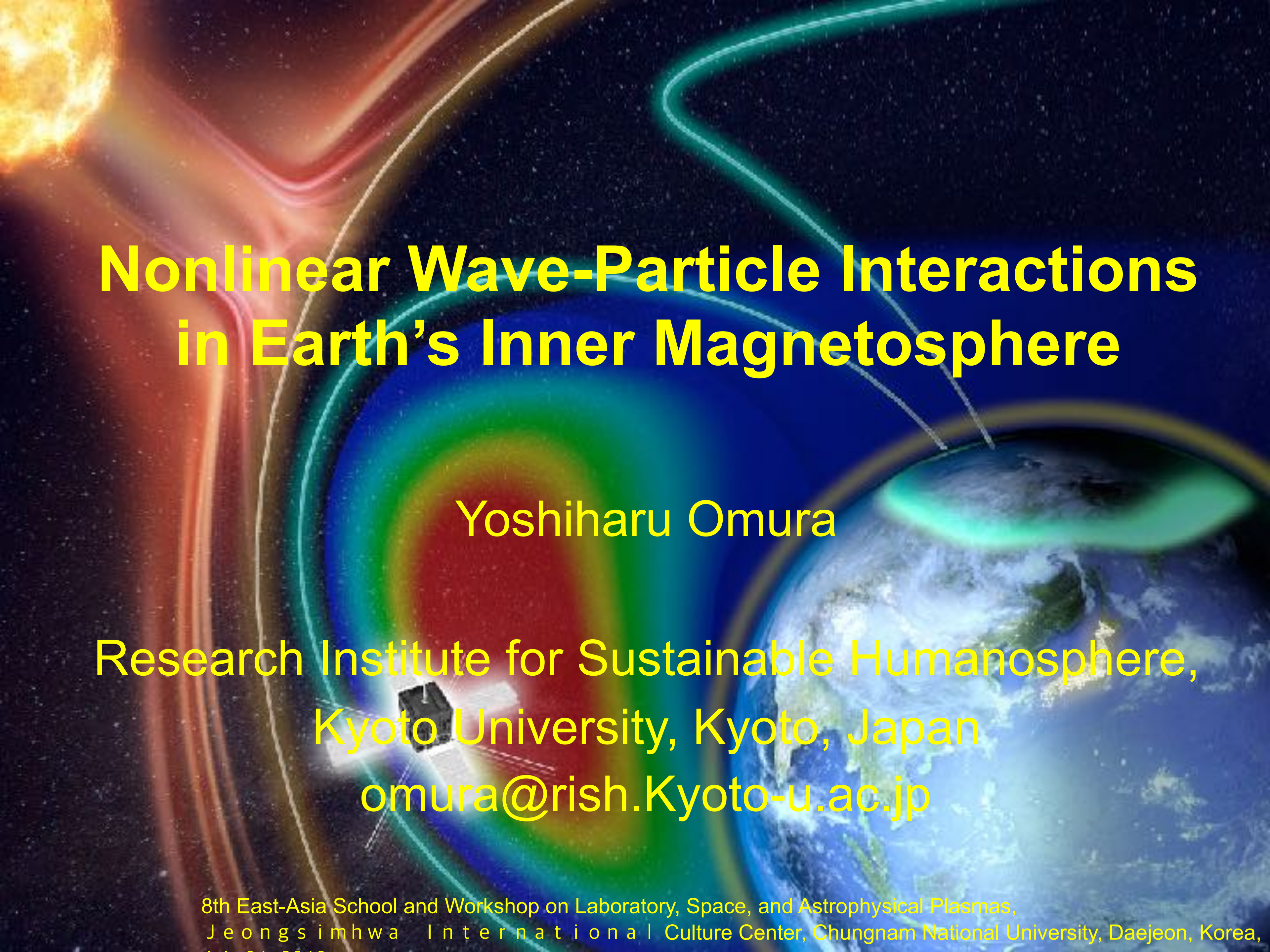

Nonlinear Wave-Particle Interactions in Earth's Inner Magnetosphere

Yoshiharu Omura

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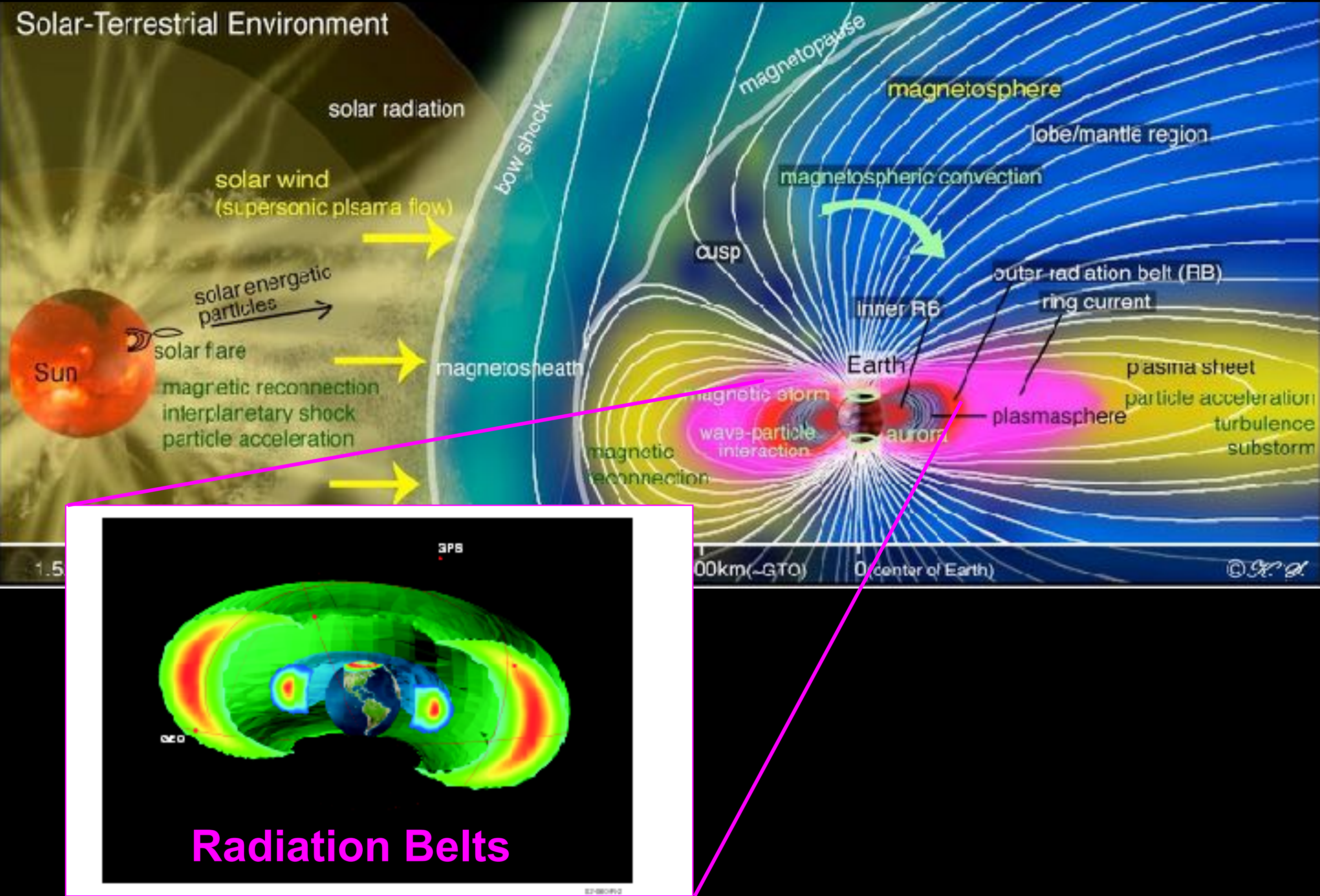
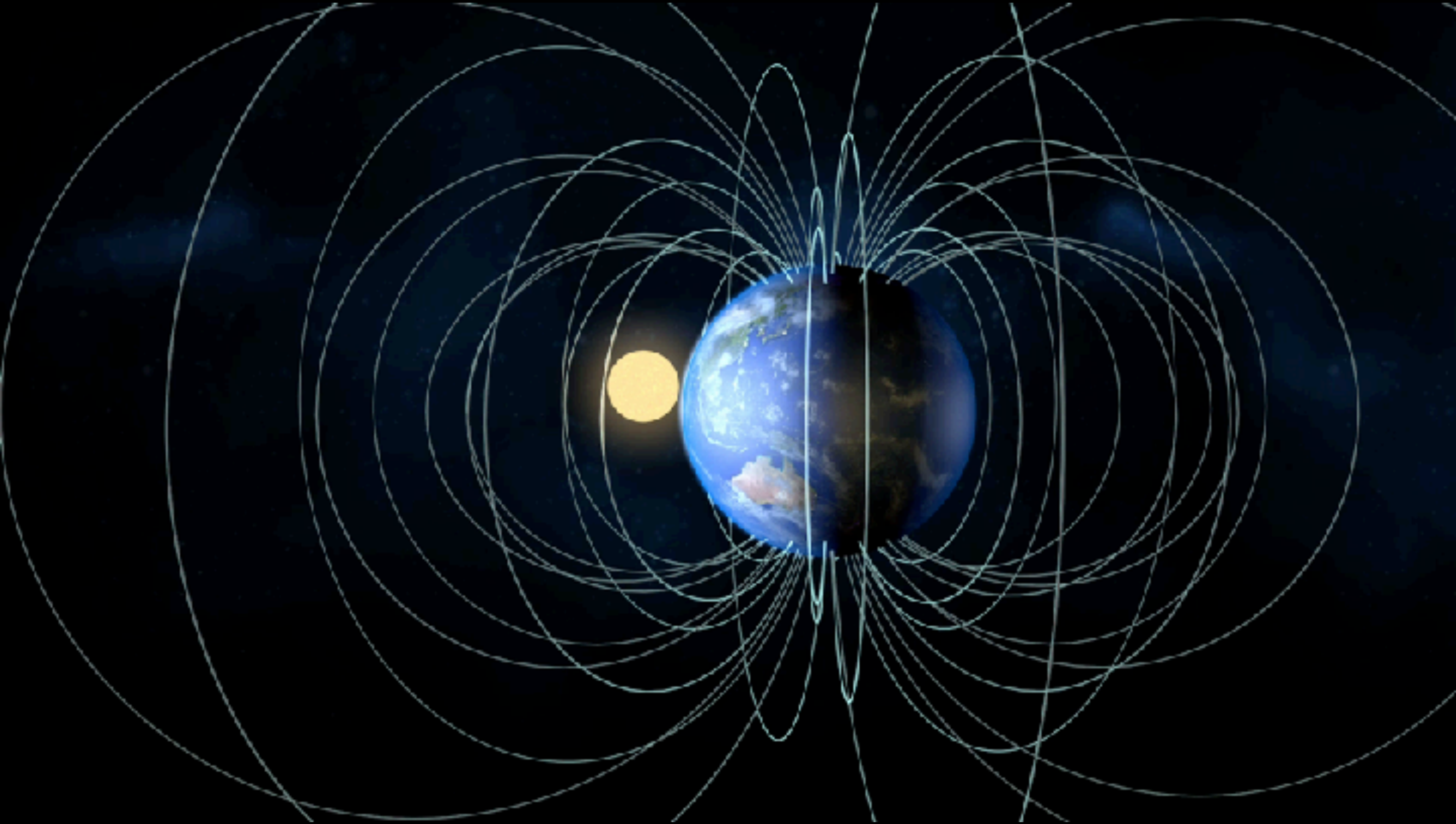
8th East-Asia School and Workshop on Laboratory, Space, and Astrophysical Plasmas,
Jeongsimhwa International Culture Center, Chungnam National University, Daejeon, Korea,

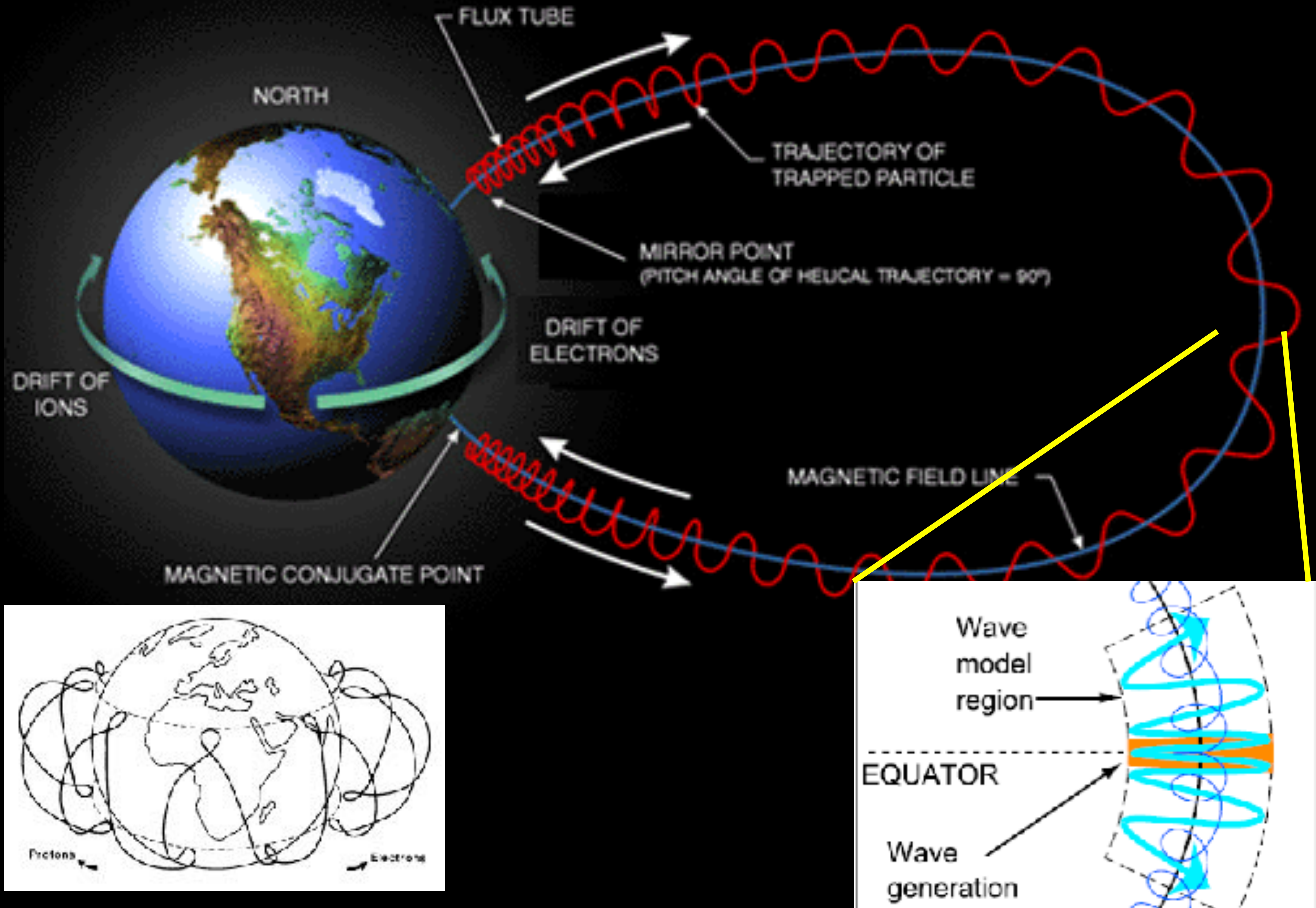
Geospace

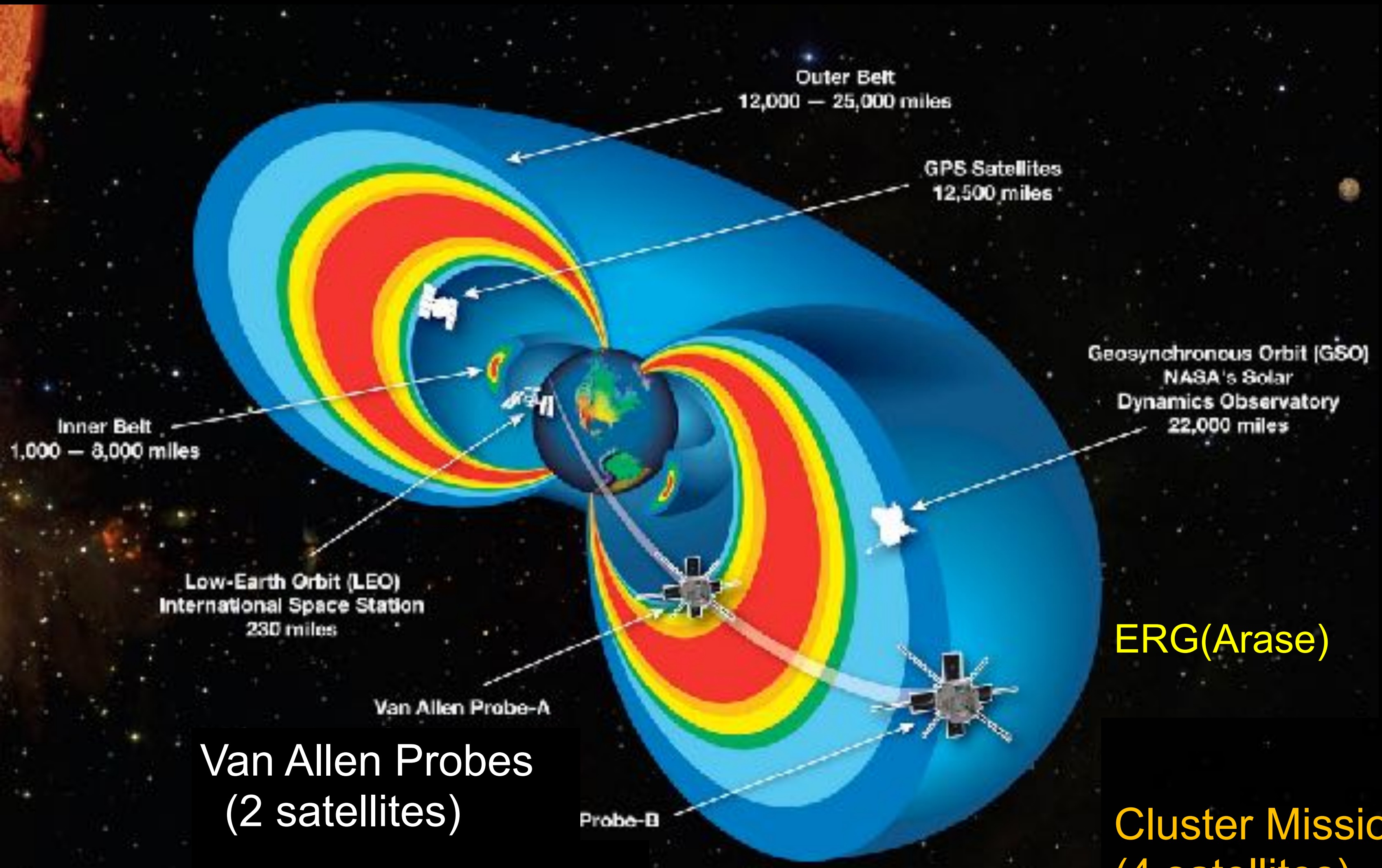


Radiation Belts



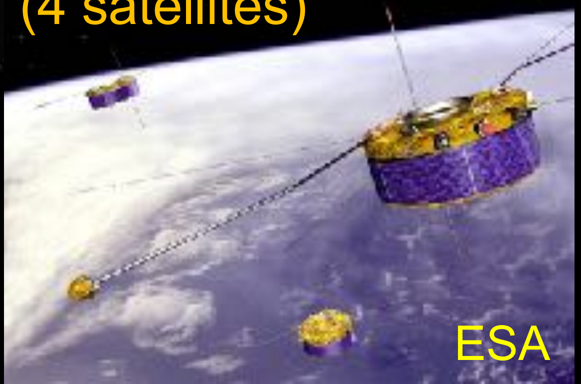
Motion of Charged Particles in Dipole Magnetic Field





ERG(Arase)

Van Allen Probes
(2 satellites)

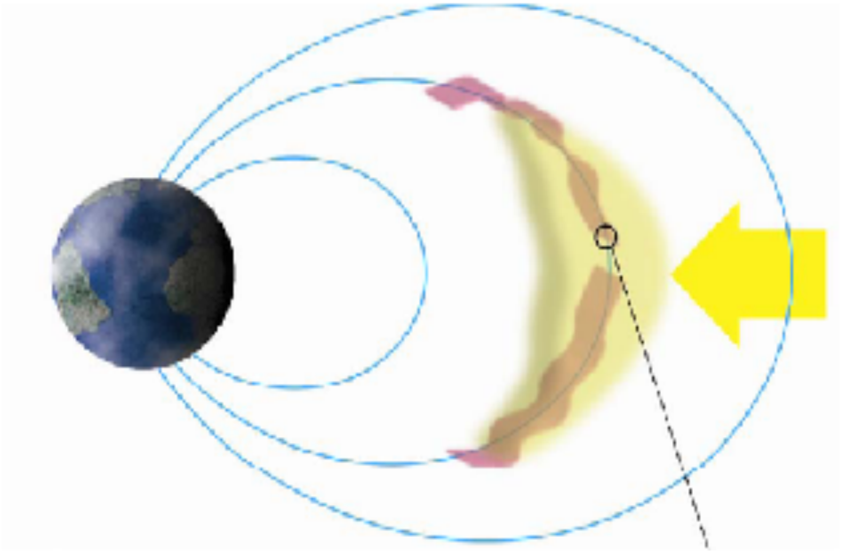


Cluster Mission
(4 satellites)

NASA

ESA

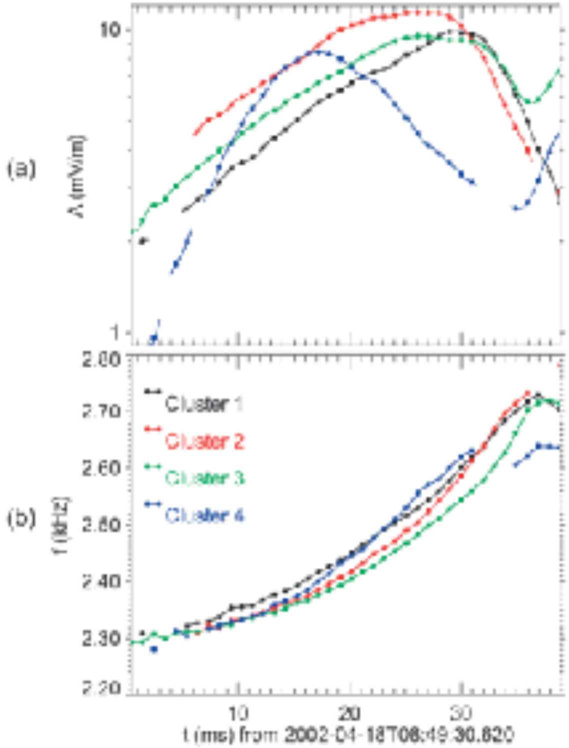
Chorus Emission due to Injection of Energetic Electrons

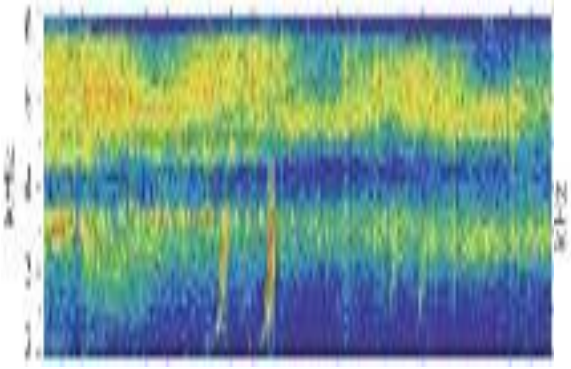


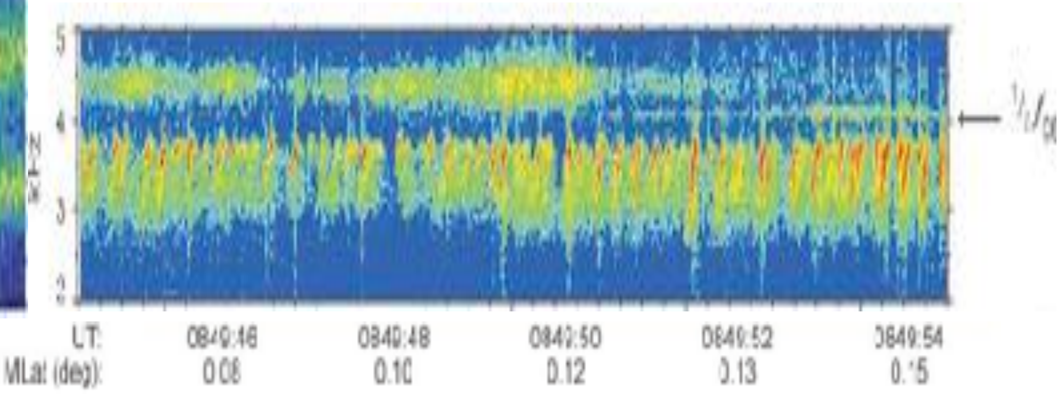
10 – 100 keV

$$\frac{T_\perp}{T_\parallel} > 1$$

Cluster observation







[*Santolik, Gurnett, Pickett, Parrot, Cornilleau-Wehrlin, JGR*, 2003]

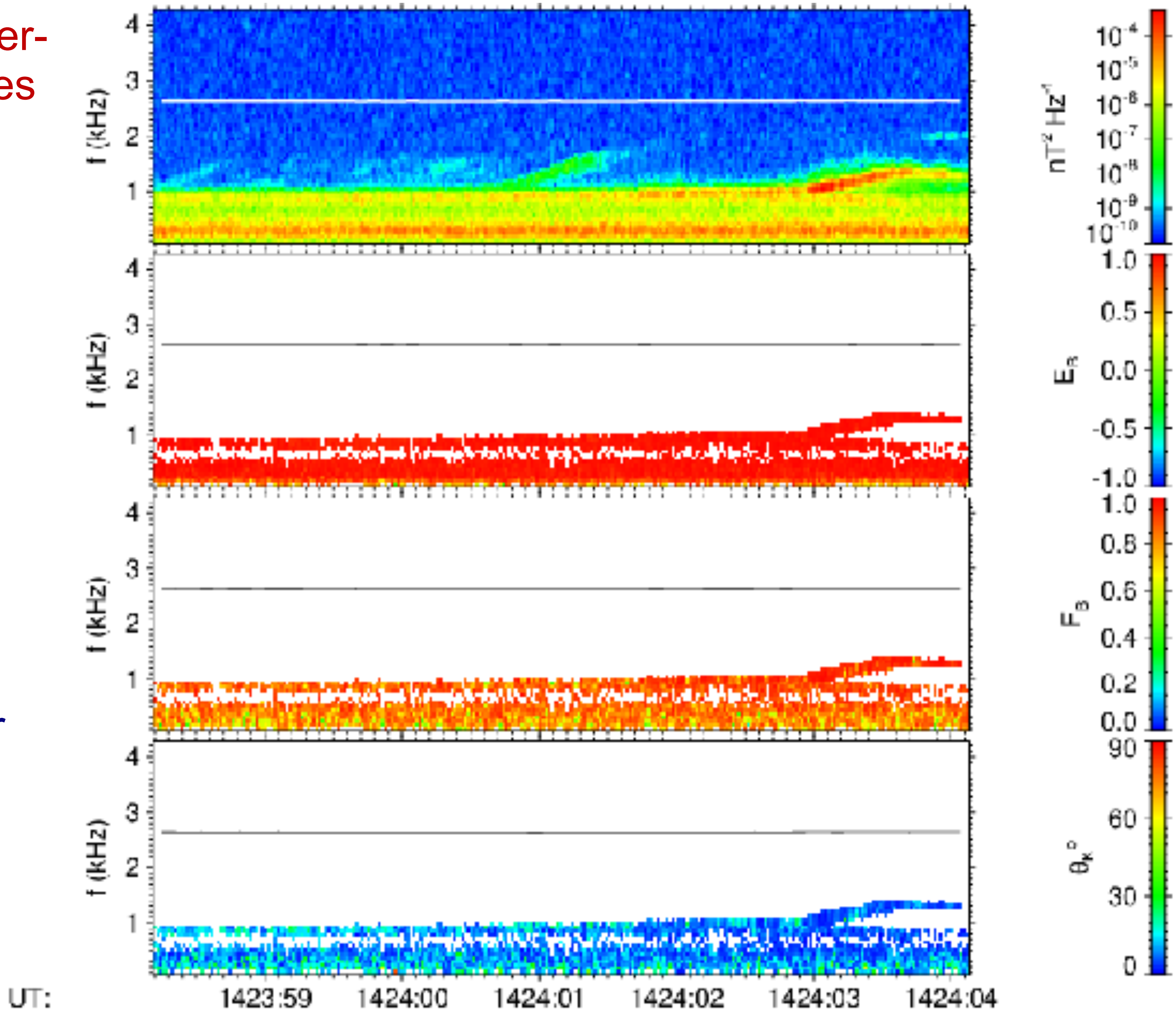
Wave Observation, Van Allen Probe B, 2 July 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



[*Kletzing*, 2014]

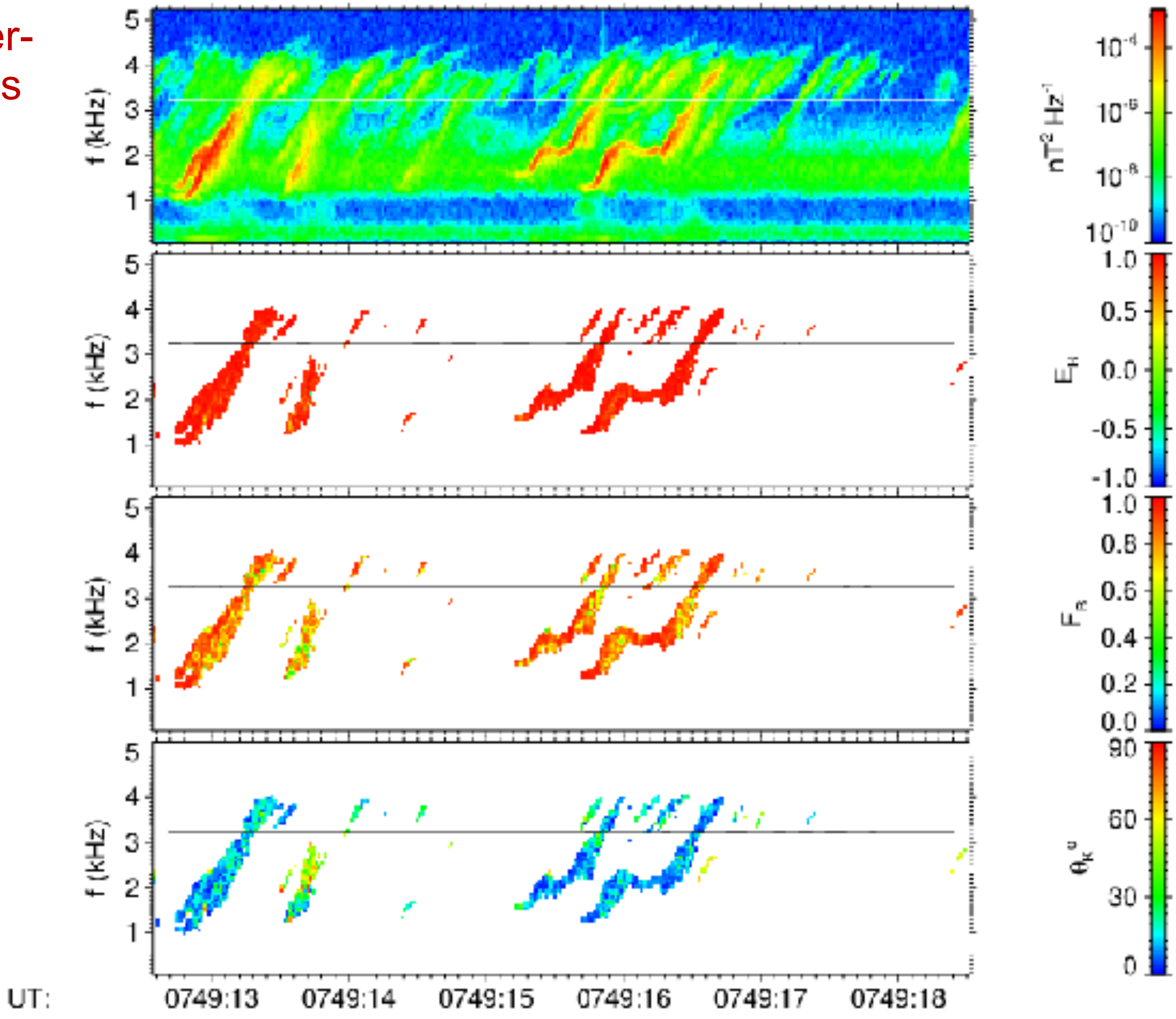
Wave Observation, Van Allen Probe A, 14 April 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



[*Kletzing*, 2014]

