RG flows with enhanced infrared supersymmetry

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Introduction

Symmetry is an important quantity which characterizes QFT.

We usually define a theory in UV and try to analyze the RG flow and its IR theory.

(Suppose we have a nontrivial fixed point in IR, then)
Can the symmetry in UV still characterize the IR theory?
Or is the IR symmetry same as the UV symmetry?

— in many cases yes, but

the IR symmetry could be different from the UV symmetry.

Susy enhancement

We consider enhancement of supersymmetry in 4d supersymmetric QFTs along a renormalization group flow.

Few example is known for supersymmetry in 4d:

- N=1 SU(n) theory (with gauge coupling g) with three adjoint chirals with superpotential $W = h\, \Phi_1\Phi_2\Phi_3$ → N=4
- N=1 Lagrangian theories where a coupling constant is set to infinity → N=2 E_6, E_7 and $R_{0,N}$ theories **[Gadde-Razamat-Willet, Agarwal-KM-Song]**

… 3d examples **[Dongmin Gang's talk]**

N=1 SU(2) adjoint SQCD w/ N_f=1 coupled to a gauge-singlet **[KM-Song]**

↓

N=2 Argyres-Douglas theory

The main point, which has not been fully studied so far, is the coupling with (gauge-singlet) chiral fields. **[Seiberg, Leigh-Strassler]**

This example shows that coupling singlets leads to an interesting IR fixed point. Other example… **[Kim-Razamat-Vafa-Zafrir]**

Argyres-Douglas theory

- was originally found at a special point on the Coulomb branch of N=2 SU(3) pure SYM with mutually non-local massless particles
  [Argyres-Douglas, Argyres-Plesser-Seiberg-Witten]

- with the Coulomb branch operator of scaling dimension 6/5

- **strongly coupled; no known Lagrangian description....**

Many generalizations

[Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang]
[Cecotti-Neitzke-Vafa, Bonelli-KM-Tanzini, Xie, ...]

Motivations

The UV Lagrangian enables us to compute the partition functions of the SCFTs, e.g., superconformal index in the full generality **[KM-Song, Agarwal-KM-Song]**

N=2 theories describe interesting physics as a toy model, and are also related to theories in other dimensions and various areas of mathematical physics:

- integrable systems **[Gorsky et al., Nekarasov-Shatashvilli]**
- 2d CFTs **[Alday-Gaiotto-Tachikawa]**
- 2d chiral algebra **[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]**
- Topological string theory **[Aganagic et al., Eguchi-Kanno]**
- M-theory **[Gaiotto, Gaiotto-Moore-Neitzke]**

Results

- **We find Lagrangian descriptions for the following theories:**

H_0, H_1, (A_1,A_k), and (A_1, D_k)

more results [Agarwal-Sciarappa-Song]

- **We compute SCIs in full generality of the above theories.**

Lagrangians for H_0 and H_1 theories are simply SU(2) SQCD with one adjoint and two fundamental chiral multiplets with some superpotential coupling with gauge-singlets.

- **(We classify all relevant deformations from W=0 fixed point and find various new N=1 SCFT.)**

→ [Jaewon's talk]

