AdS_6/CFT_5 in Type IIB Part II: Dualities, tests and applications

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with E. D'Hoker, M. Gutperle, A. Karch, C. Marasinou, A. Trivella, O. Varela, O. Bergman, D. Rodríguez-Gómez, M. Fluder

Higher-dimensional CFTs are interesting:

- mysterious, non-Lagrangian
- interesting implications for $d \leq 4$
- $-$ string/M-theory connections \ldots

5d SCFTs: often fixed points for asymptotically safe gauge theories, large classes engineered in Type IIB via 5-brane webs Recap

Any planar 5-brane junction realizes 5d SCFT on intersection point

Characterized entirely by external 5-brane charges. May or may not have gauge theory deformations.

Supergravity solutions for fully localized 5-brane intersections, constructed directly as "near-horizon" limit:

Recap

Supergravity solutions for fully localized 5-brane intersections, constructed directly as "near-horizon" limit:

- $AdS_6 \times S^2 \times \Sigma$ with $\Sigma = \mathrm{disc}$ 5-branes emerge at poles on $\partial \Sigma$
- parametrized entirely by residues =conserved 5-brane charges

$$
-\mathrm{AdS}_6 + 16 \text{ suse} = F(4)
$$

Naturally fit to string theory picture. "Large N ": all (p_i,q_i) large.

Recap

Claim: poles \sim groups of 5-branes that are unconstrained by s-rule. Constrained junctions realized by bringing 7-branes into web:

Supergravity: $\Sigma =$ disc with puncture(s), $[p, q]$ 7-brane monodromy

Position of puncture on $\Sigma \leftrightarrow$ choice of face in which 7-brane is placed with no 5-branes attached(?)

AdS_6/CFT_5 in Type IIB

Goal of this talk: convincing case for proposed identifications and dualities, first lessons on 5d SCFTs

AdS_6/CFT_5 in Type IIB

Outline

- Testing AdS_6/CFT_5 with stringy operators
- $-$ Holographic S^5 partition functions and EE, $\#$ d.o.f.
- QFT partition functions: Precision test of AdS_6/CFT_5
- (Massive) spin-2 fluctuations

– Testing AdS_6/CFT_5 with strings –

Testing AdS_6/CFT_5 with strings

5-brane picture: gauge invariant operators from strings and string junctions connecting external 5-branes

Supergravity: operators with $\Delta = \mathcal{O}(N) \leftrightarrow$ probe strings or string junctions in supergravity background $(1 \ll \Delta \ll \mathcal{F}(S^5)).$

Strategy: study gauge theory deformations, identify stringy BPS operators, extrapolate to SCFT, compare to supergravity

Stringy operators in $+_{N,M}$ theories

 $[N] - SU(N)^{M-1} - [N]$ $SU(N)^2 \times U(1)_{B}^{M} \times U(1)_{I}^{M-1}$ $\subset SU(N)^2\times SU(M)^2\times U(1)$

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M_j^i = (x^{(1)} \cdots x^{(M)})_j^i \qquad (\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}, \mathbf{1}) \qquad \Delta = \frac{3}{2}M \qquad Q = \frac{1}{2}M
$$

$$
B^{(k)} = \det(x^{(k)}) \qquad \subset (\mathbf{1}, \mathbf{1}, \mathbf{M}, \bar{\mathbf{M}}) \qquad \Delta = \frac{3}{2}N \qquad Q = \frac{1}{2}N
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 $M^i_j \sim \mathsf{F1}$ between D5, $\ B^{(k)} \subset \mathsf{D1}$ between NS5. S-dual quiver deformation realizes all D1 operators, subset of F1 operators.

global AdS $_6/\textsf{CFT}_5$ on $\mathbb{R}\times S^4$ AdS_6 $\partial_t \leftrightarrow$ dilation: $\Delta = \mathcal{H}$

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Action and EOM for (p, q) string along t and Σ :

$$
S_{(p,q)} = -2T \int d^2 \xi f_6 \rho |w'| \sqrt{\mathfrak{q} \mathcal{M} \mathfrak{q}} \qquad \mathfrak{q} \mathcal{M} \mathfrak{q} = e^{2\phi} \begin{pmatrix} p \\ q \end{pmatrix} \begin{pmatrix} 1 & -\chi \\ -\chi & \chi^2 + e^{-4\phi} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}
$$

$$
0 = \frac{\bar{w}''}{\bar{w}'} - \frac{w''}{w'} + (\bar{w}' \partial_{\bar{w}} - w' \partial_{w}) \ln \left(f_6^2 \rho^2 \mathfrak{q}^T \mathcal{M} \mathfrak{q} \right)
$$

