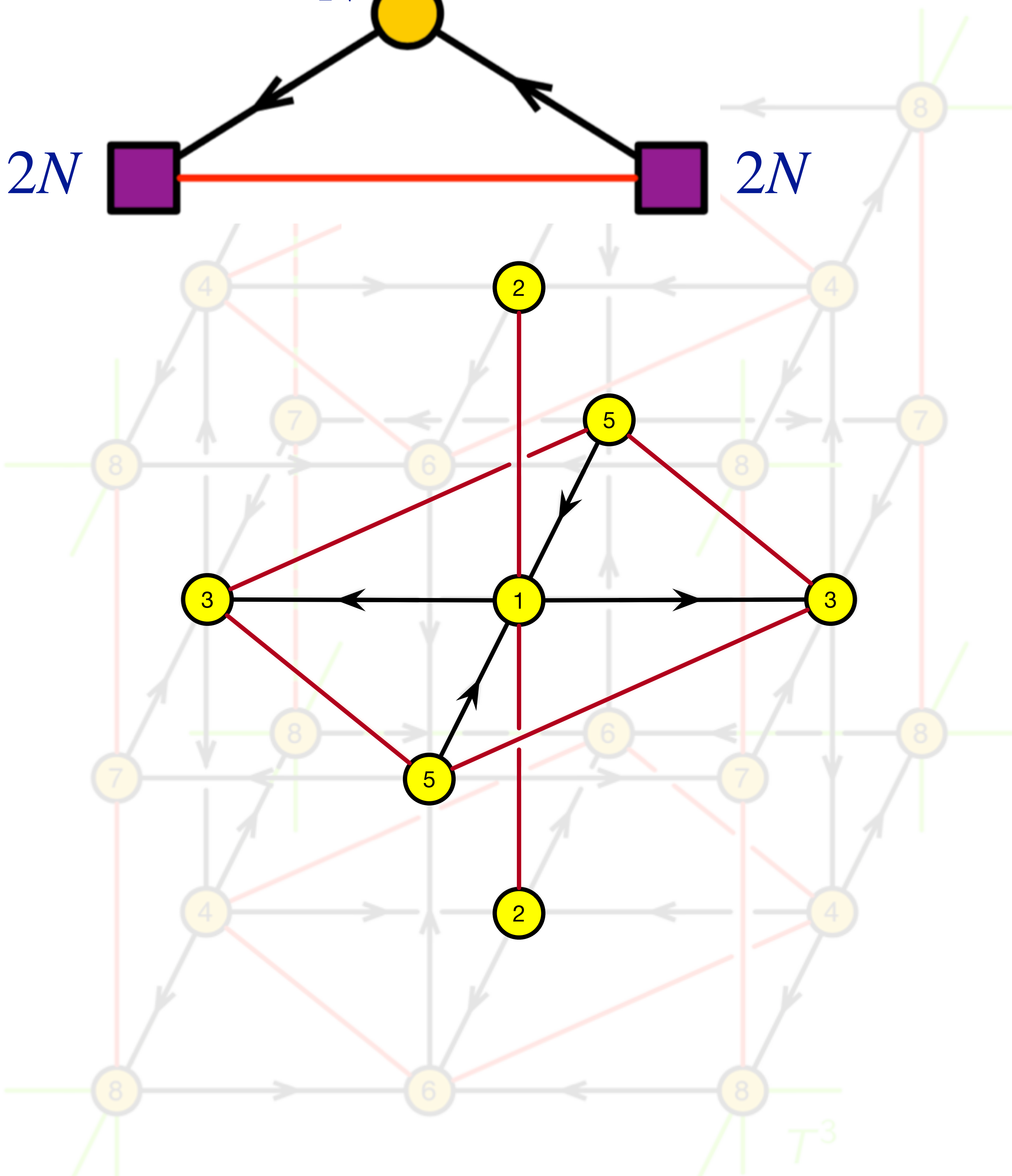


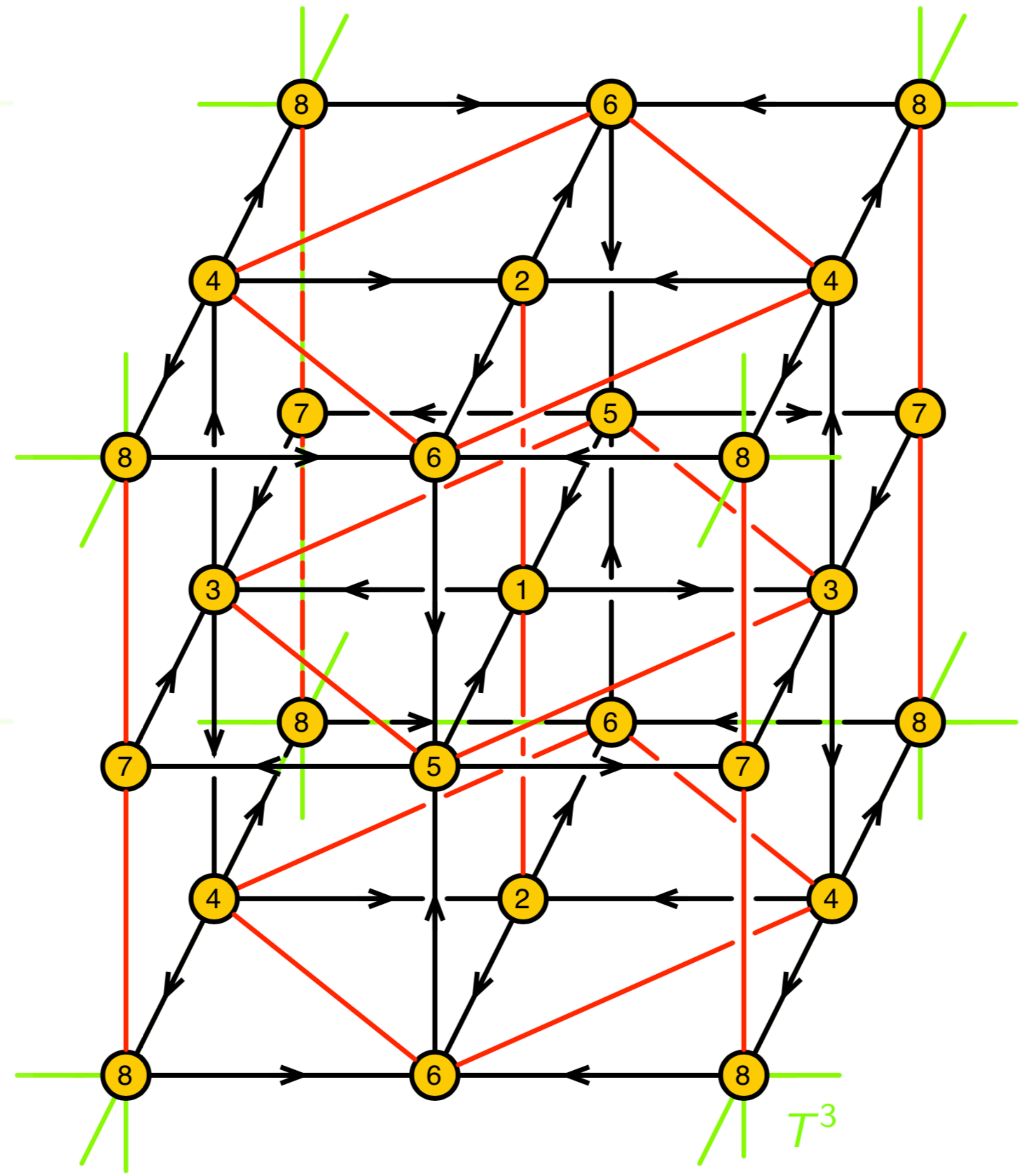


$U(2N) \to U(N) \times U(N)$

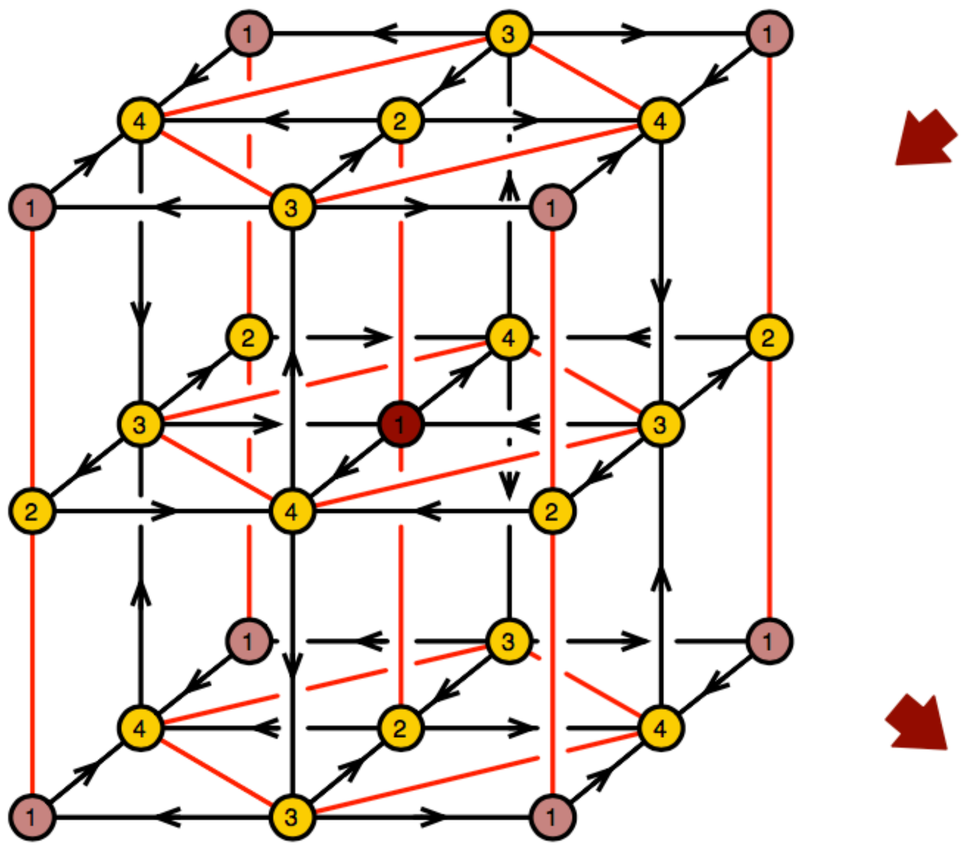
for each flavor node