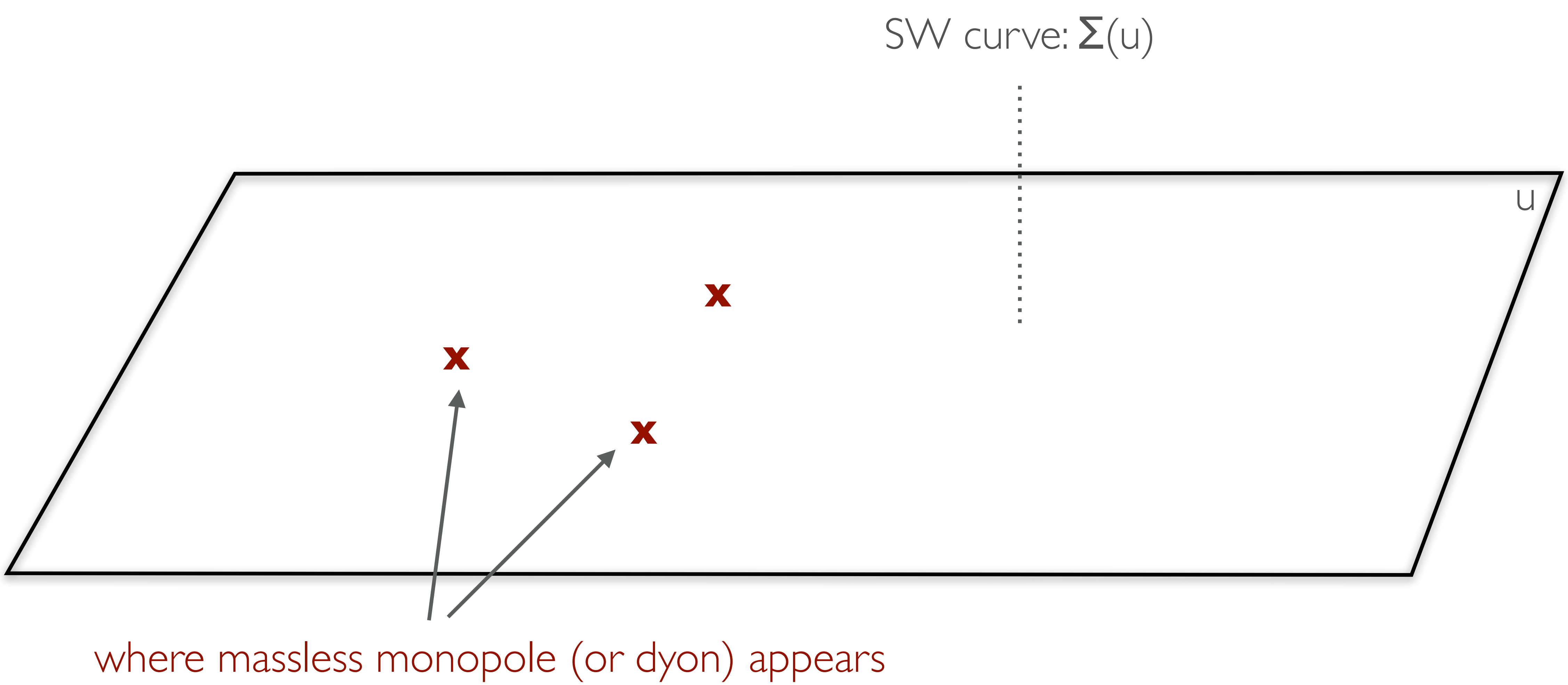
Plan of talk

- **Review of N=2 AD theories**
- **A Lagrangian for AD theory**
- **N=1 deformations of N=2 SCFTs**

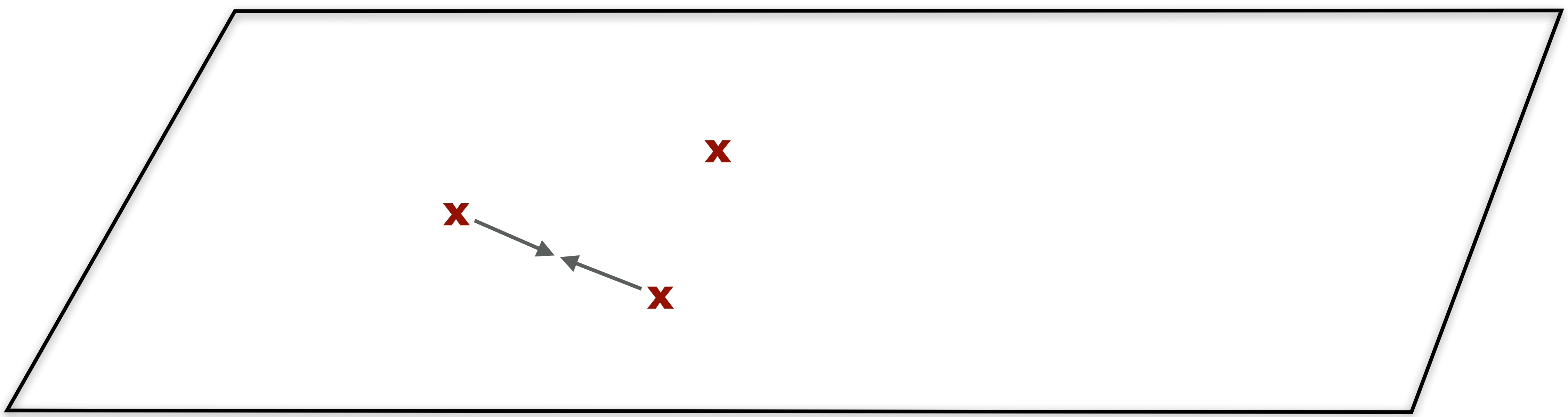
Review of N=2 AD theories

N=2 physics on Coulomb branch

The effective theory on Coulomb branch is determined by the so-called Seiberg-Witten curve and (one-form)



Conformal point



In some limits of parameters, two or more points
where mutually non-local massless particle appears collides.

