



Multiband chorus in the Earth's magnetosphere

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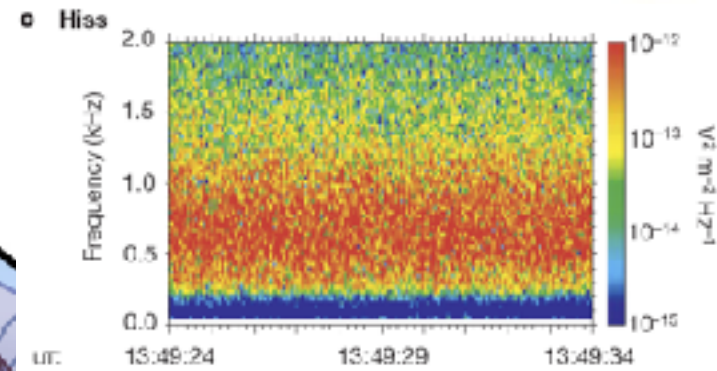
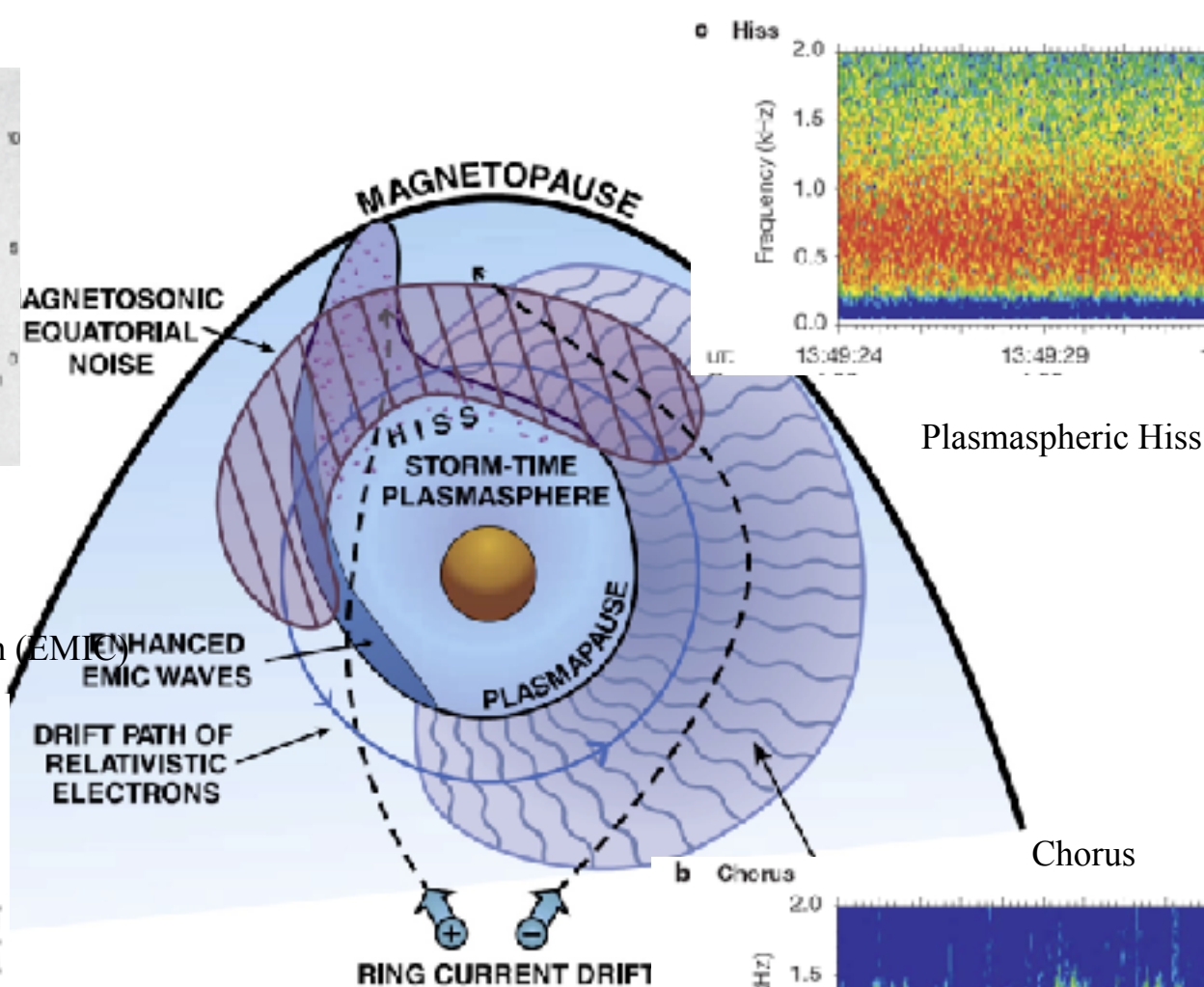
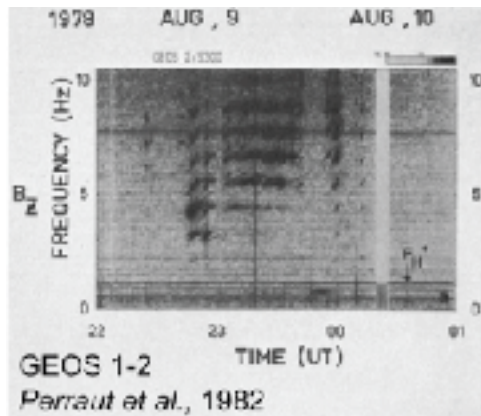


- Plasma waves in magnetosphere
- Chorus waves
- Multiband chorus:
 - Case study, Simulations, and Statistical analysis
- Summary

