AdS_6/CFT_5 in Type IIB Part II: Dualities, tests and applications

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Strings, Branes and Gauge Theories APCTP, July 2018

arXiv: 1606.01254, 1611.09411, 1703.08186, 1705.01561, 1706.00433, 1802.07274, 1805.11914, 1806.07898, 1806.08374

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Introduction

Higher-dimensional CFTs are interesting:

- mysterious, non-Lagrangian
- interesting implications for $d \leq 4$
- string/M-theory connections ...



5d SCFTs: often fixed points for asymptotically safe gauge theories, large classes engineered in Type IIB via 5-brane webs

Recap

Any planar 5-brane junction realizes 5d SCFT on intersection point



Characterized entirely by external 5-brane charges. May or may not have gauge theory deformations.



Supergravity solutions for fully localized 5-brane intersections, constructed directly as "near-horizon" limit:



Recap

Supergravity solutions for fully localized 5-brane intersections, constructed directly as "near-horizon" limit:



- $\begin{array}{l} \ AdS_6 \times S^2 \times \Sigma \ \text{with} \ \Sigma = \text{disc} \\ \text{5-branes emerge at poles on} \ \partial \Sigma \end{array}$
- parametrized entirely by residues
 =conserved 5-brane charges

$$-\operatorname{AdS}_6 + 16 \text{ susies} = F(4)$$

Naturally fit to string theory picture. "Large N": all (p_i, q_i) large.

Recap

Claim: poles \sim groups of 5-branes that are unconstrained by *s*-rule. Constrained junctions realized by bringing 7-branes into web:



Supergravity: $\Sigma = \text{disc with puncture(s)}, [p,q]$ 7-brane monodromy

Position of puncture on $\Sigma \leftrightarrow$ choice of face in which 7-brane is placed with no 5-branes attached(?)

$\mathsf{AdS}_6/\mathsf{CFT}_5$ in Type IIB

Goal of this talk: convincing case for proposed identifications and dualities, first lessons on 5d SCFTs

$\mathsf{AdS}_6/\mathsf{CFT}_5$ in Type IIB

Outline

- Testing AdS_6/CFT_5 with stringy operators
- Holographic S^5 partition functions and EE, # d.o.f.
- QFT partition functions: Precision test of $\mathsf{AdS}_6/\mathsf{CFT}_5$
- (Massive) spin-2 fluctuations

– Testing AdS_6/CFT_5 with strings –

Testing AdS_6/CFT_5 with strings

5-brane picture: gauge invariant operators from strings and string junctions connecting external 5-branes



Supergravity: operators with $\Delta = \mathcal{O}(N) \leftrightarrow$ probe strings or string junctions in supergravity background $(1 \ll \Delta \ll \mathcal{F}(S^5))$.

Strategy: study gauge theory deformations, identify stringy BPS operators, extrapolate to SCFT, compare to supergravity

Stringy operators in $+_{N,M}$ theories



$$\begin{split} [N] &- SU(N)^{M-1} - [N] \\ SU(N)^2 \times U(1)^M_B \times U(1)^{M-1}_I \\ &\subset SU(N)^2 \times SU(M)^2 \times U(1) \end{split}$$

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$$M_{j}^{i} = (x^{(1)} \cdots x^{(M)})_{j}^{i} \qquad (\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}, \mathbf{1}) \qquad \Delta = \frac{3}{2}M \qquad Q = \frac{1}{2}M$$
$$B^{(k)} = \det(x^{(k)}) \qquad \subset (\mathbf{1}, \mathbf{1}, \mathbf{M}, \bar{\mathbf{M}}) \qquad \Delta = \frac{3}{2}N \qquad Q = \frac{1}{2}N$$

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 $M_j^i \sim$ F1 between D5, $B^{(k)} \subset$ D1 between NS5. S-dual quiver deformation realizes all D1 operators, subset of F1 operators.







global AdS₆/CFT₅ on $\mathbb{R} \times S^4$ AdS₆ $\partial_t \leftrightarrow$ dilation: $\Delta = \mathcal{H}$





global AdS₆/CFT₅ on $\mathbb{R} \times S^4$ AdS₆ $\partial_t \leftrightarrow$ dilation: $\Delta = \mathcal{H}$