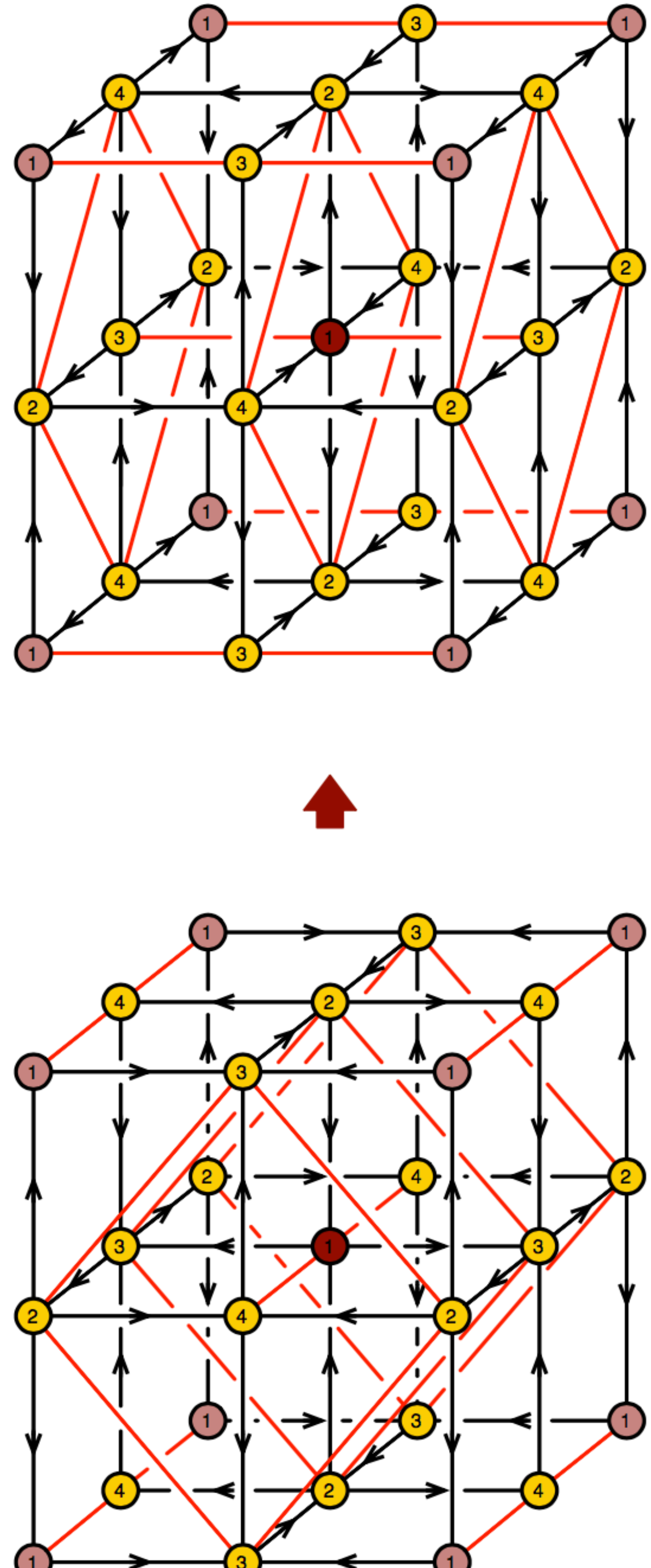


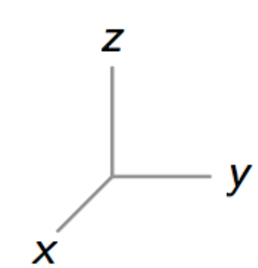




Triality loop





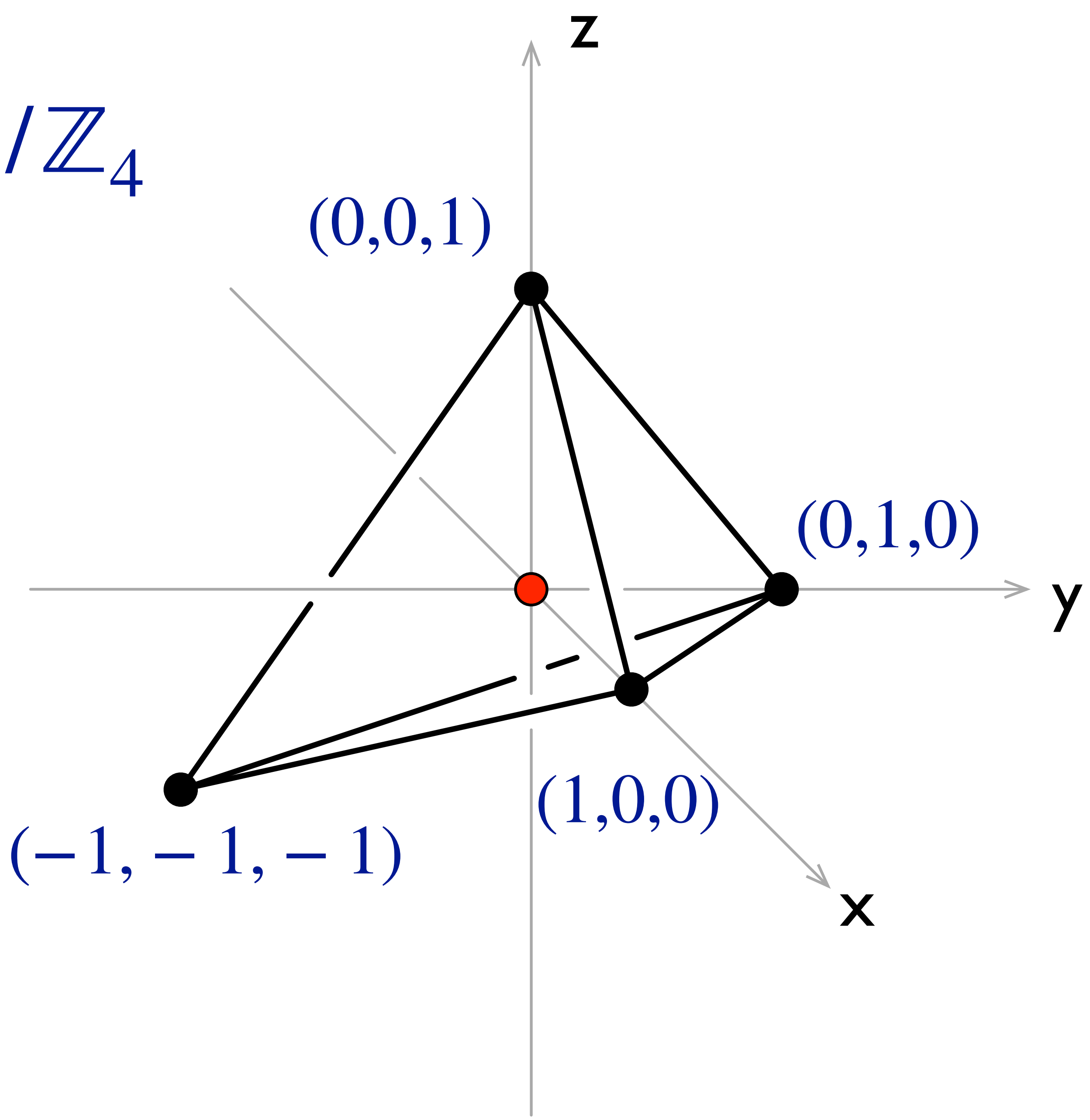


First example of triality without flavor nodes

Mirror
(T-dual)

