
Introduction to Tokamak Core Turbulence

Part I.

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Outline

Properties of Tokamak Core Turbulence
Implications on Tokamak Confinement Scaling

Self-organized Structures in Torus

Radially Elongated Eddys
Zonal Flows

Emphasis:

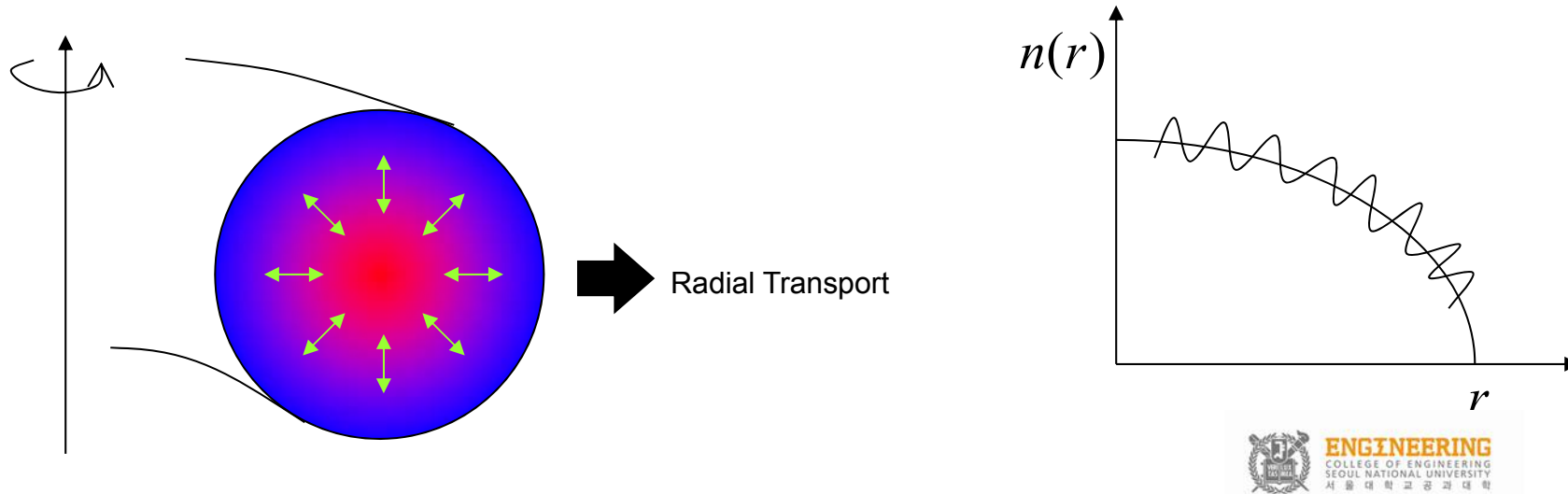
Study of underlying Physics Mechanisms leading to
Paradigm Shift

Outline

Properties of Tokamak Core Turbulence

Microinstabilities in Tokamaks

- Tokamak transport is usually anomalous, even in the absence of large-scale magneto-hydro-dynamic (MHD) instabilities.
- Caused by small-scale collective instabilities driven by gradients in temperature, density,
- Instabilities saturate at low amplitude due to nonlinear mechanisms
- Particles $\mathbf{E} \times \mathbf{B}$ drift radially due to fluctuating electric field



Confinement gets worse with increasing Turbulence Level

Global confinement scales with **core** turbulence level

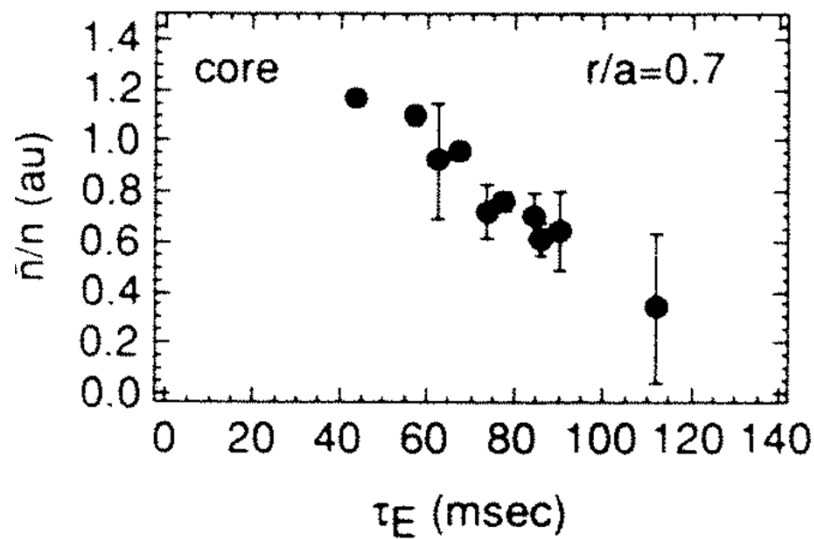
Equipe TFR & A. Truc, NF (1986)

Brower NF (1987) TEXT

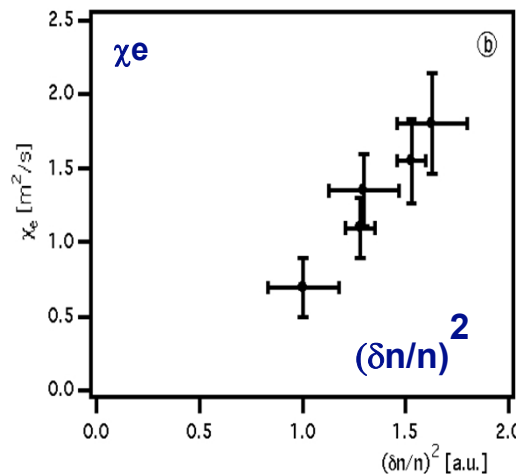
Paul et al, PoF (1992) TFTR

R=2.5m, a=0.89m

Durst et al, PRL (1993)



Local confinement also scales with turbulence level



Tore Supra

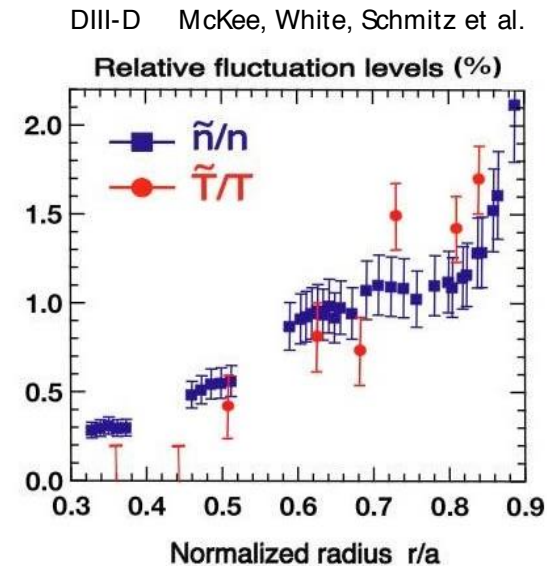
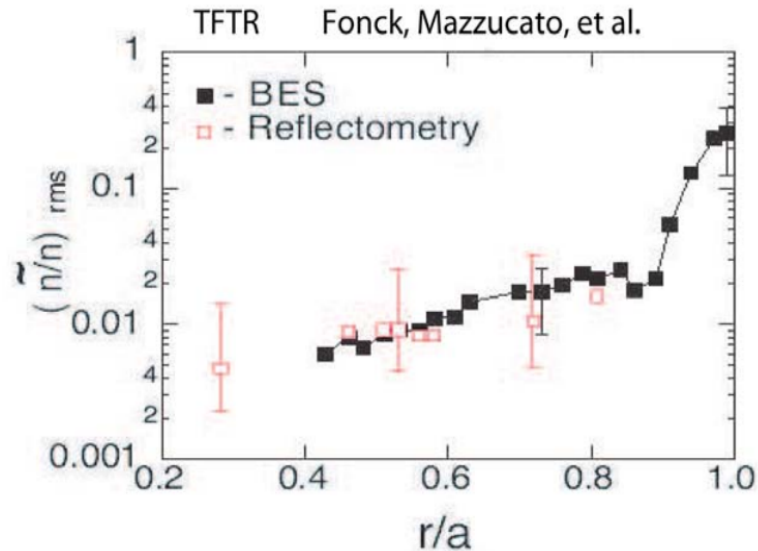
R=2.4m, a=0.7m

Laviron et al., IAEA (1996)

Zou et al., PRL (1995)

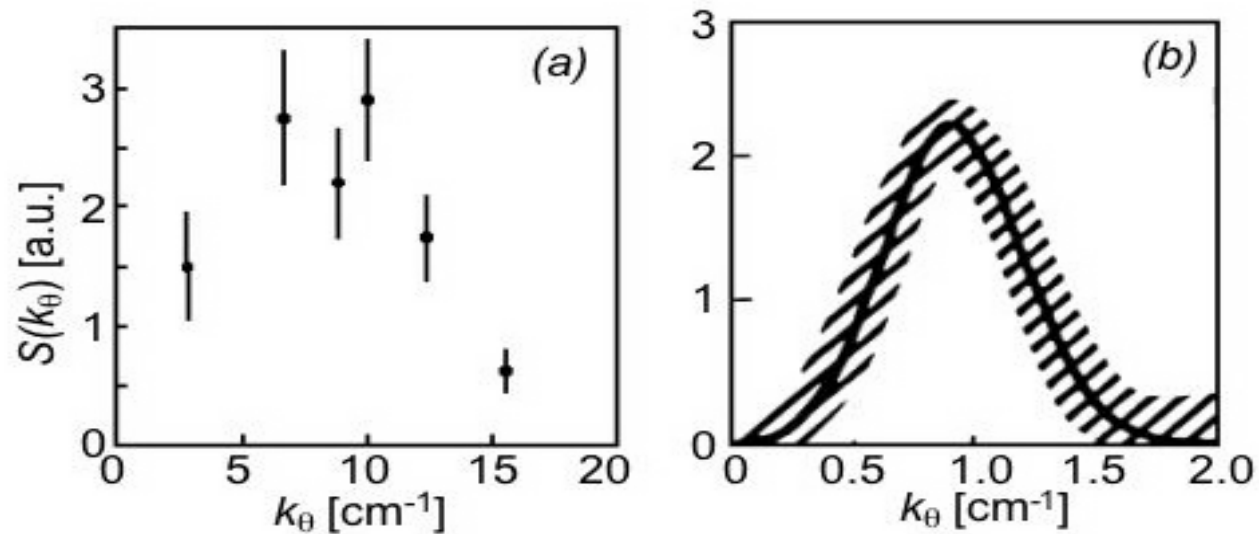
Hoang et al, Nuc. Fus. (1998)

Amplitude of Tokamak Microturbulence



- Relative fluctuation amplitude $\delta n / n_0$ at core typically less than 1%
- At the edge, it can be greater than 10%
- Confirmed in different machines using different diagnostics

k-spectra of tokamak micro-turbulence



$k_\theta \rho_i \sim 0.1 - 0.2$

-from Mazzucato et al., PRL '82 (μ -wave scattering on ATC)
Fonck et al., PRL '93 (BES on TFTR)

-similar results from

TS, ASDEX, JET, JT-60U and DIII-D

Properties of Tokamak Core Microturbulence

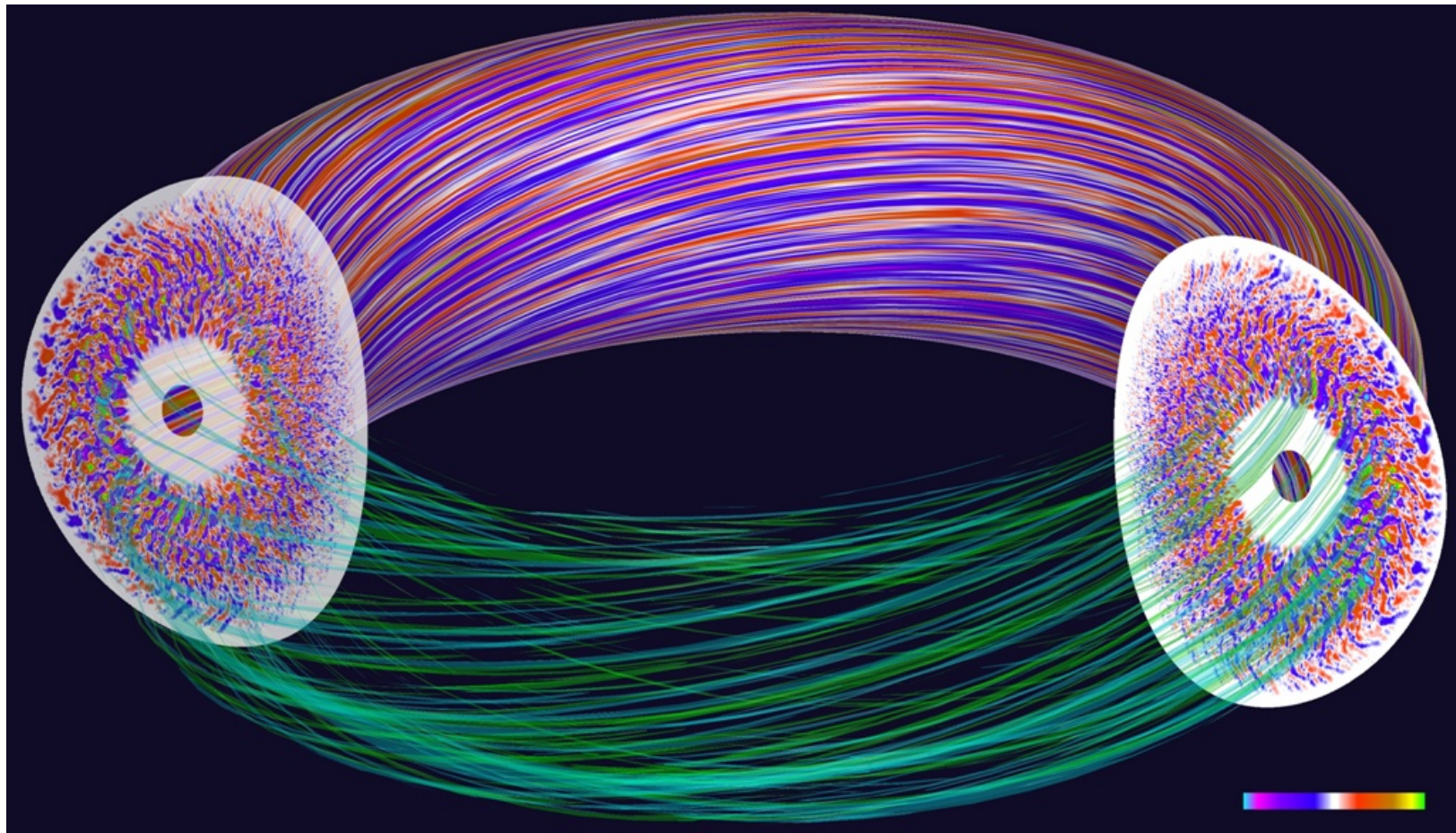
from Measurements

- $\delta n / n_0 \sim 1\%$
- $k_r \rho_i \sim k_\theta \rho_i \sim 0.1 - 0.2$
- $k_{\parallel} < 1/qR \ll k_{\perp}$: Rarely measured
- $\omega - \mathbf{k} \cdot \mathbf{u}_E \sim \Delta\omega \sim \omega_{*pi}$

Broad-band \Rightarrow Strong Turbulence

Sometimes Doppler shift dominates in rotating plasmas

Contours of Density Fluctuations Exhibit Turbulence Structure



Fully Developed Ion Temperature Gradient (ITG) Driven Turbulence:
from Gyrokinetic Particle Simulations by S. Ethier, W. Wang et al.,

Outline

Properties of Tokamak Core Turbulence

**Implications on Tokamak Confinement Scaling
with respect to Machine Size**

Ion Temperature Gradient(ITG) Instability

- High T_i for fusion plasma \rightarrow study of ∇T_i and heat transport.

∇T_i : excites ion acoustic wave

+ ∇n_0 : influences growth rate

- **Derivation of ITG Dispersion Relation**

Assumptions:

$$\begin{aligned} T_{i0} &= T_0(x) & \mathbf{B} &= B_0 \hat{z} & \omega &\ll k_{\parallel} v_{\text{th},e} & k_{\perp}^2 \rho_i^2 &\ll 1 \\ T_i &= T_0 + \delta T_i & & & \omega &\gg k_{\parallel} v_{\text{th},i} & & \end{aligned}$$

1. Continuity equation (linearized)

$$\frac{\partial}{\partial t} \delta n + \delta \mathbf{u}_E \cdot \nabla n_0 + n_0 \nabla_{\parallel} \delta u_{\parallel} + n_0 \cancel{\nabla_{\perp} \cdot \delta \mathbf{u}_{\text{pol}}} = 0$$

Ion Temperature Gradient(ITG) Instability

2. Equation of motion for ions

$$M_i \frac{\partial}{\partial t} \delta u_{\parallel} = -|e| \nabla_{\parallel} \delta \phi - \frac{1}{n_0} \nabla_{\parallel} \delta p_i$$

assume $k_{\perp}^2 \rho_s^2 \ll 1$

where $\rho_s = c_s / \Omega_{ci}$ and $c_s = \sqrt{T_e / M_i}$

3. Pressure Evolution :

Γ : Adiabatic exponent for equation of state $\rightarrow \frac{p}{\rho^{\Gamma}} = \text{const}$

$$\rightarrow \frac{\partial}{\partial t} \delta p_i + \delta \mathbf{u}_E \cdot \nabla p_0 + \Gamma n_0 \nabla_{\parallel} \delta u_{\parallel} = 0$$

- Here, “adiabatic” implies “thermal insulation”



- Perturbed quantities $\propto \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{x}) \rightarrow \frac{\partial}{\partial t} \Rightarrow -i\omega, \nabla_{\parallel} \Rightarrow ik_{\parallel}$

Ion Temperature Gradient(ITG) Instability

- Characteristic spatiotemporal scales:

Length:

$$\left. \frac{\partial T_i(x)}{\partial x} \right|_{x=x_0} = -\frac{1}{L_{Ti}} T_i(x) \Big|_{x=x_0}$$

$$p_i = n_0 T_i \rightarrow \frac{1}{L_{pi}} = \frac{1}{L_{Ti}} + \frac{1}{L_n}, \quad \tau \equiv \frac{T_e}{T_i}$$

Frequency:

$$\omega_{*e} = \frac{k_y \rho_s}{L_n} c_s > 0$$

$$\omega_{*pi} = -\frac{k_y \rho_i}{L_{pi}} v_{Ti} < 0$$

$$\omega_{*Ti} = -\frac{k_y \rho_i}{L_{Ti}} v_{Ti} < 0$$

- Governing equations (1~3) :

$$-i\omega \frac{\delta n}{n_0} + i\omega_{*e} \frac{|e| \delta \phi}{T_e} + ik_{\parallel} c_s \left(\frac{\delta u_{\parallel}}{c_s} \right) = 0$$

$$-i\omega \frac{\delta u_{\parallel}}{c_s} + ik_{\parallel} c_s \frac{|e| \delta \phi}{T_e} + ik_{\parallel} c_s \left(\frac{T_i}{T_e} \right) \frac{\delta p_i}{p_0} = 0$$

$$-i\omega \frac{\delta p_i}{p_0} - i \frac{\omega_{*pi}}{\tau} \frac{|e| \delta \phi}{T_e} + ik_{\parallel} c_s \Gamma \frac{\delta u_{\parallel}}{c_s} = 0$$

$$\rightarrow \text{with } \frac{\delta n}{n_0} = \frac{|e| \delta \phi}{T_e}, \quad \underbrace{\begin{bmatrix} -i(\omega - \omega_{*e}) & ik_{\parallel} c_s & 0 \\ ik_{\parallel} c_s & -i\omega & ik_{\parallel} c_s \frac{1}{\tau} \\ -i \frac{\omega_{*pi}}{\tau} & ik_{\parallel} c_s \Gamma & -i\omega \end{bmatrix}}_{\text{Determinant is 0 for non-trivial solution}} \begin{bmatrix} \delta n/n_0 \\ \delta u_{\parallel}/c_s \\ \delta p_i/p_0 \end{bmatrix} = 0$$

Determinant is 0 for non-trivial solution

Ion Temperature Gradient(ITG) Instability

- ITG Instability Dispersion Relation:

$$1 - \frac{\omega_{*e}}{\omega} - \frac{k_{\parallel}^2 c_s^2}{\omega^2} \left(1 + \frac{\Gamma}{\tau} - \frac{\omega_{*pi}}{\omega} \right) = 0$$

(from determinant = 0)

- Limiting Cases of Dispersion Relation

1. $\tau \rightarrow \infty, L_{Ti} \rightarrow \infty$

we recover $1 - \frac{\omega_{*e}}{\omega} - \frac{k_{\parallel}^2 c_s^2}{\omega^2} = 0.$

(No $k_{\perp} \rho_s$ term because we assumed $k_{\perp}^2 \rho_s^2 \ll 1$)

2. $1/L_{Ti} \nearrow, 1/L_n \nearrow$

Then, $\omega_{*e}, |\omega_{*pi}| \nearrow$. The 2nd and 5th terms of dispersion relation become dominant.

$$\frac{\omega_{*e}}{\omega} \simeq \frac{k_{\parallel}^2 c_s^2}{\omega^2} \frac{\omega_{*pi}}{\omega} = -\frac{\omega_{*e}}{\omega} \frac{k_{\parallel}^2 c_s^2}{\omega^2} \frac{1 + \eta_i}{\tau}$$

here, $\eta_i \equiv L_n/L_{Ti}$

$$\Rightarrow \omega^2 = -\left(\frac{1 + \eta_i}{\tau}\right) k_{\parallel}^2 c_s^2. \quad \therefore \text{growth rate } \underline{\gamma = \left(\frac{1 + \eta_i}{\tau}\right)^{1/2} k_{\parallel} c_s}$$

ITG Instability in Uniform Magnetic Field is a Negative Compressibility Wave

- **ITG dispersion relation:** $\frac{\omega^2}{k_{\parallel}^2} = - \left(\frac{1 + \eta_i}{\tau} \right) c_s^2$
 - **Soundwave-like** when $(1 + \eta_i) / \tau \gg 1$, k_{\parallel} very small, and $\omega / k_{\parallel} \gg v_{th,i}$.

