Symmetry, Self-Organization and the Large Scale Dynamo

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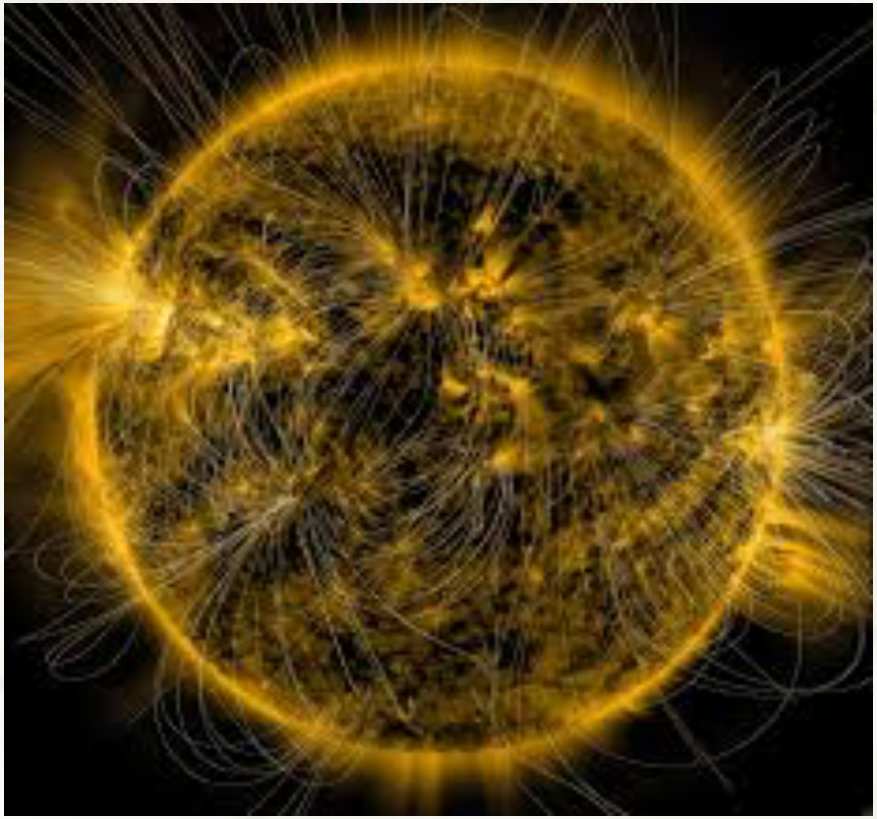


