Bootstrapping the 3D minimal $\mathcal{N} = 1$ superconformal field theories

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Modern conformal bootstrap

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High Energy Physics - Theory

Bounding scalar operator dimensions in 4D CFT

Riccardo Rattazzi, Vyacheslav S. Rychkov, Erik Tonni, Alessandro Vichi

(Submitted on 1 Jul 2008 (v1), last revised 11 Dec 2008 (this version, v2))

In an arbitrary unitary 4D CFT we consider a scalar operator \phi, and the operator \phi^2 defined as the lowest dimension scalar which appears in the OPE \phi\times\phi with a nonzero coefficient. Using general considerations of OPE, conformal block decomposition, and crossing symmetry, we derive a theory-independent inequality [\phi^2] \leq f([\phi]) for the dimensions of these two operators. The function f(d) entering this bound is computed numerically. For d->1 we have f(d)=2+O(\sqrt{d-1}), which shows that the free theory limit is approached continuously. We perform some checks of our bound. We find that the bound is

Conformal Field Theory

Two-point and three-point functions are fixed conformal symmetry

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta_\phi}}$$

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\rangle = \frac{\lambda_{123}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}|x_3-x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

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Four-point functions are fixed up to a function of cross ratio

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{f(u,v)}{x_{12}^{2\Delta_\phi}x_{34}^{\Delta_\phi}}$$

where

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Conformal Field Theory

Consider Operator Product Expansion

$$\phi_i(x)\phi_j(y) = \sum_a \lambda_{ija} C_a(x-y, \partial_y)\mathcal{O}^a$$

where $\mathcal{O}^a$ is a (quasi-)primary operator.

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$$\begin{aligned}
&\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \\
&= \sum_{\mathcal{O}\in\phi\times\phi} \lambda^2_{12\mathcal{O}} C_a(x_1-x_2, \partial_2) C_b(x_3-x_4, \partial_4)\langle \phi(x_3)\phi(x_4)\rangle \\
&= \sum_{\mathcal{O}\in\phi\times\phi} x_{12}^{-\Delta_\phi} x_{34}^{-\Delta_\phi} \lambda^2_{12\mathcal{O}} \times g_{\Delta_{\mathcal{O}}, l_{\mathcal{O}}}(u, v)
\end{aligned}$$

Conformal Block [Dolan,Osborn '01,'04]

$$g_{\Delta_{\mathcal{O}},l_{\mathcal{O}}}(u,v) = k_{\Delta+l}(z)k_{\Delta-l}(\bar{z} + z \leftrightarrow \bar{z}) \qquad \text{in D=2}$$

$$g_{\Delta_{\mathcal{O}},l_{\mathcal{O}}}(u,v) = \frac{z\bar{z}}{z-\bar{z}}(k_{\Delta+l}(z)k_{\Delta-l-2}(\bar{z}) - z \leftrightarrow \bar{z}) \qquad \text{in D=4}$$

where $u = z\bar{z}, v = (1-z)(1-\bar{z})$ and

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One can forget about Lagrangian and describe a CFT merely by its spectrum and OPE coefficient. Is that all?

Four-point functions have crossing symmetry

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle,$$

which leads to

$$u^{-\Delta_\phi} \sum_{\mathcal{O}\in\phi\times\phi} \lambda^2_{\phi\phi\mathcal{O}} \times g_{\Delta_{\mathcal{O}},l_{\mathcal{O}}}(u,v) = v^{-\Delta_\phi} \sum_{\mathcal{O}\in\phi\times\phi} \lambda^2_{\phi\phi\mathcal{O}} \times g_{\Delta_{\mathcal{O}},l_{\mathcal{O}}}(v,u)$$

Conformal Bootstrap

Define convolved conformal block $F_{\Delta,l} = u^{-\Delta_\phi} g_{\Delta,l}(u,v) - v^{-\Delta} g_{\Delta,l}(v,u)$, we get

$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 F_{\Delta_{\mathcal{O}},l_{\mathcal{O}}}(z,\bar{z}) = 0$$

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Assumption

All $l = 0$ primaries operators in $\phi \times \phi$ OPE has scaling dimension $\Delta \geq \Delta_0$.

Suppose

$$\begin{aligned} & F_{0,0} > 0, \\ & F_{\Delta,0} > 0, \quad \text{when } \Delta > \Delta_0, \\ \text{and} \quad & F_{\Delta,l} > 0, \quad \text{when } \Delta > l + 2 (\text{Unitary bound}), \end{aligned}$$

then the assumption is excluded!

