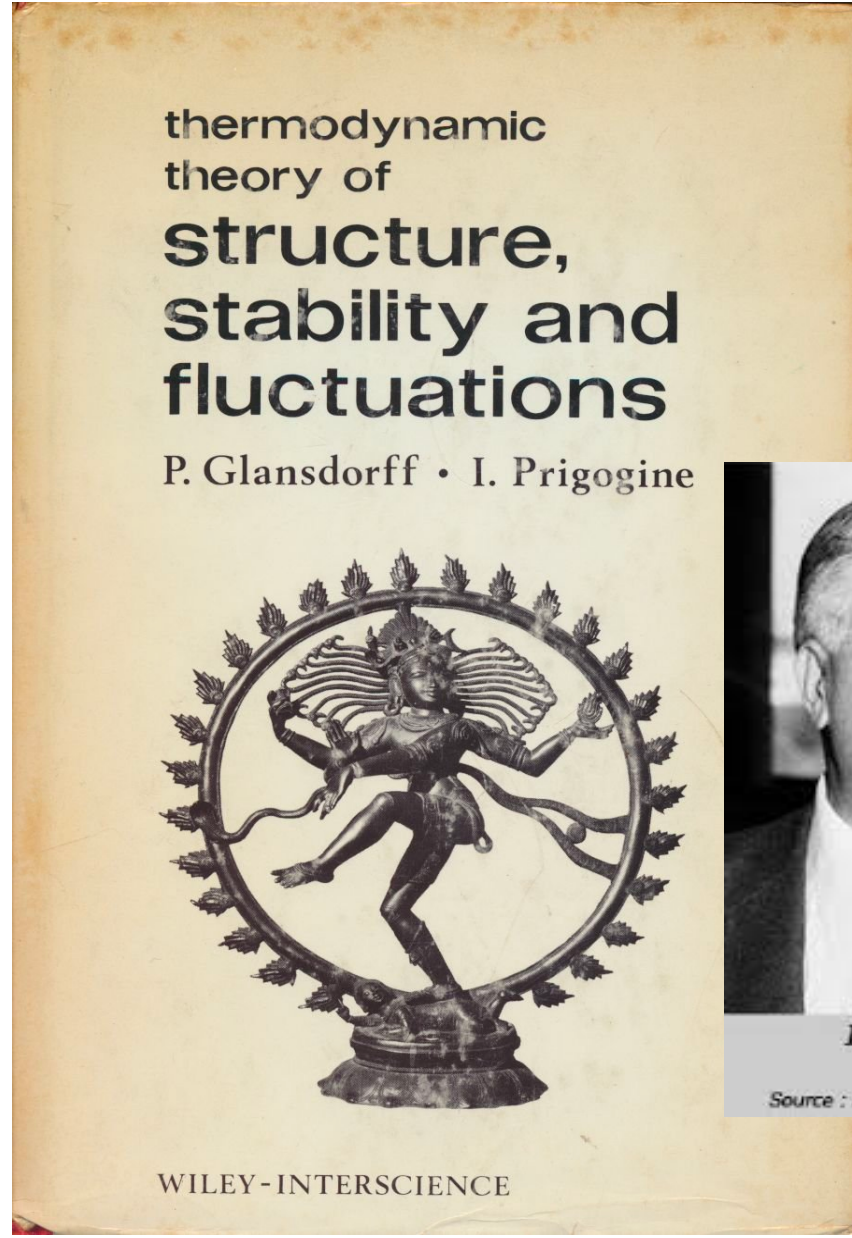


생명현상의 열역학

조정효
(KIAS)

2018 생명물리 여름학교, 7월 10일, 포항

Order under fluctuations

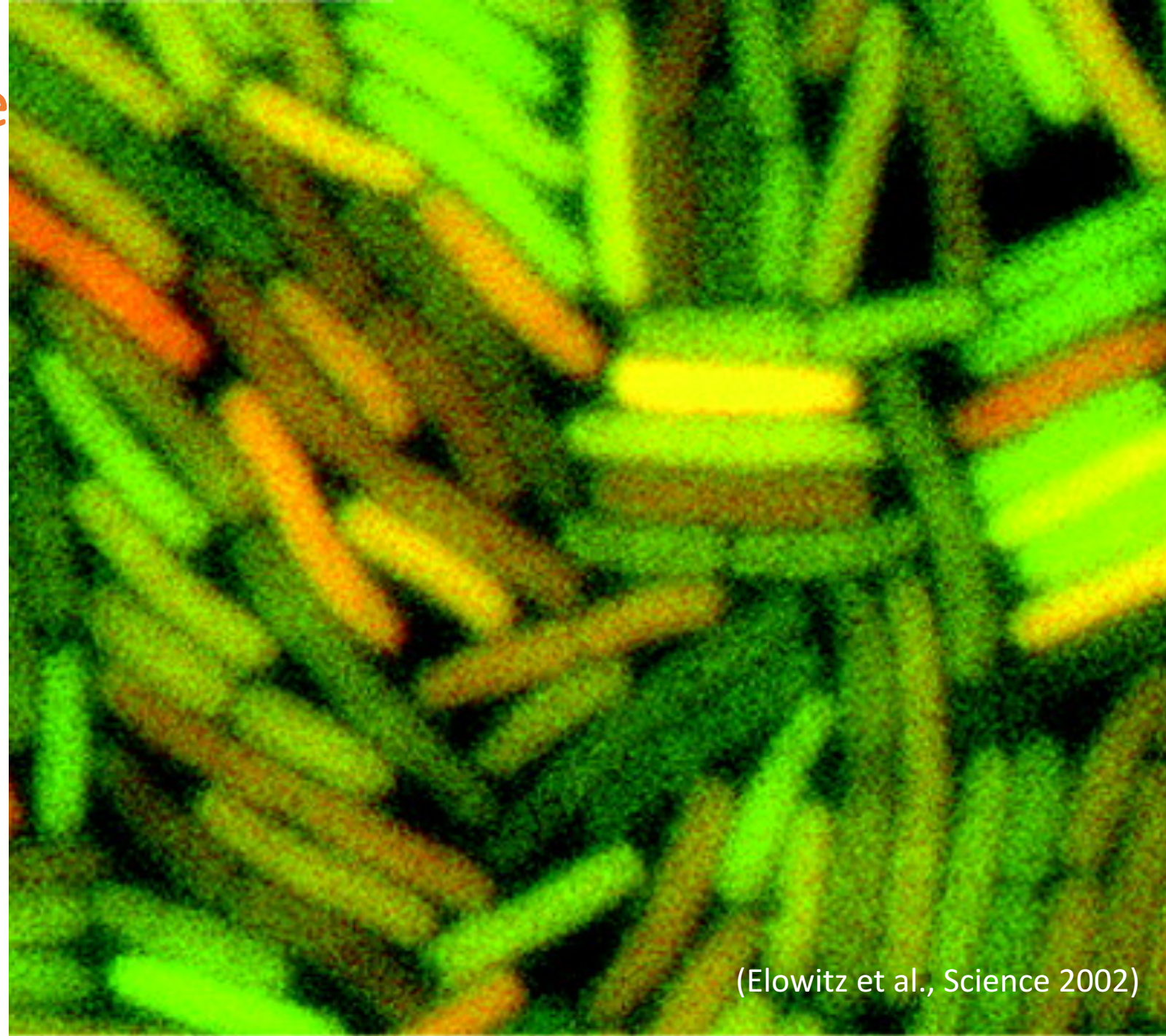
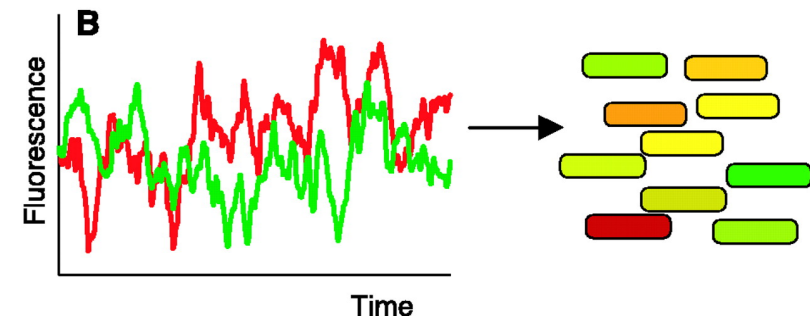
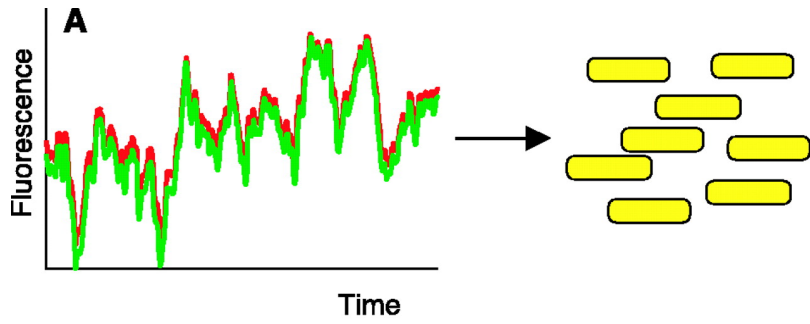


Nature/Nurture/Chance

Dizygotic twins

Monozygotic twins

Clonal cells



(Elowitz et al., Science 2002)

Information processing under noisy environment

“Uncertainty”

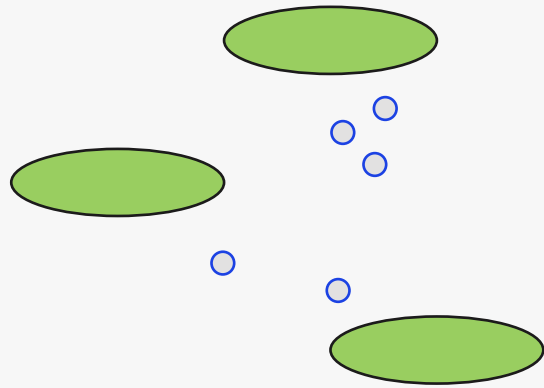
Hidden degrees of freedom

Fundamentally stochastic

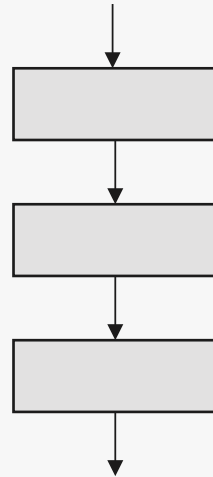
Nonlinearity (chaos)

How living systems overcome uncertainty (fluctuations)?