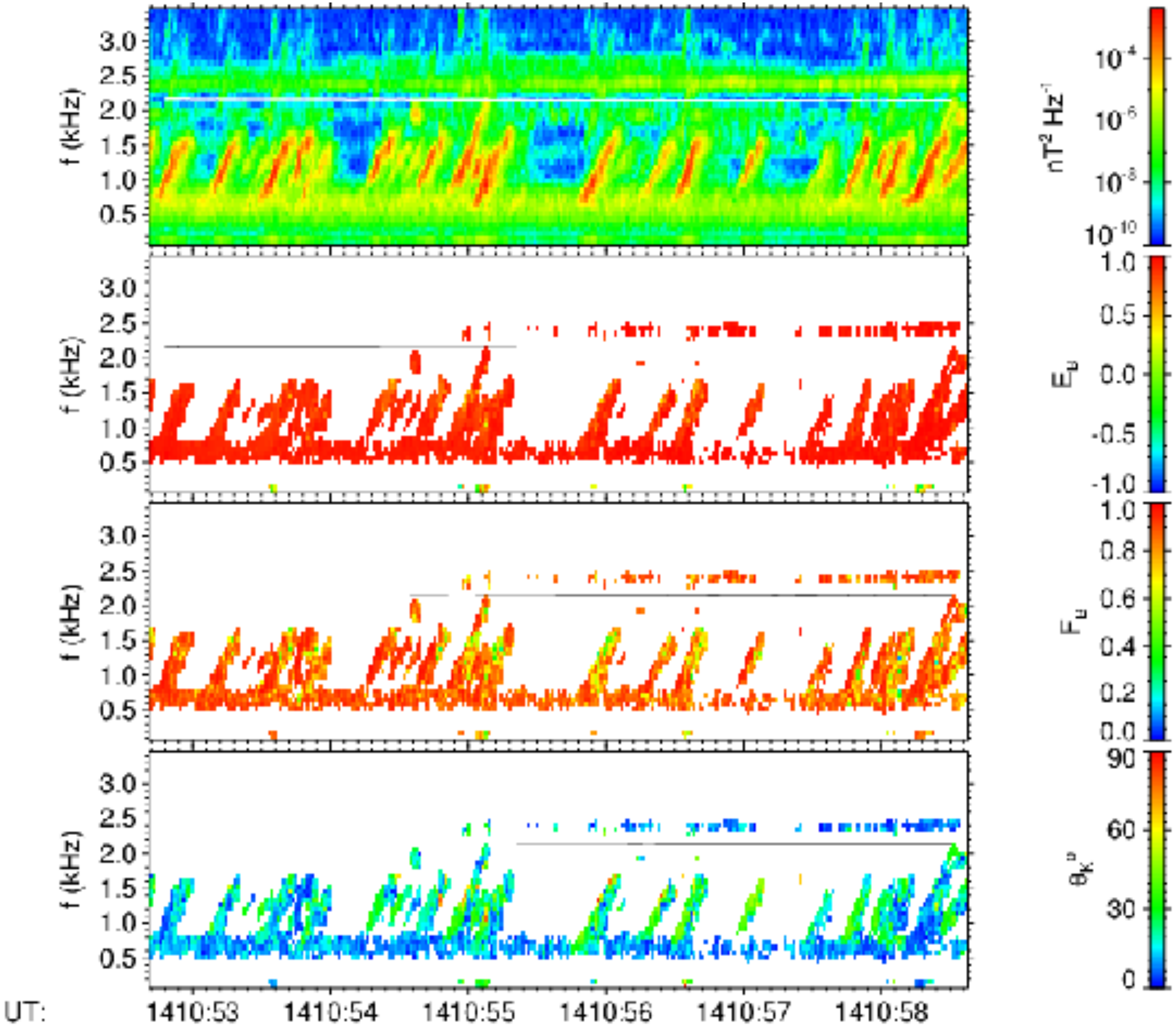
Wave Observation, Van Allen Probe A, 14 Nov 2012

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

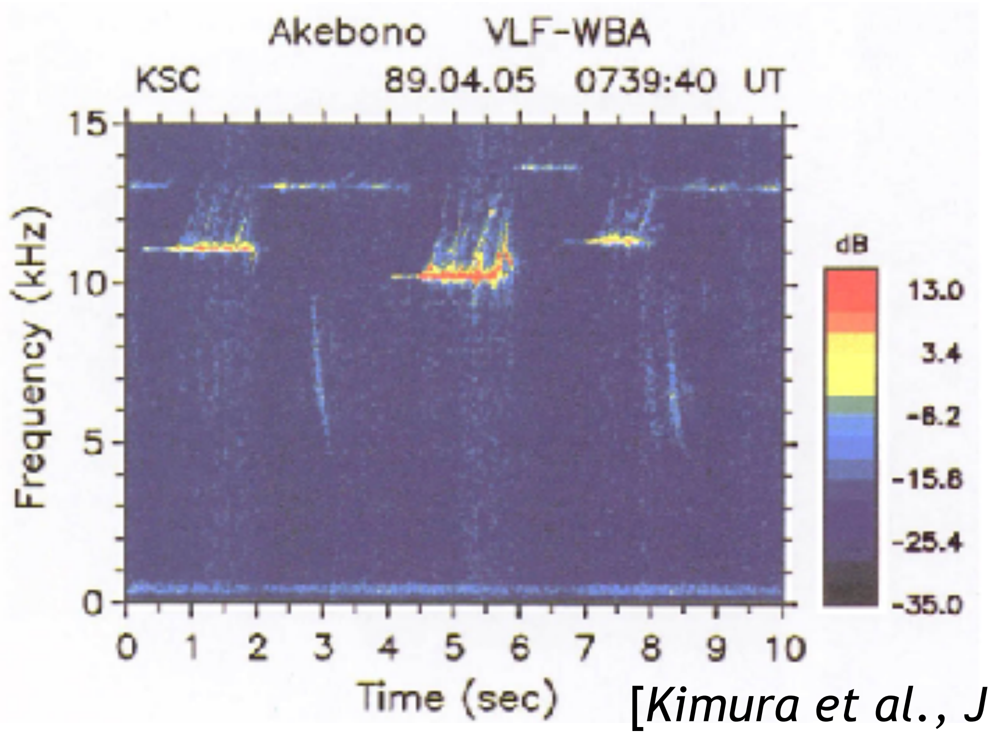
planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



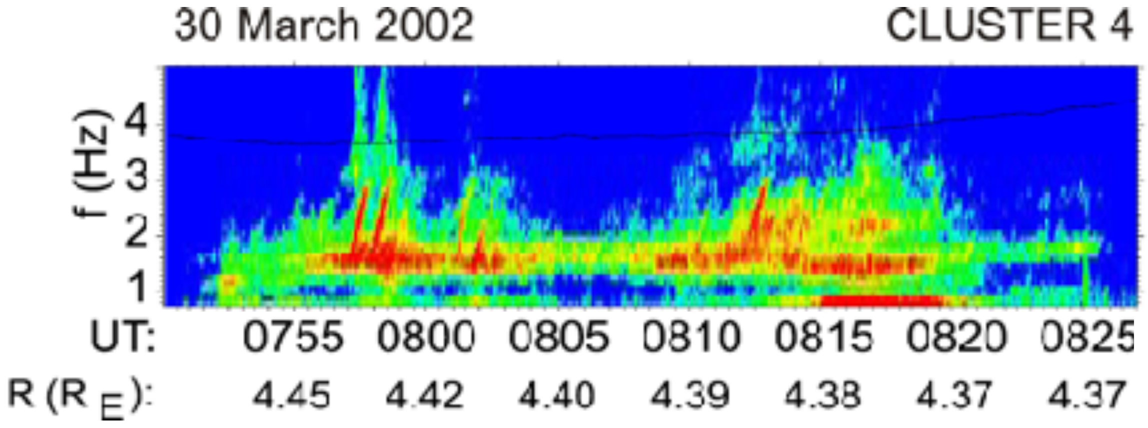
[*Santolik et al. GRL* 201

Whistler-mode Triggered Emissions

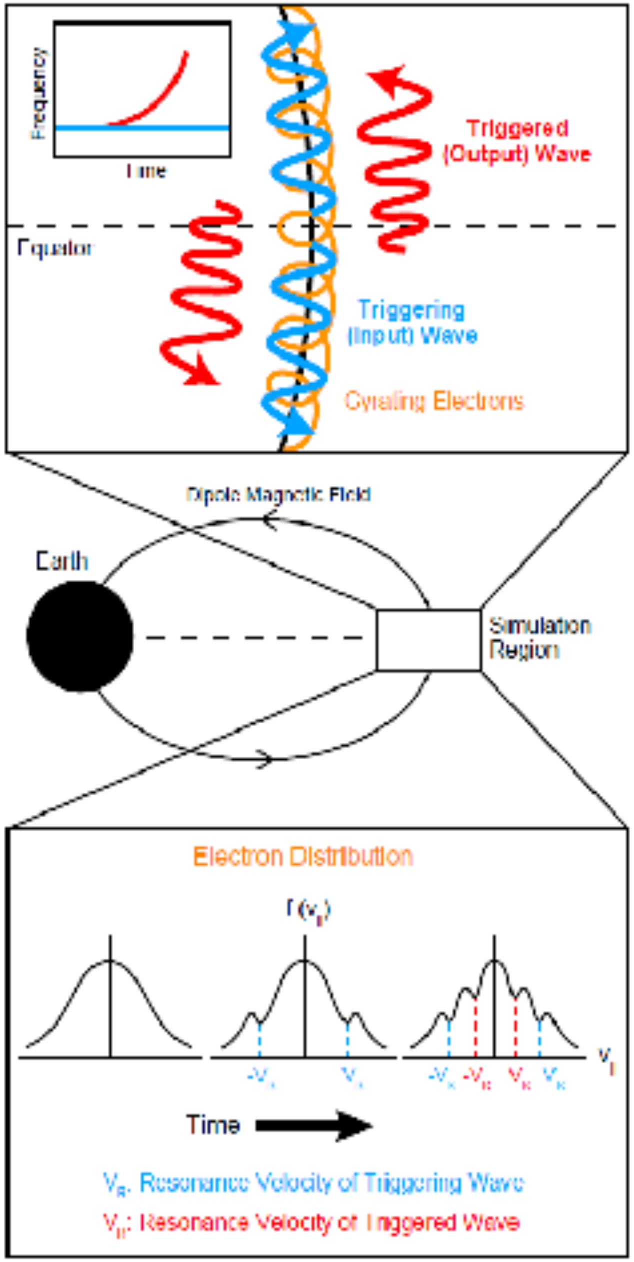


[*Kimura et al., JGG,* 1990]

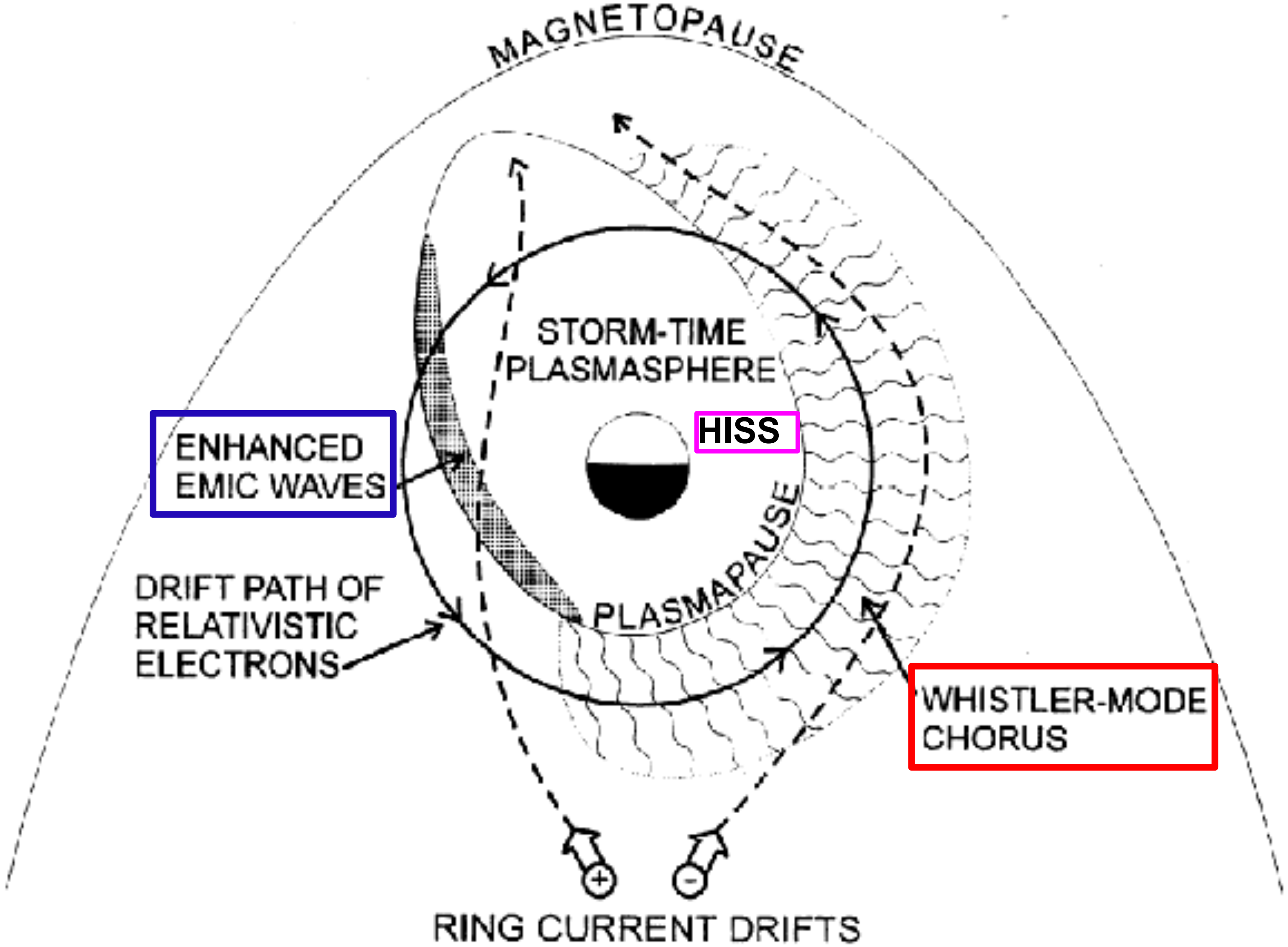
EMIC Triggered Emissions



[*Pickett et al., GRL,*



Wave-Particle Interactions in Earth's Inner Magnetosphere

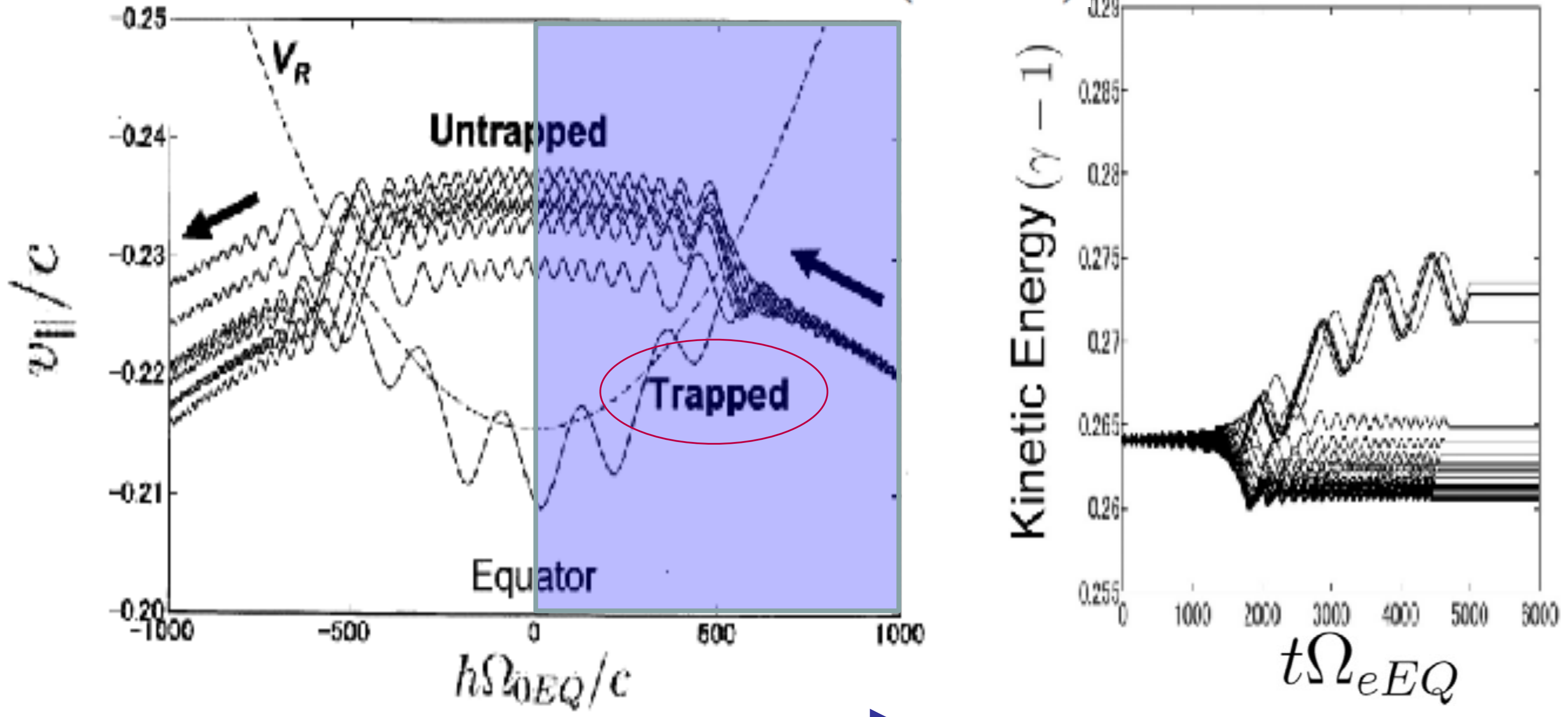


[*Summers et al.*, *JGR*, 1998]

Test Particle Simulation of Nonlinear Wave Trapping and Acceleration of Trapped Particles (10 – 100 keV)

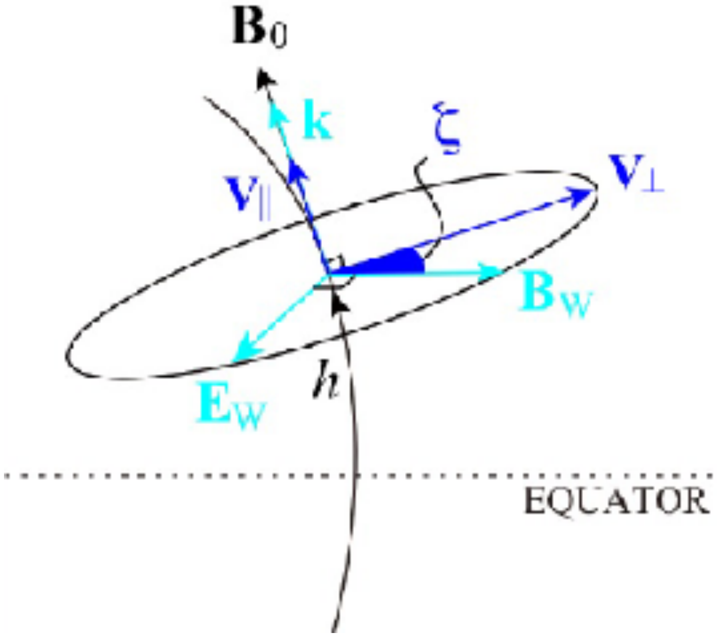
$$\boxed{\frac{\partial \omega}{\partial t} = 0}$$

$$\omega - k v_{\parallel} = \Omega_e(h) / \gamma \quad \Rightarrow \quad V_R = \frac{1}{k}\left(\omega - \frac{\Omega_e}{\gamma}\right)$$



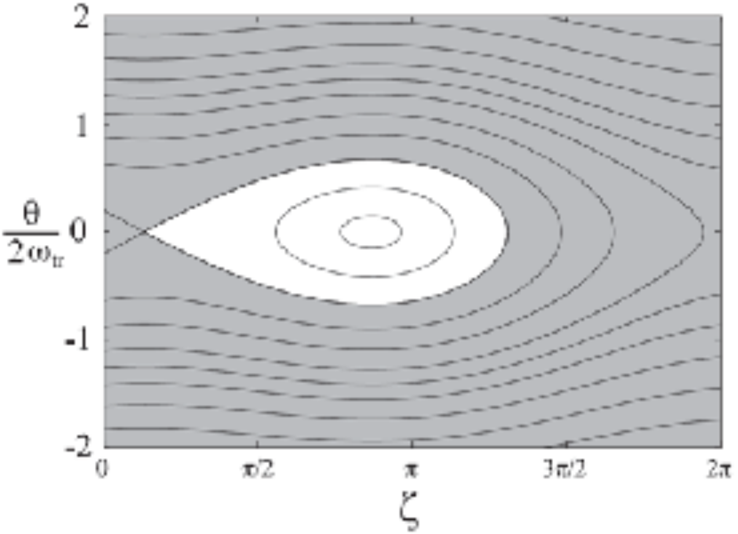
[*Omura and Summers, JGR*, 2006]

Nonlinear Dynamics of Resonant Electrons



$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \theta$$
$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_t^2(\sin\zeta + S)$$

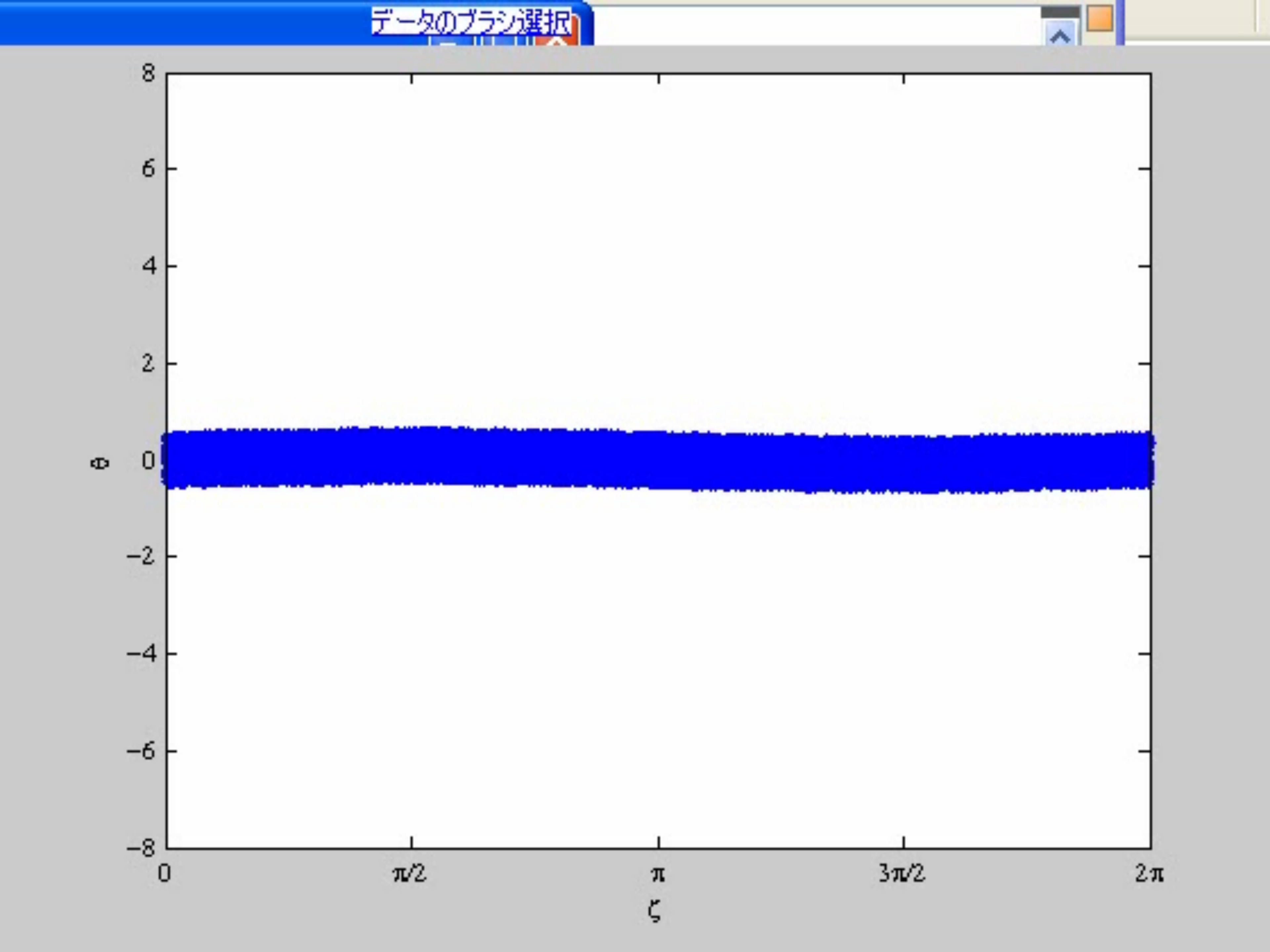
$$\theta = k(v_{\|} - V_R)$$

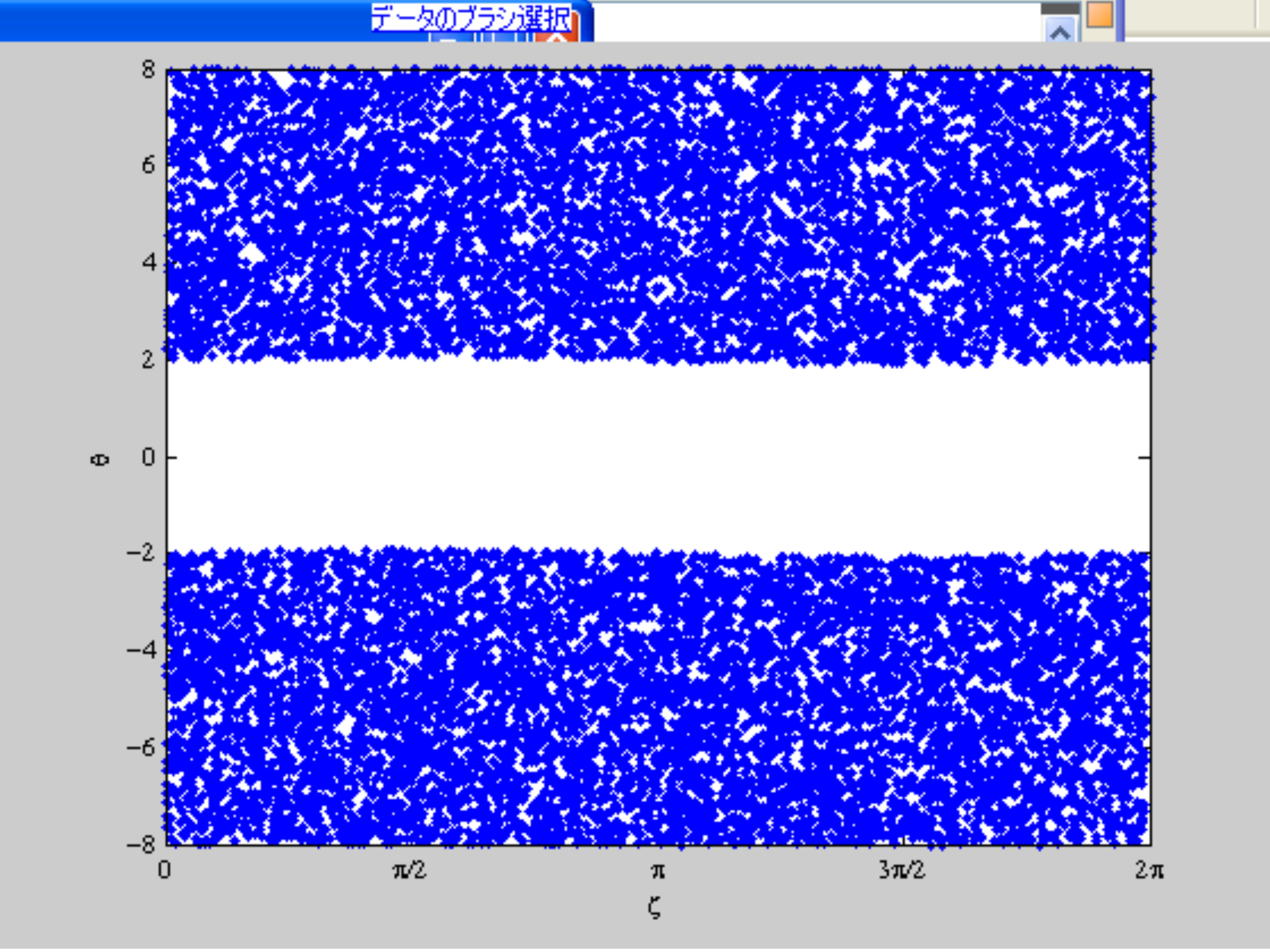


Inhomogeneity Factor

$$S = -\frac{1}{s_0\omega\boxed{\Omega_w}}\left(s_1\boxed{\frac{\partial\omega}{\partial t}} + cs_2\boxed{\frac{\partial\Omega_e}{\partial h}}\right)$$

[*Omura et al.*, *JGR*, 2008]


データのブラシ選択
8
6
4
2
0
−2
−4
−6
−8
θ
0
π/2
π
3π/2
2π
ζ


データのブラシ選択
8
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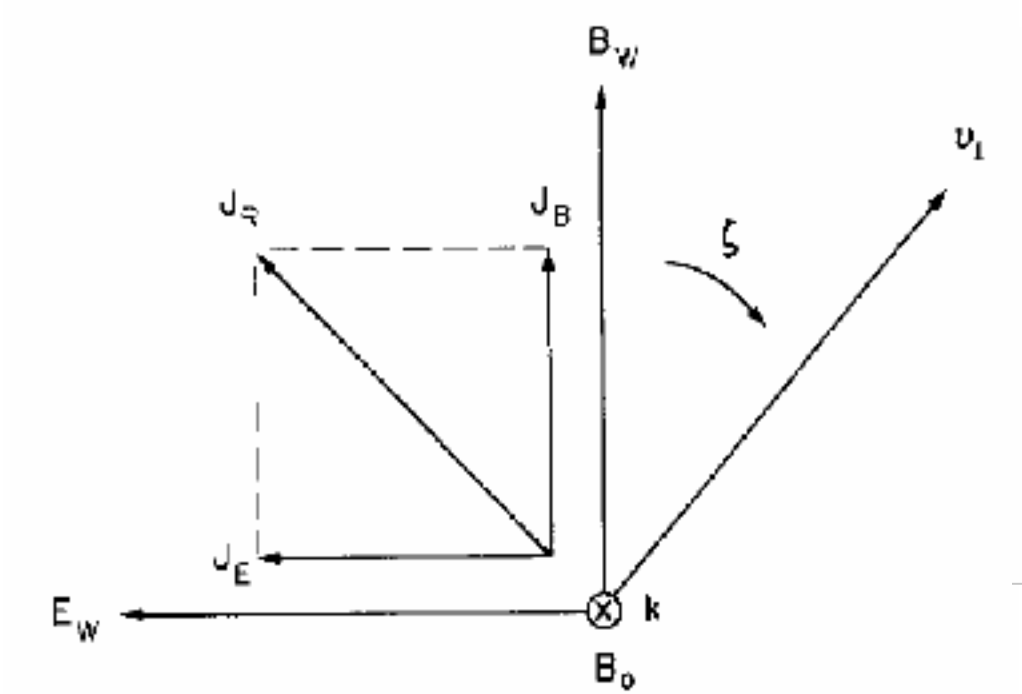
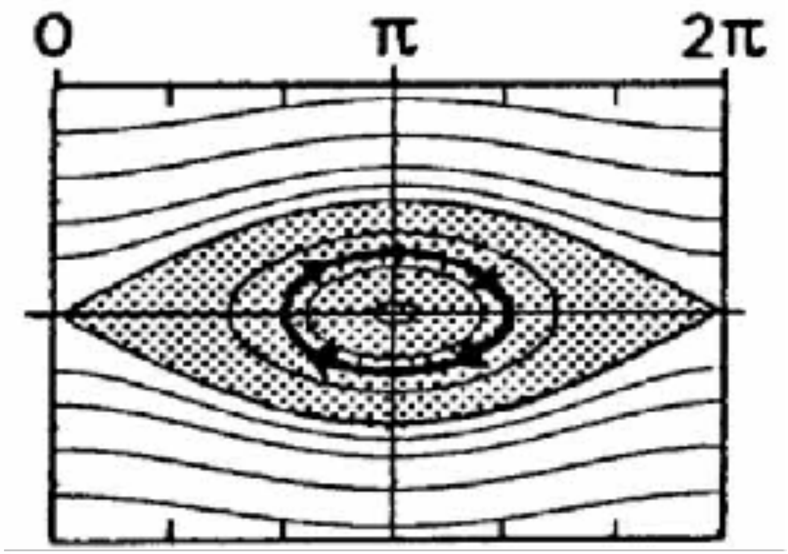
π
3π/2
2π
ζ