Conformal bootstrap

In more complicated cases, we need to consider linear functional acting on $\sum_{\mathcal{O}\in\phi\times\phi}\lambda_{\mathcal{O}}^2 F_{\Delta,l}(z,\bar{z})=0$.

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&\alpha(F_{0,0}(z,\bar{z}))=1,\\
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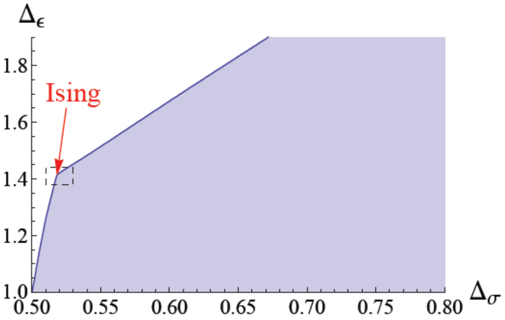
A simple basis is $\alpha=\sum\alpha_{mn}\partial_z^m\partial_{\bar{z}}^n$, and the problem could be studied using "SDPB". [Simmons-Duffin '15]

Conformal bootstrap

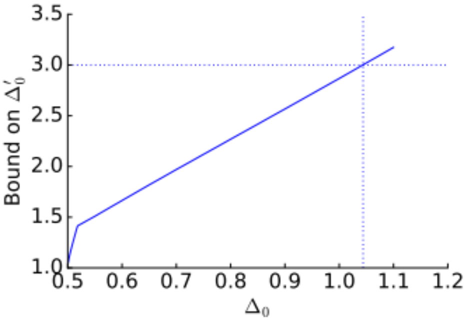
Applied to 3D Ising model, one get



[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin,Vichi '12]

Constraining critical exponents

Same plot at wider range [Nakayama, Ohtsuki '16]



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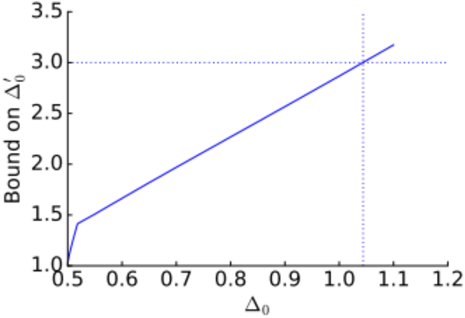

One need to allow ϵ to appear in $\epsilon \times \epsilon$ OPE.
This simple tells us that for any 2nd order phase transition that could be reached without fine-tunning, the critical exponents need to satisfy

$$\nu > 0.511$$

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A model of special interest is called model studied is call JQ model [Sandvik '07]. It describes the quantum phase transition from Neel phase to VBS pahse, with lattice Hamiltonian:

$$H = -J \sum_{\langle ij \rangle} P_{ij} - Q \sum_{\langle ijkl \rangle} P_{ij} P_{kl}$$

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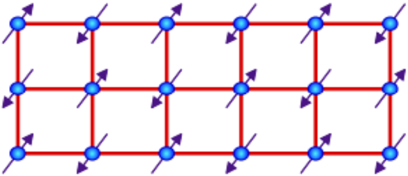

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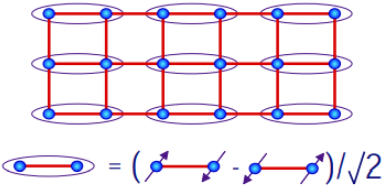
VBS phase:



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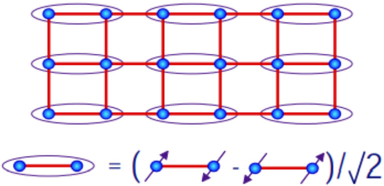

It is believed that these model would flow to the IR critical point of scalar-QED_3.

$$\mathcal{L} = \frac{1}{e^2} F_{\mu\nu} F^{\mu\nu} + |D\Phi_I|^2 + m^2 |\Phi_I|^2 + \lambda |\Phi_I|^4$$

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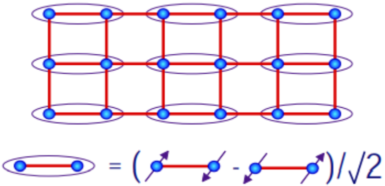

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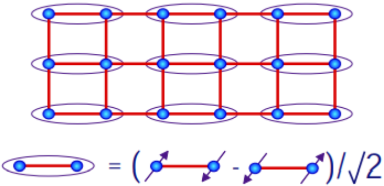

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The theory has $SU(N) \times U(1)$ symmetry, where $U(1)$ is the monopole charge. Large N calculation can be performed. The fate of the small N fixed points are not clear.