- **Soundwave in gas:** $\left. \begin{aligned} \rho_0 \frac{\partial}{\partial t} \delta u &= - \frac{\partial}{\partial z} \delta p \\ \frac{\partial}{\partial t} \delta p &= - \Gamma p_0 \frac{\partial}{\partial z} \delta u \end{aligned} \right\} \rightarrow$ from $\frac{\partial}{\partial t} \delta \rho + \rho_0 \frac{\partial}{\partial z} \delta u = 0$ and $\frac{p}{\rho^\Gamma} = \text{const}$
 - Dispersion Relation: $\frac{\omega^2}{k_{\parallel}^2} = \frac{\Gamma p_0}{\rho_0}$
 - Adiabatic compressibility: $\kappa_s = -V^{-1} \left(\frac{\partial V}{\partial p} \right)_s$
 - From the equation of state, $pV^\Gamma = p_0 V_0^\Gamma$. and so, $p = p_0 V_0^\Gamma V^{-\Gamma}$

$$\left(\frac{\partial p}{\partial V} \right)_s = -\Gamma p_0 V_0^\Gamma V^{-\Gamma-1} \text{ and } V \left(\frac{\partial p}{\partial V} \right)_s = -\Gamma p$$

Therefore, $\kappa_s \equiv -V^{-1} \left(\frac{\partial V}{\partial p} \right)_s = \left[-V \left(\frac{\partial p}{\partial V} \right)_s \right]^{-1} = (\Gamma p)^{-1} \rightarrow \left(\frac{\omega}{k} \right)^2 = (\rho_0 \kappa_s)^{-1}$

ITG Instability in Uniform Magnetic Field is a Negative Compressibility Wave

$$\left(\frac{\omega}{k}\right)^2 = (\rho_0 \kappa_s)^{-1}$$

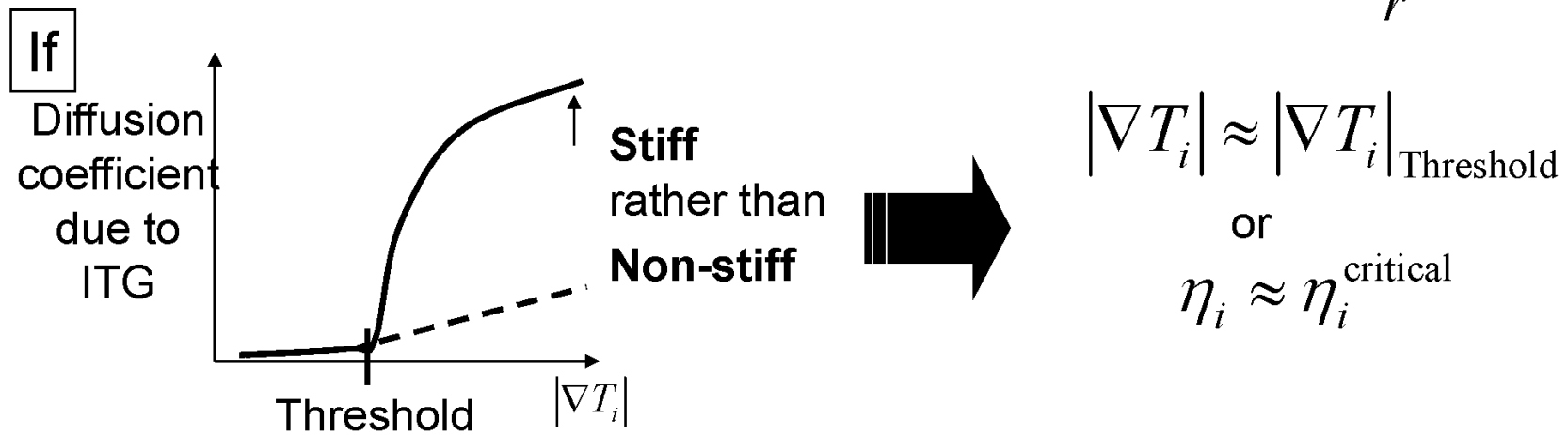
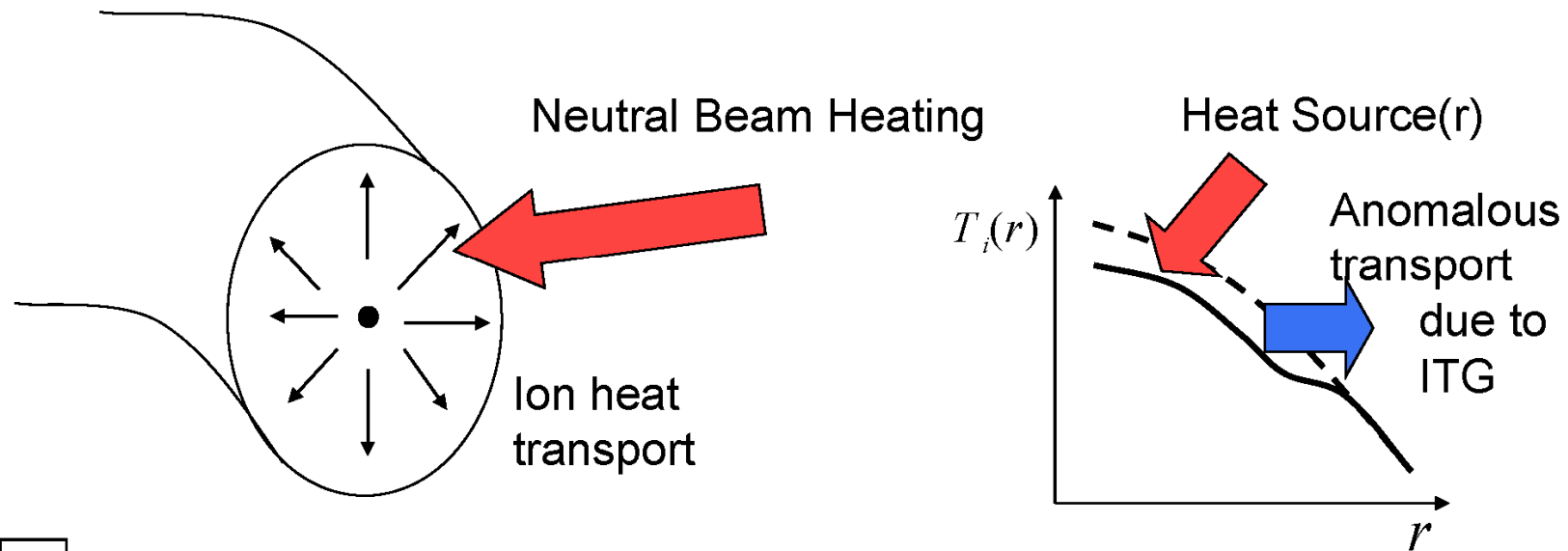
Soundwave dispersion
relation

$$\frac{\omega^2}{k_{\parallel}^2} = - \left(\frac{1 + \eta_i}{\tau}\right) c_s^2$$

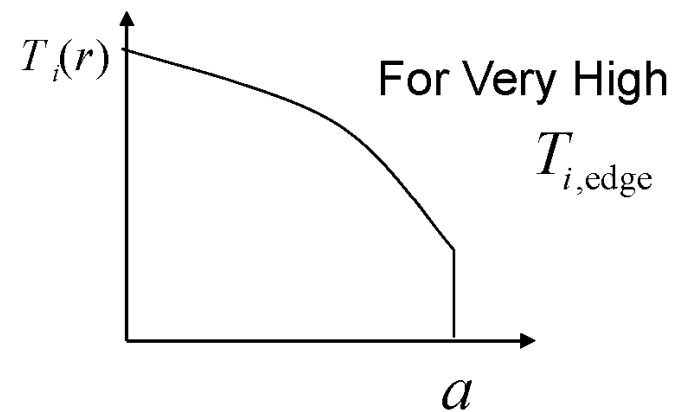
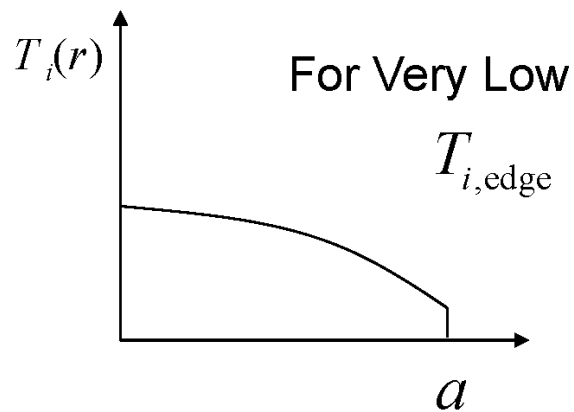
ITG instability
dispersion relation

- Soundwave depends on compressibility κ_s .
- Effective “compressibility” of the ITG instability is “negative”.

∴ The instability mechanism for ITG is a **“negative compressibility”**.



So if $T_i(r) \propto \text{const} \cdot n_0(r)^2$ is the marginality profile, and n_0 is given, $T_i(0)$ is almost uniquely determined by $T_{i,\text{edge}}$



- ❖ In this extreme limit, $T_i(0)$ (better be high for fusion) is mostly determined by $T_i(a)$, not much by transport in the core (since it's so rapid, throws away excess heat which will raise $T_i(r)$ above marginality.)
- ❖ This is (very simplified) reason why ITER needs to achieve H-mode plasmas in which $T_{i,edge}$ (pedestal) is high.

Spatial Structure of Microturbulence

A. Role of Magnetic Geometry

So far in this lecture series, we've discussed microinstabilities in the context of "local theory", i.e., for given values of Macroscopic Parameters, $n_0(x = x_0)$, $\frac{dn_0}{dx}(x = x_0)$, \vec{B}_0 (uniform), etc.

with
$$\delta\phi(x, y, z, t) = \sum_{\mathbf{k}, \omega} \delta\phi_{\mathbf{k}, \omega} e^{i(k_x x + k_y y + k_z z) - i\omega t}$$

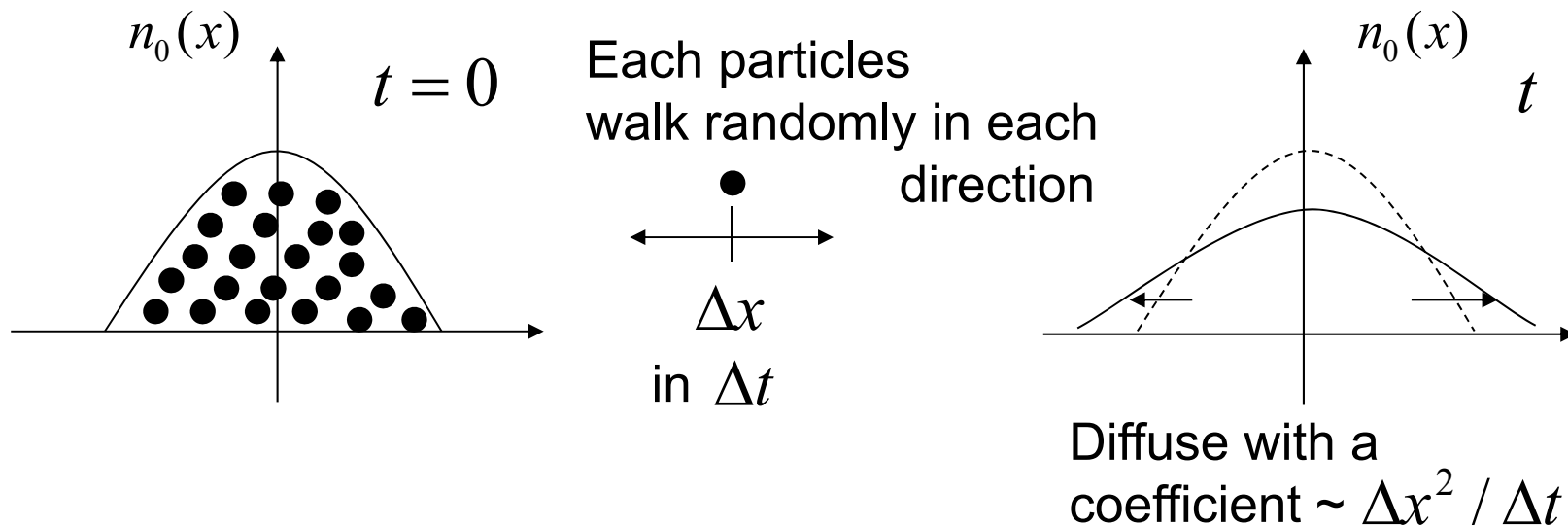
Independent of x , while there are $n_0(x)$, $T_i(x)$ profiles so on.

A challenge is to find more realistic and relevant representation of tokamak microinstabilities.

Very rough estimation of the anomalous transport coefficient

D_{Turb} using dimensional analysis based on

“Random Walk” argument



Since anomalous transport is caused by fluctuating δv_x due to microinstabilities in plasmas, we can argue

$$\Delta x \sim \frac{1}{k_x}, \quad \Delta t \sim \omega_{\text{decorrelation time}}^{\text{Turb}-1} \sim \gamma_{\text{linear}}^{-1}$$

Then,

$$D_{\text{Turb}} \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\gamma_{\text{lin}}}{k_x^2} \sim \frac{\omega_*}{k_x^2} \sim \frac{k_y}{k_x^2} \frac{\rho_i}{L} \left(\frac{cT_i}{eB} \right)$$

$L \sim a$, L_n for drift waves, L_{Ti} for ITG turbulence, so on

It's obvious that depending on the choice of k_x and k_y ,

D_{Turb} scaling has many possibilities.

If one takes a practical approach of using values of k_x and k_y from experimental measurements, $k_x, k_y \propto \rho_i^{-1}$ (where the spectrum peaks)

Then

$$D_{\text{Turb}} \sim \left(\frac{\rho_i}{L} \right) \left(\frac{cT_i}{eB} \right) : \propto \frac{cT}{eB} \text{ is called the "Bohm" scaling.}$$

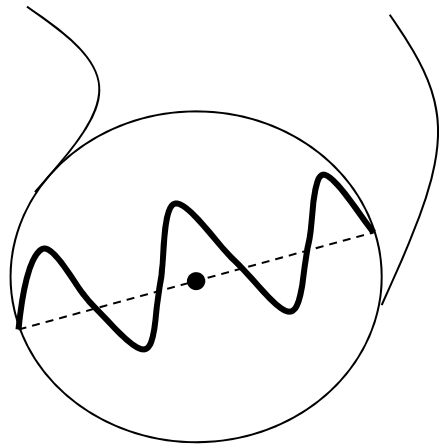
Since it's reduced by a factor $\left(\frac{\rho_i}{L} \right) \ll 1$, "gyroBohm" scaling

While it's more common to get “gyroBohm” scaling from simple local theory, most experiments in tokamaks exhibited results which are closer to “Bohm” scaling rather than “gyroBohm” scaling, especially for ion thermal transport (χ_i) in L-mode plasmas. It's very important to achieve a thorough understanding of “size-scaling” of D_{Turb} , for prediction to larger devices in the future.

$$\begin{array}{l}
 D_{\text{Bohm}} \propto \left(\frac{cT_i}{eB} \right) \\
 \text{or} \\
 D_{\text{gyroBohm}} \propto \left(\frac{\rho_i}{a} \right) \left(\frac{cT_i}{eB} \right) \quad ?
 \end{array}$$

Then, what scales of k_x , and k_y can give us D_{Bohm} ?

“Bohm” came from experimental observations on very early basic devices (i.e., small). Then, even drift wave type instabilities have relatively low mode numbers.



Eg., Quantization condition

$$k_x a \sim N_x \pi \quad (N_x, M_y \sim O(1) \text{ integer})$$


$$k_y a \sim M_y \pi$$

$$\rightarrow D_{\text{Turb}} \sim \frac{k_y}{k_x^2} \frac{\rho_i}{L} \left(\frac{cT_i}{eB} \right) \sim \left(\frac{cT_i}{eB} \right) !$$

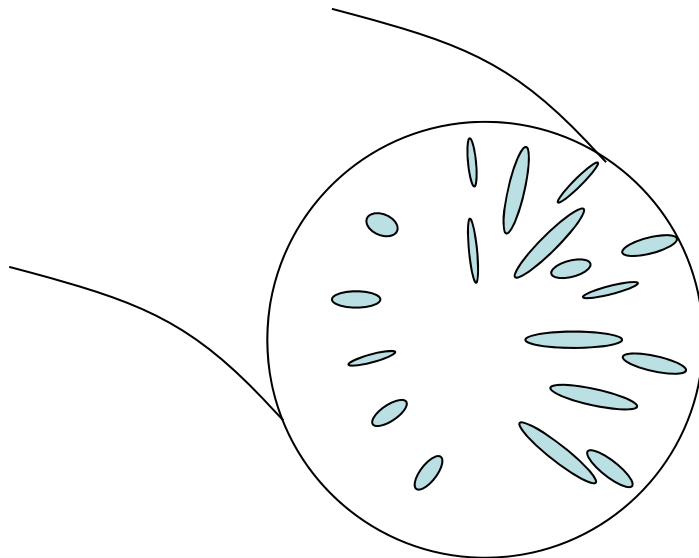
We learn that if $\lambda_x, \lambda_y \propto a$ (system size)

one can get “Bohm” scaling of transport.

Then, what happens to present day larger tokamaks? say $a \approx 100 \rho_i$

- From B.E.S.
Microwave Scatt.
etc.  Eddy size $\sim \lambda_x, \lambda_y \sim$ several ρ_i

- From Nonlinear Gyrokinetic Simulations



$$\lambda_x, \lambda_y \ll a$$

So we want to know **what physics mechanisms determine** dominant λ_x and λ_y (**eddy size** to be more precise).

➔ “Nonlocal Analysis” is required to find

“spatial structure of micro-turbulence.”

Linear theory limit



“eigenmode structure of microinstabilities”

❖ **Toroidal Geometry**, taking into account of

$$|\vec{B}| \sim B_\phi \propto \frac{1}{R} = \frac{1}{R_0 + \underline{\underline{r \cos \theta}}}$$

❖ In the end, **Self-Organization** or **Self-Regulation**

determines the spatial structure of tokamak micro-turbulence

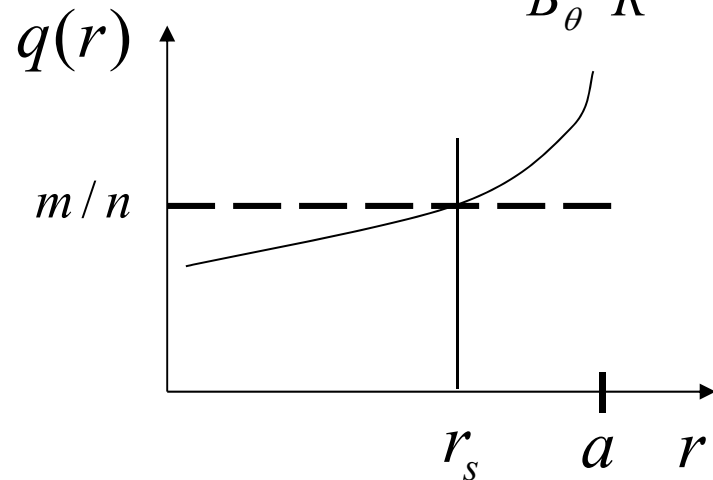
So far, in slab

$$\begin{aligned} x &\longrightarrow r \\ y &\longrightarrow r\theta \\ z &\longrightarrow R\phi \end{aligned}$$

but \vec{B} has both
 B_ϕ (toroidal) components
 B_θ (poloidal)

$$\delta\phi(r, \theta, \phi) = \sum_{n,m} \delta\phi_{n,m}(r) e^{i(m\theta - n\phi)} \quad \text{“pitch of fluctuation”}$$

$$q(r) \cong \frac{B_\phi}{B_\theta} \frac{r}{R} \quad \text{determines pitch of } \vec{B} \quad n, m \in \mathbf{Z}_+$$



If two coincide, $q(r_s) = \frac{m}{n}$.

This radial location $r = r_s$ is called the
 “mode rational surface.”

$$\begin{array}{ccc}
 e^{i(m\theta - n\phi)} & \longrightarrow & k_\theta = \frac{m}{r}, \quad k_\phi = -\frac{n}{R} \\
 \searrow & & \\
 e^{i(k_y y + k_z z)} & & \vec{B} = B_\phi \hat{\phi} + B_\theta \hat{\theta}
 \end{array}$$

$$\therefore k_{\parallel} = \frac{\vec{k} \cdot \vec{B}}{|\vec{B}|} = \frac{m}{r} \frac{B_\theta}{B} - \frac{n}{R} \frac{B_\phi}{B} = \frac{B_\theta}{rB} (m - nq(r))$$

★ $k_{\parallel} = 0$ at $r = r_s$, $(q(r_s) = \frac{m}{n})$.

m, n fixed but $q(r)$ & therefore k_{\parallel} varies with r .

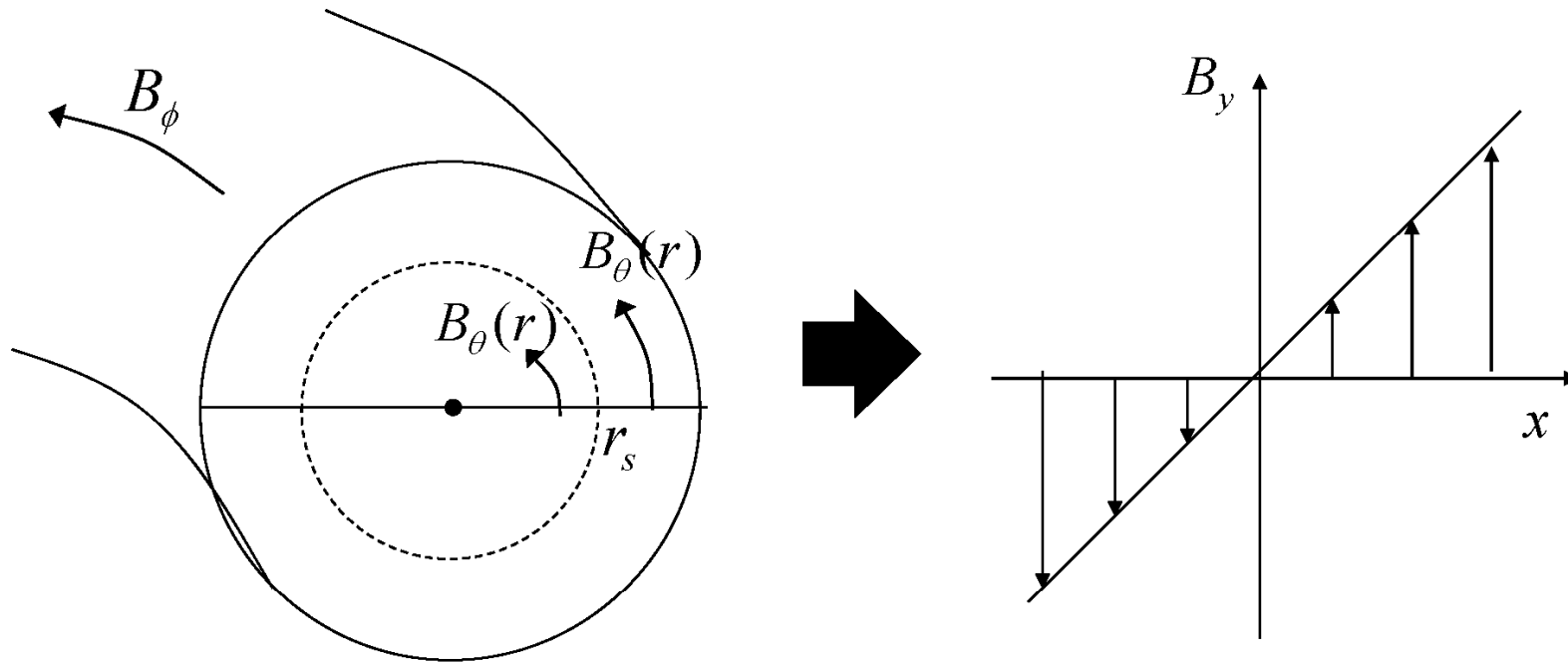
Expanding $q(r) = q(r_s) + (r - r_s) \left(\frac{\partial q}{\partial r} \right) (r_s) + \dots$

$k_{\parallel}(r - r_s) = k_{\parallel}(x)$ increases with x ,

flips sign across r_s (or $x = 0$).

➔ Near mode rational surface $\vec{B} = B(\hat{z} + \frac{x}{L_s} \hat{y})$ (3.9)

where $L_s = \frac{qR}{\hat{s}}$, $\hat{s} = \frac{r}{q} \frac{dq}{dr}$ magnetic shear.



“Sheared Slab Geometry”

In sheared slab geometry (with one rational surface),

$$k_{\parallel} = \frac{k_y}{L_s} x \quad , \quad k_x^2 \rightarrow -\frac{\partial^2}{\partial x^2}$$

k_z has been shifted away.

“ $\delta\phi(\vec{x}, t) = \sum_{k_y} \delta\phi_{k_y, \omega}(x) e^{i(k_y y - \omega t)}$ “ : This mode refers to
 “a single helicity fluctuation”
 (n, m) with $\frac{m}{n} \rightarrow q(r_s)$

Radial Mode Structure of Drift Wave

Lecture I.
 Local theory \longrightarrow $\left(1 + \rho_s^2 k_{\perp}^2 - \frac{\omega_{*e}}{\omega} - \frac{k_{\parallel}^2 C_s^2}{\omega^2} \right) \delta\phi_{\vec{k}, \omega} = 0$

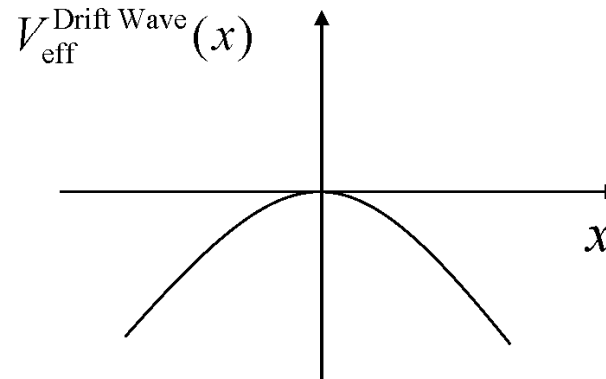
In sheared slab :

$$\left(1 + \rho_s^2 k_y^2 + \rho_s^2 \frac{\partial^2}{\partial x^2} - \frac{\omega_{*e}}{\omega} - \frac{C_s^2}{\omega^2} \frac{k_y^2}{L_s^2} x^2 \right) \delta\phi_{\vec{k}, \omega} = 0 \quad (3.10)$$

★ Weber Eqn. : familiar from Single Harmonic Osc. In Q.M.