Wave Equations

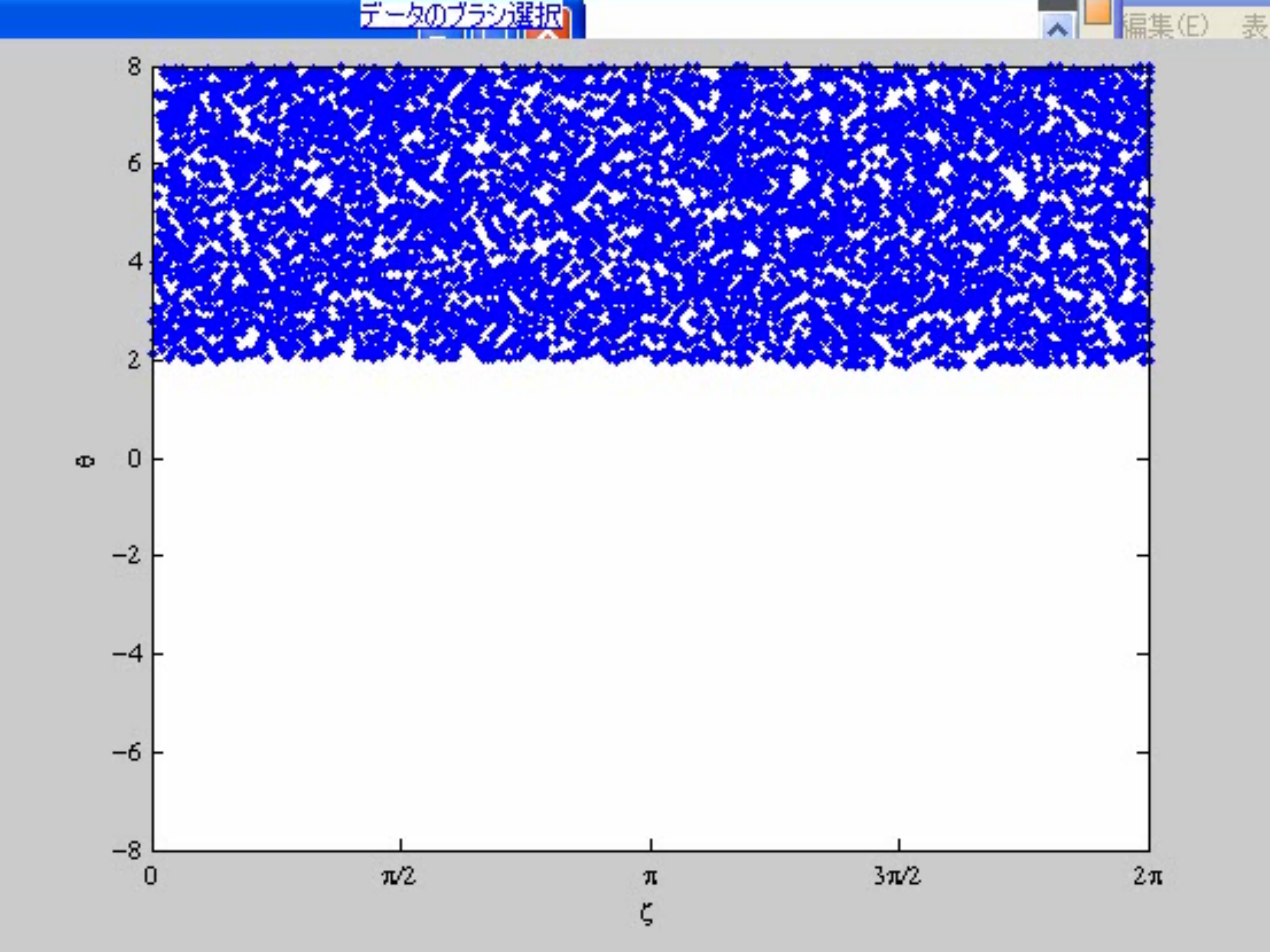
$$\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E$$

$$c^2k^2 - \omega^2 - \frac{\omega\omega_{pe}^2}{\Omega_e - \omega} = \mu_0 c^2 k \frac{J_B}{B_w}$$





$J_E > 0$ $J_E < 0$


データのブラシ選択
編集(E)
表
8
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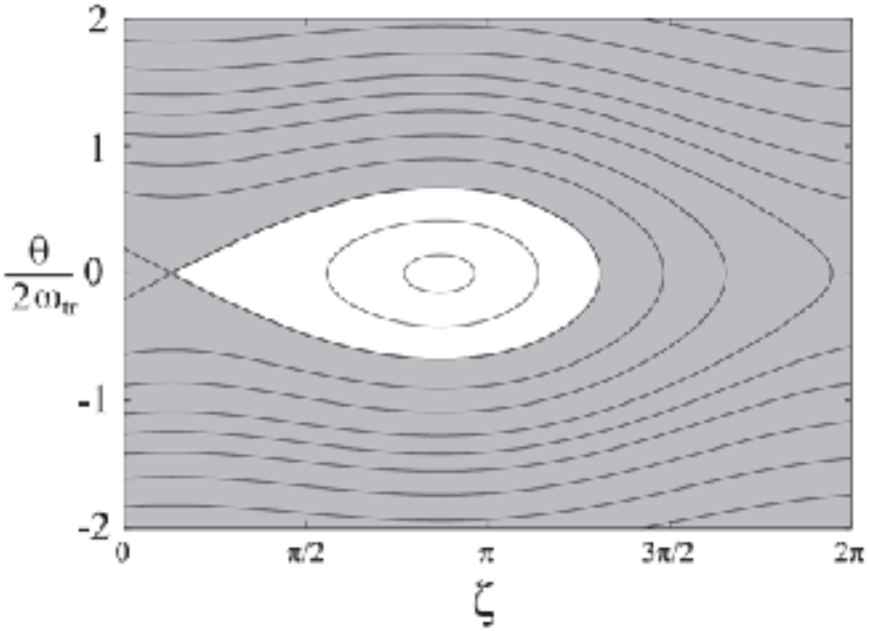
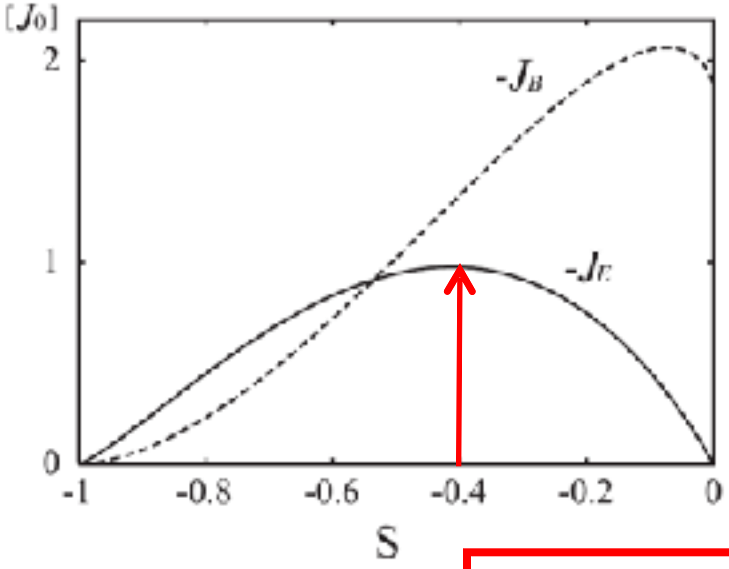
−8
Φ
0
π/2
π
3π/2
2π
ζ

Nonlinear Wave Growth due to Formation of Electromagnetic Electron Hole

$$\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E$$

$$c^2 k^2 - \omega^2 - \frac{\omega \omega_{pe}^2}{\Omega_e - \omega} = \mu_0 c^2 k \frac{J_B}{B_w}$$





$$S = -\frac{1}{s_0 \omega \Omega_w}\left(s_1 \frac{\partial \omega}{\partial t} + c s_2 \cancel{\frac{\partial \Omega_e}{\partial h}}\right)$$

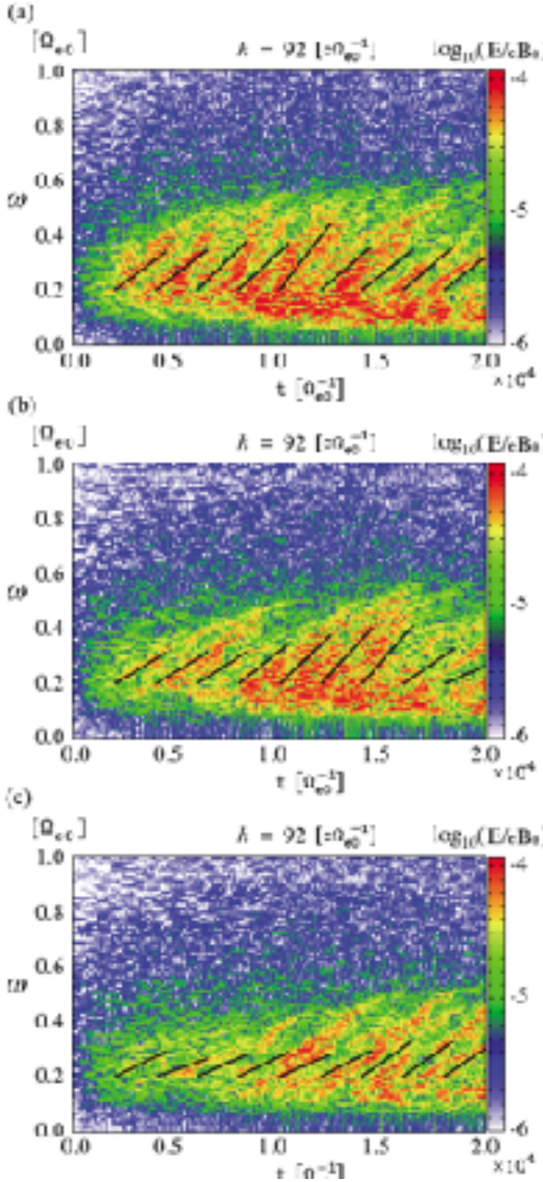
Maximum -J_E

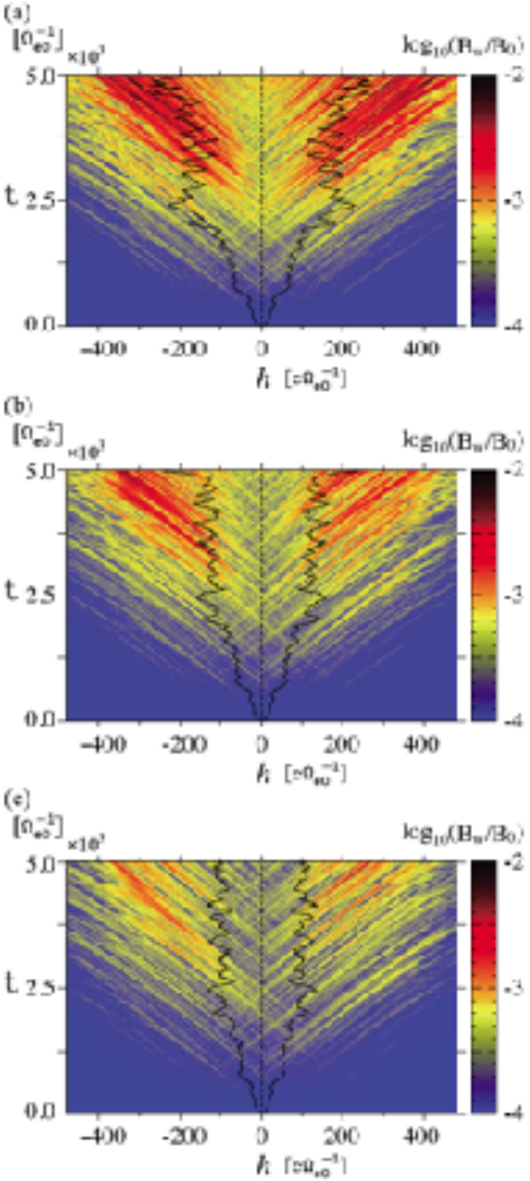
$$\boxed{S_{EQ} = -0.4}$$

$$\boxed{\frac{\partial \omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1} \Omega_w}$$

[*Omura et al.*, *JGR*, 2008]

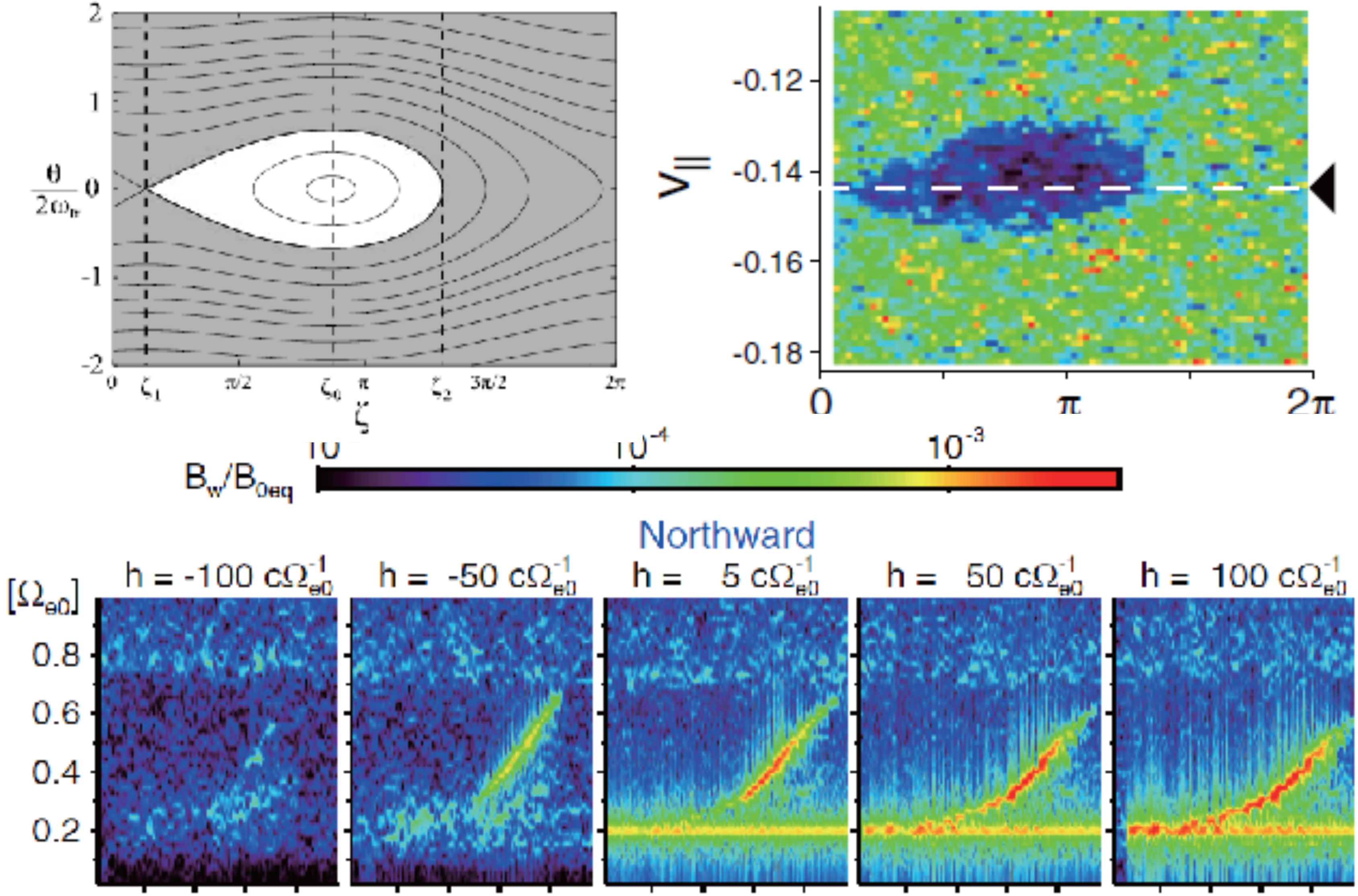
Frequency Sweep Rate Dependence on Wave Amplitude at Equator, which is controlled by Energetic Electron Density





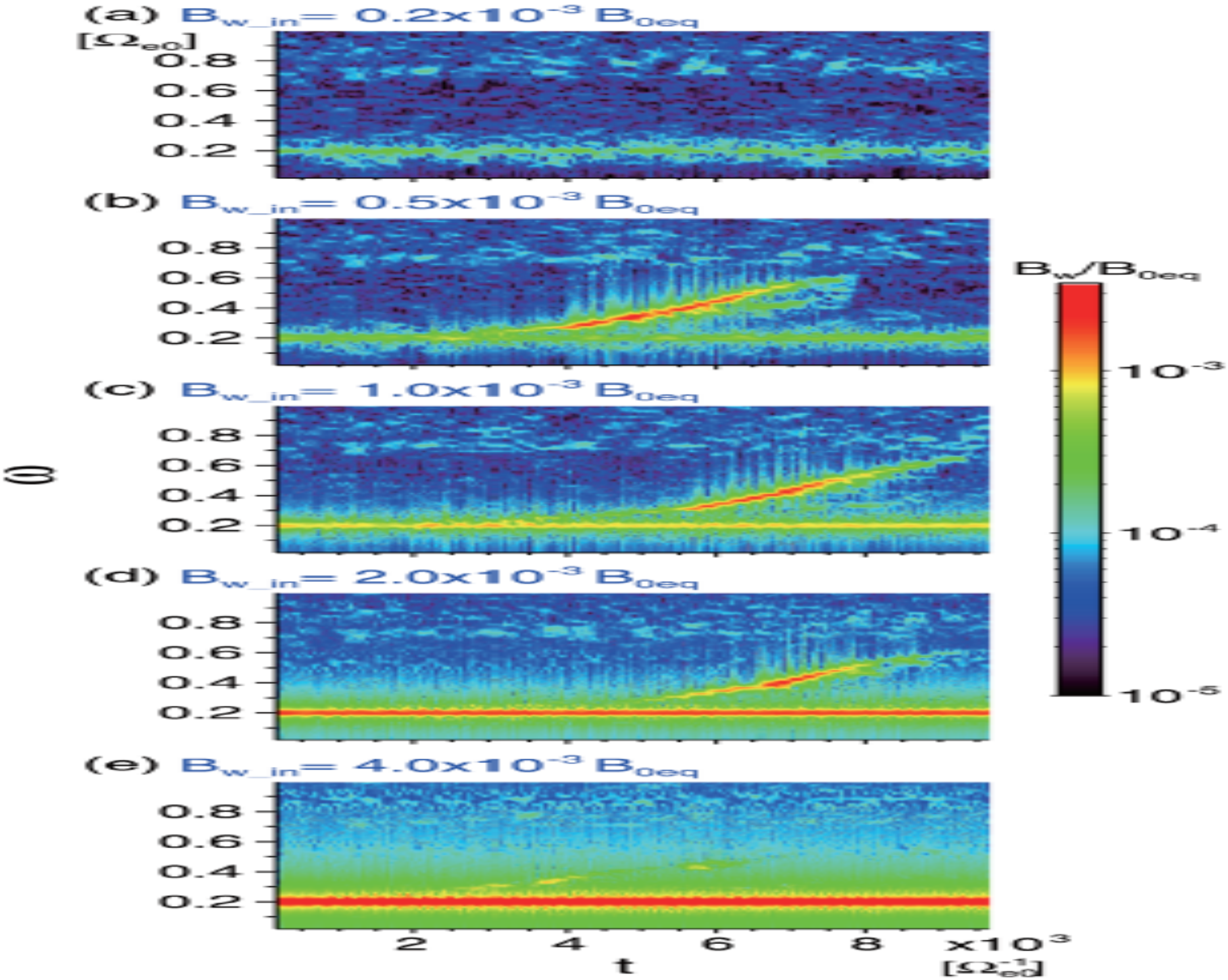
[*Katoh and Omura*, 2011]

Electron Hole for Nonlinear Wave Growth



[*Hikishima and Omura, JGR,* 2012]

Rising-Tone Emissions Triggered by Waves with Different Amplitudes



[*Hikishima and Omura*, *JGR* 2012]

Linear Dispersion Relation

$$c^2k^2 - \omega_0^2 - \frac{\omega_0\omega_{pe}^2}{\Omega_e - \omega_0} = 0$$

Nonlinear Dispersion Relation

$$c^2k^2 - \omega^2 - \frac{\omega\omega_{pe}^2}{\Omega_e - \omega} = \mu_0 c^2 k \frac{J_B}{B_w}$$

Assuming $\omega_1 << \omega_0$ where $\omega = \omega_0 + \omega_1$

Nonlinear Frequency Shift

$$\omega_1 = -\frac{\mu_0 V_g}{2}\frac{J_B}{B_w}$$ (> 0 : Electron Hole)

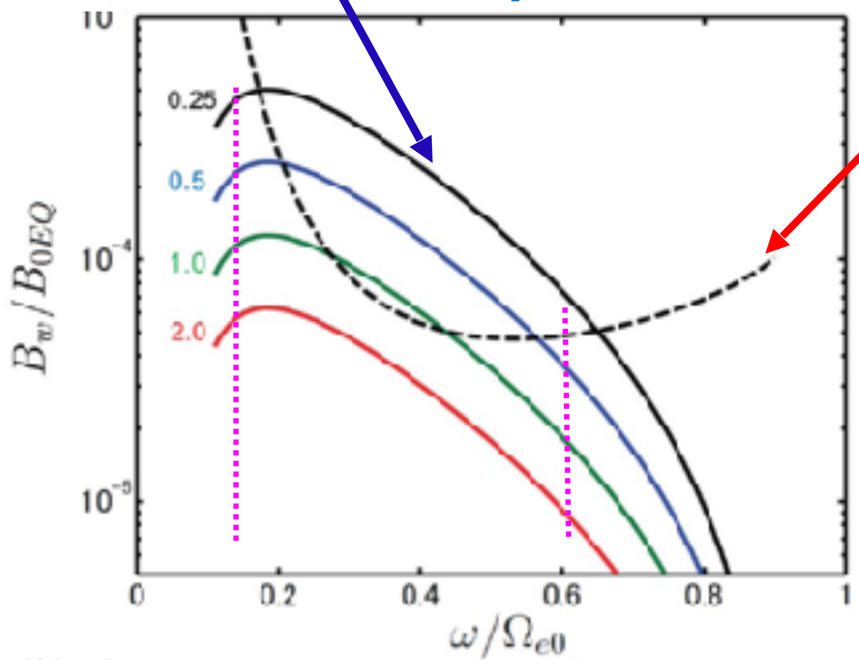
Optimum Wave Growth Condition

$$\boxed{\frac{\omega_1}{T_N} \sim \frac{\partial\omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1}\Omega_w}$$

Nonlinear Transition Time

$$\boxed{\tau = T_N/T_{tr}} \qquad T_{tr} = \frac{2\pi}{\omega_{tr}} = \frac{2\pi}{\delta}\left(\frac{m_0\gamma}{kV_{\perp 0}eB_w}\right)^{1/2}$$

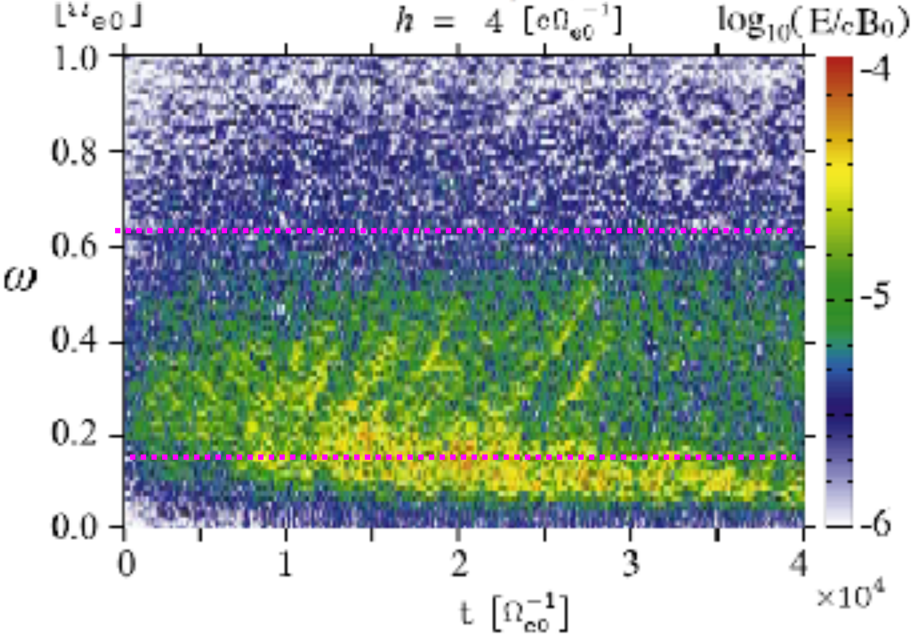
Optimum Wave Amplitude and Threshold Amplitude

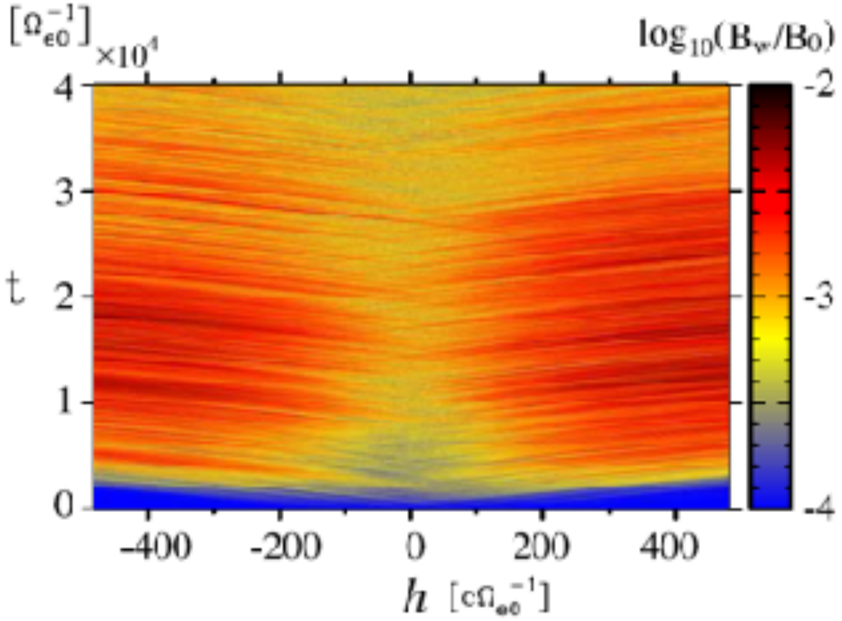


Electron Hybrid Simulation
[*Katoh and Omura, 2007*]

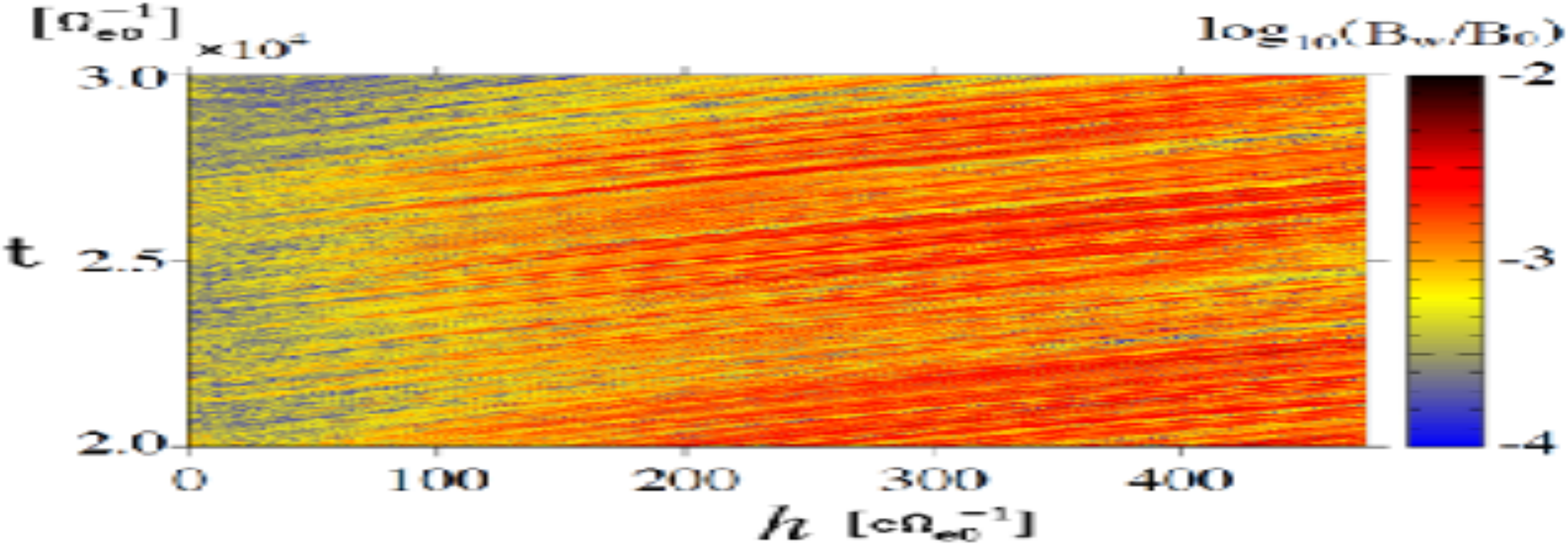
$$\tau = T_N/T_{tr}$$

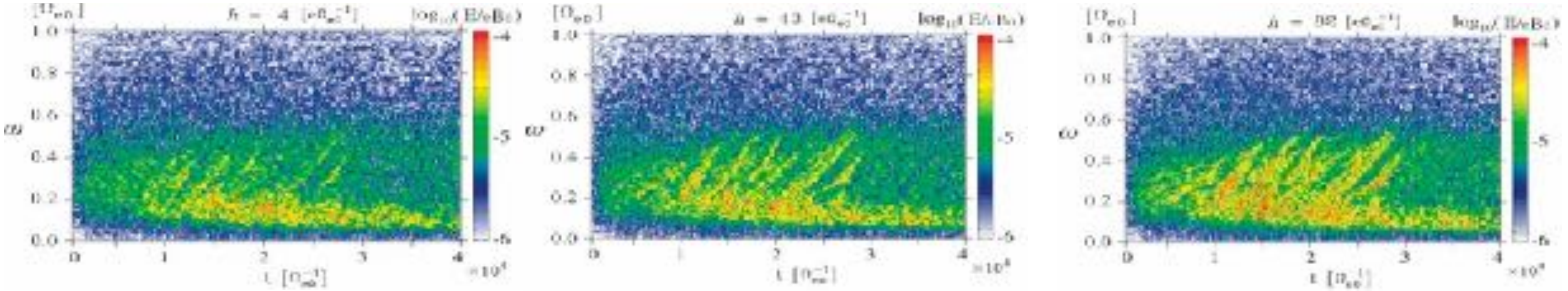
$$\tau = 0.25 \sim 0.5$$

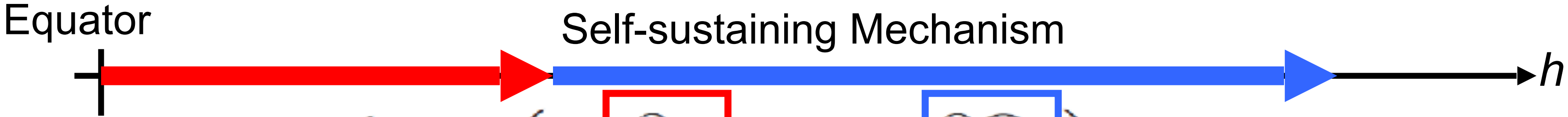




Nonlinear Wave Growth through Propagation : Convective Instability



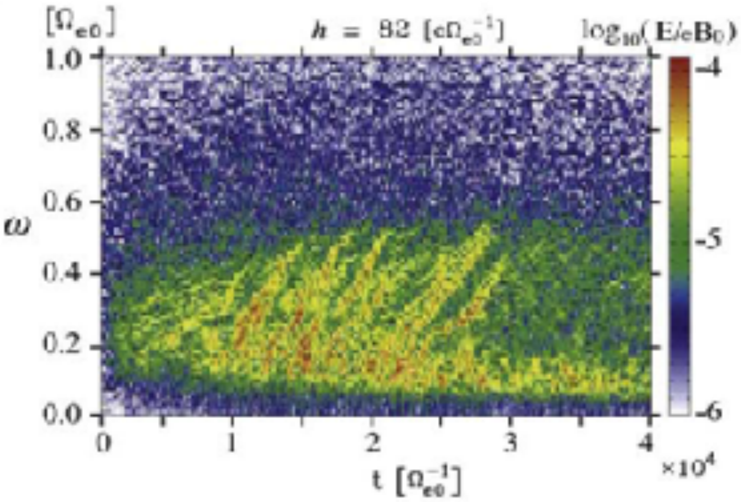




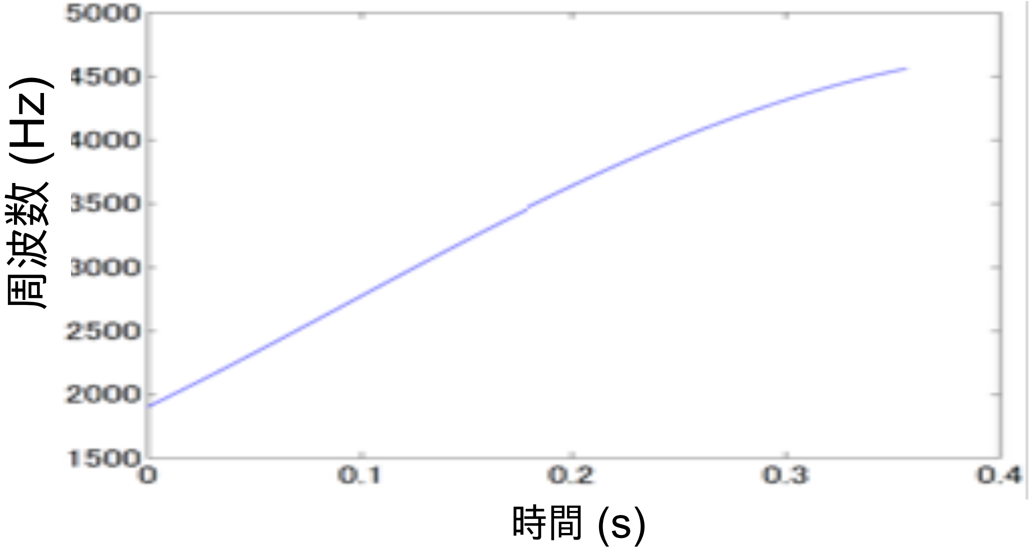
$$S = -\frac{1}{s_0 \omega \boxed{\Omega_w}} \left(s_1 \boxed{\frac{\partial \omega}{\partial t}} + c s_2 \boxed{\frac{\partial \Omega_e}{\partial h}} \right) \sim -0.4$$