Constraining critical exponents

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See Dongmin's talk for a similar story in SCFT setup.

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Liquid-gas transition vs Magnetization

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$$O \propto \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

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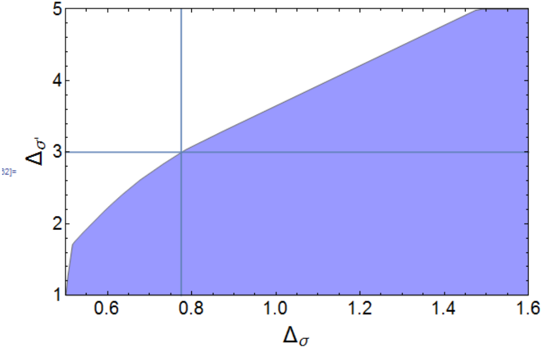
The SO(5) singlet operator must be irrelevant!

Emergent symmetry



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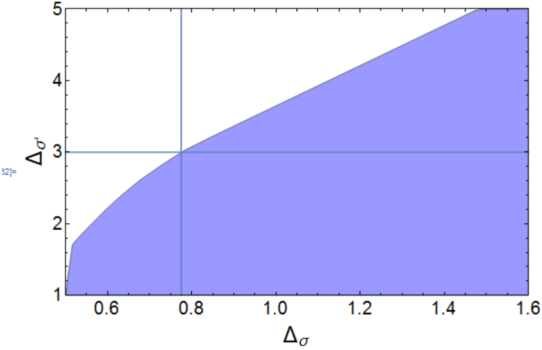

Bootstrap tells that $\Delta > 0.775$ or $\eta > 0.55$. This result was first discussed in [Nakayama, Ohtsuki '16].

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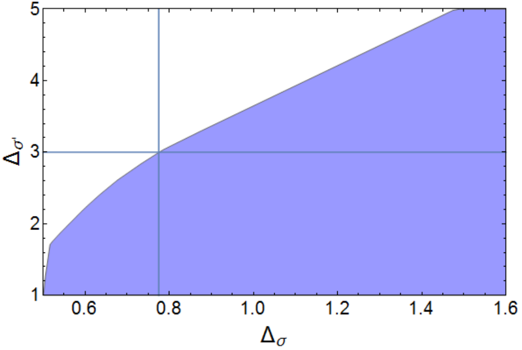

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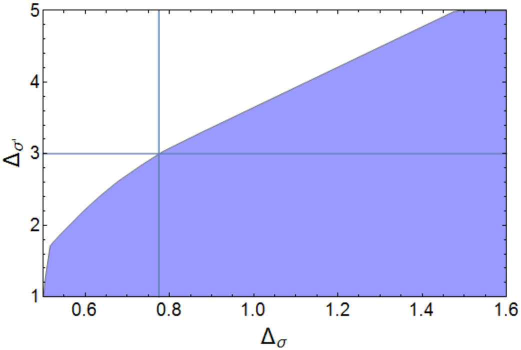

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Notice that there is indeed a paper claiming that the phase transition is 1st order [Chen, Huang, Deng, Kuklov, Prokofev, Svistunov '13].

Emergent SUSY

The candidate theory is simply

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \bar{\psi}\partial\!\!\!/\psi + \frac{\lambda_1}{2}\sigma\bar{\psi}\psi + \frac{\lambda_2^2}{8}\sigma^4.$$

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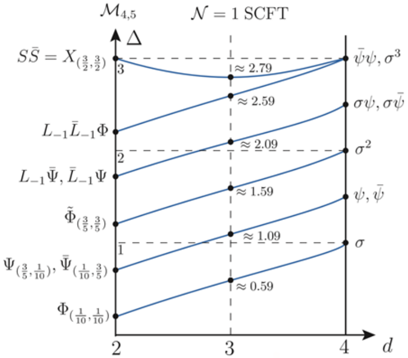
It was argued in [Grover, Sheng, Vishwanath '15] that it can be realized at the boundary of a 3+1D topological superconductor. Emergent SUSY is critical for experimental realization.