With Amir Jafari and Ben Jackel

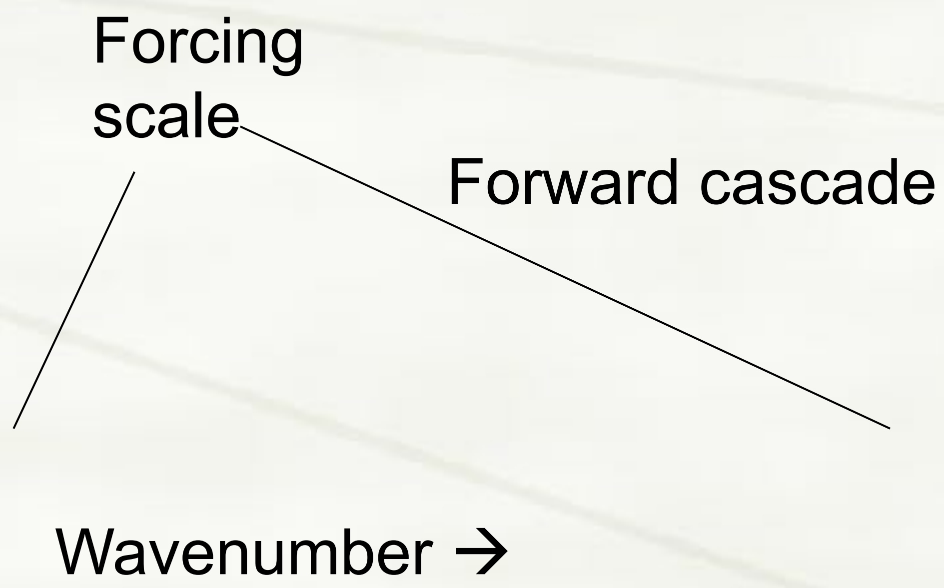
Outline

- What is the Large Scale Dynamo Problem?
- Magnetic helicity and the inverse cascade in MHD turbulence
- The large scale magnetic helicity flux
- The large scale dynamo
- Applications to stars, accretion disks and galaxies
- Summary

What is the Problem?

- How do large scale magnetic fields grow, and what sets their saturation limit? This is the **Large Scale Dynamo** problem.
- How does this work in stars, planets, accretion disks, and galaxies?
- This is an inverse cascade (or self-organization) problem. It is distinct from the small scale dynamo problem, which describes the growth of magnetic energy on eddy scales or below.