Frequency and Amplitude Variation of Chorus: Model 1

Chorus Emissions Reproduced by Particle Simulation



Reproducing chorus sound

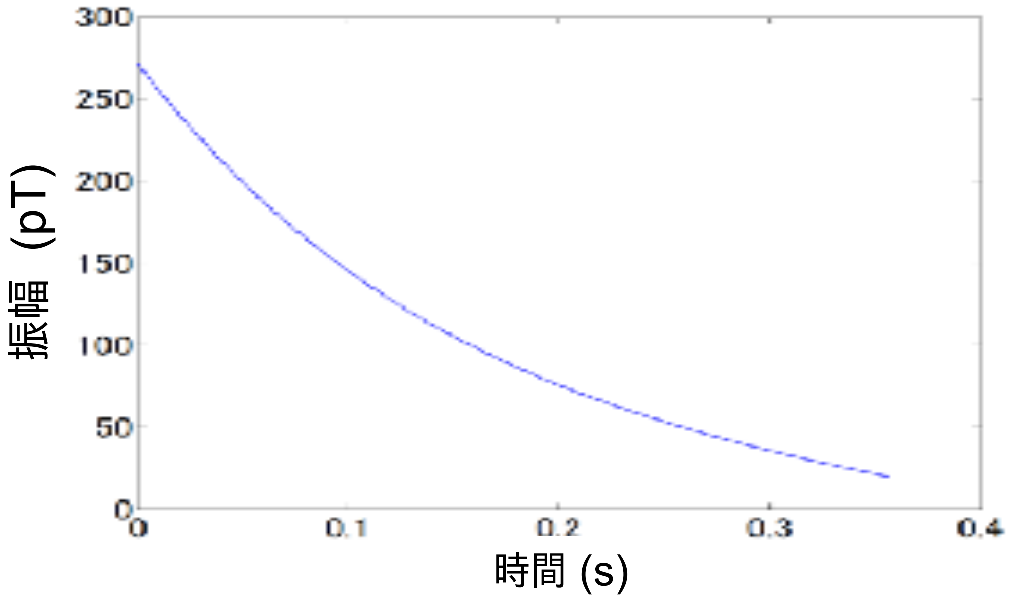


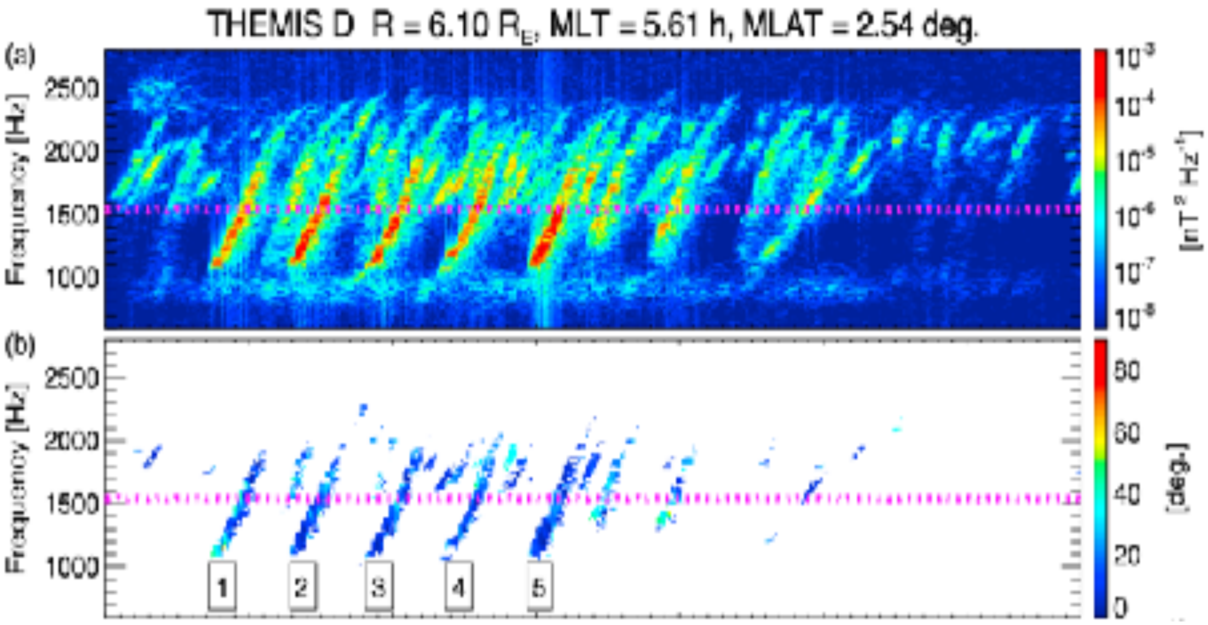
Chorus Equations

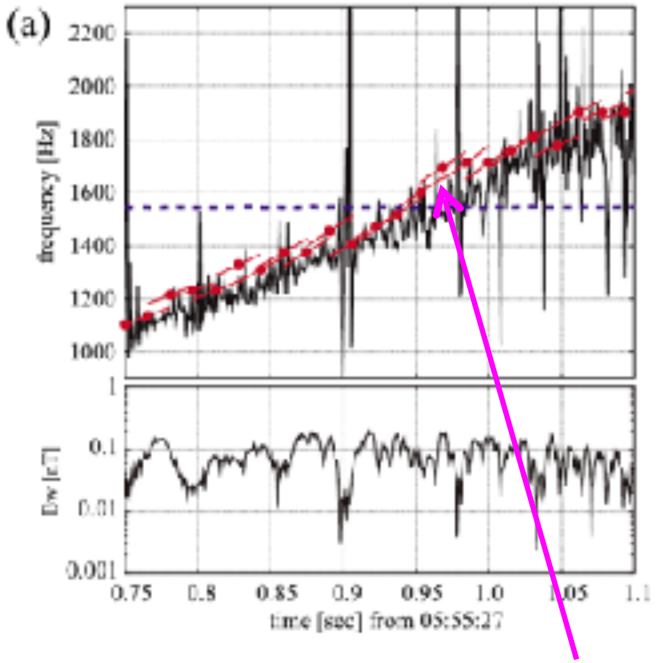
$$\frac{\partial\tilde{\omega}}{\partial\tilde{t}} = \frac{2s_0}{5s_1}\tilde{\omega}\tilde{\Omega}_{w0}.$$

$$\frac{\partial\tilde{\Omega}_{w0}}{\partial\tilde{t}} = \tilde{V}_g\left[\frac{Q\tilde{\omega}_{ph}^2}{2\tilde{U}_{t\|}}\left(\frac{\chi\tilde{V}_{\perp 0}}{\pi\gamma}\right)^{3/2}\left(\frac{\xi\tilde{\Omega}_{w0}}{\tilde{\omega}}\right)^{1/2} \times\exp\left(\frac{\gamma^2\tilde{V}_R^2}{2\tilde{U}_{t\|}^2}\right)\quad\frac{5s_2\tilde{a}}{s_0\tilde{\omega}}\right].$$

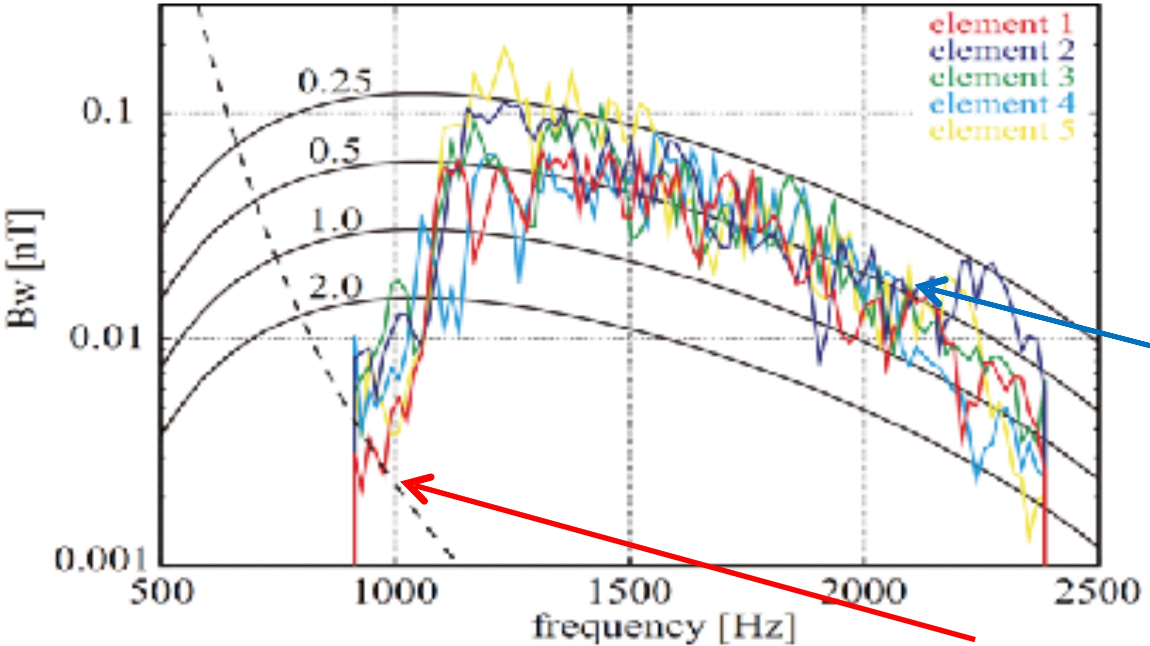
[*Omura et al., JGR,* 2008, 2009]







$$\frac{\partial\omega}{\partial t} = \frac{0.4 s_0 \omega}{s_1}\Omega_w$$

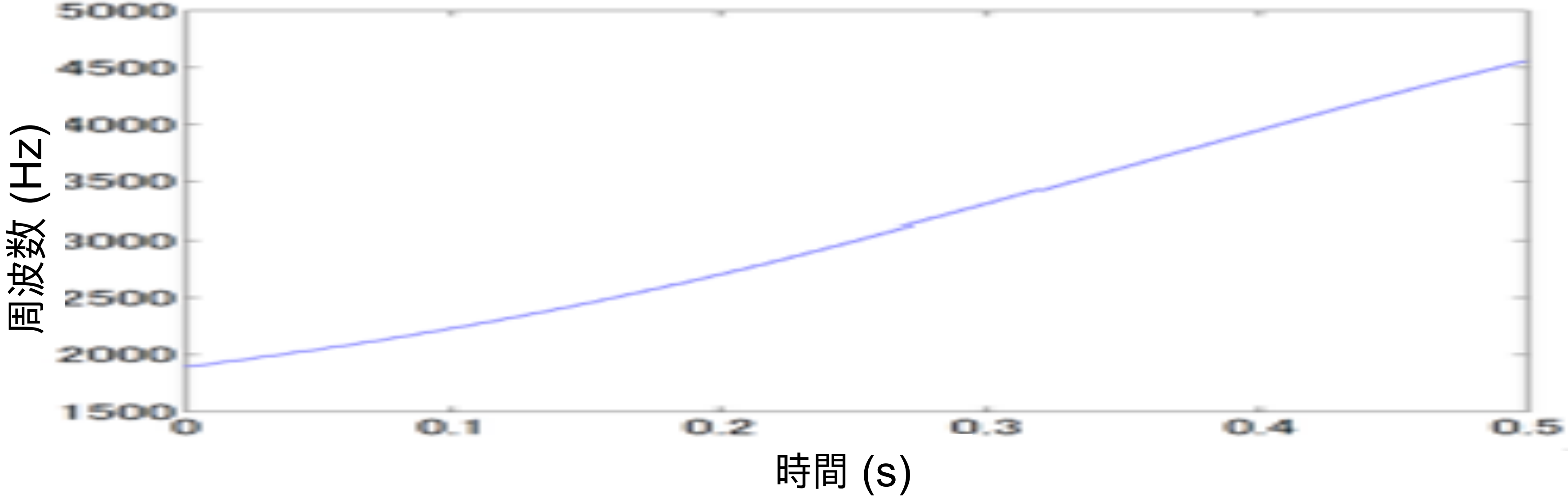


Optimum Amplitude

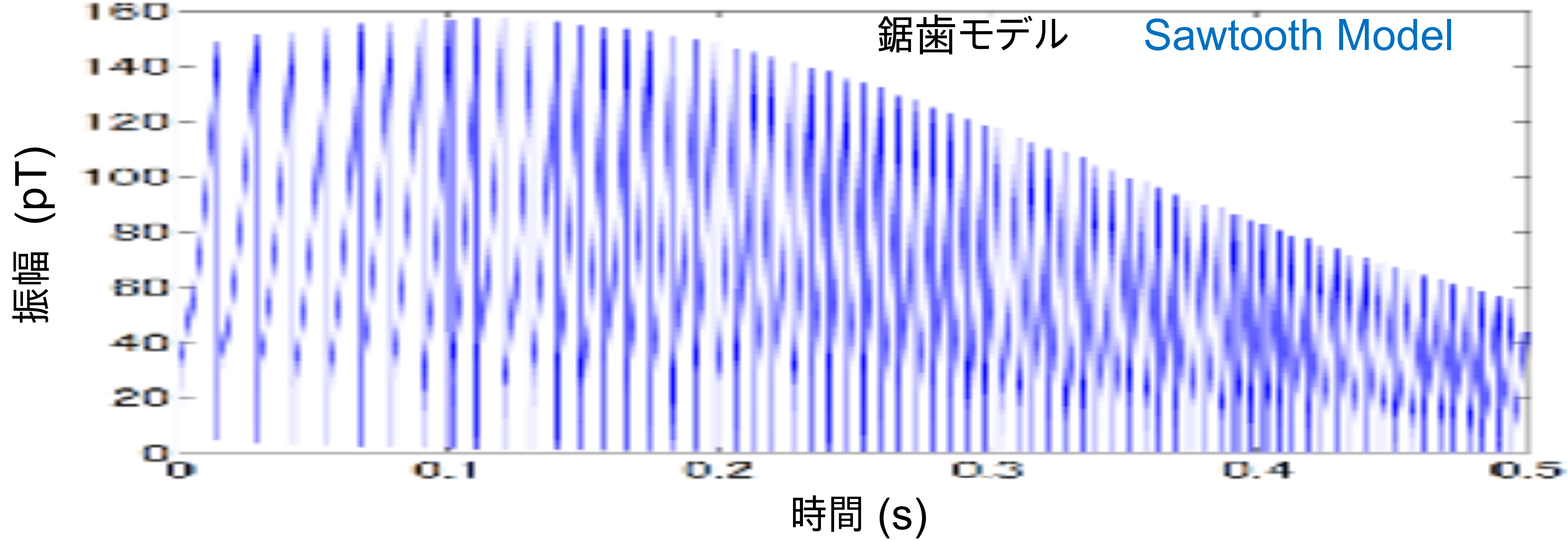
Threshold Amplitude

[*Kurita et al., JGR, 2012*]

Frequency and Amplitude Variation of Chorus: Model 2



Reproducing chorus sound



鋸歯モデル Sawtooth Model

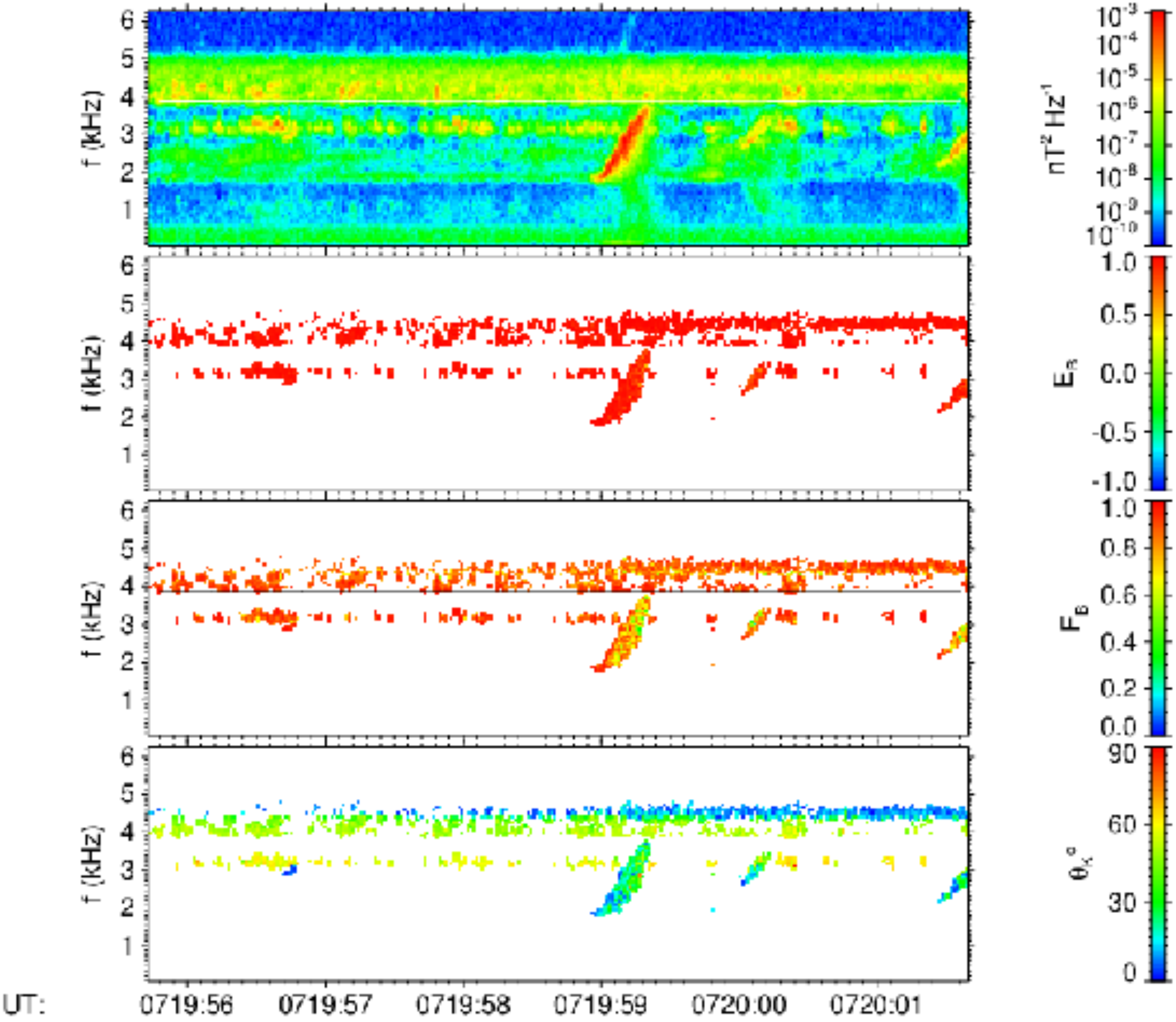
Wave Observation, Van Allen Probe A, 8 June 2014

sum of the power-spectral densities of magnetic components

ellipticity of the magnetic field polarization

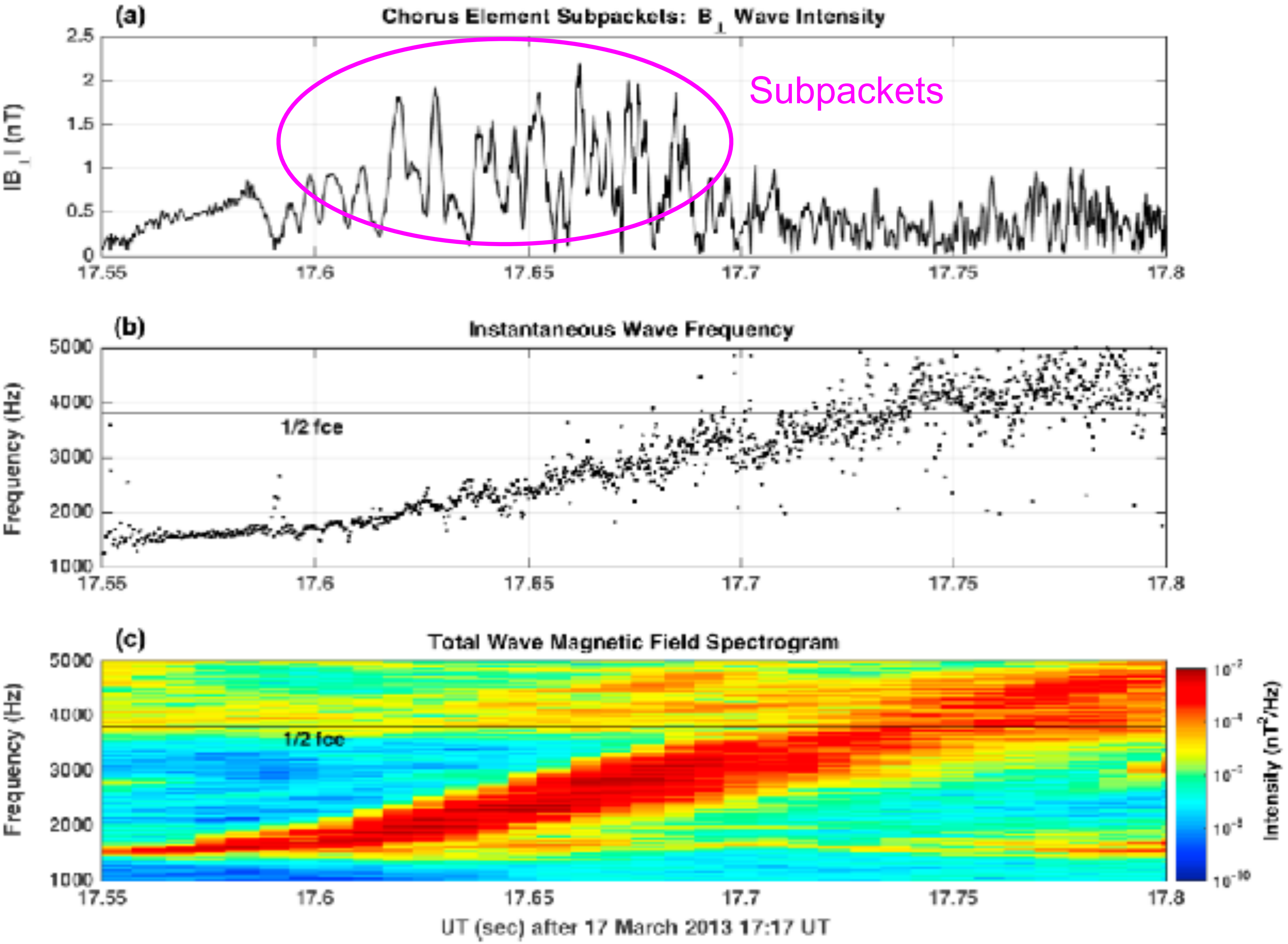
planarity of the magnetic field polarization

angle between the wave vector and the background magnetic field



[*Kletzing*, 2014]

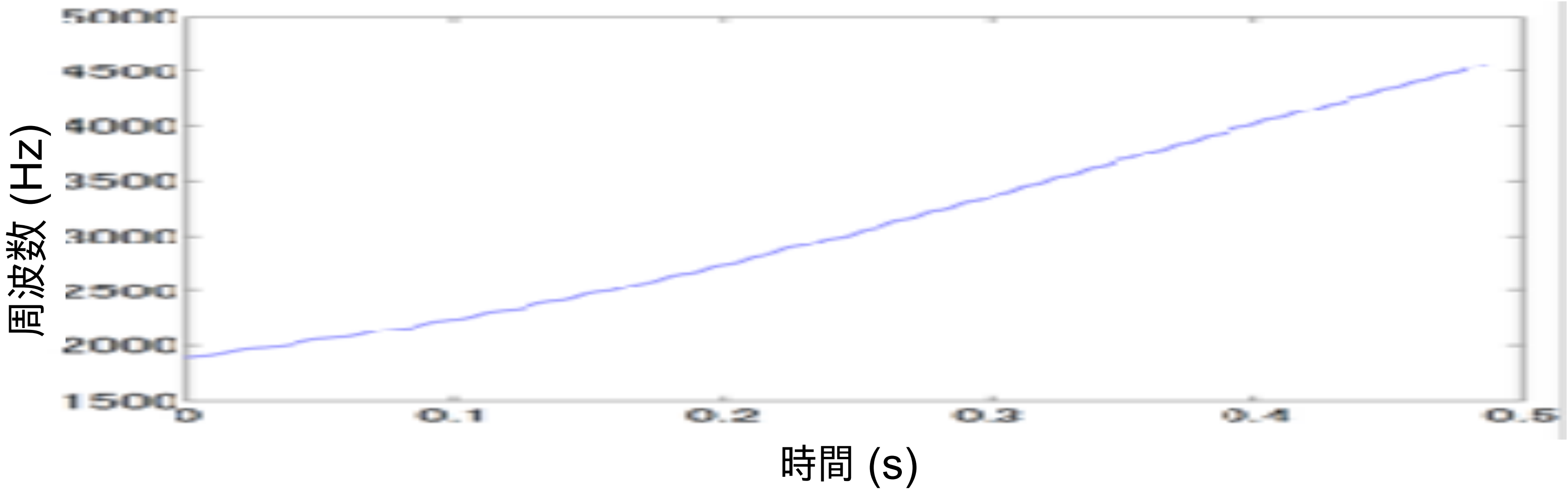
Subpacket Structure in Chorus Element

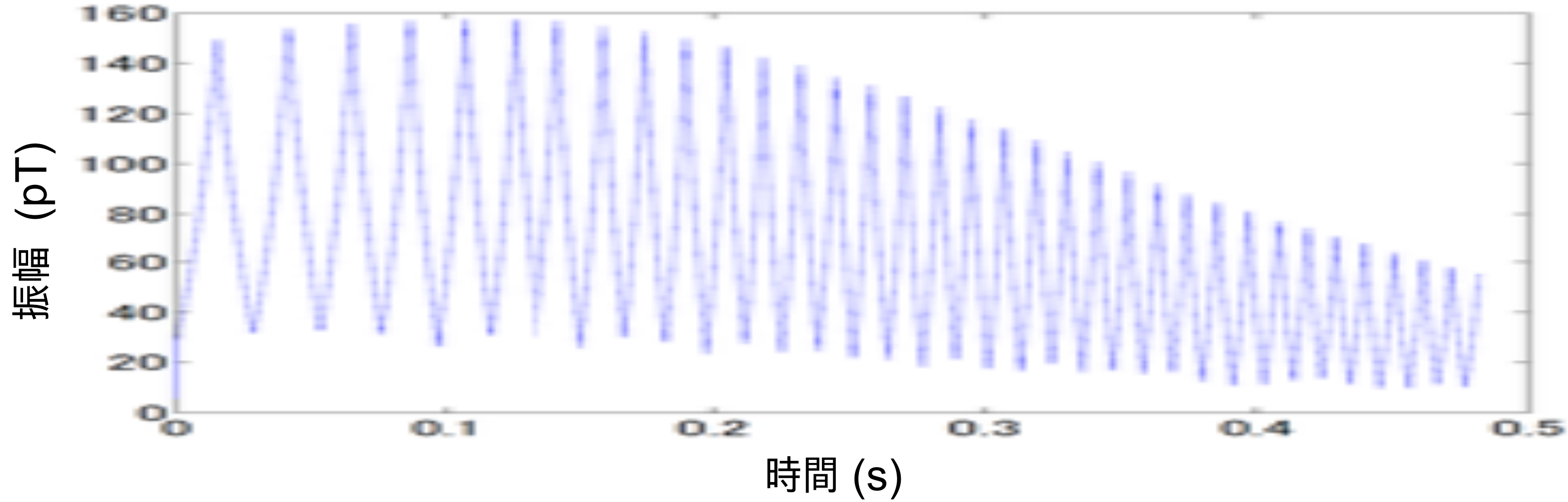


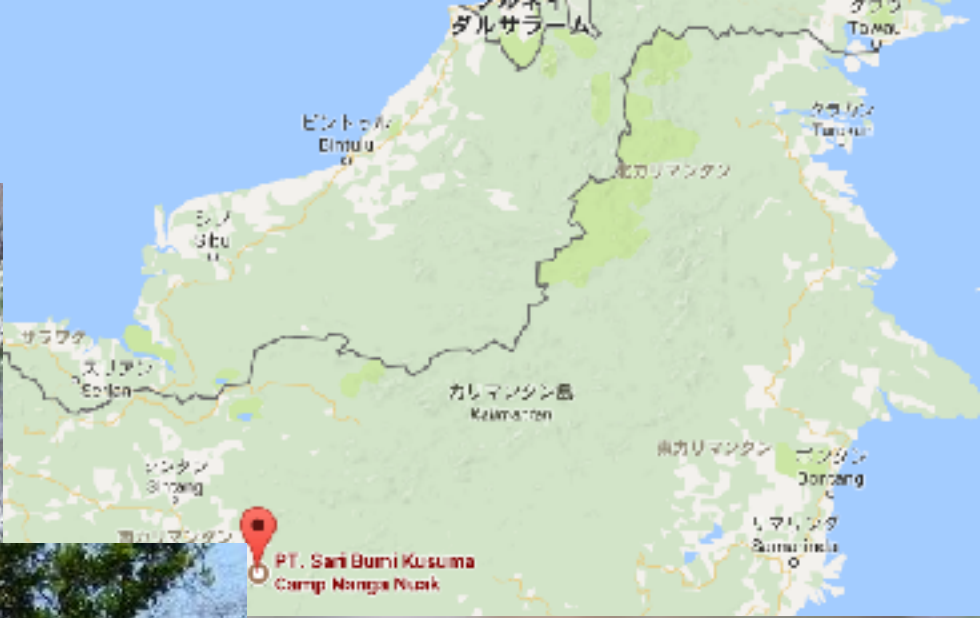
[*Foster et al.*, *JGR*, 2017]

Frequency and Amplitude Variation of Chorus: Model 3

Reproducing chorus sound

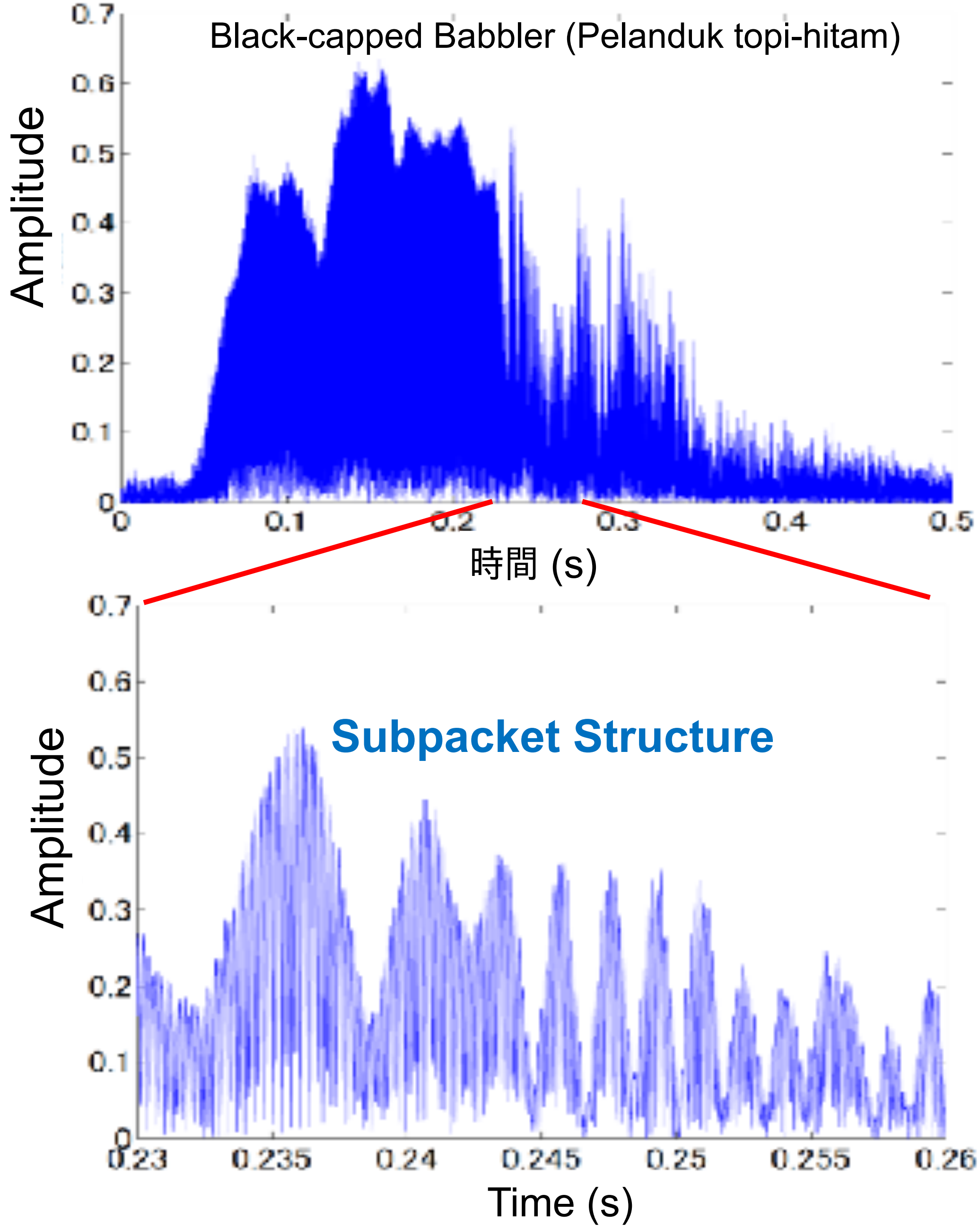




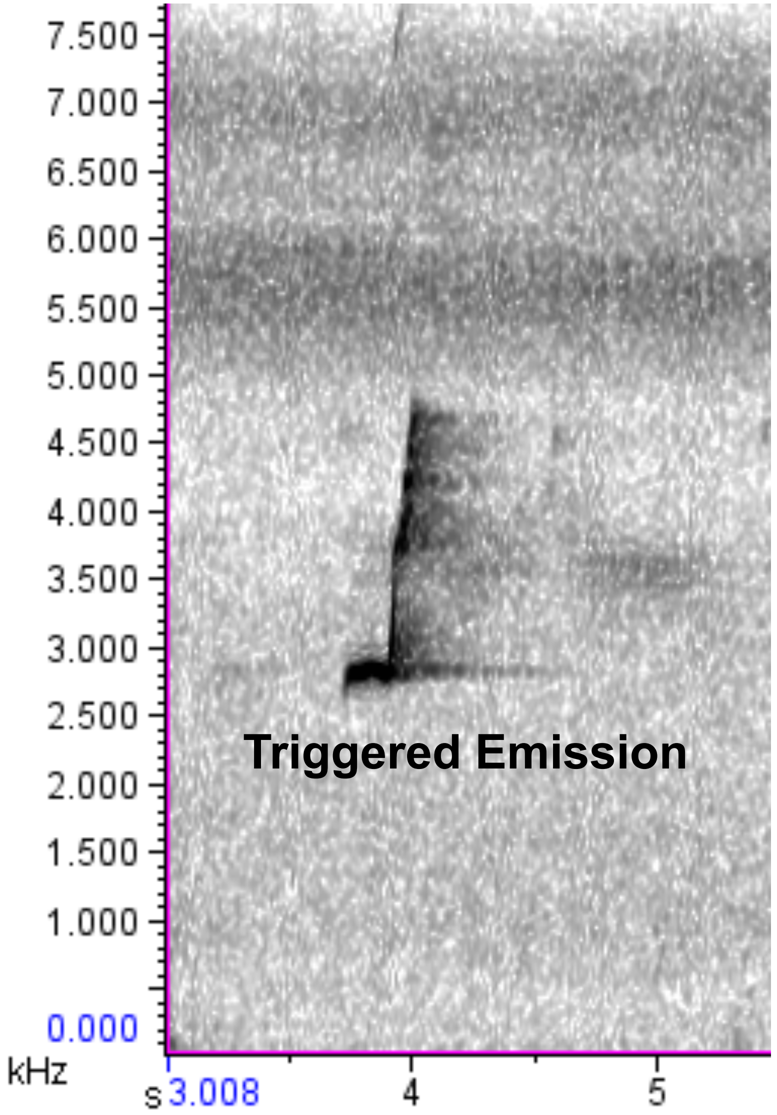

ダルサラーム
Tawau
ビントゥル
Bintulu
タラカン
Tarakan
北カリマンタン
シブ
Sibu
サラワク
スリアン
Serian
カリマンタン島
Kalimantan
東カリマンタン
ボンタン
Bontang
シンタン
Sintang
西カリマンタン
PT. Sari Bumi Kusuma
Camp Nanga Nuak
サマリンダ
Samarinda

