Critically balanced turbulence in fusion-grade plasmas

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at Chungnam National University

8th East-Asia School and Workshop on Laboratory, Space and Astrophysical Plasmas

NFP Lab. Nuclear Fusion&Plasma Lab

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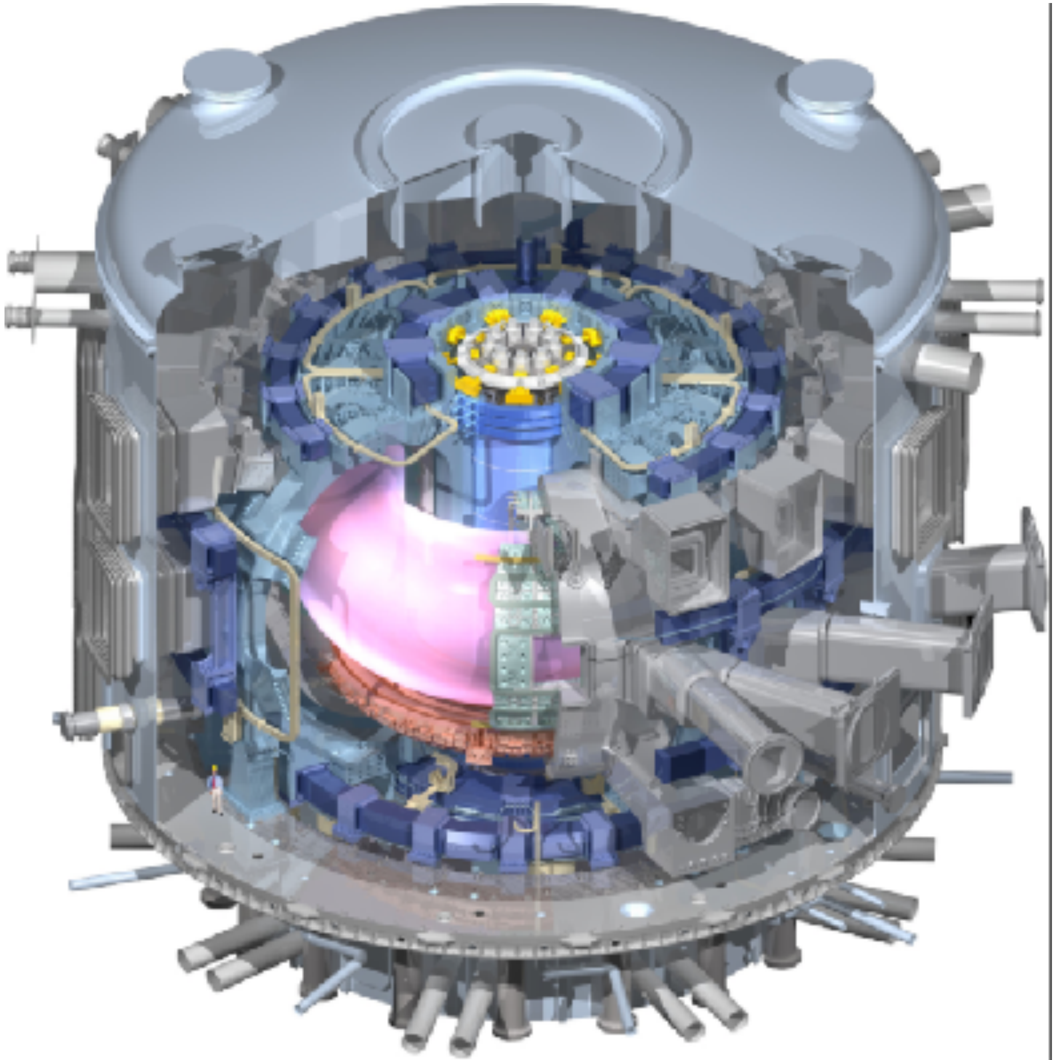
Contents

FUSION ENERGY

Fusion Reactor: ITER

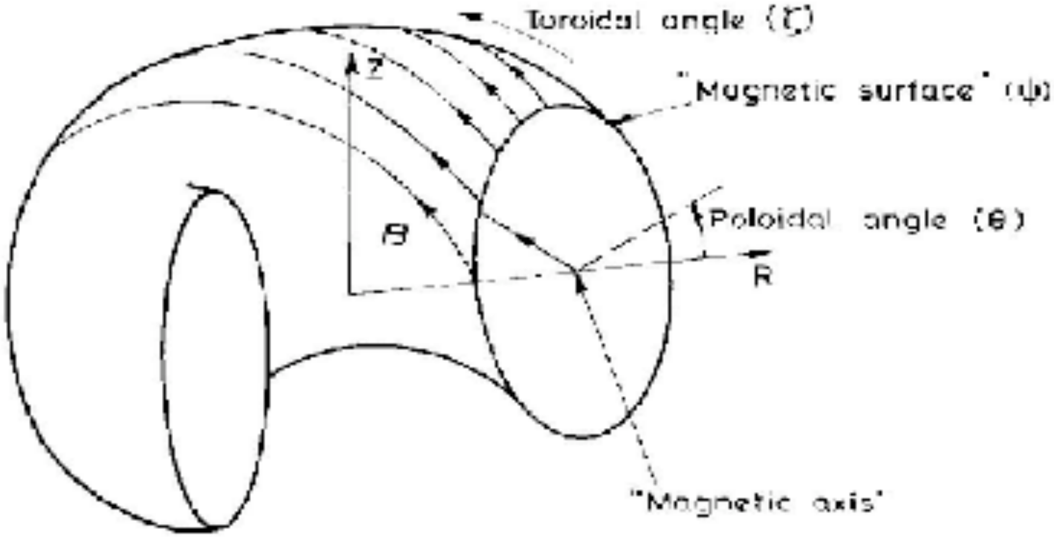


]

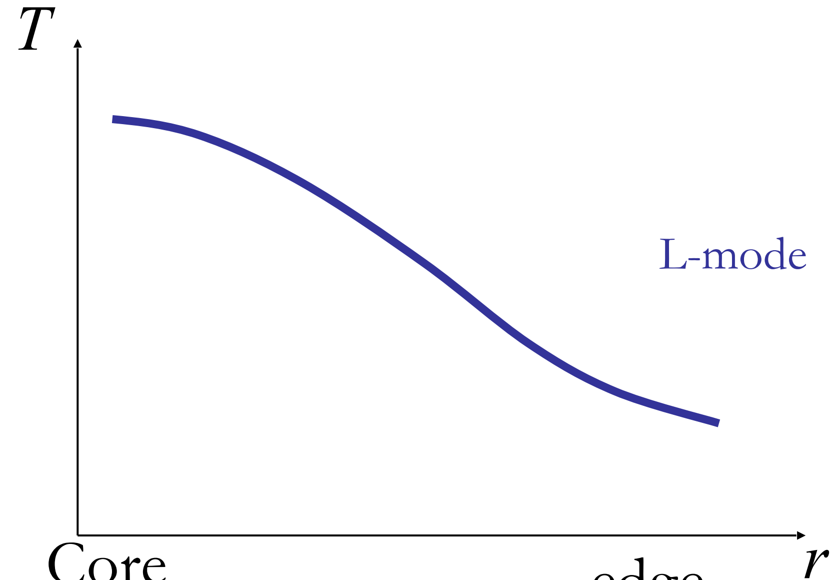
@ about

Simplifying the picture to obtain a physical picture

Simplified picture for the interested issues

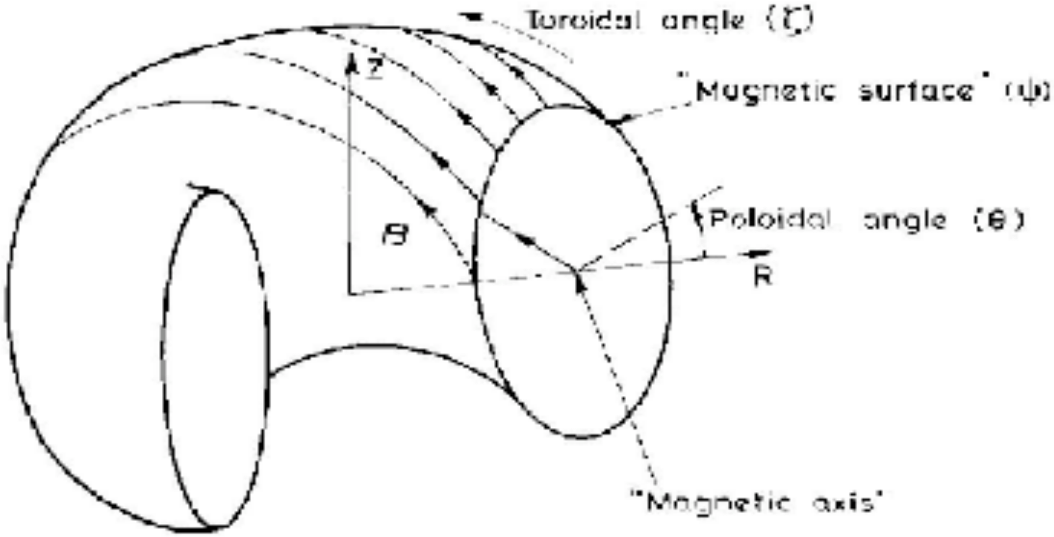


Courtesy of J. Connor



- Plasma confined by magnetic field
- Temperature profile:
  hot in the core, **cold** at the edge