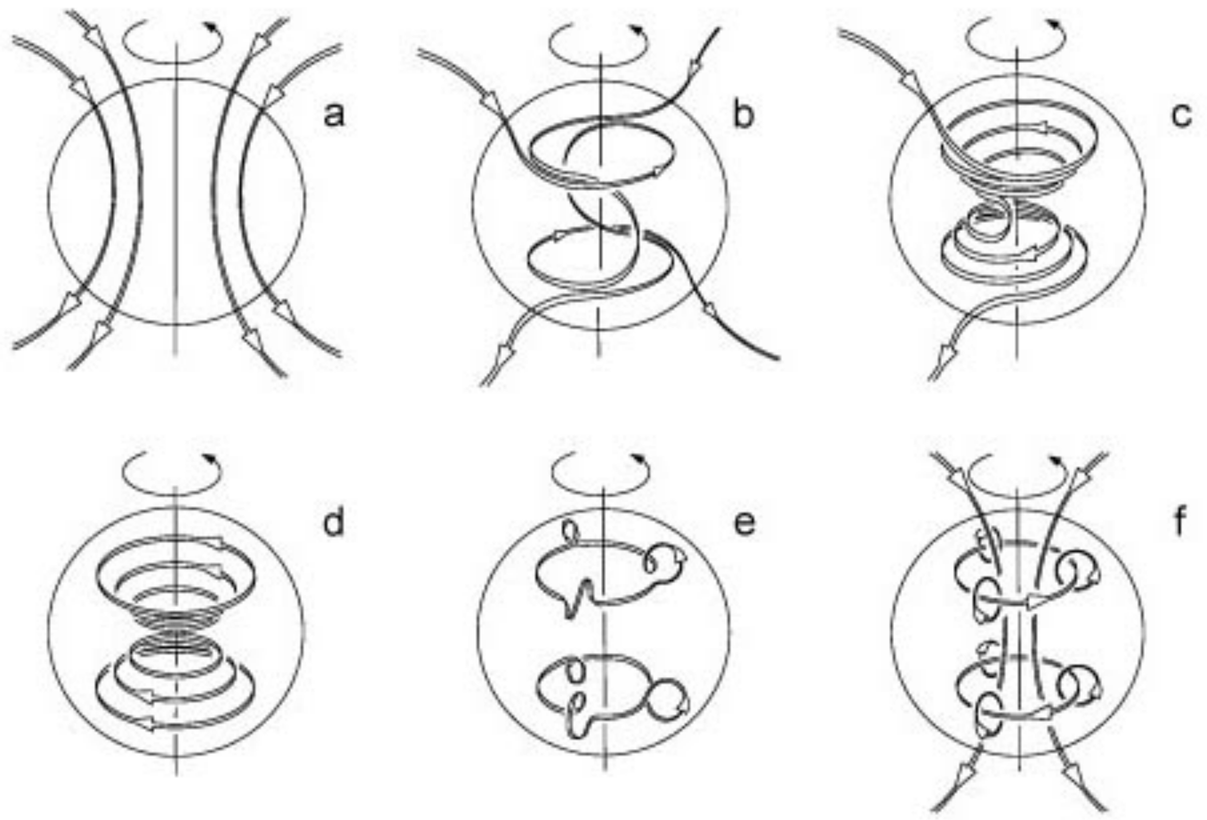
Turbulent Power Spectrum



Energy cascades to small scales and is dissipated.
Inverse cascades require some extra symmetry.

2D turbulence promotes large scale circulation

Large scale magnetic fields are an example of self-organization from turbulence in the presence of rotation and/or shear



Love, J. J., 1999. Astronomy & Geophysics, 40, 6.14-6.19.

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We need some equations!

- A highly conducting fluid, i.e. most astrophysical plasmas, has

$$\vec{E} \approx -\vec{v} \times \vec{B}$$

- Consequently the induction equation becomes

$$\partial_t \vec{B} \approx \nabla \times \left(\vec{v} \times \vec{B} \right)$$

We will ignore small resistive terms. If they are nonzero then in the presence of strong turbulence their functional form and amplitude are irrelevant.

That they are not zero is relevant.

How do we move from the dynamo picture to a dynamical model?

- Mean field dynamo theory - the large scale field is considered to be a dynamical object affected by some average of turbulent, eddy scale effects.

$$\partial_t \vec{B} = -\left(\vec{V}\cdot\nabla\right)\vec{B} + \left(\vec{B}\cdot\nabla\right)\vec{V} + \nabla\times\left\langle \vec{v}\times\vec{b}\right\rangle$$

advection

stretching

Coherent electric field due to correlated eddies

What is the electromotive force?

- We can estimate the correlations involved by assuming a balance between its time derivative and turbulent decorrelation i.e.

$$= \partial \left\langle \vec{\ } \times \vec{\ } \right\rangle - \left\langle \vec{\ } \times \vec{\ } \right\rangle / \tau$$

Pouquet et al. 1976

$$\left\langle \vec{v} \times \vec{b} \right\rangle \approx \left\langle \partial_t \vec{v} \times \vec{b} \right\rangle \tau_c + \left\langle \vec{v} \times \partial_t \vec{b} \right\rangle \tau_c$$

- Leaving out a few steps (and some details), we can write this result in terms of scalars (schematically) as

Electromotive force = [(current helicity tensor– kinetic helicity tensor) x B x]

+ [drift terms (e.g. buoyancy)]+[dissipative terms]

current helicity $\equiv \vec{j} \cdot \vec{b}$ ←Related to a conserved quantity (magnetic helicity) – a damping term?

kinetic helicity $\equiv \vec{v} \cdot \left(\nabla \times \vec{v}\right)$ ← Not conserved, imposed by the environment, usually taken to be the driving term.

What is the kinetic helicity?

- This is a pseudo-scalar. Its value is nonzero iff there is symmetry breaking in all three directions. Differential rotation breaks symmetry in the rφ plane. So we estimate the kinetic helicity as

$$h_k \sim \frac{v^2}{L}(\Omega \tau_c)$$

- In an accretion disk this is

$$h_k \sim \frac{\langle v^2 \rangle}{H}$$

What is the magnetic helicity (tensor)?

The magnetic helicity, $\vec{A}\cdot\vec{B}$, is a *locally* conserved topological quantity. It is the "twistiness" of the magnetic field. In the coulomb gauge the current helicity is roughly proportional to it. It can be moved around, but not destroyed. If we separate out the contribution from the eddy scales, $h \equiv \langle \vec{a}\cdot\vec{b} \rangle$ we can show that

$$\partial_t h + 2\vec{B}\cdot\langle \vec{v}\times\vec{b} \rangle = -\nabla\cdot\vec{j}_h$$

Since h appears in the second term, there is a time scale for relaxing to a stationary state $\sim V_A{}^2 k_{||}{}^2 \tau_c$

Alpha suppression (continued)

- *If* we can neglect the RHS of this equation, then running a dynamo produces an accumulation of current helicity which counteracts the kinetic helicity and turns the dynamo off when the large scale field is still weak. (Gruzinov and Diamond 1994)
- The **first** effect of magnetic helicity is that the purely kinematic dynamo dies young.