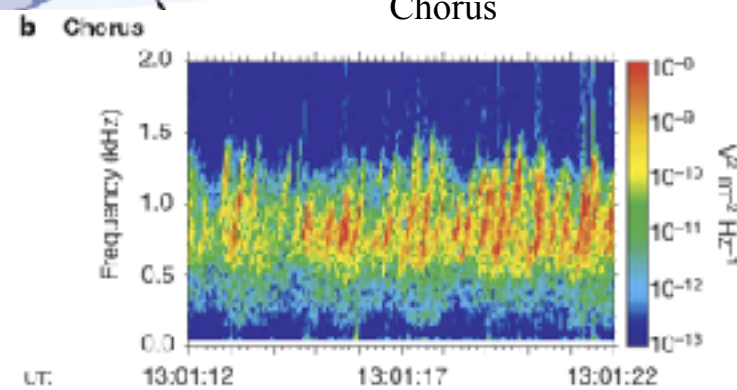
Plasma waves in magnetosphere



Magnetosonic waves



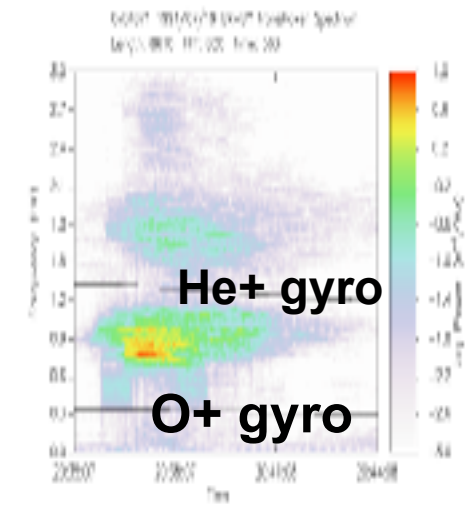
Plasmaspheric Hiss



Chorus

Thorne, GRL, 2010

Electromagnetic Ion Cyclotron (EMIC) waves

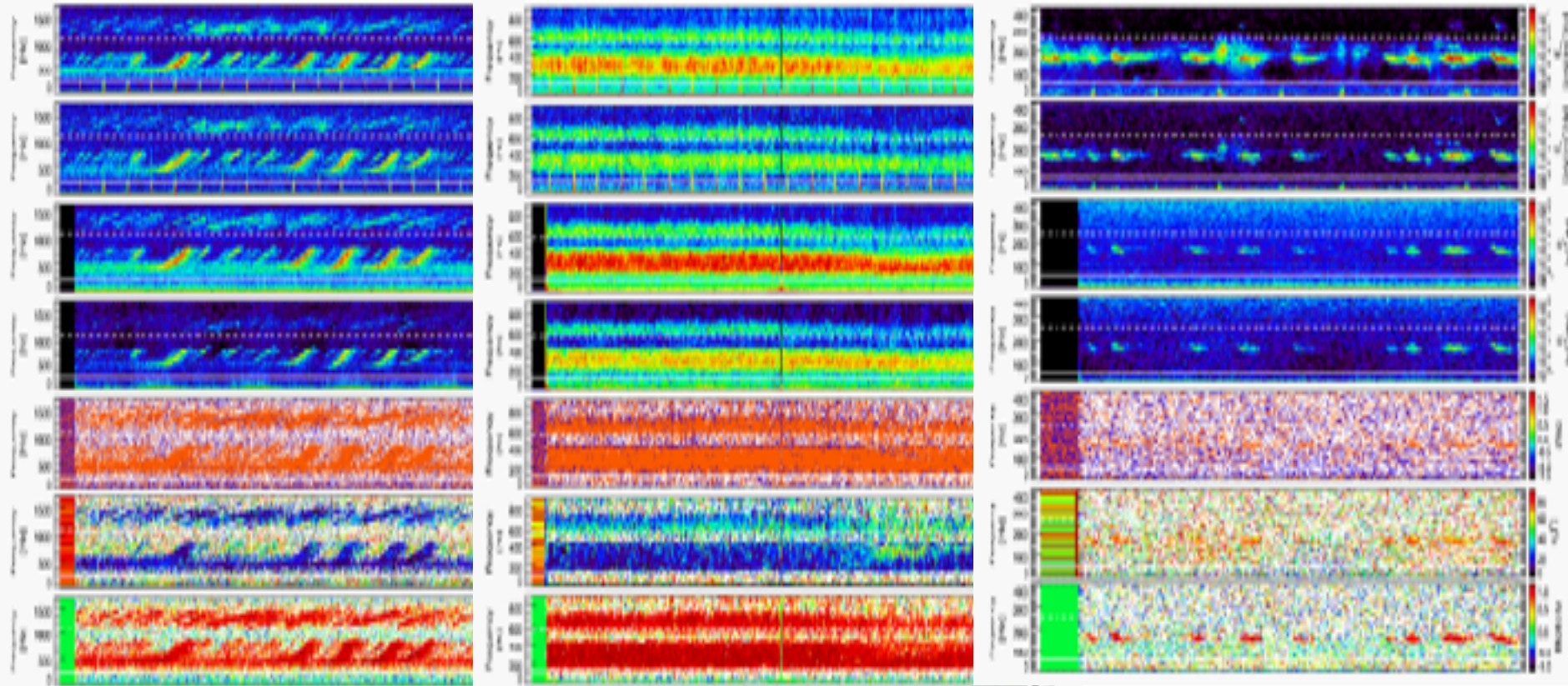




Rising tone

Hiss-like emission

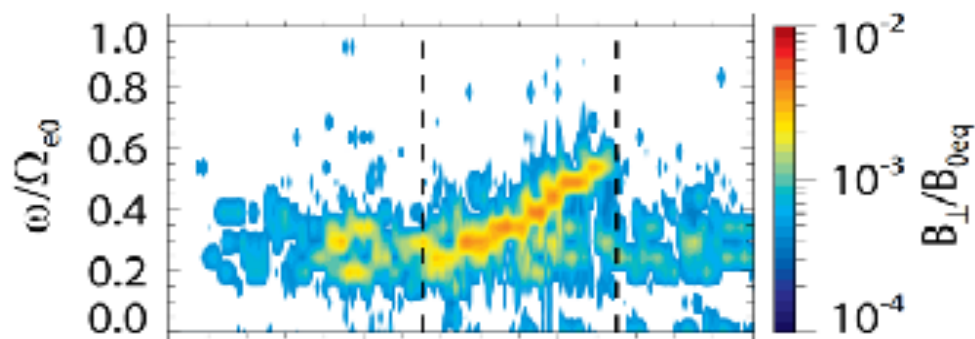
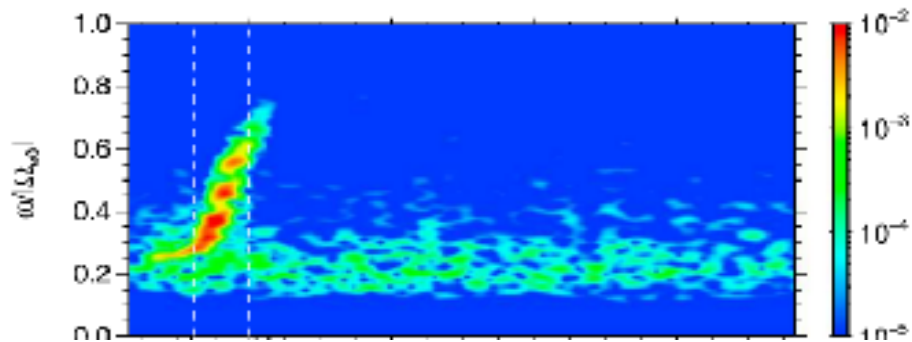
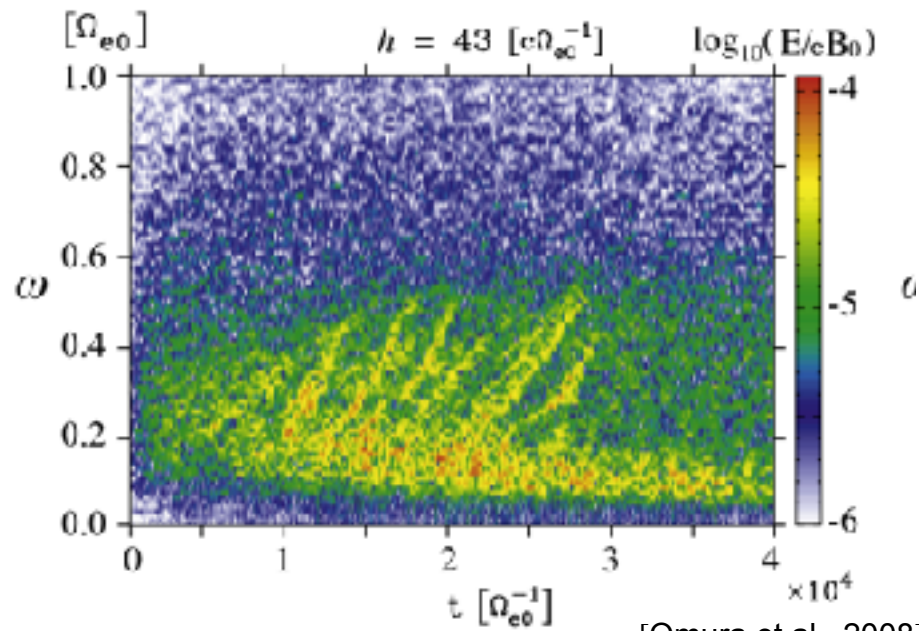
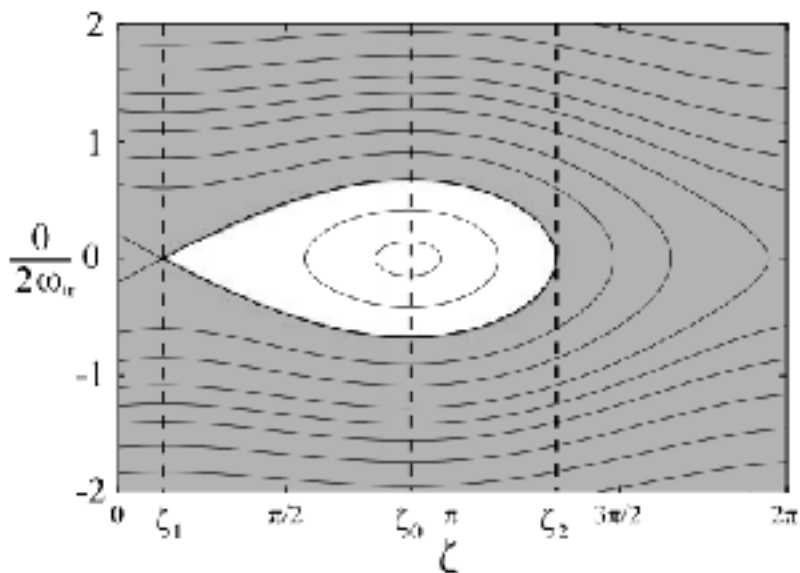
falling tone



- ◆ Frequency chirping
- ◆ Power gap at about $0.5f_{ce}$



Nonlinear trapping



[Tao, 2014]

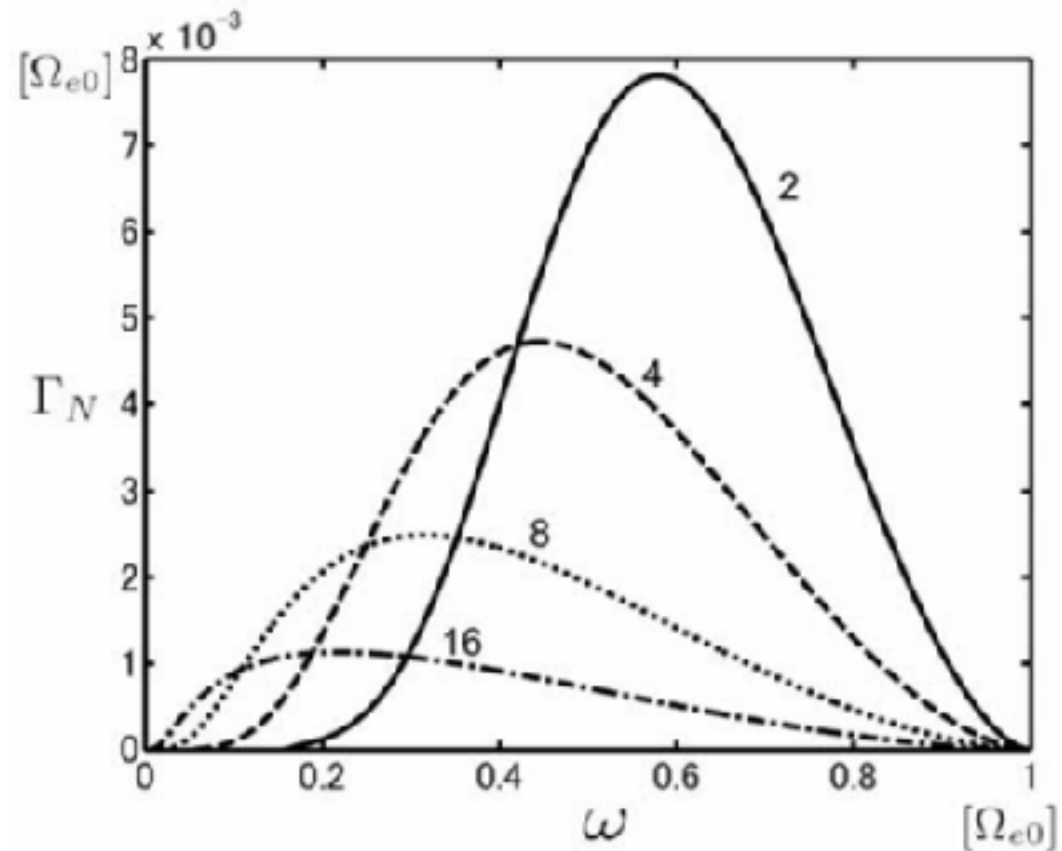
[Ke et al., 2017]



Nonlinear growth rate (taking into account of the inhomogeneous of background magnetic field):

$$\Gamma_N = \frac{Q\omega_{pe}^2}{2} \left(\frac{\xi}{\Omega_w \omega} \right)^{1/2} \frac{V_E}{U_{\perp}} \left(\frac{V_{\perp 0} \delta}{c\pi\gamma} \right)^{3/2} \exp\left(-\frac{\gamma^2 V_E^2}{2U_{\parallel}^2} \right)$$

Upper band wave is just the extension of lower band.