In this case, there appear a fixed point where the theory is described by an SCFT **[Argyres-Douglas]**
[Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang]

Argyres-Douglas theory

- **dimension of Coulomb branch operator is 6/5 !**
- **the central charges are given by [Aharony-Tachikawa]**

$$a = \frac{43}{120}, \quad c = \frac{11}{30}$$

- **this value of c saturates the bound of nontrivial N=2 SCFT [Liendo-Ramirez-Seo]**

Recent developments

Argyres-Douglas theory and its generalization are strongly coupled, so basically it is difficult to analyze, but

- classifications **[Argyres-Lotito-Lu-Martone] [Cecotti-Neitzke-Vafa, Xie, Xie-Yau, Wang-Xie, Argyres-Martone, Caorsi-Cecotti]**
- chiral algebra **[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees, Cordova-Shao, Creutzig, Song-Xie-Yan]**
- conformal bootstrap **[Cornagliotto-Lemos-Liendo]**
- BPS quivers and spectral network **[Alim-Cecotti-Cordova-Espahbodi-Rastogi-Vafa, Gaiotto-Moore-Neitzke, KM-Park-Yan]**
- superconformal index **[Buican-Nishinaka, Cordova-Shao, Song]**

A Lagrangian for AD theory

Let us consider the following N=1 theory with **SU(2) vector multiplets** and **with the following chiral multiplets**:

	q	q'	ϕ	M
SU(2)	**2**	**2**	**adj**	**1**
$U(1)_{R0}$	1/2	-5/2	1	6
$U(1)_{\mathscr{F}}$	1/2	7/2	-1	-6

with the superpotential

$$W = \phi q q + M \phi q' q'$$

Central charges

The central charges of the SCFT are determined from the anomaly coefficients of the IR R-symmetry: **[Anselmi-Freedman-Grisaru-Johansen]**

$$a = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}^3 - \mathrm{Tr}R_{\mathrm{IR}}), \quad c = \frac{1}{32}(9\mathrm{Tr}R_{\mathrm{IR}}^3 - 5\mathrm{Tr}R_{\mathrm{IR}})$$

In our case, the IR R-symmetry is a combination of two U(1)'s. Thus consider the following

$$R_{\mathrm{IR}}(\epsilon) = R_0 + \epsilon\mathcal{F}$$

The true R symmetry is determined by maximizing trial central charge **[Intriligator-Wecht]**

$$a(\epsilon) = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}(\epsilon)^3 - \mathrm{Tr}R_{\mathrm{IR}}(\epsilon))$$

Decoupling issue

The trϕ^2 operator hits the unitarity bound (Δ<1). We interpret this as being decoupled. Thus we subtract its contribution from central charge, and re-a-maximize

[cf. Kutasov-Parnachev-Sahakyan]

$$\text{Tr}\phi^2, \; M, \; \ldots$$

dimension 6/5

$$\epsilon = \frac{13}{15}, \quad a = \frac{43}{120}, \quad c = \frac{11}{30}$$

A way to pick up the interacting part is by introducing a chiral multiplet X to set trϕ^2=0: $\delta W = X\phi^2$

$$a_{\text{chiral}}(r) = -a_{\text{chiral}}(2-r)$$

In the end, the Lagrangian which flows to the Argyres-Douglas theory (H_0 theory) is

$$W = \phi q q + M\phi q' q' + X\phi^2$$

Chiral ring of H_0

We had the following chiral operators

$$\mathrm{tr}\phi q^2, \quad \mathrm{tr}\phi q q', \quad \mathrm{tr} q q', \quad \mathrm{tr}\phi q'^2, \quad X, \quad M$$

The F-term conditions are

$$0 = qq + Mq'^2 + 2X\phi, \quad 0 = \mathrm{tr}\phi q'^2, \quad 0 = \phi q, \quad 0 = M\phi q', \quad 0 = \mathrm{tr}\phi^2.$$