- Magnetic helicity is a robustly conserved quantity in MHD turbulence.
- If we minimize the energy in a system while conserving magnetic helicity we find a “Taylor state” with large scale magnetic structure.
- Magnetic helicity is the key to understanding the inverse cascade in turbulent MHD systems.
- If H=0 we can still get interesting effects if a system generates a separation of positive and negative H.

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What is the magnetic helicity flux? *(and is it =0?)*

- The eddy scale magnetic helicity has a flux given by

$$\vec{j}_h = \left\langle \vec{a} \times \left(\vec{v} \times \vec{b} + \nabla \varphi \right) \right\rangle - 2S_{ij} \left\langle \vec{b} \nabla^{-2} \left(a_{i,j} \right) \right\rangle$$

- Follow previous approach and estimate it by balancing growth with turbulent decorrelation.
- We can use this to calculate the leading order terms (and we have done so in some limites) but we also note that a simple dimensional estimate is…..

$$j_h \sim \langle v^2 \rangle \langle b^2 \rangle \left(S \text{ or } \Omega \times \text{ eddy shape factors} \right) \tau_c^2$$

- If rotation or shear is large the shape factors are significant (i.e. small).
- This is a pseudo-vector (does not reverse under parity). In a thin disk it will point along the vertical axis, aligned with rotation and shear.
- In other cases (stars) it will have components aligned with shear and rotation when they are weak, but strong anisotropy allows the net flux to point in other directions.

What effect does j_h have on the electromotive force?

- We can use our previous estimate for h_k and ask how long it will take for the accumulation magnetic helicity to produce a current helicity which will dominate over h_k. It takes about **one** eddy turnover time.
- The **second** effect of magnetic helicity is that there is no kinematic dynamo.
- This is true for all(?) astrophysical dynamos.

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- This implies that the electromotive force is (*magnetic nulls modify helicity flux):

$$\langle v \times b \rangle = \left[I - \frac{\langle RR \rangle \cdot BB}{B \cdot \langle RR \rangle \cdot B} \right] \cdot \langle v \times b \rangle - \frac{\langle RR \rangle \cdot B}{2 B \cdot \langle RR \rangle \cdot B} \nabla \cdot j_h$$

- Here *R* is a linear operator equivalent to multiplying by a unit vector in phase space. The matrix <*RR*> is the average weighted by the kinetic or magnetic energy.
- It describes the average large scale eddy shape, which in almost all applications will not be isotropic.

- The resulting dynamo growth rate is

$$\Gamma_{dynamo} \approx \left[S \frac{\nabla \cdot \left(? \, J_h \right)}{B^2 L} \right]^{1/2} \sim \frac{b}{B} \frac{v}{L} \left(S \Omega \right)^{1/2} \tau_c$$

- In general the implication is that dynamo growth is fast when the field is weak, and slows until magnetic field loss or dissipation balances growth. When the shear is weak, the growth rate scales linearly with the geometric mean of shear and rotation.

So the **third** effect of magnetic helicity is to produce a large scale dynamo, in the presence of shear and turbulence, with non-exponential growth, starting much faster than the kinetic dynamo estimate and slowing down as it saturates.

Caveats? When does this fail?

- Simple estimates suggest that the kinetic helicity is dominated by the current helicity after ***one*** eddy turn over time. There is no kinematic dynamo in nature.
- When the large scale field is very weak, magnetic helicity accumulates while the large scale field grows randomly.
- At late times, near magnetic nulls, the turbulent transport of magnetic helicity is important in preventing singularities in h.