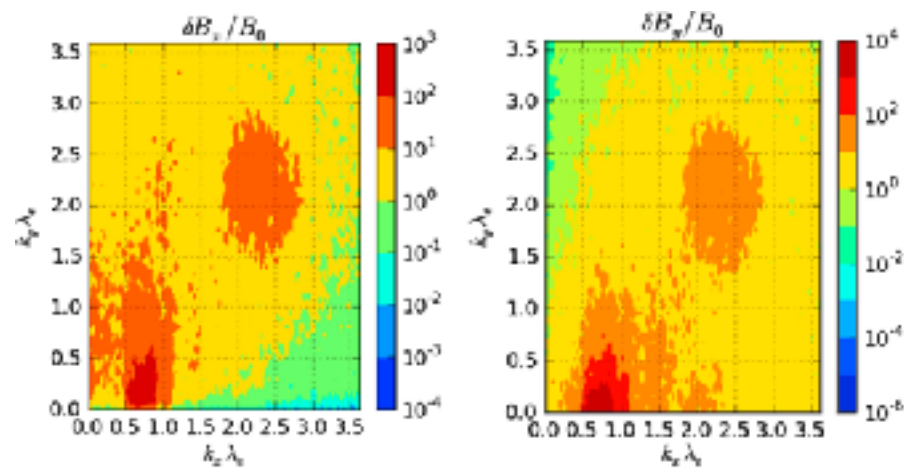
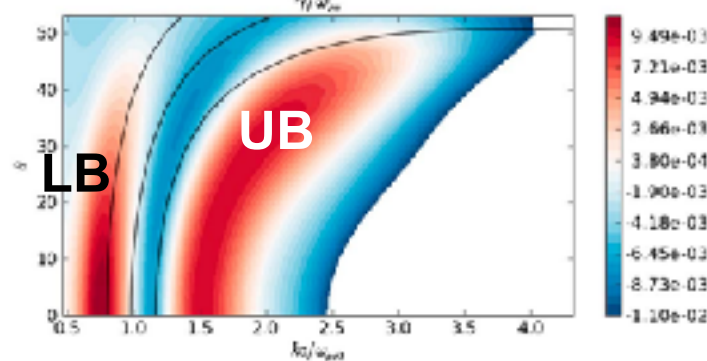
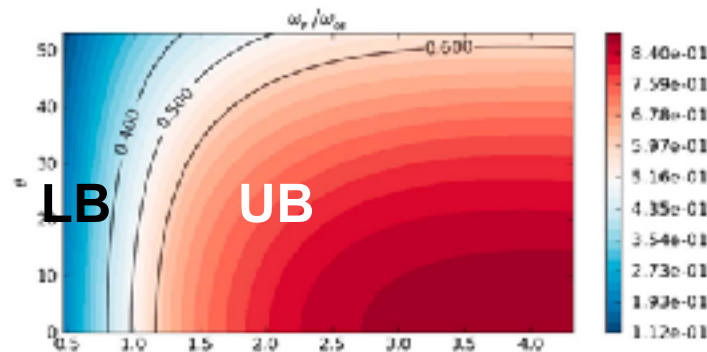
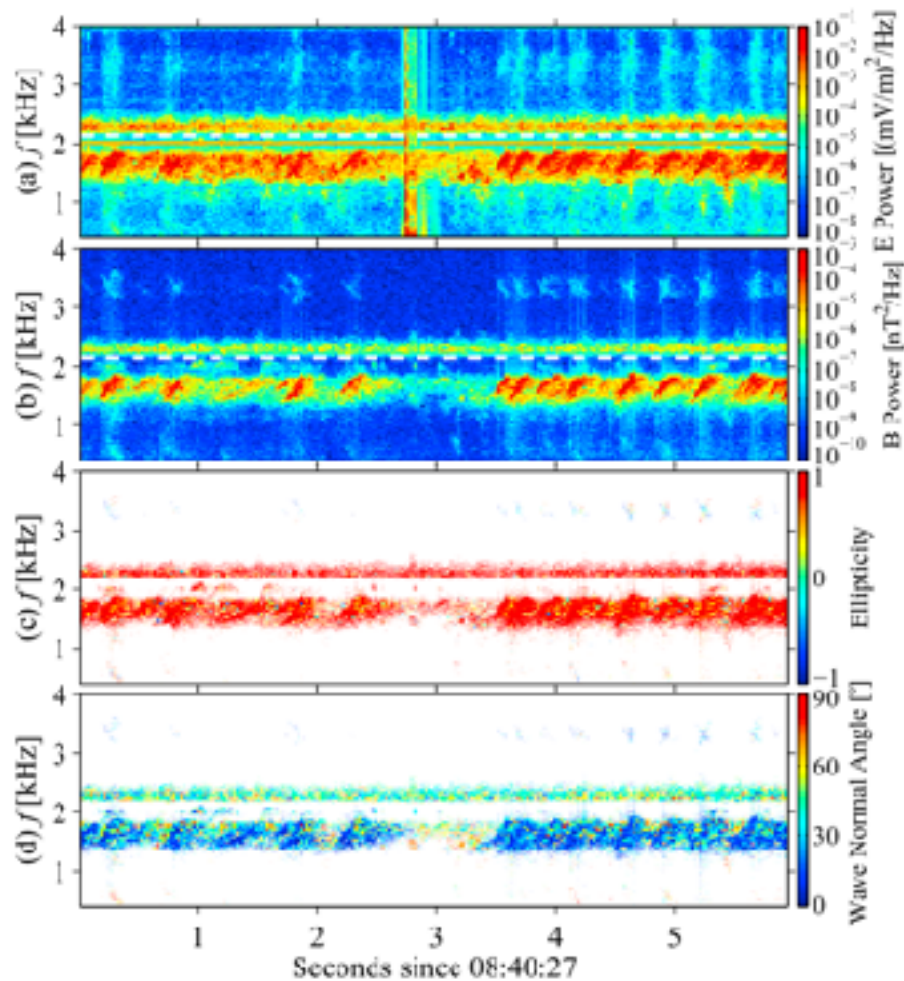


$$\omega_{pe}/\Omega_{e0} = 2, 4, 8, 16$$

[Omura et al., 2009]



RBSP-A



[Fu et al., 2014]



Power gap



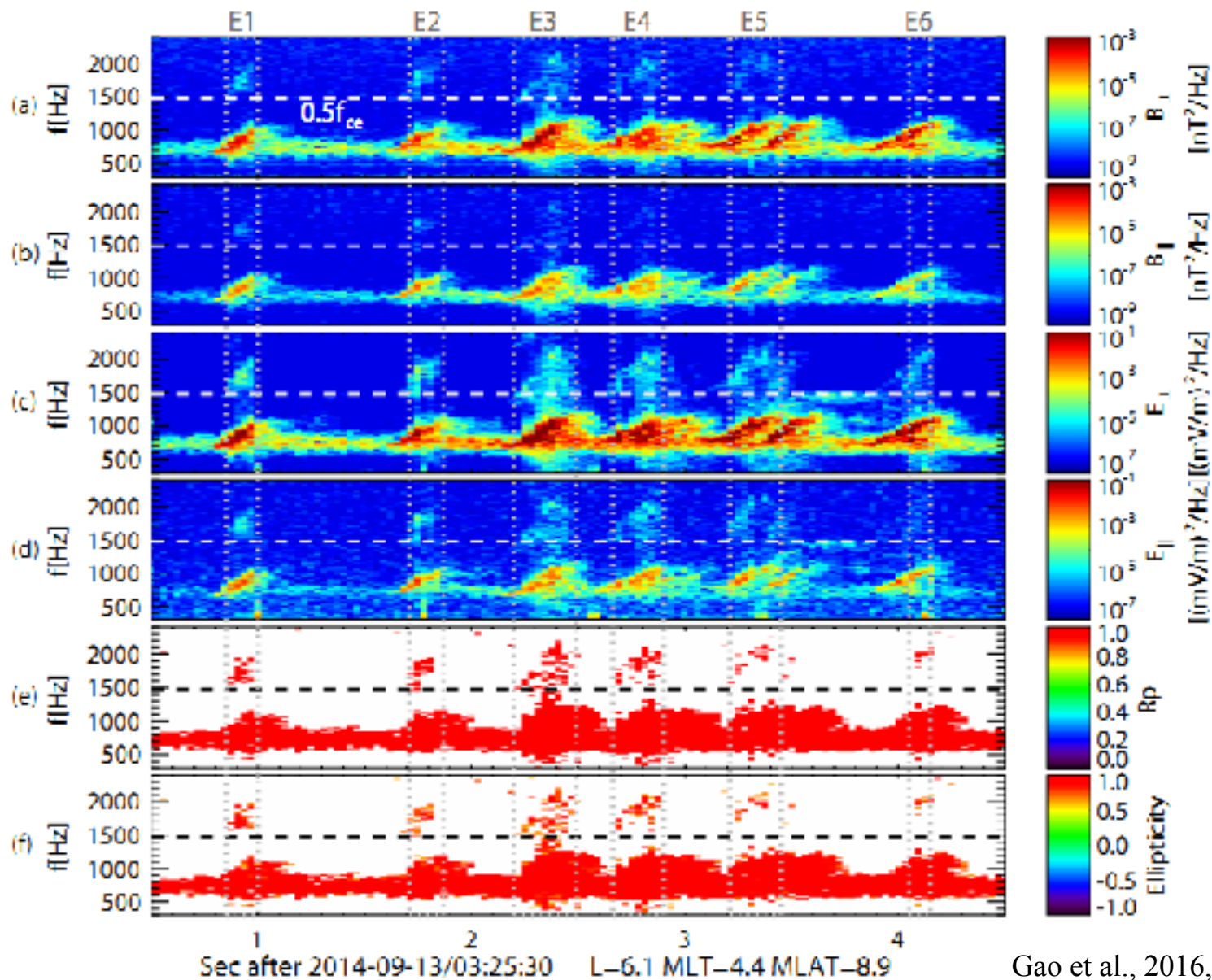
**Discrete power distribution
in frequency**