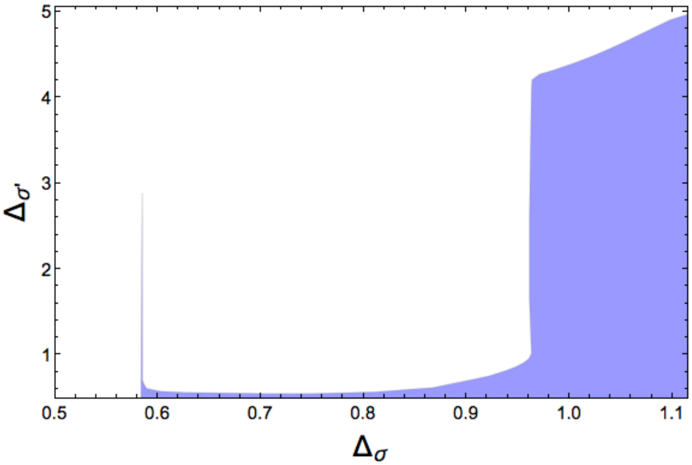
Emergent SUSY



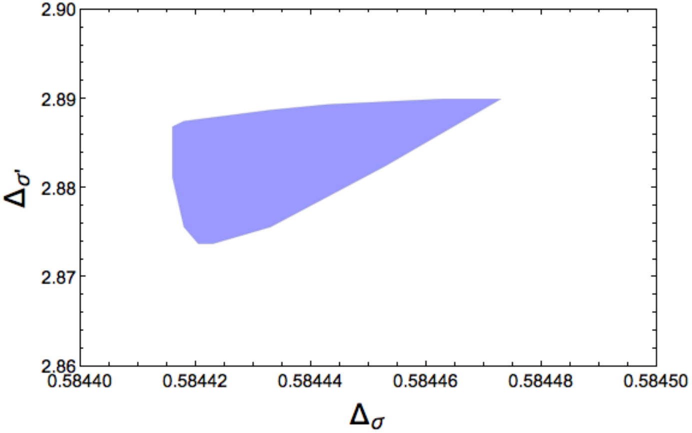
[Fei, Giombi, Klebanov, Tarnopolsky '16]

Emergent SUSY

Imposing emergent SUSY in numerical bootstrap, we get [Rong, Su '18]



Assuming the spectrum to contain only two T-parity even scalar, we get a bootstrap island



details of the calculation

Bootstrap equation for Ising model

$$\sum_{O^+} \begin{pmatrix} \lambda_{\sigma\sigma O} & \lambda_{\epsilon\epsilon O} \end{pmatrix} \vec{V}_{+,\Delta,\ell} \begin{pmatrix} \lambda_{\sigma\sigma O} \\ \lambda_{\epsilon\epsilon O} \end{pmatrix} + \sum_{O^-} \lambda^2_{\sigma\epsilon O} \vec{V}_{-,\Delta,\ell} = 0,$$

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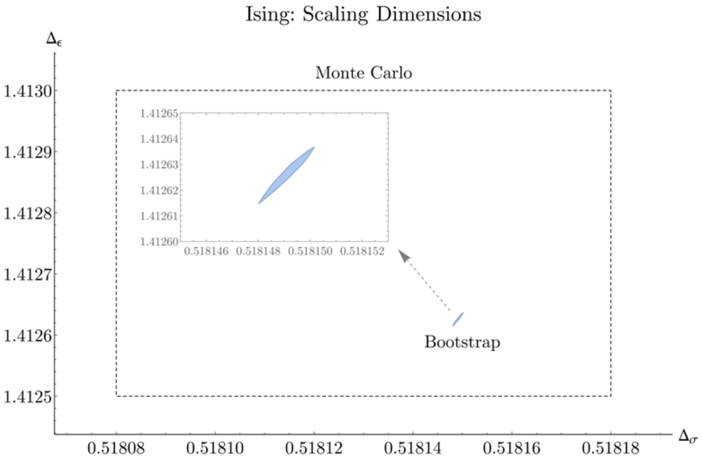
where
$$\vec{V}_{+,\Delta,\ell} = \begin{pmatrix} \begin{pmatrix} F^{\sigma\sigma,\sigma\sigma}_{-,\Delta,\ell}(u,v) & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & F^{\epsilon\epsilon,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{-,\Delta,\ell}(u,v) & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\sigma\sigma,\epsilon\epsilon}_{+,\Delta,\ell}(u,v) & 0 \end{pmatrix} \end{pmatrix}, \quad \vec{V}_{-,\Delta,\ell} = \begin{pmatrix} 0 \\ 0 \\ F^{\sigma\epsilon,\sigma\epsilon}_{-,\Delta,\ell}(u,v) \\ (-1)^{\ell} F^{\epsilon\sigma,\sigma\epsilon}_{-,\Delta,\ell}(u,v) \\ -(-1)^{\ell} F^{\epsilon\sigma,\sigma\epsilon}_{+,\Delta,\ell}(u,v) \end{pmatrix},$$

Assuming the spectrum to contain one Z_2 even and one Z_2 odd relevant operators, we get [Kos, Poland, Simmons-Duffin, Vichi '16]