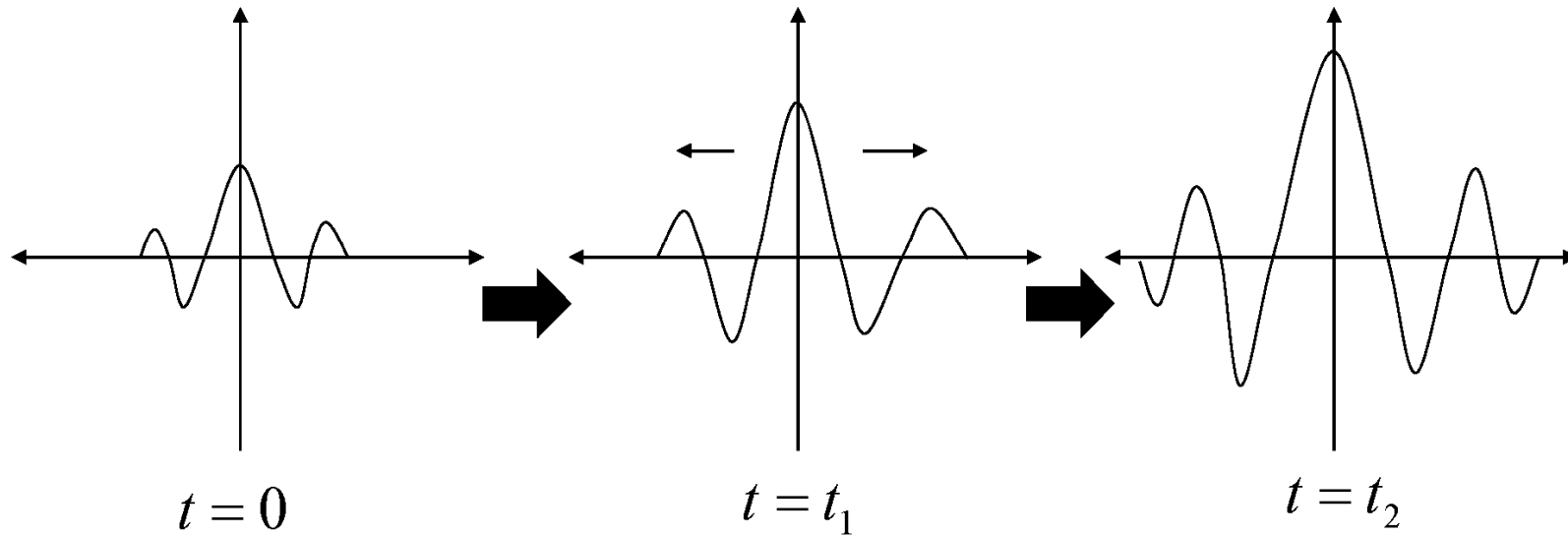
$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = (E - V_{\text{SHO}}(x))\psi = \left(E - \frac{1}{2} m \omega^2 x^2 \right) \psi \quad (3.11)$$

➔ We know how get Eigenfunctions and Eigenvalues of this eqn.
 One tricky point is that we have an antiwell potential (hill)
 rather than potential well, for $|\text{Re}(\omega)| \gg |\text{Im}(\omega)|$.



➔ Physically meaningful solution should satisfy the causality condition.

i.e., $\lim_{|x| \rightarrow \infty} |\delta\phi(x)| \rightarrow 0$ for $\text{Im}(\omega) > 0$
 unstable solution.



for unstable solution

while fluctuation grows locally in time,

at a given time, it should decay in radius as $|x| \rightarrow \infty$

Group Velocity : $v_{gx} \equiv \frac{\partial \omega}{\partial k_x}$ As $|x| \rightarrow \infty$

$$\frac{\partial \omega}{\partial k_x} > 0 \quad \text{for } x > 0$$

$$\frac{\partial \omega}{\partial k_x} < 0 \quad \text{for } x < 0$$

Out of two (mathematically) possible solutions
we should choose the upper one!

$$\begin{cases} \sim \exp\left(-\frac{ik_y C_s}{2\omega L_s \rho_s} x^2\right) \\ \sim \exp\left(+\frac{ik_y C_s}{2\omega L_s \rho_s} x^2\right) \end{cases}$$

Eigenvalue : $\omega = \omega_{*e} \left(\frac{1}{1+k_y^2 \rho_s^2} - \underbrace{\frac{i(2\ell_x+1)L_n}{1+k_y^2 \rho_s^2 L_s}}_{\text{magnetic shear-induced damping}} \right)$: radial quantum number
 $\ell_x = 0, 1, \dots$

magnetic shear-induced damping

$$\Delta x(\sim \lambda_x) \sim \sqrt{\frac{L_s \omega_{*e} \rho_s}{k_y C_s}} \sim \sqrt{\frac{L_s}{L_n}} \rho_s$$

$\hat{s} \curvearrowright \Rightarrow \Delta x \curvearrowright$: i.e., magnetic shear localizes the mode within the device
(not determined by B.C. at the WALL).

➡ get “gyroBohm” scaling. (if it were unstable by additional mechanism, eg., trapped electrons)

One can also extend local theory of “ITG” to sheared slab geometry.

(negative compressibility
acoustic mode)

Analysis is slightly more complicated than e⁻ DW, but can be reduced to Weber-Eqn. [*Coppi, Rosenbluth & Sagdeev, PF 10, 582 (1967)*].

It's noteworthy that an elaborate nonlinear mode coupling theory in sheared slab yielded (rather than from dimensional analysis we're discussing).

$$\chi_i^{\text{ITG}} \propto \text{gyroBohm}$$

[*G.S. Lee & P.H. Diamond, Phys. Fluids 29, 3291 (1986)*]

“Nonlocal “ kinetic theory can also be pursued in sheared slab geometry :

- Local kinetic theory predicted [K&P] Eq. (3.7)

$$\eta_i \geq \frac{2}{1 + 2b_i \left(1 - \frac{I_1(b_i)}{I_0(b_i)} \right)} \quad \text{for ITG excitation.}$$

Since $\eta_i \equiv \frac{L_n}{L_{Ti}}$, it predicts instability for very weak $\frac{1}{L_{Ti}}$ for flat density profile ($L_n \rightarrow \infty$)

NO GOOD for that limit.

I don't recall a credible analytic local ITG onset condition in the flat density limit.

- For sheared slab, flat density case,

$$\frac{L_s}{L_{Ti}} \geq 1.9 \left(\frac{T_i}{T_e} + 1 \right) \quad \text{is the onset condition}$$

[Hahm & Tang, PFB '89]

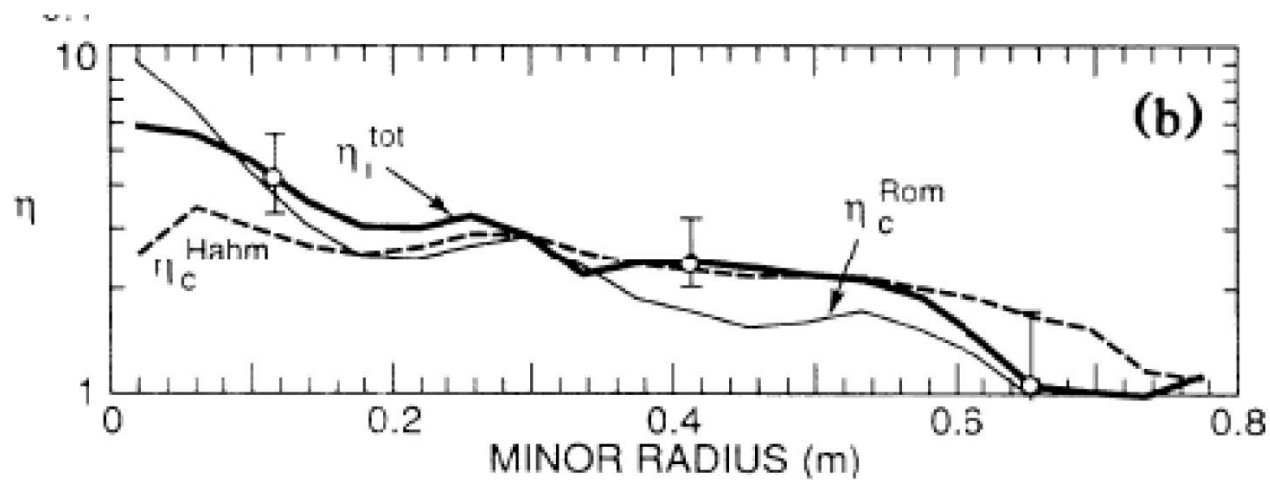
“ $\frac{T_i}{T_e} > 1$ strong magnetic shear favorable for ITG stability !”

- In toroidal geometry : $\omega - k_{\parallel} v_{\parallel}$ resonance should be generalized to $\omega - k_{\parallel} v_{\parallel} - \omega_{di}$ resonance (ω_{di} is from ∇B & Curvature drift)

If one keeps only $\omega - \omega_{di}$ resonance, (ignoring $k_{\parallel} v_{\parallel}$)

$$\frac{R}{L_{Ti}} \geq \frac{4}{3} \left(\frac{T_i}{T_e} + 1 \right) \quad \text{[Romanelli, PFB '89]}$$

→ Comparisons to TFTR



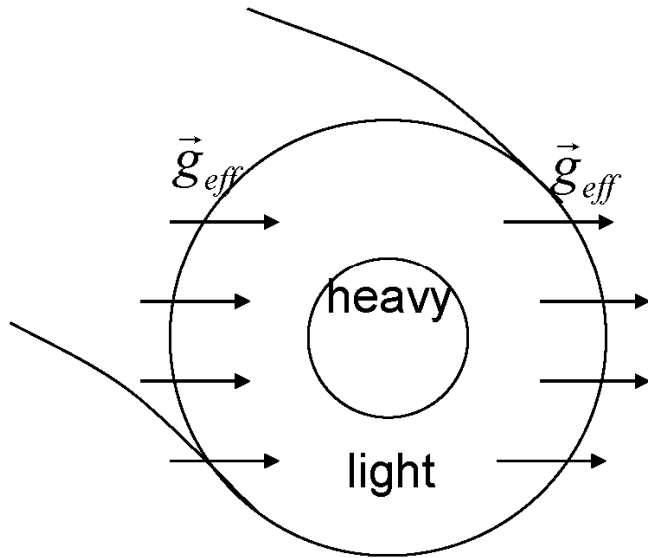
Comparison of measured η_i^{tot}
 with the theoretical estimates η_c^{Rom} and η_c^{Hahm}

[S. Scott et al., PRL 29, 531 (1990)]

Role of Toroidal Geometry

Uniform \vec{B} model \longrightarrow Sheared Slab Model \longrightarrow "Toroidal Geometry"

$$\vec{B} = B_0 \left(\hat{z} + \frac{x}{L_s} \hat{y} \right)$$



\hat{y} still a symmetry : $\frac{1}{R}$ dependence
 direction

m : \longrightarrow poloidal angle
 No longer symmetry direction

n : a good quantum #



For given n ,
 many " m " harmonics
 should couple
 (even in linear theory)

$$|\vec{B}| \propto \frac{1}{R} = \frac{1}{R_0 + r \cdot \cos^2 \theta}$$

Before looking for a good representation of fluctuation decomposition in torus, let's consider a simplest nontrivial example of ITG mode in torus (with : can be unrelated to “negative compressibility acoustic” ITG.)

It has an “interchange” or “Rayleigh-Taylor” character.

➡ must be localized at bad curvature (low B field) side.

→ motivate a “local” theory at bad curvature side.

$|\vec{B}| \propto \frac{1}{R}$ ➡ particles drift in vertical direction (∇B & curvature drift)

We'll get to details later, but

$$v_{\nabla B, \text{Curv}} \propto \bar{v}_{\nabla B, \text{Curv}} \frac{v_{\parallel}^2 + \mu B}{v_{Ti}^2}$$

An important consequence of this “energy dependent” particle drift

➡ δn couples to δT_i !

Recall, in uniform \vec{B}_0 ➡ $\partial_t \delta n + \delta \vec{u}_E \cdot \vec{\nabla} n_0 + n_0 \nabla_{\parallel} \delta u_{\parallel} \simeq 0$

Now with nonuniform \vec{B}_0 : take moments of linearized GK eqn in torus.

$$\left(\frac{\partial}{\partial t} + v_{\parallel} \nabla_{\parallel} + \underbrace{\vec{v}_d \cdot \vec{\nabla}}_{\substack{\uparrow \\ \text{most obvious addition due to toroidal effects}}} \right) \delta f + \left(\frac{c}{B} \vec{\nabla} \langle \delta \phi \rangle \times \hat{b} \cdot \nabla - \frac{q}{m} \nabla_{\parallel} \langle \delta \phi \rangle \frac{\partial}{\partial v_{\parallel}} \right) F_0 = 0$$

Apply

$$\int d^3 \vec{v} : \quad \text{noting that} \quad \vec{v}_d = \vec{v}_{d,th} \left(\frac{v_{\parallel}^2 + \mu B}{v_{Ti}^2} \right)$$

Then,

$$\frac{\partial}{\partial t} \delta n_i + \delta \vec{u}_E \cdot \vec{\nabla} n_0 + \frac{n_0}{T_i} \bar{\omega}_{di} \delta T_i + n_0 \nabla_{\parallel} \delta u_{\parallel} + \dots = 0$$

To focus on “interchange” physics, take $k_{\parallel} \rightarrow 0$ (as done by Romanelli in kinetic regime)
and take a simple “flat density” limit.

$$\Rightarrow \frac{\partial}{\partial t} \delta n_i + \frac{n_0}{T_i} \bar{\omega}_{di} \delta T_i \simeq 0$$

$$\bar{\omega}_{di} \equiv -\frac{cT_i}{eBR} k_y$$

(at bad curvature side)

&

Take simplest ∇T_i evolution eqn.

$$\frac{\partial}{\partial t} \delta T_i + \delta u_E \cdot \nabla T_0 \simeq 0$$

(assumed $\omega > \bar{\omega}_{di}$, but $|\omega_{*Ti}| > \omega$)

(2x2) \Rightarrow

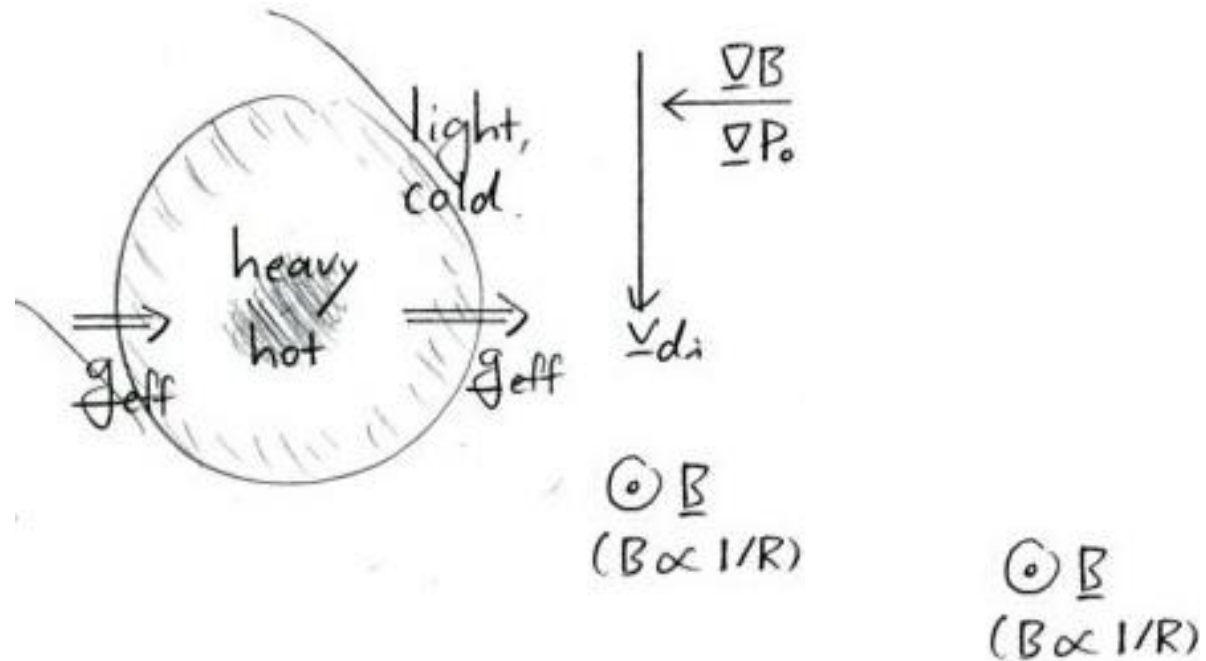
$$\boxed{\omega^2 = -\frac{T_e}{T_i} |\bar{\omega}_{di} \omega_{*Ti}|}$$

—— (simplified)
Toroidal ITG

bad-curvature coupled to ∇T_i

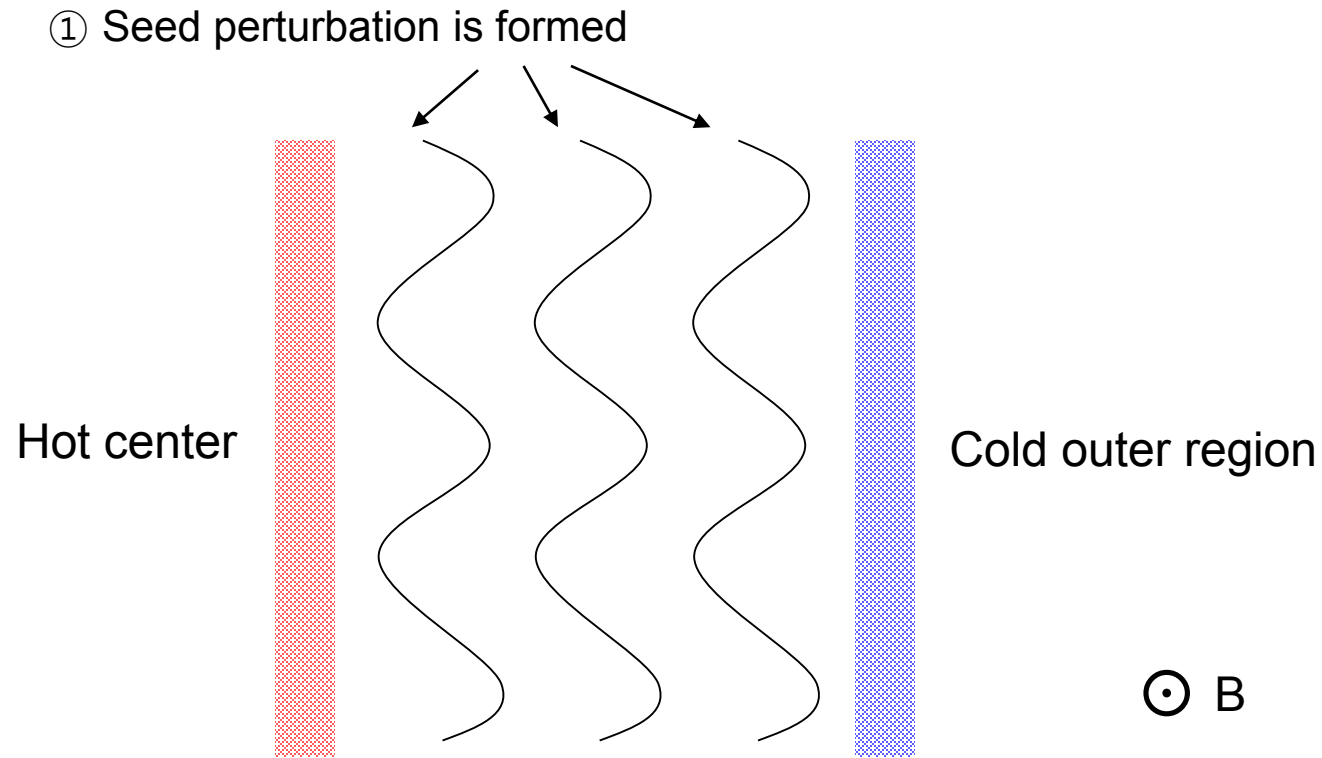
Basic Properties of ITG Instability in Toroidal Geometry

- Unstable ITG: related to Rayleigh-Taylor instability.
(contrast to negative compressibility ITG in slab geometry)



Basic Properties of ITG Instability in Toroidal Geometry

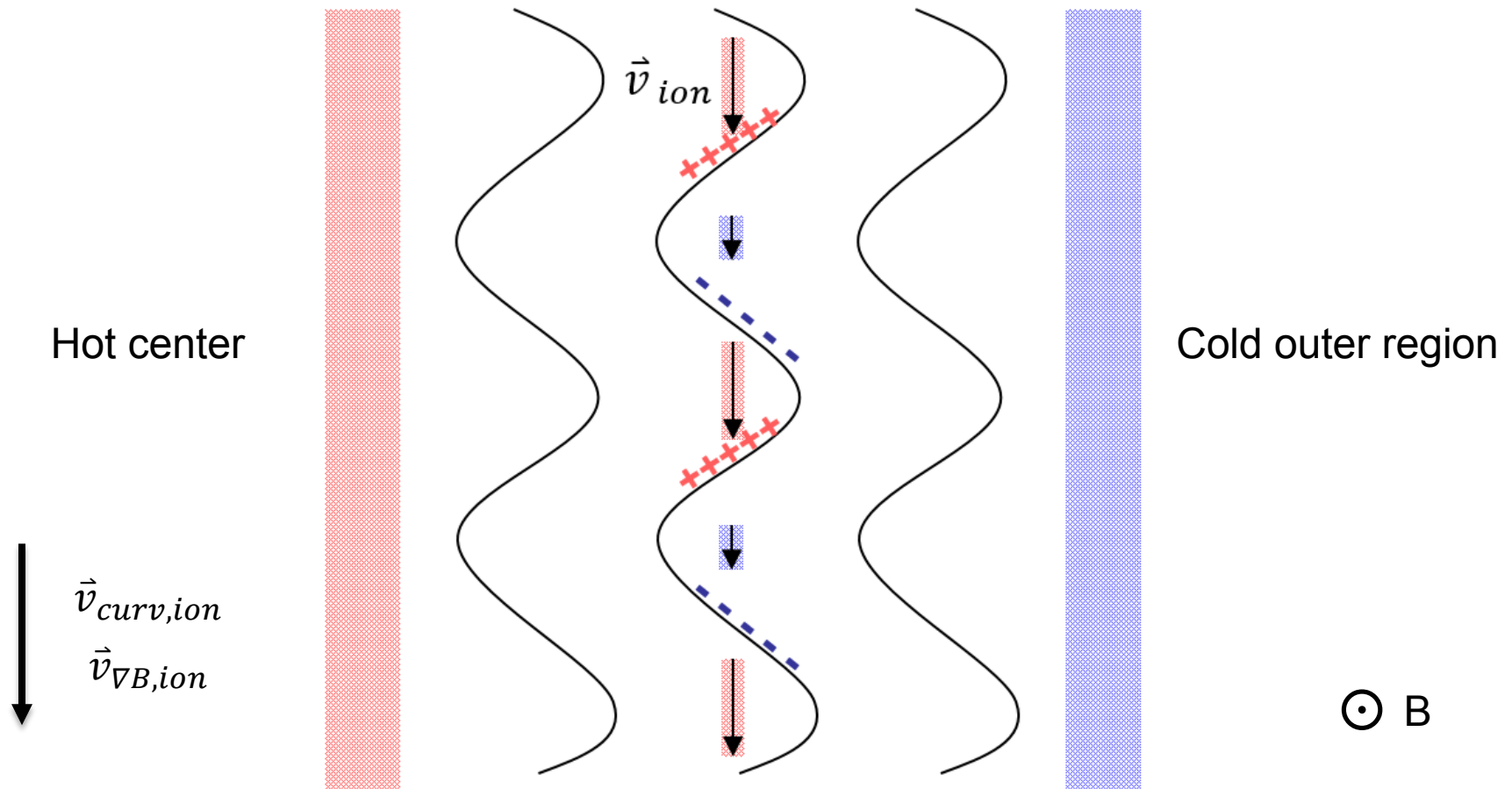
- **Physical Picture of Rayleigh-Taylor ITG Instability in 4 Steps**