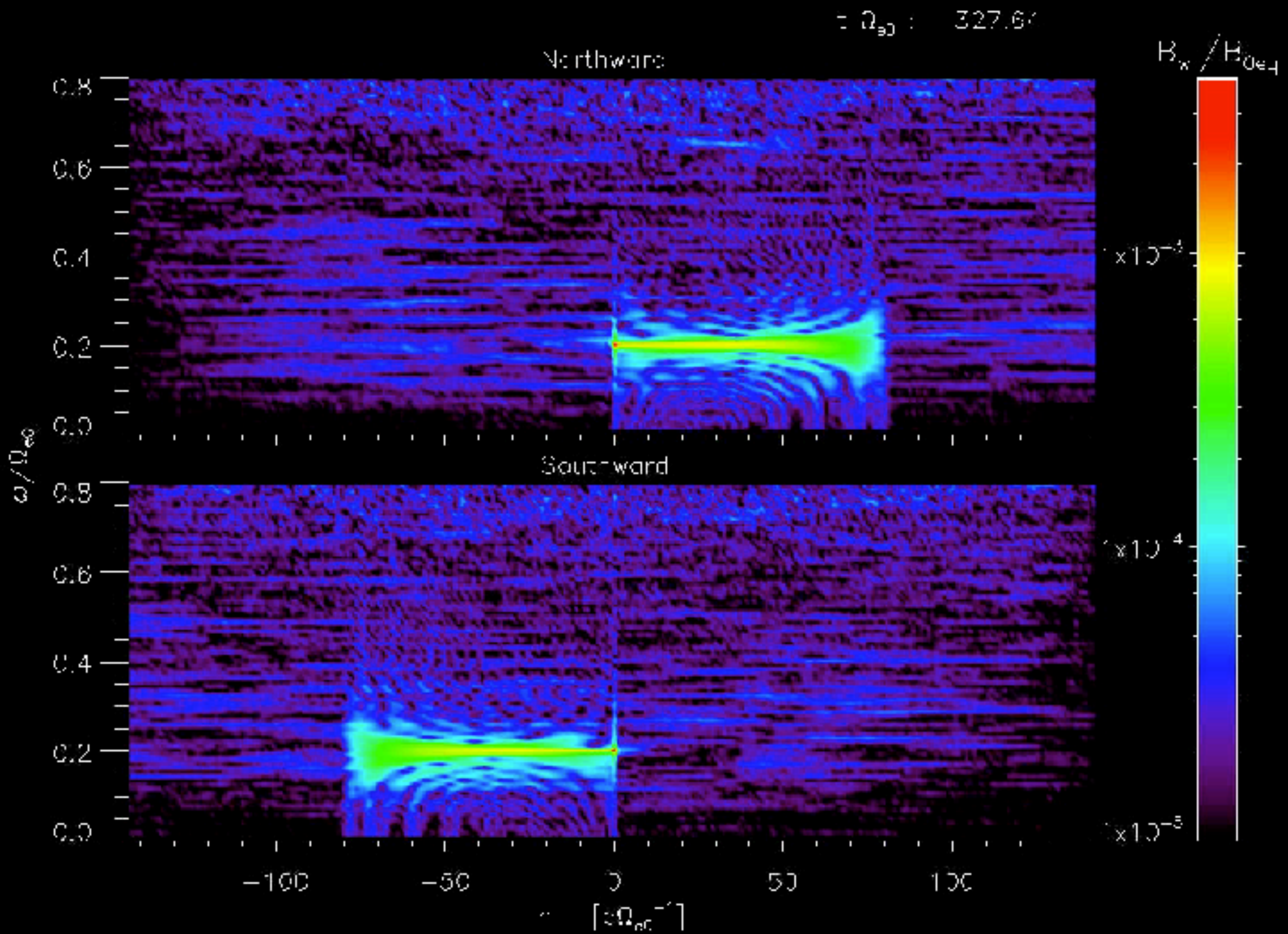
[藤田素子 提供]

Subpacket Structure in Dawn Chorus in Indonesia

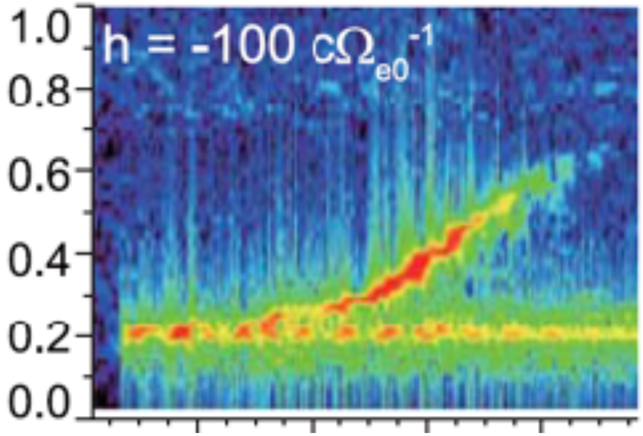


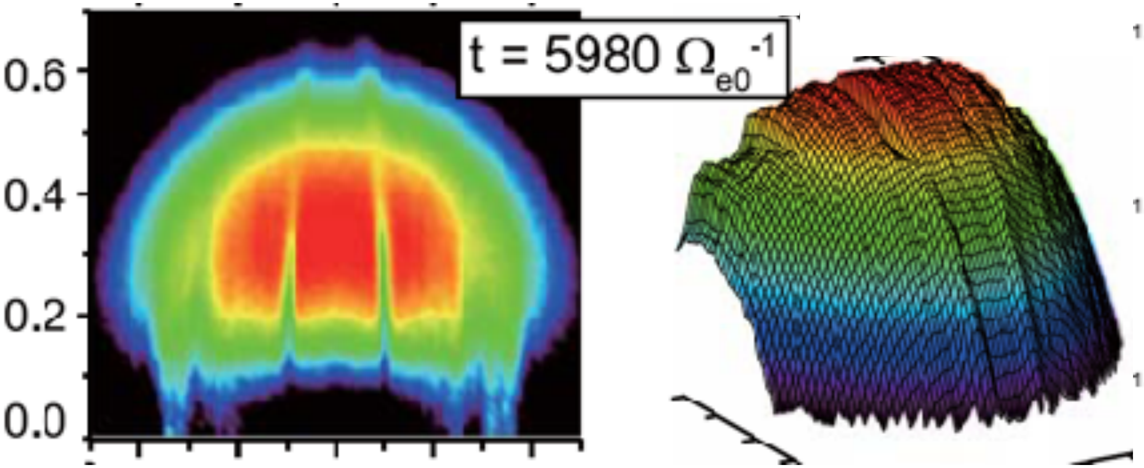
Dawn Chorus in Indonesia

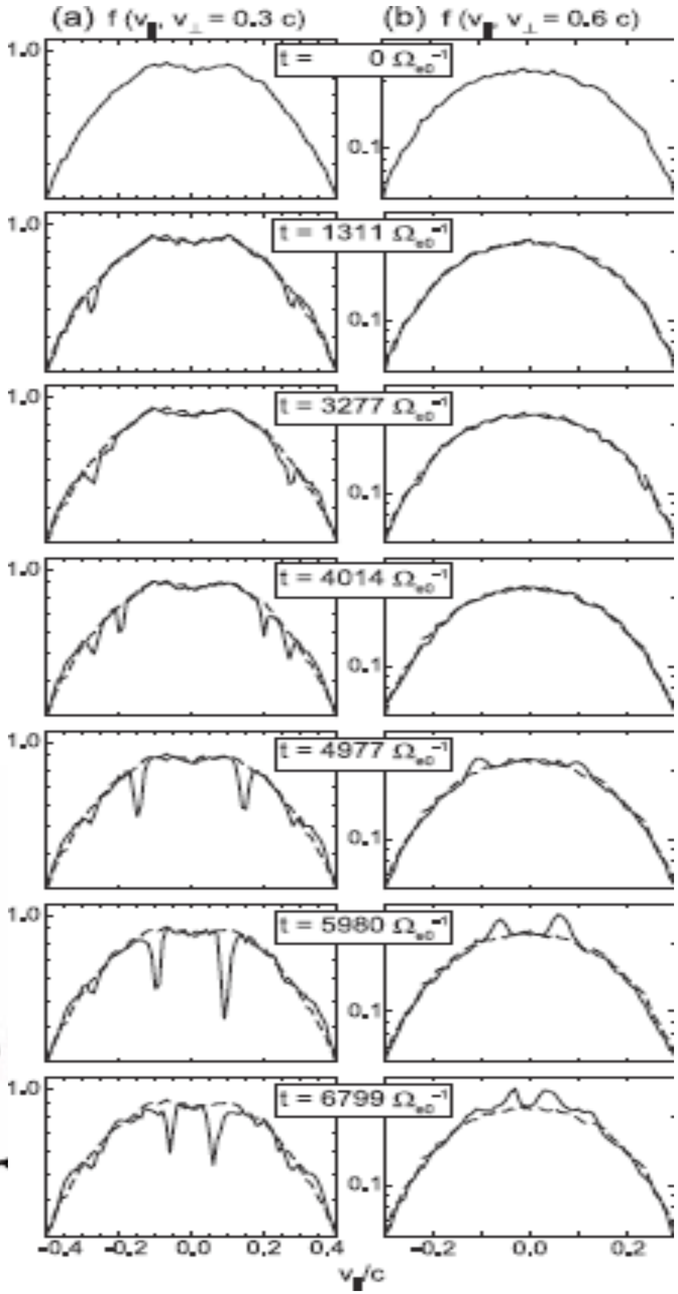



t Ω_e0 : 327.67
Northward
Southward
ω/Ω_e0
0.8
0.6
0.4
0.2
0.0
−100
−50
0
50
100
x [cΩ_ec^-1]
B_w / B_0eq
1x10^-3
1x10^-4
1x10^-5

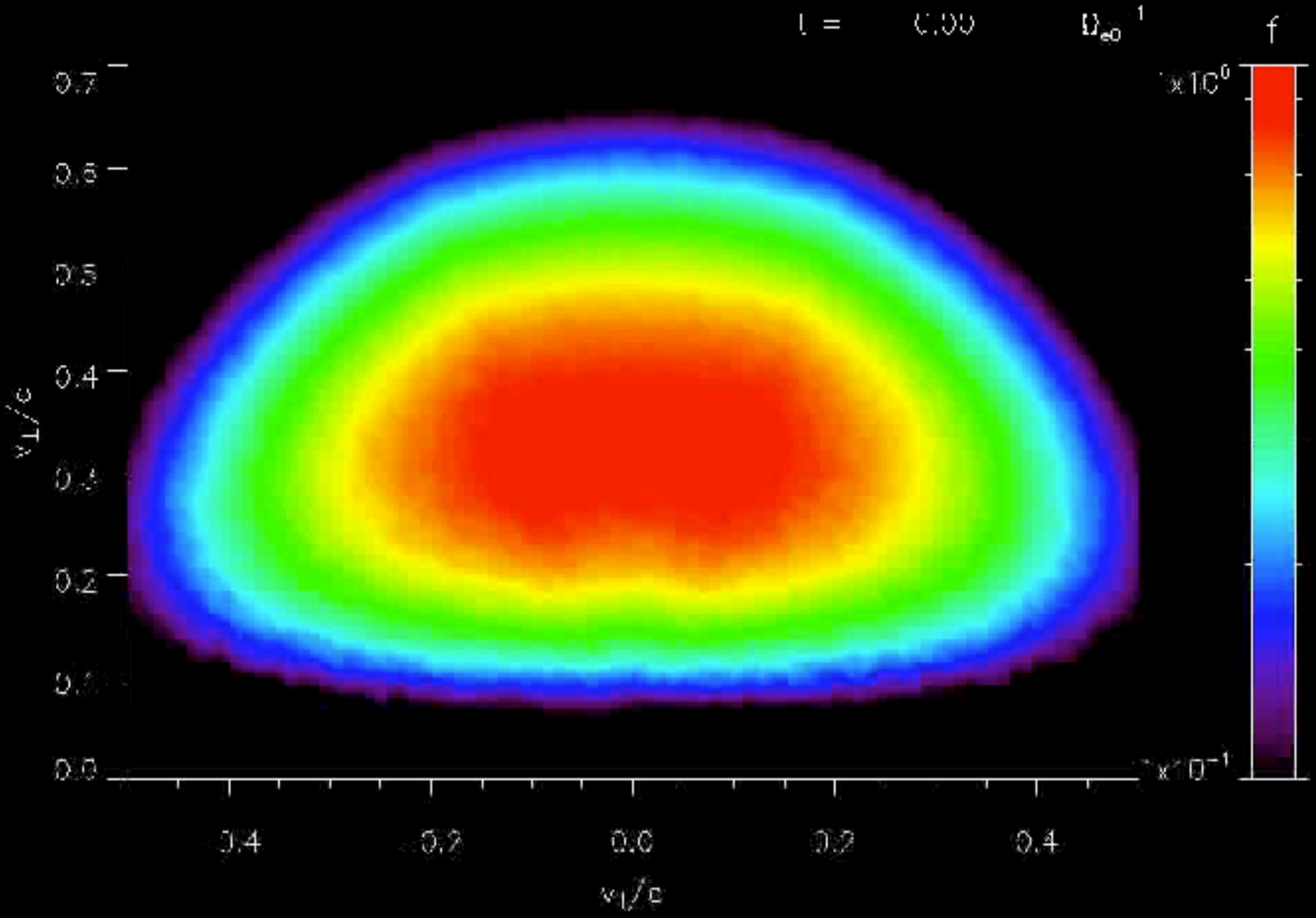
Formation of Electron Hole and Bump



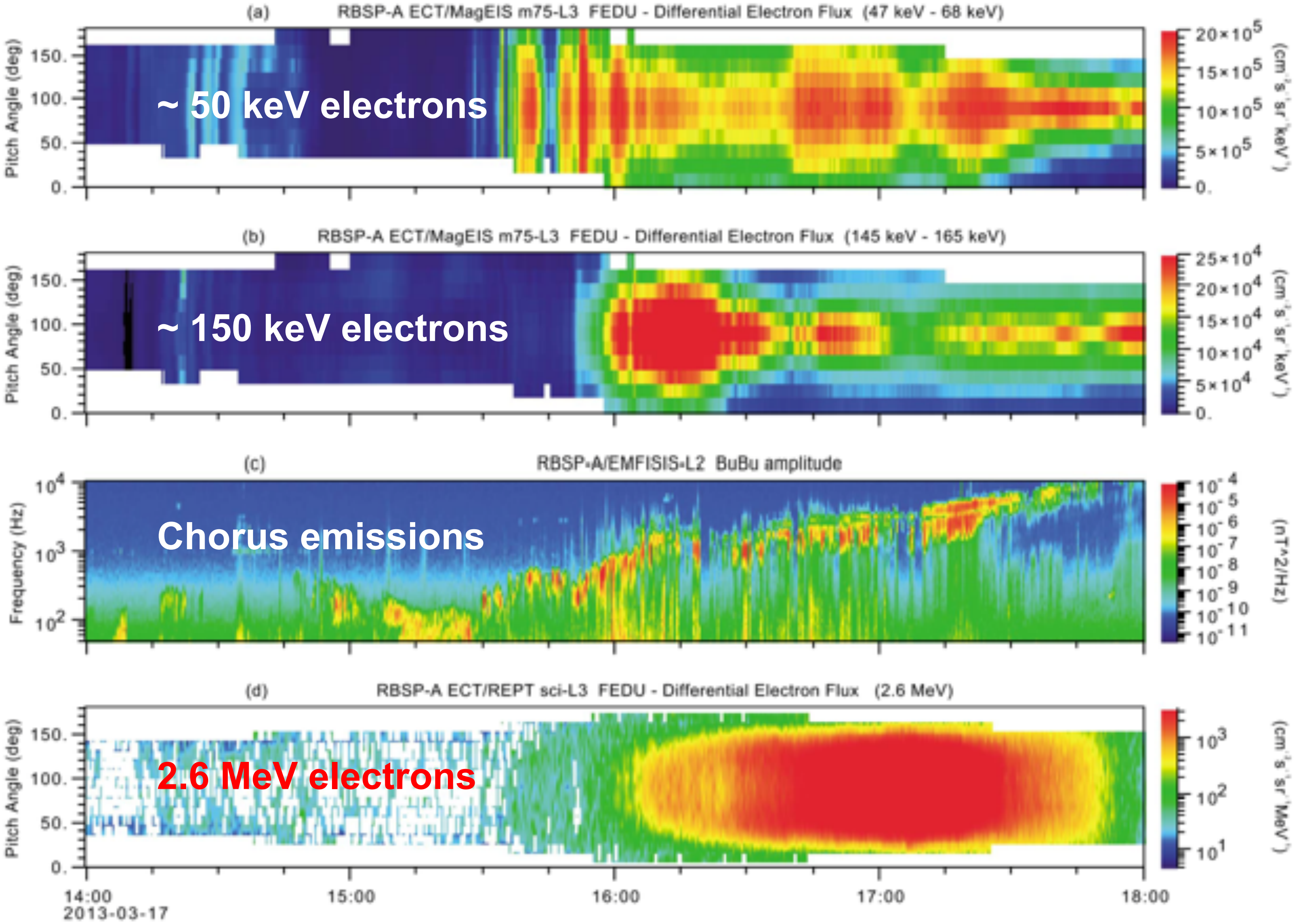




[*Hikishima et al., JGR, 2010*]


t = 0.00 Ω_{e0}^{-1}
f
1x10^0
1x10^-1
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$v_\perp/c$
-0.4
-0.2
0.0
0.2
0.4
$v_\parallel/c$

Rapid-acceleration of MeV Electrons

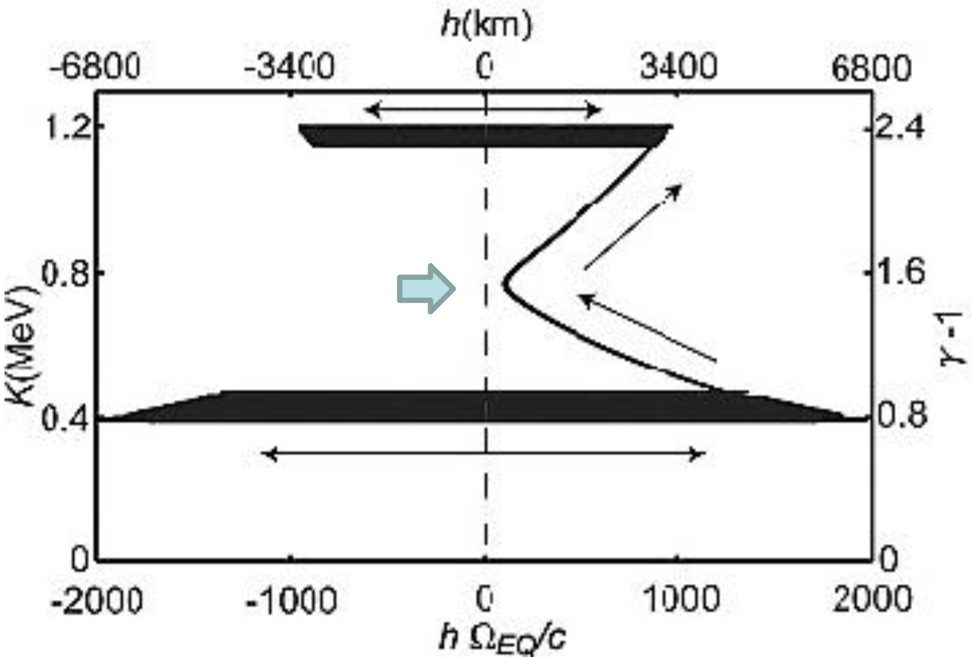


[*Foster et al.*, 2017]

Two Nonlinear Acceleration Processes

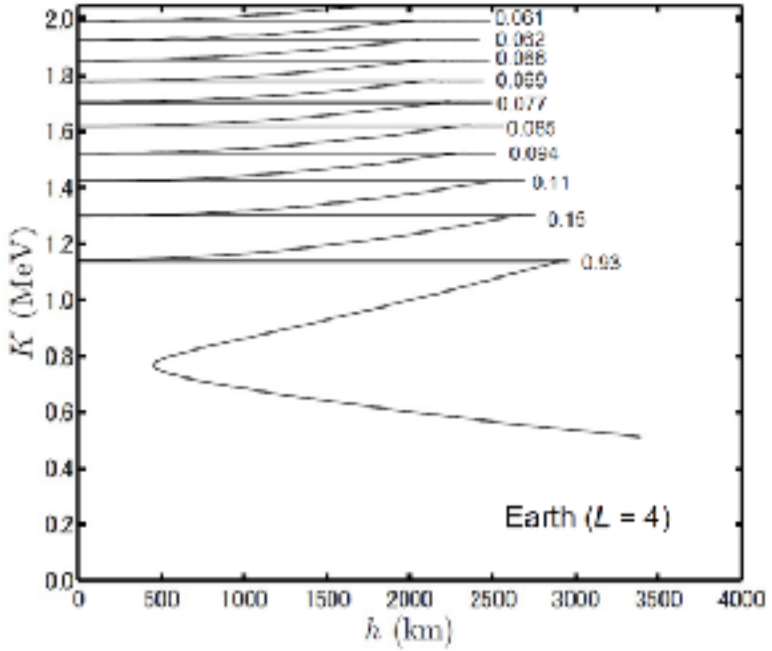
$$V_R = \frac{\omega - \Omega_{ce}/\gamma}{k} \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

Relativistic Turning Acceleration（RTA)



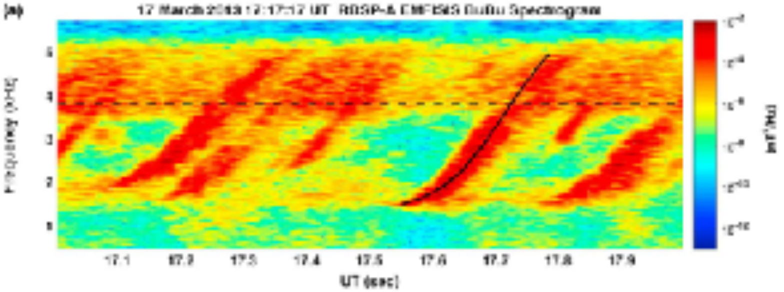
[*Omura,et al., 2007*]

Ultra-Relativistic Acceleration (URA)

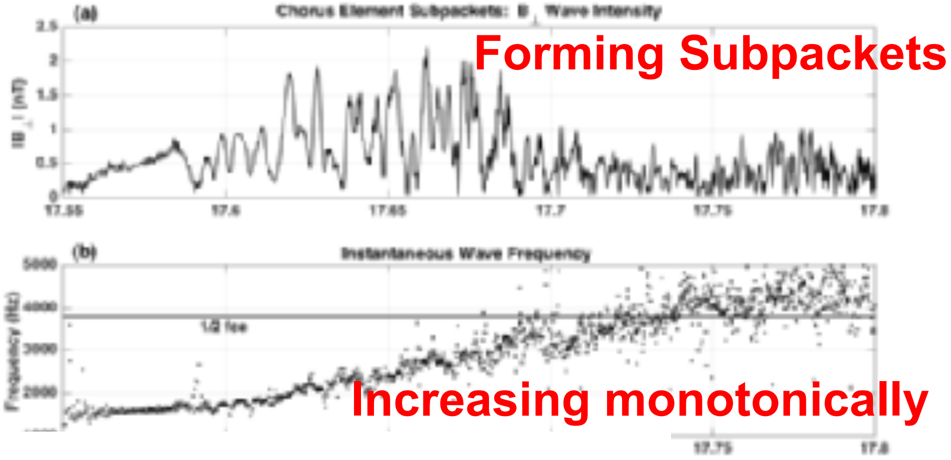


[*Summers and Omura, 2007*]

Chorus Waves

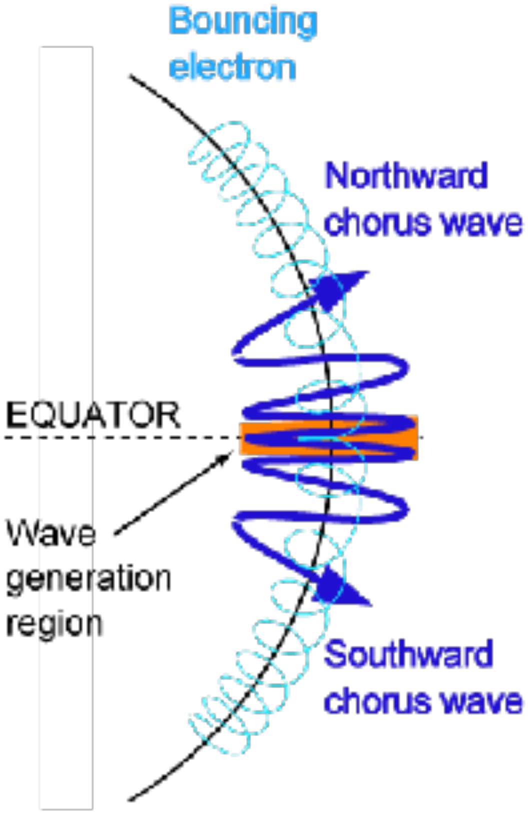


[*Foster et al.*, 2017]



Forming Subpackets

Increasing monotonically



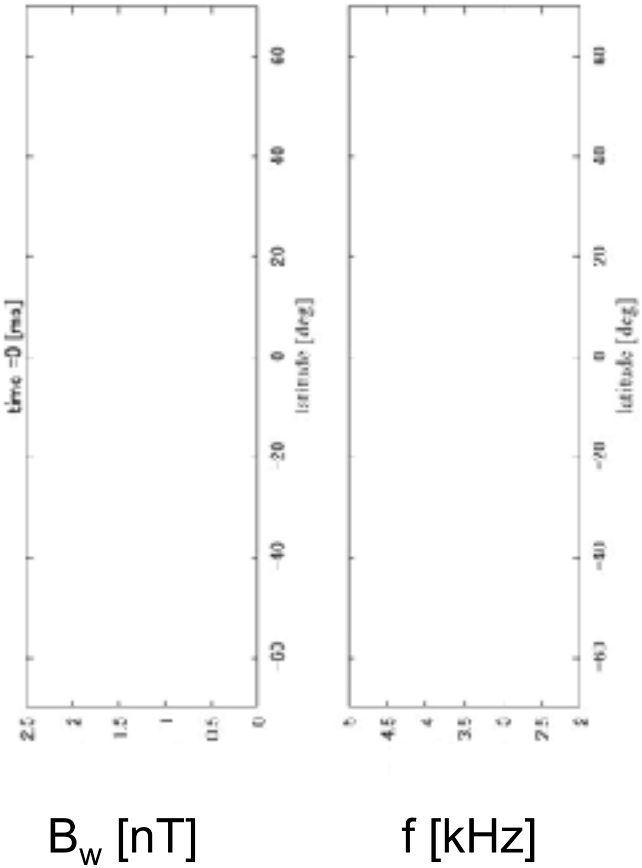
Bouncing electron

Northward chorus wave

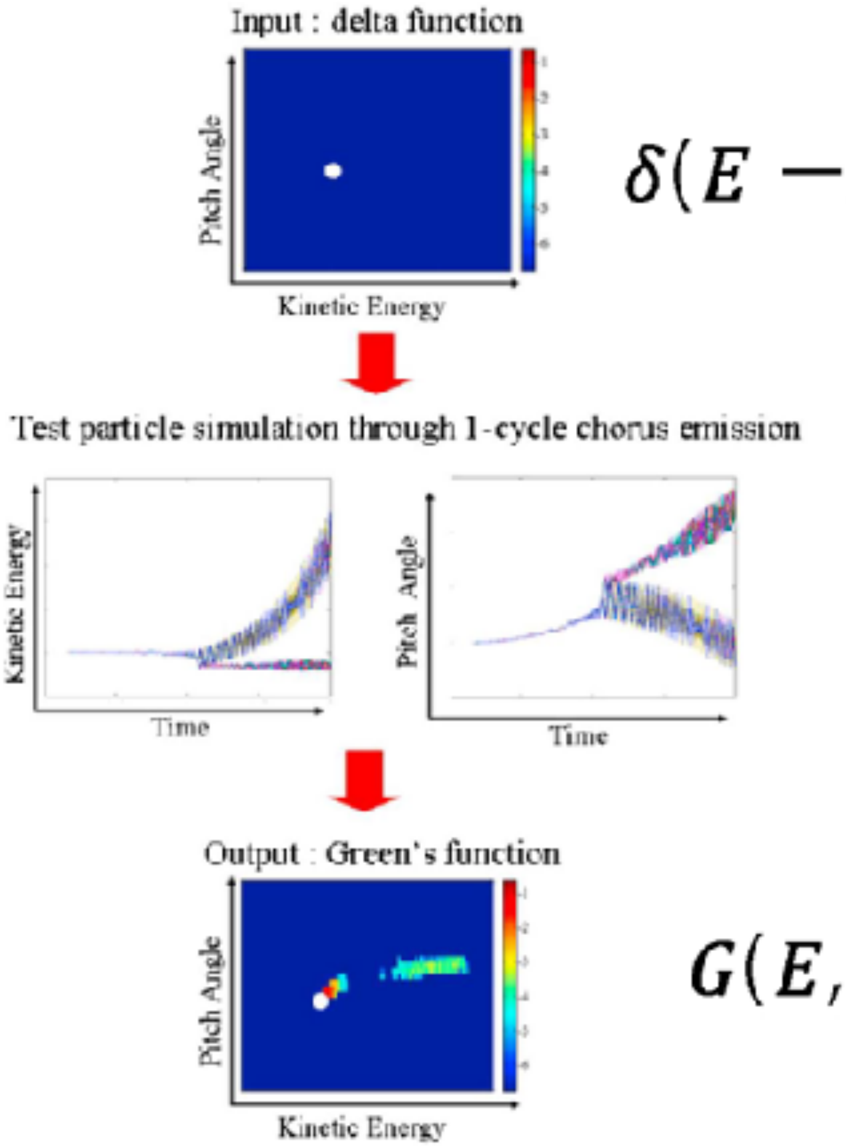
EQUATOR

Wave generation region

Southward chorus wave



Numerical Green’s Functions


Input : delta function

Test particle simulation through 1-cycle chorus emission

Output : Green's function

$$\delta(E - E_0, \alpha - \alpha_0)$$

×

$$C \quad \boxed{m_e \frac{d(\gamma \boldsymbol{v})}{dt} = -e\left[\boldsymbol{E}_w + \boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{B}_w)\right]}$$

||

$$G(E, E_0, \alpha, \alpha_0)$$

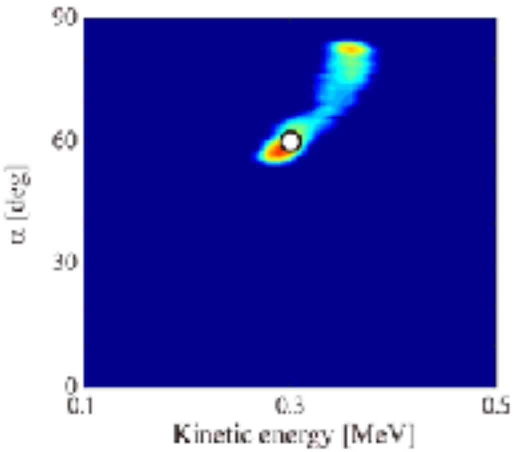
[*Omura et al.*, 2015]