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Constrains from SUSY

- $\Delta_\epsilon = \Delta_\sigma + 1$, since $\Sigma = \sigma + \bar{\theta}\psi + \bar{\theta}\theta\epsilon$, Constrains from SUSY

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The line $\Delta_\epsilon = \Delta_\sigma + 1$ intersect with single correlator bound at $\Delta_\sigma \approx 0.565$, which is a lower bound for Δ_σ.

details of the calculation

To get the OPE relation, we use the result of [Park '99].

$$\langle \mathcal{O}^{(l)}(x_1,\theta_1,\eta_1)\Sigma(x_2,\theta_2)\Sigma(x_3,\theta_3)\rangle = \frac{t(\mathrm{X}_1,\Theta_1,\eta_1)}{x_{12}^{2\Delta_\phi-\Delta_\mathcal{O}-l}x_{13}^{2\Delta_\phi-\Delta_\mathcal{O}-l}x_{23}^{\Delta_\mathcal{O}+l}},$$

$$x_{12}^\mu = x_1^\mu + x_2^\mu + \mathrm{i}\bar{\theta}_1\gamma_\mu\theta_2, \quad \mathrm{x}_{12\pm} = x_{12}^\mu\gamma_\mu \pm \mathrm{i}\frac{1}{2}\bar{\theta}_{12}\theta_{12}, \quad \theta_{12} = \theta_1 - \theta_2,$$

$$\mathrm{X}_1 = \frac{1}{2}(\mathrm{x}_{31+}^{-1}\mathrm{x}_{23-}\mathrm{x}_{21+}^{-1} + \mathrm{x}_{21+}^{-1}\mathrm{x}_{23+}\mathrm{x}_{31-}^{-1}), \quad \Theta_1 = \mathrm{i}(\mathrm{x}_{21+}^{-1}\theta_{21} - \mathrm{x}_{31+}^{-1}\theta_{31}).$$

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X_1 and Θ_1 are in tangent space of point 1. Under superconformal transformation, they transform as $(x_1,\theta_1) \xrightarrow{g} (x_1',\theta_1')$

$$\Theta_1' = \Omega^{-1/2}(x_1,\theta_1;g)L^{-1}(x_1,\theta_1;g)\Theta_1$$
$$X'^\mu_1 = \Omega^{-1}(x_1,\theta_1;g){R^\mu}_\nu{}^{-1}(x_1,\theta_1;g)X_1^\nu$$

details of the calculation

There are four tensor structures

$$\begin{aligned}
\mathcal{B}_+^{(l)} &: (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^l, \\
\mathcal{B}_-^{(l)} &: \bar{\Theta}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^l (\mathrm{tr}[\mathrm{X}_1^2])^{-1/2}, \\
\mathcal{F}_+^{(j)} &: \bar{\eta}_1 \mathrm{X}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^{j-1/2} (\mathrm{tr}[\mathrm{X}_1^2])^{-3/4}, \\
\mathcal{F}_-^{(j)} &: \bar{\eta}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^{j-1/2} (\mathrm{tr}[\mathrm{X}_1^2])^{-1/4}.
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\mathcal{B}_{+}^{(l)} &: (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^l, \\
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\mathcal{F}_{-}^{(j)} &: \bar{\eta}_1 \Theta_1 (\bar{\eta}_1 \mathrm{X}_1 \eta_1)^{j-1/2} (\mathrm{tr}[\mathrm{X}_1^2])^{-1/4}.
\end{aligned}
$$

The super multiplets contains

$$
\mathcal{B}_{+/-}^{(l)} : [l]_{\Delta}^{+/-} \xrightarrow{Q} [l \pm 1/2]_{\Delta+1/2} \xrightarrow{Q} [l]_{\Delta+1}^{-/+}, \qquad \text{with } l = \text{integer},
$$

$$
\mathcal{F}_{+/-}^{(j)} : [j]_{\Delta} \xrightarrow{Q} \begin{matrix} [j-1/2]_{\Delta+1/2}^{+/-} \\ [j+1/2]_{\Delta+1/2}^{-/+} \end{matrix} \xrightarrow{Q} [j+1]_{\Delta+1}. \quad \text{with } j = \text{half integer}.
$$

details of the calculation

For example, since $\mathcal{O} = O_+ + \ldots + \bar{\theta}\theta O_-$, we have

$$\begin{aligned}\langle \mathcal{O}(x_1,\theta_1)\Phi(x_2,\theta_2)\Phi(x_3,\theta)\rangle = & \langle O_+\sigma\sigma\rangle \\ & +\bar{\theta}_2\theta_2\bar{\theta}_3\theta_3\langle O_+\epsilon\epsilon\rangle \\ & +\bar{\theta}_1\theta_1\bar{\theta}_2\theta_2\langle O_-\epsilon\sigma\rangle \\ & \ldots\end{aligned}$$