Thus, **the generators in the chiral ring** are only

$$\mathrm{tr} q q', \quad M$$

(moduli space of X is uplifted quantum mechanically)

dim = 11/5, 6/5 → **form N=2 Coulomb branch operator multiplet**

Localization computations

- One can get the superconformal index in full generality which agrees with the results in some limits **[KM-Song, Agarwal-KM-Song] [Evtikhiev]**

- Other partition functions **[Fredrickson-Pei-Yan-Ye, Gukov, Fluder-Song]**

- 3d reduction and mirror quiver **[Benvenuti-Giacomelli]**

N=1 deformations of N=2 SCFTs

N=1 deformation

Suppose we have an N=2 SCFT ***T*** with **non-Abelian flavor symmetry F.**

[Gadde-KM-Tachikawa-Yan, Agarwal-Bah-KM-Song]
[Agarwal-Intriligator-Song]
cf. [Heckman-Tachikawa-Vafa-Wecht]

Then let us

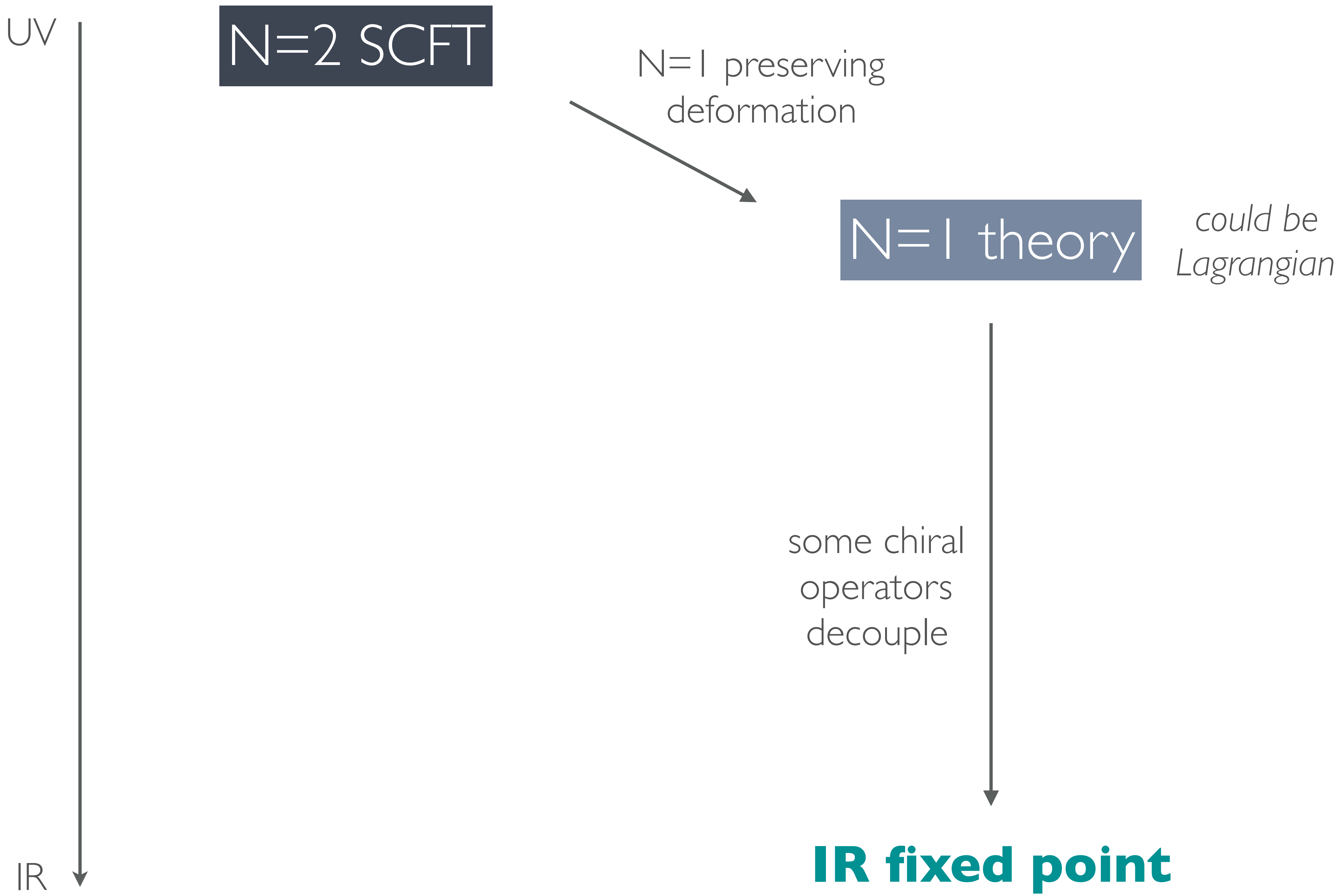
- **couple N=1 chiral multiplet M in the adjoint rep of F by the superpotential**