D1 global AdS $_6 /$ CFT $_5$ on $\mathbb{R} \times S^4$ AdS_6 $\partial_t \leftrightarrow$ dilation: $\Delta = \mathcal{H}$

F1 between D5 poles, D1 between NS5 poles:

$$
\Delta_{F1} = \frac{3}{2}M \qquad \qquad \Delta_{D1} = \frac{3}{2}N
$$

Solve EOM, scaling dimensions match field theory exactly

R-charge of string states from coupling to $SU(2)$ bulk gauge field fluctuation dual to R -symmetry current $(S^2$ isometries).

Relevant part from KK ansatz $+ SU(2)$ gauge invariance:

 $dx^{\mu} \rightarrow dx^{\mu} + K_I^{\mu} A^I$ $K_I \sim S^2$ Killing vector fields $\delta C_{(2)} = dC \wedge f_I A^I + \dots$ $df_I = \star_{S^2} K_I$

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$$
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$$

 (p, q) strings couple to $\delta C_{(2)}$ through WZ term

$$
Q_{(p,q)} = T \int_{\Sigma_{(p,q)}} (p \operatorname{Re}(d\mathcal{C}) - q \operatorname{Im}(d\mathcal{C}))
$$

$$
\implies Q_{\text{F1}} = \frac{1}{2}M \qquad Q_{\text{D1}} = \frac{1}{2}N \qquad \qquad \checkmark
$$

Stringy operators in T_N [Benini, Benvenuti, Tachikawa '09]

ext. 5-branes can be connected with D1-F1- $(1, -1)$ triple string junction

$$
\Delta = \frac{3}{2}(N-1) \qquad Q = \frac{1}{2}(N-1)
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4d version in [Gaiotto, Maldacena] (N, N, N) of $SU(N)^3$ global symmetry

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Triple junction in supergravity:

$$
z_{F1} = \xi
$$
 $z_{D1} = e^{\pi i/3} \xi$ $z_{(1,1)} = e^{2\pi i/3} \xi$
\n $\Delta = \frac{3}{2} N$ $Q = \frac{1}{2} N$

Agrees with T_N operator at large N. $\checkmark\checkmark$

Stringy operators in $+_{N,M,k}$

$$
[N] - (N)^{M - \frac{N}{k} - 1} - (N) - (N - k) - (N - 2k) - \dots - (k)
$$

\n|*y*
\n|*k*

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$$

\n
$$
[k]
$$

$$
\mathcal{O}_{\tilde{b}}^a = [x_1 \cdots x_{M - \frac{N}{k}} y]_{\tilde{b}}^a \qquad \Delta = \frac{3}{2} \left(M - \frac{N}{k} + 1 \right) \qquad Q = \frac{1}{3} \Delta
$$

$$
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D1, F1 in supergravity agree on Δ and Q at large N, M \checkmark

Spectrum of high-dimension operators

Similar stories for Y_N , $X_{N,M}$, \neq_N

Precise quantitative agreement of field theory analyses and supergravity results on scaling dimensions at large N

Confirms brane junction interpretation for solutions without and for solutions with monodromy.

Holographic S^5 partition functions and entanglement entropies, $#$ d.o.f.

First steps in AdS/CFT: Holographic interpretation consistent? Warped product, poles, renormalization?

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Computations free from conceptual subtleties – No divergences from poles, no finite contributions.