How about a toy model?

- If we use this expression for the magnetic helicity flux for turbulence in a box, the dynamo growth rate drops as the magnetic field grows.
- Eventually turbulent dissipation can compete with the dynamo and we reach saturation.
- At this point we get a crude estimate of the saturation Alfven speed.

Characteristic Toroidal Magnetic Fields

- For slow rotators (like a galaxy or our sun) this leads to

$$B_T \sim \rho^{1/2} L (S\Omega)^{1/2}$$

- This ignores the role of magnetic buoyancy, which in real objects pushes out the magnetic field, and turbulent density pumping, which pushes the field down. The net effect is that the "box size" is a few pressure scales heights, i.e.

$$B_T \sim \rho^{1/2} L_P (S\Omega)^{1/2}$$

Buoyancy?

- A diffuse large scale field will have a negligible effect on the local dynamics, but magnetic field pressure will fluctuate in the local eddies.
- Concentration of magnetic fields will be buoyant, rising with a speed ~δ gτ or

$$V_{buoyant} \sim \frac{\delta\left(B^2\right)}{4\pi\rho}\frac{\tau_{corr}}{L_{pressure}} \sim \frac{D_{turb}}{L_{pressure}}$$

- Buoyant losses scale like turbulent diffusion (also like turbulent pumping)
- A detailed calculation shows that buoyancy wins in disks.

What if.....

- There is a strong background magnetic field?

-- We get a similar contribution but with

$$\langle b^2 \rangle \to \frac{\left(\vec{k} \cdot \vec{B}\right)^2}{k^2}$$

In this case the numerator is always the square of the dynamical rate and the magnetic helicity flux does not increase with the strength of the background field.

What if.....

Rotation is fast ($\Omega\tau_c \gg 1$)?

-- The turbulence is stretched in the vertical direction by that same factor. The terms that contribute to rotationally driven magnetic helicity flux all have a factor of

$$\frac{k_z^2}{k^2}$$

The rotationally driven magnetic helicity flux decreases as rotation increases. Total growth rate in this limit scales as $(S/\Omega)^{1/2}$

What if…..

Shear is strong($S_c \gg 1$)?

--Turbulent eddies are sheared so that the radial wavelength and radial field components decrease as the shear increases. Since every term depends on the square of the radial wavelength, or

$\langle b_r^2 \rangle$, or $\langle v_r^2 \rangle$ this means that the magnetic helicity flux is inversely proportional to the shear.

The total dynamo growth rate is independent of shear.

Characteristic Magnetic Fields For Fast Rotation or Shear

- More realistically, rotation will usually be at least as strong as shear. Even the shear term will scale inversely with the rotation rate.
- Consequently, in the limit of strong rotation and shear the α-Ω dynamo depends on the ratio of shear to rotation.
- In this limit the saturated magnetic field is

$$B_T \sim \rho^{1/2} L_P \tau_c^{-1} \left(S/\Omega \right)^{1/2}$$

If we take the fast rotation limit for a periodic box model, the energy dissipated in the magnetic field (B^2 times the turbulent mixing rate) is equal to v^2/τ_c.

That is, the magnetic field energy is limited by the energy in the turbulent cascade, even though most of the energy in the magnetic field comes from the shear.

For a star, with an effective box size linked to the pressure scale height, the magnetic “luminosity” is a set fraction of the total luminosity – everything scales with the convective flux.

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Stars

- Stars range from fast rotators (young) to slow rotators (old)
- This theory predicts a linear relation between the buried toroidal field (or starspot field) and the rotational frequency for slow rotators
- And saturation for fast rotators.