$$W = \mathrm{tr}\mu M$$

- **give a nilpotent vev to M (which is specified by the embedding ρ: SU(2)→F), which breaks F.** $W = \sum_j \mu_{j,j} M_{j,-j}$

  (For F=SU(N), this is classified by a partition of N or Young diagram.)

This gives IR theory $T_{IR}[T, \rho]$, which is generically N=1 supersymmetric.


UV
N=2 SCFT
N=1 preserving deformation
N=1 theory
could be Lagrangian
some chiral operators decouple
IR fixed point
IR

T = SU(2) w/ 4 flavors

In this case, F = SO(8)

We consider the principal embedding of SO(8), the vev which breaks SO(8) completely.

The adjoint rep decomposes as

28 → 3, 7, 7, 11

$$M_{1,-1}, M_{3,-3}, M'_{3,-3}, M_{5,-5}$$

→ after integrating out the massive fields,
we get the superpotential

$$W = \phi qq + M_1\phi^2 qq' + M_3 qq' + M_5\phi q'q' + M'_3\phi^3 q'q',$$

T = SU(2) w/ 4 flavors

Other choices of embeddings:

- **$[5,1^3]$, $[4,4]$** (with SU(2)) → **H_1 theory** (SU(2) flavor symmetry)

$$a = \frac{11}{24}, \quad c = \frac{1}{2}$$

- **$[3^2,1^2]$** (with U(1)xU(1)) → **H_2 theory** (SU(3) flavor symmetry)

$$a = \frac{7}{12}, \quad c = \frac{2}{3}$$

- other embeddings → N=1 SCFTs

H_1 theory

By the deformation procedure one can obtain **SU(2) gauge theory with the following chiral multiplets**:

	(q, q')	ϕ	M
SU(2)	**2**	**adj**	**1**
$U(1)_{R0}$	-1	1	4
$U(1)_{\mathcal{F}}$	2	-1	-4
$SU(2)_f$	**2**	**1**	**1**

with the superpotential

$$W = X\phi^2 + Mqq'$$

This theory flows to the H_1 theory with central charges

$$a = \frac{11}{24}, \;\; c = \frac{1}{2}$$

Chiral ring of H_1

By using the F-term conditions one can show that the chiral ring is generated by the following operators

$$M, \quad \mathcal{O}_i$$

where

$$\mathcal{O}_1 = \mathrm{tr}\phi q q, \quad \mathcal{O}_0 = \mathrm{tr}\phi q q', \quad \mathcal{O}_{-1} = \mathrm{tr}\phi q' q',$$

M is the lowest component of the Coulomb branch operator multiplet, and Oi are the moment map operator of flavor SU(2). Indeed, they satisfy the relation

$$M \cdot \mathcal{O}_i \sim 0, \quad \mathcal{O}_1 \cdot \mathcal{O}_{-1} \sim \mathcal{O}_0^2$$

Conditions for N=2 susy

For principal embedding: we conjecture that the condition for *T* to have enhancement of supersymmetry in the IR is as follows:

- F is of ADE type
- 2d chiral algebra satisfies the Sugawara condition:

[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

$$\frac{\dim F}{c} = \frac{24h^\vee}{k_F} - 12$$

- **rank-one theories H_1, H_2, D_4, E_6, E_7, E_8 → H_0**
- **SU(N) SQCD with 2N flavors → (A_1, A_{2N})**
- **Sp(N) SQCD with 2N+2 flavors → (A_1, A_{2N+1})**
- **(A_1, D_k) theory → (A_1, A_{k-1})**
- **some quiver gauge theories → (A_N, A_L)**

[Agarwal-Sciarappa-Song, Benvenuti-Giacomelli]

N=2? on Coulomb branch

From the Argyres-Douglas theory viewpoint, one can go to the Coulomb branch by turning on