Large number's law

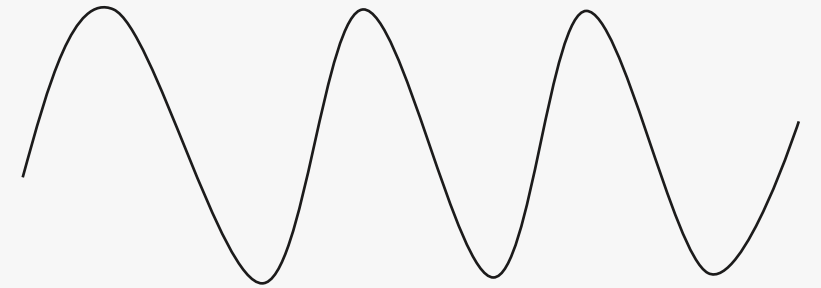


Quorum sensing



Signal cascade

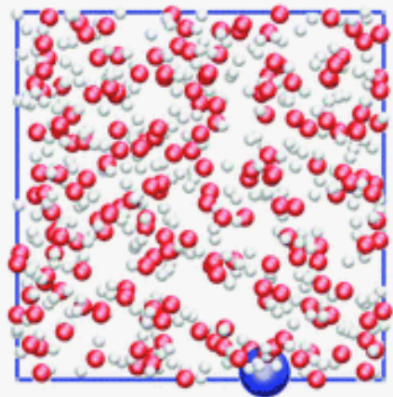
Time (predictability)



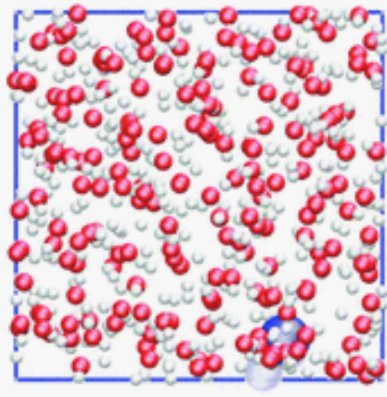
Oscillation

Noise is sometimes useful

Source for energy



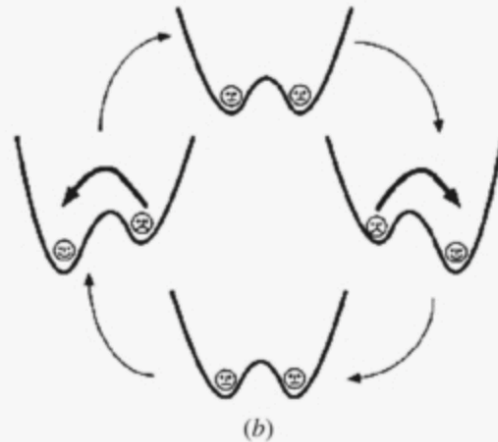
(a) $t = 0$ ps



(b) $t = 2$ ps

Molecular diffusion

(Murdachaew et al., Phys. Chem. Chem. Phys., 2012)



(b)

Stochastic resonance

(Yang et al., J. Phys. A, 2009)

Source for evolution



Mutation

50,000 generation of *E. coli* since 1988
(Richard Lenski)

**Non-equilibrium
thermodynamics**

Jarzynski, Crooks, ...

What is life?

**Stochastic
thermodynamics**

Sekimoto, Seifert, ...

**Information
thermodynamics**

Ueda, Sagawa, ...

THE JOURNAL OF CHEMICAL PHYSICS **139**, 121923 (2013)



Statistical physics of self-replication

Jeremy L. England

*Department of Physics, Massachusetts Institute of Technology, Building 6C, 77 Massachusetts Avenue,
Cambridge, Massachusetts 02139, USA*

(Received 28 April 2013; accepted 1 August 2013; published online 21 August 2013)

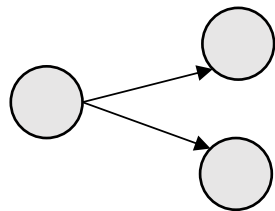
PHYSICAL REVIEW X **6**, 021036 (2016)

Statistical Physics of Adaptation

Nikolay Perunov, Robert A. Marsland, and Jeremy L. England

*Department of Physics, Physics of Living Systems Group, Massachusetts Institute of Technology, Floor 6,
400 Tech Square, Cambridge, Massachusetts 02139, USA*

(Received 23 December 2014; revised manuscript received 27 March 2016; published 16 June 2016)



Irreversible, entropy-producing, non-equilibrium processes

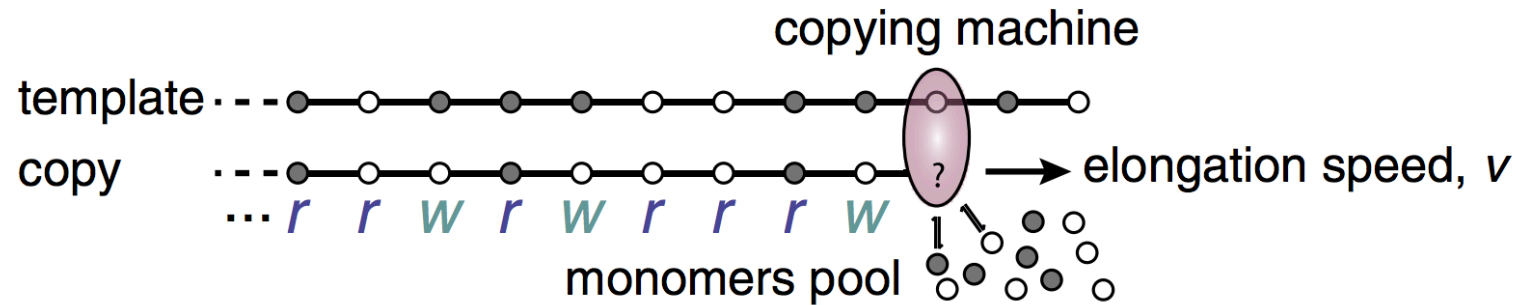
Thermodynamics of Error Correction

Pablo Sartori¹ and Simone Pigolotti²

¹*Max Planck Institute for the Physics of Complex Systems,
Noethnitzer Strasse 38, 01187 Dresden, Germany*

²*Departament de Física, Universitat Politècnica de Catalunya, Edifici Gaia,
Rambla Sant Nebridi 22, 08222 Terrassa, Barcelona, Spain*

(Received 23 April 2015; revised manuscript received 25 September 2015; published 10 December 2015)



one mistake for every 10^7 nucleotides added

Free Energy Cost of Reducing Noise while Maintaining a High Sensitivity

Pablo Sartori^{1,*} and Yuhai Tu²

¹*Max Planck Institute for the Physics of Complex Systems, Noethnitzer Strasse 38, 01187 Dresden, Germany*

²*IBM T.J. Watson Research Center, 1101 Kitchawan Road, Yorktown Heights, New York 10598, USA*

(Received 27 May 2015; published 8 September 2015)

Fluctuation dissipation theorem under equilibrium conditions

Free energy dissipation under non-equilibrium conditions

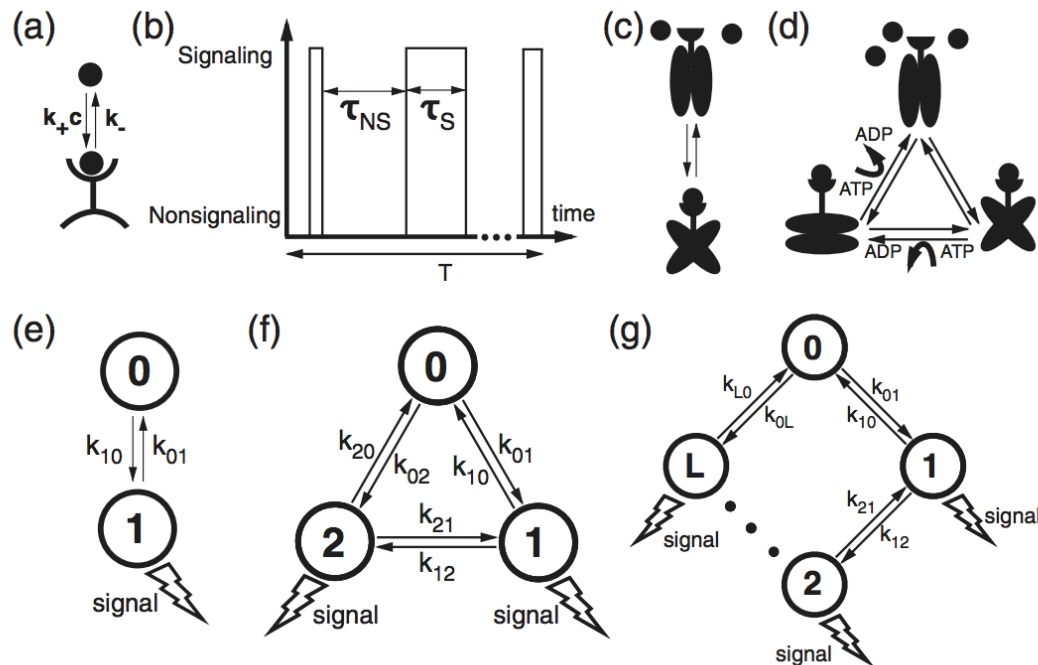
Thermodynamics of Statistical Inference by Cells