Simplified picture for the interested issues



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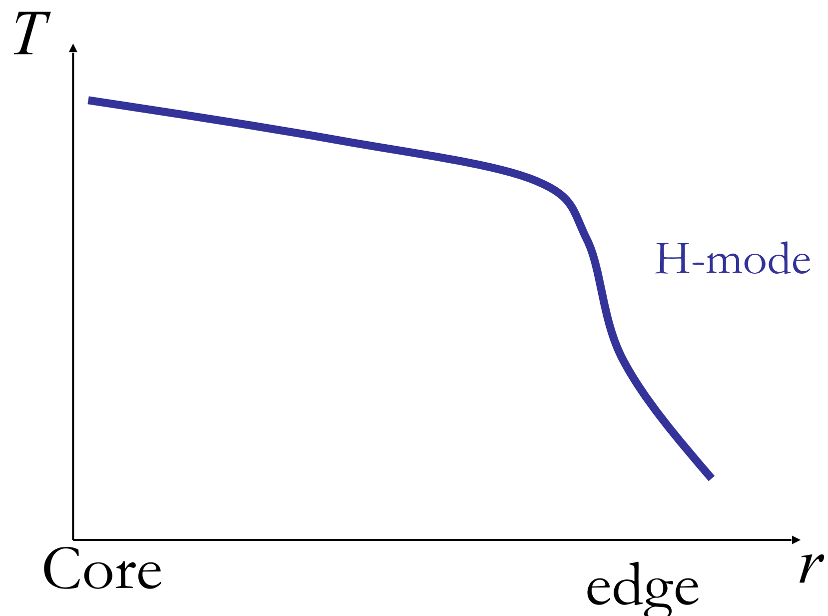
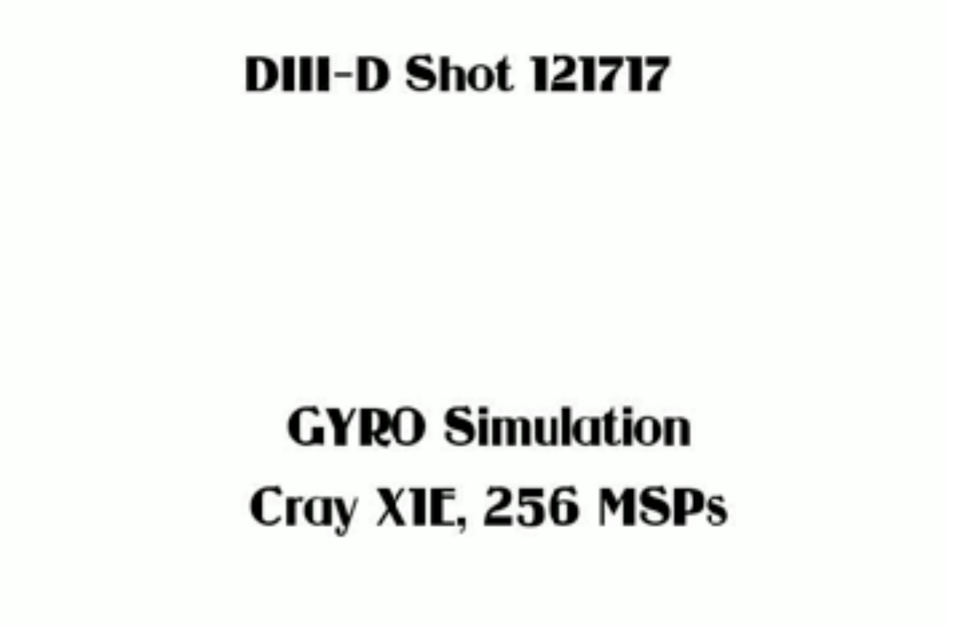
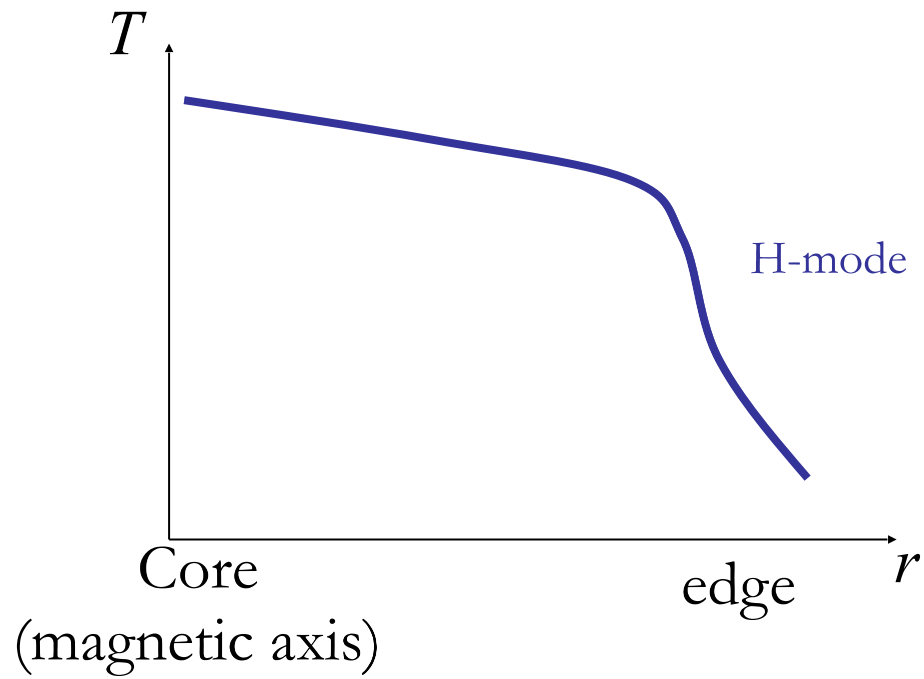


- Plasma confined by magnetic field
- Temperature profile:
  hot in the core, **cold** at the edge
- **Goal:**
  - Maintain the profile (confinement)
  - Make it as sharp as possible

Plasma turbulence seems to relax the sharp profile



A gyrokinetic simulation of ITG turbulence in DIII-D
(J. Candy, R. Waltz @GA



- **Question**: *What relaxes the profiles?*
- Pressure gradient is a source of **free energy**

✓ **microinstability** at Larmor scales

✓ **TURBULENCE**

Thus, turbulence is the enemy. In order to kill it, we must understand it.

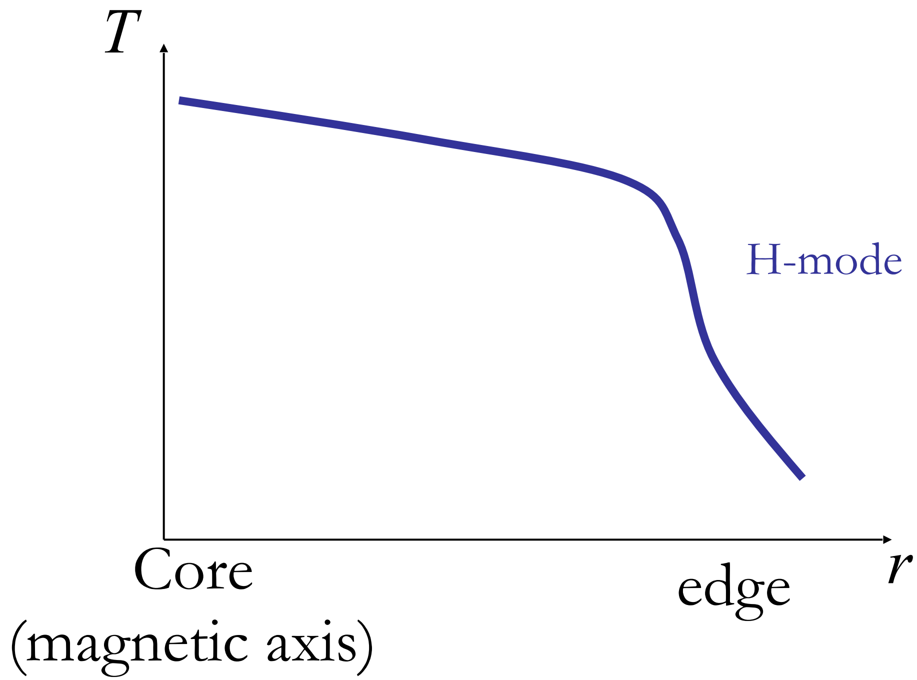
(Intellectually, it is our friend as we are fascinated by it and admire it for its complexity and ingenuity even as it tests our wits and foils our attempts to control it.)

Our challenge:

✓ Multiscale plasma dynamics & theory of microscale turbulence

✓ Some effective theory for macroscale transport

✓ Effect of macroscale profiles on turbulence (closing the loop)



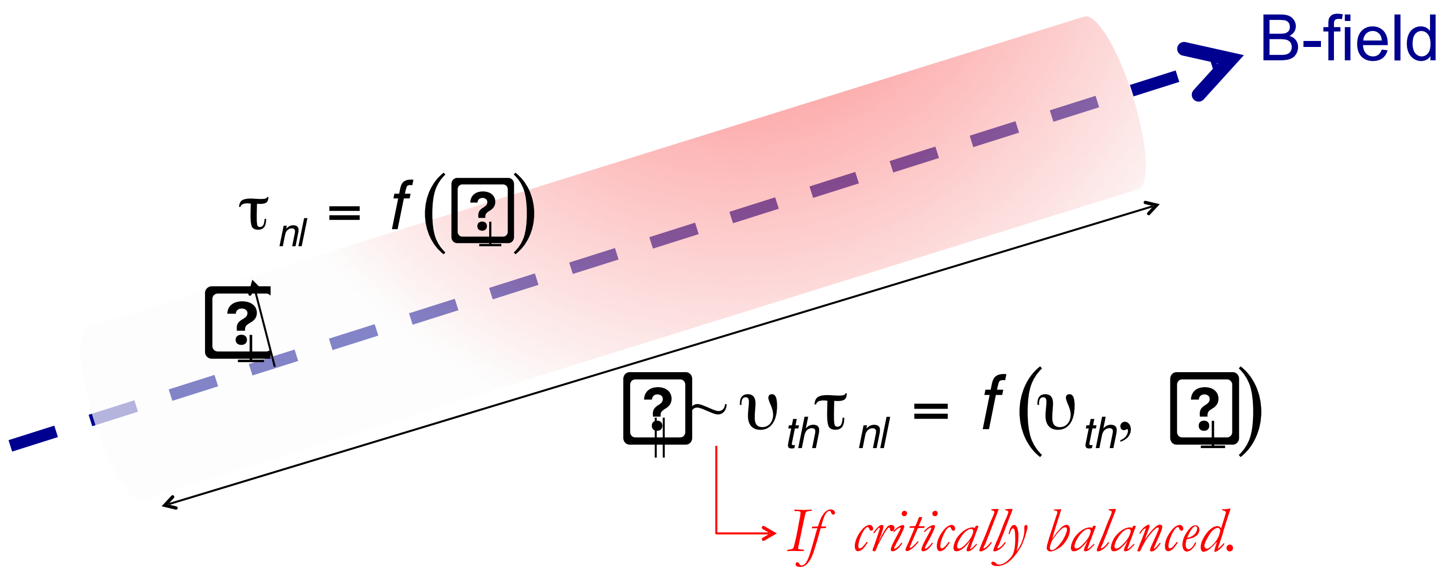
- What is the spatial structure of turbulent "eddies"? Amplitude vs. scale.
- How does it change under shear? Does amplitude decrease? And/or shape changes?
- What combination of rotation profile and magnetic geometry gives minimum transport? (Careful: shear can drive turbulence!)