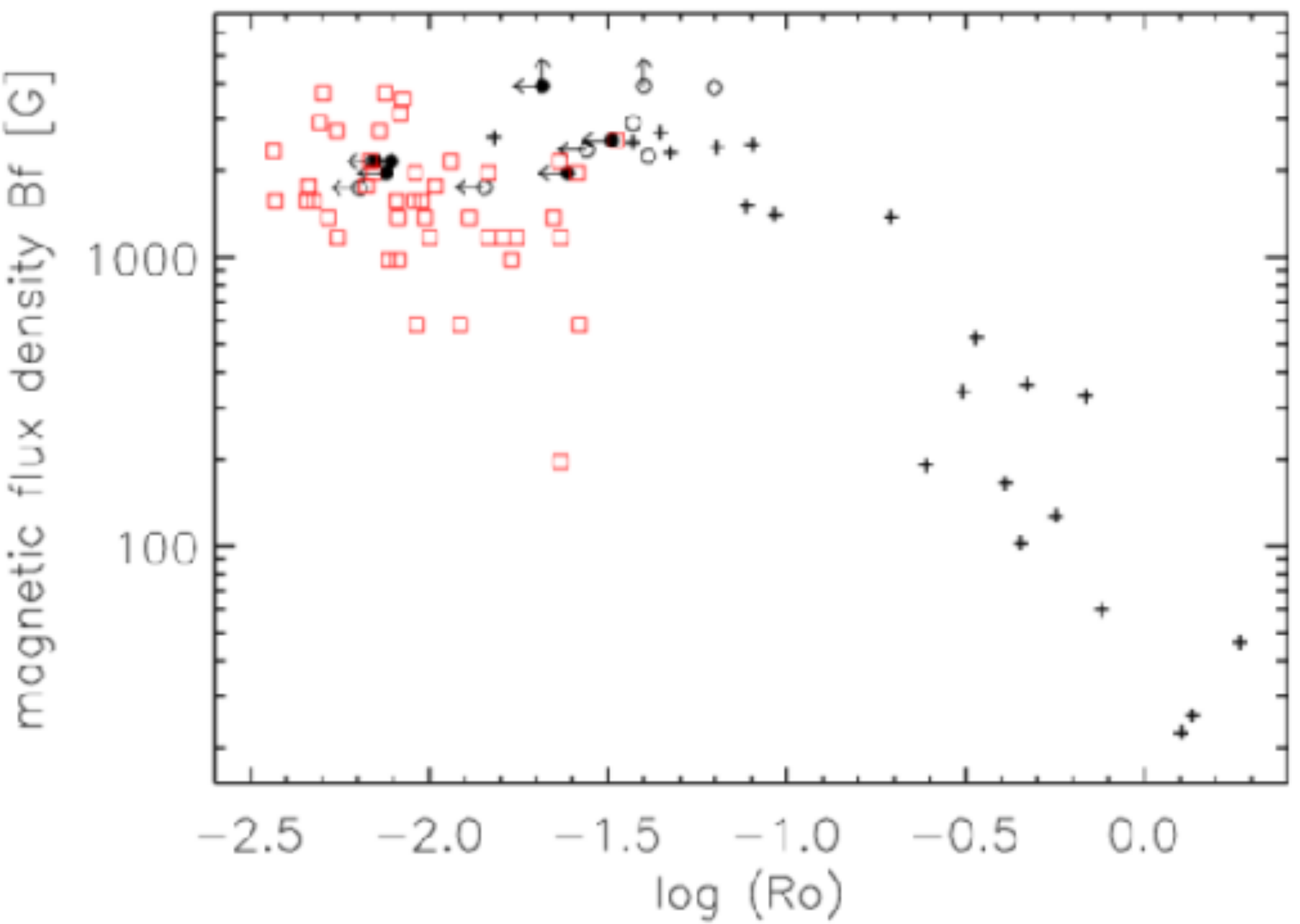


Figure 19: Magnetic fields as a function of Rossby number. Crosses are sun-like stars Saar (1996a, 2001), circles are M-type of spectral class M6 and earlier (see Reiners *et al.*, 2009a). For the latter, no period measurements are available and Rossby numbers are upper limits (they may shift to the left hand side in the figure). The black crosses and circles follow the rotation-activity relation known from activity indicators. Red squares are objects of spectral type M7 – M9 (Reiners and Basri, 2010) that do not seem to follow this trend ($\tau_{\rm conv}$ = 70 d was assumed for this sample).