Alex H. Lang,^{1,*} Charles K. Fisher,¹ Thierry Mora,² and Pankaj Mehta^{1,†}

¹Physics Department, Boston University, Boston, Massachusetts 02215, USA

²Laboratoire de physique statistique, CNRS, UPMC and École normale supérieure, 75005 Paris, France

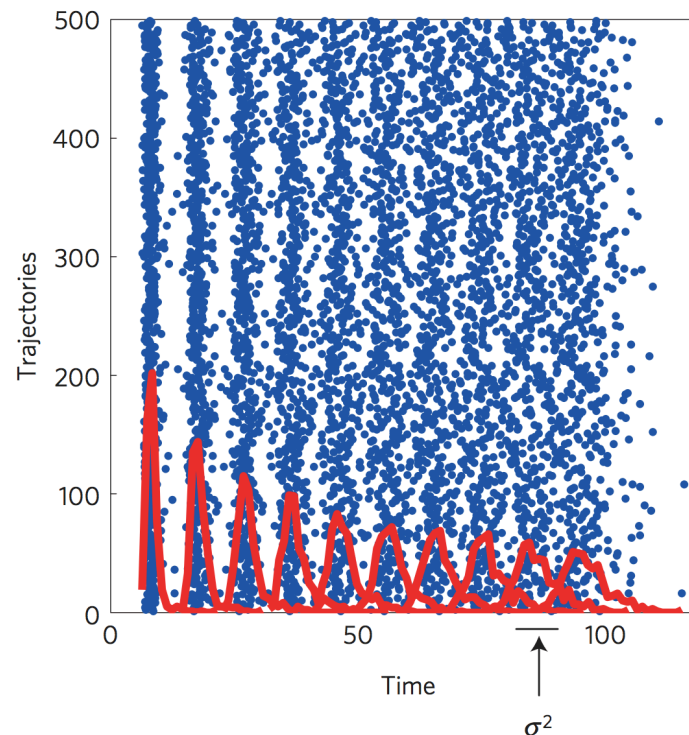
(Received 10 June 2014; published 3 October 2014)



Energetic cost for accurate signaling

The free-energy cost of accurate biochemical oscillations

Yuansheng Cao¹, Hongli Wang¹, Qi Ouyang^{1,2*} and Yuhai Tu^{3*}



Energetic cost for accurate oscillations

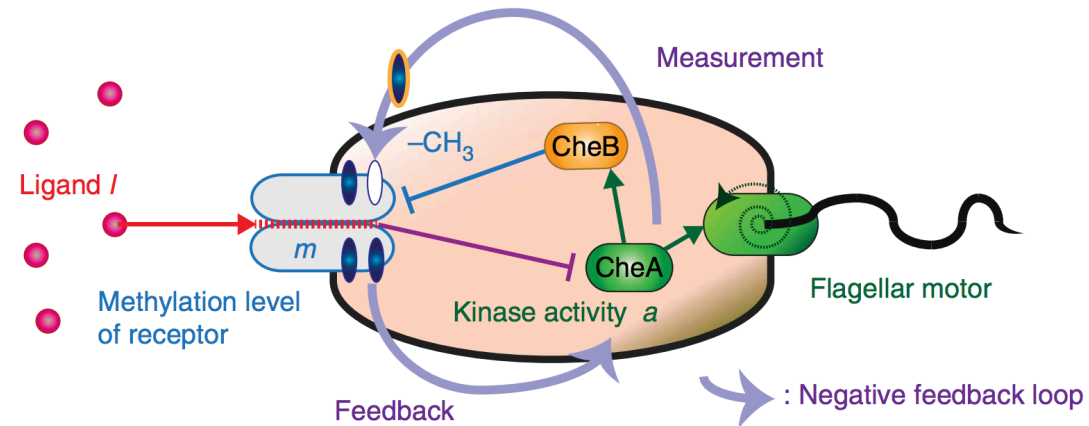
ARTICLE

Received 2 Jul 2014 | Accepted 12 May 2015 | Published 23 Jun 2015

DOI: 10.1038/ncomms8498

Maxwell's demon in biochemical signal transduction with feedback loop

Sosuke Ito^{1,†} & Takahiro Sagawa^{2,†}



Information entropy

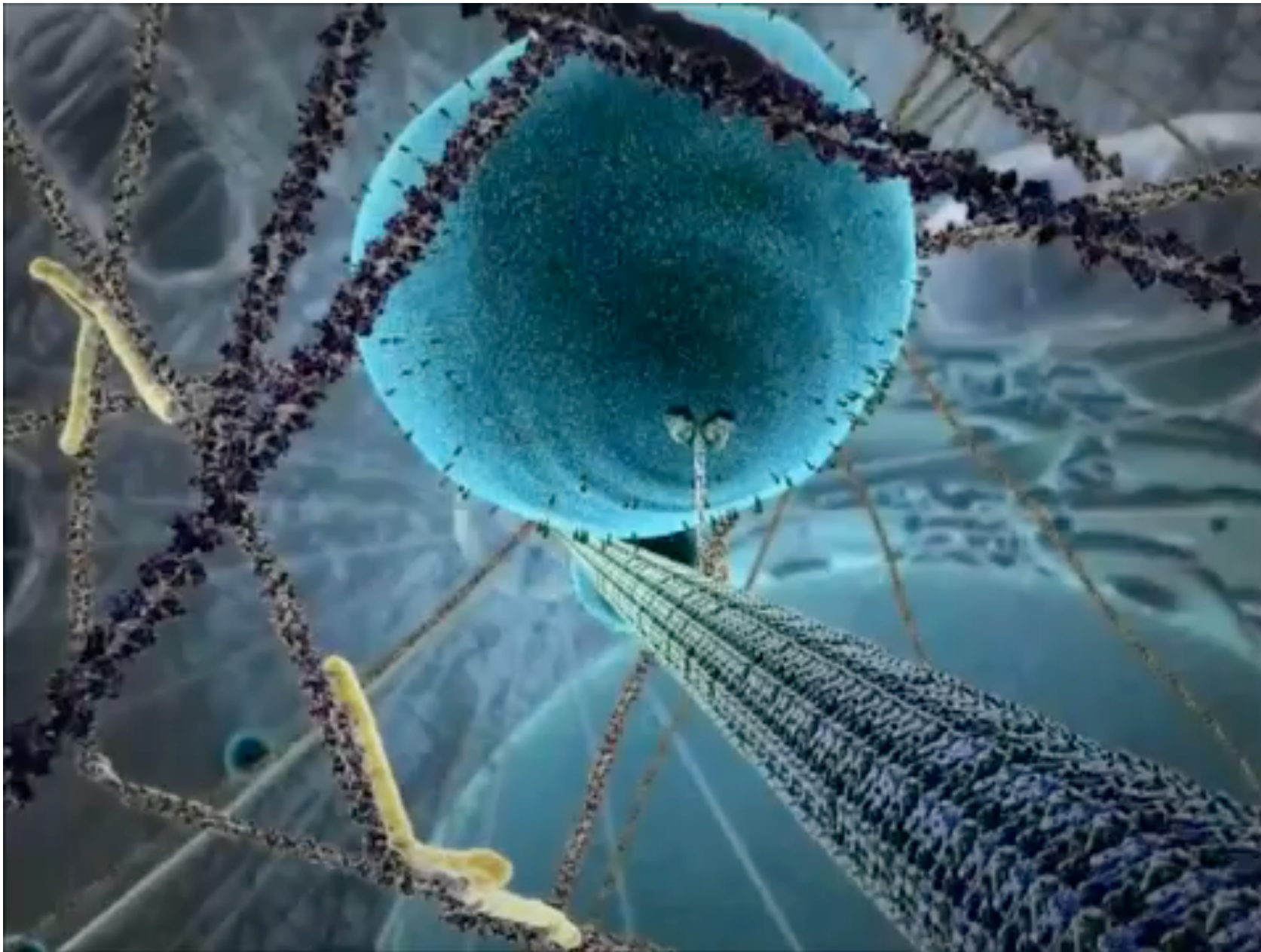
Stochastic Thermodynamics of Learning

Sebastian Goldt* and Udo Seifert

II. Institut für Theoretische Physik, Universität Stuttgart, 70550 Stuttgart, Germany

(Received 11 July 2016; revised manuscript received 12 October 2016; published 6 January 2017)