Action and EOM for (p,q) string along t and Σ :

$$\begin{split} S_{(p,q)} &= -2T \int d^2 \xi f_6 \rho |w'| \sqrt{\mathfrak{q} \mathcal{M} \mathfrak{q}} \qquad \mathfrak{q} \mathcal{M} \mathfrak{q} = e^{2\phi} \begin{pmatrix} p \\ q \end{pmatrix} \begin{pmatrix} 1 & -\chi \\ -\chi & \chi^2 + e^{-4\phi} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \\ 0 &= \frac{\bar{w}''}{\bar{w}'} - \frac{w''}{w'} + \left(\bar{w}' \partial_{\bar{w}} - w' \partial_w \right) \ln \left(f_6^2 \rho^2 \mathfrak{q}^T \mathcal{M} \mathfrak{q} \right) \end{split}$$





global AdS_6/CFT_5 on $\mathbb{R} \times S^4$ $AdS_6 \ \partial_t \leftrightarrow dilation: \ \Delta = \mathcal{H}$

F1 between D5 poles, D1 between NS5 poles:

$$\Delta_{\rm F1} = \frac{3}{2}M \qquad \qquad \Delta_{\rm D1} = \frac{3}{2}N$$

Solve EOM, scaling dimensions match field theory exactly

R-charge of string states from coupling to SU(2) bulk gauge field fluctuation dual to R-symmetry current (S^2 isometries).

Relevant part from KK ansatz + SU(2) gauge invariance:

 $dx^{\mu} \to dx^{\mu} + K_{I}^{\mu} A^{I} \qquad K_{I} \sim S^{2} \text{ Killing vector fields}$ $\delta C_{(2)} = d\mathcal{C} \wedge f_{I} A^{I} + \dots \qquad df_{I} = \star_{S^{2}} K_{I}$

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(p,q) strings couple to $\delta C_{(2)}$ through WZ term

$$Q_{(p,q)} = T \int_{\Sigma_{(p,q)}} \left(p \operatorname{Re}(d\mathcal{C}) - q \operatorname{Im}(d\mathcal{C}) \right)$$

$$\implies Q_{\rm F1} = \frac{1}{2}M \qquad \qquad Q_{\rm D1} = \frac{1}{2}N \qquad \qquad \checkmark\checkmark$$

Stringy operators in T_N



[Benini,Benvenuti,Tachikawa '09]

ext. 5-branes can be connected with D1-F1-(1, -1) triple string junction

$$\Delta=\frac{3}{2}(N-1) \qquad Q=\frac{1}{2}(N-1)$$

 $(\mathbf{N}, \mathbf{N}, \mathbf{N})$ of $SU(N)^3$ global symmetry 4d version in [Gaiotto,Maldacena]

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Triple junction in supergravity:

$$z_{\rm F1} = \xi$$
 $z_{\rm D1} = e^{\pi i/3} \xi$ $z_{(1,1)} = e^{2\pi i/3} \xi$
 $\Delta = \frac{3}{2}N$ $Q = \frac{1}{2}N$

Agrees with T_N operator at large N.



[Benini,Benvenuti,Tachikawa '09]

Stringy operators in $+_{N,M,k}$



$$[N] - (N)^{M - \frac{N}{k} - 1} - (N) - (N - k) - (N - 2k) - \dots - (k)$$
$$|y$$
$$[k]$$

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$$[k]$$
$$\mathcal{O}_{\tilde{b}}^{a} = [x_{1} \cdots x_{M - \frac{N}{k}} y]_{\tilde{b}}^{a} \qquad \Delta = \frac{3}{2} \left(M - \frac{N}{k} + 1\right) \qquad Q = \frac{1}{3} \Delta$$

$$\mathcal{O}_{(i)} = \det x_i$$
 $\Delta = \frac{3}{2}N$ $Q = \frac{1}{2}N$

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D1, F1 in supergravity agree on Δ and Q at large N, M

Spectrum of high-dimension operators

Similar stories for Y_N , $X_{N,M}$, $\not\prec_N$



Precise quantitative agreement of field theory analyses and supergravity results on scaling dimensions at large ${\cal N}$

Confirms brane junction interpretation for solutions without and for solutions with monodromy.

Holographic S^5 partition functions and entanglement entropies, # d.o.f.

First steps in AdS/CFT: Holographic interpretation consistent? Warped product, poles, renormalization?

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Computations free from conceptual subtleties – No divergences from poles, no finite contributions.