Contents

- ➢ Defining problems in fusion-grade plasmas
  - ✓ Why do we have turbulence in fusion-grade plasmas?
  - ✓ Why is the turbulence issue in fusion-grade plasmas?
  - ✓ What are the questions we need to find answers? (for building an economical fusion power plant)
- ➢ Critically balanced turbulence
  - ✓ Concept and its consequence
  - ✓ Observations
- ➢ Summary

Critically balanced turbulence

- Critical balance
  - ✓ If a medium can support parallel (to the B-field) propagation of waves (and/or particles) and nonlinear interactions in the perpendicular direction, the turbulence in such a medium would normally be "critically balanced," meaning that the characteristic time scales of propagation and nonlinear interaction would be comparable to each other and (therefore) to the correlation time of the fluctuations.



Critically balanced turbulence has been observed in …

- Gyrokinetica numerical simulation of ion-temperature-gradient turbulence
  - ✓ Barnes et al. PRL, 107, 115003 (2011)
- Theory
  - ✓ Goldreich et al. Astrophys. J. 438, 763 (1995)
  - ✓ Cho et al. Astrophys. J. 615, L41 (2004)
  - ✓ Schekochihin et al. Astrophys. J. Suppl. 182, 310 (2009)
  - ✓ Nazarenko et al. J. Fluid Mech. 677, 134 (2011)
- Observations and simulations of MHD
  - ✓ Horbury et al. PRL 101, 175005 (2008)
  - ✓ Podesta, Astrophys. J. 698, 986 (2009)
  - ✓ Wicks et al. Mon. Not. R. Astron. Soc. 407, L31 (2010)
  - ✓ Cho et al. Astrophys. J. 539, 273 (2000)
  - ✓ Maron et al. Astrophys. J. 554, 1175 (2001)
  - ✓ Chen et al. Mon. Not. R. Astron. Soc. 415, 3219 (2011)
- Kinetic plasma turbulence in space
  - ✓ Cho et al. Astrophys. J. 615, L41 (2004)
  - ✓ TenBarge et al. Phys. Plasmas 19, 055901 (2012)

Use various time scales for turbulence characteristics

- Fluctuations in a magnetized toroidal plasma are subject to a number of *physical effects*, which can be classified in terms of various *time scales*:

1. Drift time (associate with the temperature/density gradients): $\boldsymbol{\tau_*}$
2. Particle streaming time (along the B-field as it takes around the torus): $\boldsymbol{\tau_{st}}$
3. Turbulence correlation time (in the moving frame, i.e., Lagrangian approach): $\boldsymbol{\tau_c}$
4. Nonlinear time (of the fluctuations advected across the B-field by fluctuating v_E): $\boldsymbol{\tau_{nl}}$
5. Magnetic drift time: $\boldsymbol{\tau_M}$
6. Collision (ion) time: $\boldsymbol{\nu_i^{-1}}$
7. Shear (perpendicular) time of the plasma rotation: $\boldsymbol{\tau_{sh}}$ $\boldsymbol{(\gamma_E^{-1})}$

Some thoughts on turbulecne: what IF…

$$\tau_{*i}^{-1} = \frac{\rho_i}{?_y}\frac{\upsilon_{th,i}}{L_{T_i}};\ \tau_{st}^{-1} = \frac{\upsilon_{th,i}}{\Lambda} \sim \frac{\upsilon_{th,i}}{?_\parallel};\ \tau_c$$

Assume $?_\parallel \sim \Lambda$ (the connection length)

What IF (so far mere assumptions)…

$\tau_c \sim \tau_{*i}$ At the "energy injection scale", the linear driving time is comparable to the turbulence correlation time.

$\tau_c \sim \tau_{st}$ Critical balance

Then, we have

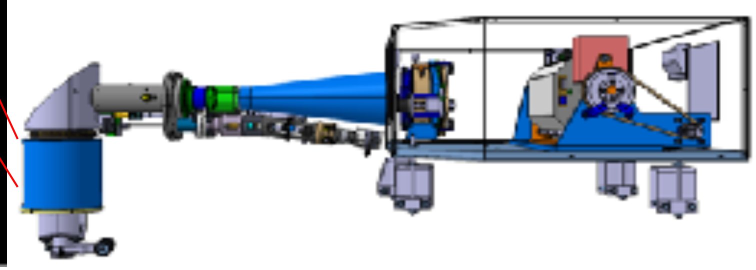
$\tau_{*i} \sim \tau_{st}$ whose consequence is $\dfrac{?_y}{\rho_i} \sim \dfrac{\Lambda}{L_T}$

We found $?_y$ based on local equilibrium quantities!!!

All starts from careful OBSERVATIONS!

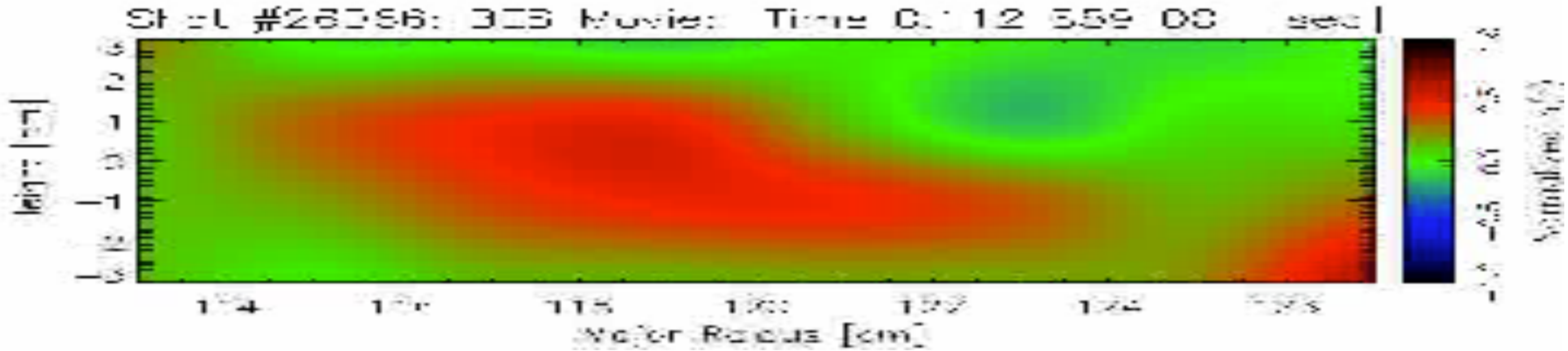
From MAST tokamak

2D BES system

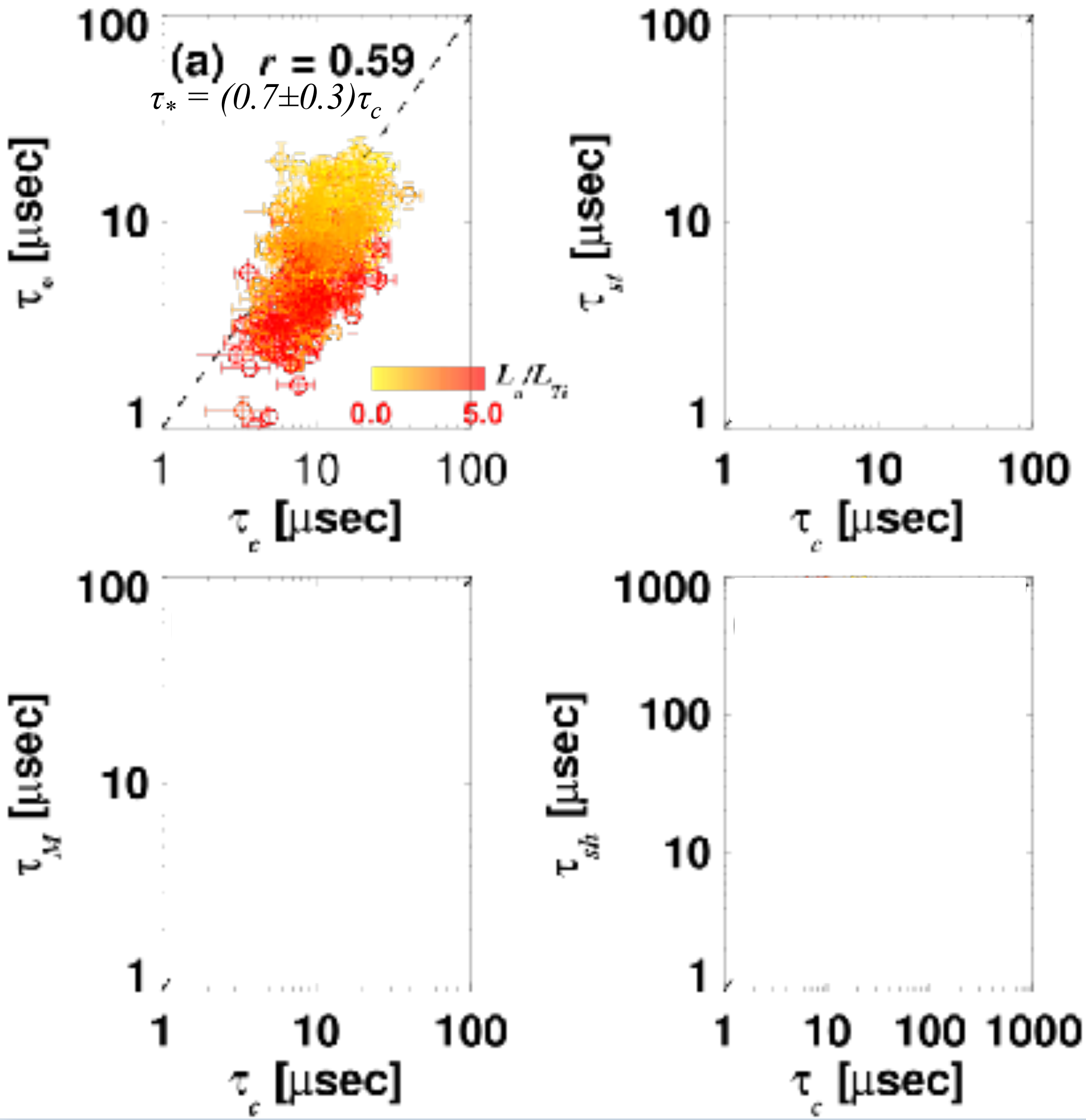


An example of the observation by 2D BES system

Channels: 8 (radial) x 4 (poloidal)
Radial span: ~ 16 cm (dR = 2 cm): scannable
Poloidal span: ~ 8cm (dZ = 2 cm)
dt: 0.5 microsecond