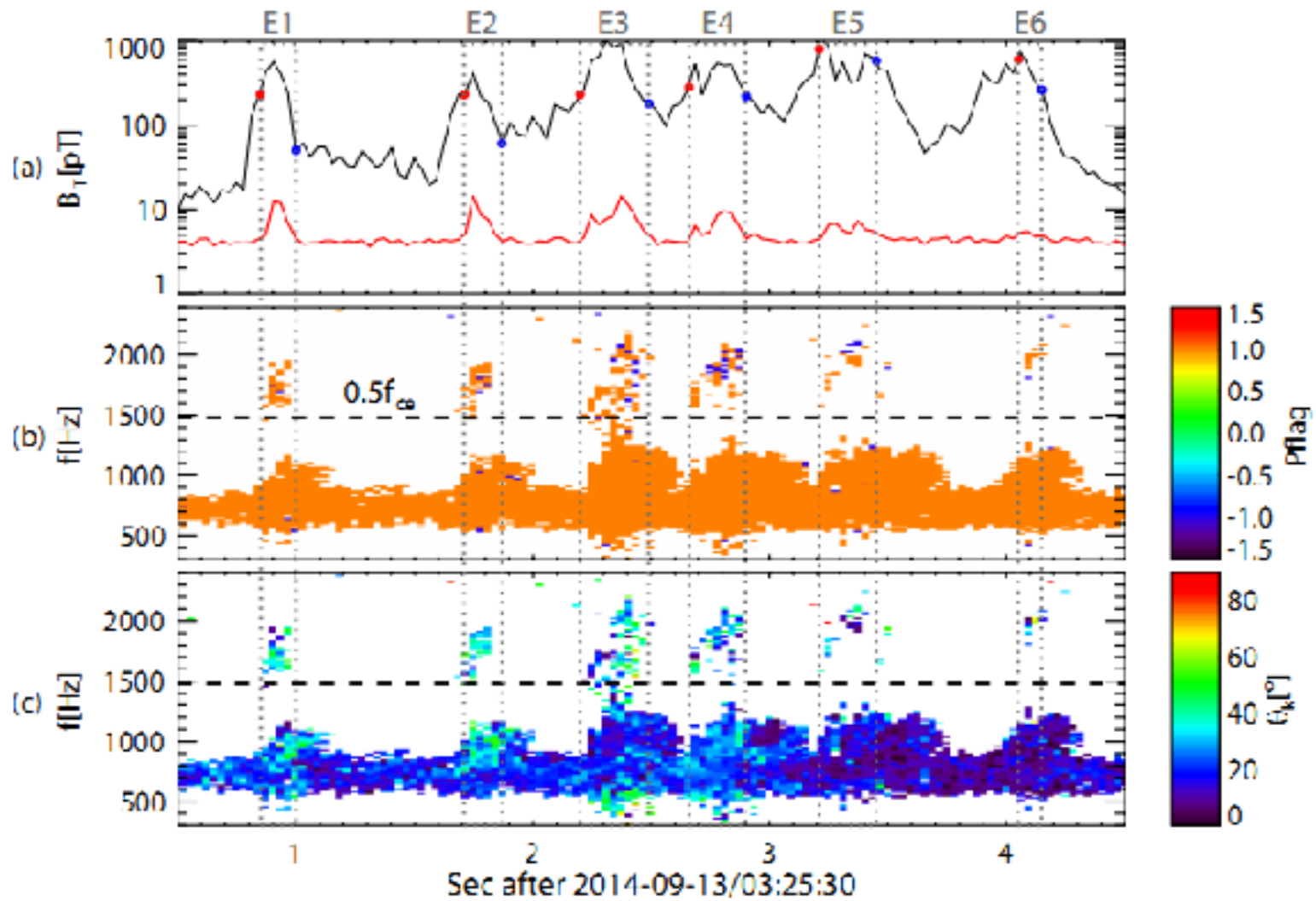


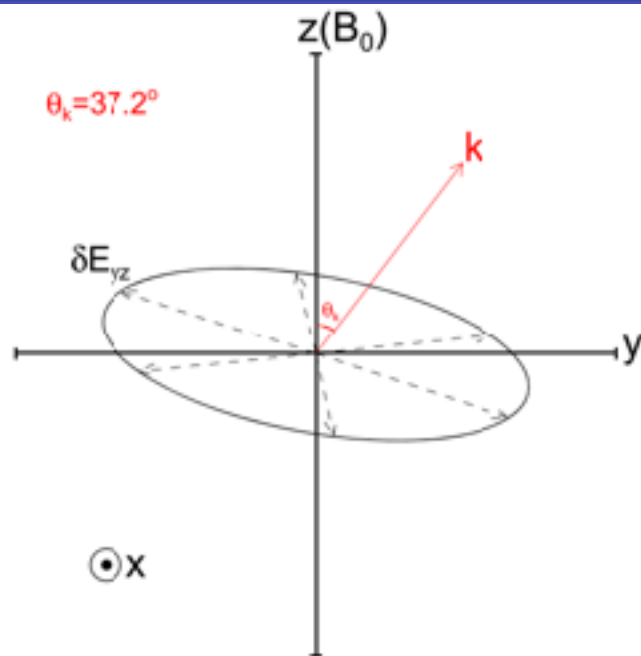
**EMIC waves
MS waves**



Chorus waves







Bicoherence index: measure the amount of phase coupling that occurs among three wave modes.

$$\frac{|\langle E_z(f_1) B_x(f_2) B_x^*(f_3) \rangle|^2}{\langle |E_z(f_1) B_x(f_2)|^2 \rangle \langle |B_x^*(f_3)|^2 \rangle}$$