EE factorizes into geometric and theory-dependent part

$$S_{\rm EE} = \frac{1}{4G_{\rm N}} \times \operatorname{Area}(\gamma_4) \times \frac{8 \operatorname{Vol}_{\mathrm{S}^2}}{3} \int_{\Sigma} d^2 w \; \kappa^2 \mathcal{G}$$

 S^5 partition function from renormalized on-shell action

$$S_{\text{IIB}}^{\text{E}} = -\frac{5\text{Vol}_{\text{S}^2}}{3G_{\text{N}}}\text{Vol}_{\text{AdS}_6,\text{ren}} \sum_{\ell \neq k, m \neq n} Z^{[\ell k]} Z^{[mn]} \\ \times \int_{-\infty}^{r_\ell} dx \ln \left| \frac{x - r_k}{r_\ell - r_k} \right| \ln \left| \frac{x - r_m}{r_m - r_n} \right| \frac{1}{x - r_m}$$

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Related as expected, $S_{\rm EE}({
m disc}) \big|_{\rm finite} = - {\cal F}(S^5)$

1

General scaling of # d.o.f.

In general $\mathcal{F}(S^5)$ depends on all 5-brane charges in junction



Simple behavior under homogeneous rescaling of all charges:

$$N_i \to \alpha N_i \quad \forall i \qquad \Longrightarrow \qquad \mathcal{F}(S^5) \to \alpha^4 \mathcal{F}(S^5)$$

Compare $\alpha^{5/2}$ for USp(N) theory from D4/D8/O8 [Jafferis,Pufu]

Examples



5d T_N theory w/ gauge theory deformation $N - SU(N-1) \times \cdots \times SU(2) - 2$

[Benini,Benvenuti,Tachikawa '09;Bergman,Zafrir '14]

$$\mathcal{F}_{\text{sugra}}(S^5) = -\frac{27}{8\pi^2}\,\zeta(3)N^4$$

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NS5/D5 intersection:

[Aharony, Hanany, Kol '97]

$$N - SU(N)^{M-1} - N$$

$$\mathcal{F}_{sugra}(S^5) = -\frac{189}{16\pi^2}\,\zeta(3)N^2M^2$$



Field theory partition functions and precision tests of AdS_6/CFT_5

Expected large-N simplifications: instantons exponentially suppressed, saddle point approximation exact

Strategy: study gauge theory deformation to obtain SCFT results. 5d SYM partition function on (squashed) S^5 [Källen et al.; H.-C. Kim, S. Kim;Imamura;Lockhart,Vafa]

Expected large-N simplifications: instantons exponentially suppressed, saddle point approximation exact

Remaining challenge for zero instanton partition functions:

$$\mathcal{Z}_{0} = \int_{-\infty}^{\infty} \prod_{i,j} \mathrm{d}\lambda_{i}^{(j)} \exp\left(-\mathcal{F}\right)$$

Gauge group node becomes effectively continuous parameter at large N. Scaling of $\lambda_i^{(j)}$ not necessarily homogeneous.

Numerical evaluation [Herzog,Klebanov,Pufu,Tesileanu]: Replace saddle point eq. by set of particles w/ coordinates $\lambda_i^{(j)}$ in potential \mathcal{F}

$$\frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}} = 0 \qquad \longrightarrow \qquad \frac{\partial \mathcal{F}}{\partial \lambda_i^{(j)}(t)} = -\frac{d\lambda_i^{(j)}(t)}{dt}$$

1.

Equilibrium configurations are solutions to saddle point equation. Approximate solutions from late-time behavior of $\lambda_i^{(j)}(t)$.

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 \rightarrow Zero instanton saddle point approximation to $\mathcal{F}(S^5)$, conformal central charge C_T from squashing deformations [Chang,Fluder,Lin,Wang]

Explicit numerical results for $S^{5}% ^{2}$ partition functions for

 $T_N: \quad 2 \le N \le 52 \quad +_{N,M}: \quad 2 \le N, M \le 30$

Computationally more expensive conformal central charges:

 $T_N: \quad 2 \le N \le 22 \quad +_{N,M}: \quad 2 \le N, M \le 15$

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 $+_{N,M}$ consistency check: SCFT partition functions obtained from quiver and S-dual quiver should agree



Relative "error" $\rightarrow 0$ for large N, M. Below 1% for $N, M \geq 16$

Results for T_N :



Numerical results as dots, degree-4 polynomial from least-squares fit as dashed line. F_{S^5} on the left, C_T on the right.

Clear quartic scaling, as predicted holographically.