Newton polynomial

$\mathbb{C}^4/\mathbb{Z}_4$



$$P(x,y,z) = x + y + z + \frac{1}{xyz}$$

$(x, y, z \in \mathbb{C}^*)$

In general, $P(x,y,z) = \sum_{\vec{v}\in V} c_{\vec{v}} x^{v_1} y^{v_2} z^{v_3}$

Brane brick in IIA string theory

Brane bricks are D4-branes wrapping the 3-torus spanned by

$$(\arg(x), \arg(y), \arg(z))$$

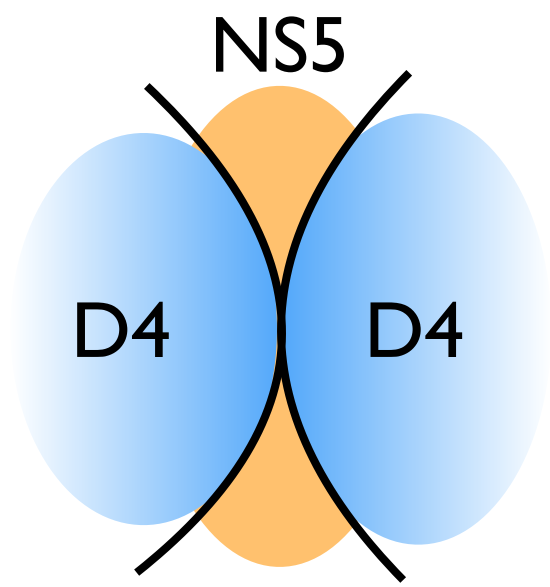
The D4-branes are cut into pieces by an NS5-brane wrapping

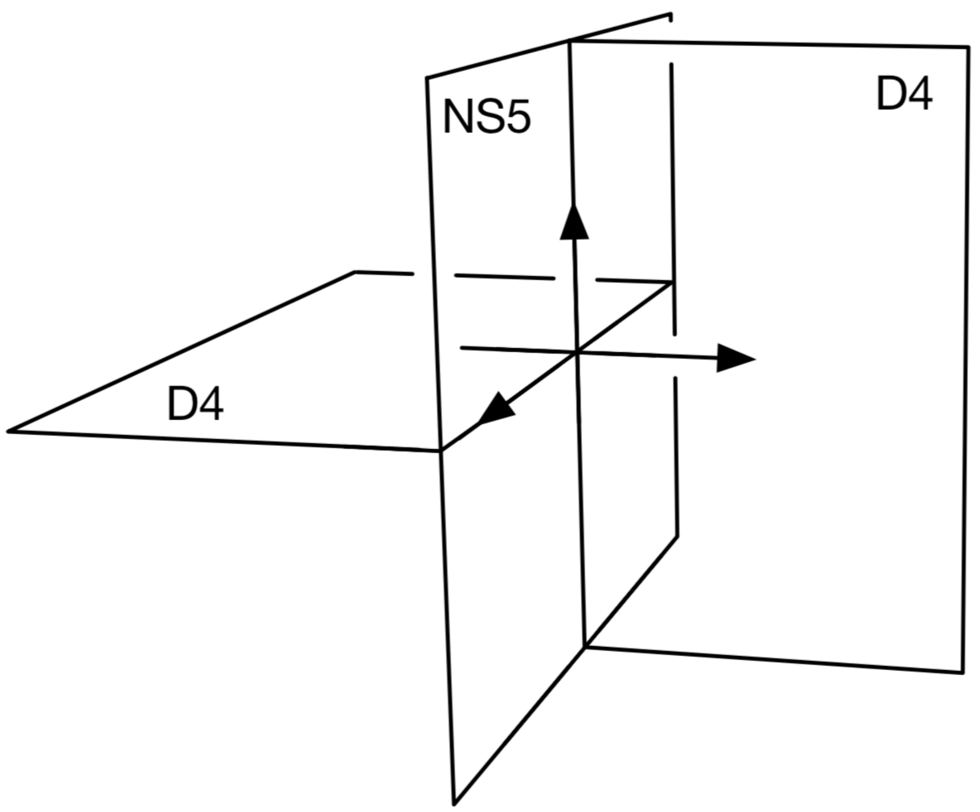
$$P(x, y, z) = 0 \quad \subset \quad (\mathbb{C}^*)^3$$