Turbulence correlation time vs. drift time



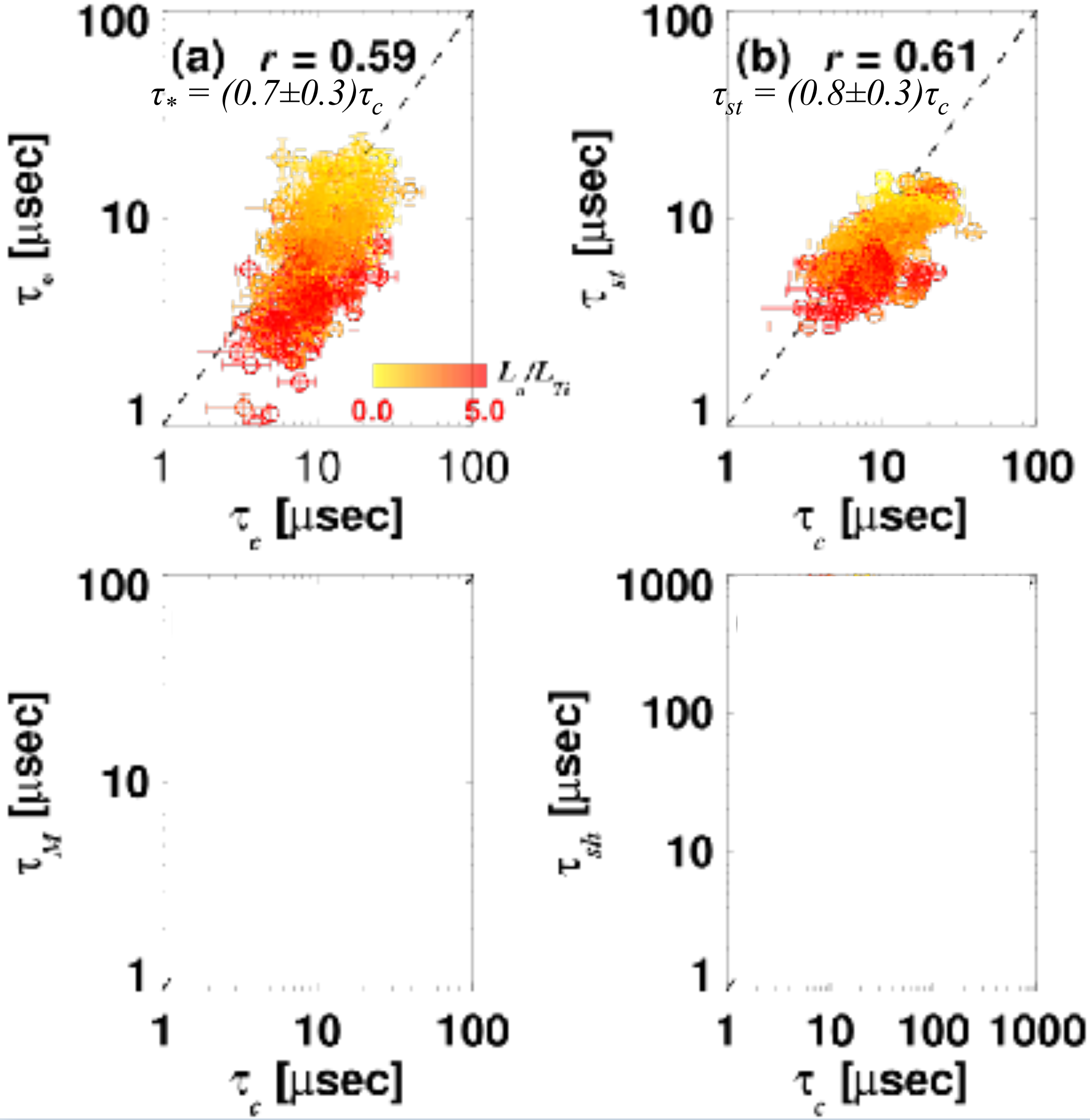
$$\tau_{*i,e}^{-1} = \frac{\rho_{i,e}}{?_y}\frac{\upsilon_{thi,e}}{L_{Ti,e}};\ \tau_{*n}^{-1} = \frac{\rho_i}{?_y}\frac{\upsilon_{thi}}{L_n}$$

$$\tau_* = \min\{\tau_{*i},\ \tau_{*n}\}$$

$\tau_c \sim \tau_*$ → the measured turbulence knows *the linear drive!*

Note:
For $\tau_c \leq 10\mu s$, $\tau_{*e} \sim \tau_{*i}$
For $\tau_c \geq 10\mu s$, $\tau_{*n} \sim \tau_{*i}$
Thus, we cannot rule out ion-scale electron drive. However, we find no clear correlation of τ_{*e} with τ_c, or with any of the other time scales discussed later.

Turbulence correlation time vs. parallel streaming time

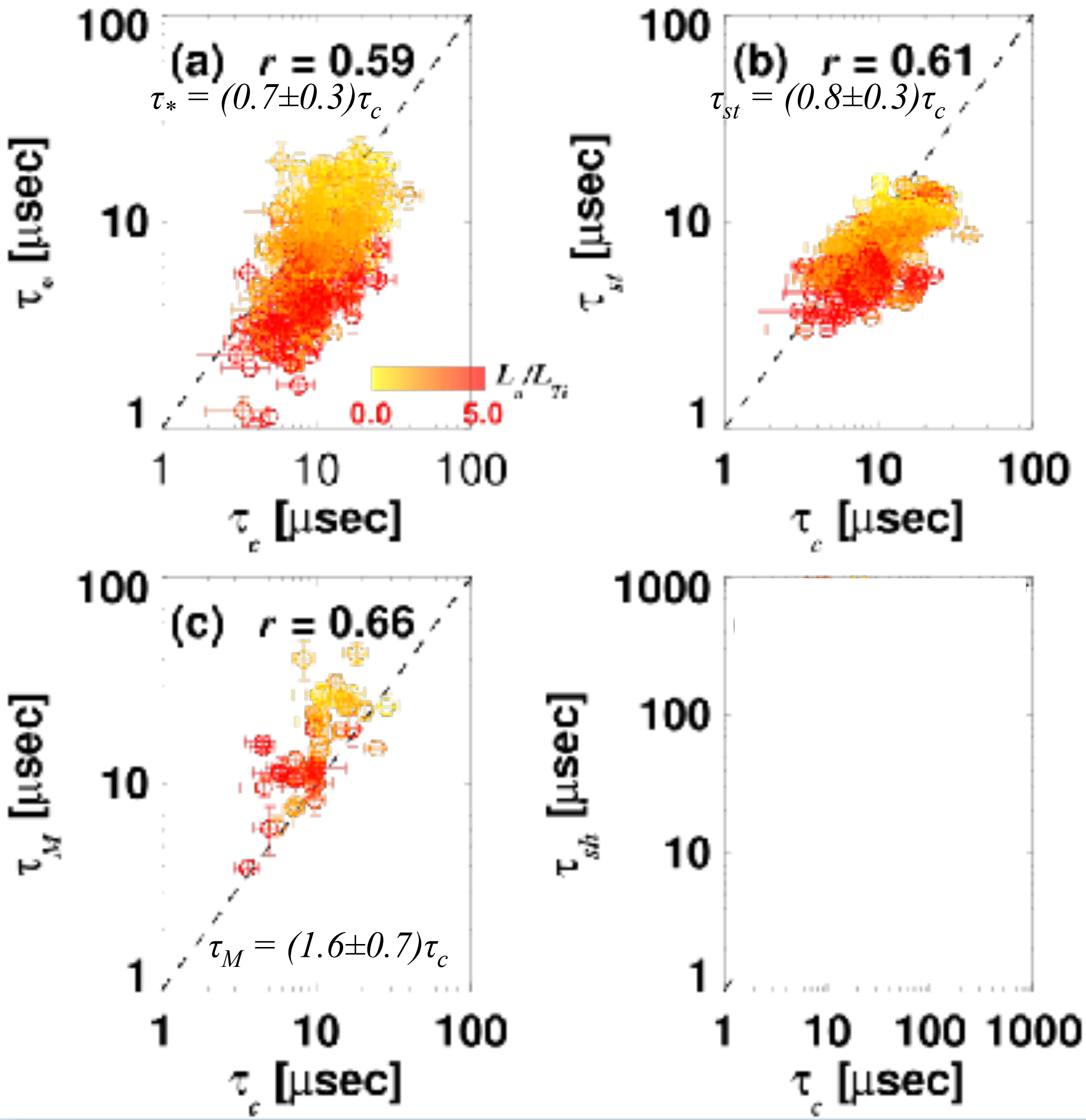


$$\tau_{st}^{-1} = \frac{\upsilon_{thi}}{\Lambda} = \frac{\upsilon_{thi}}{\pi r}\frac{B_p}{B} \sim \frac{\upsilon_{thi}}{?}$$

$\boldsymbol{\tau_c \sim \tau_{st}}$ → the measured turbulence knows *the parallel system size. (perhaps through the critical balance?)*

Note: Although, this result cannot be used to state (conclusively) that we have 'critical balance', the result is at least consistent with it.