Sectional view of tokamak plasma in “bad curvature” region

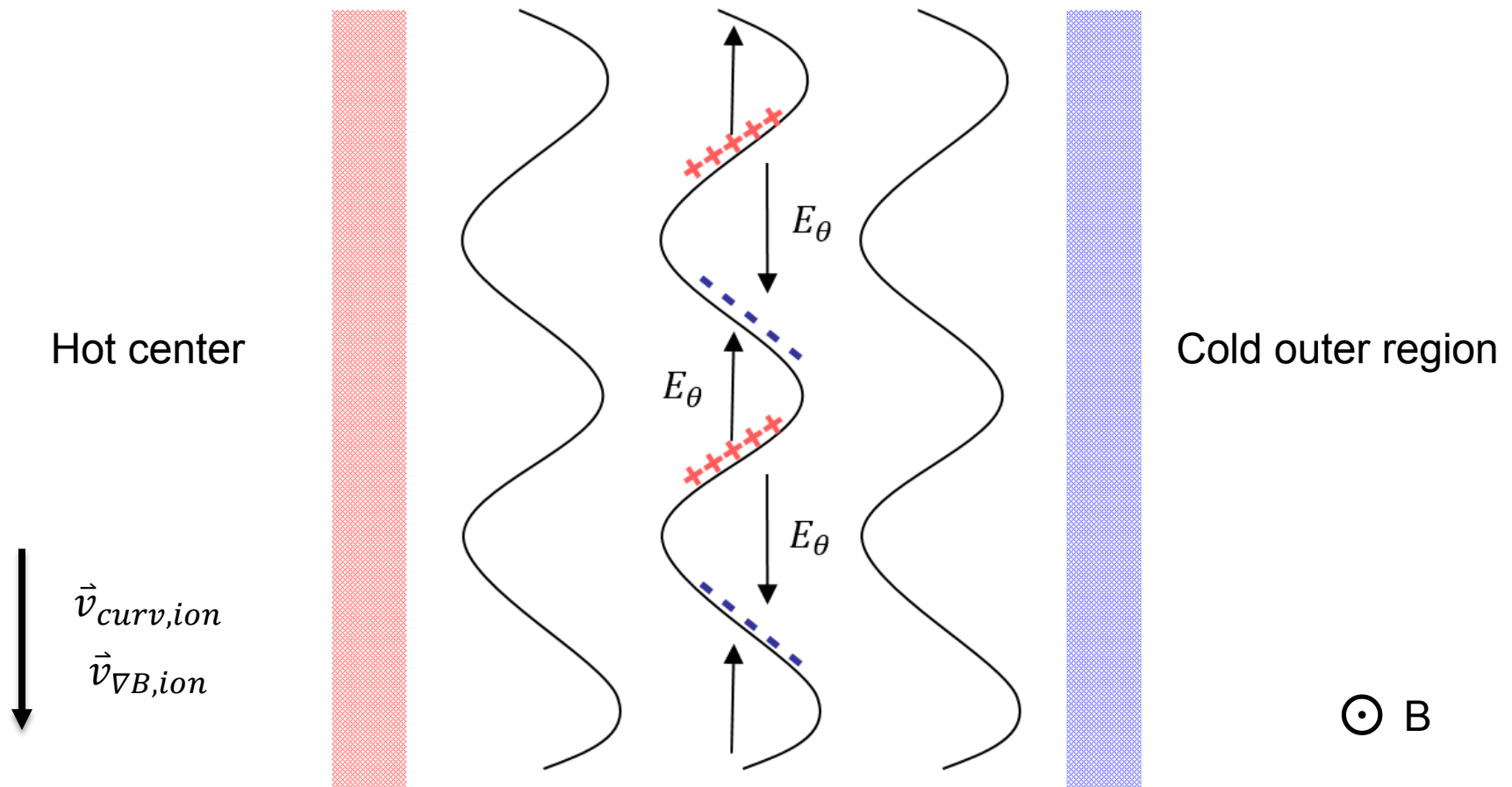
Basic Properties of ITG Instability in Toroidal Geometry

- ② Higher energy ions drift downwards faster, leads to charge separation.



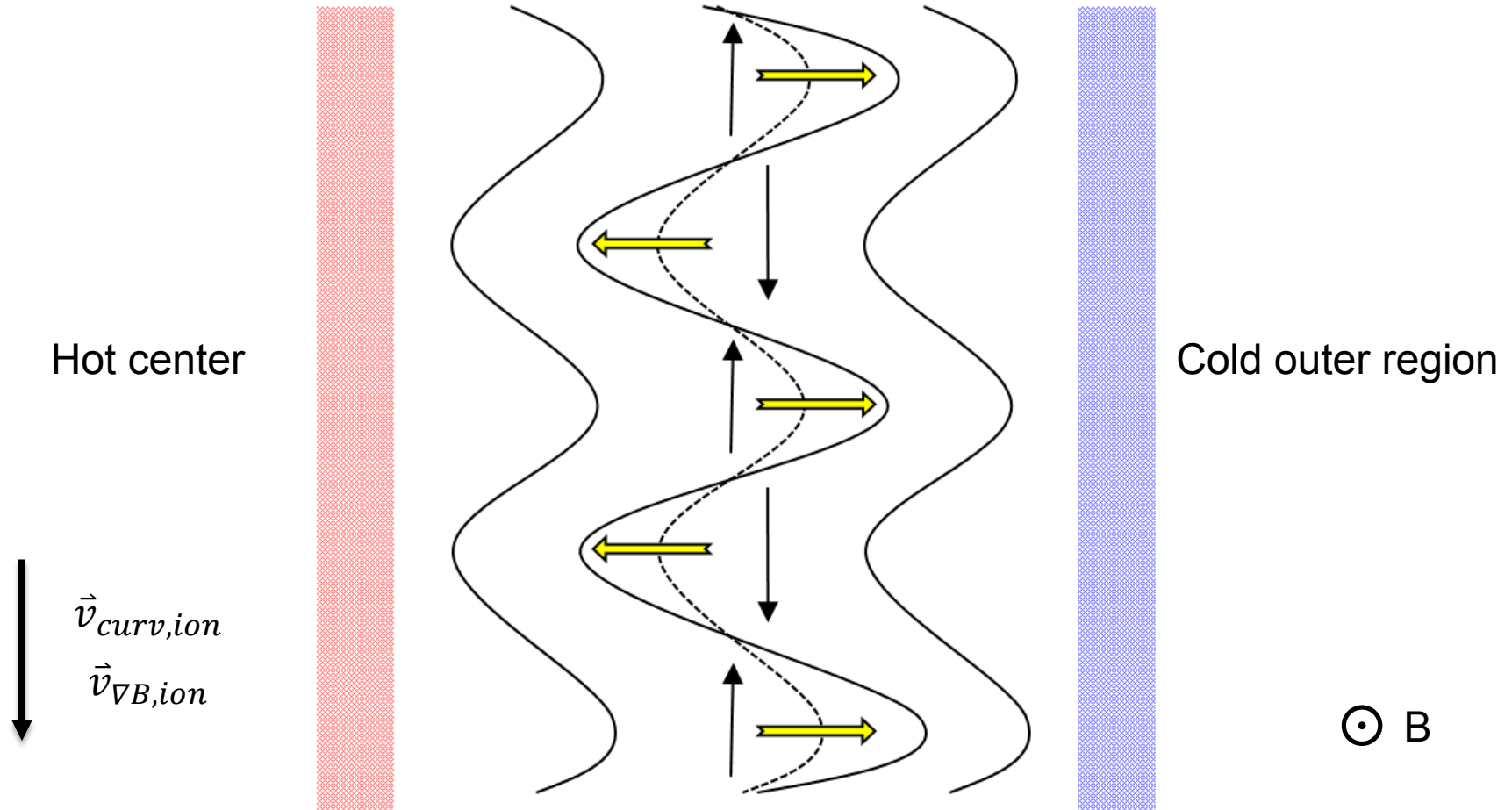
Basic Properties of ITG Instability in Toroidal Geometry

③ E_θ is induced from charge separation.



Basic Properties of ITG Instability in Toroidal Geometry

- ④ Particles drift in $\vec{E}_\theta \times \vec{B}_T$ direction, and perturbation gets amplified.



This is another very illuminating limiting (but relevant) case.

Further readable physical discussion in M.A. Beers et al., Ph.D Thesis

Princeton U. '95.

Both in this fluid limit & analytic kinetic derivation of Romanelli, PFB'89

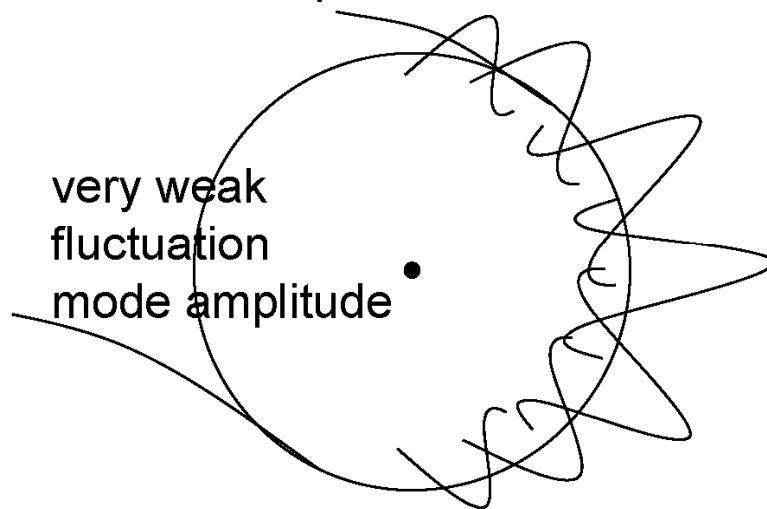
$k_{\parallel} \rightarrow 0$ has been assumed.

$$\omega - \omega_{di} - k_{\parallel} v_{\parallel}$$

resonance

This is incompatible with fluctuation localized in bad-curvature side

i.e., "ballooning" mode structure



$$" k_{\parallel} \approx \frac{1}{qR} "$$

Recap :

- ⊛ In sheared slab : $k_{\parallel} = \frac{k_y}{L_s} x$, some fluctuations can be localized near mode rational surface in radius
(small $k_{\parallel} \rightarrow$ minimize magnetic shear-induced damping
ion-Landau damping)
& Extended along \vec{B}

➔ “flute-like fluctuations”

- ⊛ But “ballooning” fluctuations localized at bad curvature side.

$$\text{➔ } k_{\parallel} \approx \frac{1}{qR}$$

What’s their radial extent? → Next Lecture.

Radially Elongated Eddy is a Natural Structure

Since

- Poloidal direction no longer symmetric in torus.
- Poloidal harmonics couple to form a Global Eigenmode.

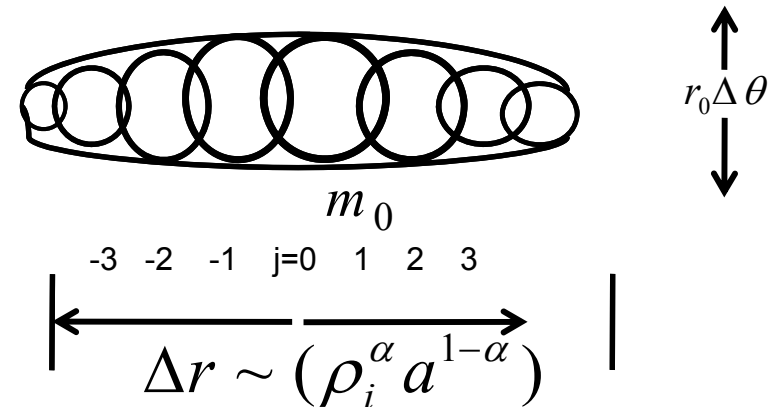
Radially elongated eddy



“Streamers”

Cf. This is a linear theory-based simple illustration.

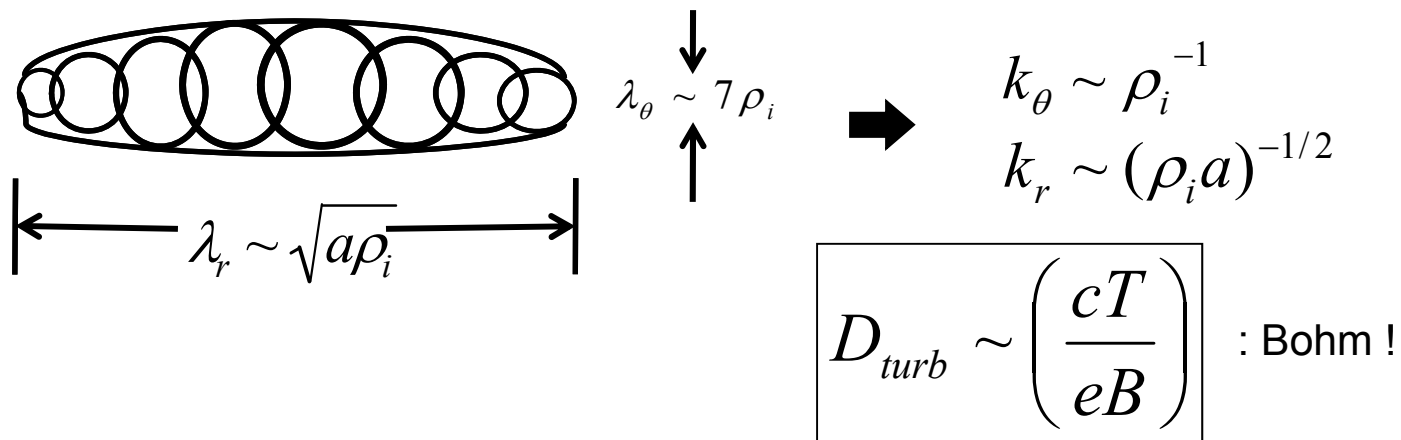
Some strongly prefer “nonlinear” explanation.



**Radially Elongated Eddys extract free energy efficiently,
and minimize convective (vector) nonlinearity
which increases with k_r**

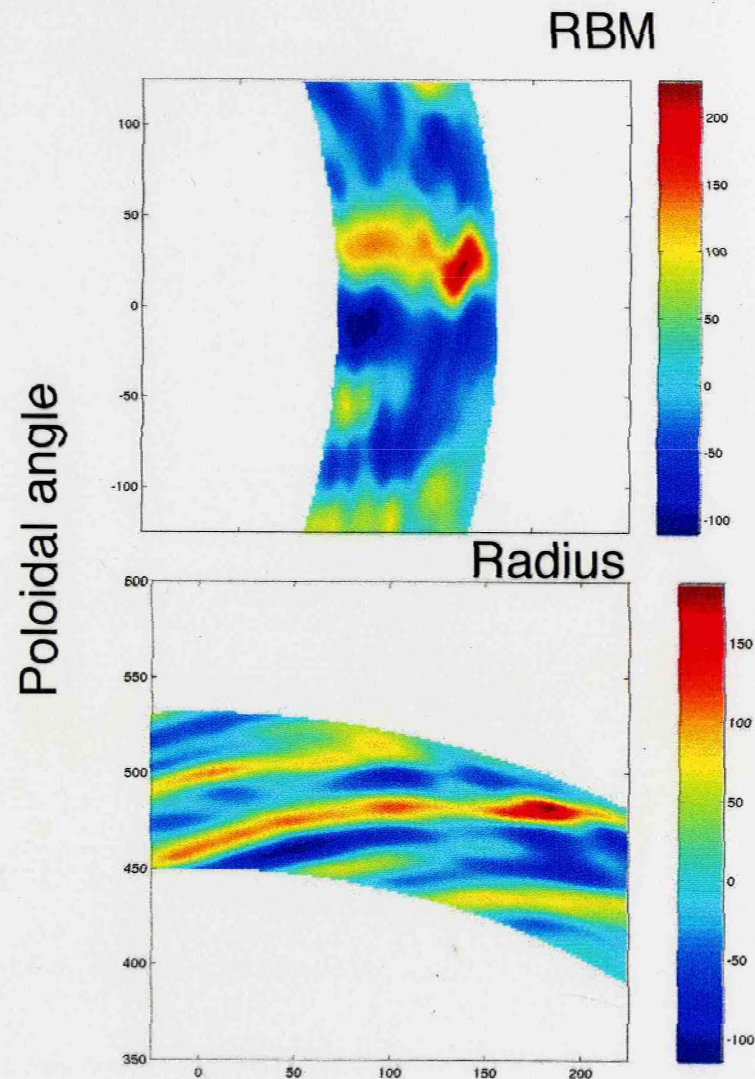
From
$$D_{turb} \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\gamma}{k_r^2} \sim \frac{\omega_*}{k_r^2} \sim \left(\frac{k_\theta}{k_r^2 \rho_i} \right) \frac{\rho_i}{L} \left(\frac{cT_i}{eB} \right)$$

Radially Elongated Eddys transport heat very efficiently ! :



3D Structure of Streamers

- Maps of the flux in poloidal planes.
- Elongated structures in the radial direction: **streamers**.
- Aligned with the direction of field lines.

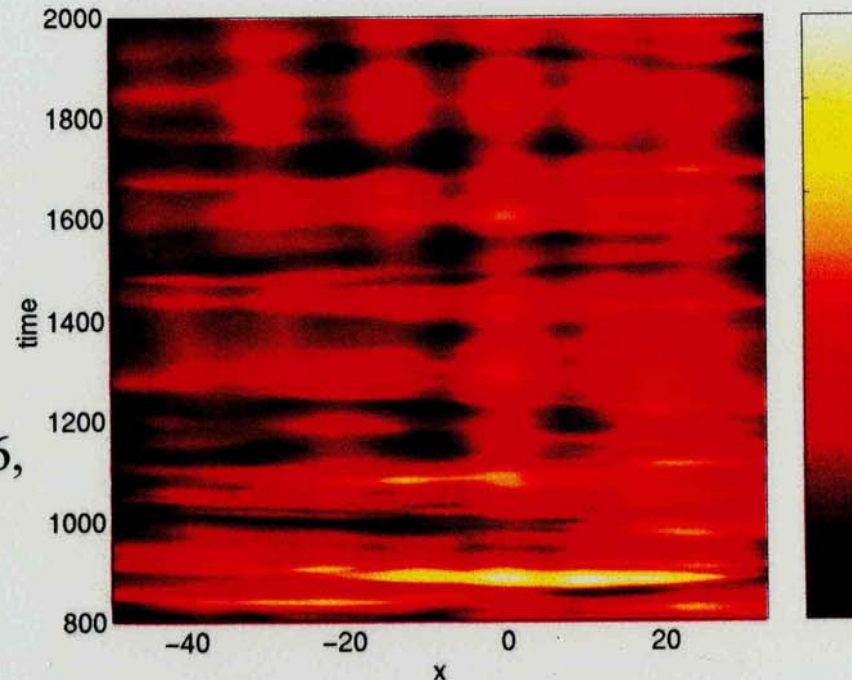


Bursty Transport

- Diamond and Hahm 95: **profile relaxations** at all spatial and time scales (avalanches).

- Observed in many turbulence simulations (Carreras 96, Sarazin and Gendrih 98, Garbet and Waltz 98, Beyer et al. 99,...)

Beyer et al 99



Flux vs. r and t

Self-Organized-Criticality Model of Tokamak Transport

- Construction of Heat Flux Expression from consideration of symmetry and conservation law
 - rather than quasi-linear expression from specific linear instability (ITG, TEM, ETG, ..., any AE)
- Earliest and Simplest Model for MFE [Diamond and Hahm, PoP '95]
 - Sand Pile model by [Hwa and Kadar, PRA '92]

Joint Reflection Symmetry

$$Q[\delta T] = \frac{\lambda}{2}(\delta T)^2 - \chi_{Ne0}\partial_x\delta T + \text{higher order terms}$$

invariant under

$$\begin{cases} x \rightarrow -x \\ \delta T \rightarrow -\delta T \end{cases}$$

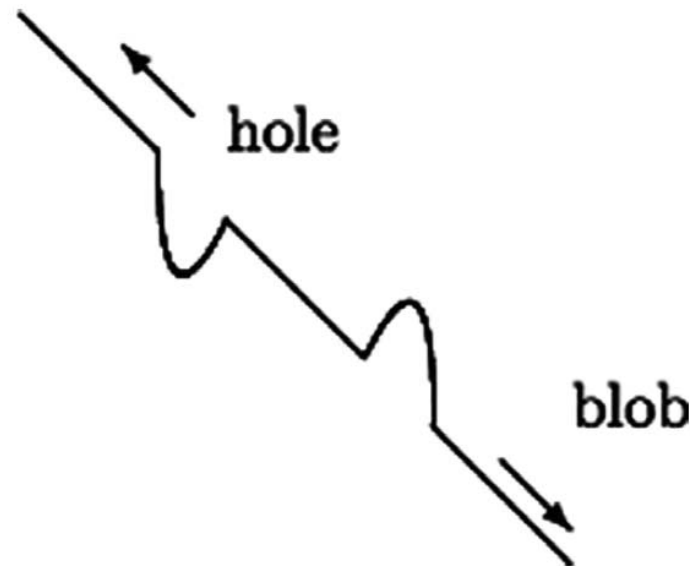


FIG. 4. Schematics for blobs propagating outward and holes propagating inward. These are allowed to have net transport down the gradient. To have both solutions, equation must be invariant under the simultaneous transformation of $x \rightarrow -x$ and $\delta T \rightarrow -\delta T$.