where $f_3 = f_1 + f_2$

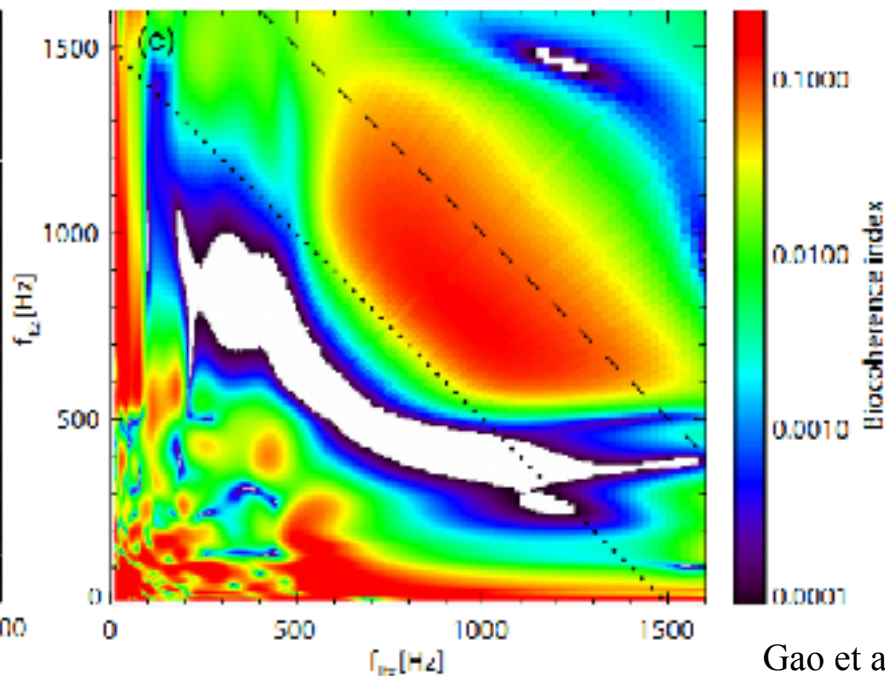
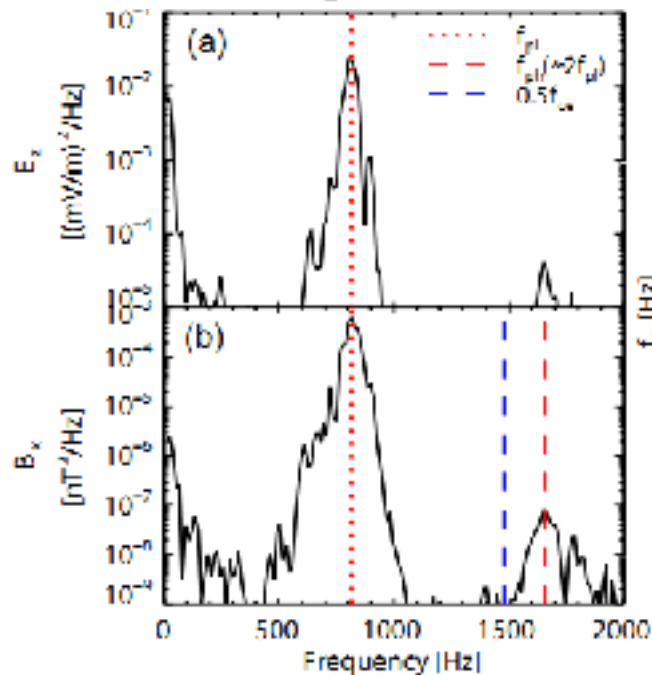
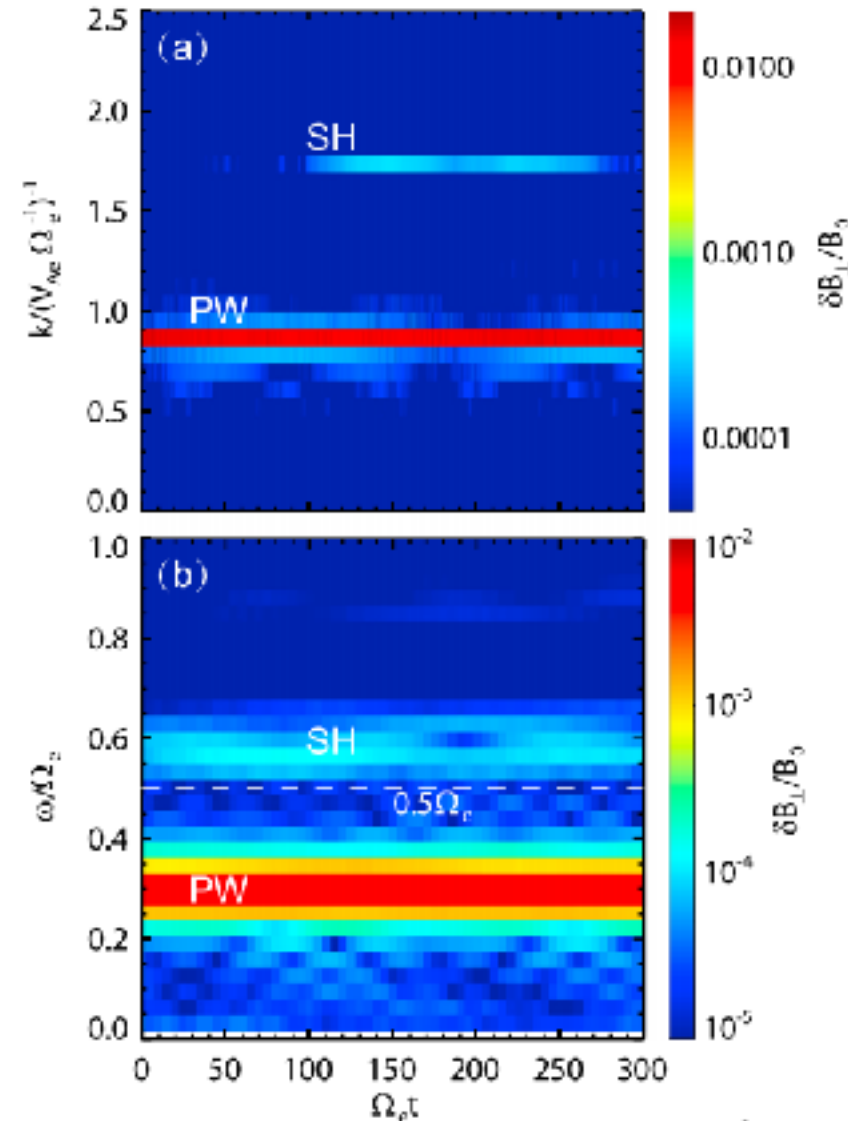
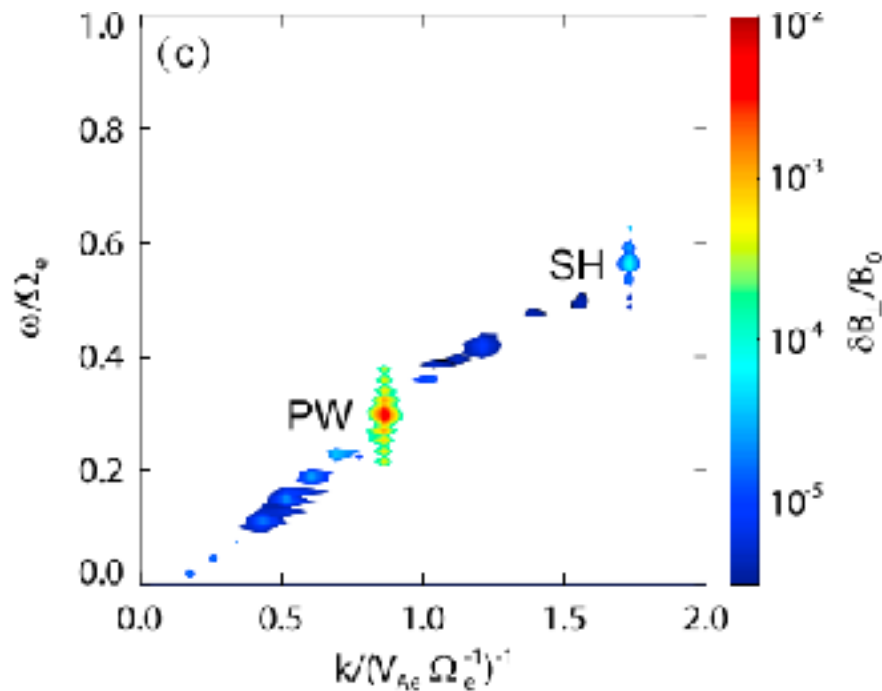
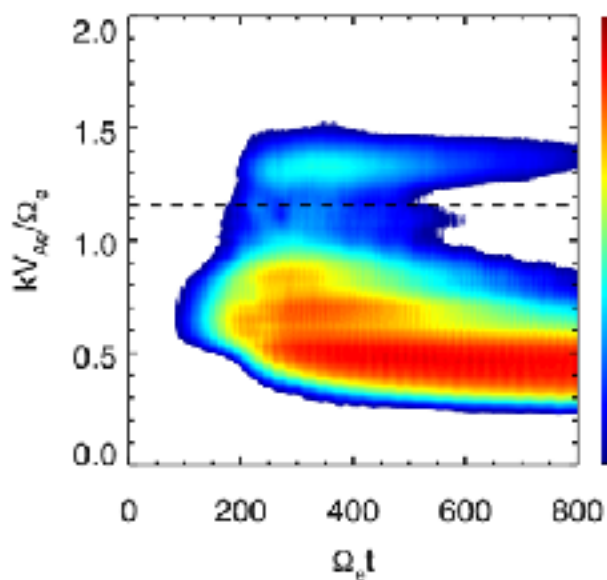




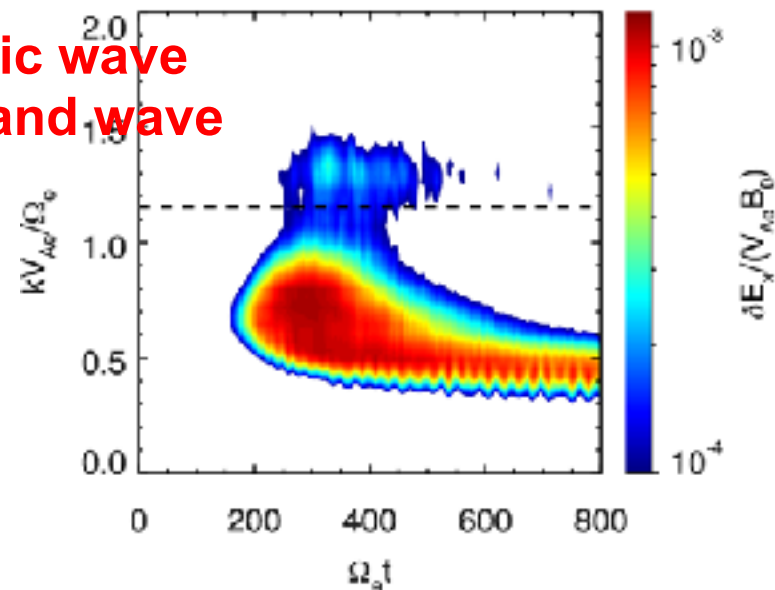
Table 1. Some Initial Parameters in the Simulation Model^a

Parameter	Value	Parameter	Value
n_c/n_0	0.9	m_p/m_e	1836
v_{xc}/V_{Ae}	6×10^{-2}	ω_0/Ω_e	0.3
n_h/n_0	0.1	$k_0 V_{Ae}/\Omega_e$	0.86
v_{th}/V_{Ae}	0.4	l	45°
v_{tp}/V_{Ae}	1.4×10^{-3}	$\delta B_{yz}/B_0$	0.02