A heuristic T-duality argument shows

D1 probing CY4 $\longrightarrow$ D4 ending on NS5

A local picture

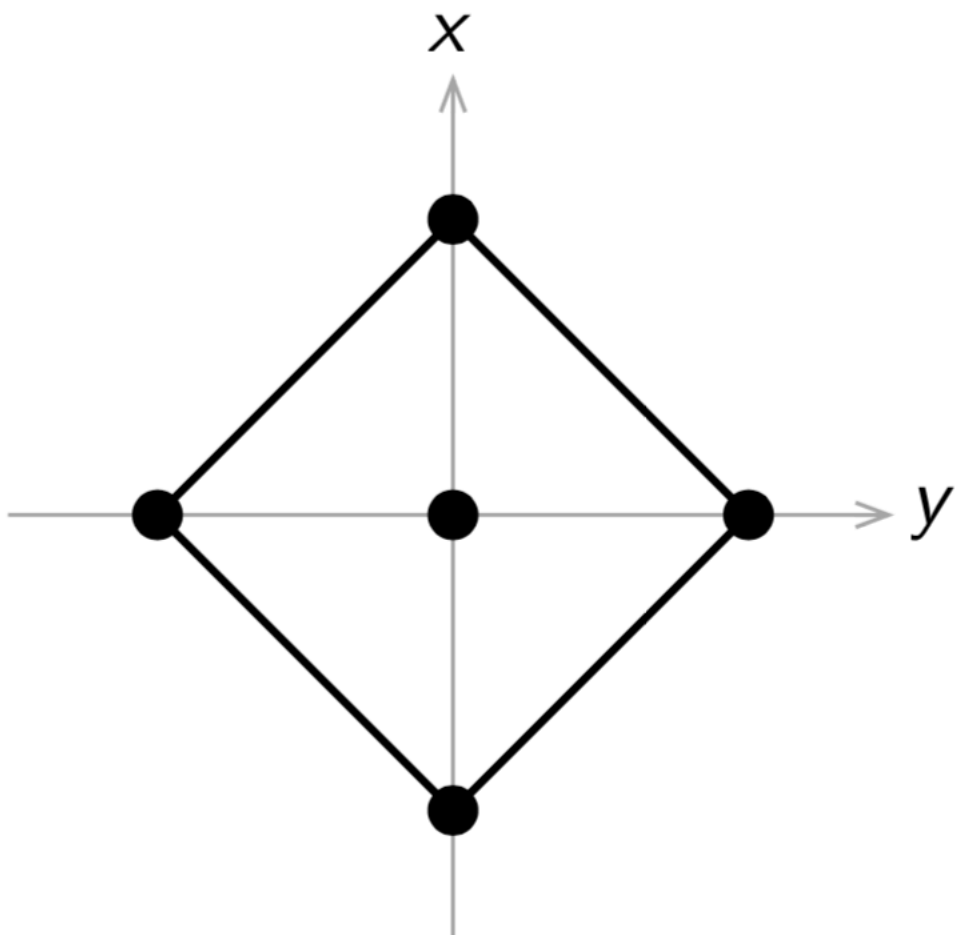


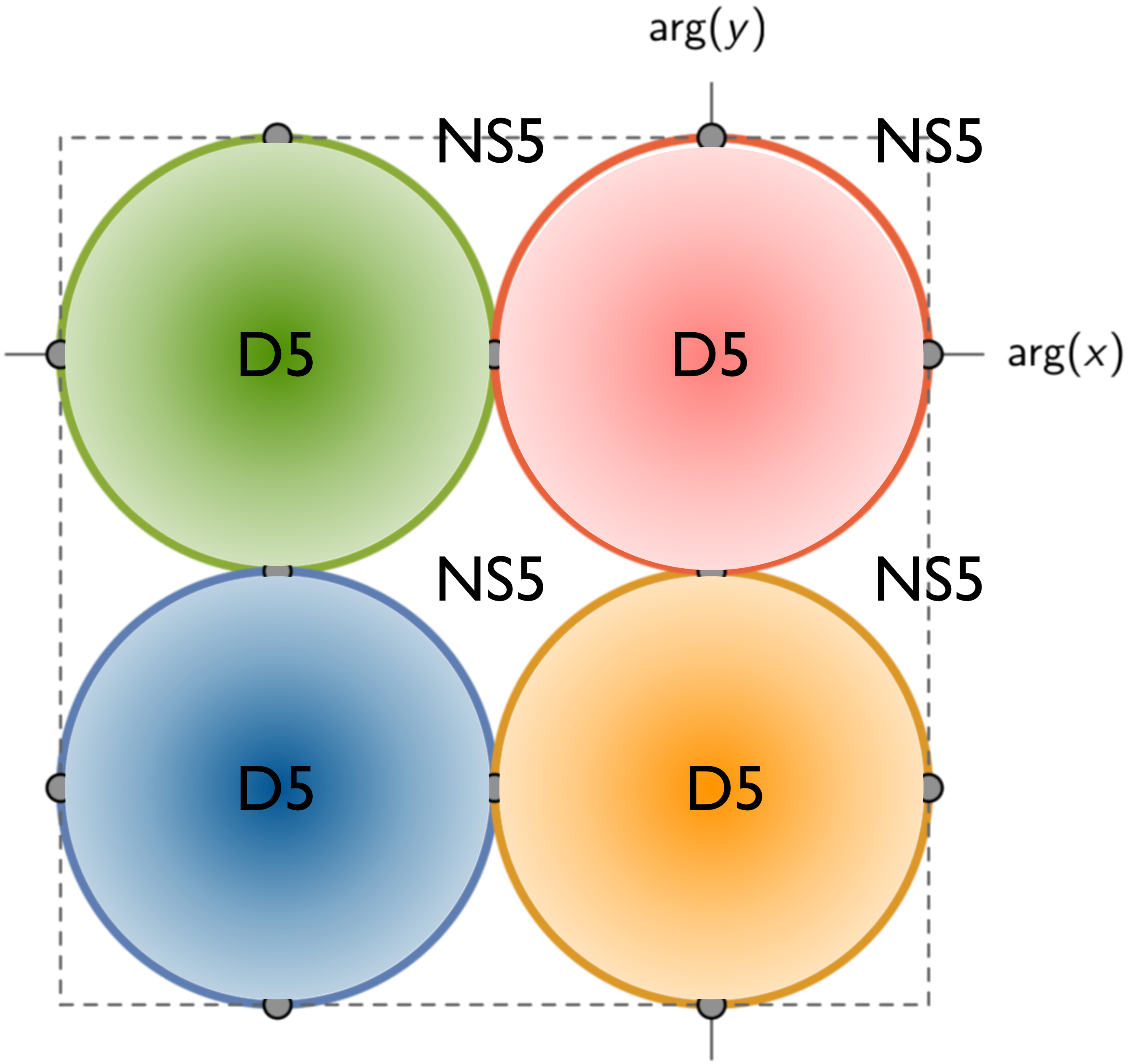


Gauge groups ~ D4 branes (3-ball) ending on NS5

Matter fields ~ intersection between two adjacent
D4-brane boundaries (2-sphere) in NS5

A brane-tiling example





$$P(x,y) = x + \frac{1}{x} + i\left(y + \frac{1}{y}\right)$$

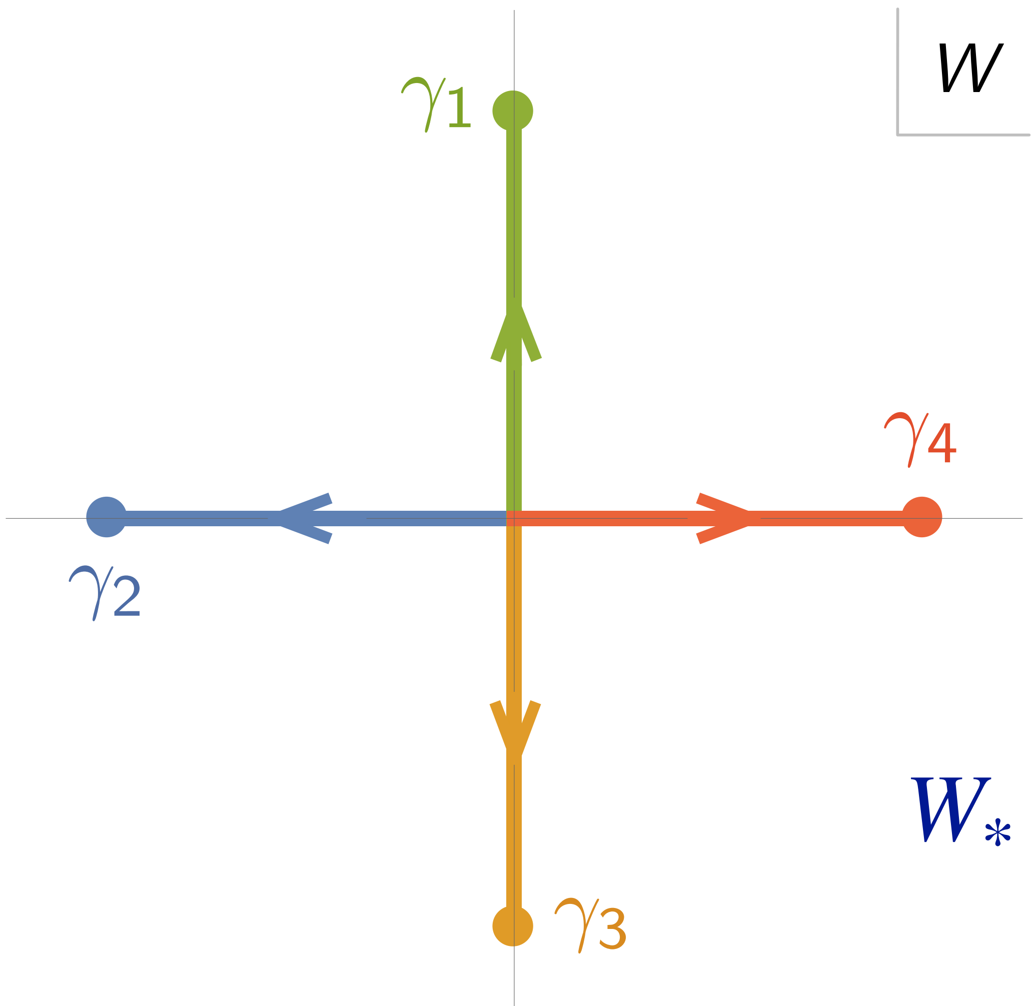
Picard-Lefschetz theory

The centers of the D4 brane bricks (gauge groups)