Frequency Spectra of Heat Transport Events

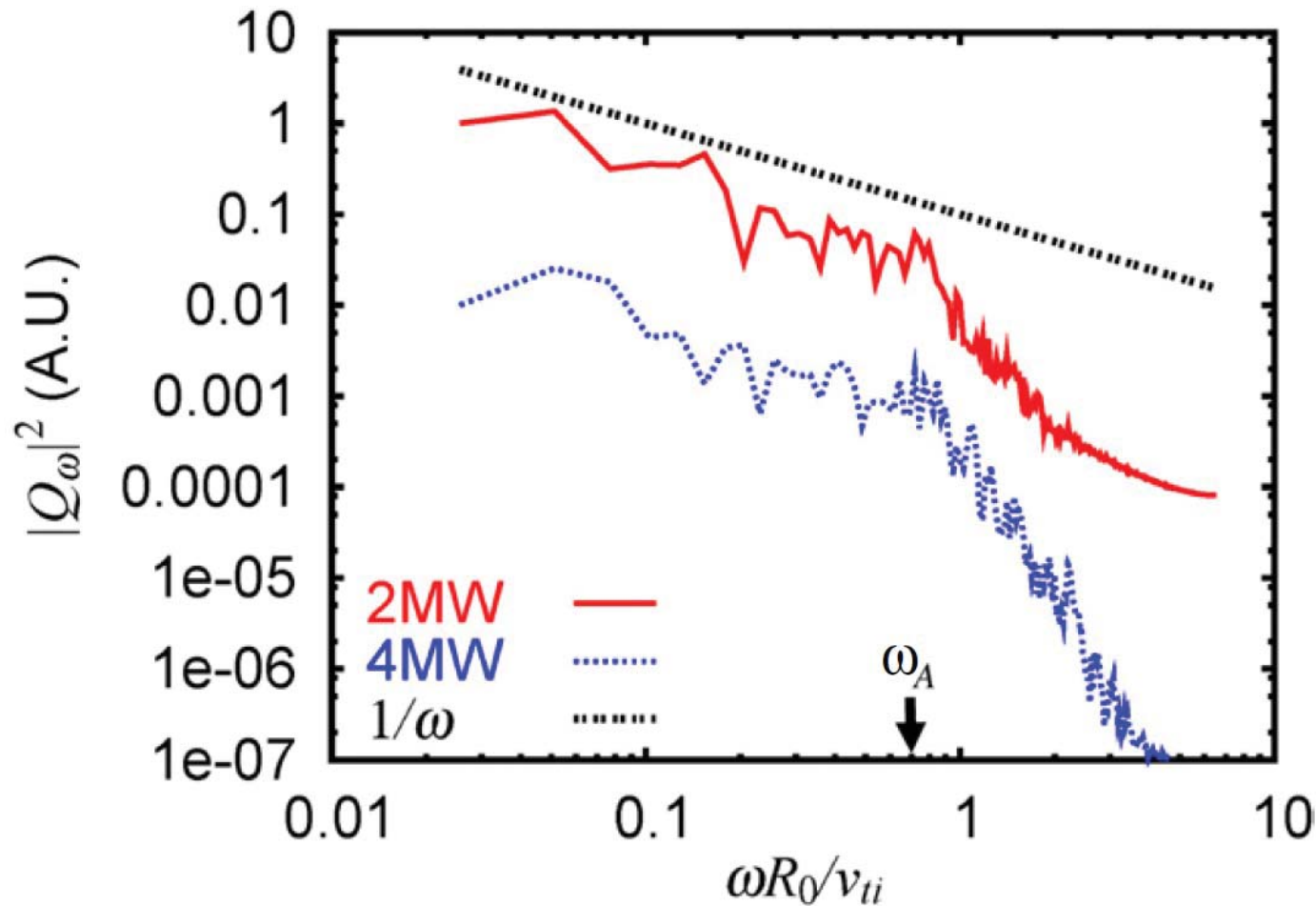
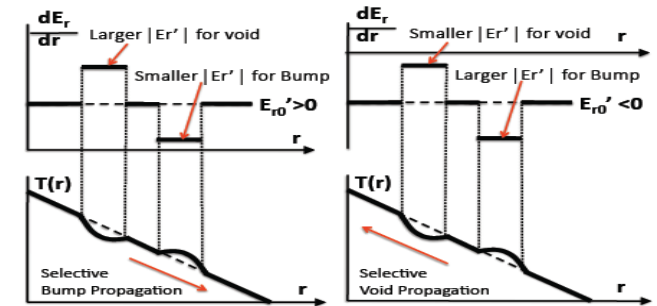
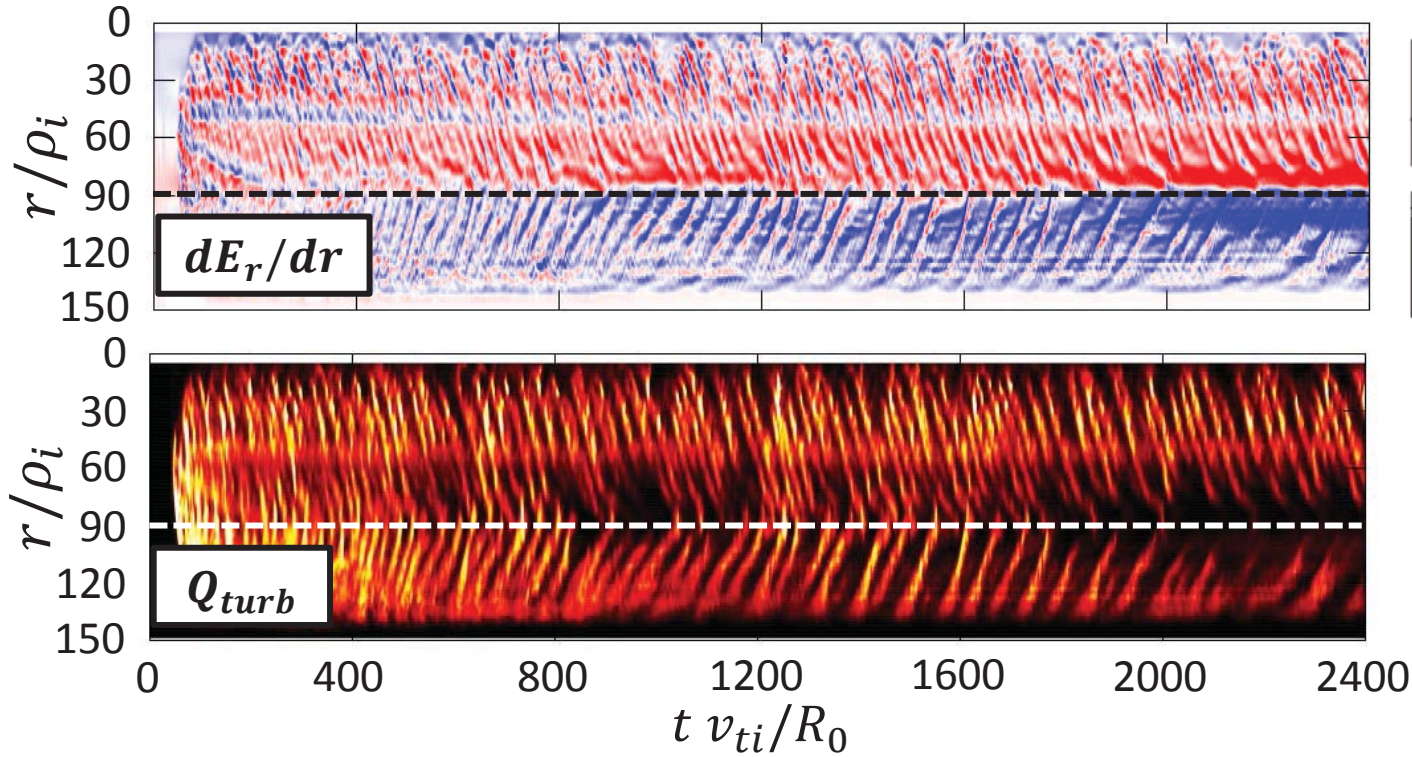


Figure 12. The power spectrum of the turbulent heat flux Q averaged over source free regions ($r/a = 0.5-0.9$). The spectra in low frequency region show $1/f$ type spectra.

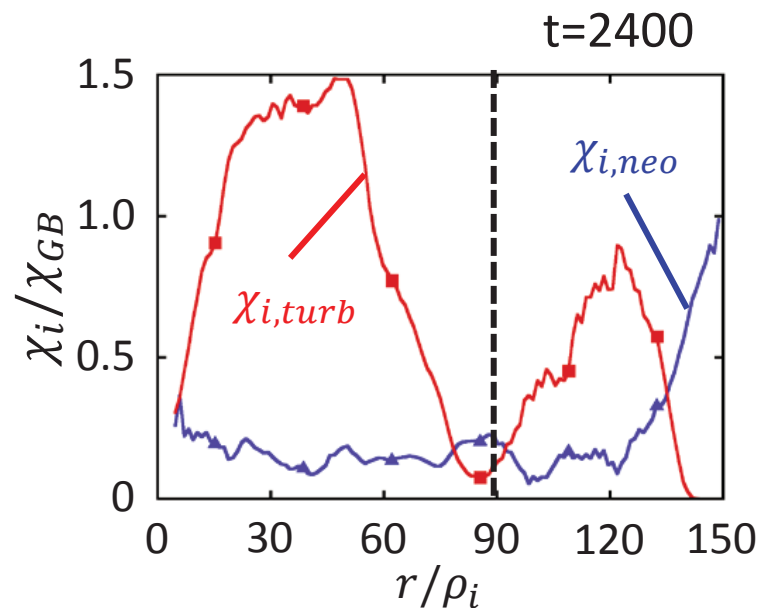
[Y. Idomura et al., Nucl. Fusion 49, 065029 (2009)]

Avalanches exhibit Joint Reflection Symmetry?



[Y. Idomura, *et al.* Nucl. Fusion, **49**, 065029 (2009).]

[M. Kikuchi and M. Azumi, Rev. Mod. Phys. **84**, 1807 (2012).]



- ✓ Clear correlation between the sign of E_r shear and the direction of avalanches can be observed.
- ✓ Strong E_r shear triggered by toroidal rotation in outer region suppresses the turbulence, leading to an **ITB formation**.

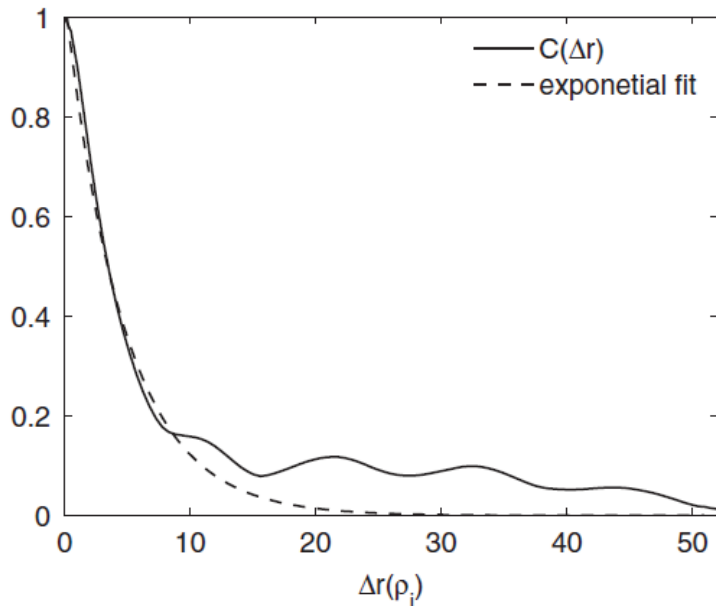
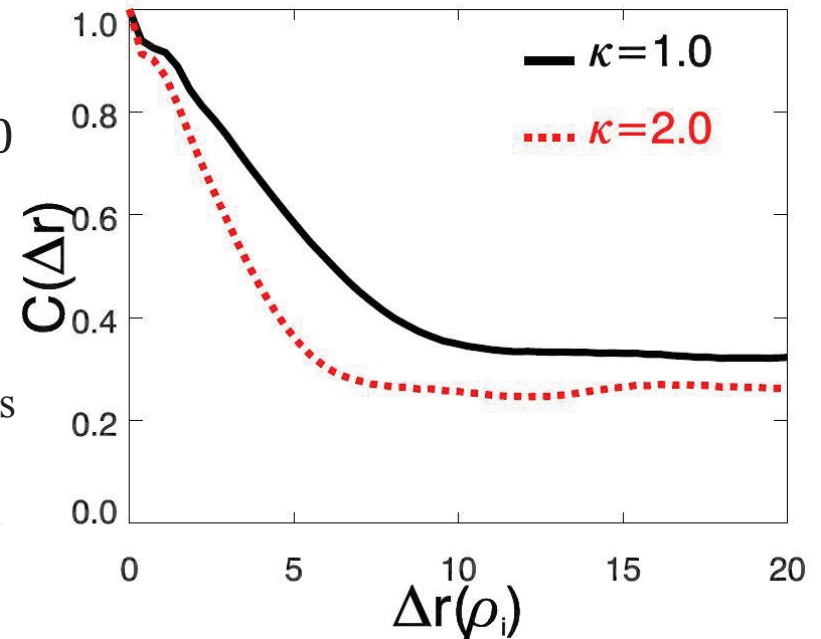
Radial Correlation Function

Two point correlation function

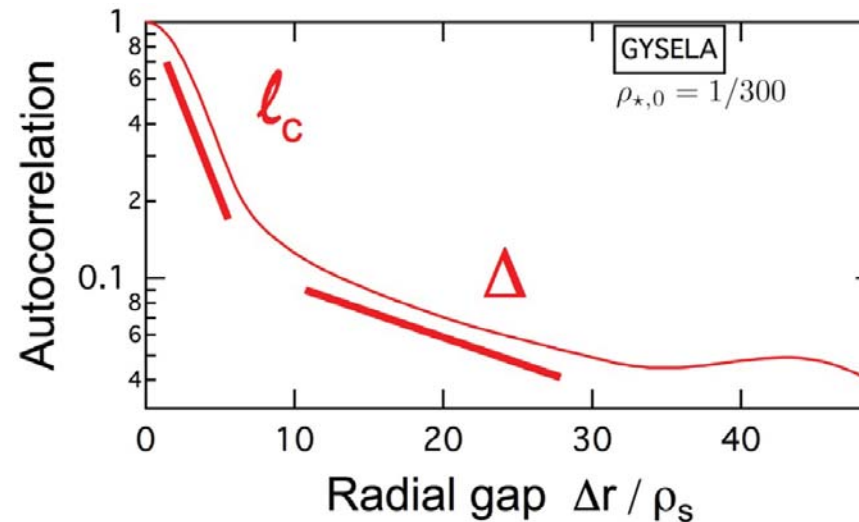
$$C_{r\zeta}(\Delta r, \Delta\zeta) = \frac{\langle \delta\phi(r+\Delta r, \zeta+\Delta\zeta)\delta\phi(r, \zeta) \rangle}{\sqrt{\langle \delta\phi^2(r+\Delta r, \zeta+\Delta\zeta) \rangle \langle \delta\phi^2(r, \zeta) \rangle}}, \text{ at } \theta = 0$$

By taking maxima along the ridge of $C_{r\zeta}(\Delta r, \Delta\zeta)$
we obtain $C_r(\Delta r)$

from GKPSP simulations
of TEM turbulence
Qi, Kwon, Hahm and Yi

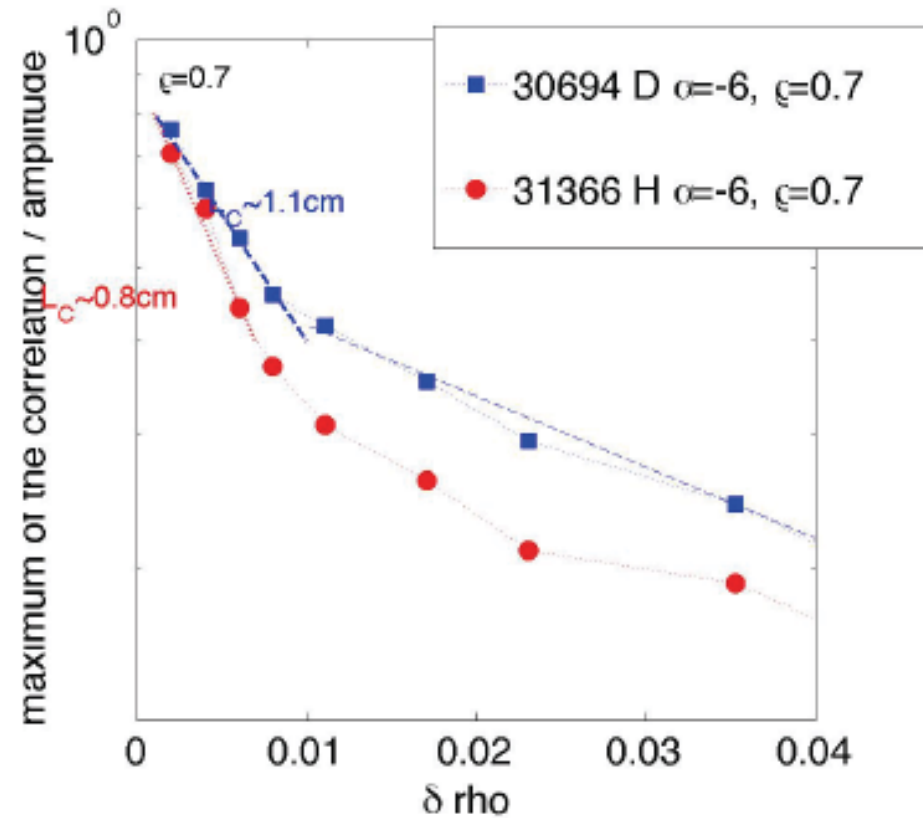
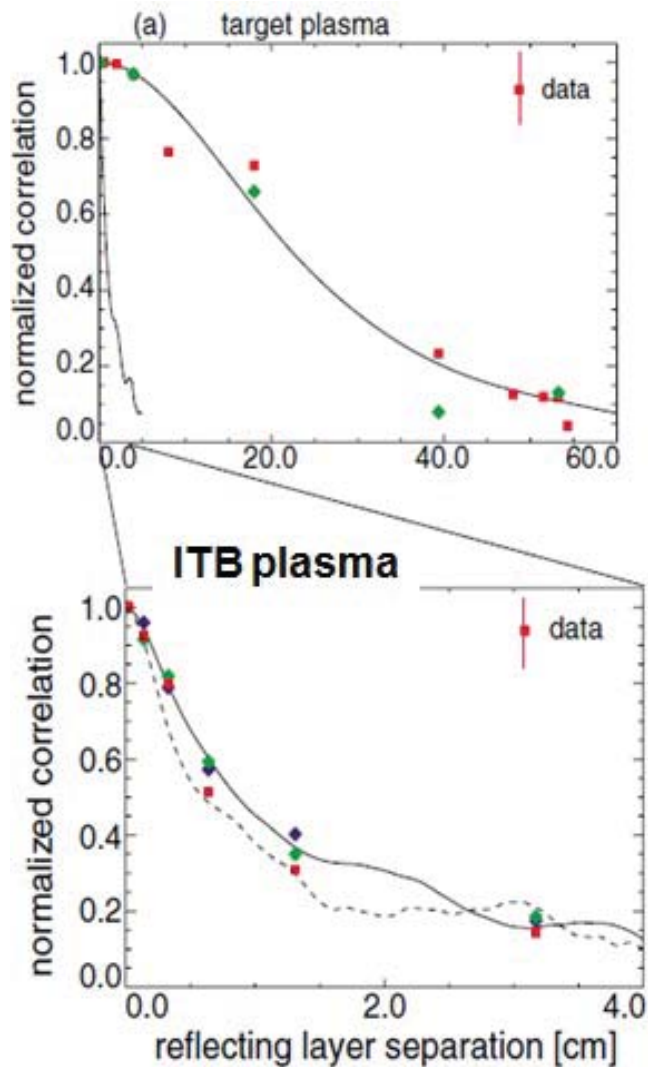


from Yong Xiao and Zhihong Lin.
PRL 103, 085004 (2009)



from GYSELA simulations of ITG turbulence
by G. Dif-Pradalier et al., NF 57, 066026 (2017)

Mesoscale Turbulence may be major contributor to transport



from P.Hennequin et al.
42nd EPS Conference on Plasma physics(2015)

Radial correlation measured by a
reflectometry in JT-60U plasmas
(R. Nazikian, et al, PRL 2005)

Partial Summary

Radially Elongated Eddys (Streamers) can be formed in toroidal geometry and transport heat efficiently.

---> Bohm Scaling of Confinement ~ Experimental Trends

Why not sufficient ?

Recall that from experimental measurements:

Eddy size $\sim \lambda_x, \lambda_y \sim \text{several } \rho_i$

Introduction to Tokamak Core Turbulence

Part II. Role of Self-generated Zonal Flows

T.S. Hahm

Seoul National University, Seoul, KOREA

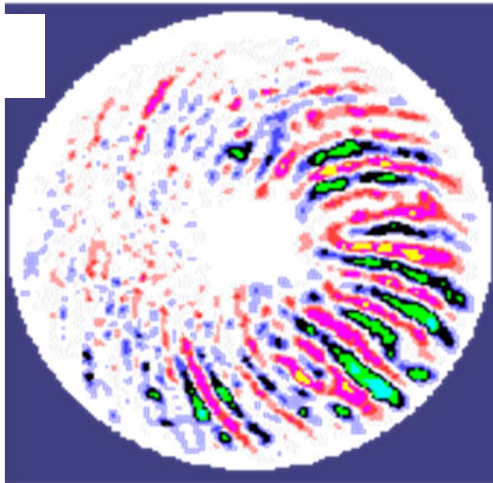
“8th East-Asian School and Workshop on Laboratory, Space, Astrophysical Plasmas”, presented at Chungnam Univ. Daejeon, July 30, 2018



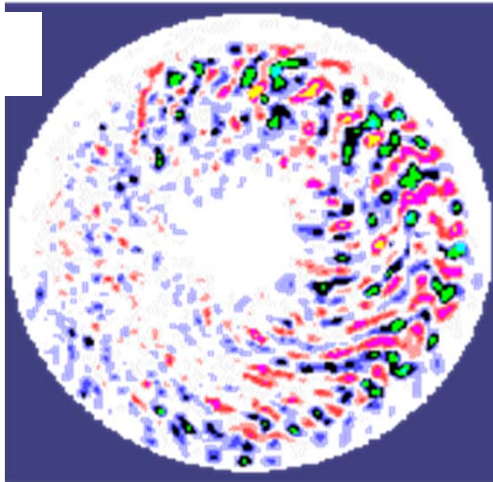
Sheared Zonal Flow Regulates Turbulent Eddy Size and Transport

[Lin, Hahm, Lee et al., Science 1998]

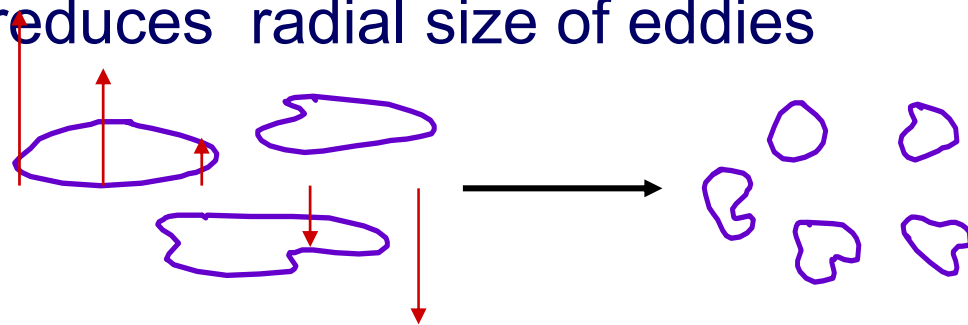
No flow



With flow



. Self-generated ExB zonal flow reduces radial size of eddies



. Breakup of radially elongated structures reduces transport

- Externally driven ExB Shear Flows were used before for the direct control of the turbulence

Role of E x B Shear in Reducing Turbulence

- Flow shear decorrelation in cylinder [Biglari-Diamond-Terry, Phys. Fluids-B '90]

$$\omega_E > \Delta \omega_T$$

- Turbulence quenching in gyrofluid simulation [Waltz-Kerbel-Milovich, Phys. Plasmas '94]

$$\omega_E > \gamma_{lin}$$

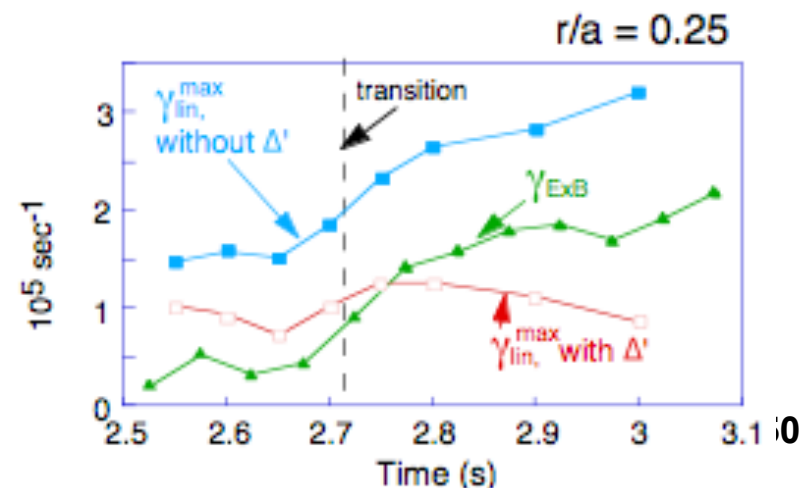
- ExB Shearing Rate in **General Toroidal Geometry** [Hahm-Burrell, Phys. Plasmas '95]

$$\omega_E = \frac{\Delta r_0}{\Delta \ell_{\perp}} \frac{(RB_{\theta})^2}{B} \frac{\partial}{\partial \psi} \left(\frac{E_r}{RB_{\theta}} \right)$$

- Made possible by developments of **Experimental Diagnostics** for E_r and B_{θ} (Motional Stark Effects, Charge Exchange Recombination Spectroscopy,)
- Useful Rule of Thumb for Indication of the importance of ExB shear

- Widely used for experimental results analysis (TFTR, DIII-D, JET, JT60-U, AUG, TEXTOR, NSTX, MAST, LHD, W7-AS,) with linear growth calculations via various gyrokinetic codes.

E. Synakowski et al., Phys. Plasmas '97 ----->



$E \times B$ Shearing Rate in Toroidal Geometry

PPPL

Hahm and Burrell, Phys. Plasmas **2**, 1648 (1995)

$$\omega_E = \frac{\Delta r_0}{\Delta l_{\perp}} \frac{(RB_{\theta})^2}{B} \left| \frac{\partial}{\partial \psi} \left(\frac{E_r^{(0)}}{RB_{\theta}} \right) \right|$$

- $\frac{\partial}{\partial \psi} \left(\frac{E_r^{(0)}}{RB_{\theta}} \right)$:

From u_{θ} and ∇P_i : (TFTR ERS, H-mode)

or from u_{ϕ} (DIII-D NCS, VH; JET OS, RS...)