Energetic cost for learning



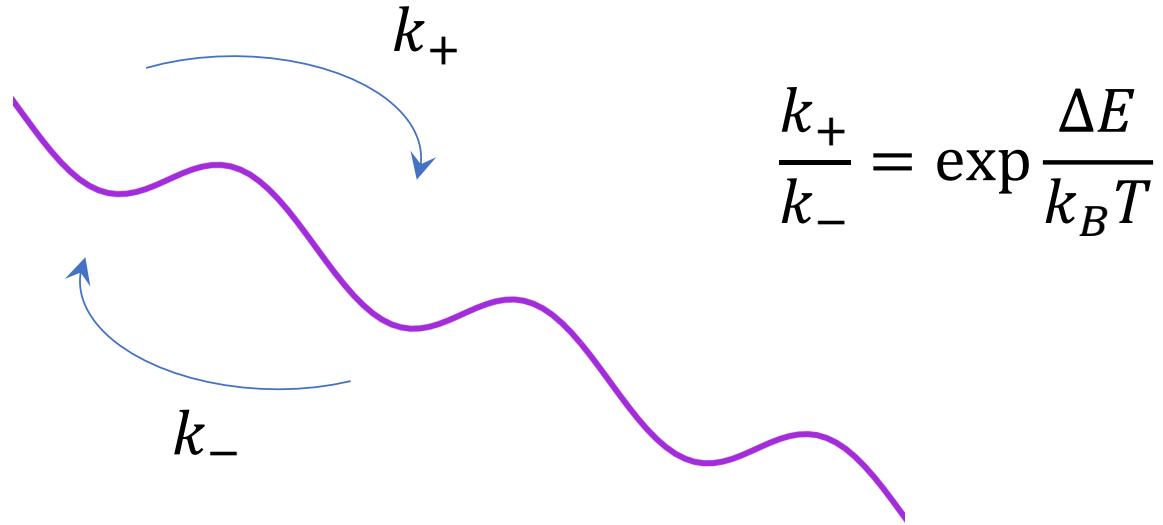
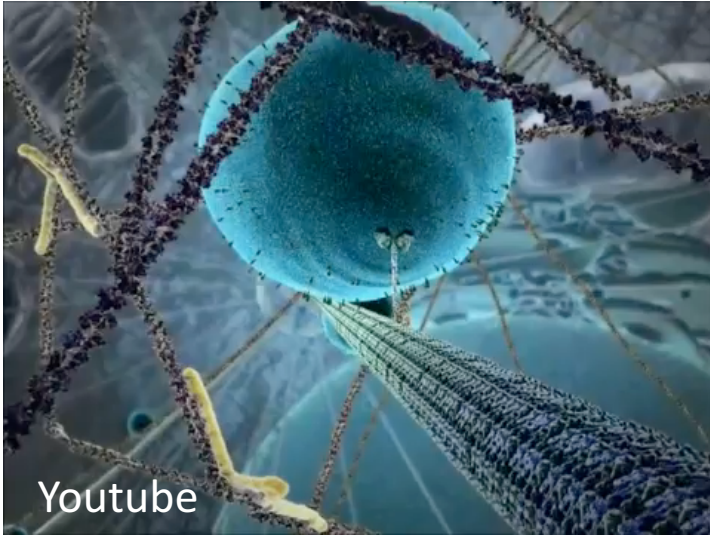
Dynamics

Chemical & mechanical mechanisms

Function (accurate delivery)

Energetic cost?

Thermodynamic uncertainty relation (2015)



$$\langle X(t) \rangle = (k_+ - k_-)t$$

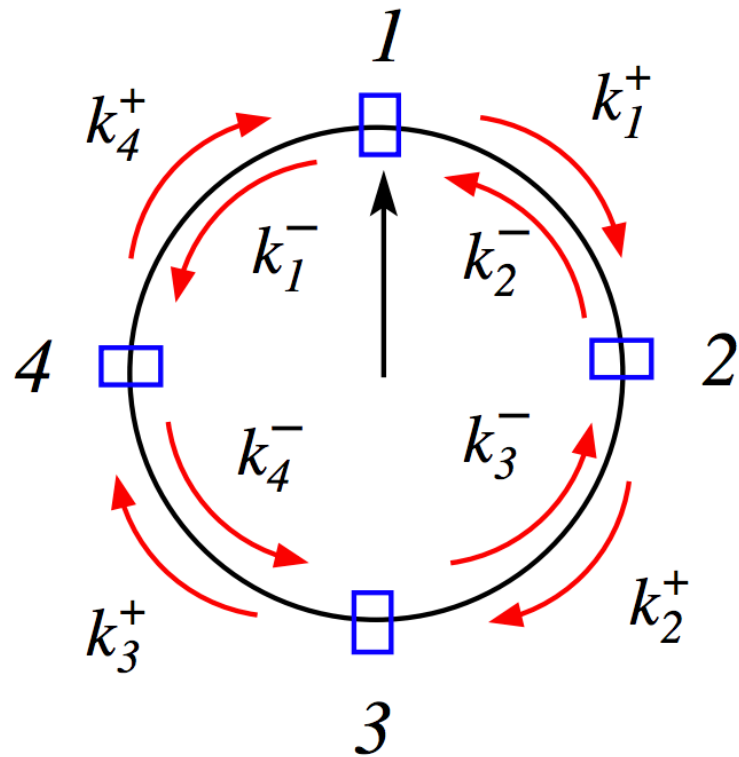
$$\langle \delta X(t)^2 \rangle = (k_+ + k_-)t$$

$$\langle q(t) \rangle = (k_+ - k_-)t k_B T \log \frac{k_+}{k_-}$$

$$\begin{aligned} Q &= \langle q(t) \rangle \frac{\langle \delta X(t)^2 \rangle}{\langle X(t) \rangle^2} \\ &= k_B T \frac{(k_+ + k_-)}{(k_+ - k_-)} \log \frac{k_+}{k_-} \geq 2k_B T \end{aligned}$$

(Barato and Seifert, PRL 2015)

Brownian clock



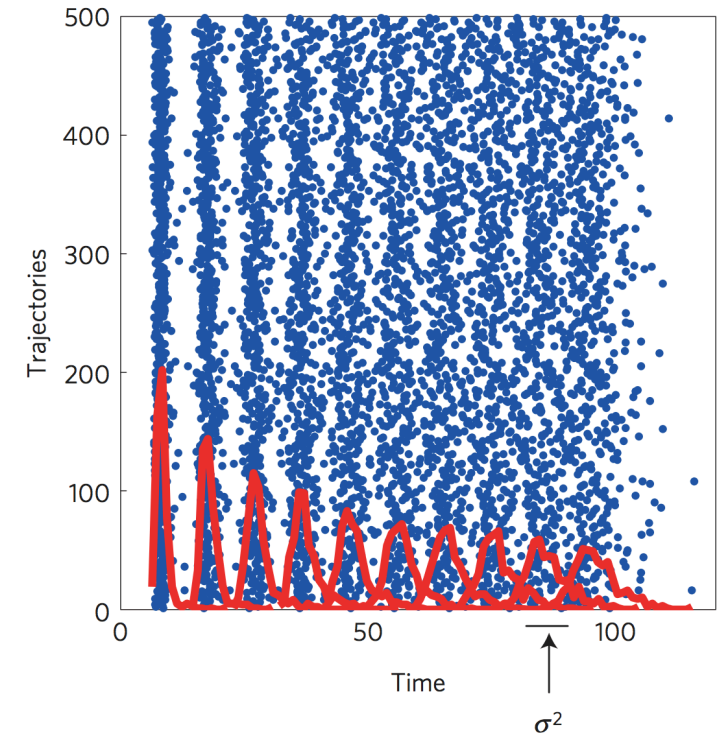
(Barato and Seifert, PRX 2016)

$$k_+ = k_1^+ k_2^+ k_3^+ k_4^+$$

$$k_- = k_1^- k_2^- k_3^- k_4^-$$

$$Q = \langle q(t) \rangle \frac{\langle \delta\theta^2 \rangle}{\langle \theta \rangle^2} \geq 2k_B T$$

θ : displacement, time, product number, heat, ...



Noisy oscillator

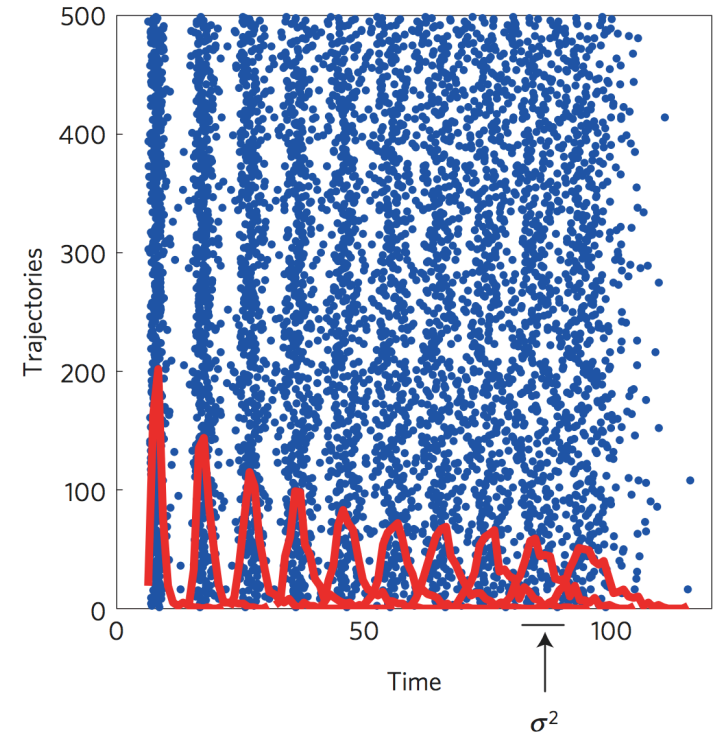
Langevin equation

$$\frac{d\theta}{dt} = \omega + \eta(t)$$

$$\langle \eta(t) \rangle = 0$$

$$\langle \eta(t)\eta(t') \rangle = 2D\delta(t - t')$$

$$D = \mu k_B T$$



$$\langle \theta(t) \rangle = \omega t$$

$$\langle \delta\theta(t)^2 \rangle = 2Dt$$

$$\langle q(t) \rangle = \left\langle \int_0^t F(\tau) \circ \frac{d\theta}{d\tau} d\tau \right\rangle = \omega^2 t$$