- vev of Coulomb branch operator $\langle \mathscr{O} \rangle = u$
- relevant coupling: $\delta \mathscr{L} = c \int d^2\theta_1 d^2\theta_2 U$
- mass deformation: $\delta \mathscr{L} = m \int d^2\theta_1 \mu_0, \quad (\mu_0 : \text{moment map operator})$

One can study the physics on the IR Coulomb branch from the Lagrangian viewpoint: for the H_1 theory, the above deformations correspond to adding

$$W = X\phi^2 + uqq' + cX + m\phi qq'$$

The theory with superpotential

$$W = uqq' + m\phi qq'$$

has been studied by **[Intriligator-Seiberg]**. They found the theory is in **N=1 Coulomb branch** parametrized by $v = \langle \mathrm{tr}\phi^2 \rangle$, whose curve is given by

$$y^2 = x^3 - vx^2 + \frac{1}{4}u\Lambda^3 x - \frac{1}{64}m^2\Lambda^6$$

Adding the terms $X\phi^2 + cX$ sets the vev $v = \langle \mathrm{tr}\phi^2 \rangle$ to -c. Thus the N=1 curve is now

$$y^2 = x^3 + cx^2 + \frac{1}{4}u\Lambda^3 x - \frac{1}{64}m^2\Lambda^6$$

which is indeed the same as the Seiberg-Witten curve of the N=2 H_1 theory after the redefinition of the parameters.

Discussions

- What is the precise conditions for susy enhancement?
- Why susy enhancement??
- Holographic dual of the RG flow with the enhanced susy.
- string/M-theory realization?

Discussion

Experimentally, something interesting (symmetry enhancement) happens in the IR when we **flip** the relevant operators:

SU(2) w/ adjoint + 2 fundamentals and W=0 (a=0.4525…, c=0.4986..)

At this fixed point, there are four relevant deformations

$$\mathcal{O}_1 = Q^2\phi, \ \mathcal{O}_2 = Q\tilde{Q}\phi, \ \mathcal{O}_3 = Q\tilde{Q}, \ \mathcal{O}_4 = \tilde{Q}^2\phi$$

We can deform by these or flip these by adding a term like

$$W = M_i \mathcal{O}_i$$

We get various fixed points with interesting properties.

[Jaewon's talk]

Superconformal index

Now we had Lagrangian theories which flow to SCFTs in the IR. **Thus the superconformal indices of the latter can be simply computed from the matter content.**

The index of our N=1 theory is defined by

$$I = \mathrm{Tr}_{\mathcal{H}_{S^3}} (-1)^F p^{j_1+j_2-R/2} q^{j_1-j_2-R/2} \xi^{\mathcal{F}}$$

where j_1 and j_2 are rotation generators of the maximal torus $U(1)_1$ and $U(1)_2$ of $SO(4)=SU(2)_1 \times SU(2)_2$ and R and $\mathcal{F}$ is the generators of the $U(1)_R$ and $U(1)_{\mathcal{F}}$.

(If S^3 is described by equation $|x_1|^2+|x_2|^2=1$, j_1+j_2 and j_1-j_2 rotate x_1 and x_2 by phase.)

Index of H_0 theory

For instance one could calculate the index of the Argyres-Douglas (H_0) theory from the Lagrangian:

$$I = \kappa \frac{\Gamma((pq)^3 \xi^{-6})}{\Gamma((pq)^1 \xi^{-2})} \oint \frac{dz}{2\pi i z} \frac{\Gamma(z^{\pm}(pq)^{\frac{1}{4}} \xi^{\frac{1}{2}}) \Gamma(z^{\pm}(pq)^{-\frac{5}{4}} \xi^{\frac{7}{2}}) \Gamma(z^{\pm 2,0}(pq)^{\frac{1}{2}} \xi^{-1})}{\Gamma(z^{\pm 2})}$$

ξ : fugacity for $\mathrm{U}(1)_{\mathcal{F}}$

(We subtract the contributions of the decoupled operators!)

We substitute $\xi \to t^{\frac{1}{5}} (pq)^{\frac{3}{10}}$ for the correct IR R symmetry. After that

- basically one can compute the integral
- Coulomb index limit (pq/t=u, p,q,t→0): $I_C = \frac{1}{1 - u^{\frac{6}{5}}}$
- Macdonald limit (p→0) agrees with the index by **[Cordova-Shao, Song]**