- $\frac{(RB_{\theta})^2}{B}$: In-out Asymmetry

Evidence from DIII-D, Pronounced for STs

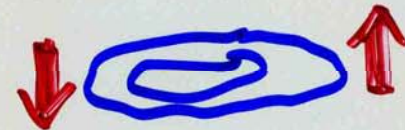
Rettig

Stambaugh

- $\frac{\Delta r_0}{\Delta l_{\perp}}$: Eddy shape dependence

Typically assumed to be 1

Stronger shearing for radially elongated eddy



E x B Flow Shear is well-known to reduce Turbulence and Turbulence-driver Transport :

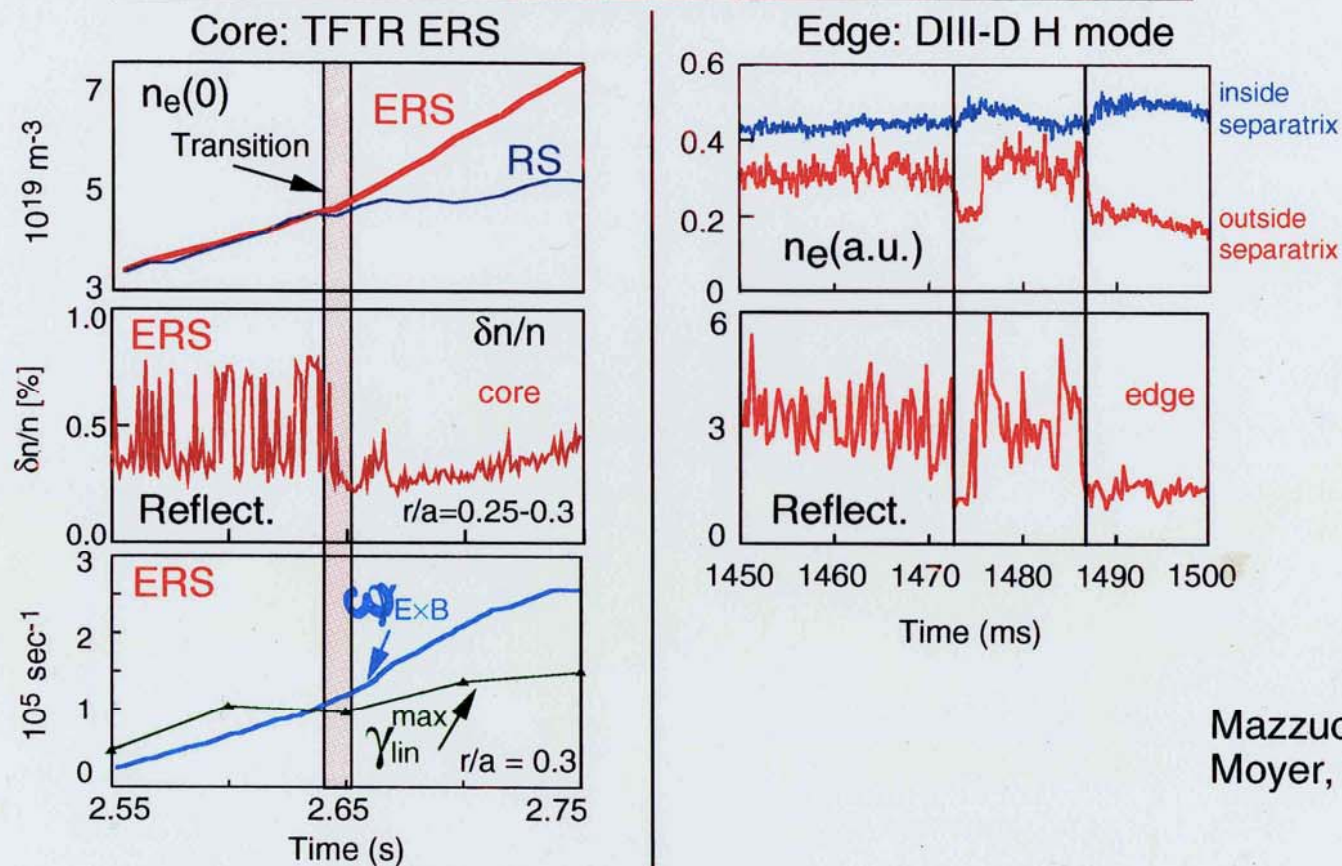
Theoretically it can occur via

- Reduction in fluctuation amplitude
- Reduction in radial correlation length (eddy size)
- Elimination of large Transport Events
- Shift I cross phase between transported quantity ($\delta n, \delta T_i, \dots$)

and transporter ($\delta v_r = \left(\frac{c}{B} \hat{b} \times \nabla \delta \phi \right) \cdot \hat{e}_r$)

(Flux: $\Gamma = \langle \delta n \delta v_r \rangle$)

Density fluctuations are reduced across confinement bifurcations in both the core and the edge



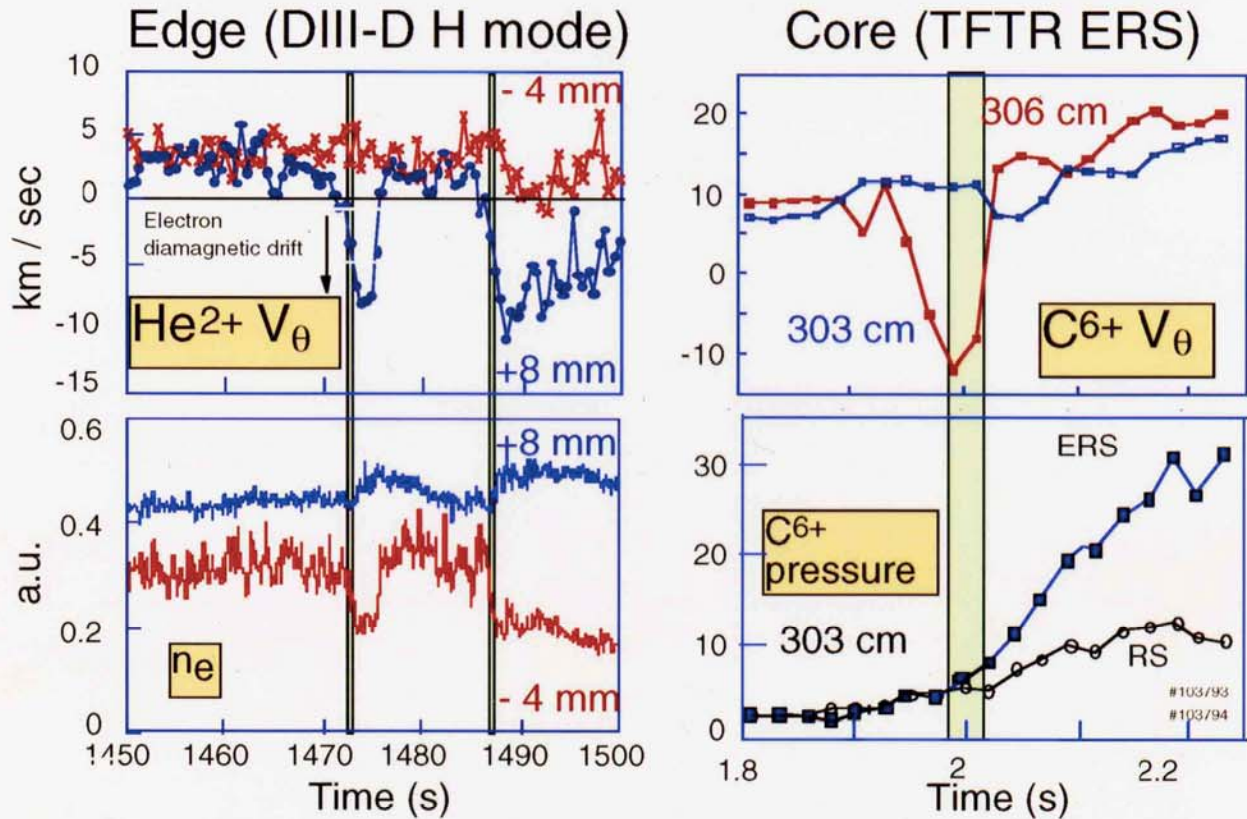
Mazzucato, 1996
Moyer, 1995

- In core, $\omega_{ExB}/\gamma_{lin}$ increases with rising ∇p
 \Rightarrow positive feedback

- → Closeness of shearing and growth rates at transition should be treated with caution

• BDT: $\Delta\omega_T$
 \approx factor of 2 from Gyrofluid simulation

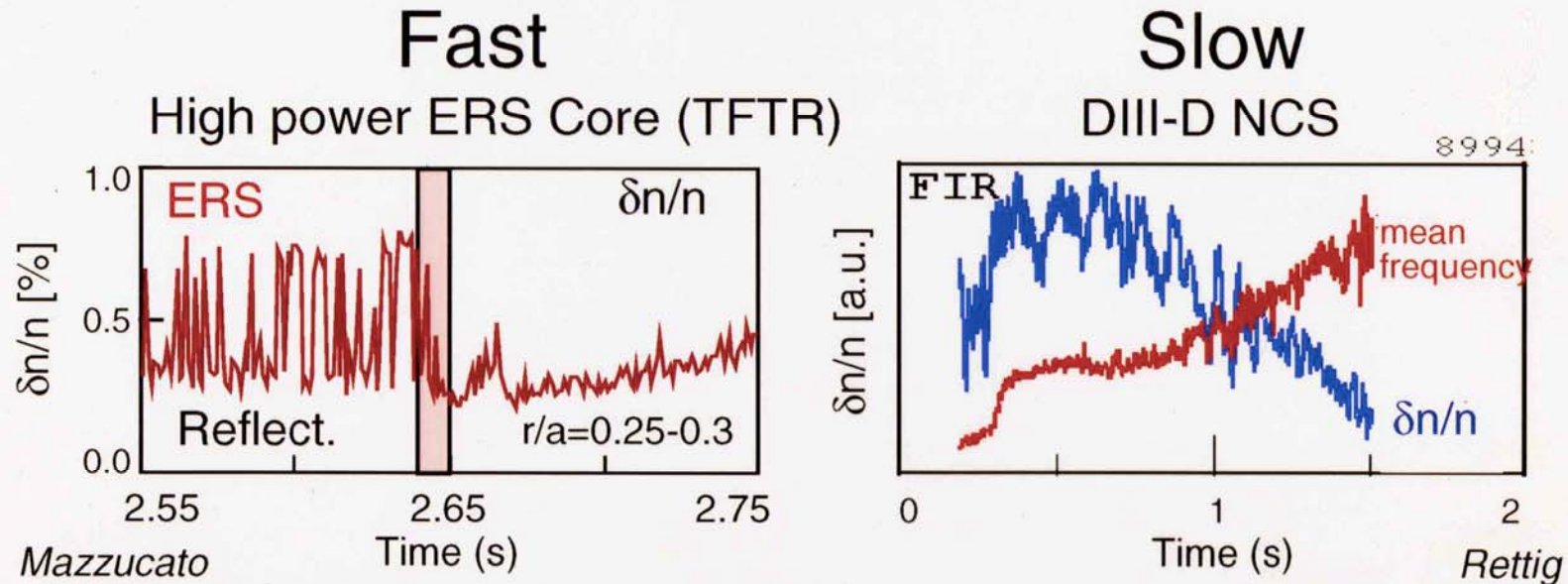
E_r shear layer bifurcation starts before confinement improvement in the TFTR ERS core and DIII-D edge



For both: large negative $\Delta V_\theta \Rightarrow$ large negative ΔE_r
 ∇p eventually dominates E_r in force balance
 $\omega_{E \times B} / \gamma_{lin}^{max} \sim 10$ at peak ΔV_θ for ERS

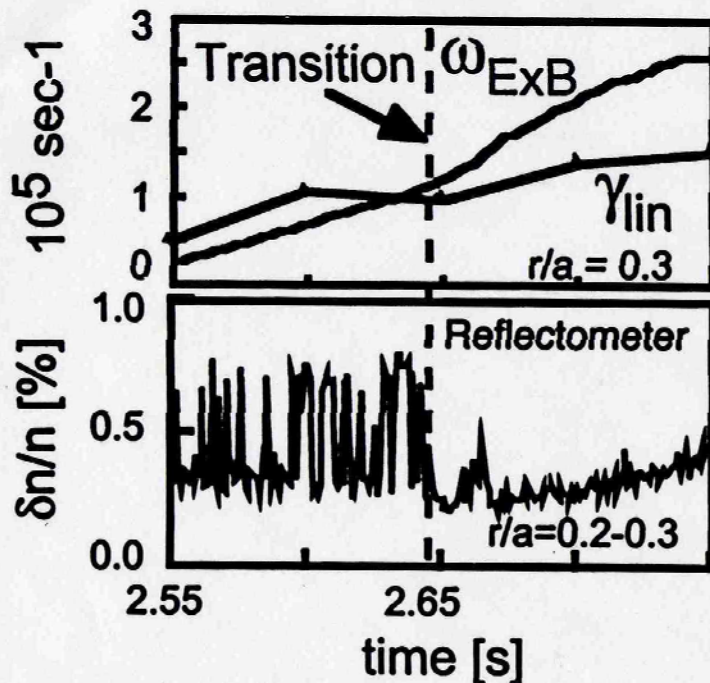


Core transport and fluctuation reduction can have very different time scales

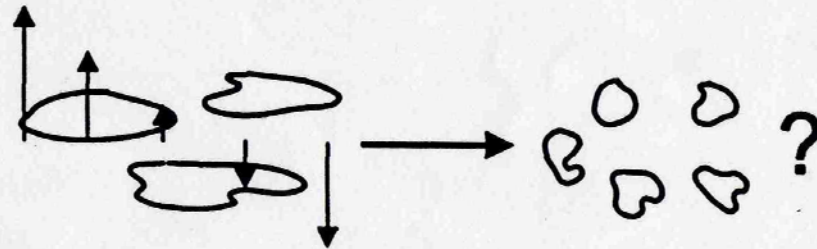


- Difference in time scale may be due to fast bootstrap with ∇p -dominated bifurcation vs. competition between ∇p and V_ϕ in DIII-D case
- DIII-D: fluctuation, transport reduction during time when V_ϕ shear is slowly increasing
- TFTR: fluctuation, transport change is “single step” in character

Transition to Enhanced Confinement Regime is Correlated with Suppression of Core Fluctuations in TFTR



- Theory predicts fluctuation suppression when rate of shearing (ω_{ExB}) exceeds rate of growth (γ_{lin})
- Outstanding issue: Is suppression accompanied by radial decorrelation?

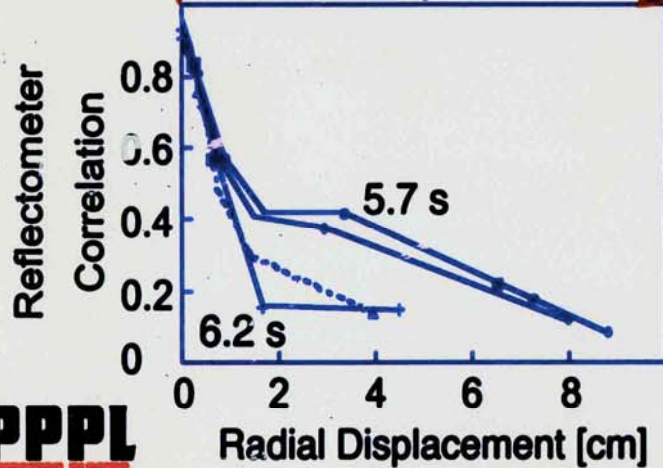
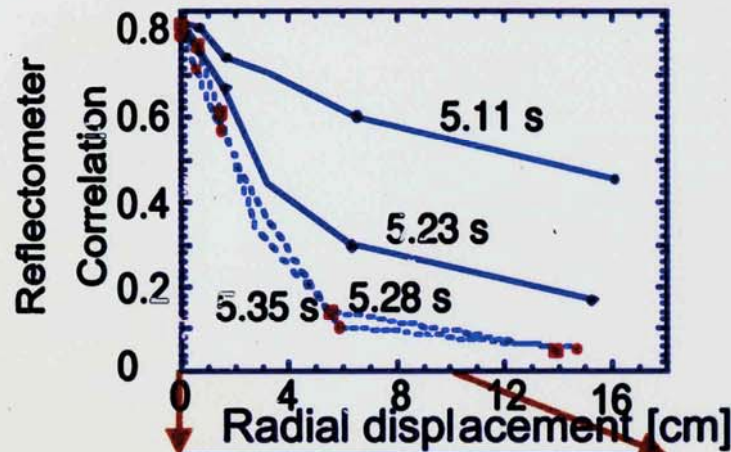


- Similar suppression observed on JET (X-mode reflectometer) and DIII-D (FIR Scattering)

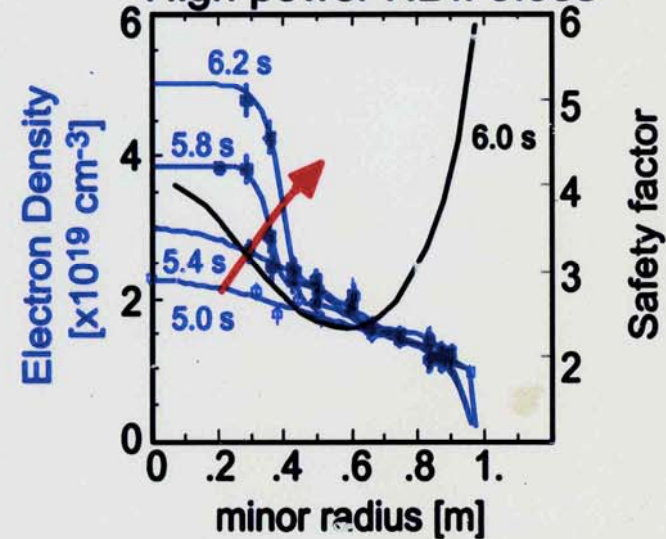
Hahm, Burrell, Phys. Plas. 1995, E. Mazzucato et al., PRL 1996.



Dramatic Reduction of Radial Correlation Length in ITB of JT-60U: Are We at The Limit of Our Spatial Resolution?



ITB Density Evolution:
High power NBI: 5.05s



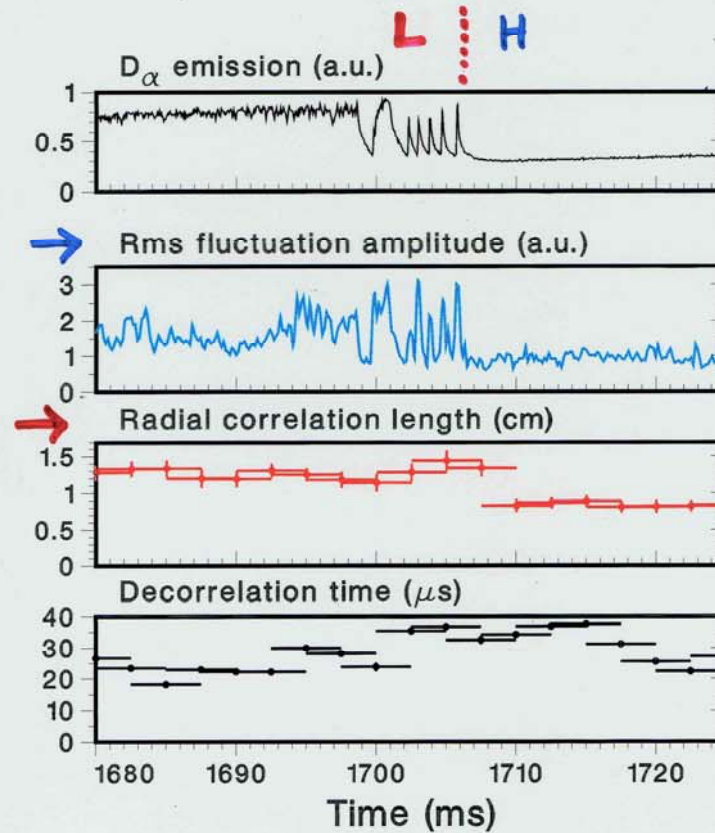
- Very long correlation lengths before ITB formation: $L_r/\rho_i \gg 1$!
- Very short correlation lengths inside ITB: close to resolution limit?

• **Weak change in fluctuation level**



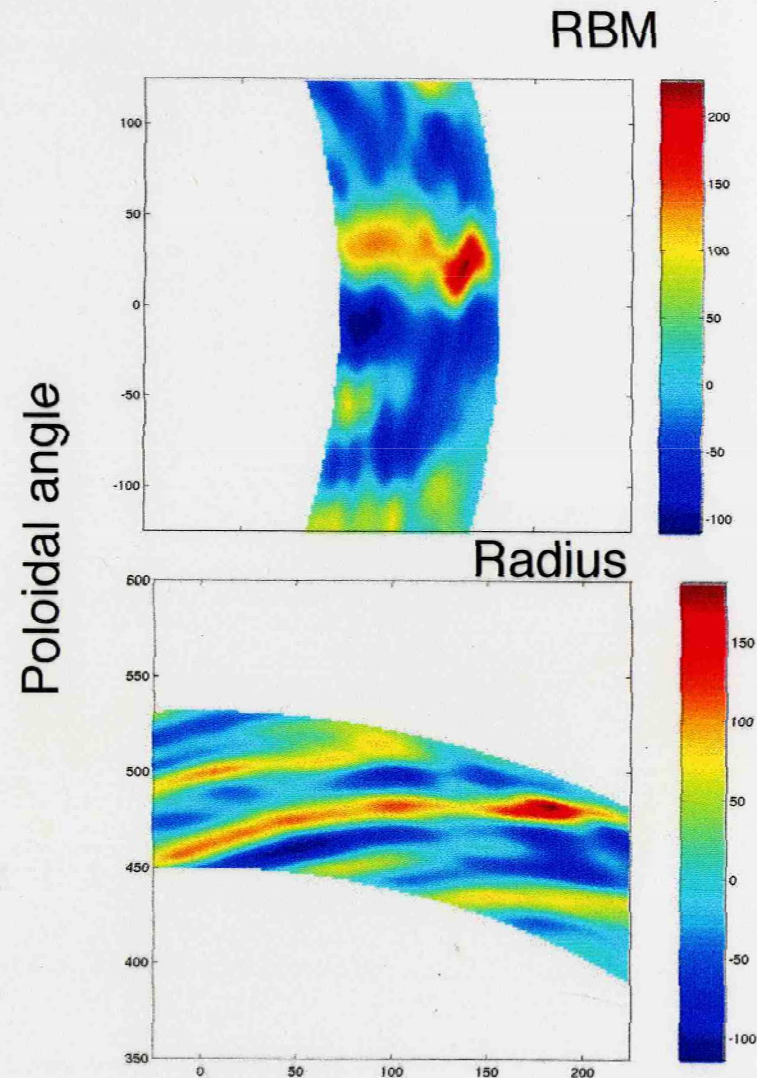
- PCI Measurements at DIII-D Edge show that δn and Δr decrease at L-H transition

Coda, Porkolab & Burrell: Phys. Lett. A. 2000.



3D Structure of Streamers

- Maps of the flux in poloidal planes.
- Elongated structures in the radial direction: **streamers**.
- Aligned with the direction of field lines.

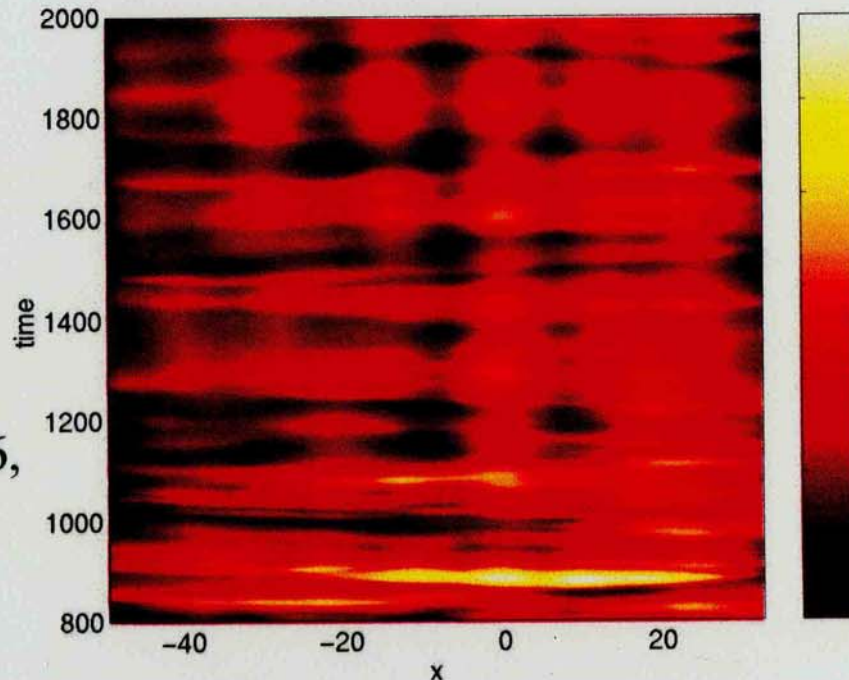


Bursty Transport

- Diamond and Hahm 95: **profile relaxations** at all spatial and time scales (avalanches).