EE factorizes into geometric and theory-dependent part

$$
S_{\rm EE} = \frac{1}{4G_{\rm N}} \times \text{Area}(\gamma_4) \times \frac{8\text{Vol}_{\rm S^2}}{3} \int_{\Sigma} d^2w \ \kappa^2 \mathcal{G}
$$

 S^5 partition function from renormalized on-shell action

$$
S_{\text{IIB}}^{\text{E}} = -\frac{5\text{Vol}_{\text{S}^2}}{3G_N} \text{Vol}_{\text{AdS}_6,\text{ren}} \sum_{\ell \neq k,m \neq n} Z^{[\ell k]} Z^{[mn]}
$$

$$
\times \int_{-\infty}^{r_{\ell}} dx \ln \left| \frac{x - r_k}{r_{\ell} - r_k} \right| \ln \left| \frac{x - r_m}{r_m - r_n} \right| \frac{1}{x - r_n}
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$$

Related as expected, $S_{\rm EE}(\mathrm{disc})\big|_{\rm finite}=-\mathcal{F}(S^5)$ $)$

General scaling of $#$ d.o.f.

In general $\mathcal{F}(S^5)$ depends on all 5-brane charges in junction

Simple behavior under homogeneous rescaling of all charges:

$$
N_i \to \alpha N_i \quad \forall i \qquad \Longrightarrow \qquad \mathcal{F}(S^5) \to \alpha^4 \mathcal{F}(S^5)
$$

Compare $\alpha^{5/2}$ for $USp(N)$ theory from D4/D8/O8 [Jafferis, Pufu]

Examples

 \mathbb{Z}_m^N 5d T_N theory w/ gauge theory deformation $N-SU(N-1)\times\cdots\times SU(2)-2$

[Benini,Benvenuti,Tachikawa '09;Bergman,Zafrir '14]

$$
\mathcal{F}_{\text{sugra}}(S^5) = -\frac{27}{8\pi^2} \zeta(3) N^4
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 N 5d T_N theory w/ gauge theory deformation $N-SU(N-1)\times\cdots\times SU(2)-2$

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$$

NS5/D5 intersection: [Aharony, Hanany, Kol '97]

$$
N-SU(N)^{M-1}-N
$$

$$
\mathcal{F}_{\text{sugra}}(S^5) = -\frac{189}{16\pi^2} \zeta(3) N^2 M^2
$$

Field theory partition functions and precision tests of AdS_6/CFT_5

Strategy: study gauge theory deformation to obtain SCFT results. 5 d SYM partition function on (squashed) S^5 [Källen et al.; H.-C. Kim, S. Kim;Imamura;Lockhart,Vafa]

Expected large- N simplifications: instantons exponentially suppressed, saddle point approximation exact

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Remaining challenge for zero instanton partition functions:

$$
\mathcal{Z}_0 = \int_{-\infty}^{\infty} \prod_{i,j} d\lambda_i^{(j)} \exp(-\mathcal{F})
$$

Gauge group node becomes effectively continuous parameter at large N . Scaling of $\lambda_i^{(j)}$ $i^{(J)}$ not necessarily homogeneous.

Numerical evaluation [Herzog,Klebanov,Pufu,Tesileanu]: Replace saddle point eq. by set of particles w/ coordinates $\lambda_i^{(j)}$ $i^{(J)}$ in potential ${\cal F}$

$$
\frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}} = 0 \qquad \longrightarrow \qquad \frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}(t)} = -\frac{d \lambda_i^{(j)}(t)}{dt}
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Equilibrium configurations are solutions to saddle point equation. Approximate solutions from late-time behavior of $\lambda_i^{(j)}$ $\binom{U}{i}(t)$.

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 \rightarrow Zero instanton saddle point approximation to $\mathcal{F}(S^5)$, conformal central charge C_T from squashing deformations [Chang, Fluder, Lin, Wang]

Explicit numerical results for S^5 partition functions for

 $T_N: 2 \le N \le 52 +_{N,M}: 2 \le N, M \le 30$

Computationally more expensive conformal central charges:

 $T_N: \t2 \le N \le 22 \t +_{N,M}: \t2 \le N, M \le 15$

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 $+N_M$ consistency check: SCFT partition functions obtained from quiver and S-dual quiver should agree

Relative "error" $\rightarrow 0$ for large N, M. Below 1% for $N, M \geq 16$ \checkmark

Results for T_N :

Numerical results as dots, degree-4 polynomial from least-squares fit as dashed line. F_{S5} on the left, C_T on the right.

Clear quartic scaling, as predicted holographically.