In summary, the OPE rations are

\	Even channel	Odd channel
$\mathcal{B}_+$	$\frac{\lambda_{OFF}}{\lambda_{O\phi\phi}} = \frac{(-1-l-\Delta O+2\,\Delta\phi)\,(l-\Delta O+2\,\Delta\phi)}{2\,\Delta\phi\,(-1+2\,\Delta\phi)}$	$\left(\frac{\lambda_{PF\phi}}{\lambda_{O\phi\phi}}\right)^2 = \frac{(-1+\Delta O)\,(-1-l+\Delta O)\,(l+\Delta O)}{4\,(-1+2\,\Delta O)\,\Delta\phi\,(-1+2\,\Delta\phi)}$
$\mathcal{B}_-$	$\frac{\lambda_{PFF}}{\lambda_{P\phi\phi}} = \frac{(-3-l+\Delta O+2\,\Delta\phi)\,(-2+l+\Delta O+2\,\Delta\phi)}{2\,\Delta\phi\,(-1+2\,\Delta\phi)}$	$\left(\frac{\lambda_{OF\phi}}{\lambda_{P\phi\phi}}\right)^2 = \frac{(-1-l+\Delta O)\,(l+\Delta O)\,(-1+2\,\Delta O)}{(-1+\Delta O)\,\Delta\phi\,(-1+2\,\Delta\phi)}$
$\mathcal{F}_+$	$\frac{\lambda_{O_{j+1}\,FF}}{\lambda_{O_{j+1}\,\phi\phi}} = \frac{(1+j-\Delta j+2\,\Delta\phi)\,(-2+j+\Delta j+2\,\Delta\phi)}{2\,\Delta\phi\,(-1+2\,\Delta\phi)}$	$\left(\frac{\lambda_{O_j\,F\phi}}{\lambda_{O_{j+1}\,\phi\phi}}\right)^2 = \frac{(1+j)\,(\Delta j-2-j)\,(j+\Delta j)}{2\,(1+2\,j)\,\Delta\phi\,(-1+2\,\Delta\phi)}$
$\mathcal{F}_-$	$\frac{\lambda_{O_j\,FF}}{\lambda_{O_j\,\phi\phi}} = \frac{(-1-j-\Delta O+2\,\Delta\phi)\,(-4-j+\Delta O+2\,\Delta\phi)}{2\,\Delta\phi\,(-1+2\,\Delta\phi)}$	$\left(\frac{\lambda_{O_{j+1}\,F\phi}}{\lambda_{O_j\,\phi\phi}}\right)^2 = -\frac{(1+2\,j)\,(2+j-\Delta j)\,(j+\Delta j)}{2\,(1+j)\,\Delta\phi\,(-1+2\,\Delta\phi)}$

$\phi=\sigma$ and $F=\epsilon$ for super Ising model.

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$\mathcal{B}_-$	$\frac{\lambda_{PFF}}{\lambda_{P\phi\phi}} = \frac{(-3-l+\Delta O+2\Delta\phi)(-2+l+\Delta O+2\Delta\phi)}{2\Delta\phi(-1+2\Delta\phi)}$	$\left(\frac{\lambda_{OF\phi}}{\lambda_{P\phi\phi}}\right)^2 = \frac{(-1-l+\Delta O)(l+\Delta O)(-1+2\Delta O)}{(-1+\Delta O)\Delta\phi(-1+2\Delta\phi)}$
$\mathcal{F}_+$	$\frac{\lambda_{O_{j+1}FF}}{\lambda_{O_{j+1}\phi\phi}} = \frac{(1+j-\Delta j+2\Delta\phi)(-2+j+\Delta j+2\Delta\phi)}{2\Delta\phi(-1+2\Delta\phi)}$	$\left(\frac{\lambda_{O_j F\phi}}{\lambda_{O_{j+1}\phi\phi}}\right)^2 = \frac{(1+j)(\Delta j-2-j)(j+\Delta j)}{2(1+2j)\Delta\phi(-1+2\Delta\phi)}$
$\mathcal{F}_-$	$\frac{\lambda_{O_j FF}}{\lambda_{O_j\phi\phi}} = \frac{(-1-j-\Delta O+2\Delta\phi)(-4-j+\Delta O+2\Delta\phi)}{2\Delta\phi(-1+2\Delta\phi)}$	$\left(\frac{\lambda_{O_{j+1}F\phi}}{\lambda_{O_j\phi\phi}}\right)^2 = -\frac{(1+2j)(2+j-\Delta j)(j+\Delta j)}{2(1+j)\Delta\phi(-1+2\Delta\phi)}$

$\phi=\sigma$ and $F=\epsilon$ for super Ising model.