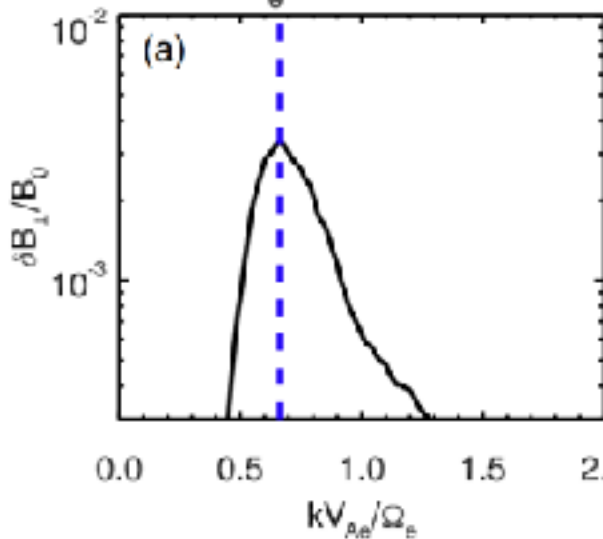




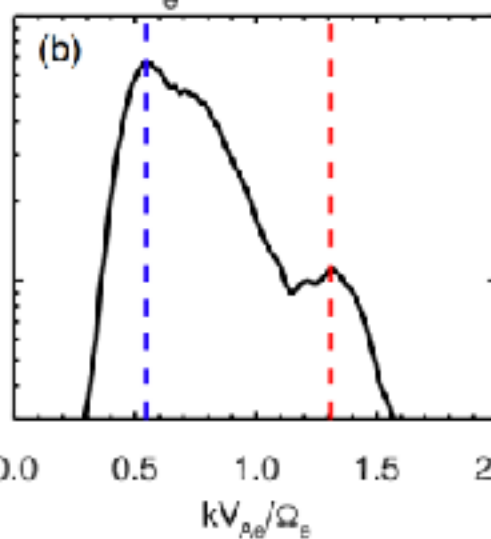
HW: harmonic wave
LW: lower-band wave



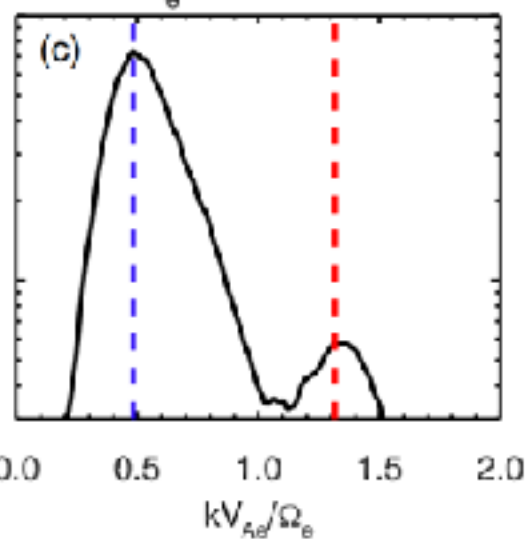
$\Omega_e t = 170-190$



$\Omega_e t = 290-310$



$\Omega_e t = 590-610$





$$\partial_t n_e + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$m_e (\partial_t \mathbf{v}_e + \mathbf{v}_e \cdot \nabla \mathbf{v}_e) = -e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B})$$

$$\nabla \cdot \mathbf{E} = -\frac{1}{\epsilon_0} (n_e - n_0) e$$

$$\nabla \times \mathbf{B} = -\mu_0 n_e e \mathbf{v}_e + \frac{1}{c^2} \partial_t \mathbf{E}$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\begin{cases} n_e = n_0 + (\delta n' + \delta n'') \\ \mathbf{v}_e = (\delta v'_{\parallel} + \delta v''_{\parallel}) \hat{\mathbf{x}} + (\delta v'_{\perp y} + \delta v''_{\perp y}) \hat{\mathbf{y}} + (\delta v'_{\perp z} + \delta v''_{\perp z}) \hat{\mathbf{z}} \\ \mathbf{E} = (\delta E'_{\parallel} + \delta E''_{\parallel}) \hat{\mathbf{x}} + (\delta E'_{\perp y} + \delta E''_{\perp y}) \hat{\mathbf{y}} + (\delta E'_{\perp z} + \delta E''_{\perp z}) \hat{\mathbf{z}} \\ \mathbf{B} = B_0 (\cos \theta \hat{\mathbf{x}} + \sin \theta \hat{\mathbf{y}}) + (\delta b'_{\perp y} + \delta b''_{\perp y}) \hat{\mathbf{y}} + (\delta b'_{\perp z} + \delta b''_{\perp z}) \hat{\mathbf{z}} \end{cases}$$



Linearized but keep up to the second order

Dispersion relation of whistler with θ

$$\begin{aligned} & \left(\frac{1}{c^2 \Omega_e} \omega^2 k - \frac{\Omega_e}{V_{Ae}^2} k \right) \left[\left(\cos \theta k^2 - \frac{\cos \theta}{c^2} \omega^2 \right)^2 - \left(\frac{1}{c^2 \Omega_e} \omega^3 - \frac{1}{\Omega_e} \omega k^2 - \frac{\Omega_e}{V_{Ae}^2} \omega \right)^2 \right] \\ & - \left(\sin \theta k^2 - \frac{1}{c^2} \sin \theta \omega^2 \right) \frac{1}{c^2} \sin \theta \omega k \left(\frac{1}{c^2 \Omega_e} \omega^3 - \frac{1}{\Omega_e} \omega k^2 - \frac{\Omega_e}{V_{Ae}^2} \omega \right) = 0 \end{aligned}$$



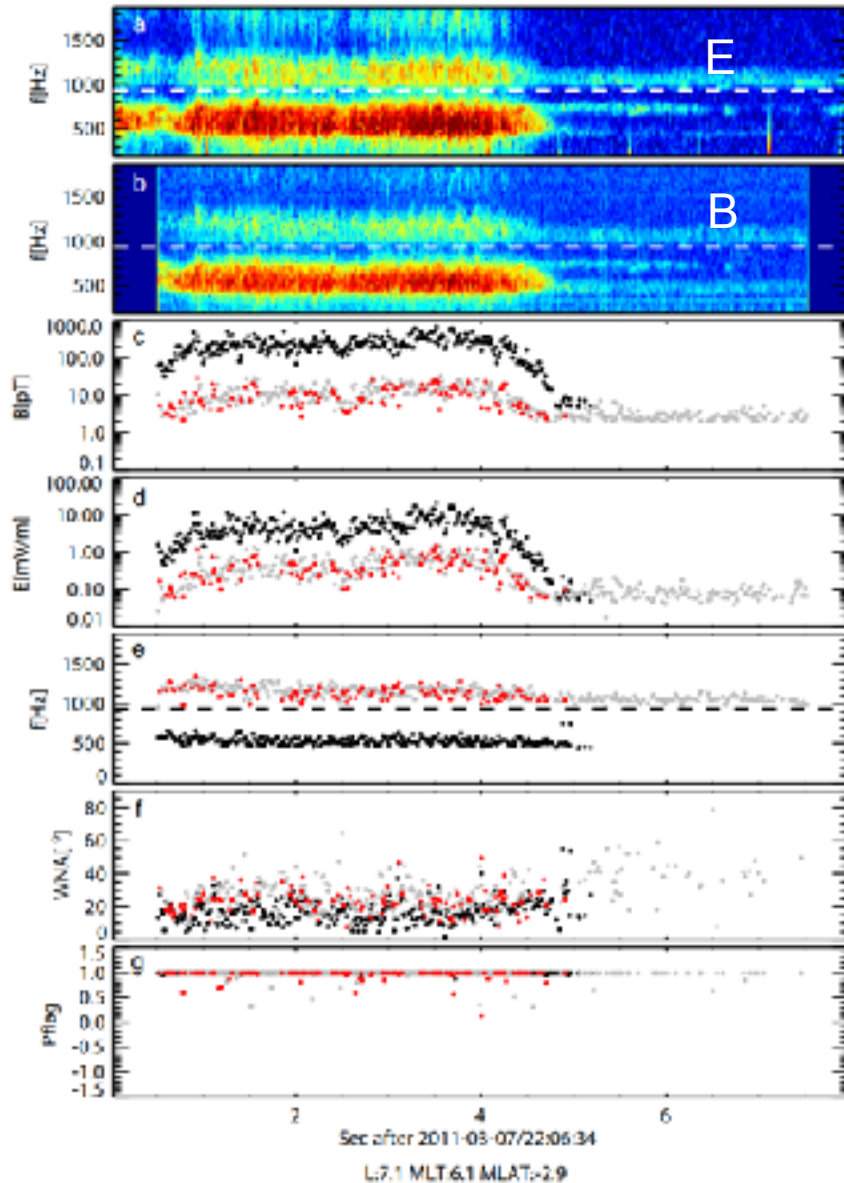
$$\begin{bmatrix} i\frac{8}{c^2\Omega_e}\omega^2k - i\frac{2\Omega_e}{V_{Ae}^2}k & 4\sin\theta k^2 - \frac{4}{c^2}\sin\theta\omega^2 & 0 \\ 0 & 4\cos\theta k^2 - \frac{4\cos\theta}{c^2}\omega^2 & \begin{bmatrix} -i\frac{8}{c^2\Omega_e}\omega^3 + i\frac{8}{\Omega_e}\omega k^2 \\ +i\frac{2\Omega_e}{V_{Ae}^2}\omega \end{bmatrix} \\ \frac{4}{c^2}\sin\theta\omega k & \begin{bmatrix} i\frac{8}{c^2\Omega_e}\omega^3 - i\frac{8}{\Omega_e}\omega k^2 \\ -i\frac{2\Omega_e}{V_{Ae}^2}\omega \end{bmatrix} & 4\cos\theta k^2 - \frac{4\cos\theta}{c^2}\omega^2 \end{bmatrix} \begin{bmatrix} \delta E''_{\parallel} \\ \delta b''_{1y} \\ \delta b''_{1z} \end{bmatrix} = NL$$