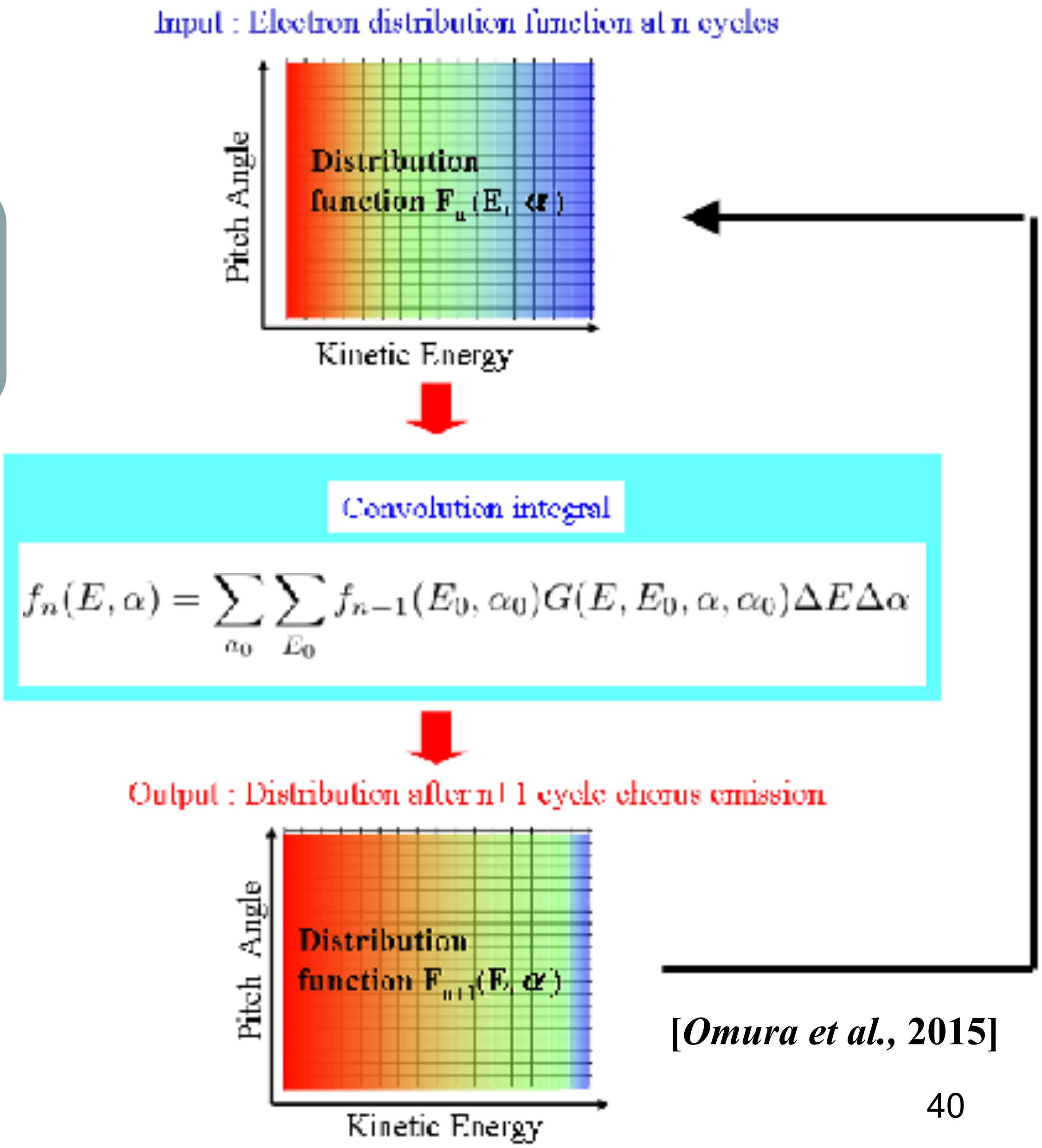
Numerical Green's Function Method

Test particle simulation:
Linearity

$$C[f_a + f_b] = C[f_a] + C[f_b]$$
$$wC[f_a] = C[wf_a]$$

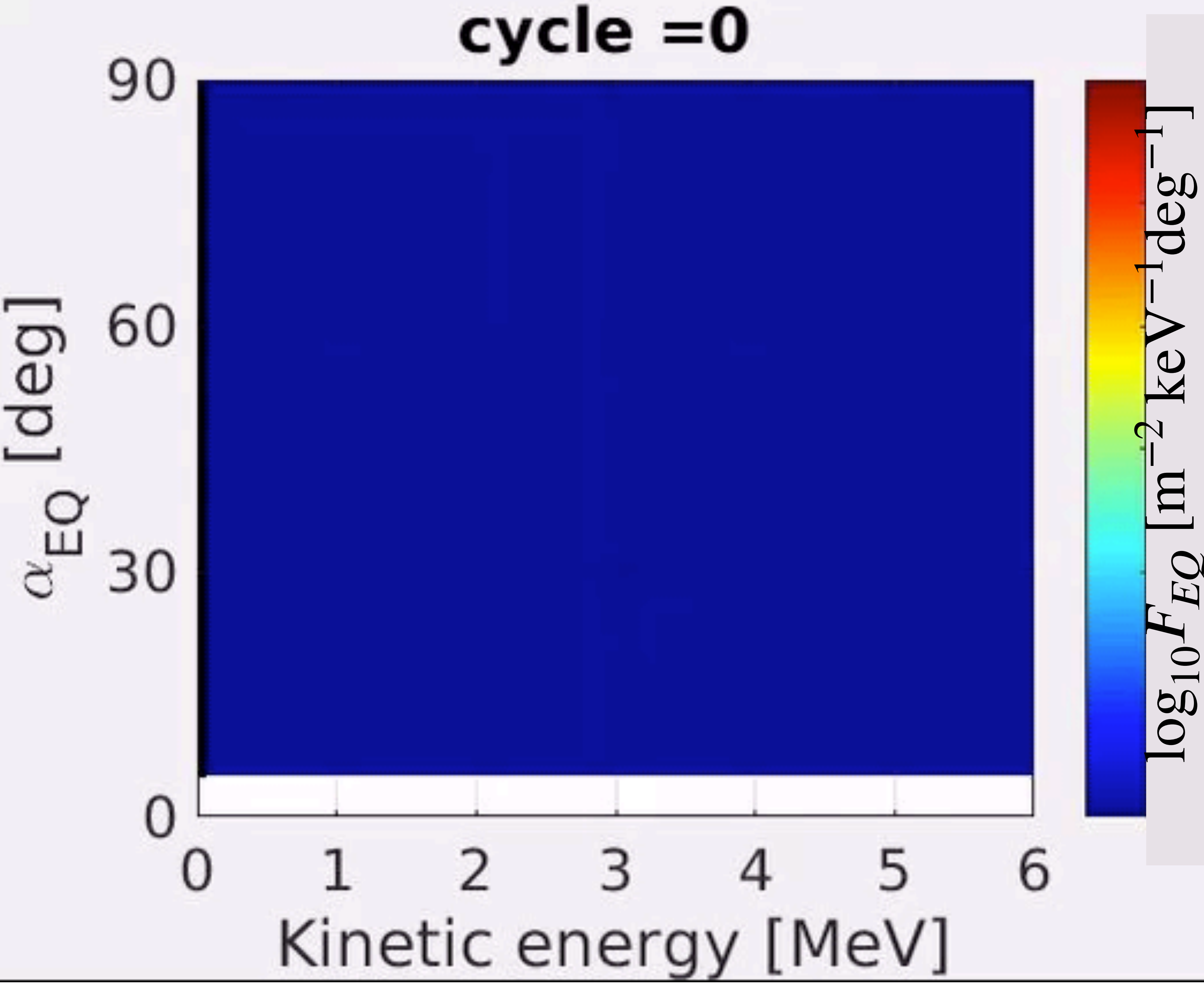
Green's function



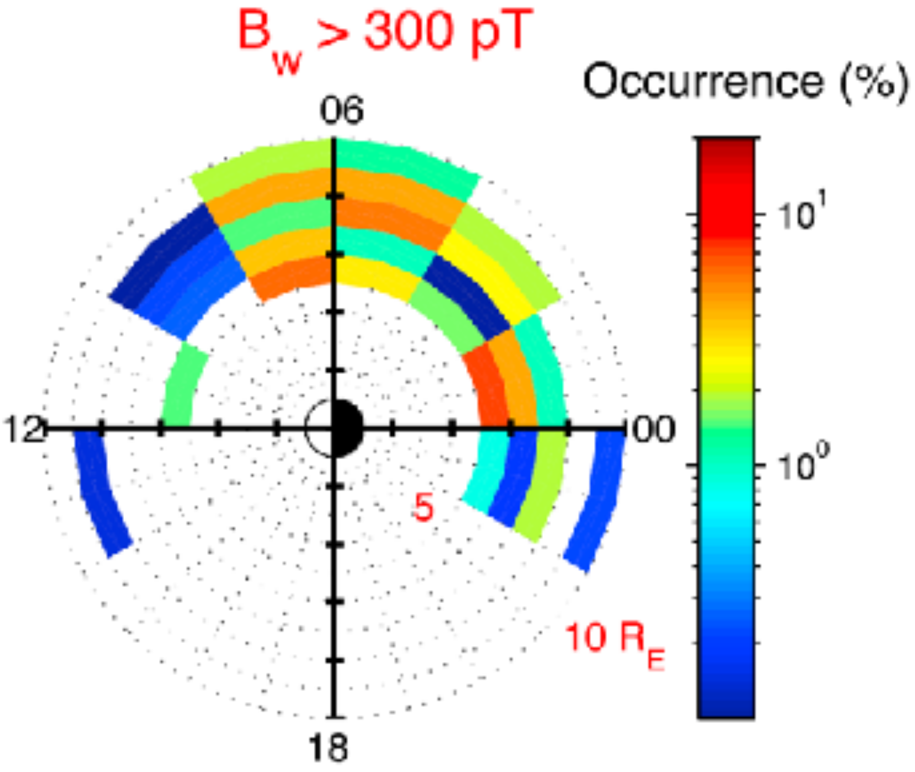


$$f_n(E,\alpha) = \sum_{\alpha_0}\sum_{E_0} f_{n-1}(E_0,\alpha_0)G(E,E_0,\alpha,\alpha_0)\Delta E\Delta\alpha$$

[*Omura et al.*, 2015]

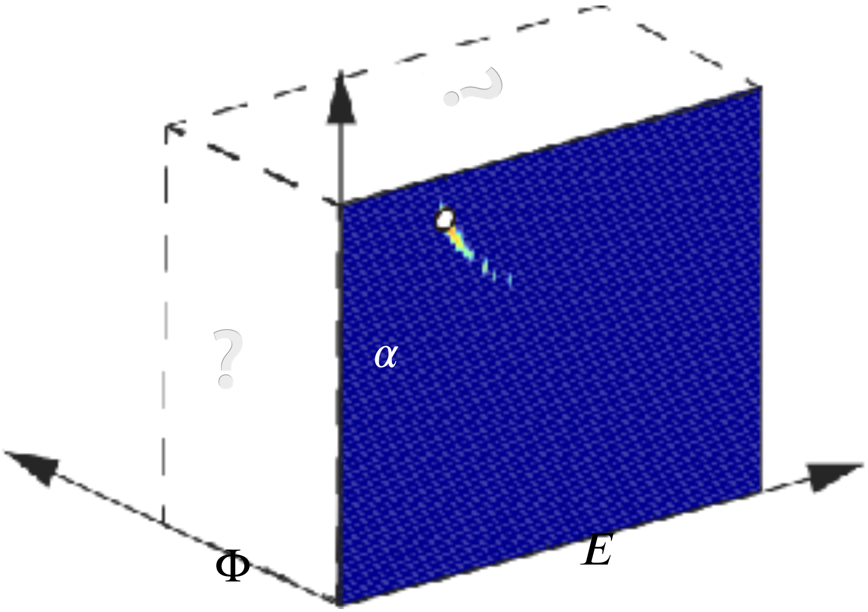

cycle =0
90
60
30
0
α_{EQ} [deg]
0
1
2
3
4
5
6
Kinetic energy [MeV]
$\log_{10} F_{EQ}$ [m^{-2} keV^{-1}deg^{-1}]

Longitudinal Dependency

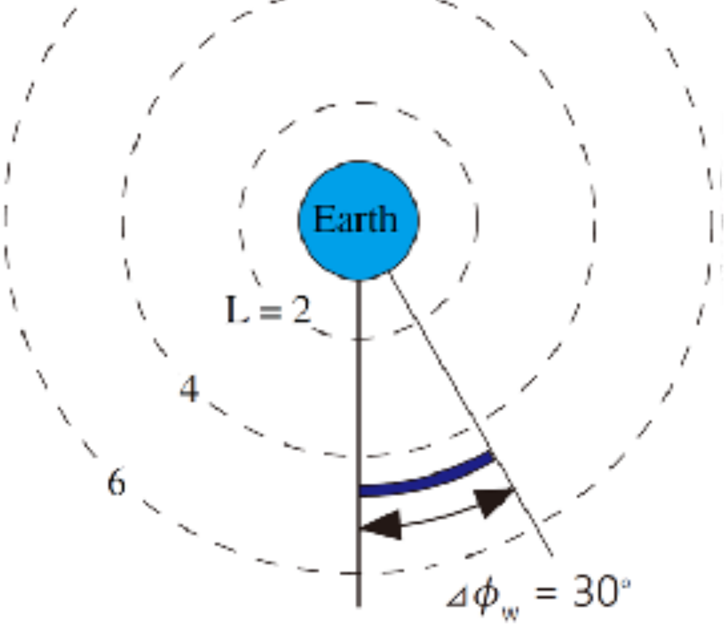


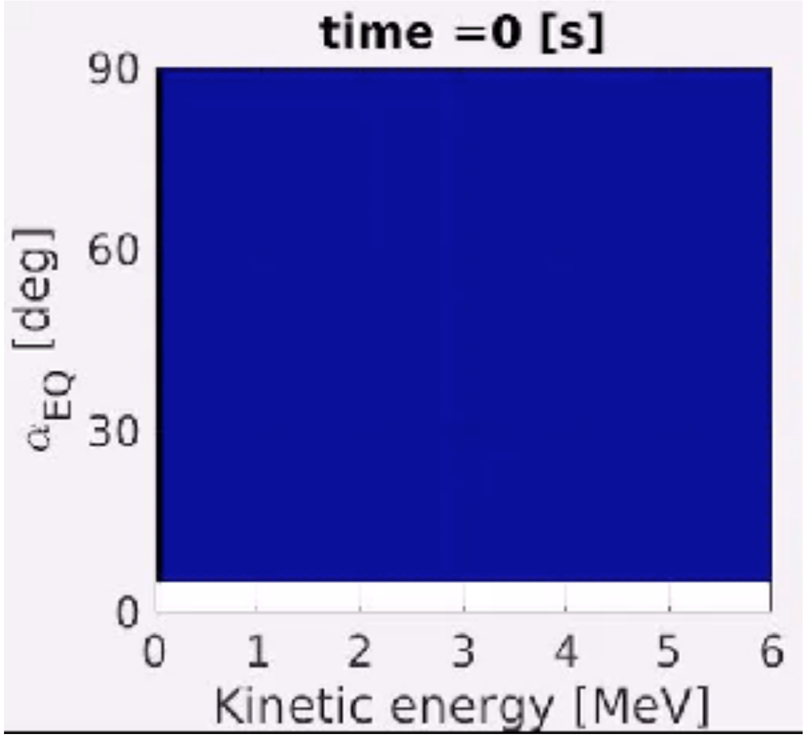
[*Li et al., JGR,* 2011]

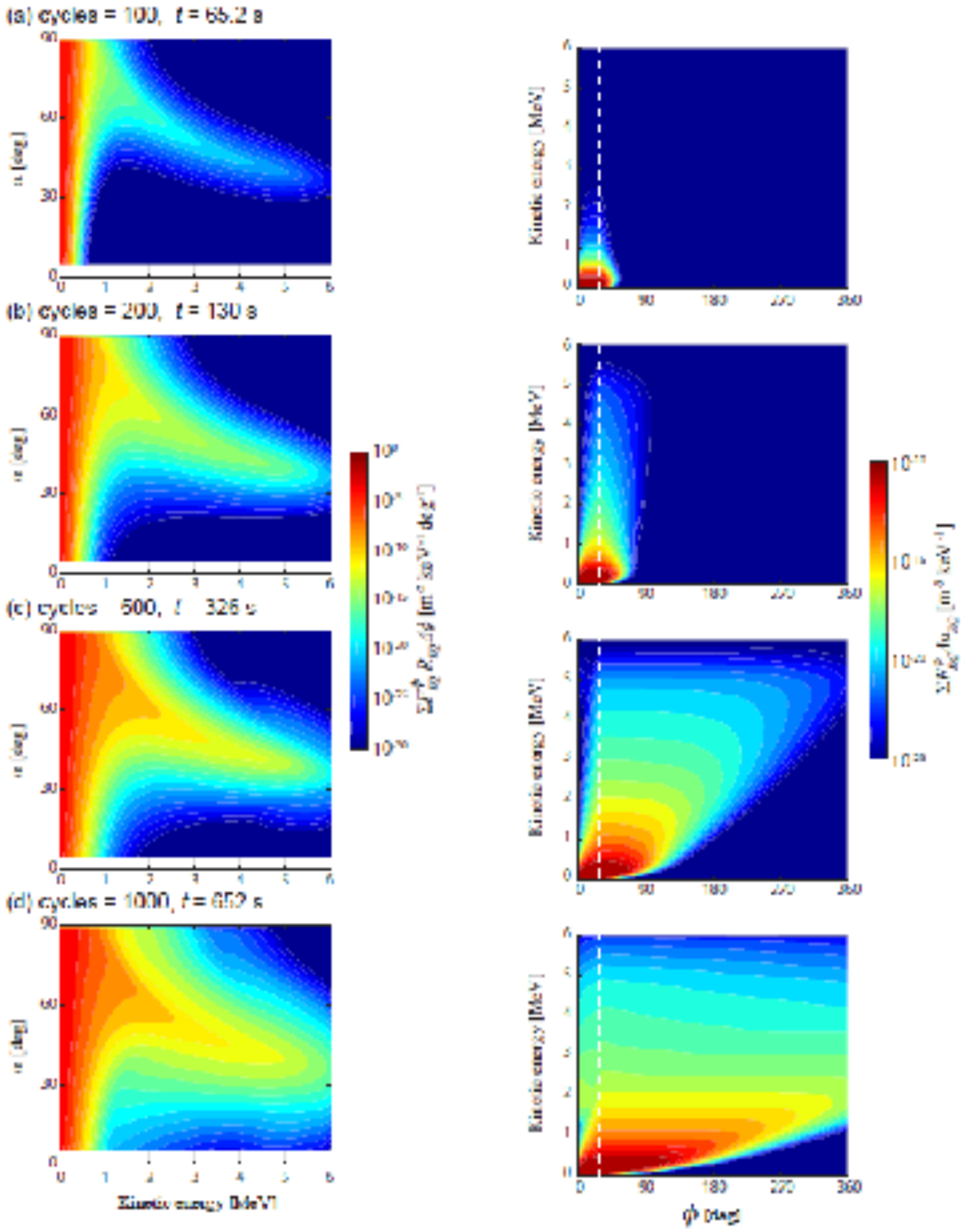
$$G(E, E_i, \alpha, \alpha_j, \Phi, \Phi_k)$$



Constant influx: 10 – 30 keV electrons

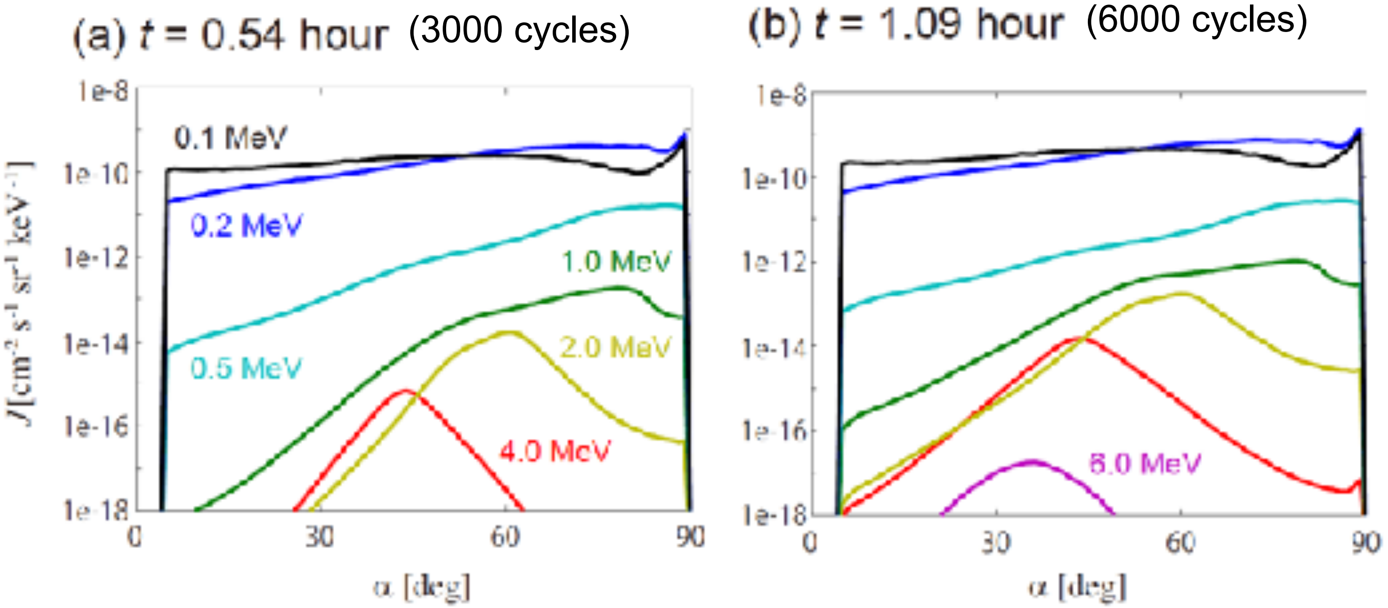


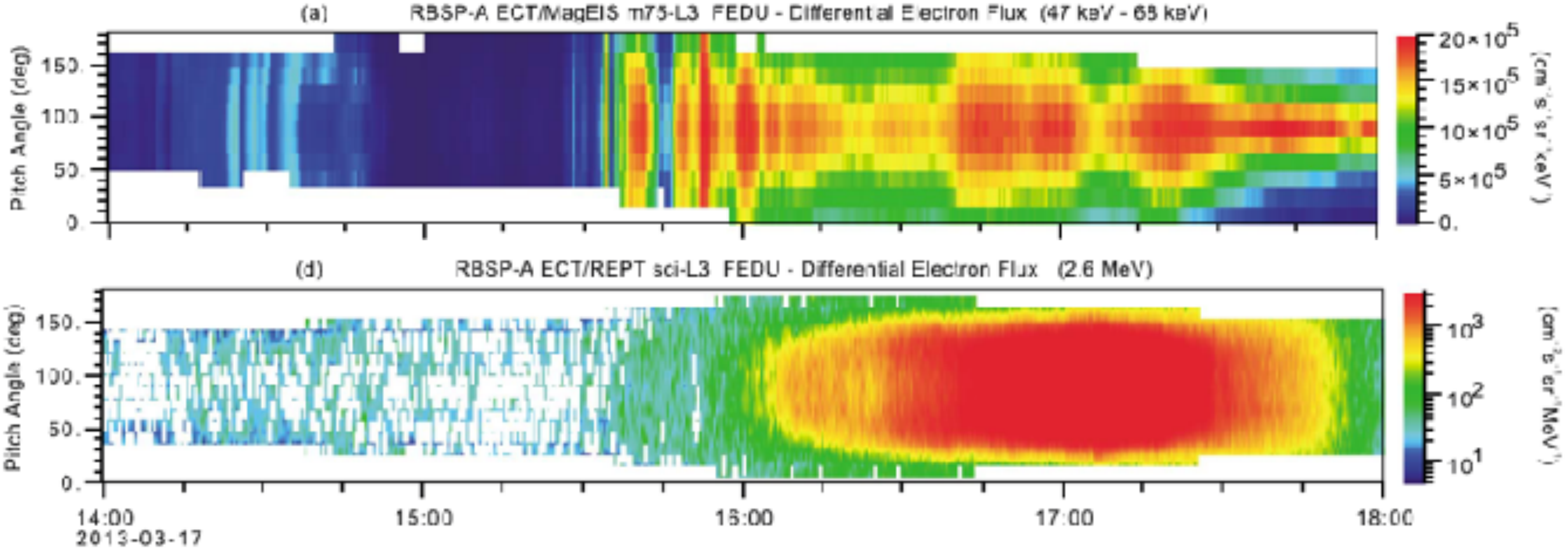




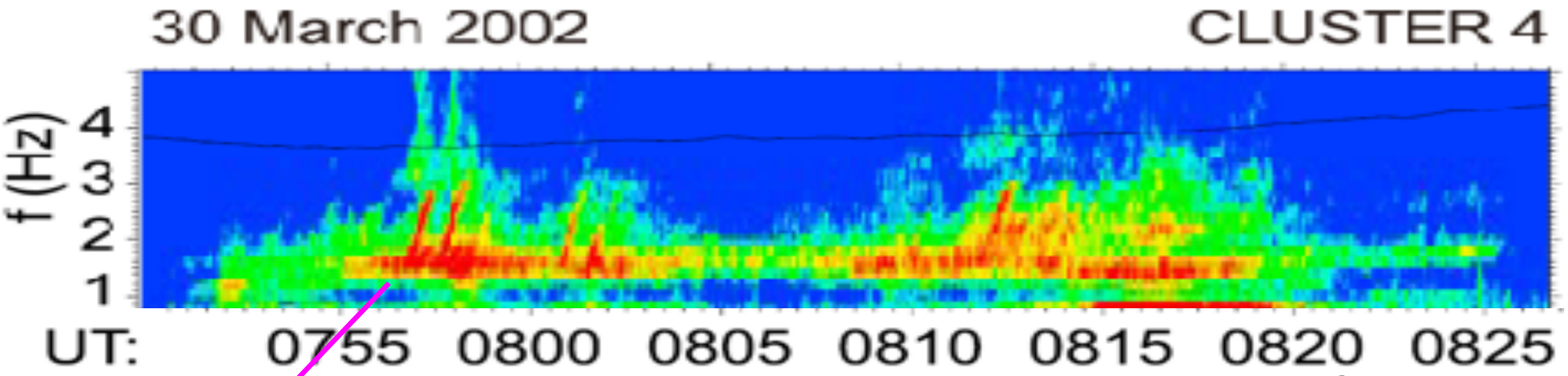
[*Kubota and Omura, JGR*, 2018]

Differential Electron Flux (Simulation and Observation)



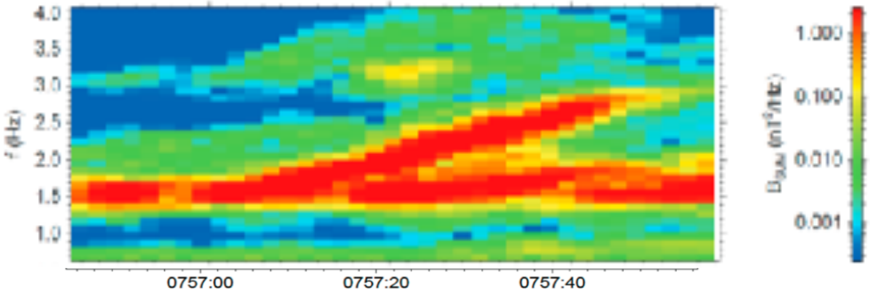


EMIC Rising-tone Emissions



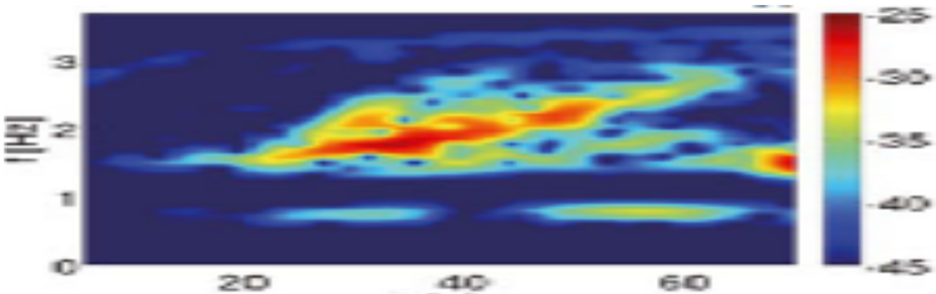
[*Pickett et al., GRL, 2010*]

Nonlinear Wave Growth Theory



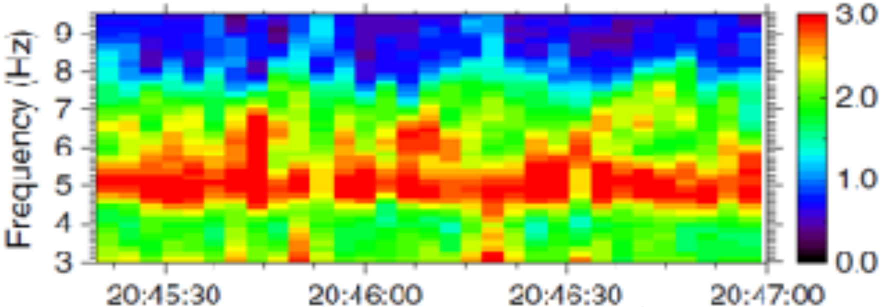
[*Omura et al.,JGR,* 2010]

Hybrid Code Simulations



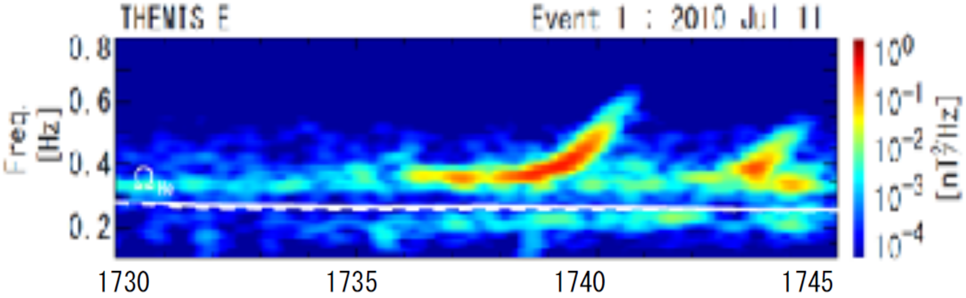
[*Shoji and Omura, JGR,* 2011,2012, 2013,2014]

Radiation Belt Slot Region



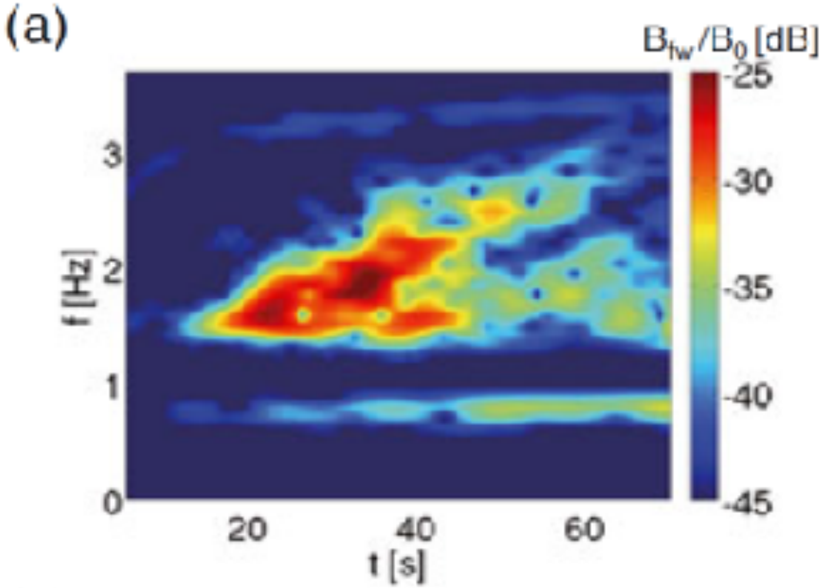
Akebono [*Sakaguchi et al.,GRL,* 2013]

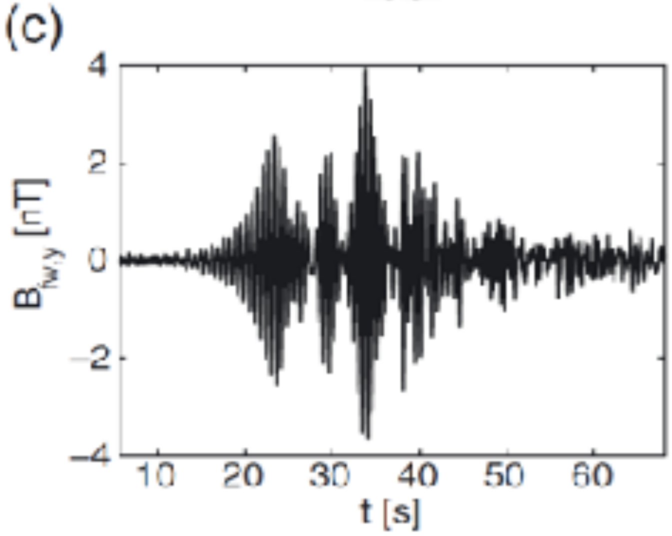
Outside Plasmapause



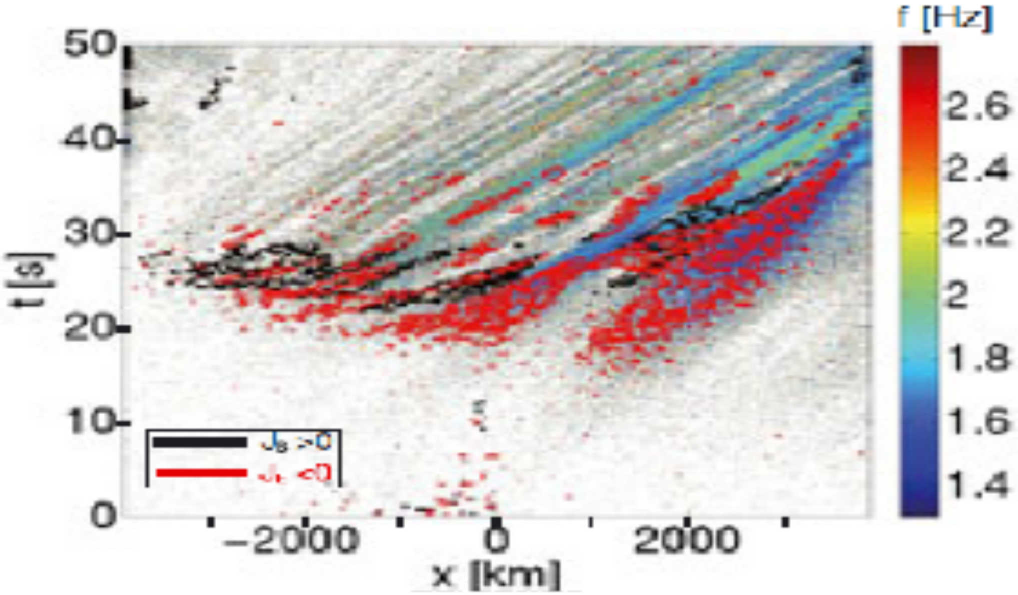
[*Nakamura et al., JGR,* 2014]