are located at the critical points: $dP = 0$

$$P(x,y,z) = x + y + z + \frac{1}{xyz}$$

$\mathbb{C}^4/\mathbb{Z}_4$



$$dP = 0$$

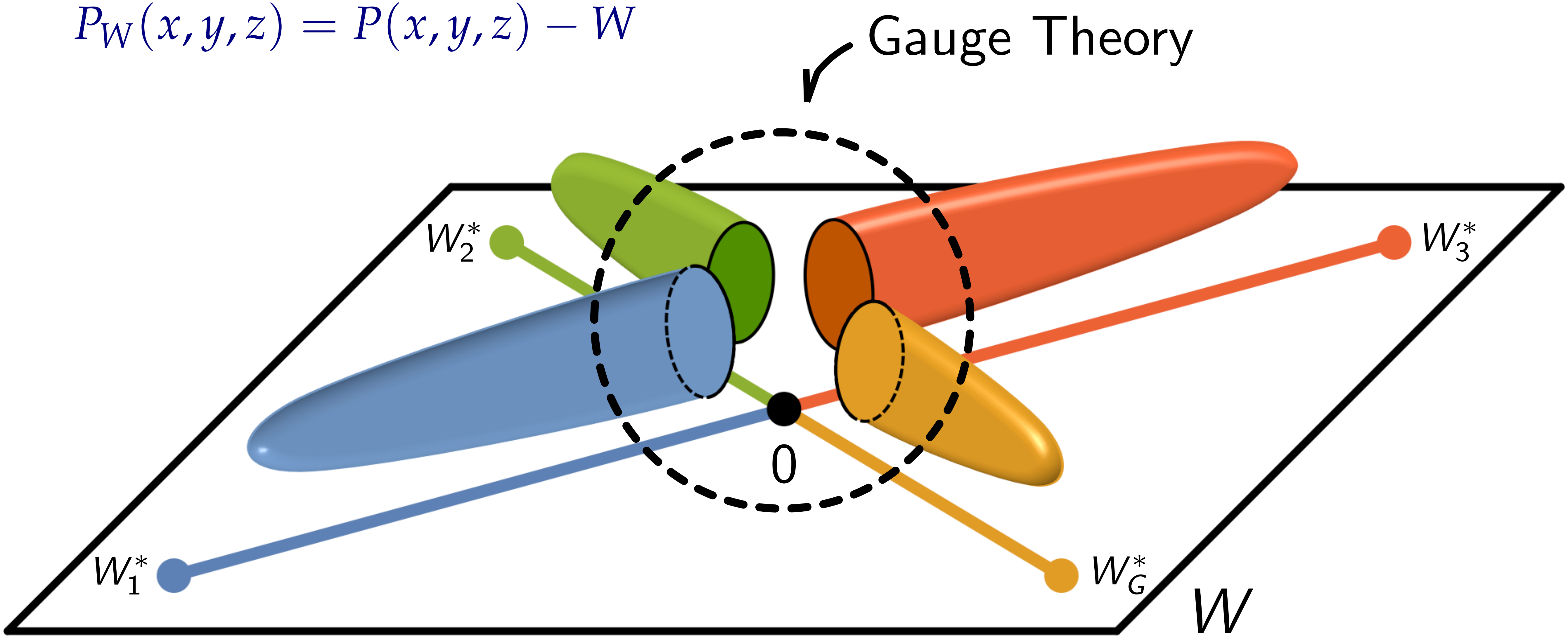
$$x_* = y_* = z_* = e^{k\pi i/2}$$

$$(k = 0, 1, 2, 3)$$

$$W_* = P(x_*, y_*, z_*)$$

Brane bricks in the W-plane

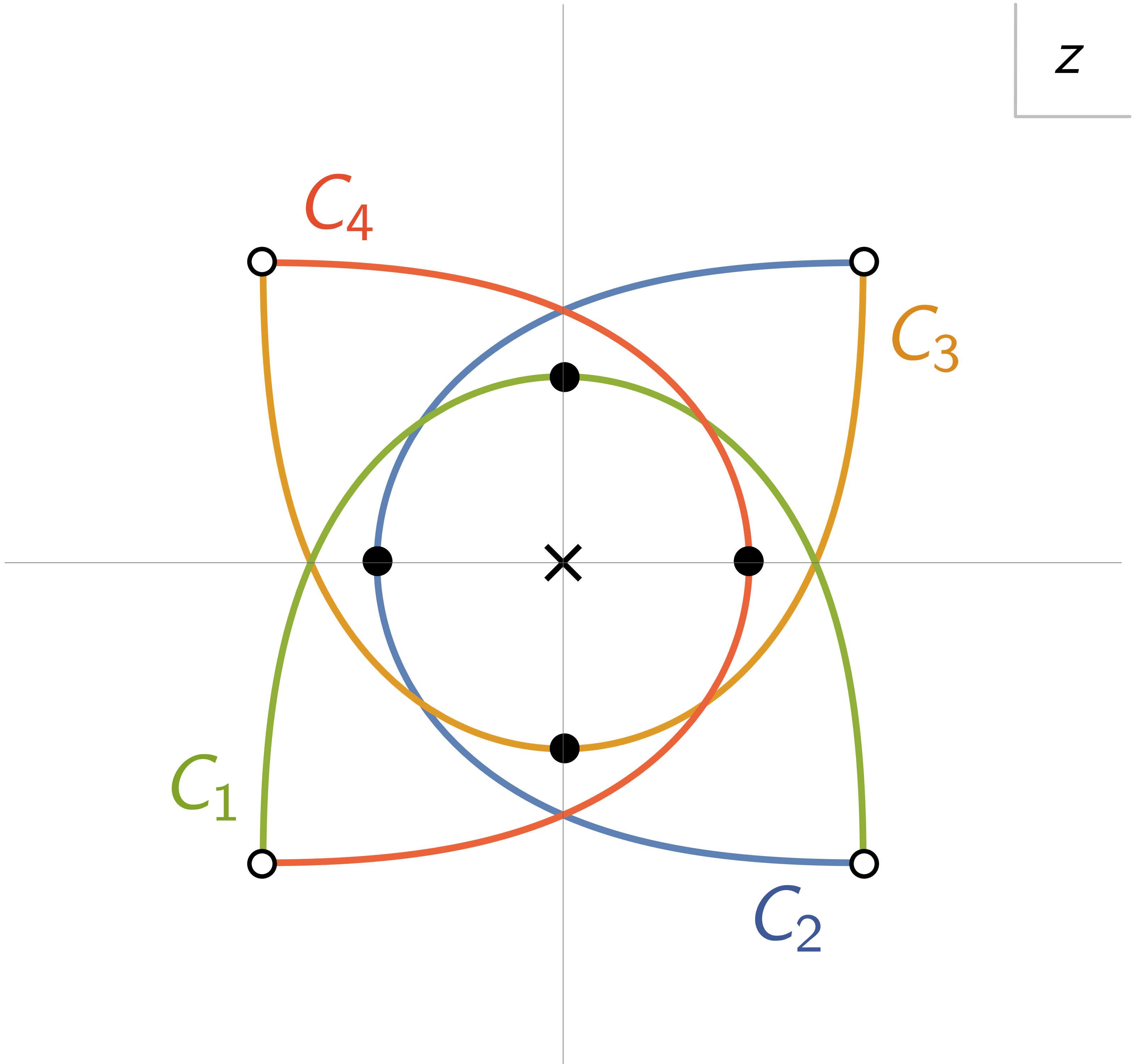
$$P_W(x,y,z) = P(x,y,z) - W$$



Bi-fundamental matter fields arise from intersections among boundaries of the bricks at $W = 0$.

Tomography

2-spheres as arcs on the (x or y or z)-plane



More on Picard-Lefschetz theory

Triality fits nicely with Picard-Lefschetz monodromy

Bi-fundamental matter fields …

1) intersections among boundaries of the bricks at $W = 0$.

2) "instanton paths" in an auxiliary SUSY-QM
in which P is the super-potential.
(the paths lie outside the surface $P = 0$.)

It takes some work to distinguish chirals from Fermis.

What about the E/J interaction terms?

Elliptic Genus

Elliptic genus

$$\mathcal{I}_{\mathrm{R/NS}}(q;x) = \mathrm{Tr}_{\mathrm{R/NS}}[(-1)^F q^{H_L} \bar{q}^{H_R} \prod_i x_i^{f_i}]$$

↑

Chemical potential (fugacity)
for global symmetries

$$q = e^{2\pi i \tau}$$

R: Witten index $\quad T^d$

NS: superconformal index $\quad S^1 \times S^{d-1}$

Elliptic genus - UV

Localization computation of gauge theory

Roughly, $\mathcal{I} = \int_{\text{BPS}} Z_{\text{1-loop}}$

[Gadde,Gukov,Putrov 13]
[Benini,Eager,Hori,Tachikawa 13]

Complications :

- contour integral of a meromorphic function
- choice of residues : Jeffrey-Kirwan prescription

Elliptic genus - IR

Gauge theory will flow to an NLSM
whose target space is …

$U(1)$: classical moduli space X of the gauge theory
$U(N)$: symmetric product orbifold, $S^N X = X^N / S_N$

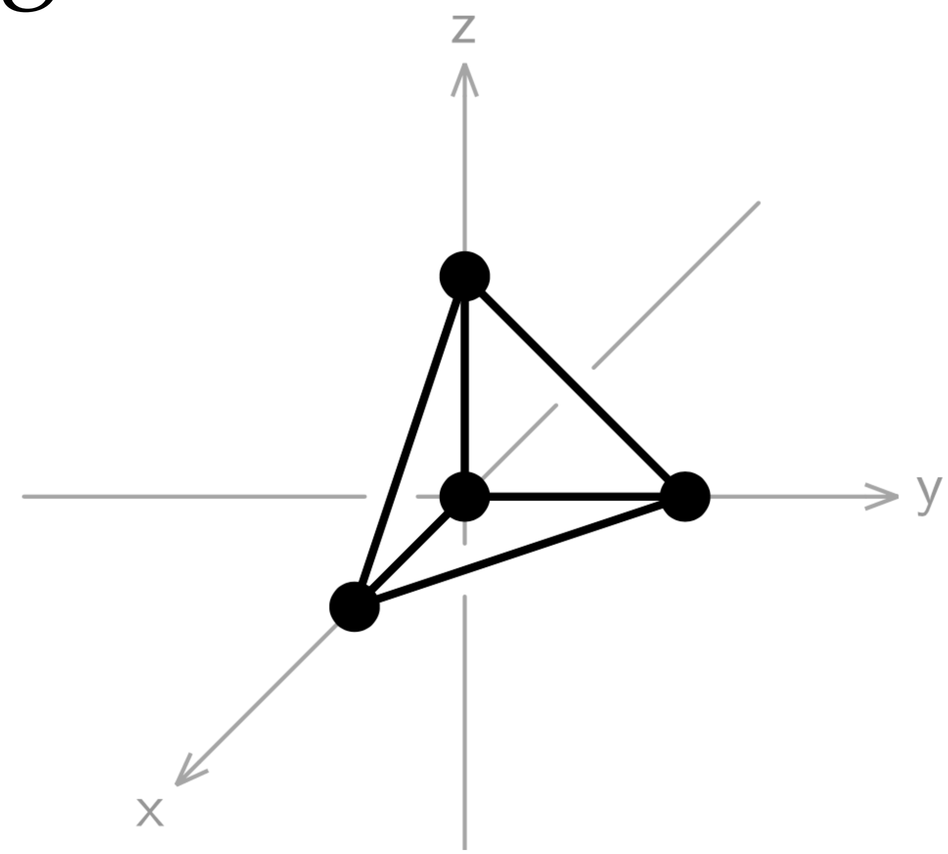
We propose a geometric formula for $U(1)$ theories,
based on triangulation of toric diagrams.