$$NL = \begin{bmatrix} \begin{bmatrix} -2\frac{B_0}{V_{Ae}^2}k^2(\delta v'_{\parallel}\delta v'_{\parallel}) + i2\frac{\Omega_e}{V_{Ae}^2}k(\delta v'_{\perp y}\delta b'_{1z} - \delta v'_{1z}\delta b'_{1y}) \\ +i2\frac{B_0}{n_0}\frac{\Omega_e}{V_{Ae}^2}\sin\theta k(\delta n'\delta v'_{1z}) - 4\frac{B_0}{n_0}\frac{1}{V_{Ae}^2}\omega k(\delta n'\delta v'_{\parallel}) \end{bmatrix} \\ \begin{bmatrix} 2\frac{B_0}{V_{Ae}^2}k^2(\delta v'_{\parallel}\delta v'_{\perp y}) + i2\frac{\Omega_e}{V_{Ae}^2}k(\delta b'_{1z}\delta v'_{\parallel}) \\ +i2\frac{B_0}{n_0}\frac{\Omega_e}{V_{Ae}^2}\cos\theta k(\delta n'\delta v'_{1z}) + 4\frac{B_0}{n_0}\frac{1}{V_{Ae}^2}\omega k(\delta n'\delta v'_{\perp y}) \end{bmatrix} \\ \begin{bmatrix} 2\frac{B_0}{V_{Ae}^2}k^2(\delta v'_{\parallel}\delta v'_{1z}) - i2\frac{\Omega_e}{V_{Ae}^2}k(\delta b'_{1y}\delta v'_{\parallel}) + 4\frac{B_0}{n_0}\frac{1}{V_{Ae}^2}\omega k(\delta n'\delta v'_{1z}) \\ +2i\frac{B_0}{n_0}\frac{\Omega_e}{V_{Ae}^2}k(\delta n'\delta v'_{\parallel}\sin\theta - \delta n'\delta v'_{\perp y}\cos\theta) \end{bmatrix} \end{bmatrix}$$

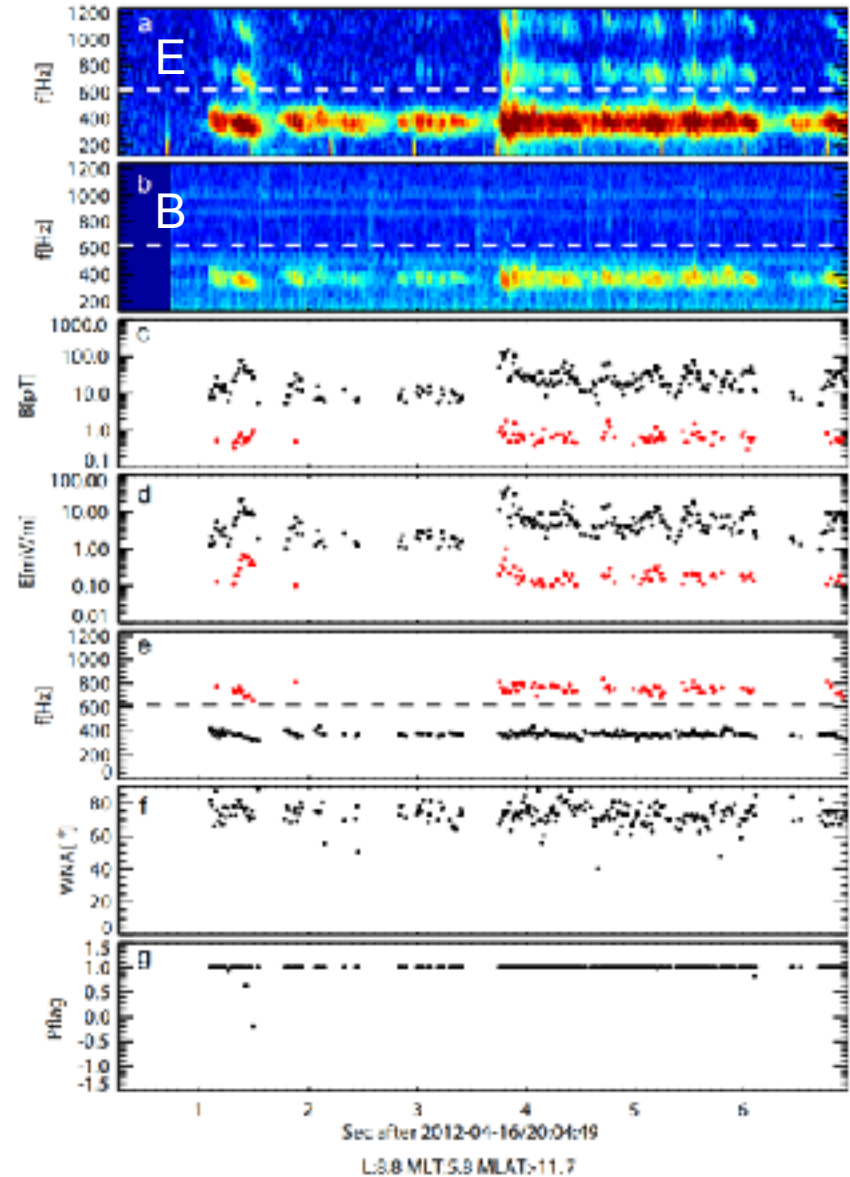
Two types of MBC

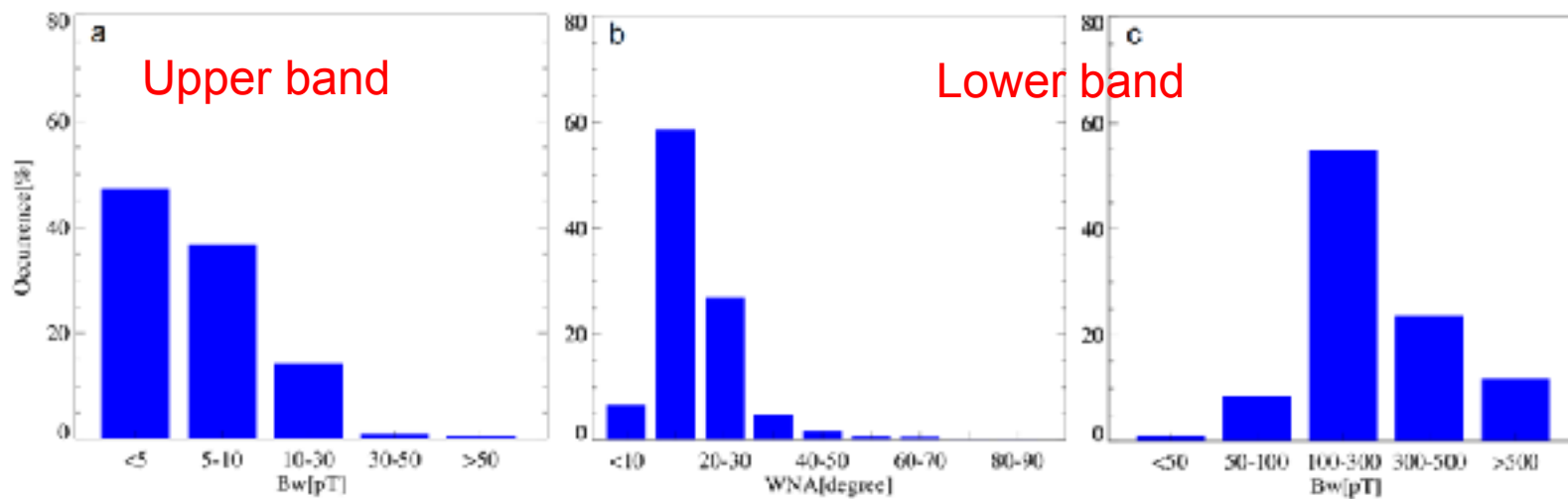
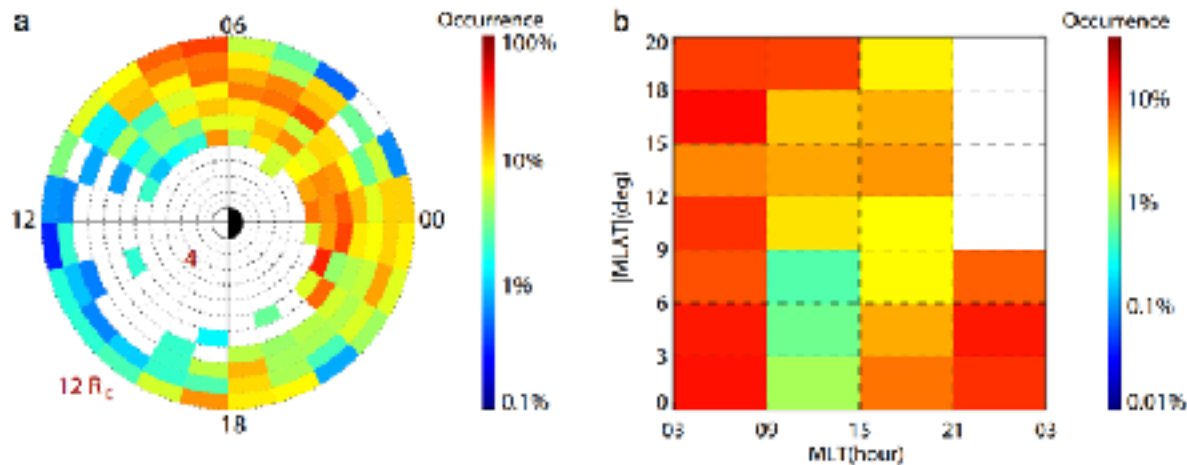