Notice in each multiplet, only one operator appears in $\sigma \times \sigma$ OPE, and only one operator appears in $\sigma \times \epsilon$ OPE.

details of the calculation

Plug the OPE ratios to the Ising bootstrap equation, we get

$$\sum_{l\,\in\,\text{even}} \lambda^2_{\mathcal{B}_+} \vec{V}^{\mathcal{B}_+}_{\Delta,l} + \sum_{l\,\in\,\text{even}} \lambda^2_{\mathcal{B}_-} \vec{V}^{\mathcal{B}_-}_{\Delta,l} + \sum_{j-1/2\,\in\,\text{even}} \lambda^2_{\mathcal{F}_+} \vec{V}^{\mathcal{F}_+}_{\Delta,j} + \sum_{j-1/2\,\in\,\text{odd}} \lambda^2_{\mathcal{F}_-} \vec{V}^{\mathcal{F}_-}_{\Delta,j} = 0,$$

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with

$$\vec{V}^{\mathcal{B}_+}_{\Delta,l} = \begin{pmatrix} F_{-,\Delta,l}{}^{\sigma\sigma,\sigma\sigma} \\ c_1^2 F_{-,\Delta,l}{}^{\epsilon\epsilon,\epsilon\epsilon} \\ c_2 F_{-,\Delta+1,l}^{\sigma\epsilon,\sigma\epsilon} \\ c_1 F_{-,\Delta,l}^{\sigma\sigma,\epsilon\epsilon} + c_2(-1)^l F_{-,\Delta+1,l}^{\epsilon\sigma,\sigma\epsilon} \\ c_1 F_{+,\Delta,l}^{\sigma\sigma,\epsilon\epsilon} - c_2(-1)^l F_{+,\Delta+1,l}^{\epsilon\sigma,\sigma\epsilon} \end{pmatrix}, \quad \vec{V}^{\mathcal{B}_-}_{\Delta,l} = \begin{pmatrix} F_{-,\Delta+1,l}^{\sigma\sigma,\sigma\sigma} \\ d_1^2 F_{-,\Delta+1,l}^{\epsilon\epsilon,\epsilon\epsilon} \\ d_2 F_{-,\Delta,l}^{\sigma\epsilon,\sigma\epsilon} \\ d_1 F_{-,\Delta+1,l}^{\sigma\sigma,\epsilon\epsilon} + d_2(-1)^l F_{-,\Delta,l}^{\epsilon\sigma,\sigma\epsilon} \\ d_1 F_{+,\Delta+1,l}^{\sigma\sigma,\epsilon\epsilon} - d_2(-1)^l F_{+,\Delta,l}^{\epsilon\sigma,\sigma\epsilon} \end{pmatrix}$$

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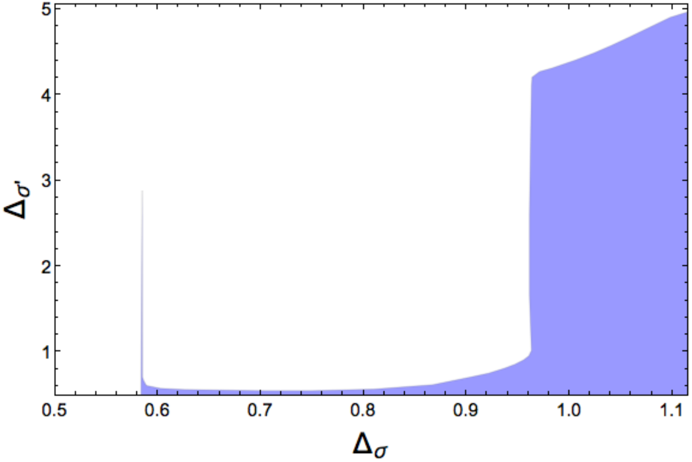
- all $\mathcal{B}_-^{(l)}$ multiplets with $l = 0$ (except for Σ) have scaling dimension bigger than 2,
- all $\mathcal{F}_+^{(j)}$ multiplets with $j = 1/2$ have scaling dimension bigger than $5/2$.