Turbulence correlation time vs. magnetic drift time



$$\tau_M^{-1} = \frac{\rho_i}{?}\frac{\upsilon_{thi}}{R}$$

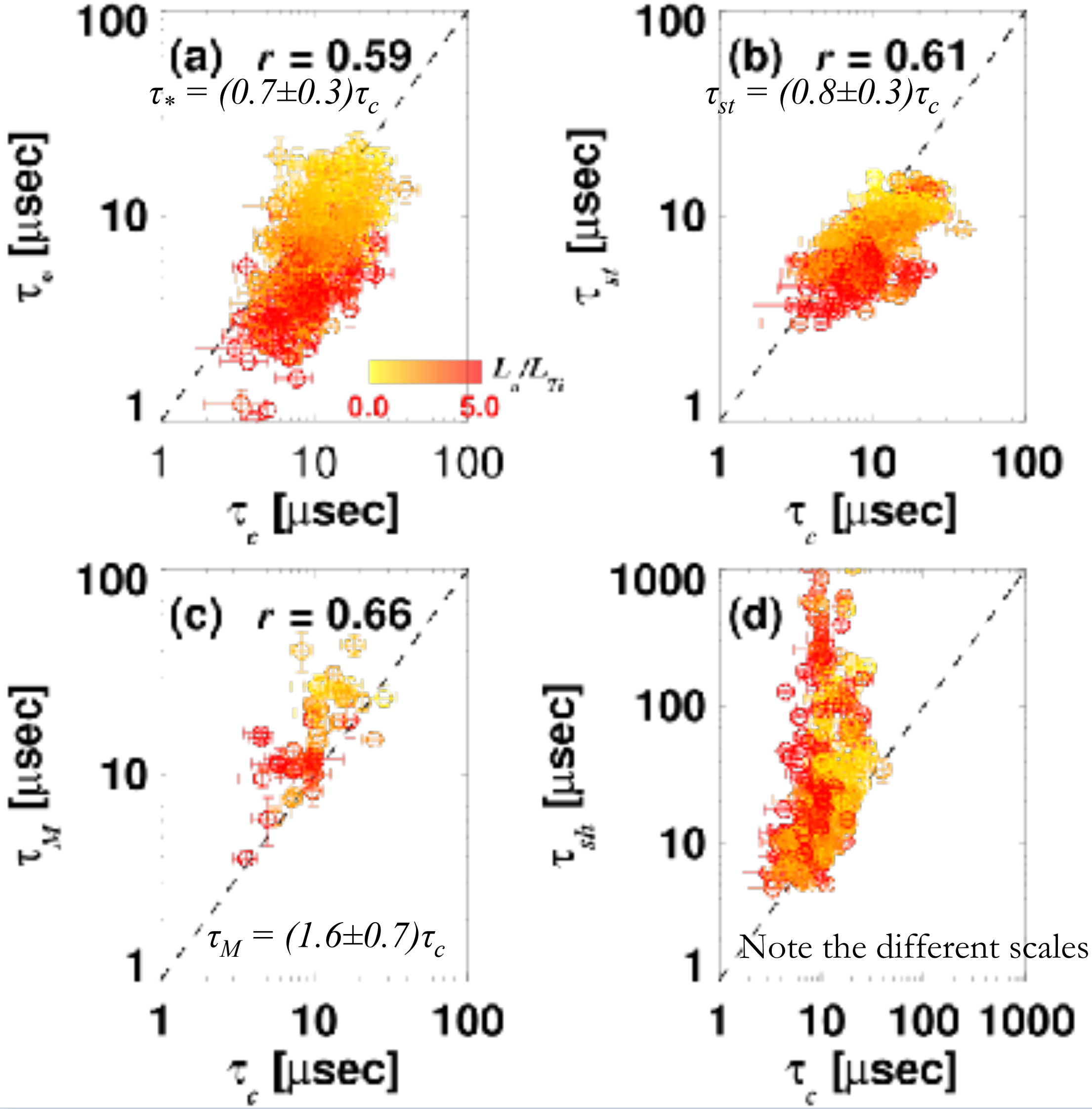
$\tau_c \sim \tau_M \rightarrow$

unexpected result. But, this sets ?

Note: As we will see later, a consequence of this result is *anisotropic structure* of turbulence in the perpendicular plane which clashes with many reported results of ? ~ ?

Spherical vs. Conventional? I don't know.

How about the perpendicular shearing time?



$$\tau_{sh}^{-1} = \frac{B_p}{B}\frac{dU_\Phi}{dr}$$

$\tau_c \nsim \tau_{sh}$ → the shearing rate was not strong enough.

Note: Most of our points satisfy $\tau_{st}/\tau_{sh}<1$ which may be the reason we have $\tau_c \sim \tau_{st}$ but *not* $\tau_c \sim \tau_{sh}$. (my conjecture on ?)

$$? = \begin{cases} \Lambda \left(= \pi r \frac{B}{B_p}\right) & \text{if } \tau_{st}\gamma_E < 1 \\ \upsilon_{th,i}\tau_{sh} \left(= \frac{\upsilon_{th,i}}{\gamma_E}\right) & \text{if } \tau_{st}\gamma_E > 1 \end{cases}$$

Summary

- We have
  - ✓ $\tau_c \sim \tau_* \sim \tau_{st} \sim \tau_M$
  - ✓ τ_c is not correlated with τ_{sh} (ν_i^{-1} is at least an order of magnitude longer than other times).

- Let's see what this means.

$$\tau_* \sim \tau_{st} \Rightarrow \frac{\rho_i}{\boxed{?}_y}\frac{\upsilon_{thi}}{L_*} \sim \frac{\upsilon_{thi}}{\Lambda} \Rightarrow \boxed{\frac{\boxed{?}_y}{\rho_i} \sim \frac{\Lambda}{L_*}} \text{ where } L_* = \min\{L_{Ti}, L_n\}, \Lambda = \boxed{\pi r \frac{B}{B_p} \sim \boxed{?}_{\parallel}}$$

This is really an assumption.

$$\tau_* \sim \tau_M \Rightarrow \frac{\rho_i}{\boxed{?}_x}\frac{\upsilon_{thi}}{R} \sim \frac{\upsilon_{thi}}{\Lambda} \Rightarrow \boxed{\frac{\boxed{?}_x}{\rho_i} \sim \frac{\Lambda}{R}}$$

- And, we have an anisotropic structure of turbulence in the perpendicular plane whose ratio is set by

$$\boxed{\frac{\boxed{?}_y}{\boxed{?}_x} \sim \frac{R}{L_*}}$$

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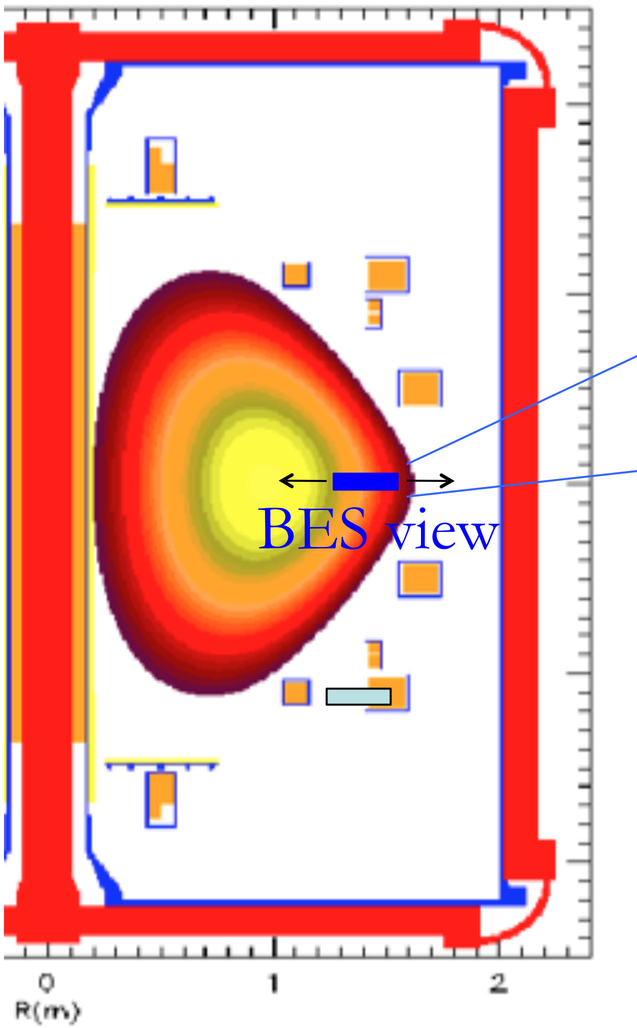
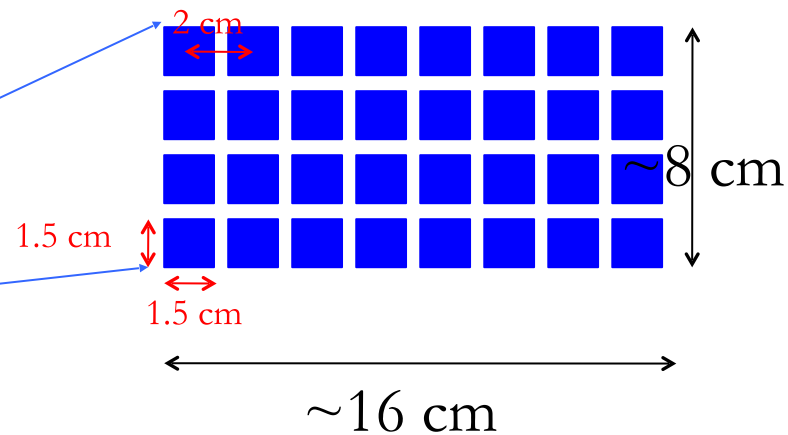
2D BES system measures local density fluctuations.