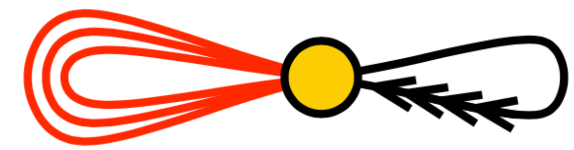
Hints taken from [Lerche][Kawai,Mohri][Martelli,Sparks,Yau]

Result: perfect agreement between UV and IR

The building block

$\mathbb{C}^4$





1 gauge + 3 Fermi + 4 chiral

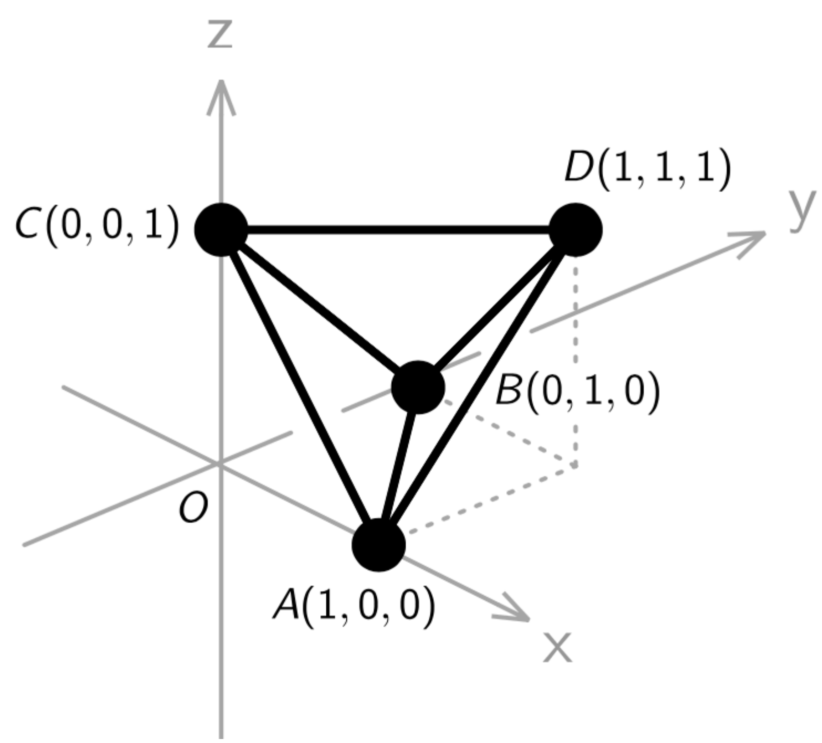
$$\mathcal{I}_{\mathbb{C}^4} = \frac{-i\eta(q)^3\theta_1(q,x)\theta_1(q,y)\theta_1(q,z)}{\theta_1(q,s_1)\theta_1(q,s_2)\theta_1(q,s_3)\theta_1(q,s_4)},$$

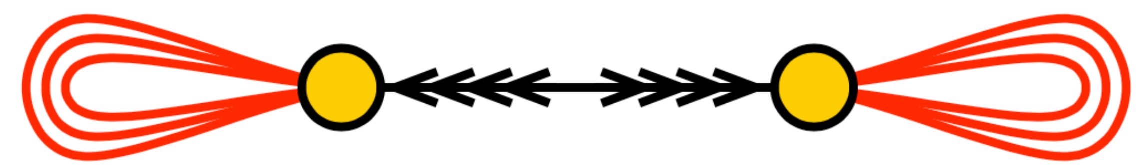
A trivial matching between

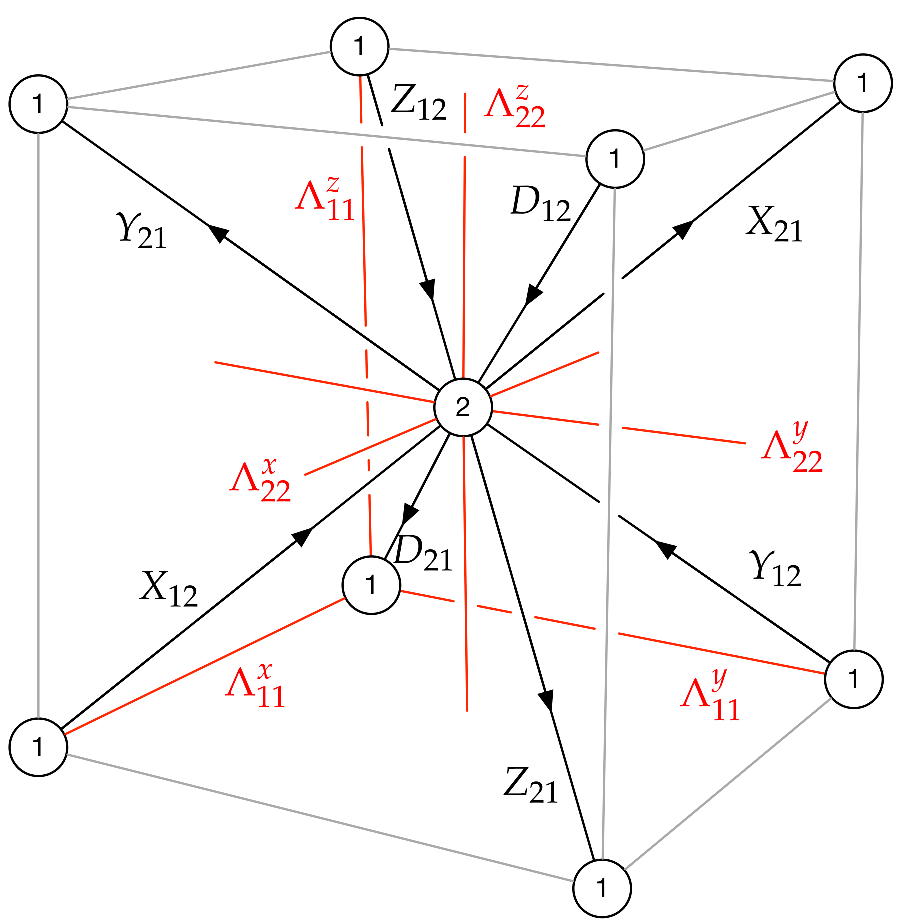
UV (gauge theory) and IR (geometry)

A non-trivial example : $\mathbb{C}^4/\mathbb{Z}_2$

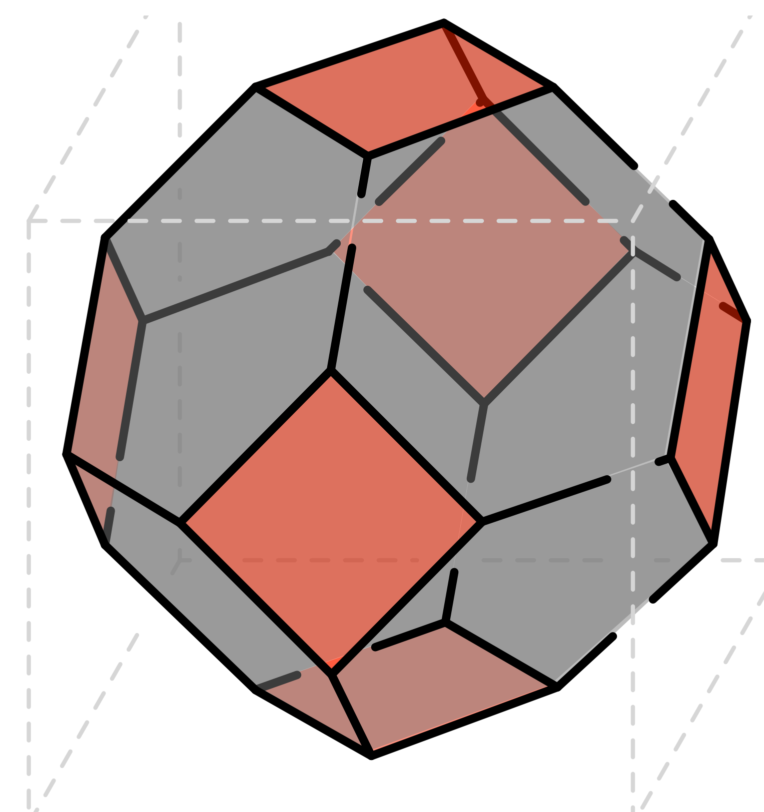
$\mathbb{C}^4/\mathbb{Z}_2$







graph dual



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Orbifold CFT method - IR

The $\mathbb{C}^4$ theory is a free CFT.