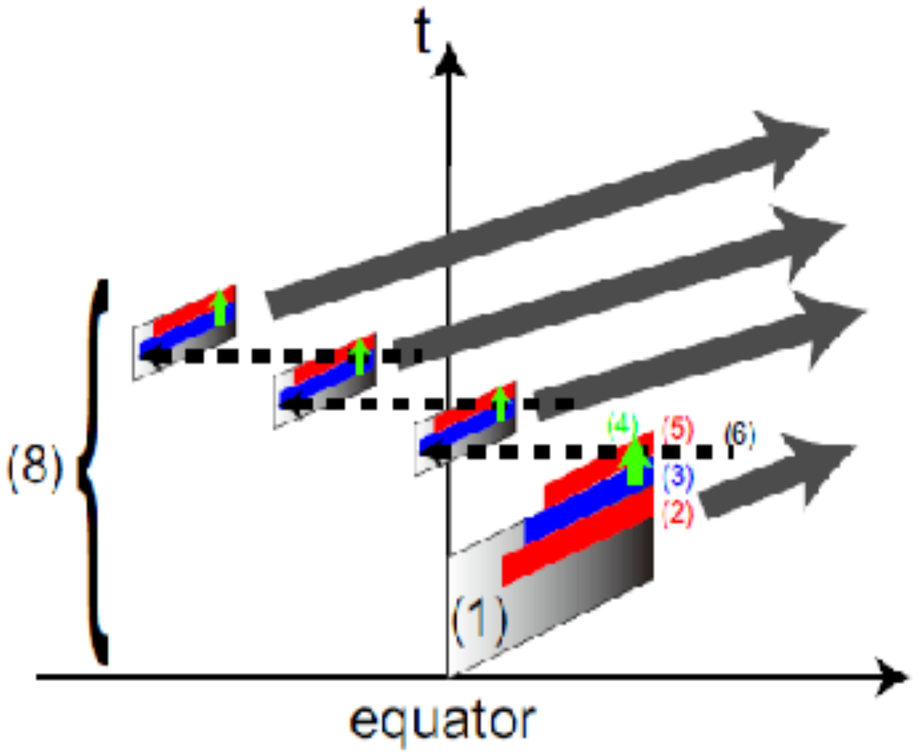
EMIC Rising Tone Emissions

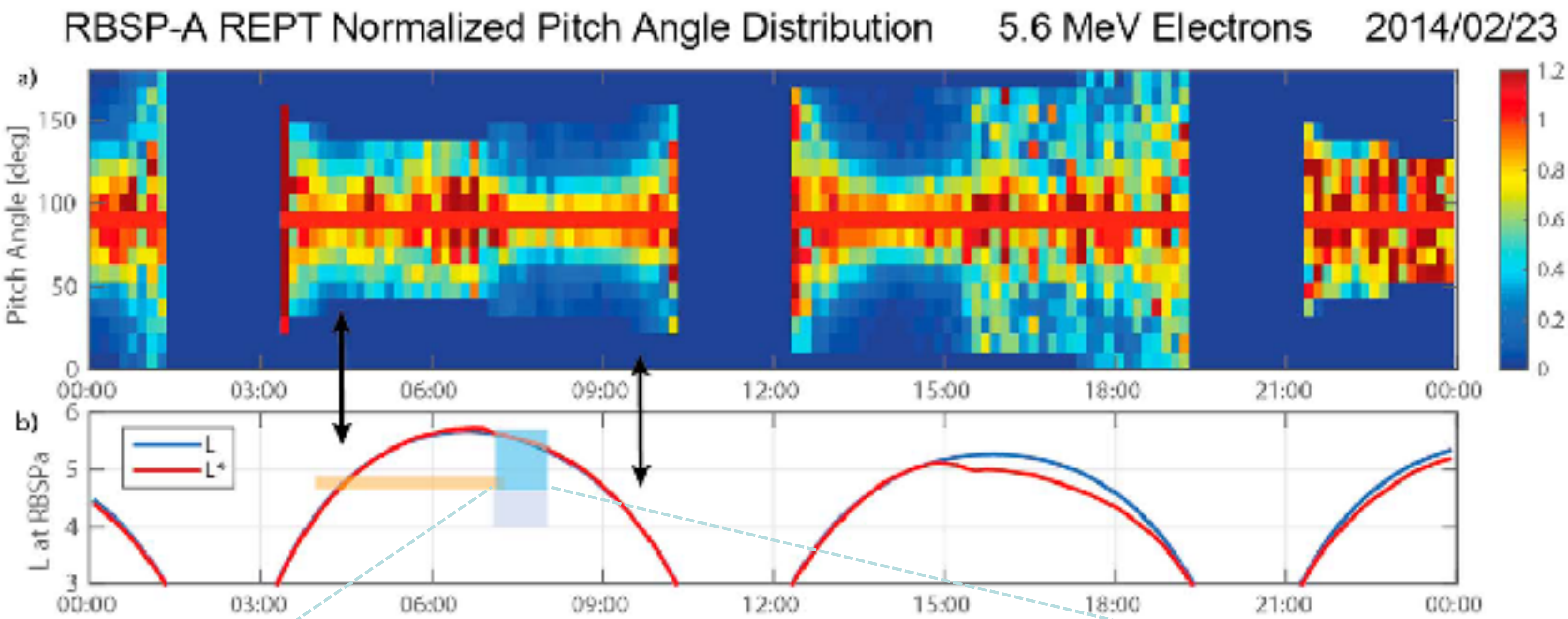




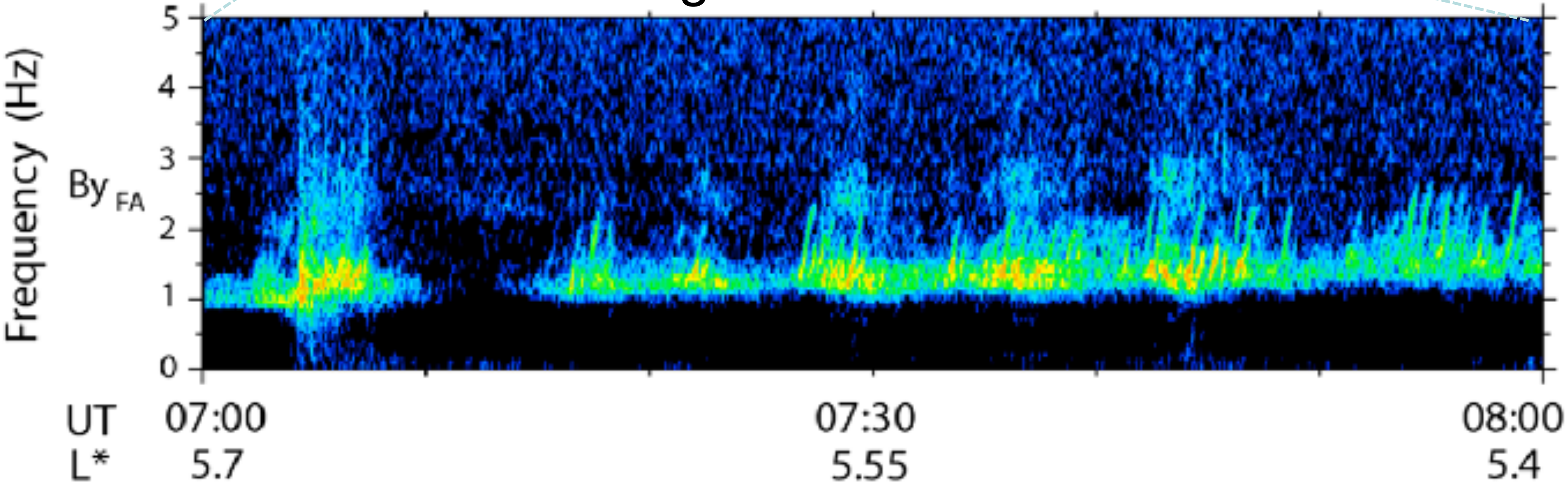
[*Shoji and Omura, JGR*, 2013]







EMIC rising-tone emissions



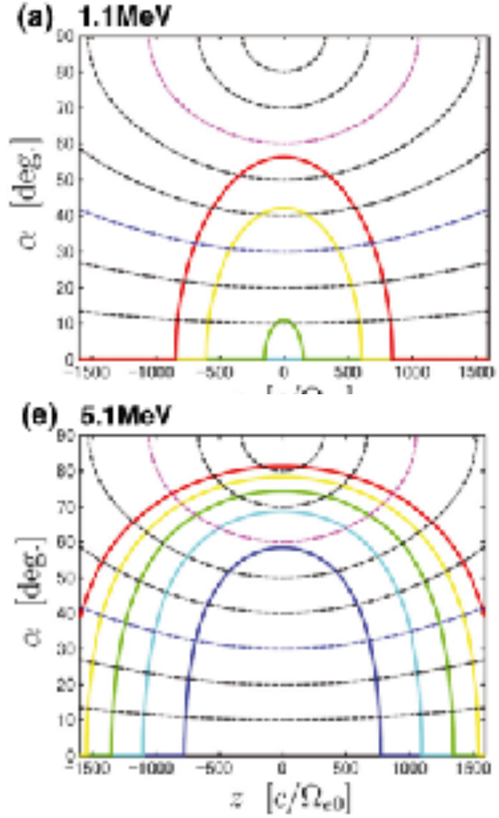
[*Engebretson et al., JGR*, 2015]

Anomalous Cyclotron Resonance

The first-order resonance condition

$$\omega - k v_{\parallel} = -\frac{\Omega_e}{\gamma}$$

$$\omega < \Omega_H << \Omega_e \qquad V_R = \frac{\Omega_e}{\gamma k}$$

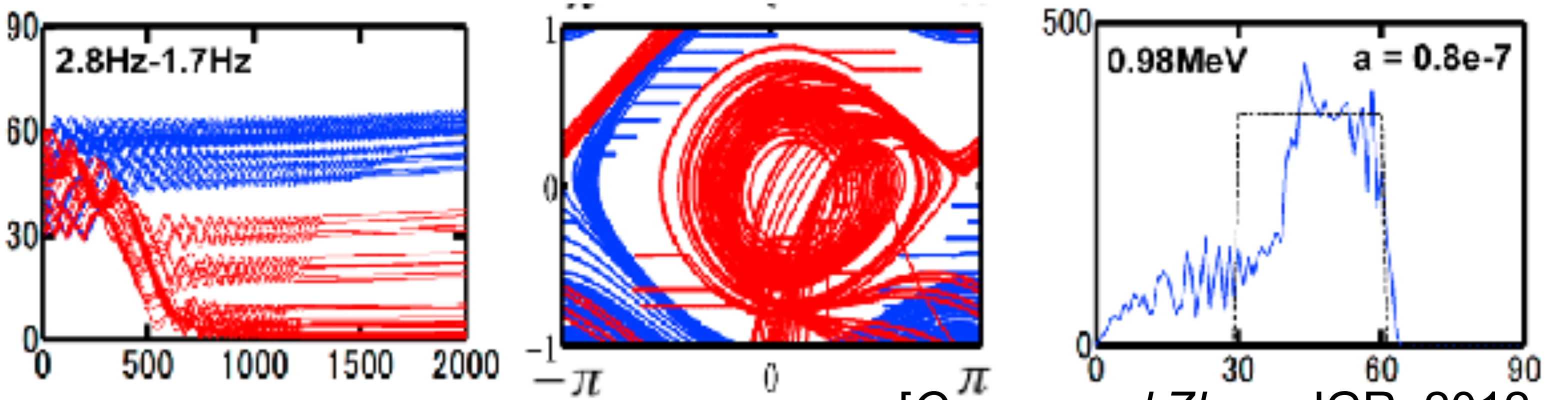


The second-order resonance condition

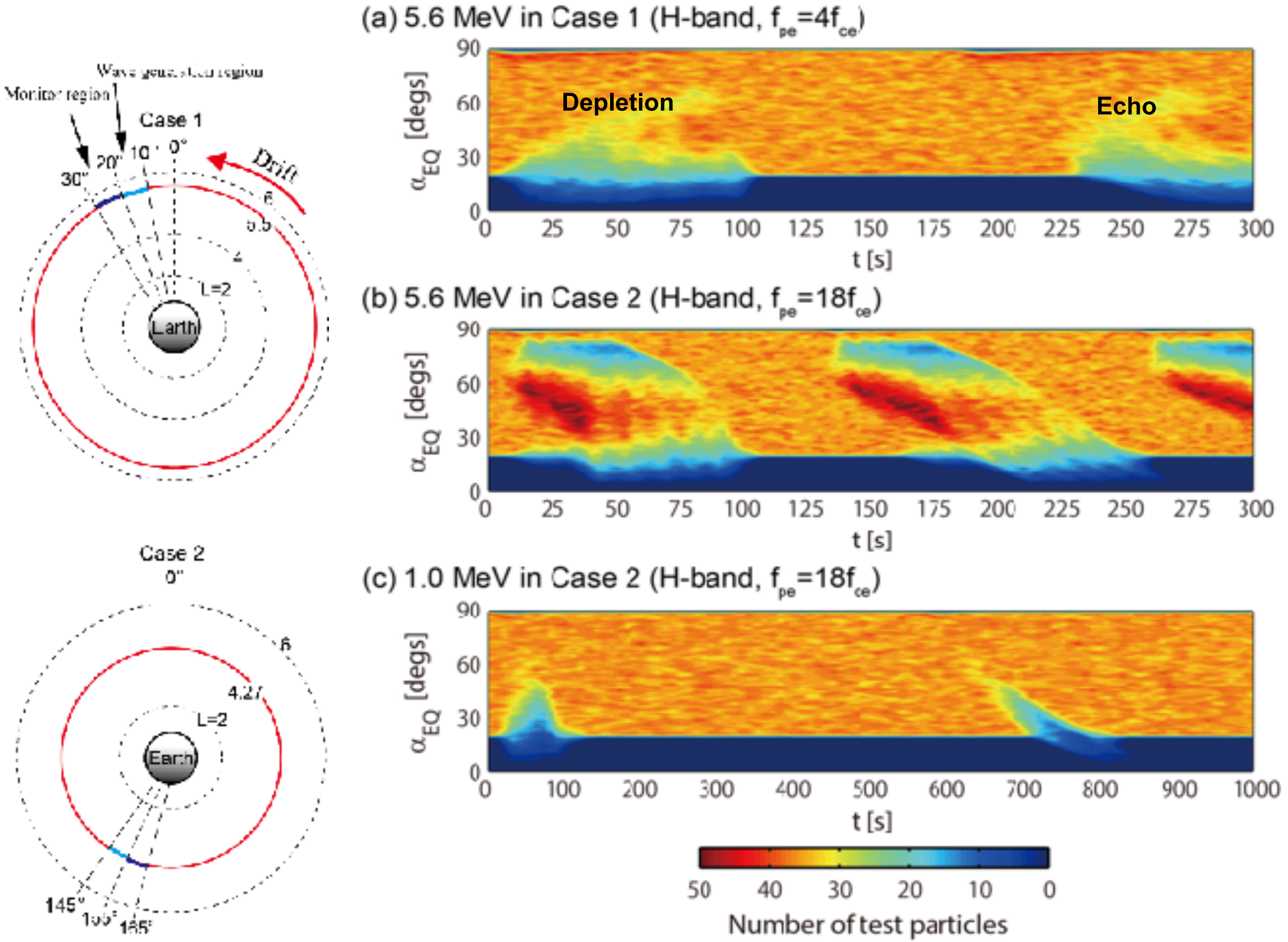
$$\frac{d\theta}{dt} = \omega_{tr}^2(\sin\zeta + S) \qquad |S| < 1$$

$$\frac{d\zeta}{dt} = -\theta \qquad S = -\frac{1}{\omega_{tr}^2}\left(s_1 \frac{\partial \omega}{\partial t} + s_2 V_p \frac{\partial \Omega_e}{\partial z}\right)$$

$$\left(\frac{d\alpha}{dt}\right)_{wave} = -\frac{1}{k v_{\perp}}\left(s_1 \frac{\partial \omega}{\partial t} + 2 s_2 V_p \Omega_{e0} a z\right)$$



[*Omura and Zhao*, JGR, 2012.2013]



(a) 5.6 MeV in Case 1 (H-band, $f_{pe}=4f_{ce}$)

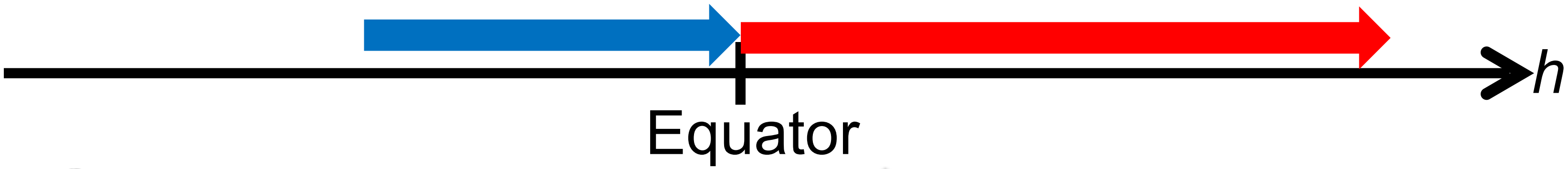
(b) 5.6 MeV in Case 2 (H-band, $f_{pe}=18f_{ce}$)

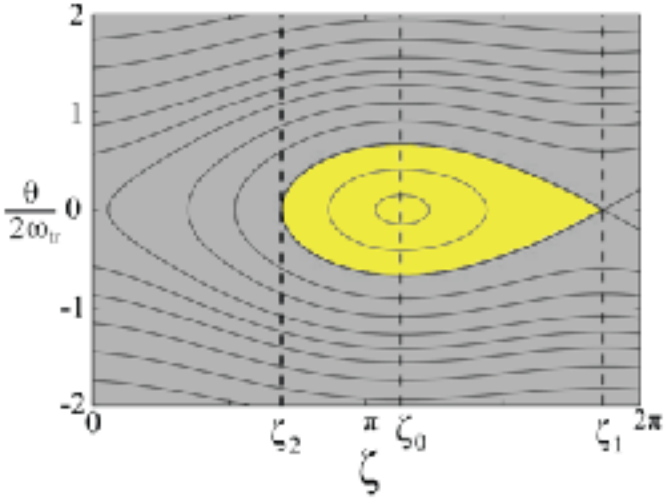
(c) 1.0 MeV in Case 2 (H-band, $f_{pe}=18f_{ce}$)

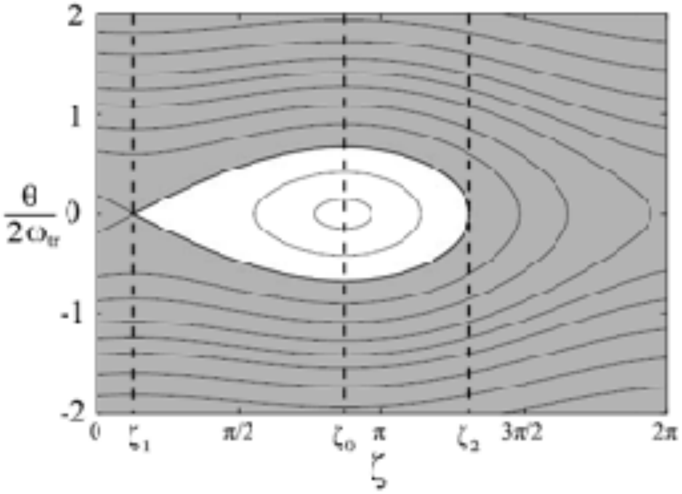
[Kubota and Omura, JGR, 2017]

Nonlinear Wave Growth near Equator

$$\partial\Omega_e/\partial h \sim 0 \qquad S = -\frac{s_1}{s_0\omega\Omega_w}\frac{\partial\omega}{\partial t}$$

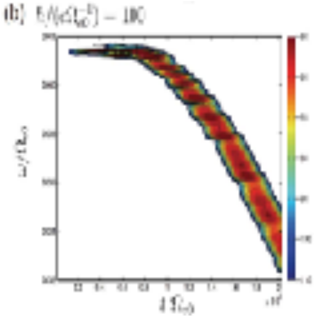






Electron Hill
for Falling Tone

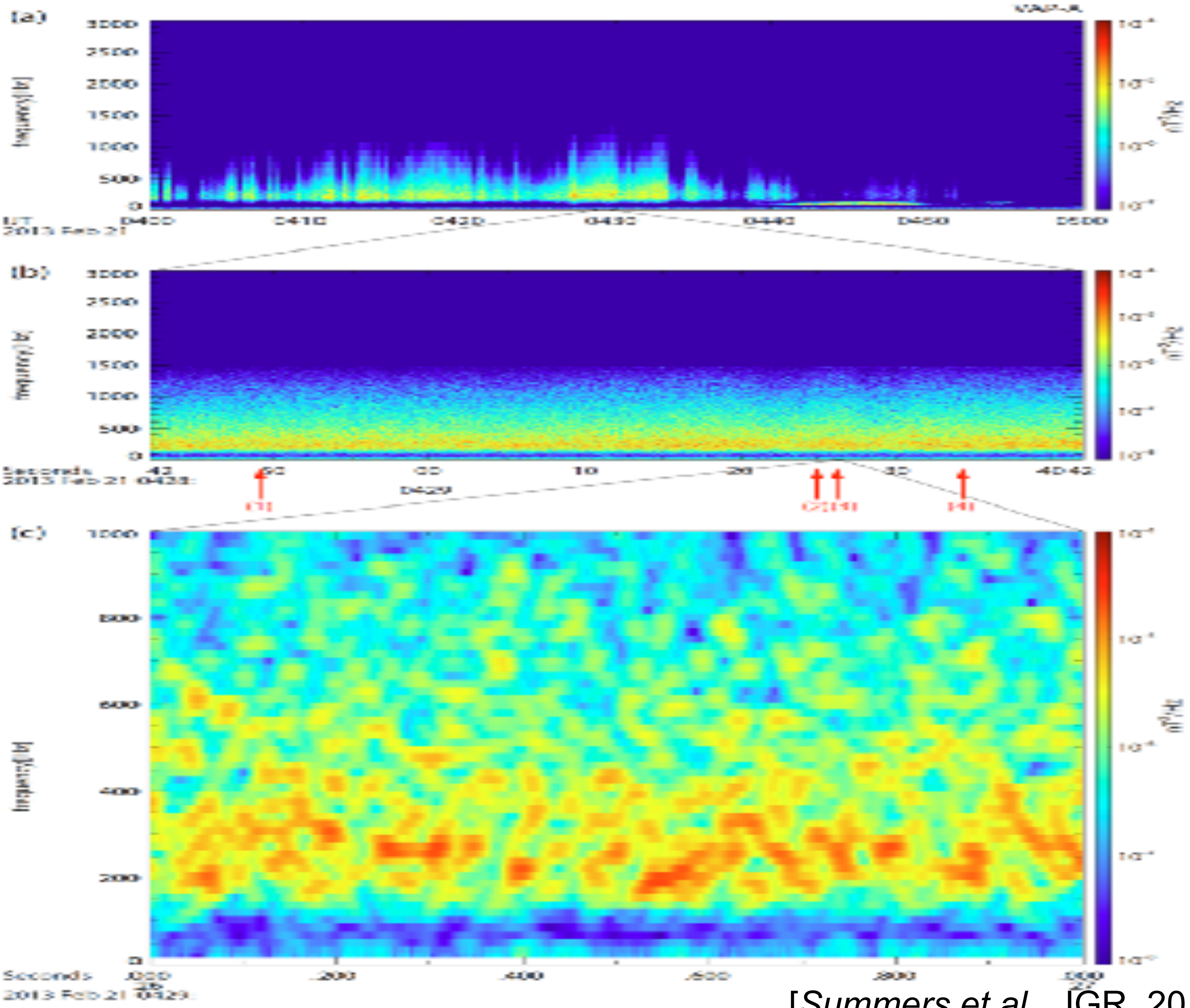
$S > 0$



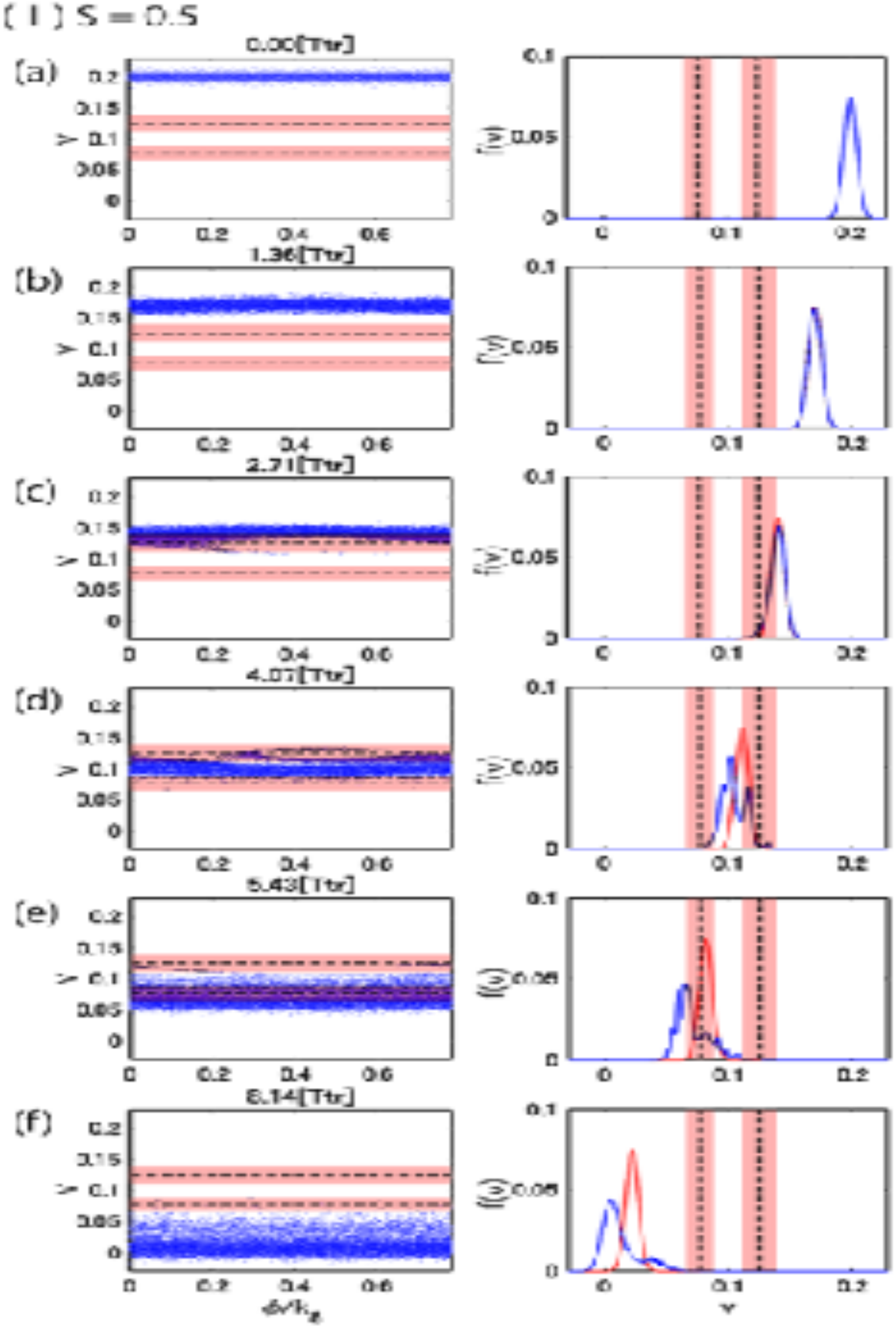
Electron Hole
for Rising Tone

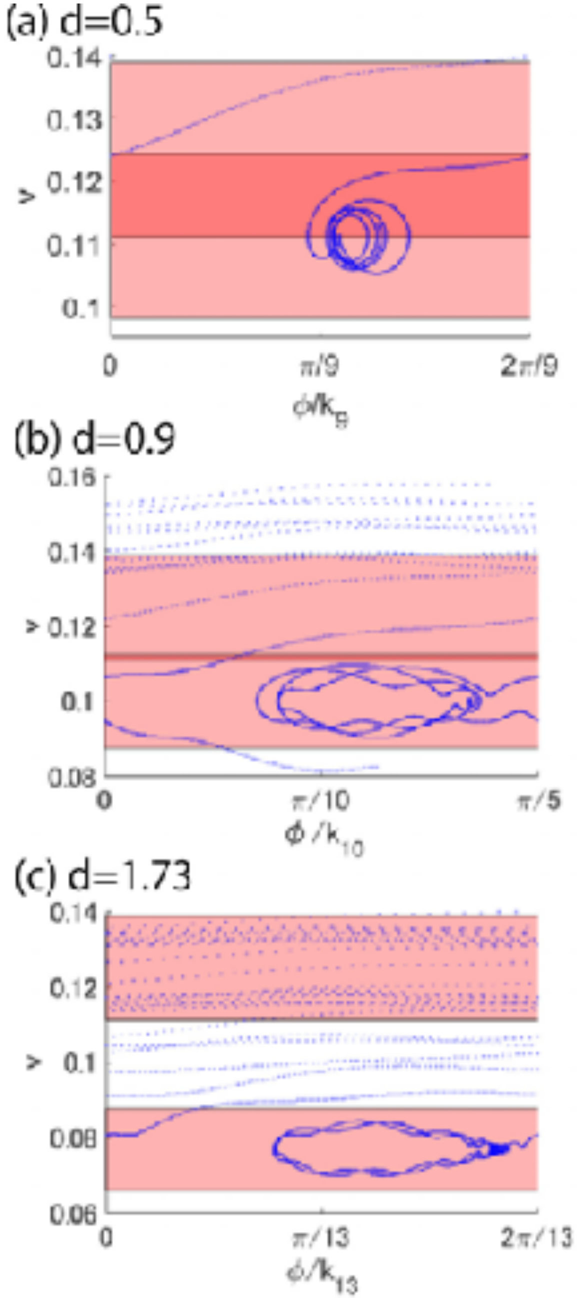
$S < 0$

[*Nunn and Omura, JGR*, 2012; *Omura, Nunn, Summers, AGU Monograph*, 2012]



[*Summers et al.*, JGR, 2014]

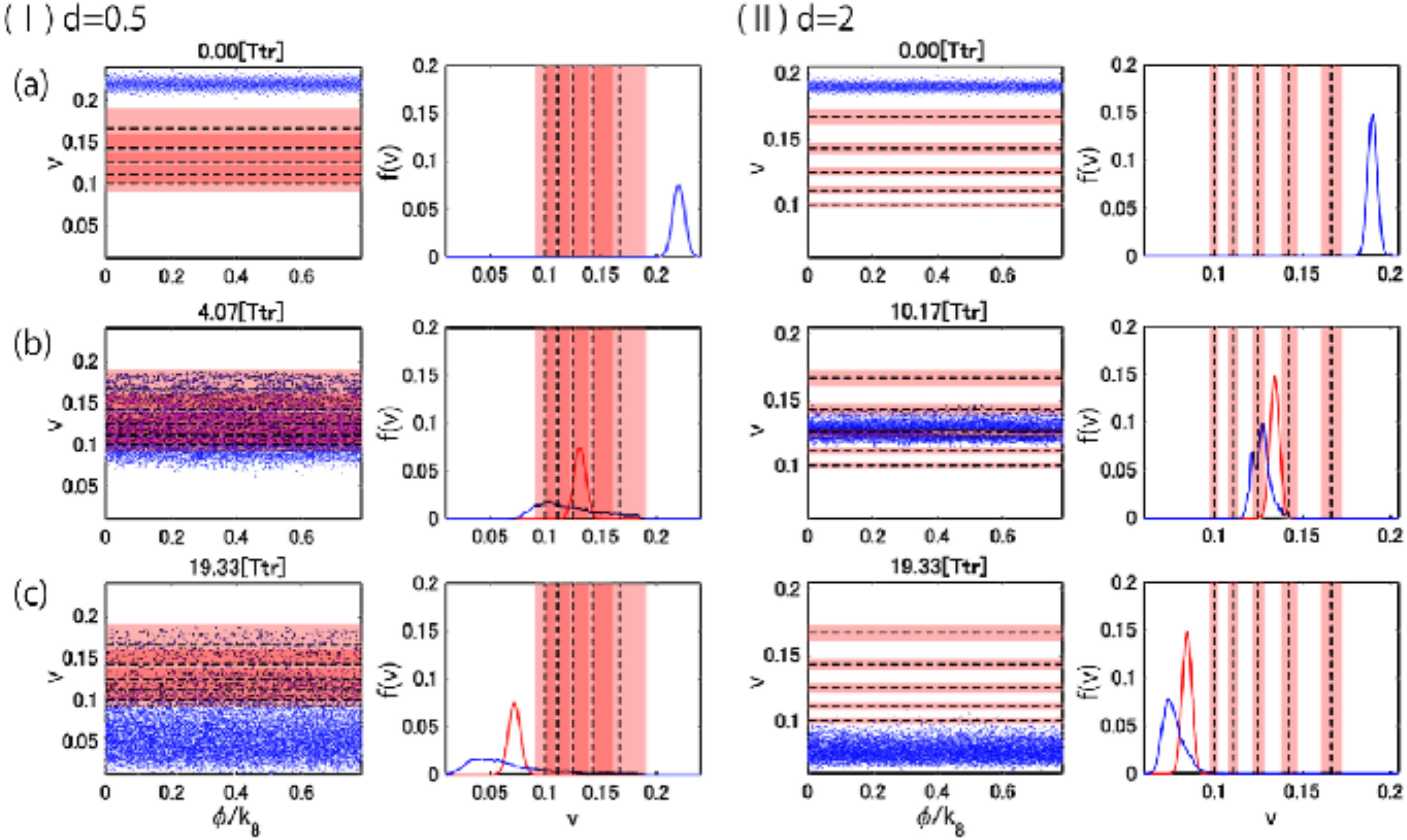




[*Tobita and Omura, PoP*, 2018]

Incoherent Waves

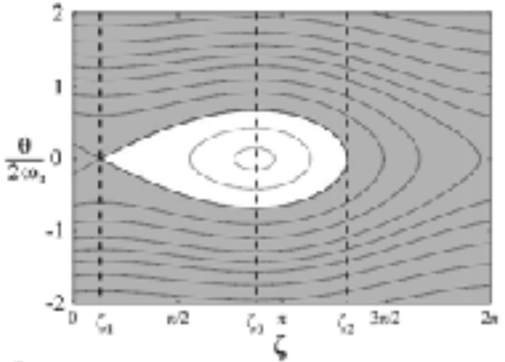
Coherent Waves



[*Tobita and Omura, PoP*, 2018]

Separability Criterion

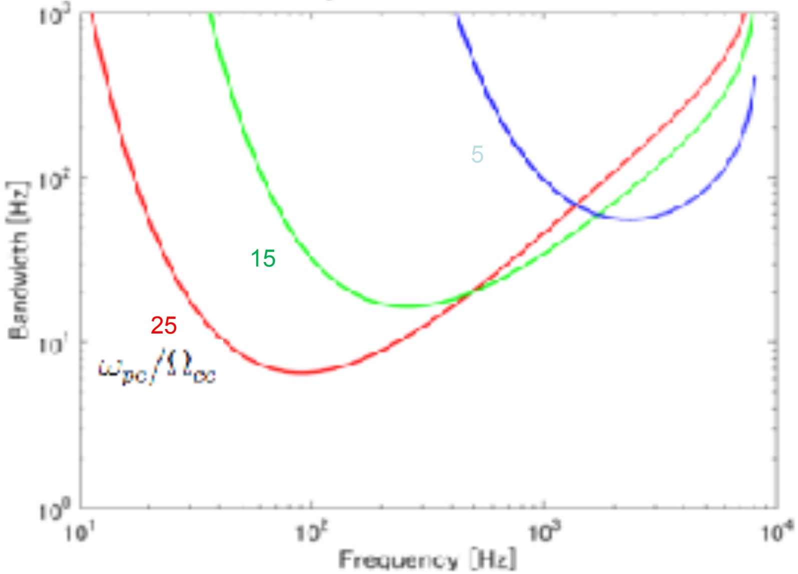
Assuming a hiss emission comprising *n* coherent waves, we have n trapping potentials in the velocity phase space.