Detector: 8 radial x 4 poloidal channels



Radially: scannable

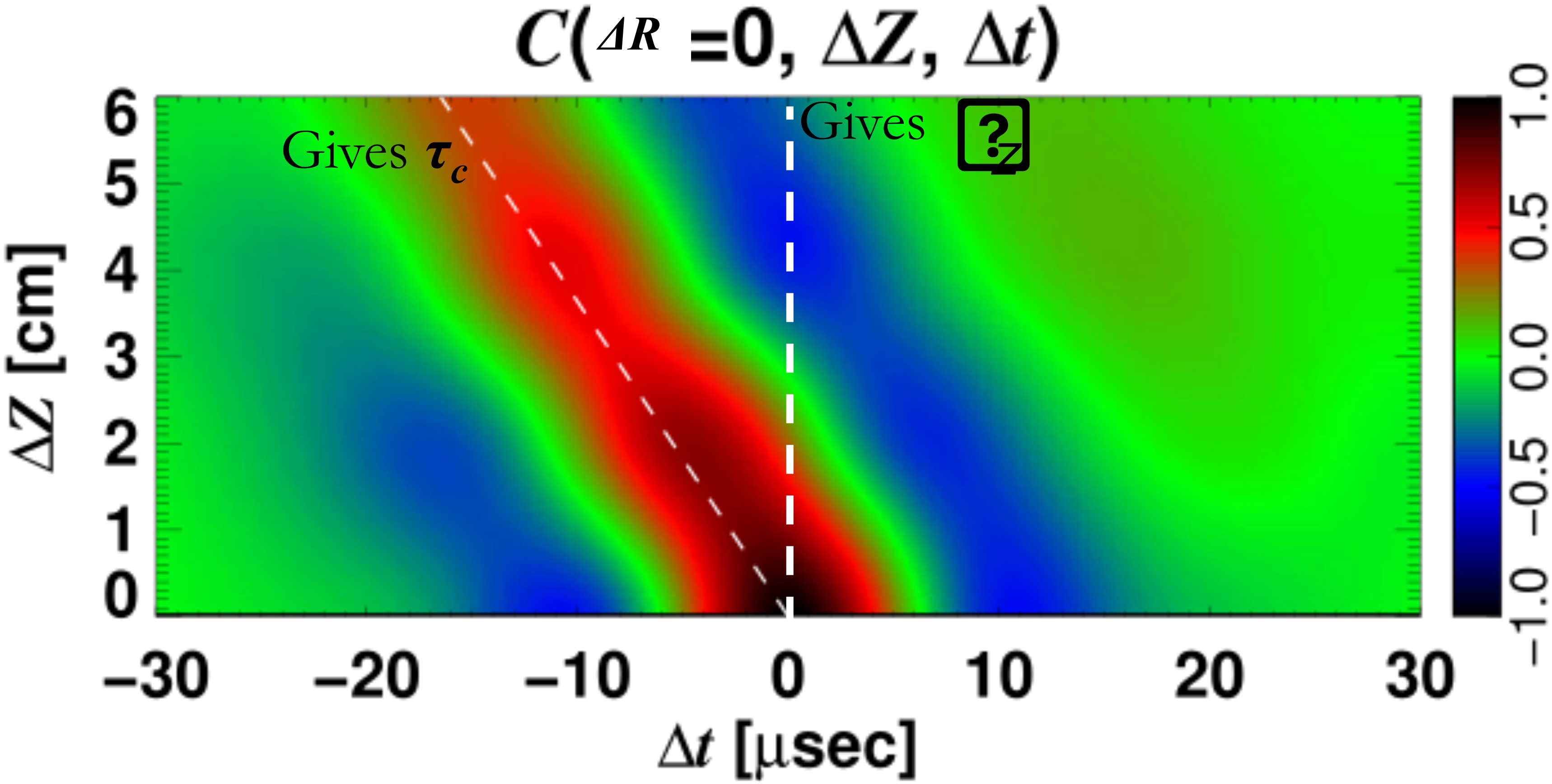
Polidally fixed (Z=-3, -1, 1, 3 cm)

k_r, $k_z < 2\pi/(2\times2\text{cm}) \sim 1.6\ \text{cm}^{-1}$, i.e. ion-scale.

But, point spread functions will correct this number.

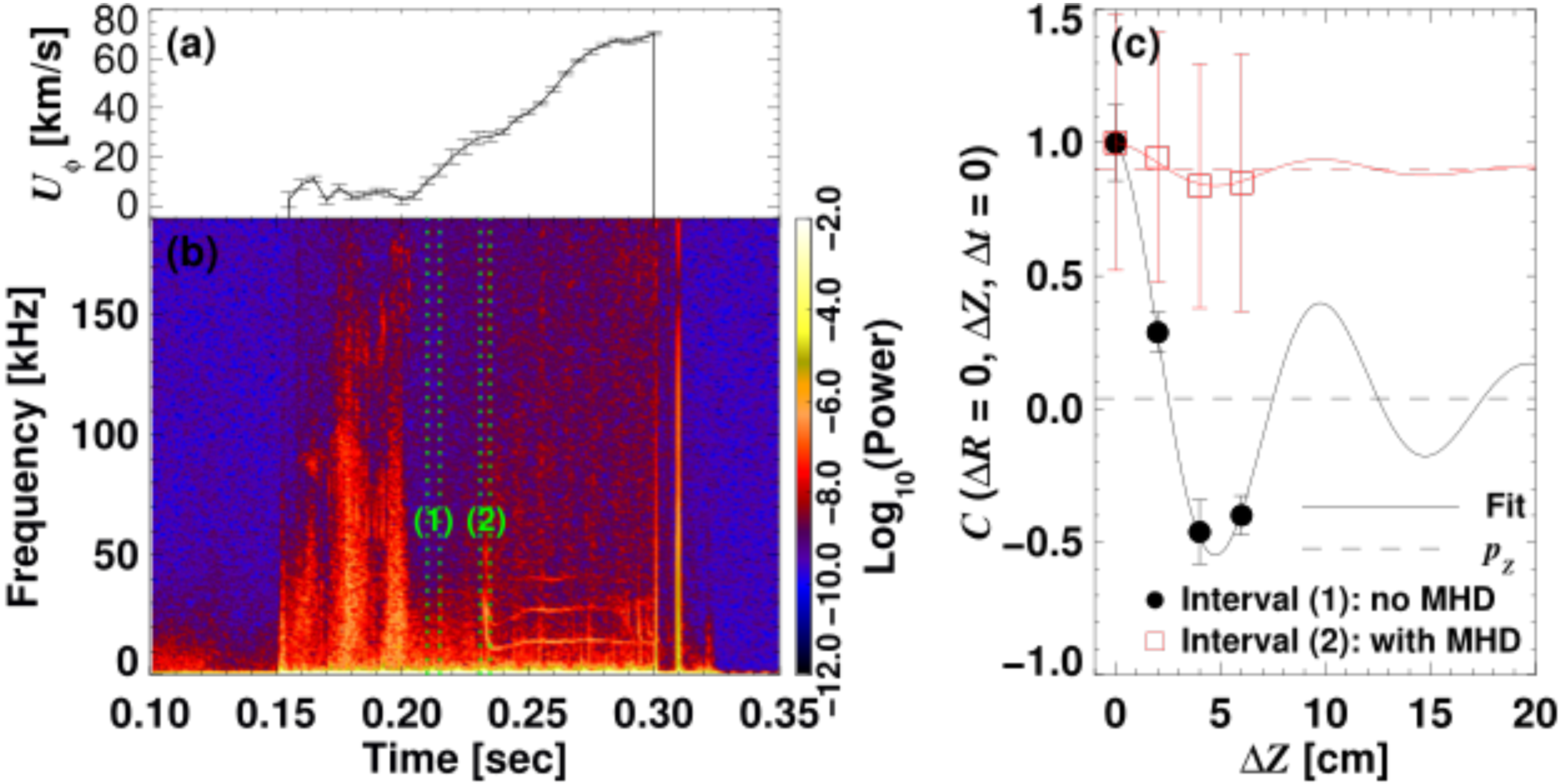
Digitization frequency: 2 MHz

Spatio-temporal correlation function



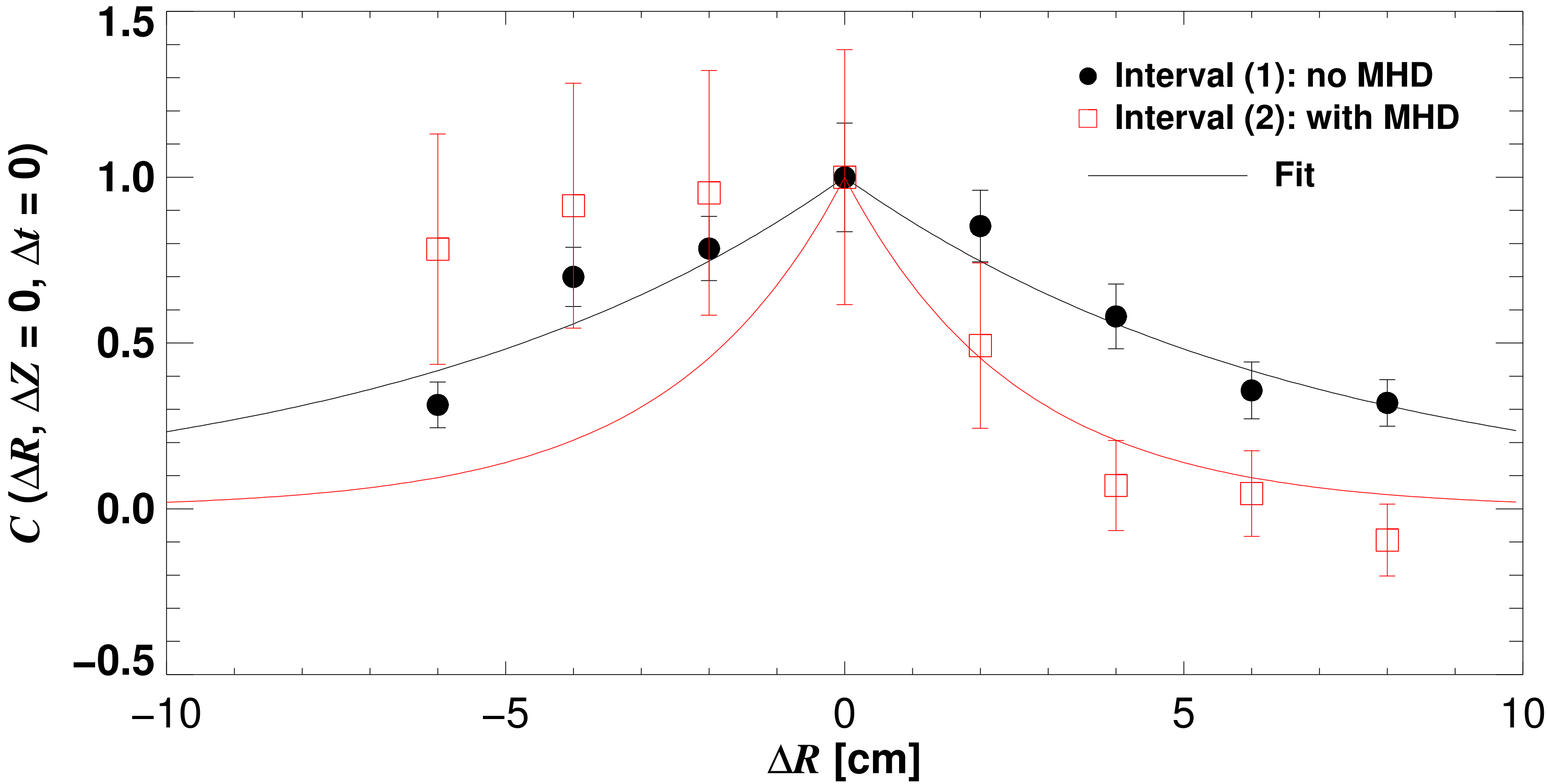
Note that $?_y = ?_z \cos\alpha$ (provided $?_{\parallel} >> ?_y \tan\alpha$)