The $\mathbb{C}^4/\Gamma$ theory is likely to flow to (free CFT)$/\Gamma$

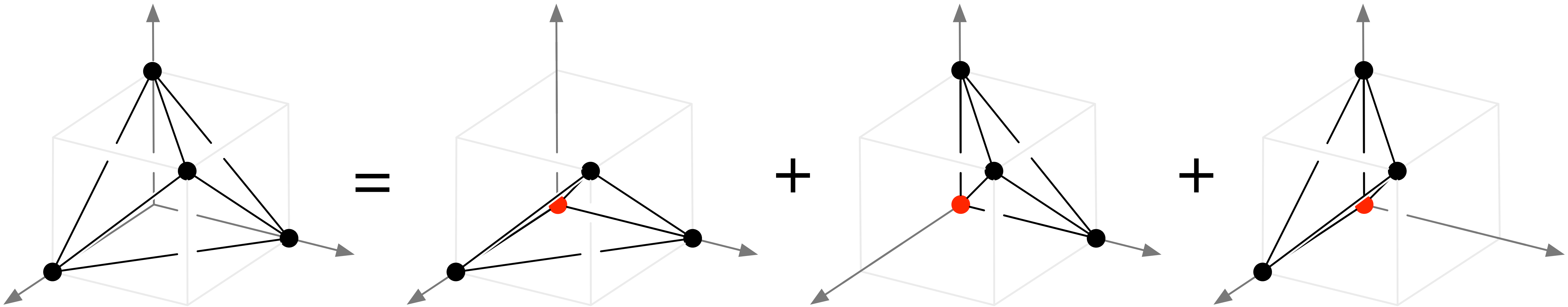
The $\mathbb{Z}_2(1,1,1,1)$ acts as

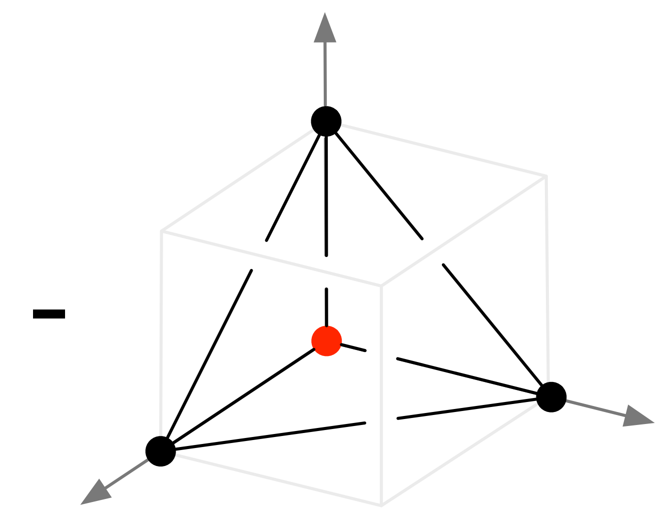
(-1) on chiral fields,

$(+1)$ on Fermi fields.

$$\mathcal{I}^{\text{orbi}} = \frac{1}{2}\sum_{a=1}^{4} \frac{(-1)^{a+1} i\eta(q)^3 \theta_1(q,x)\theta_1(q,y)\theta_1(q,z)}{\theta_a(q,\sqrt{x/yz})\theta_a(q,\sqrt{y/zx})\theta_a(q,\sqrt{z/xy})\theta_a(q,\sqrt{xyz})}$$

Geometric formula (subtraction) - IR





$$\mathcal{I}^{\mathrm{geo}} = \frac{i\eta(q)^3\theta_1(q,x)\theta_1(q,y)\theta_1(q,xy/z^2)}{\theta_1(q,z)\theta_1(q,x/z)\theta_1(q,y/z)\theta_1(q,z/xy)} + \frac{i\eta(q)^3\theta_1(q,y)\theta_1(q,z)\theta_1(q,yz/x^2)}{\theta_1(q,x)\theta_1(q,y/x)\theta_1(q,z/x)\theta_1(q,x/yz)} + \frac{i\eta(q)^3\theta_1(q,z)\theta_1(q,x)\theta_1(q,zx/y^2)}{\theta_1(q,y)\theta_1(q,z/y)\theta_1(q,x/y)\theta_1(q,y/zx)} - \frac{i\eta(q)^3\theta_1(q,xy)\theta_1(q,yz)\theta_1(q,zx)}{\theta_1(q,x)\theta_1(q,y)\theta_1(q,z)\theta_1(q,1/xyz)}$$

Gauge theory - UV

Contour integral along Jeffrey-Kirwan residue gives

$$\mathcal{I}^{\text{gauge}} = \sum_{i=1}^{4} \left[\frac{i\eta(q)^3 \theta_1(q,x)\theta_1(q,y)\theta_1(q,z)}{[\prod_{j\neq i} \theta_1(\tau, s_j/s_i)]\theta_1(\tau, s_i^2)} W(q, s_i^2) \right]$$

where we inserted an "anomaly cancelling factor"

$$W(q,u;v) = \frac{\theta_1(q,vu)\theta_1(q,v/u)}{\theta_1(q,v)^2}$$

for abelian gauge anomaly.

$(s_1, s_2, s_3, s_4) = (\sqrt{x/yz}, \sqrt{y/zx}, \sqrt{z/xy}, \sqrt{xyz})$

Beyond
Brane Brick

Vertex Operator Algebra (VOA)

M-theory lift

VOA

Vertex operator algebra (a.k.a. chiral algebra)

Spectrum : cohomology of $\overline{Q}_+$

OPE : meromorphic

"left-moving sector of CFT
coupled to SUSY vacuum
of the right-moving sector."

Elliptic genus = partition function of VOA

Why VOA ?

RG-invariant BPS sector of a (0,2) theory

With M5-branes wrapping 4-manifolds,

VOA[M_4] gives a topological invariant of M_4.

[Feign,Gukov 2018]

VOA can probe 4d gauge theories [Beem et al 2013]

VOAs with negative central charge or level are common

VOA for BBM

Spectrum known. OPE?

Elliptic genera for $U(1)$ theories have been computed.

For $\mathbb{C}^4$, the VOA is a free theory with $c = 0$.

For Lagrangian QFTs,
a systematic way to construct the VOA is known.

[Dedushenko 2015]
[Dedushenko,Gukov 2017]

VOA for $U(N)$ theory will be particularly interesting.

[Dijkgraaf,Moore,Verlinde,Verlinde 1996]

$$(b, c)^{(\lambda,\epsilon)} \quad : \quad c = -2\epsilon(6\lambda^2 - 6\lambda + 1)$$