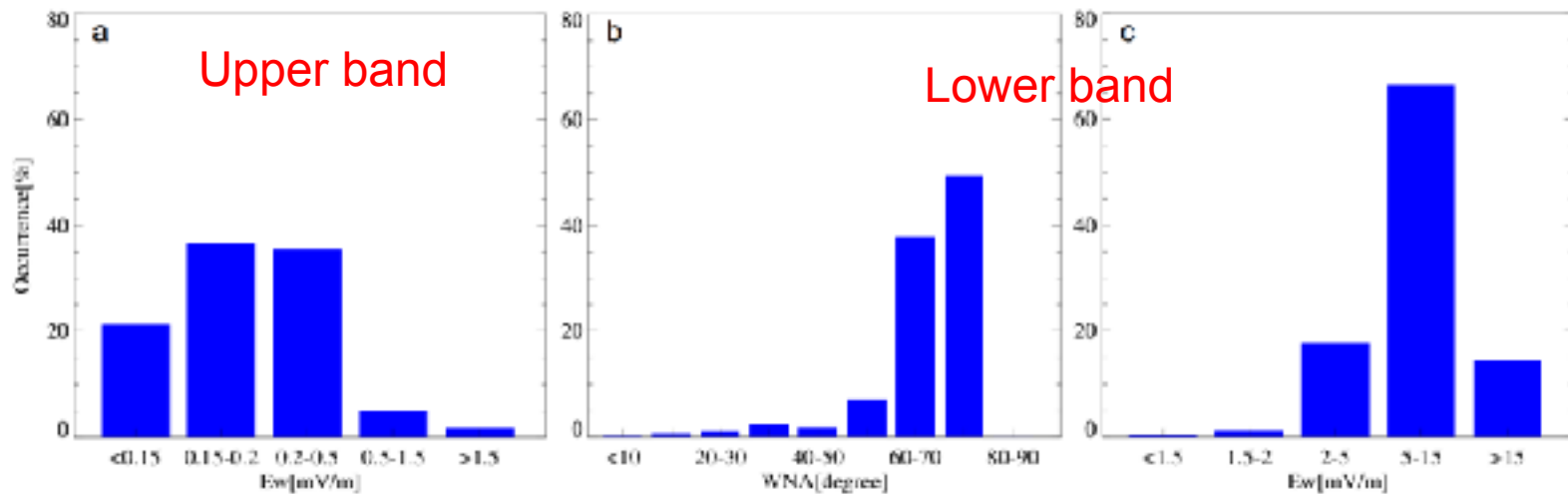
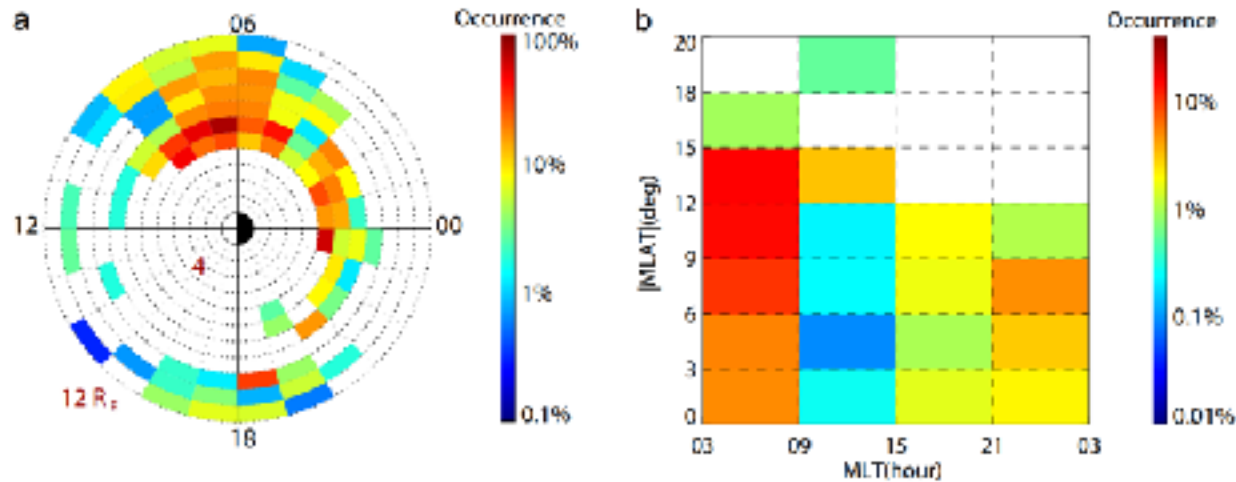
ElectroMagnetic MBC: EM-MBC

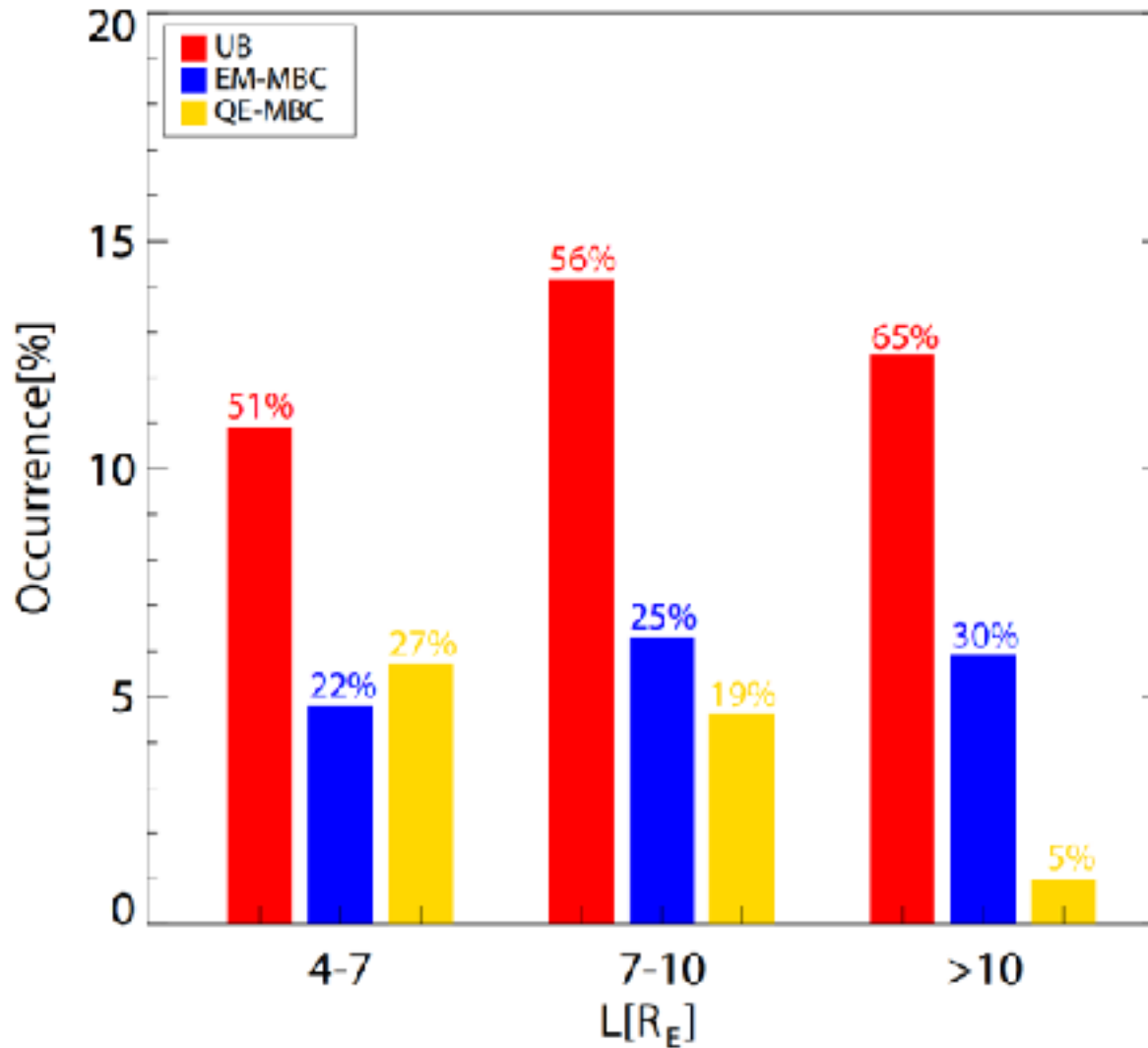


Quasi-Electrostatic MBC: QE-MBC











- Multiband chorus --- lower band cascade, reported by THEMIS and reproduced by PIC.
- Multiband chorus (EM-MBC and QE-MBC) have occupied a very large population in upper-band chorus in the magnetosphere.
- The significant role of the lower band cascade in generating upper-band chorus waves.

**Thanks for your
attention!**