Spectrogram & poloidal correlation function



Fitting function for $C(\Delta R = 0,\ \Delta Z,\ \Delta t = 0)$: $f_z(\Delta Z) = p_z + (1 - p_z)\cos\left[2\pi \frac{\Delta Z}{?_z}\right]\exp\left[-\frac{|\Delta Z|}{?_z}\right]$

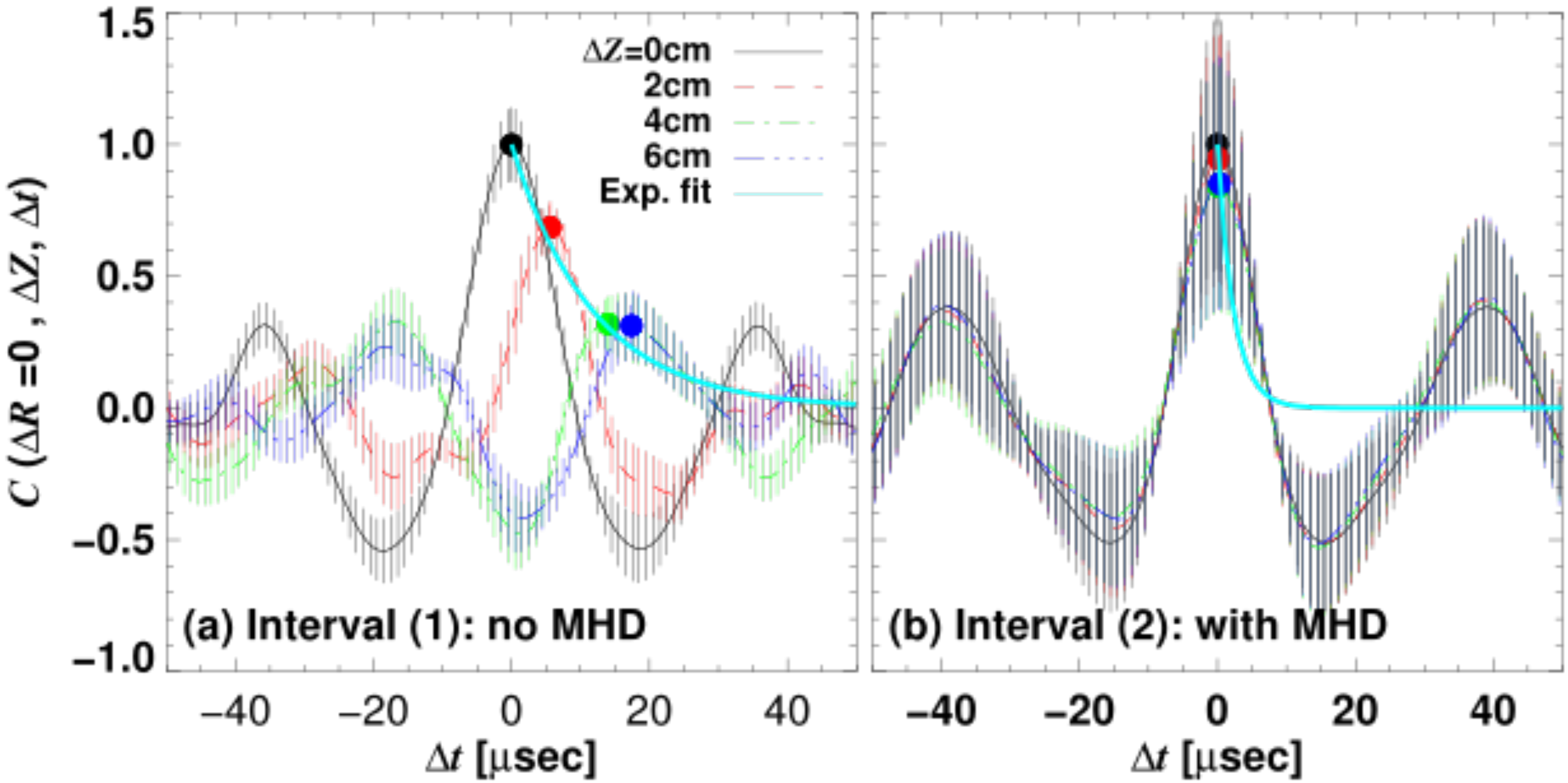
Radial correlation function



Fitting function for $C(\Delta R, \Delta Z = 0, \Delta t = 0)$: $f_R(\Delta R) = p_R + (1 - p_R)\exp\left[-\frac{|\Delta R|}{?_R}\right]$

Note that $?_x = ?_R$

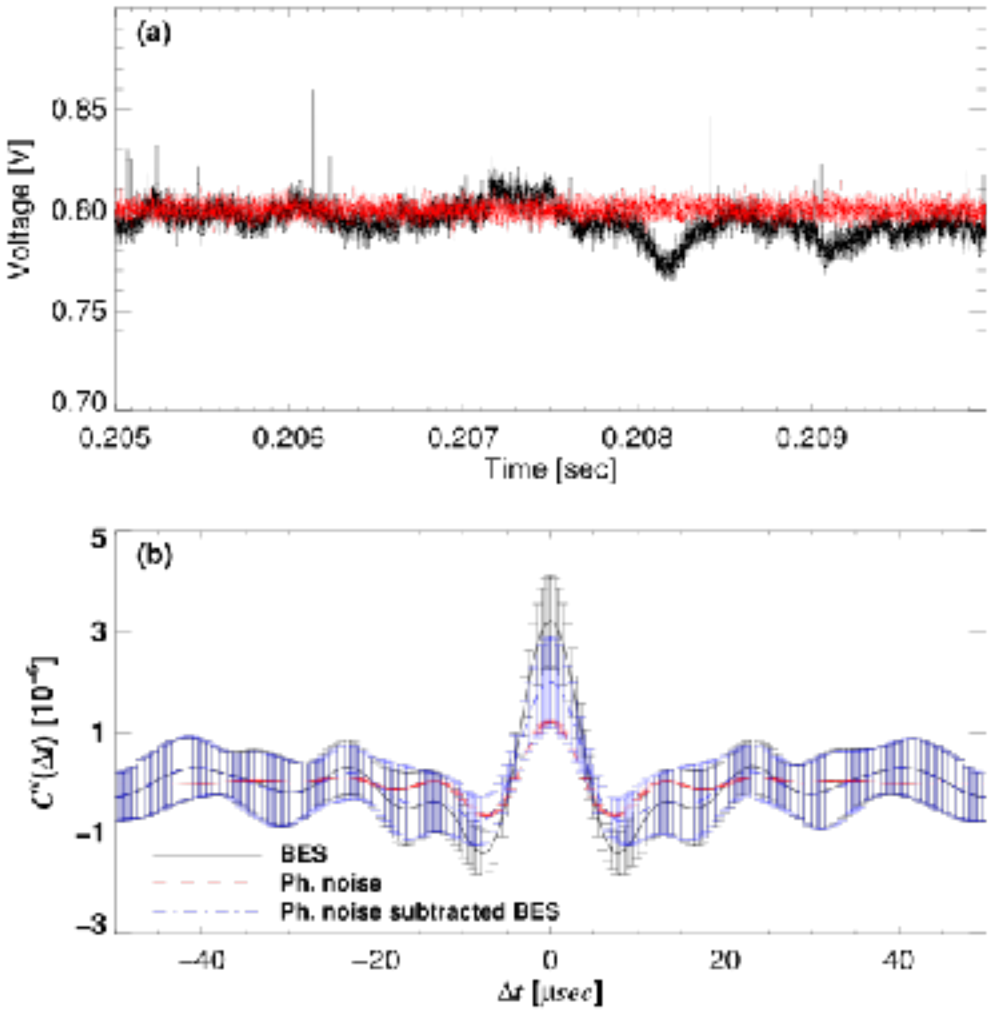
Temporal correlation function



Fitting function for $C_{peak}(\Delta R = 0,\ \Delta Z,\ \Delta t)$: $f_\tau(\Delta Z) = \exp\left[-\frac{\left|\tau_{peak}^{cc}(\Delta Z)\right|}{\tau_c}\right]$

Lagrangian approach!

Obtaining density fluctuation level **δn/n**



Photon noise is *measured* to remove it from the signal.

We discard data when the background level is significant. ('*Significant*' is a qualitative word. If you want to know how I do it, ask me at some other time as explaining it takes time.)

What about parallel correlation length?

$?_{\parallel}$

I make a bold (or could sound audacious to some of you) *assumption*, but an educated one, I believe.

I *conjecture* that "at the energy injection scale, $?_{\parallel}$ knows either *the system size* (in parallel direction) or *perpendicular shearing rate of mean flow*."

$$?_{\parallel} = \begin{cases} \Lambda \left(= \pi r \dfrac{B}{B_p} \right) & \text{if } \tau_{st}\gamma_E < 1 \\ \upsilon_{th,i}\tau_{sh} \left(= \dfrac{\upsilon_{th,i}}{\gamma_E} \right) & \text{if } \tau_{st}\gamma_E > 1 \end{cases}$$

Note: in conventional tokamak $\Lambda \sim \pi q R$

So, the physical quantities we have from experiments are

- We *measure*
  - From BES: $?_x$, $?_y$, τ_c, $\delta n/n$ ($\sim e\varphi/T_i$) and $?_{\parallel}$ from my conjecture.
  - From Thomson: n_e, T_e
  - From CXRS: U_Φ, T_i (assumed to equal the C^{6+} temperature and flow velocity.)
  - From MSE: magnetic pitch angle α
  - From pressure- and MSE-constrained EFIT: B-field information

- Using *39* double-null-diverted discharges.
  - With no pellet injection and no RMP (because I do not know what they do to turbulence.)
  - Every *5 ms*.
  - During the *MHD free periods* only (as the employed statistical techniques becomes less reliable). Aging, I do this 'quantitatively', not with my eyeballs.