- Observed in many turbulence simulations (Carreras 96, Sarazin and Gendrih 98, Garbet and Waltz 98, Beyer et al. 99,...)

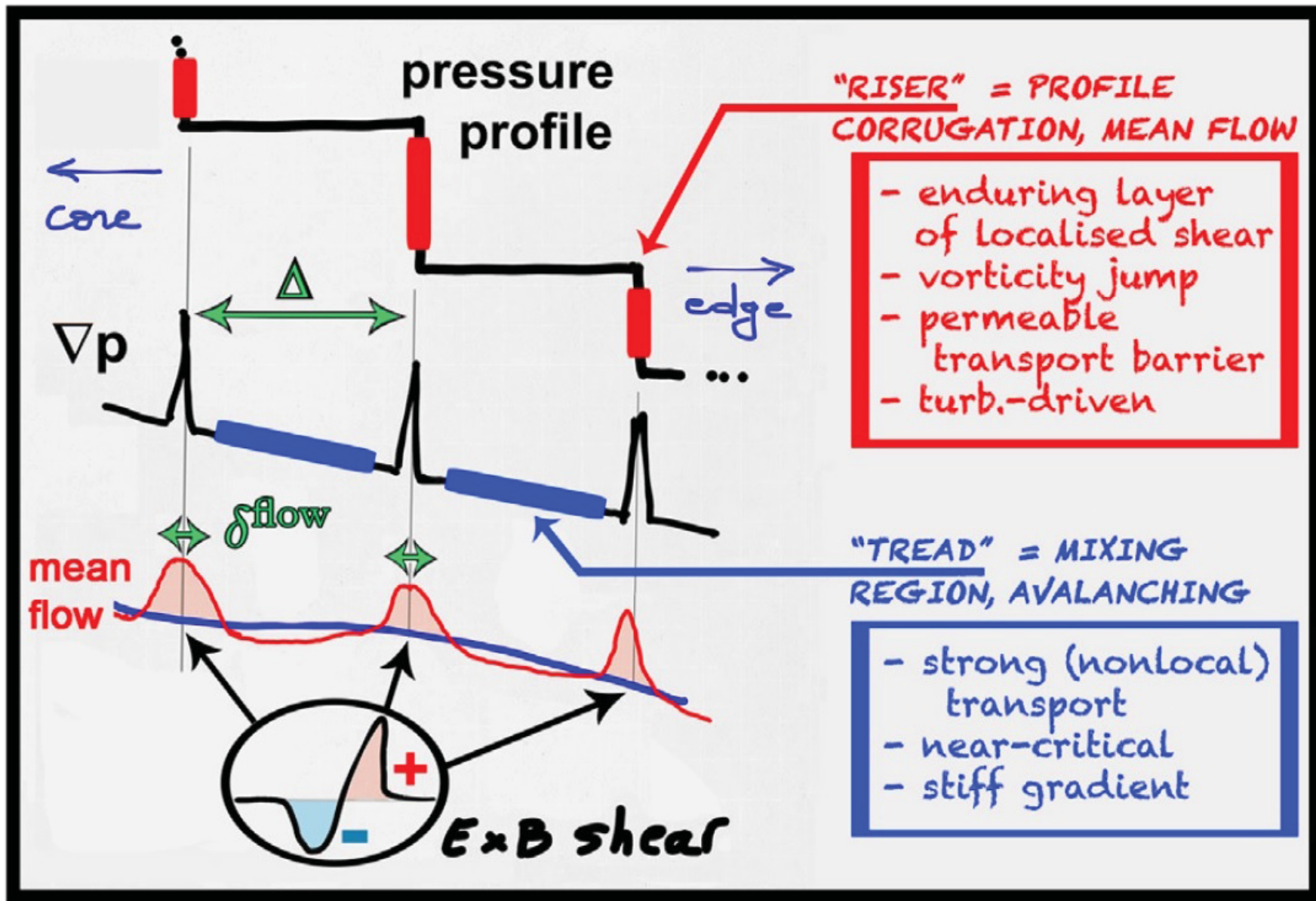
Beyer et al 99



Flux vs. r and t

Hierarchy of ExB Shear Effects

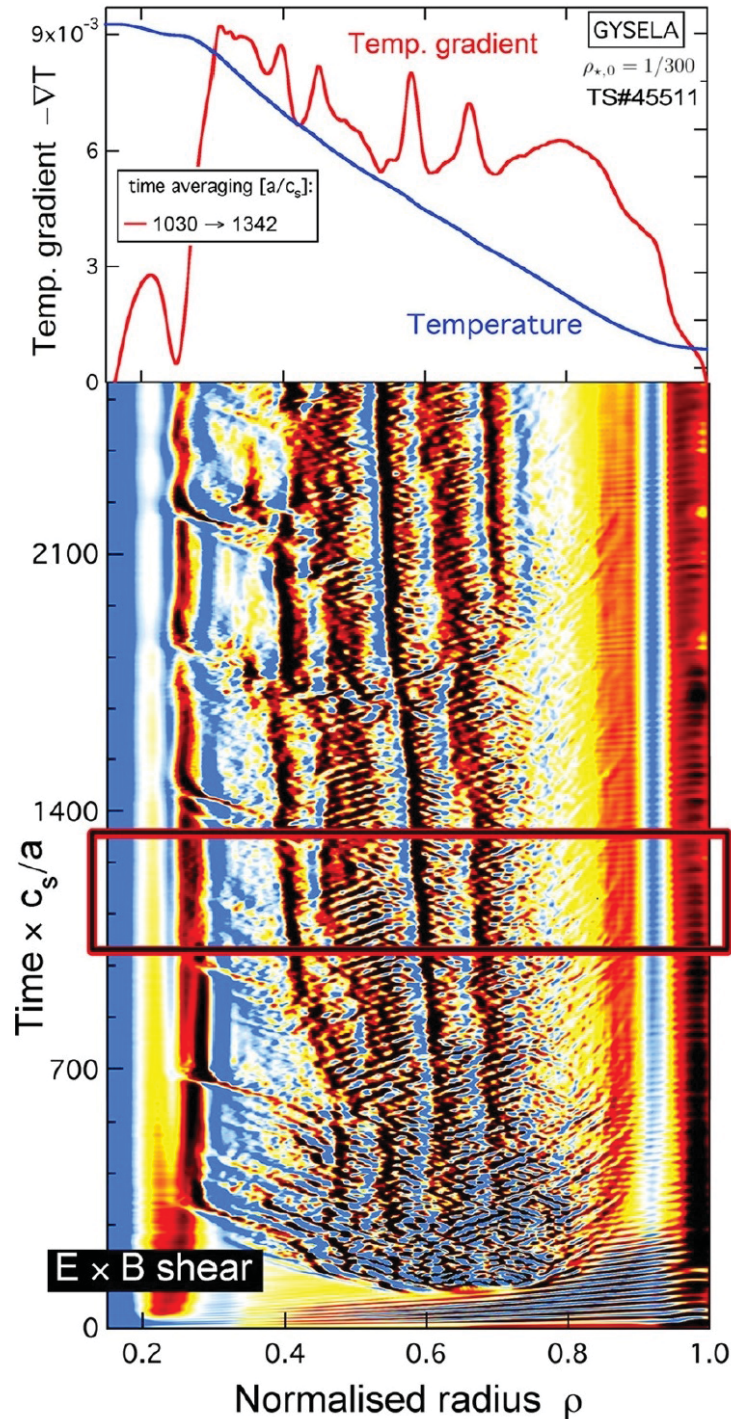
Recent Findings: ExB Staircase



Une **B**ande **D**essinée **S**cientifique

[G. Dif-Pradalier et al., Nucl. Fusion 57, 066026 (2017)]

ExB Staircase from Flux-driven ITG Simulations

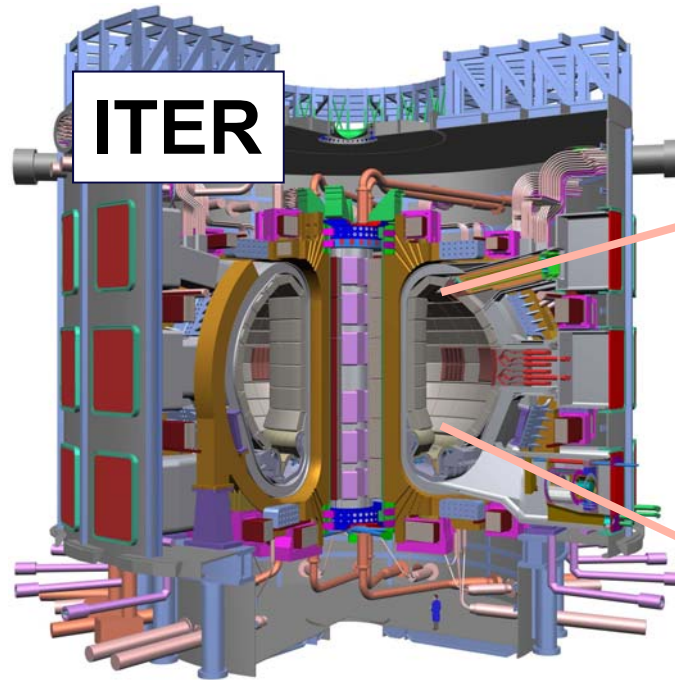
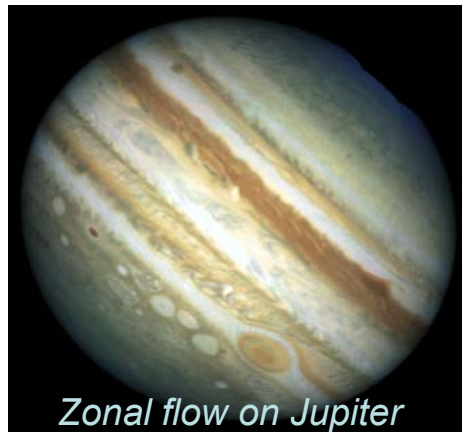


[G. Dif-Pradalier et al.,
Nucl. Fusion 57, 066026 (2017)]

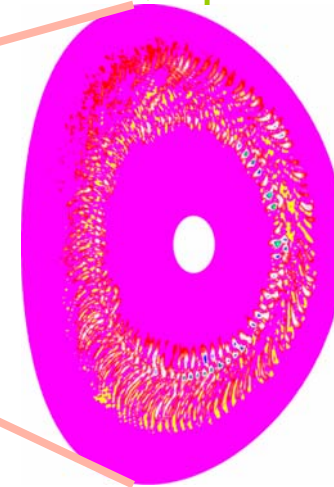
Figure 2. Three general features of the plasma staircase are visible here: (i) the mean profile corrugations here displayed on the temperature gradient, (ii) the strong, long-lived and coherent shear flows defining ‘valleys’ of hindered transport—the mean radial $\mathbf{E} \times \mathbf{B}$ shear profile is shown in figure 11 (left) and (iii) the radial transport dominated by avalanche-like events in-between the staircase steps.

What is a zonal flow?

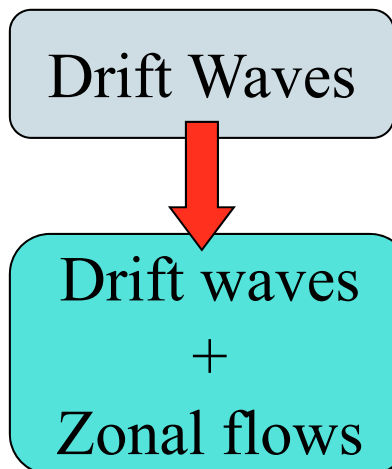
Courtesy: K. Itoh, made in Japan, edited in USA, and presented in Korea



ExB flows
 $m=n=0, k_r = \text{finite}$



From GTS



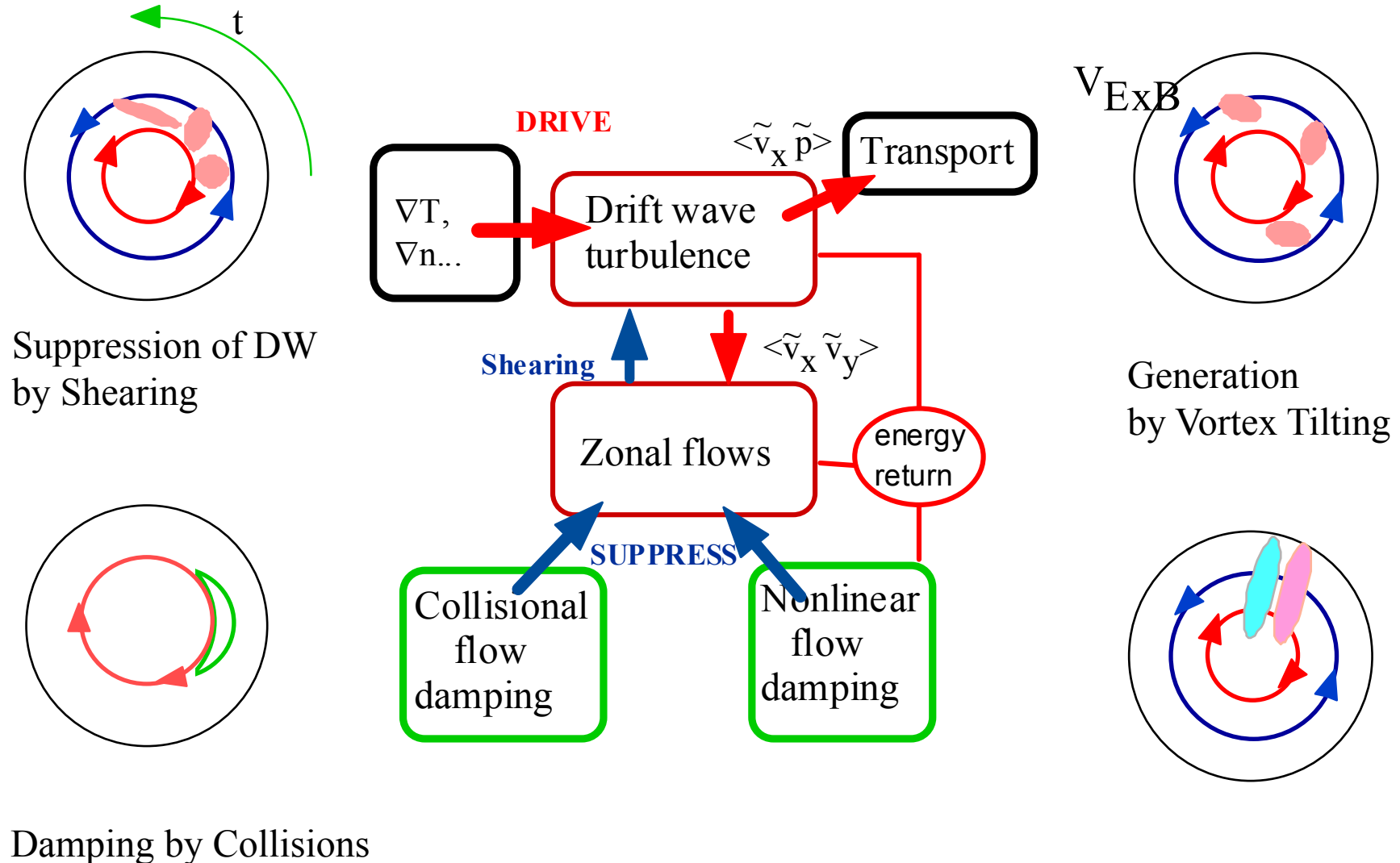
Paradigm
Change

ZFs are "modes", but:

1. No direct radial transport
2. No linear instability
3. Turbulence driven

Basic Physics of a Zonal Flow

from Diamond, Itoh, Itoh, and Hahm, "Zonal Flows in Plasma-a Review" PPCF '05



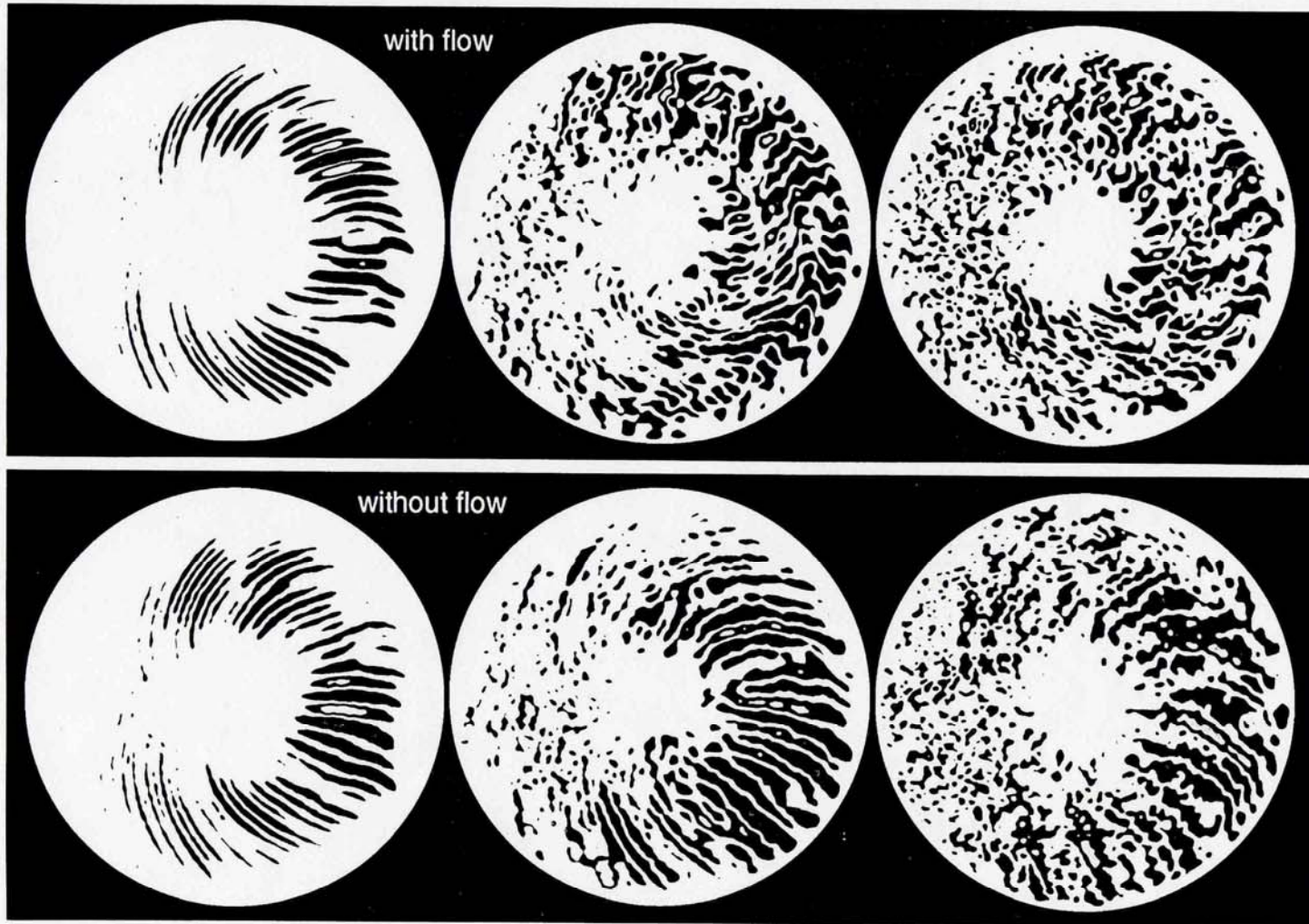
E x B Shearing by time-dependent Zonal Flow

[Hahm, Beer, Lin, et al., Phys. Plasmas '99]

- **Gyrofluid Simulations observed that instantaneous $\omega_E(t) \gg \gamma_{\text{lin}}$ while turbulence was at L-mode level and transport was anomalous.**
- **Effective E x B shearing rate has been analytically derived to take into account the time dependence of zonal flows**
- **From Gyrofluid simulation data analysis, has been observed:**
$$\omega_E^{\text{eff}} \sim \gamma_{\text{lin}}$$
- **Shearing due to high frequency comp. ZF is predicted to be ineffective for core turbulence.**
- **Gyrokinetic simulations demonstrated broadening of k_r of ITG turbulence (a symptom of eddy breaking-up) due to zonal flows quantitatively.**

Radial Correlation Length &

Turbulence Generated E X B Flows Reduce Transport in Gyrokinetic Simulation



Broadening of k_r Spectrum by Zonal Flows

PPPL

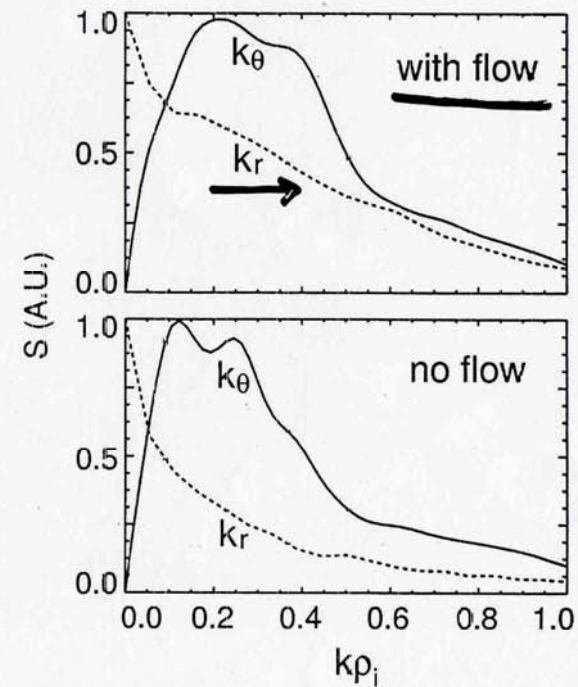
- Theory for $\mathbf{E} \times \mathbf{B}$ shear decorrelation of turbulence has been generalized to include time-dependence of zonal flows

[T. S. Hahm, M. A. Beer, Z. Lin, G. W. Hammett, W. W. Lee, and W. M. Tang, *Phys. Plasmas*, 1999]

$$\left(\frac{\Delta r_0}{\Delta r}\right)^2 = 1 + \frac{\omega_{Eff}^2}{\Delta\omega_T^2}$$

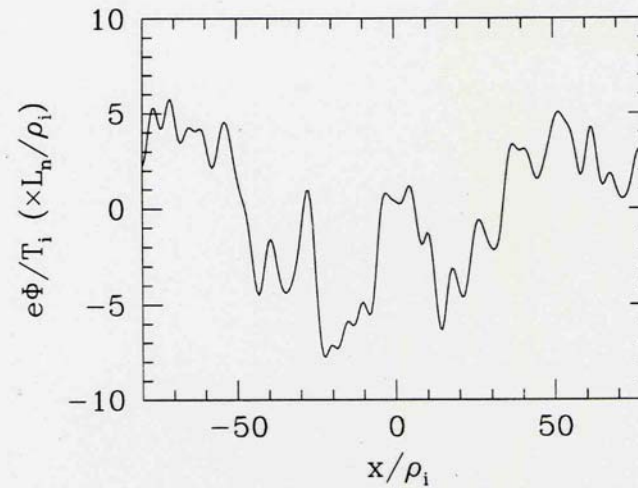
- Fast time-varying $\mathbf{E} \times \mathbf{B}$ flow is not effective in suppressing turbulence: flow pattern changes before eddies get distorted

$$\omega_{Eff} \simeq \omega_E^{(0)} \frac{\Delta\omega_T}{\sqrt{\Delta\omega_T^2 + 3\omega_f^2}}$$

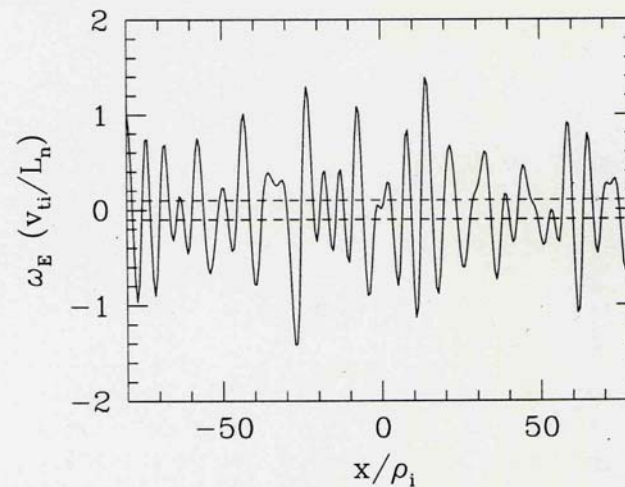


Shearing Rates from Gyrofluid Simulations

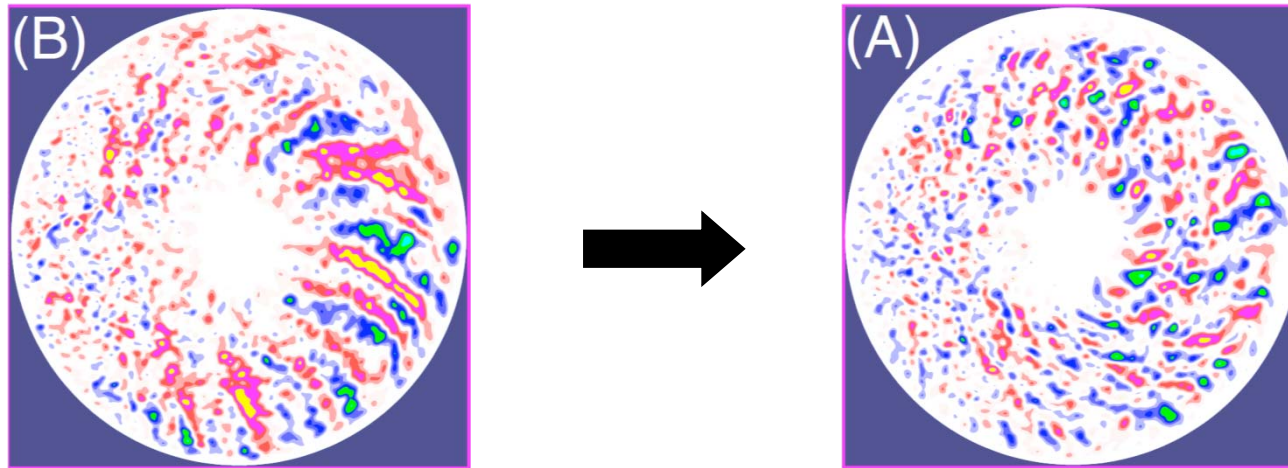
- Small-scale turbulence generated flow from gyrofluid simulation, instantaneous potential:



- Instantaneous shearing rate, ω_E , is large, but dominated by high frequency and high k_x components.