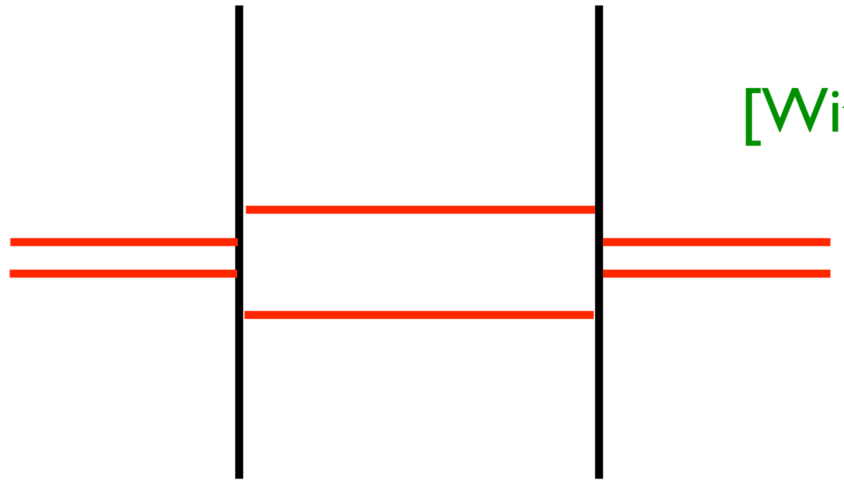
M-theory Lift

M-theory lift — old

NS5-D4 system in IIA

↓

M5 in IIA



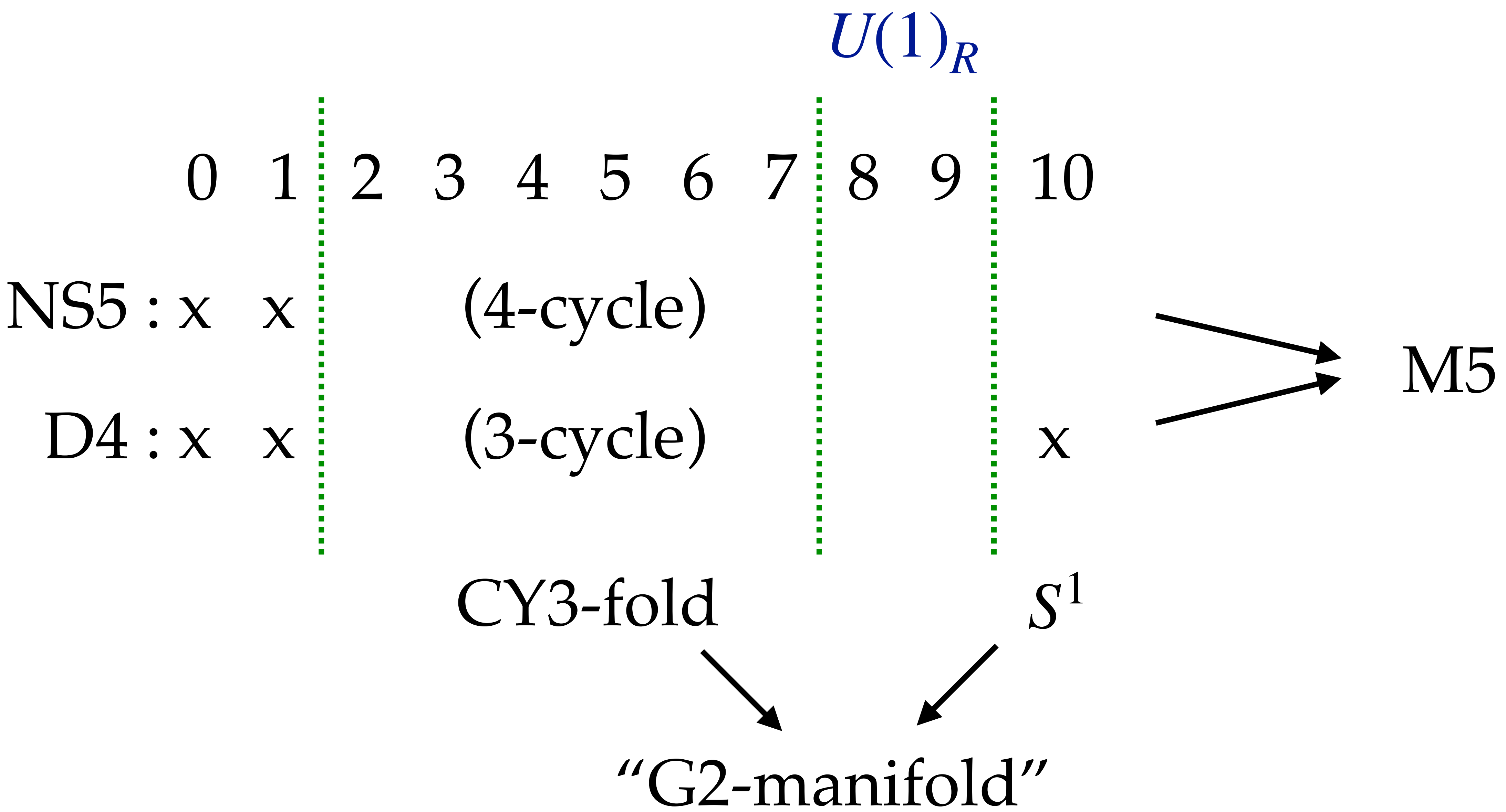
[Witten 1997]

BBM is an NS5-D4 system!

Why not take the M-theory lift?

cf. [Gadde,Gukov,Putrov 2013]

M-theory lift — new



Calibration conditions

NS5 : holomorphic

D4 : Lagrangian

M5 : co-associative

Calibrated cycles

Calabi-Yau 3-fold

Kahler 2-form J Holo : $\mathrm{vol} = \frac{1}{2} J \wedge J$

Holo 3-form Ω Lagrangian : $\mathrm{vol} = \mathrm{Re}(\Omega)$

"G2 manifold" ~ (Calabi-Yau 3-fold) $\times S^1$

Associative : $\Phi = J \wedge d\psi + \mathrm{Im}(\Omega)$

Co-associative : $\Psi = \frac{1}{2} J \wedge J + \mathrm{Re}(\Omega) \wedge d\psi$

Local models

What is the simplest NS5-D4 system?

1st attempt:

$$\mathsf{CY}_3 = \mathbb{C}^3$$

$$P(x, y, z) = x^2 + y^2 + z^2 - a^2 \quad (a \in \mathbb{R})$$

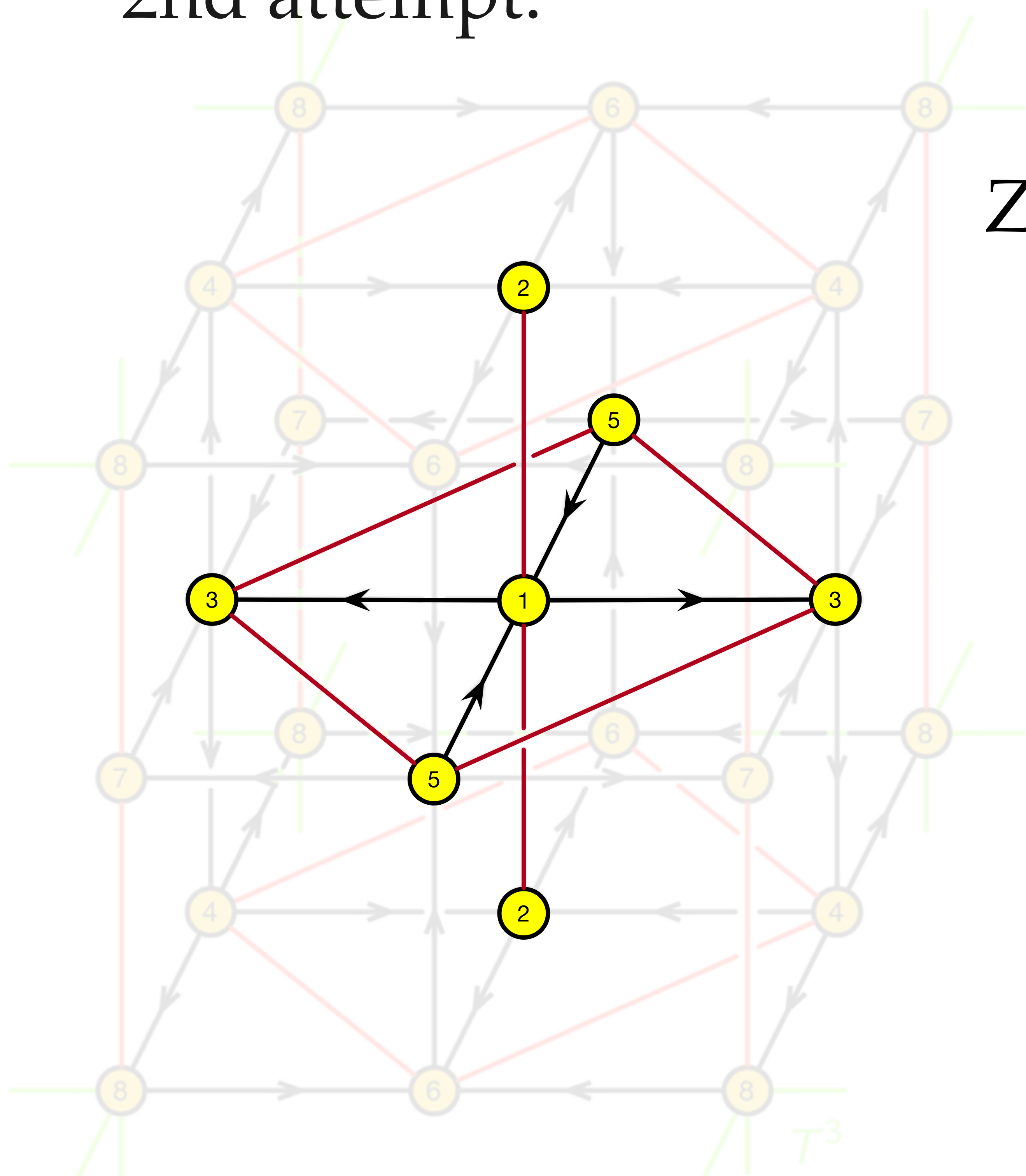
D4-brane on the real slice of $x^2 + y^2 + z^2 \leq a^2$.

A single gauge group with no matter fields …

anomalous!

Local models

2nd attempt:



Zooming into BBM :

Take the $P(x,y,z)$ of BBM,

Taylor-expand around a point.

A quartic model
under investigation.

anomaly-free!

Summary

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BBM is a large class of 2d (0,2) gauge theories.
Toric geometry makes BBM easier than generic theories.

VOA of BBM is under investigation.

By zooming into BBM,
we can obtain simple local models,
to be used as simplest examples of M-theory lift.

The local models could be used as building blocks
to construct new families of 2d (0,2) theories.

It may shed light on the $T[M_4]$ program.

Thank you