$$v = \lim_{t \rightarrow \infty} \frac{\langle \theta(t) \rangle}{t}$$

$$D = \lim_{t \rightarrow \infty} \frac{\langle \delta\theta(t)^2 \rangle}{2t}$$

$$\sigma = \lim_{t \rightarrow \infty} \frac{\langle q(t) \rangle}{t}$$

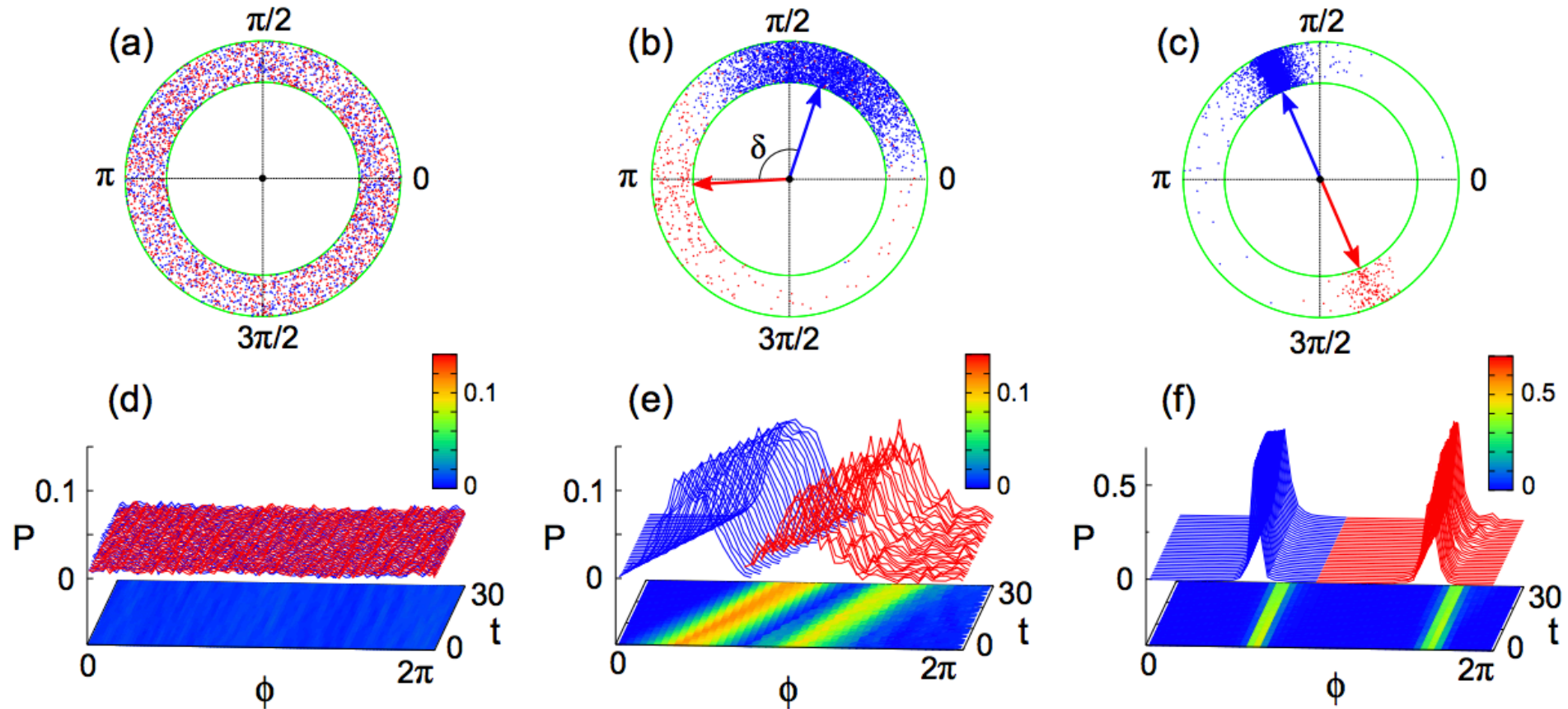
$$\begin{aligned} Q &= \langle q(t) \rangle \frac{\langle \delta\theta(t)^2 \rangle}{\langle \theta(t) \rangle^2} \\ &= \sigma \frac{2D}{v^2} = 2k_B T \end{aligned}$$

Interacting oscillators

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

Kuramoto model

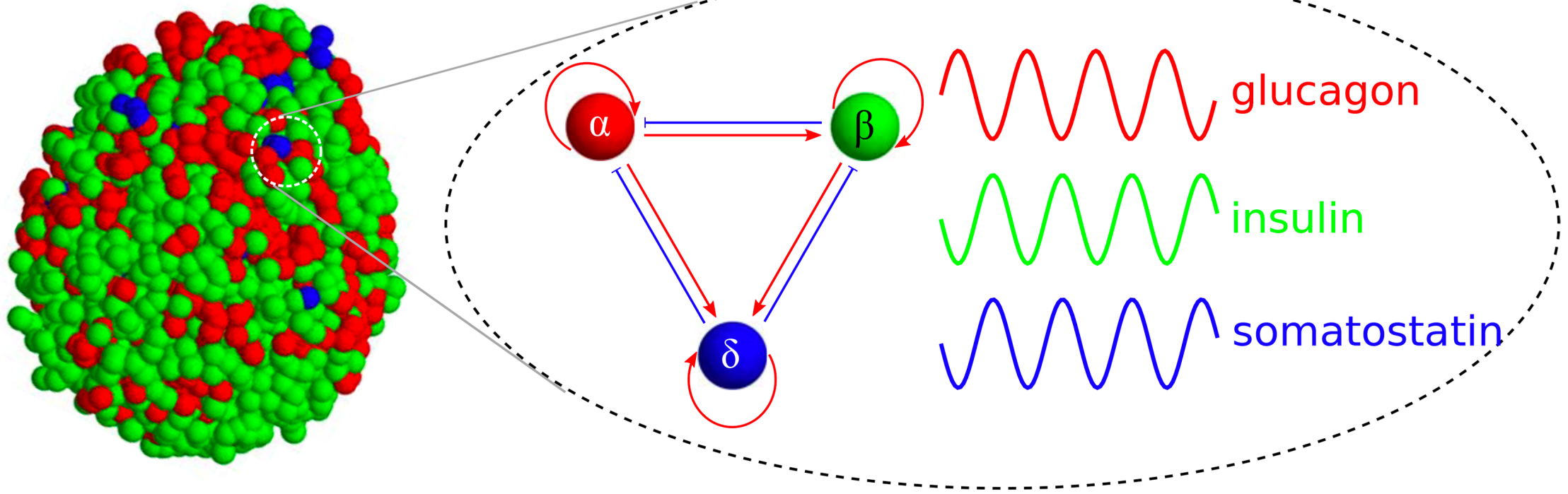
Conformist, contrarian oscillators



$$\frac{d\theta_i}{dt} = \omega_i + K_i \sum_{\langle i,j \rangle} \sin(\theta_j - \theta_i)$$

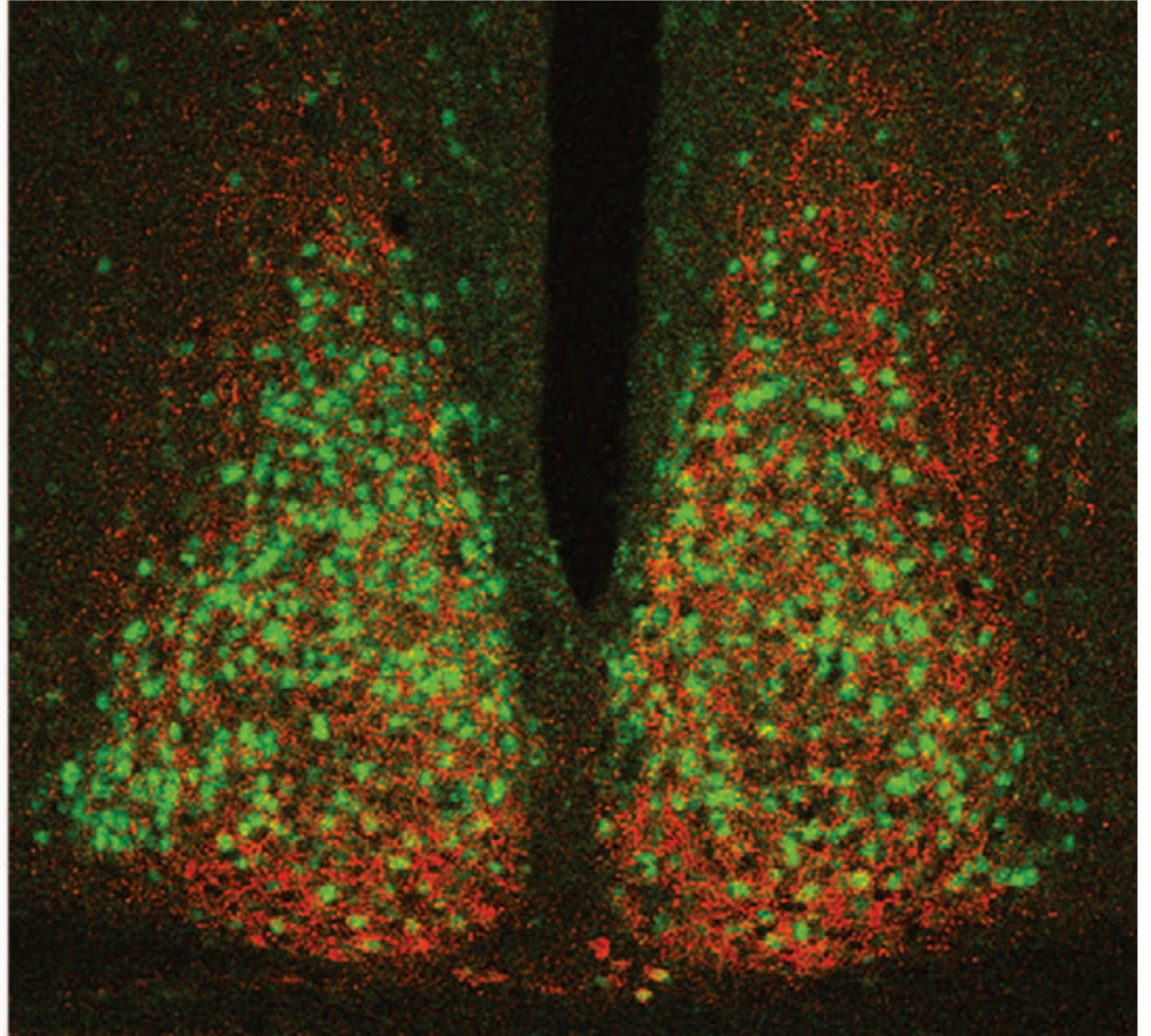
(Hoang, Jo, and Hong, PRE 2015)