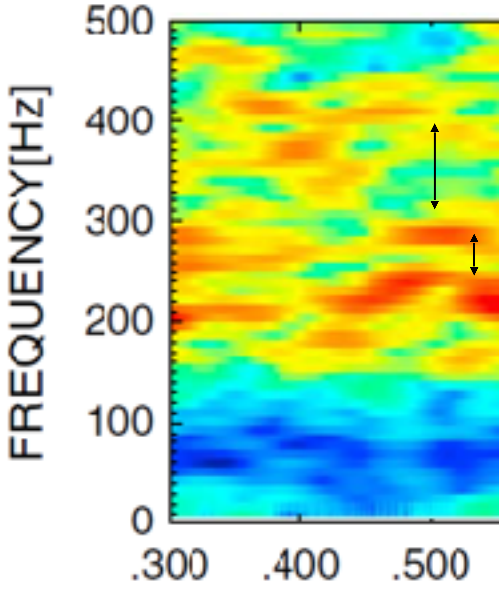


$$V_R^j - V_{tr}^j < v_\parallel < V_R^j + V_{tr}^j$$
$$(\ j = 1, 2, 3, ..., n)$$

We assume that the frequencies are separated so that there is no overlap of the trapping potentials.

$$\boxed{|V_R^{j+1} - V_R^j| \gg V_{tr}^j + V_{tr}^{j+1}}$$

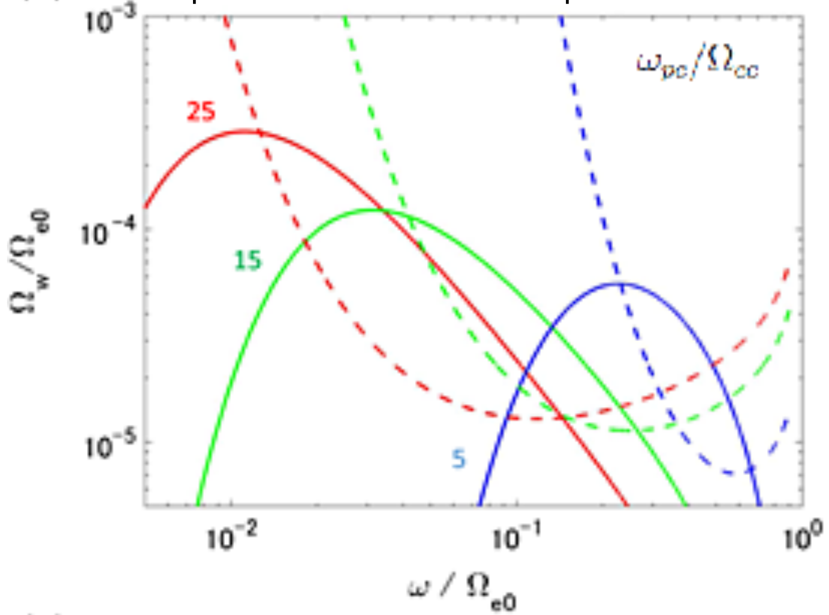




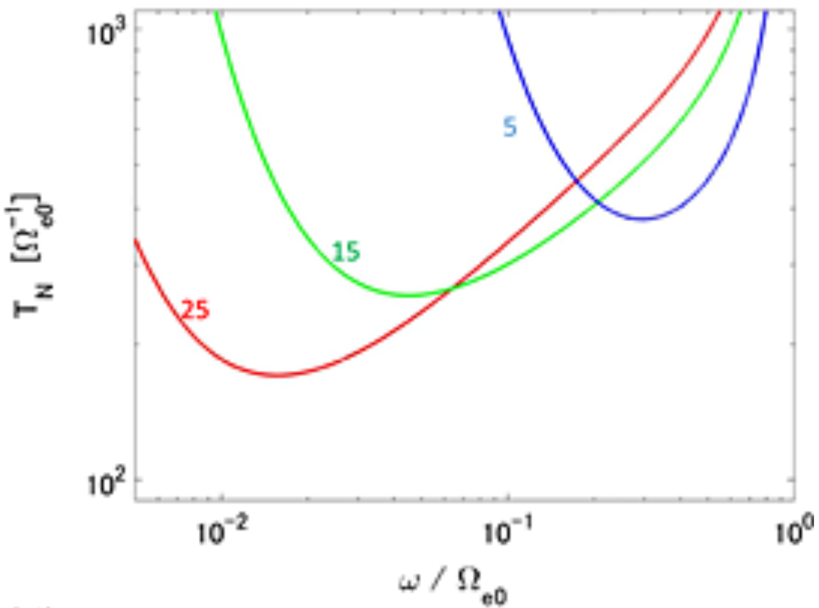
$$\Delta\omega = \left(\frac{\partial V_R}{\partial \omega}\right)^{-1} (2V_{tr})$$

$$\Delta\omega = 4\omega_{tr}\left\{1 + \frac{\Omega_e/\gamma - \omega}{\omega}\chi^2\left[\xi^2 + \frac{\Omega_e}{2(\Omega_e - \omega)}\right]\right\}^{-1}$$

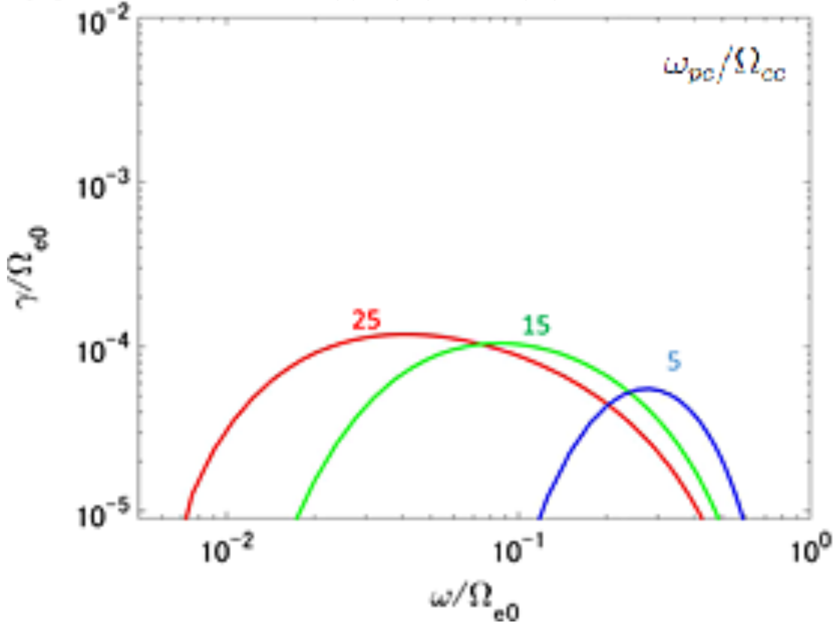
(a) Optimum and Threshold Amplitudes



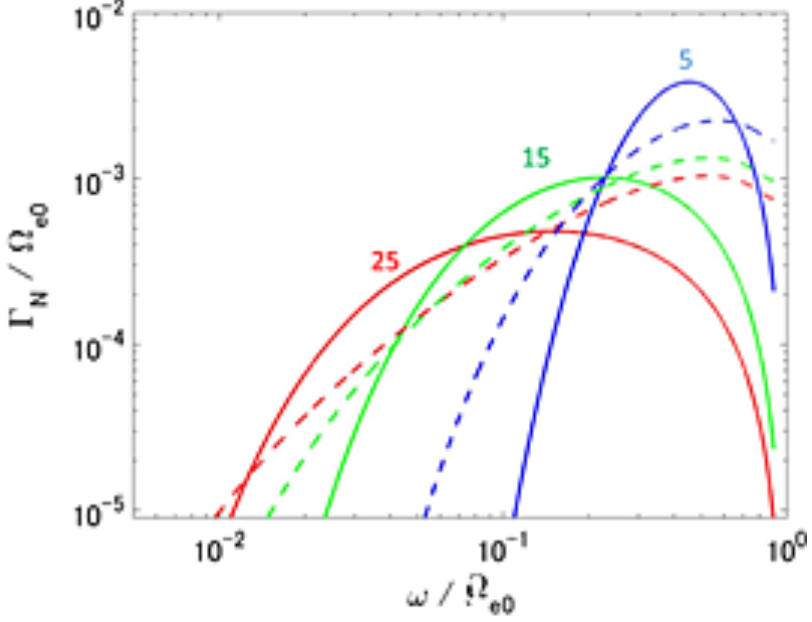
(b) Nonlinear Transition Time



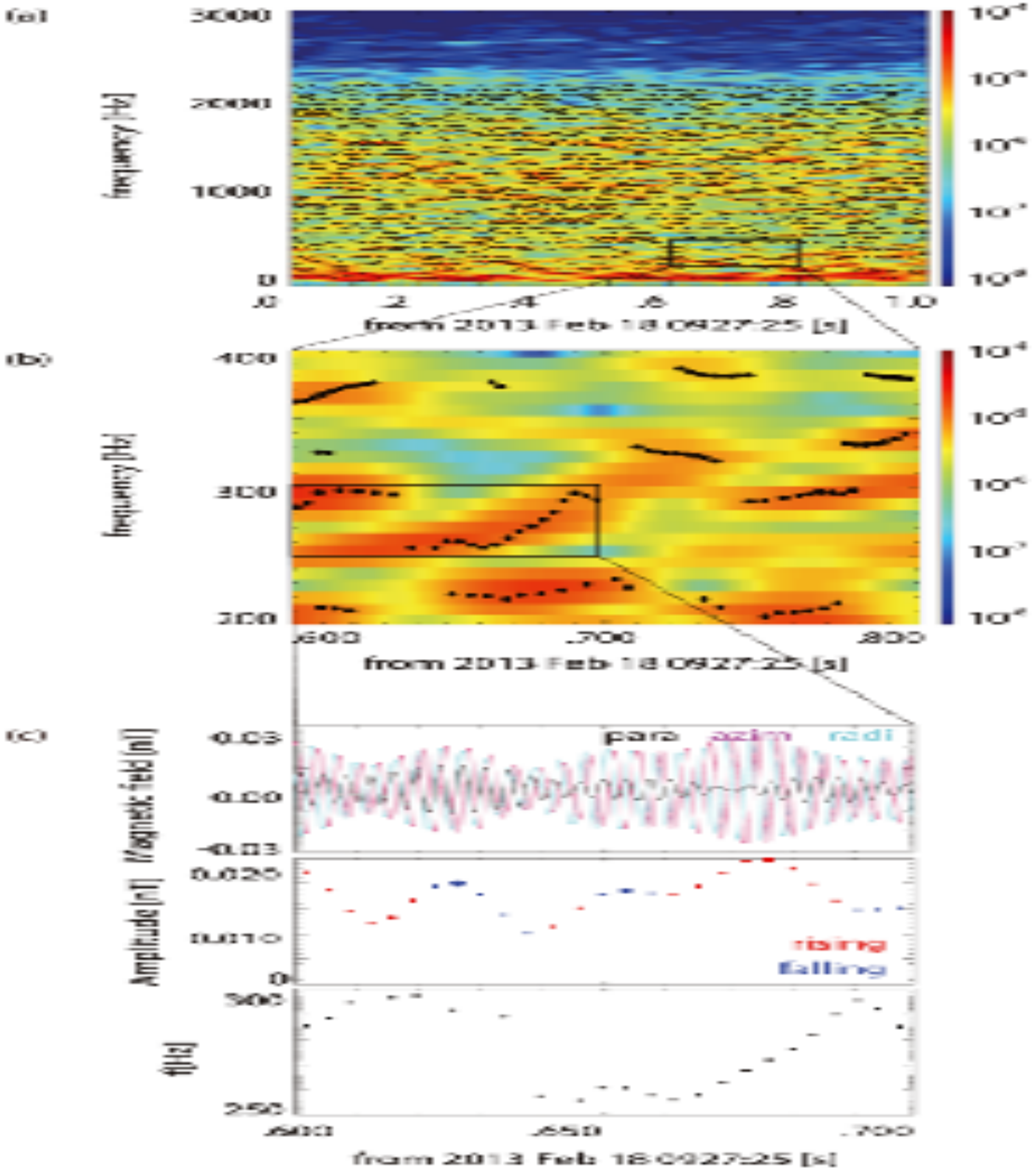
(c) Linear Growth Rate



(d) Nonlinear Growth Rate



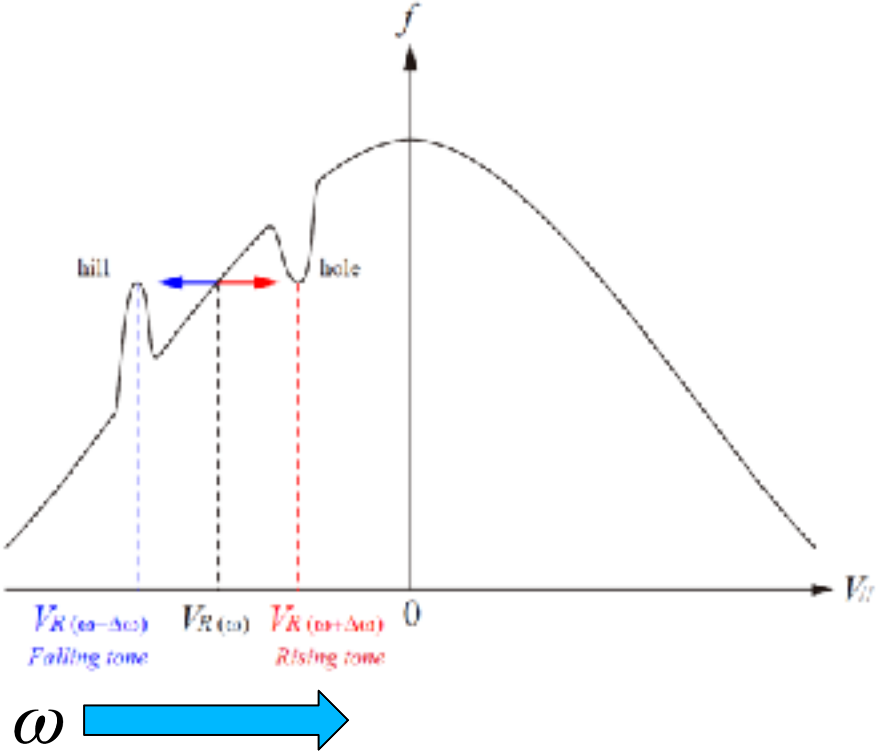
[*Omura et al., JGR*, 2015]



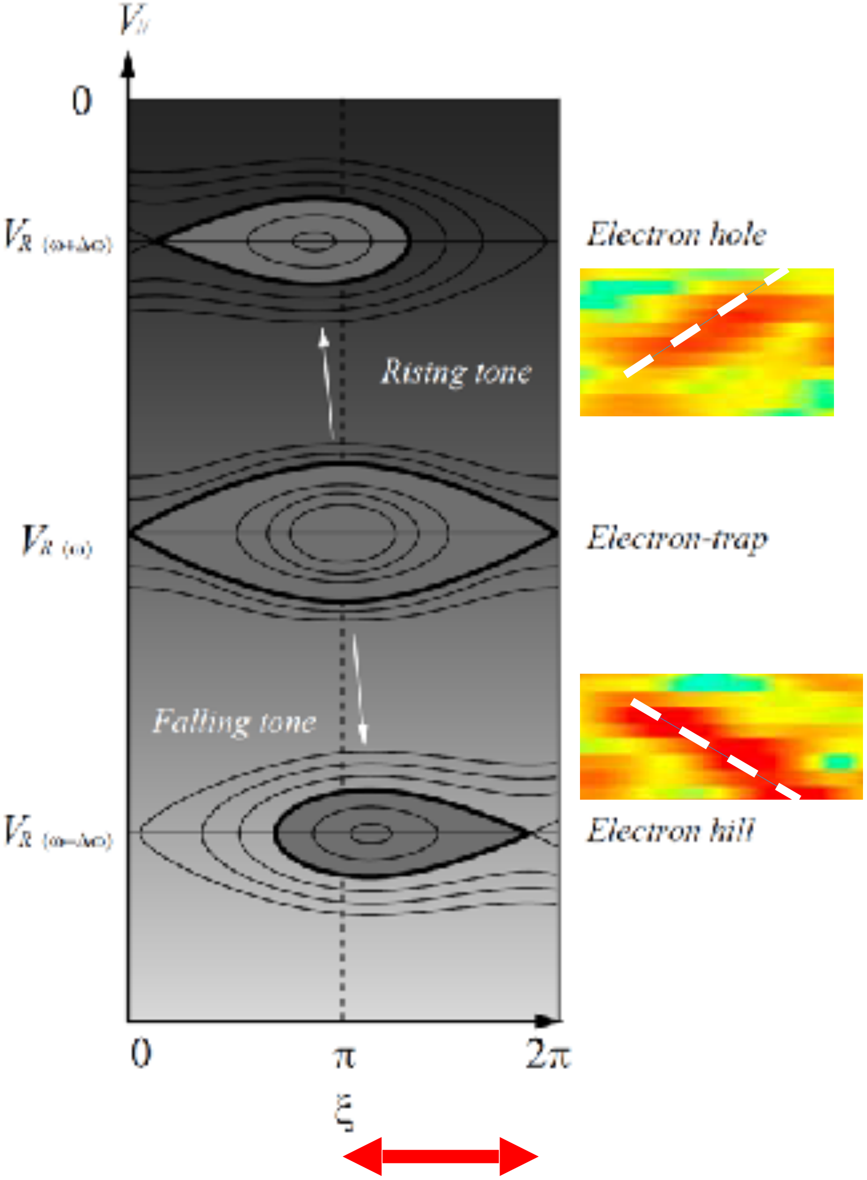
[*Nakamura et al.*, *GRL*, 2016]

Formation of Electron Hole (rising tone) and Hill (falling tone)

hiss element
= chorus subpacket



$$V_R = \frac{1}{k}\left(\omega - \frac{\Omega_e}{\gamma}\right)$$

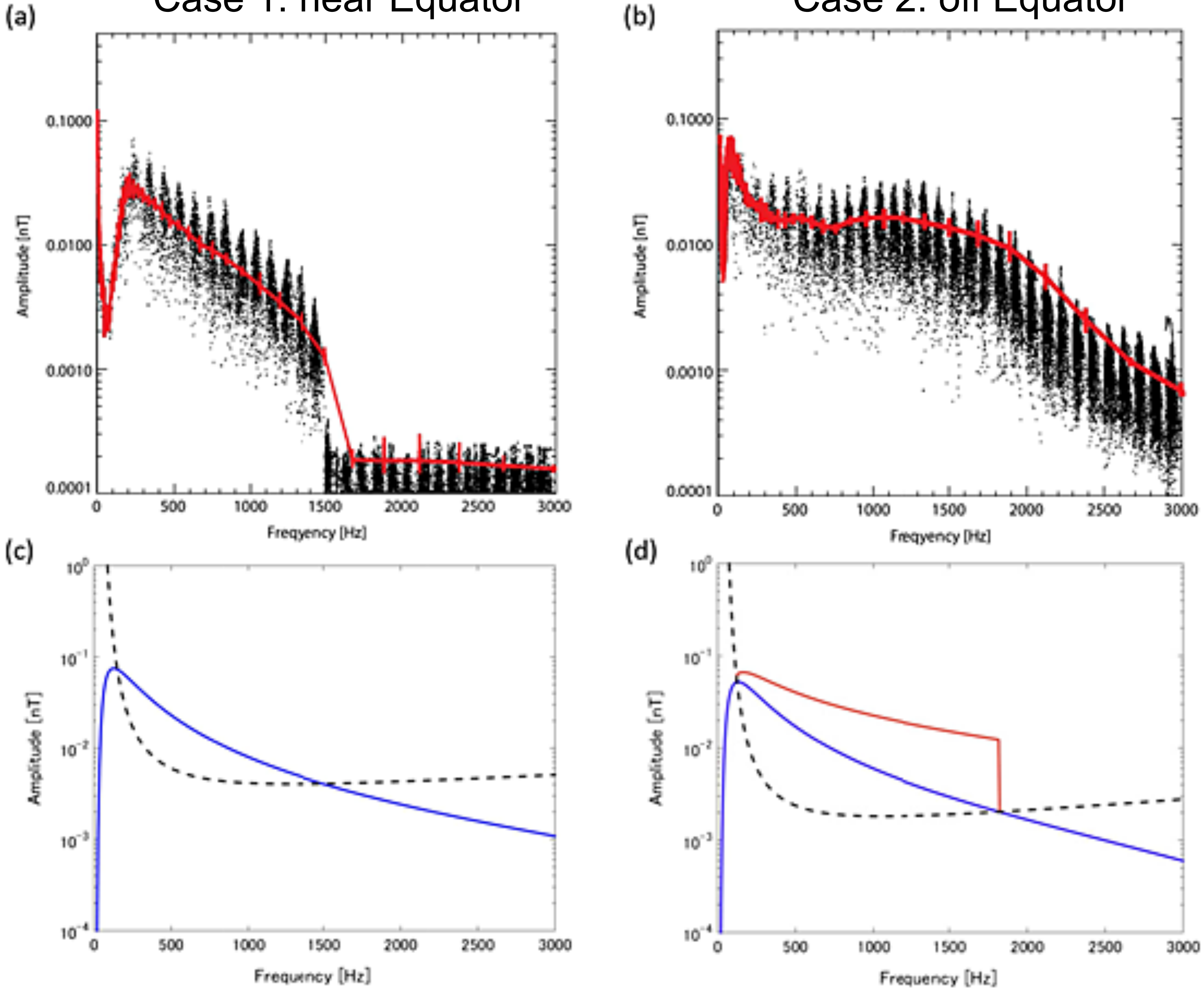


$$\boldsymbol{E_w} \cdot \boldsymbol{J_R} < 0$$

[*Omura et al.*, *JGR*, 2015]

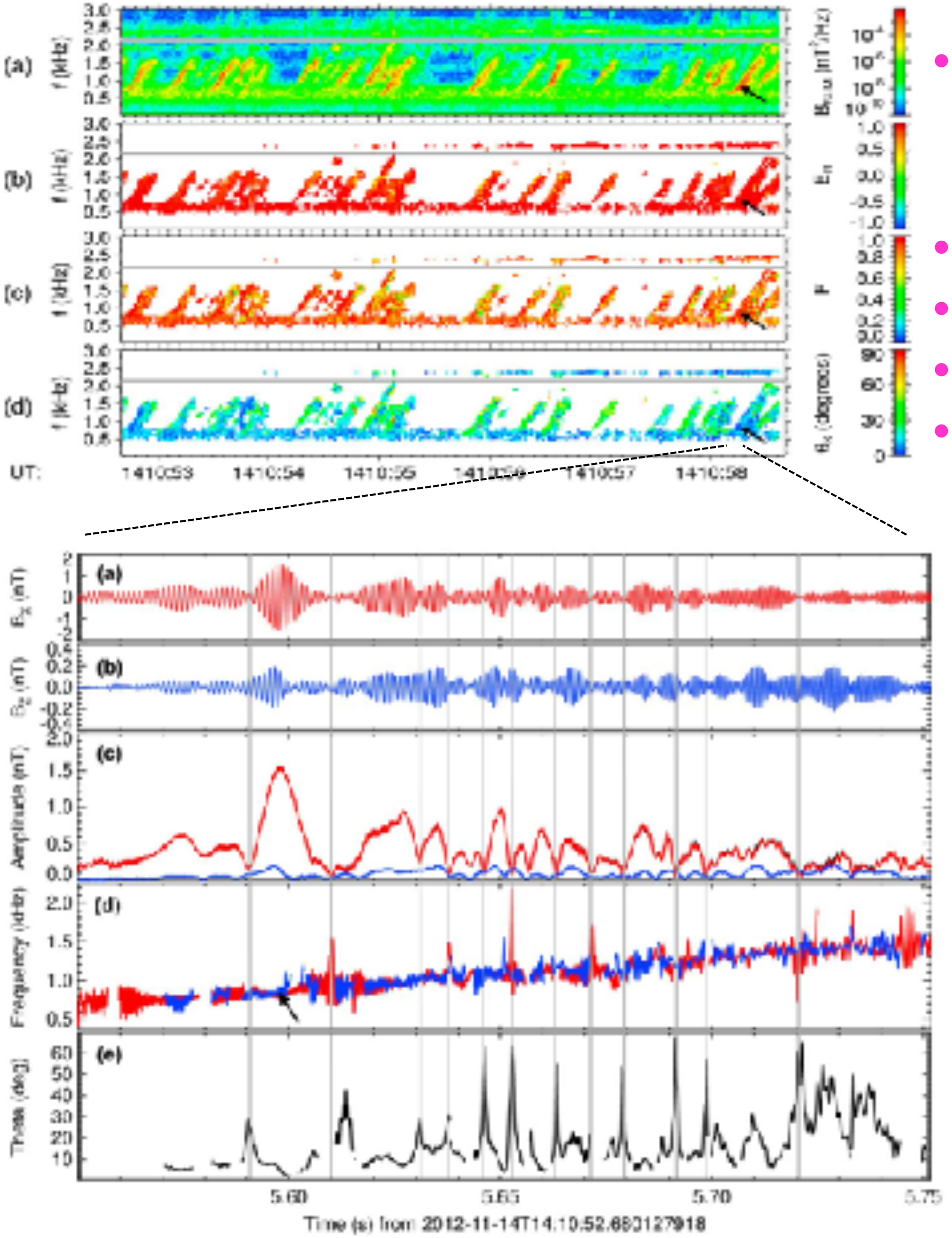
Case 1: near Equator

Case 2: off Equator

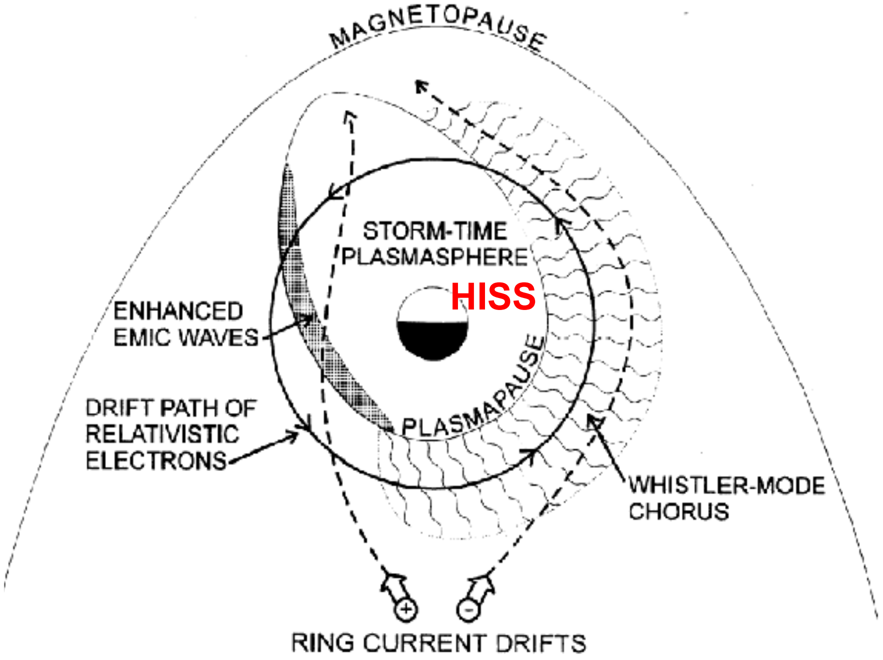


[*Omura et al.*, *JGR*, 2015]

Summary 1



- **Absolute Instability:**
  - **Rising Tone: Electron Hole**
  - **Falling Tone: Electron Hill**
- **Optimum Amplitude**
- **Threshold Amplitude**
- **Sub-packet Formation**
- **Convective Instability**



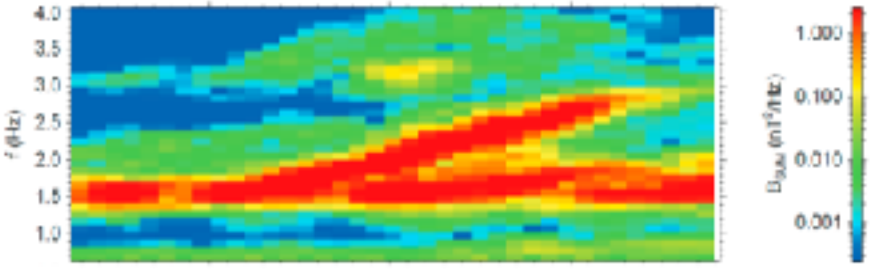
[*Summers et al., JGR,* 1998]

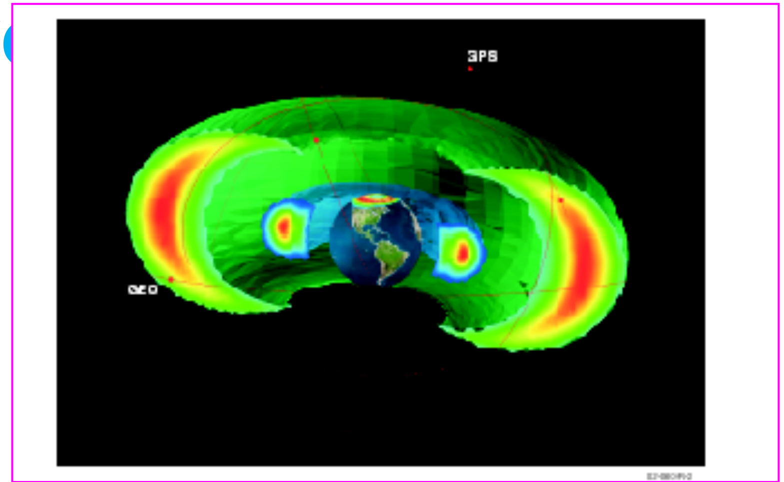
Summary 2

1. Rapid formation of the relativistic electron flux (0.5 - 6 MeV) takes place through nonlinear wave trapping by whistler-mode chorus emissions near the equator.

2. A substantial amount of relativistic electrons (0.5 – 6 MeV) is precipitated through nonlinear wav

rising-tone emissions





Review Articles

- Y. Omura, D. Nunn, and D. Summers, Generation processes of whistler-mode chorus emissions: Current status of nonlinear wave growth theory, *AGU Monograph "Dynamics of the Earth's Radiation Belts and Inner Magnetosphere"*, 10.1029/2012GM001347, 2012.

- Y. Omura, Theory and simulations of nonlinear wave-particle interactions in planetary radiation belts, *Radio Science Bulletin*, No. 349 (June) 52-58, 2014.