Nonlinear time estimation

Nonlinear term in the equation: $\delta \overset{?}{u} \cdot \nabla \delta n$

$$\tau_{nl}^{-1} = \delta \overset{?}{u} \cdot \nabla = c \frac{\overset{?}{B} \times \nabla \varphi}{B^2} \cdot \nabla = \frac{1}{\boxed{?}_x \boxed{?}_y} \frac{\upsilon_{thi}^2}{\omega_{ci}} \frac{e\varphi}{T_i} = \frac{\upsilon_{thi} \rho_i}{\boxed{?}_x \boxed{?}_y} \frac{e\varphi}{T_i}$$

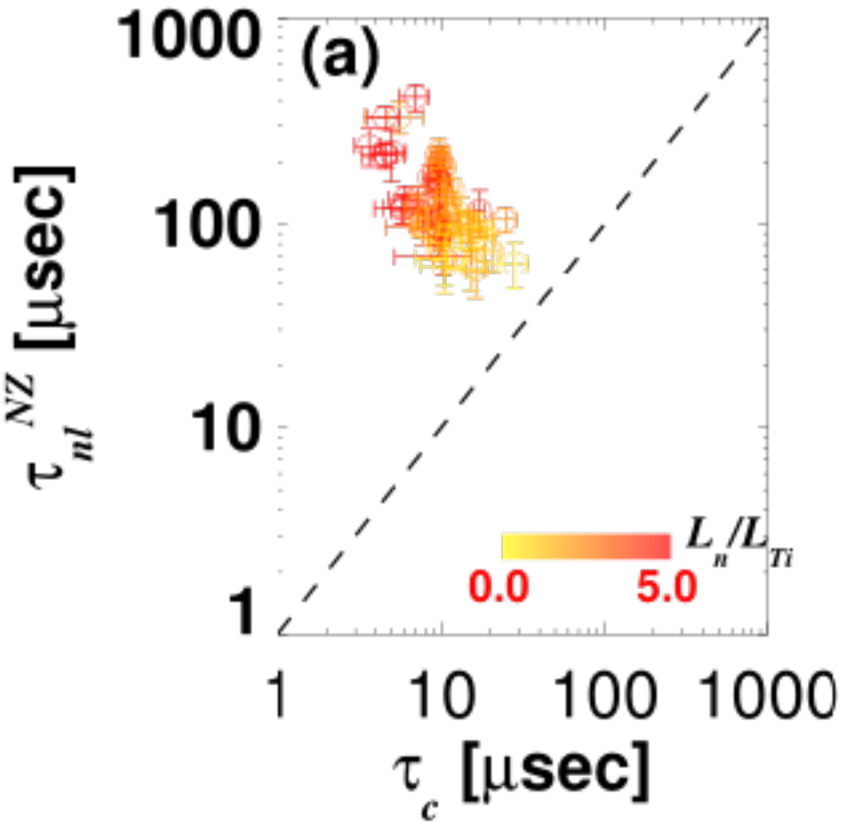
➢ Since we do not have measurements of $e\varphi/T_i$, assume $e\varphi/T_e = \delta n/n$.

1. This assumption is just for the 'magnitude only.' (surely, if they are in-phase, then we have no turbulent transport!)
2. Above assumption ignores trapped particles and, more importantly, also does *not* apply to *ion-scale zonal flows* (because $\delta n/n$ associated with zonal component φ is zero at the mid-plane).

➢ Thus, with the assumption we have

$$\left(\tau_{nl}^{NZ}\right)^{-1} = \frac{\upsilon_{thi} \rho_i}{\boxed{?}_x \boxed{?}_y} \frac{T_e}{T_i} \frac{\delta n}{n} \Longrightarrow \text{we can estimate this!}$$

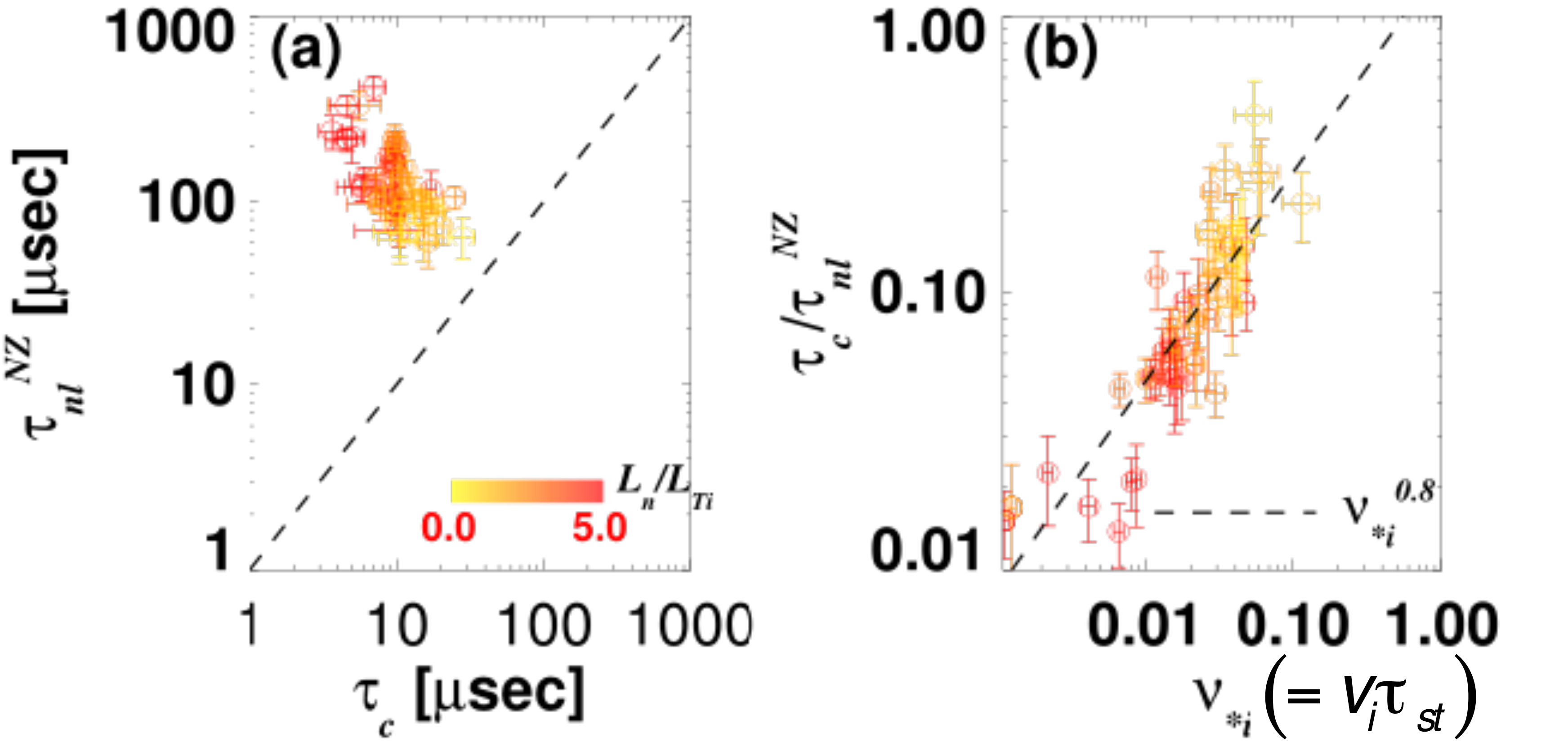
Non-zonal component of nonlinear time vs. turbulence correlation time



τ_{nl}^{NZ} is always larger than τ_c and observed to have inverse rather than direct correlation with τ_c.

- Since turbulence clearly cannot be saturated by linear physics alone, our estimation does not capture the correct nonlinear time of the system.
- So, we *conjecture* that the coupling to the zonal flows, invisible (directly) to BES, dominates over the nonlinear interaction between the drift-wave-like fluctuations represented by τ_{nl}^{NZ}.

Amplitude of zonal flow as a function of collisionality



$$\tau_c^{-1} \sim \frac{\upsilon_{thi}\rho_i}{?_x ?_y}\frac{e\varphi^{ZF}}{T_i}$$

$$\left(\tau_{nl}^{NZ}\right)^{-1} \sim \frac{\upsilon_{thi}\rho_i}{?_x ?_y}\frac{e\varphi^{NZ}}{T_i}$$

$$\Rightarrow \frac{\tau_{nl}^{NZ}}{\tau_c} = \frac{\varphi^{ZF}}{\varphi^{NZ}} \sim \nu_{*i}^{-0.8\pm0.1}$$

Estimate turbulent flux (strictly speaking for $\tau_{st}\gamma_E < 1$)

$$\ell_\parallel = \begin{cases} \Lambda & \text{if } \tau_{st}\gamma_E < 1 \\ \upsilon_{thi}\tau_{sh} & \text{if } \tau_{st}\gamma_E > 1 \end{cases} \qquad \frac{\ell_y}{\rho_i} \sim \frac{\ell_\parallel}{L_*};\quad \frac{\ell_x}{\rho_i} \sim \frac{\ell_\parallel}{R}$$

$$\tau_c^{-1} = \left(\frac{\upsilon_{thi}\rho_i}{\ell_x \ell_y}\frac{e\varphi^{ZF}}{T_i}\right) \sim \tau_*^{-1} = \left(\frac{\upsilon_{thi}\rho_i}{\ell_y L_*}\right);\quad \frac{\varphi^{ZF}}{\varphi^{NZ}} \sim \nu_{*i}^{-0.8\pm 0.1}$$

Turbulent diffusivity

$$\chi_{turb} \sim \delta u^2 \tau_c$$

where $\delta u \sim c\dfrac{\mathbf{B}\times\nabla\varphi^{NZ}}{B^2}$ (note: zonal component has no net radial transport.)

Turbulent heat flux

$$Q_i \sim nT_i\frac{\chi_i}{L_{Ti}} \text{ (very approximately!) } \propto \left(\frac{R}{L_{Ti}}\right)^3$$

Stiff transport!