Cellular oscillators



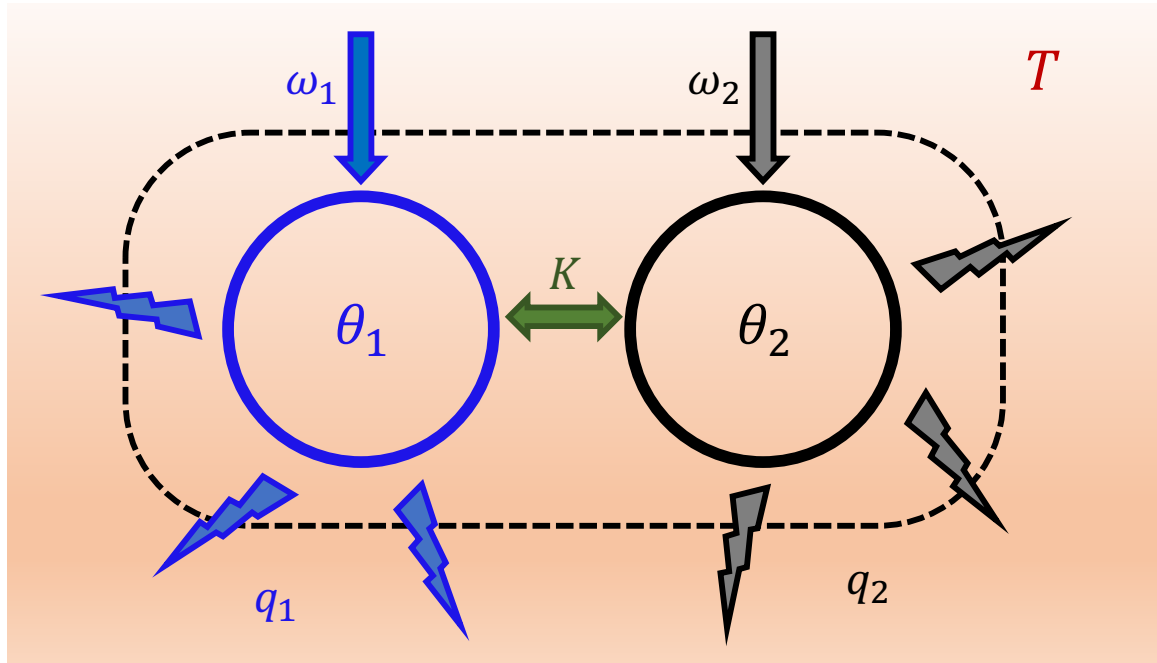
(Hoang, Hara, and Jo, PLoS ONE 2015)

Clock-cell oscillators



(Mohawk and Takahashi, Trends in Neurosciences, 2011)

Energy-accuracy trade-off of interacting subsystems



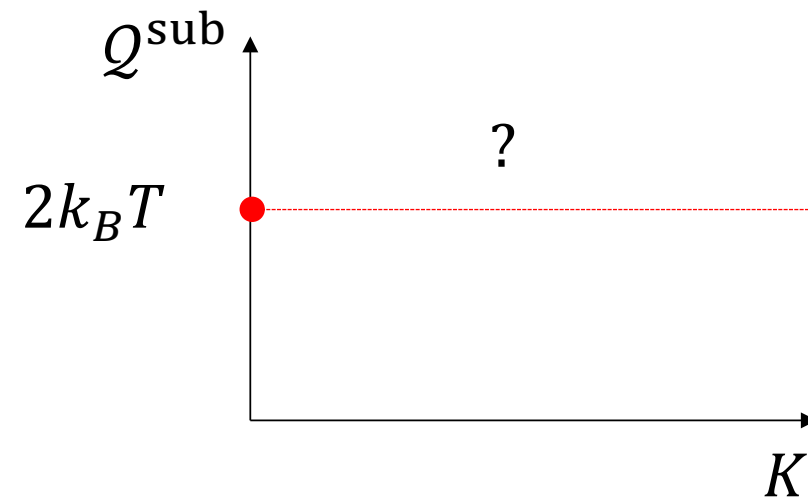
Conservative interaction

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) + \eta_i$$

$$Q = \langle q(t) \rangle \frac{\langle \delta\theta(t)^2 \rangle}{\langle \theta(t) \rangle^2}$$

$$Q^{\text{tot}} = Q(\sum_i q_i, \theta_i)$$

$$Q^{\text{sub}} = Q(q_i, \theta_i)$$



Strong coupling limit ($K \rightarrow \infty$)

$$\frac{d\theta_1}{dt} = \omega_1 + \frac{K}{2} \sin(\theta_2 - \theta_1) + \eta_1(t)$$

$$\frac{d\theta_2}{dt} = \omega_2 + \frac{K}{2} \sin(\theta_1 - \theta_2) + \eta_2(t)$$

$$\frac{d\phi_1}{dt} = 2\bar{\omega} + \xi_1(t)$$

$$\frac{d\phi_2}{dt} = \Delta\omega - K \sin \phi_2 + \xi_2(t)$$

$$\phi_1 = \theta_1 + \theta_2$$

$$\phi_2 = \theta_1 - \theta_2$$

$$\bar{\omega} = (\omega_1 + \omega_2)/2 \quad \Delta\omega = \omega_1 - \omega_2$$

$$\langle \xi_i(t) \xi_i(t') \rangle = 4D \delta(t - t')$$

$$v = \bar{\omega} \quad \sigma = \bar{\omega}^2$$

$$D = \frac{\langle \delta\theta_1^2 \rangle}{2t} = \frac{1}{2t} \frac{\langle \delta\phi_1^2 + \delta\phi_2^2 \rangle}{4}$$