Precision test of AdS_6/CFT_5

Comparison to supergravity:



Coefficients of leading terms, extracted via fit of numerical results to degree-4 polynomial, agree within 1%

Results for $+_{N,M}$: for fixed N clear quadratic scaling with M, and for fixed M clear quadratic scaling with N



Supergravity: $F_{\mathrm{S}^5} = -\frac{189}{16\pi^2} \zeta(3) N^2 M^2$ $C_T = \frac{7560}{\pi^4} \zeta(3) N^2 M^2$

Coefficients of leading terms, extracted via fit of numerical results to degree-2 polynomial in NM, agree within $2\%_{oo}$

- Massive spin-2 excitations -

Identifying fluctuations non-trivial: non-trivial background, weak symmetry constraints \rightarrow large set of coupled PDE's on $\Sigma.$

Spin-2 fluctuations exclusively from metric \rightarrow decoupling trivial. Used for warped AdS_4 fluctuations by [Bachas,Estes '11].

5d SCFTs should have conserved $T_{\mu\nu} \sim$ massless AdS₆ graviton. Decoupling of states on external 5-branes?

Massive spin-2 excitations

Metric perturbation on unit-radius AdS₆:

$$ds^{2} = f_{6}^{2} \left(ds_{AdS_{6}}^{2} + \delta g \right) + \hat{g}_{ab} dy^{a} dy^{b}$$

With $h_{\mu\nu}^{[tt]}$ transverse-traceless

$$\delta g = h_{\mu\nu}^{[tt]} \phi_{\ell m} Y_{\ell m} dx^{\mu} dx^{\nu} \qquad \Box_{AdS_6} h_{\mu\nu}^{[tt]} = (M^2 - 2) h_{\mu\nu}^{[tt]}$$

Type IIB EOM reduce to:

 $6\partial_a \left(\mathcal{G}^2 \eta^{ab} \partial_b \phi_{\ell m} \right) - \ell(\ell+1) \left(9\kappa^2 \mathcal{G} + 6|\partial \mathcal{G}|^2 \right) \phi_{\ell m} + M^2 \kappa^2 \mathcal{G} \phi_{\ell m} = 0$

Massive spin-2 excitations

Two families of *universal*, regular and normalizable solutions:

(i)
$$\phi_{\ell m} = \mathcal{G}^{\ell}$$
 $M^2 = \Delta(\Delta - 5)$ $\Delta = 5 + 3\ell$

(ii)
$$\phi_{\ell m}^{(1)} + i\phi_{\ell m}^{(2)} = \mathcal{G}^{\ell} \left(\mathcal{A}_{+} - \bar{\mathcal{A}}_{-} \right) \qquad \Delta = 6 + 3\ell$$

 $M^2 \ge 3\ell(3\ell+5)$ for regular solutions, saturated by (i). Massless graviton for $M = \ell = 0 \sim 5d$ conserved $T_{\mu\nu}$.

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 $\begin{array}{l} \text{Match } Q^4 \text{ descendants in } B_2 \text{, } A_4 \text{ multiplets with scalar primaries} \\ \Delta^{\text{prim.}}_{B_2} = 3 + 3\ell \text{ and } \Delta^{\text{prim.}}_{A_4} = 4 + 3\ell \quad \text{[Cordova,Dumitrescu,Intriligator]} \end{array}$

Universal part in spectrum of dual SCFTs. Similarities to massive IIA [Passias, Richmond '18], different in details and interpretation.

– Summary & Outlook –



Holographic interpretation of solutions appears consistent: poles/external 5-branes decouple, 5d conserved $T_{\mu\nu}$

Quantitative match of stringy operators in supergravity solutions to string theory picture and field theory predictions.

Field theory partition functions and conformal central charges match supergravity predictions for T_N and $+_{N,M}$.

First lessons: N^4 d.o.f., universal $\Delta = O(1)$ operators, stringy spectrum for theories with no suitable gauge theory deformations.

Outlook

Compelling picture for AdS_6/CFT_5 in Type IIB: large classes of solutions, coherent string theory interpretation, match to SCFTs.

Full fluctuation spectrum $\sim \Delta = O(1)$ operators. (Consistent) KK reduction to 6d gauged supergravity? Non-universal parts?

More stringy operators and op.'s corresponding to e.g. D3 branes.

Correlation functions, defects, Wilson lines, more tests, \dots Classification of large-N SCFTs?

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Thank you!