Emergent SUSY

Imposing emergent SUSY in numerical bootstrap, we get [Rong, Su '18]



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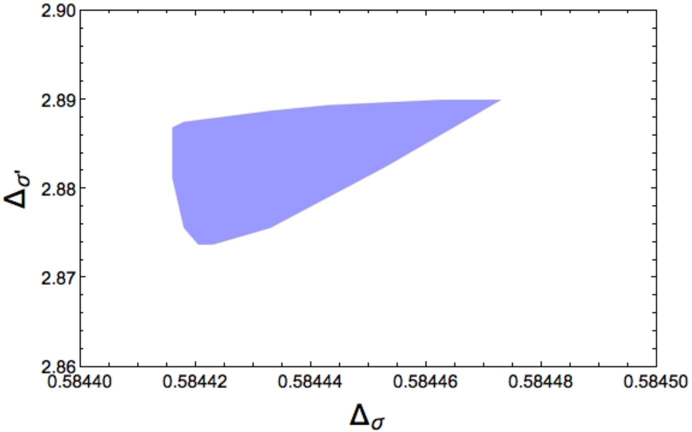

See also [Atanasov, Hillman Poland '18], where OPE constrains from $\langle \Sigma\Sigma\Sigma \rangle$ were considered.

Emergent SUSY

Descendants are important!

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By bootstrapping the OPE coefficient $\lambda^2_{\mathcal{F}_-}$, with $\mathcal{F}_-^{\Delta=5/2, j=3/2}$ being the SUSY current multiplet, we get

$$C_T^{\mathcal{N}=1}/C_T^{f.s.} \approx 1.684$$

Resumming large N series

Large N perturbation theory gives us [Gracey '93]

$$\eta_\psi = \frac{8}{3\pi^2 N} + \frac{1792}{27\pi^4 N^2} + \frac{64\left(-3402\zeta(3)+141\pi^2-668+324\pi^2\log(2)\right)}{243\pi^6 N^3} + \mathcal{O}(\frac{1}{N^4}).$$

We could use the $N = 1$ result to perform "two-sided" Padé approximation

N	4	8
large-N, Padé$_{[2,2]}$	0.0942	0.0430
large-N, Padé$_{[3,1]}$	0.1043	0.0437
$4-\epsilon$, ϵ^4, Padé$_{[2,2]}$ [Zerf, et al '17]	0.0976	0.0539
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The $N = 8$ model describes the quantum critical point of the semimetal to charge density wave order transition in graphene [Herbut '06].

What else can we do?

Recently, there has been some work on duality between $\mathcal{N} = 1$ superconformal field theories [Benini, Benvenuti '18], [Gaiotto, Komargodski, Wu '18], ...

$$\begin{array}{ccc} U(k)_{N+\frac{k}{2}-\frac{1}{2},\, N-\frac{1}{2}} & & SU(N)_{-k-\frac{N}{2}+\frac{1}{2}} \text{ with 1 flavor } P \\ \text{with 1 flavor } Q & \longleftrightarrow & \text{and a gauge-singlet } H \\ \mathcal{W} = -\frac{1}{4}\big(\sum_{i=1}^{k} Q_i Q_i^\dagger\big)^2 & & \mathcal{W} = H \sum_{i=1}^{N} P_i P_i^\dagger - \frac{1}{3} H^3 \,. \end{array}$$

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Set $N=k=1$, we get

$U(1)_{\frac{1}{2}}$ with 1 flavor Q $\mathcal{W}=-\frac{1}{4}QQ^\dagger QQ^\dagger$	$\longleftrightarrow$	WZ model with P, H $\mathcal{W}=HPP^\dagger-\frac{1}{3}H^3$.

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RHS is time-reversal invariant, while the LHS is not. For the duality to work, it is essential that there is no relevant deformation made of $O_s = PP^\dagger + \# H^2$. Such a term breaks time-reversal symmetry, which would be automatically generated on the LHS and drives the RG flow away from the fixed point.

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A two loop calculation, after Padé re-summation, gives

$$\Delta_{O_s} \approx 2 + 0.12448\epsilon - 0.12448\epsilon^2 + \mathcal{O}(\epsilon^3) \approx 2.058.$$

This needs to be confirmed by numerical bootstrap.

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When the AdS solution is maximally supersymmetric, the Kaluza-Klein modes are BPS multiplets, which are protected by supersymmetry, their scaling dimension is fixed to be some finite value.
When the AdS solution is not maximally supersymmetric, the Kaluza-Klein spectrum contain certain long multiplets, though not protected by SUSY, their scaling dimensions are finite. For $\mathcal{N} = 1$ solutions, non of the multiplets are protected (except for conserved currents)!

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Non-perturbative test of AdS/CFT?!

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- What about $\mathcal{N} = 2, 3, 4, 5, 6, 7, 8$? Descendants are important! This is just the beginning!

Thank you!