$$\langle \delta\phi_1^2 \rangle = 4Dt$$

$$\frac{d\phi_2}{dt} = \Delta\omega - K\phi_2 + \xi_2(t)$$

$$\phi_2(t) = \int_0^t d\tau e^{-K(t-\tau)} (\Delta\omega + \xi_2)$$

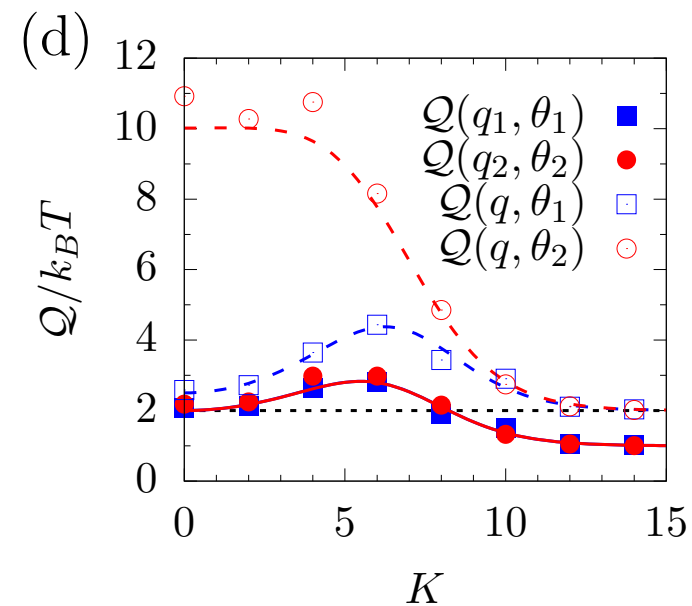
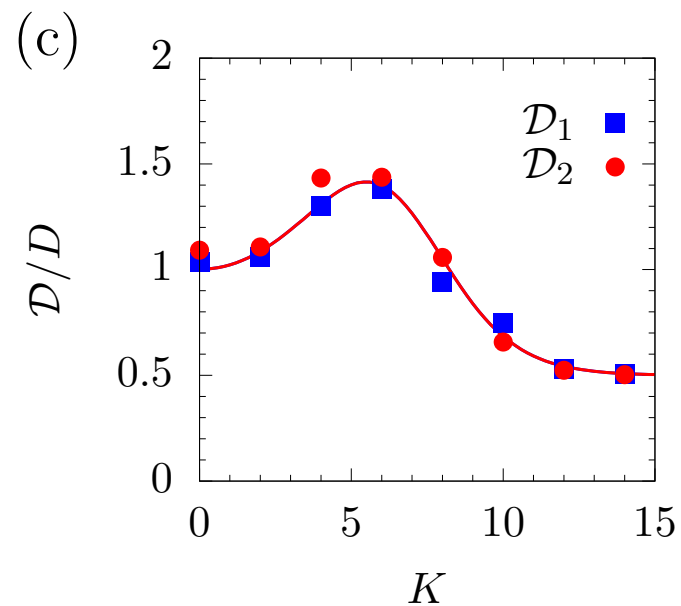
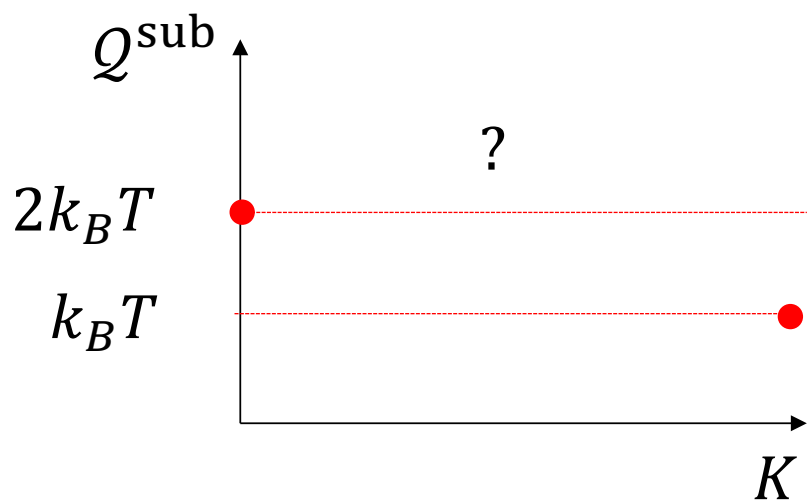
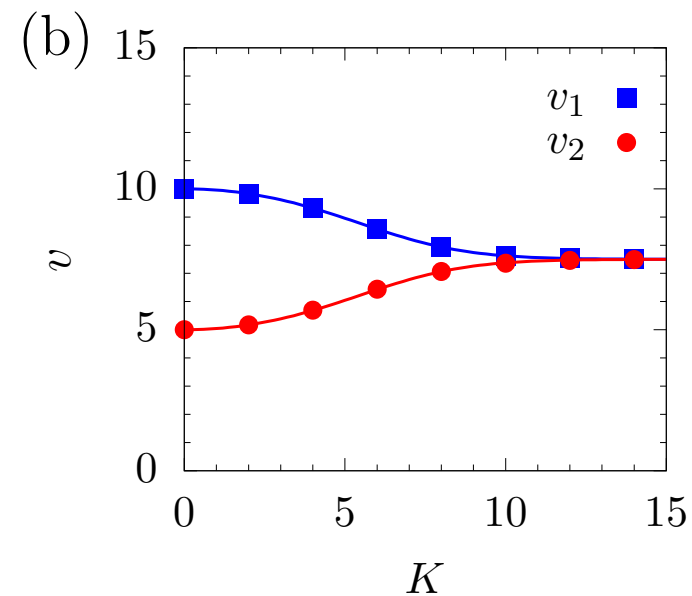
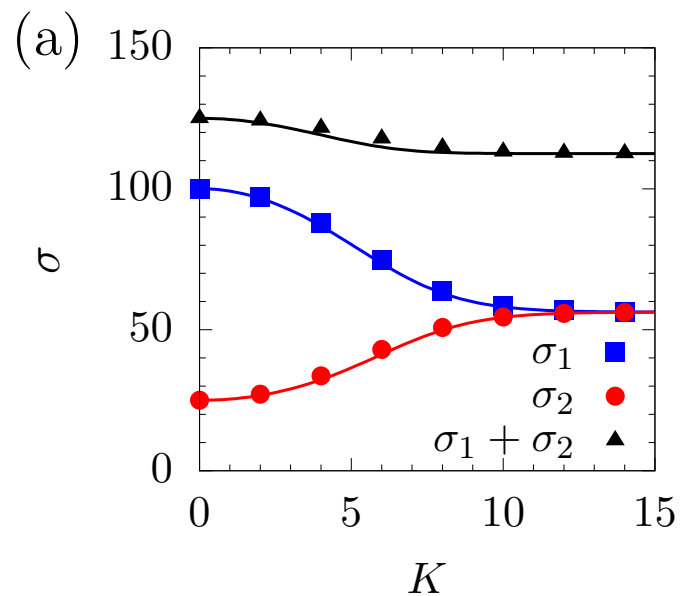
$$\langle \delta\phi_2^2 \rangle = \frac{2D}{K} (1 - e^{-2Kt})$$

$$Q^{\text{sub}} = \sigma \frac{2D}{v^2} = k_B T$$

Energy-accuracy trade-off of interacting subsystems

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) + \eta_i$$

$$Q^{\text{sub}} = \langle q_i \rangle \frac{\langle \delta\theta_i^2 \rangle}{\langle \theta_i \rangle^2}$$



Analytical approach

$$\frac{d\theta_1}{dt} = \omega_1 + \frac{K}{2} \sin(\theta_2 - \theta_1) + \eta_1(t)$$

$$\frac{d\theta_2}{dt} = \omega_2 + \frac{K}{2} \sin(\theta_1 - \theta_2) + \eta_2(t)$$

$$\frac{d\phi_1}{dt} = 2\bar{\omega} + \xi_1(t)$$

$$\frac{d\phi_2}{dt} = \Delta\omega - K \sin \phi_2 + \xi_2(t)$$

$$V(\phi_2) = -\phi_2 \Delta\omega - K \cos \phi_2$$

$$I_+(\phi_2) = \exp\left[\frac{V(\phi_2)}{2D}\right] \int_{\phi_2-2\pi}^{\phi_2} d\varphi \exp\left[-\frac{V(\varphi)}{2D}\right]$$

$$P(\theta_1, \theta_2, t)$$

$$\frac{\partial P}{\partial t} = -\frac{\partial J_1}{\partial \theta_1} - \frac{\partial J_2}{\partial \theta_2}$$

$$J_i = F_i P - D \frac{\partial P}{\partial \theta_i}$$

Fokker-Planck equation

$$v_i = \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 J_i^{ss}$$

$$\sigma_i = \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 F_i J_i^{ss}$$

$$\frac{D_{\text{eff}}}{D} = \frac{\int_0^{2\pi} d\phi_2 I_{\mp}(\phi_2) I_+(\phi_2) I_-(\phi_2)}{\left[\int_0^{2\pi} d\phi_2 I_{\mp}(\phi_2) \right]^3}$$

$$I_-(\phi_2) = \exp\left[-\frac{V(\phi_2)}{2D}\right] \int_{\phi_2}^{\phi_2+2\pi} d\varphi \exp\left[\frac{V(\varphi)}{2D}\right]$$

Many oscillators

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) + \eta_i$$

$$\frac{d\theta_i}{dt} = \omega_i + \sum_{j=1}^N M_{ij} \theta_j + \eta_i$$

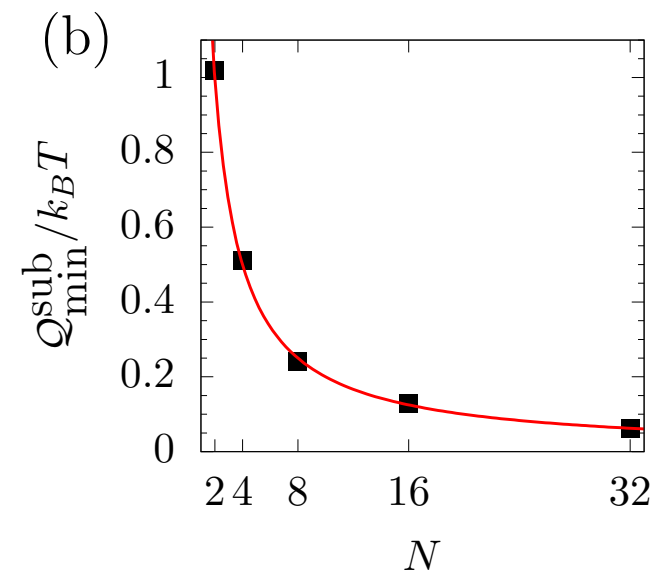
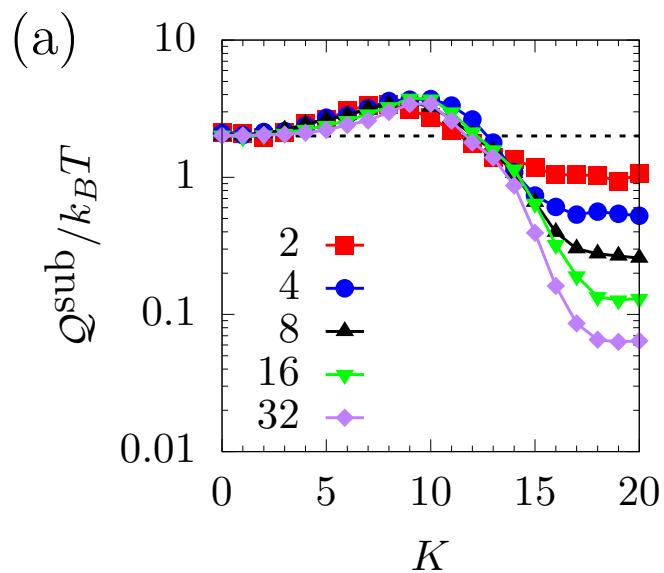
$$\mathbf{M} = \begin{bmatrix} -K & \frac{K}{N} & \frac{K}{N} & \cdots & \frac{K}{N} \\ \frac{K}{N} & -K & \frac{K}{N} & \cdots & \frac{K}{N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{K}{N} & \frac{K}{N} & \frac{K}{N} & \cdots & -K \end{bmatrix}$$