Precision test of AdS_6/CFT_5

Comparison to supergravity:

Coefficients of leading terms, extracted via fit of numerical results to degree-4 polynomial, agree within 1%

Results for $+_{N,M}$: for fixed N clear quadratic scaling with M, and for fixed M clear quadratic scaling with N

Supergravity: $F_{{\rm S}^5}=-\frac{189}{16\pi^2}$ $\frac{189}{16\pi^2}\zeta(3)N^2M^2$ $C_T = \frac{7560}{\pi^4}$ $\frac{300}{\pi^4} \zeta(3) N^2 M^2$

Coefficients of leading terms, extracted via fit of numerical results to degree-2 polynomial in NM , agree within 2 $\%$

– Massive spin-2 excitations –

Identifying fluctuations non-trivial: non-trivial background, weak symmetry constraints \rightarrow large set of coupled PDE's on Σ .

Spin-2 fluctuations exclusively from metric \rightarrow decoupling trivial. Used for warped AdS_4 fluctuations by [Bachas, Estes '11].

5d SCFTs should have conserved $T_{\mu\nu} \sim$ massless AdS₆ graviton. Decoupling of states on external 5-branes?

Massive spin-2 excitations

Metric perturbation on unit-radius AdS_6 :

$$
ds^2 = f_6^2 \left(ds^2_{AdS_6} + \delta g \right) + \hat{g}_{ab} dy^a dy^b
$$

With $h^{[tt]}_{\mu\nu}$ transverse-traceless

$$
\delta g = h_{\mu\nu}^{[tt]} \phi_{\ell m} Y_{\ell m} dx^{\mu} dx^{\nu} \qquad \Box_{AdS_6} h_{\mu\nu}^{[tt]} = (M^2 - 2) h_{\mu\nu}^{[tt]}
$$

Type IIB EOM reduce to:

 $6\partial_a\big(\mathcal{G}^2\eta^{ab}\partial_b\phi_{\ell m}\big)-\ell(\ell+1)\left(9\kappa^2\mathcal{G}+6|\partial\mathcal{G}|^2\right)\phi_{\ell m}+M^2\kappa^2\mathcal{G}\phi_{\ell m}=0$

Massive spin-2 excitations

Two families of universal, regular and normalizable solutions:

(i)
$$
\phi_{\ell m} = \mathcal{G}^{\ell}
$$
 $M^2 = \Delta(\Delta - 5)$ $\Delta = 5 + 3\ell$

(ii)
$$
\phi_{\ell m}^{(1)} + i \phi_{\ell m}^{(2)} = \mathcal{G}^{\ell} \left(\mathcal{A}_{+} - \bar{\mathcal{A}}_{-} \right) \qquad \Delta = 6 + 3\ell
$$

 $M^2 \geq 3\ell(3\ell + 5)$ for regular solutions, saturated by (i). Massless graviton for $M = \ell = 0 \sim 5d$ conserved $T_{\mu\nu}$.

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Match Q^4 descendants in B_2 , A_4 multiplets with scalar primaries $\Delta_{B_2}^{\text{prim.}}$ $\frac{\text{prim.}}{B_2} = 3 + 3\ell$ and $\Delta_{A_4}^{\text{prim.}}$ $A_4^{PIII...}=4+3\ell$ [Cordova,Dumitrescu,Intriligator]

Universal part in spectrum of dual SCFTs. Similarities to massive IIA [Passias,Richmond '18], different in details and interpretation.

– Summary & Outlook –

Holographic interpretation of solutions appears consistent: poles/external 5-branes decouple, 5d conserved $T_{\mu\nu}$

Quantitative match of stringy operators in supergravity solutions to string theory picture and field theory predictions.

Field theory partition functions and conformal central charges match supergravity predictions for T_N and $+_{N}M$.

First lessons: N^4 d.o.f., universal $\Delta = \mathcal{O}(1)$ operators, stringy spectrum for theories with no suitable gauge theory deformations.

Outlook

Compelling picture for AdS_6/CFT_5 in Type IIB: large classes of solutions, coherent string theory interpretation, match to SCFTs.

Full fluctuation spectrum $\sim \Delta = \mathcal{O}(1)$ operators. (Consistent) KK reduction to 6d gauged supergravity? Non-universal parts?

More stringy operators and op.'s corresponding to e.g. D3 branes.

Correlation functions, defects, Wilson lines, more tests, . . . Classification of large- N SCFTs?

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Thank you!