Duality of Flow Generation and Random Shearing of Eddys



$\omega_k \gg \omega_{ZF}$ \rightarrow Drift Wave Action Density, $N_{\bar{k}}$, is conserved.

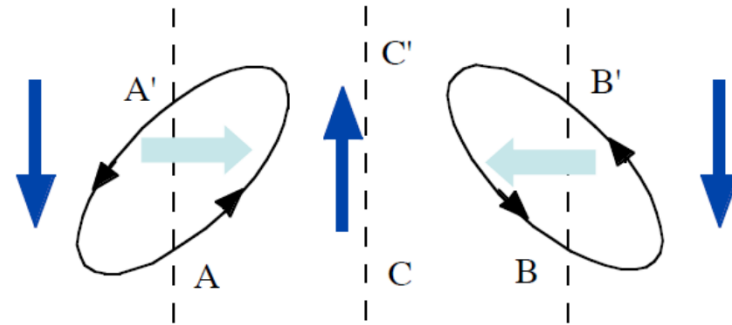
From $\omega_{DW} = \frac{k_{\theta} v_*}{1 + k_{\perp}^2 \rho_s^2}$ shearing $\rightarrow k_r^2 \nearrow \rightarrow$ Drift Wave Energy:
 $E_k = N_k \omega_k \searrow$

Since total energy conserved between ZF and Drift Wave,
 Energy for ZF generation is extracted from DWs.

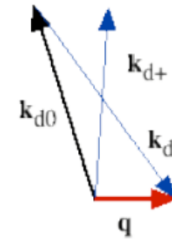
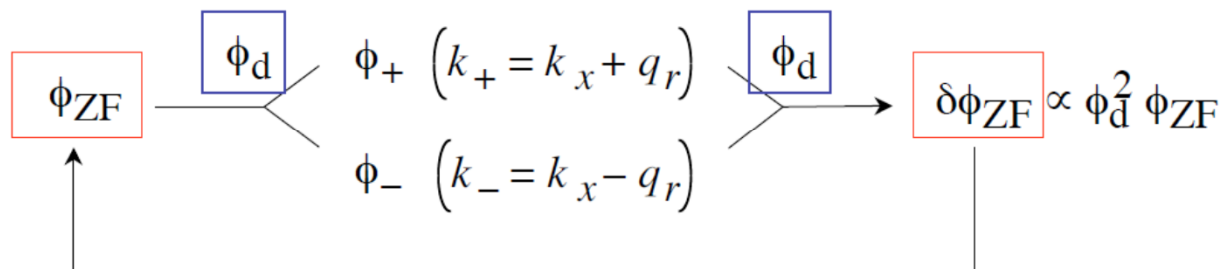
[Diamond et al., IAEA-FEC, 1994];

Generation Mechanism

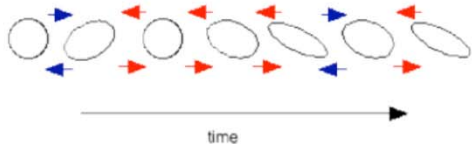
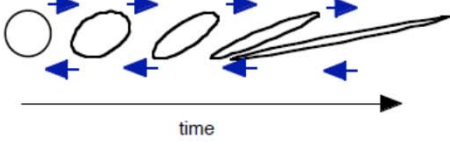
(1) Tilt of convection cell by a sheared flow



(2) Modulational Instability



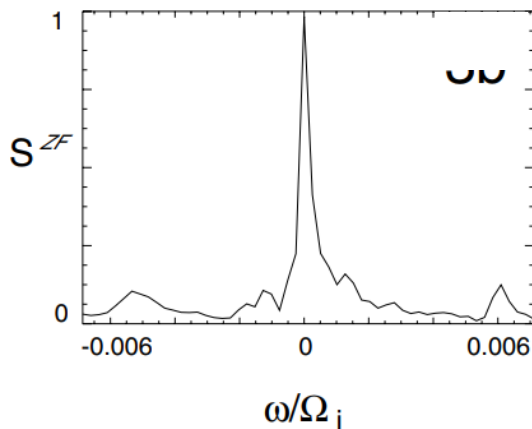
Distinction between ZF and Mean $\langle E_r \rangle$

	Zonal Flows	Mean Field $\langle E_r \rangle$
Time	can change on turbulence time scales	changes on transport time scales
Space	oscillating, complex pattern in radius $\sim 20 \rho_i$	smoothly varying
Stretching behavior k of waves	diffusive $\langle \delta k^2 \rangle \propto t$ 	ballistic $\langle \delta k^2 \rangle = t^2 k^2 V_E'^2$ 
Drive	Turbulence	equilibrium ∇p , orbit loss, external torque, turbulence, etc.

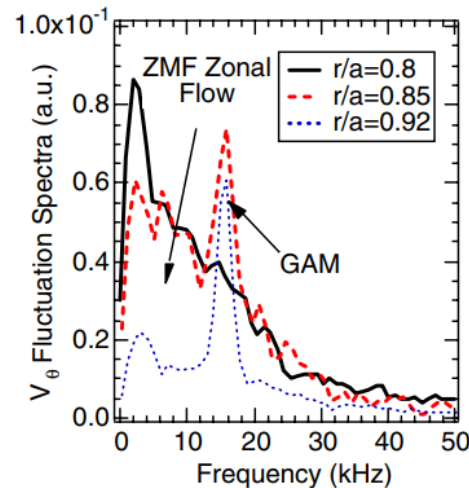
Active research on synergy between them is underway.

Characterization of Zonal Flow Properties from Simulations Motivated Experimental Measurements

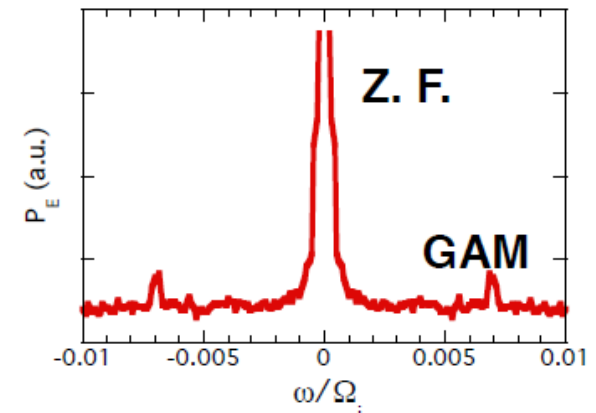
- Zonal flows regulate turbulence and transport.
- Turbulence in most cases produces zonal flows
- Characteristics have been confirmed from experiments



T.S.Hahm et al.,
PPCF (2000)
from GTC Simulation



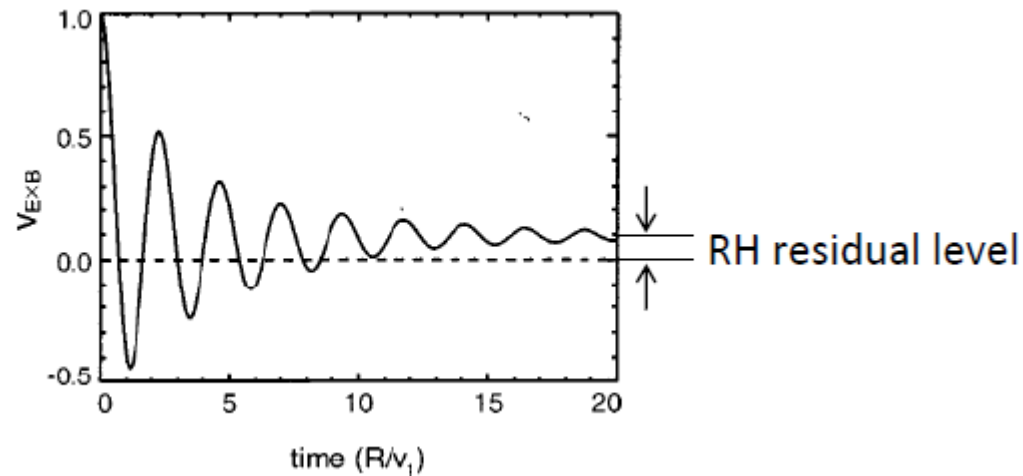
D.K. Gupta et al.,
PRL(2006)
from Tokamak(DIII-D)



A. Fujisawa et al.
PRL(2004)
from Stellarator(CHS)

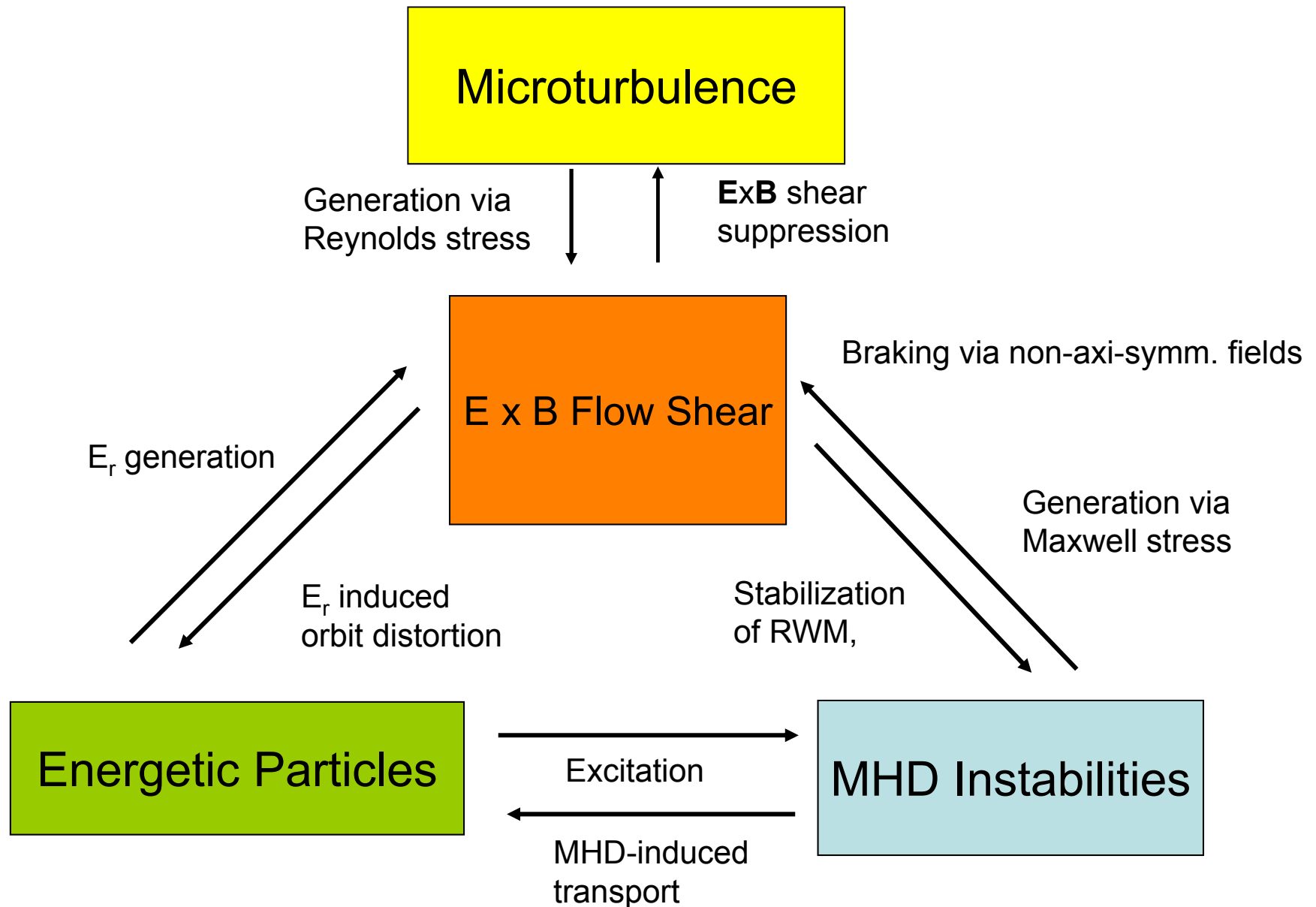
Residual Zonal Flows

- Based on gyro-Landau-fluid closure (up to mid 90's), ZF is completely damped even in collisionless plasmas
- Rosenbluth-Hinton [PRL '98] ZF undamped from Gyrokinetic theory



- Gyrokinetic codes are now benchmarked against the analytic results!
- Most transport models don't include zonal flows yet.
(exceptions : M. Nunami et al, PoP (2013), E. Narita et al., PPCF (2018))

E x B Flow Shear Plays a Central Role in Magnetic Confinement



Key Physics Mechanisms behind Size Scaling of Confinement

- **Global Toroidal ITG eigenmode**

[Horton-Choi-Tang, PF '81] [Cowley-Kulsrud-Sudan, PF B' 91]

[Romanelli-Zonca, PF B' 93][Parker-Lee-Santoro, PRL'93]

Bohm Scaling ?

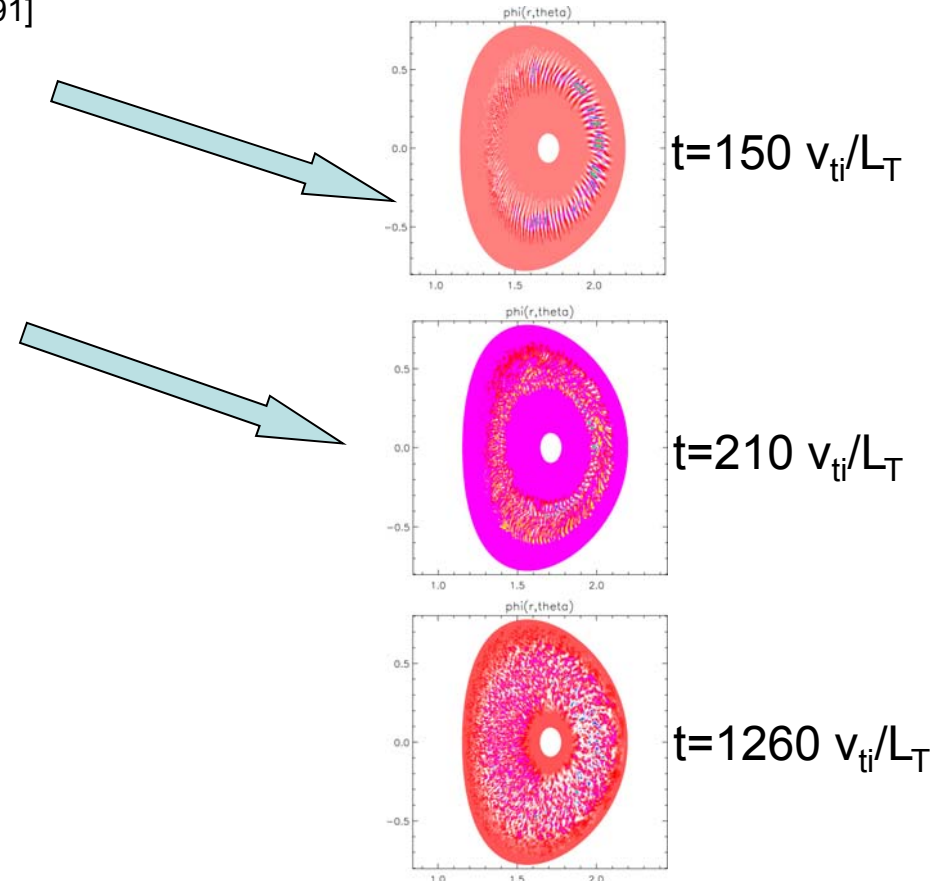
- **Self-regulation by Zonal Flows:**

[Cast of Thousands]

[Lin, Hahm, Lee et al., Science '98]

[Diamond, Itoh, Itoh and Hahm, Review in PPCF '05]

GyroBohm Scaling !



Density fluctuations from a GTS simulation of a shaped plasma with typical DIII-D core parameters

[Wang, Hahm, Lee *et al.*, PoP '07]

Summary

Zonal Flows

Huge Effect on Tokamak Confinement Scaling
with respect to Machine Size:

GyroBohm Scaling !

Not the End of Story



Outstanding Issues of Turbulent Transport in Tokamaks

- **Generalization of Fick's Law** $\Gamma_p = -D \frac{\partial n}{\partial x}$ **to Diffusion Equation**

$$\begin{pmatrix} Q_i \\ Q_e \\ \Gamma_\phi \\ \Gamma_p \end{pmatrix} = - \underbrace{\begin{pmatrix} \chi_i & \dots & \dots & \dots \\ \dots & \chi_e & \dots & \dots \\ \dots & \dots & \chi_\phi & \dots \\ \dots & \dots & \dots & D \end{pmatrix}}_{\text{off-diagonal pinch terms exist}} \begin{pmatrix} (\nabla T_i)_r \\ (\nabla T_e)_r \\ (\nabla U_\phi)_r \\ (\nabla n)_r \end{pmatrix}$$

off-diagonal pinch terms exist

- **Plasma Current Scaling of Confinement**

- $\tau_E \nearrow$ with I_p more strongly than with B_ϕ !

- **Ion Thermal Transport**

- Typically due to ITG
- One candidate: ineffective ZF shearing if geodesic acoustic sideband of ZF with $\omega_{\text{GAM}} \sim c_s/R_0$ gets stronger at the expense of main ($\omega_{\text{ZF}} \sim 0$) ZF.
- Landau damping of GAM with $\gamma_{\text{damping}} \sim -e^{-q^2/2}$ where $q \simeq rB_\phi/RB_\theta$

\therefore strong $q \rightarrow$ stronger GAM \rightarrow less shearing due to GAM if $\omega_{\text{GAM}} > \Delta\omega_{\text{Turb}}$

\rightarrow strong turbulence.

Outstanding Issues of Turbulent Transport in Tokamaks

- **Electron Thermal Transport:** dominant depending on parameters.

A. Trapped Electron Mode (TEM)

- Strong evidence from AUG ECH experiment (F. Ryter, PRL '05)
- **DTEM** (dissipative TEM) → Neo Alcator scaling $\tau_E \propto n_e a R^2$
- C-Mod GK code result: ITG more unstable than TEM
→ **KSTAR** reports different story

B. Electron Temperature Gradient Mode (ETG)

- Associated ZF relatively weak → radially elongated eddies
→ streamer could dominate
- Otherwise low level of transport with $\Delta r \sim \text{several } \rho_e$

Outstanding Issues of Turbulent Transport in Tokamaks

- **Microtearing Mode**

Instead of ExB transport due to electrostatic fluctuation, $\delta v_r = \frac{c\mathbf{b} \times \nabla\delta\phi}{B}$

transport due to magnetic flutter: $\delta v_r = \frac{\delta B_r}{B_0} v_{\parallel}$

- **Momentum Transport**

- Spontaneous/intrinsic rotation without external torque inputs from NBI/ICRH ... (even for Ohmic plasmas!)
- NBI not efficient in driving rotation in ITER. → rotation of ITER? (for resistive wall mode and turbulence reduction)

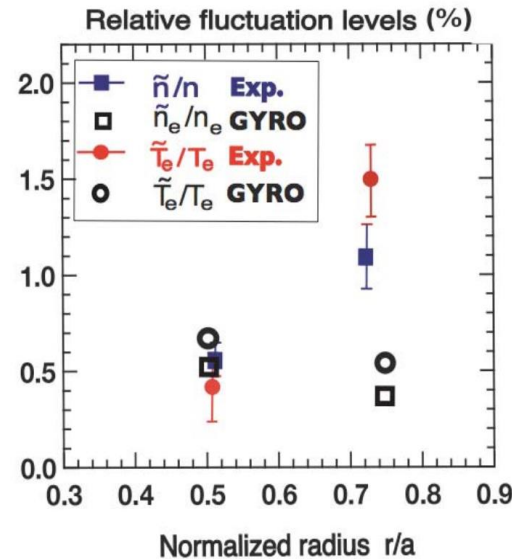
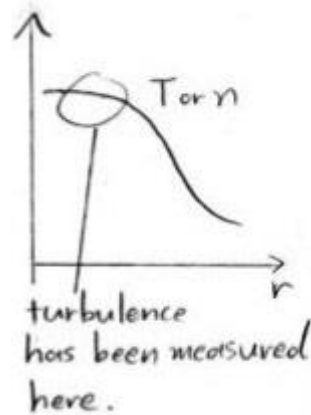
- **H-mode Transition**

$|\mathbf{v}'_{E \times B}| \nearrow \Rightarrow \text{turbulence } \delta n \searrow \Rightarrow \text{transport } \searrow$ which component, u_{θ} , u_{ϕ} or ∇p ?

- u_{θ} deviates from neoclassical prediction (DIII-D '94) even in L-mode plasmas (but deviation from u_{θ} is very small in NSTX).

Outstanding Issues of Turbulent Transport in Tokamaks

- Existence of Turbulence in the Absence of Local Drive due to Radial Gradient



- Edge-core coupling region ($0.7 < \rho = r/a < 0.85$): “**No Man’s Land**”
- “Turbulence spreading” is a candidate to resolve the short-fall problem.
- Upcoming Review Paper in JKPS:

T. S. Hahm and P. H. Diamond

"Mesoscopic Transport Events and the Breakdown of Fick's Law for Turbulent Fluxes."

Outstanding Issues of Turbulent Transport in Tokamaks

- **Isotopic Dependence of Transport**

$$\chi_{\text{GB}} \sim \frac{\rho_i c T_i}{a e B} \propto \sqrt{M_i}$$

GB scaling

≠

$$\chi_{\text{Deuterium}} < \chi_{\text{Hydrogen}}$$

Experimental result

- TFTR: tritium χ is even lower.

“Simulations & Modeling” should focus more on identification of physical mechanism rather than ever-popular case by case agreement with neighboring experiment in numbers.