$$\phi_1 = \theta_1 + \theta_2 + \cdots + \theta_N \quad \lambda_1 = 0$$

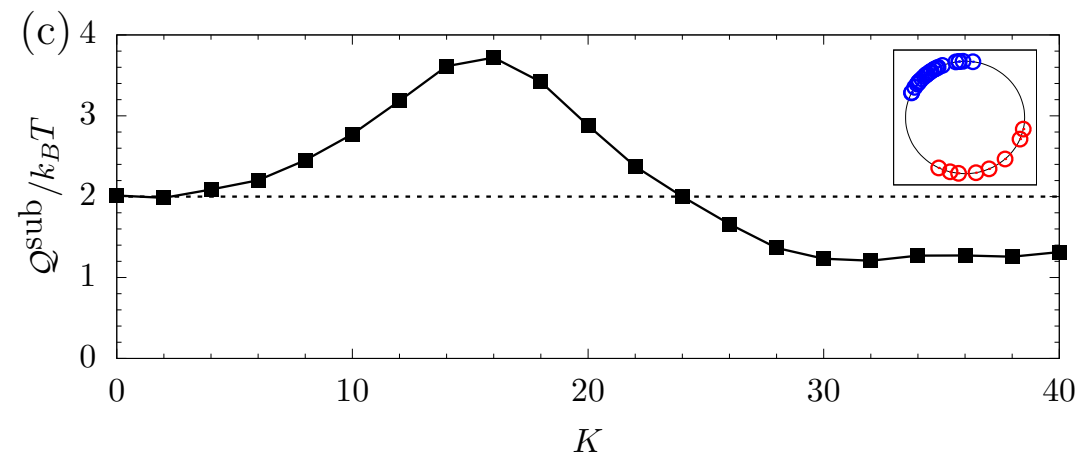
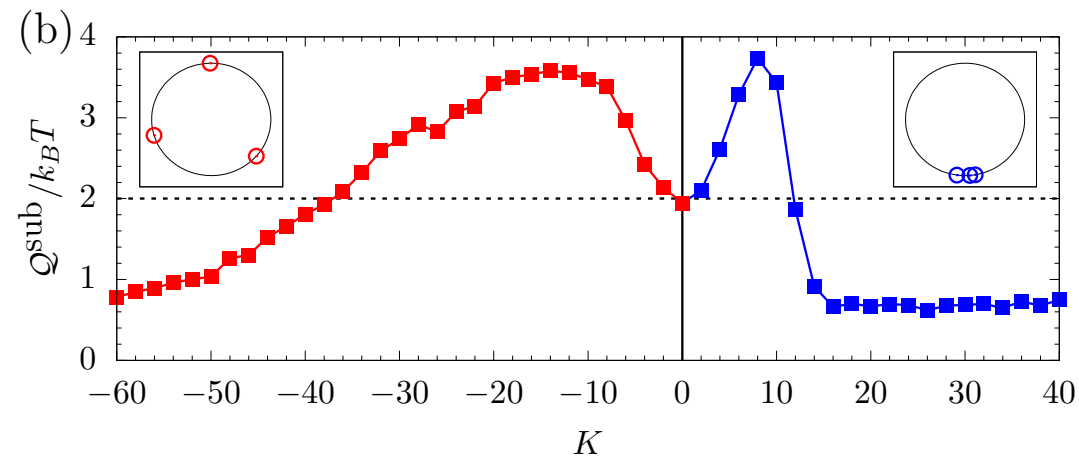
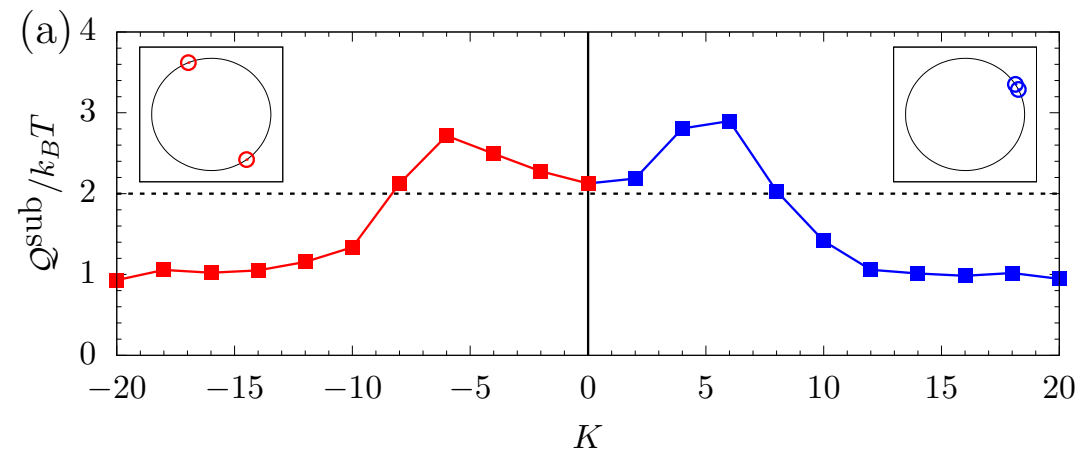
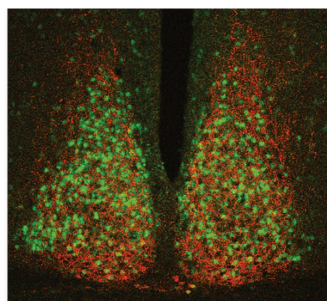
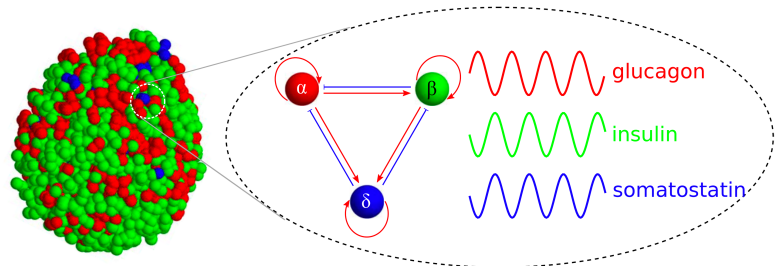
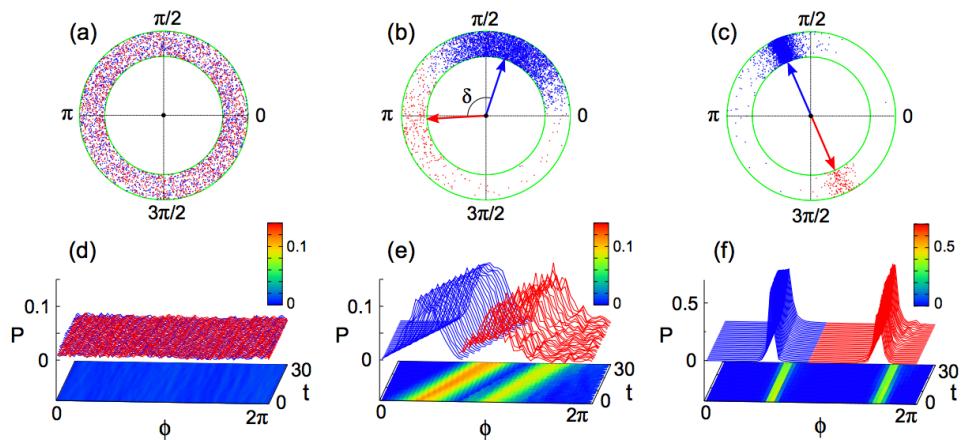
$$\phi_i = \theta_1 - \theta_i \quad \lambda_i < 0 \quad (i \neq 1)$$

$$\langle \delta\theta_1^2 \rangle = \frac{\sum_{i=1}^N \langle \delta\phi_i^2 \rangle}{N^2} \approx \frac{\langle \delta\phi_1^2 \rangle}{N^2} = \frac{2Dt}{N}$$

$$Q^{\text{sub}} = \sigma \frac{2D}{v^2} = \frac{2k_B T}{N}$$



Collective dynamics



Summary

- Non-equilibrium thermodynamics, stochastic thermodynamics, and information thermodynamics provide new languages for understanding thermodynamic (energetic and informational) aspects of rich and collective dynamics of living systems (e.g., learning, computation, memory, ...).
- Thermodynamic constraints are important for designing efficient molecular machines/robots operating under thermal fluctuations.
- The energetic cost for operational accuracy can be reduced by the cooperation between subsystems.

Thermodynamic uncertainty relation of interacting oscillators in synchrony

Sangwon Lee, Changbong Hyeon, Junghyo Jo

(Submitted on 27 Apr 2018)