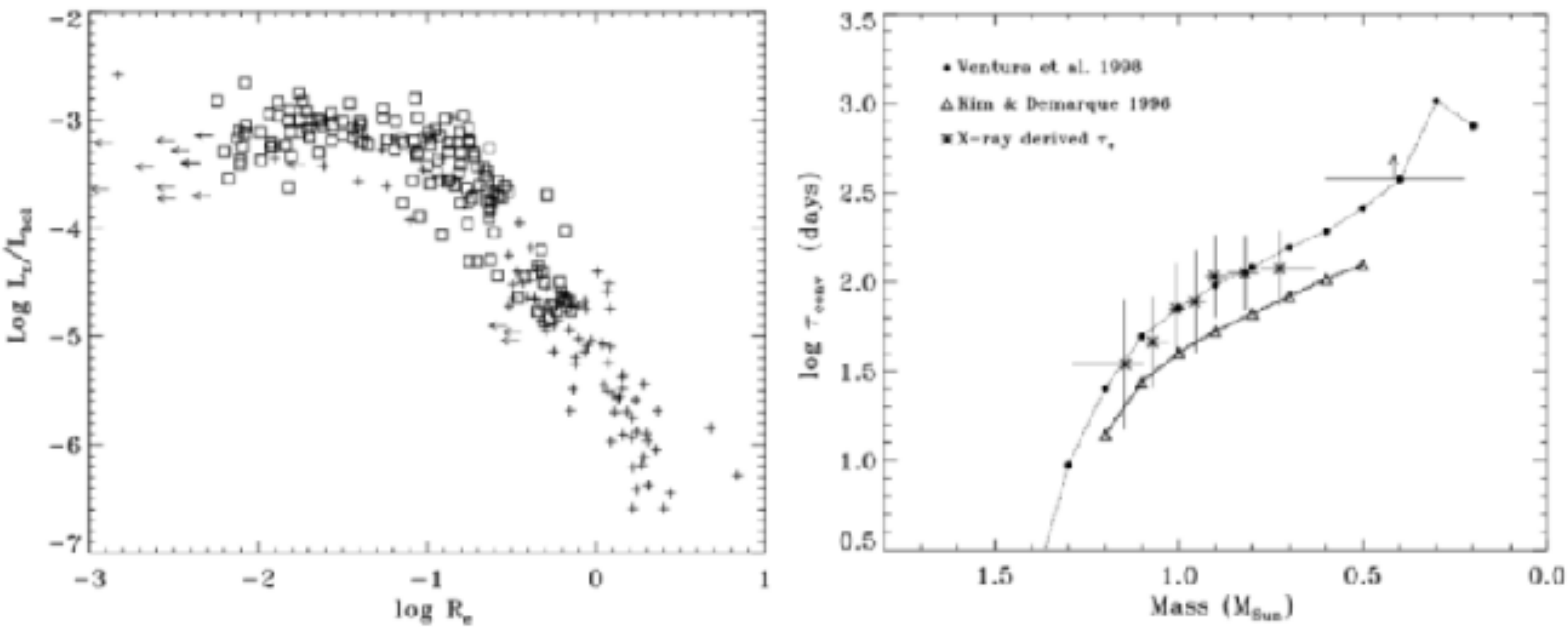


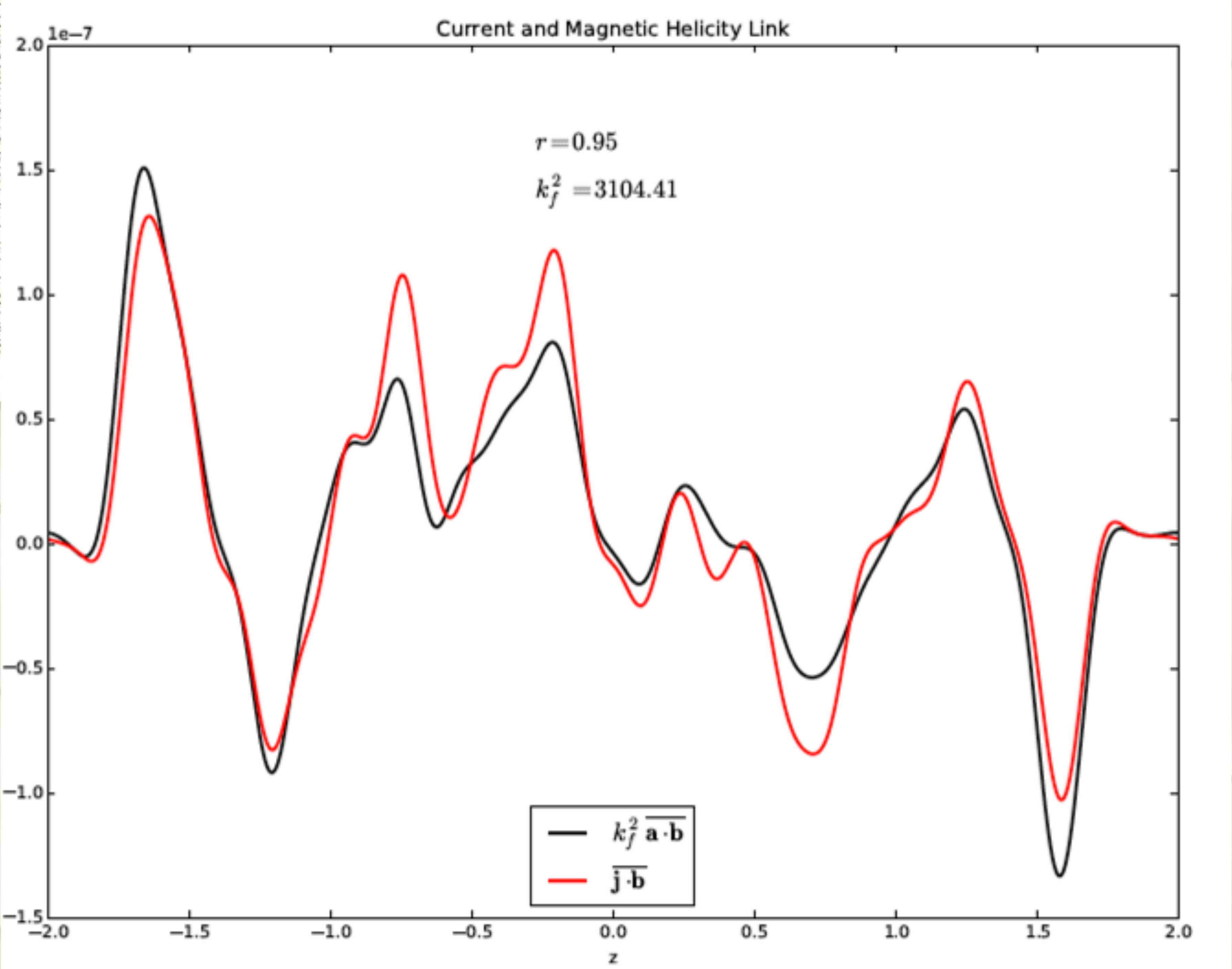
Figure 18: *Left panel:* Rotation-activity relation showing the normalized X-ray luminosity as a function of Rossby number. *Right panel:* Empirical turnover time chosen to minimize the scatter in the rotation-activity relation (from Pizzolato *et al.*, 2003, reprinted with permission © ESO).

What About the MRI?

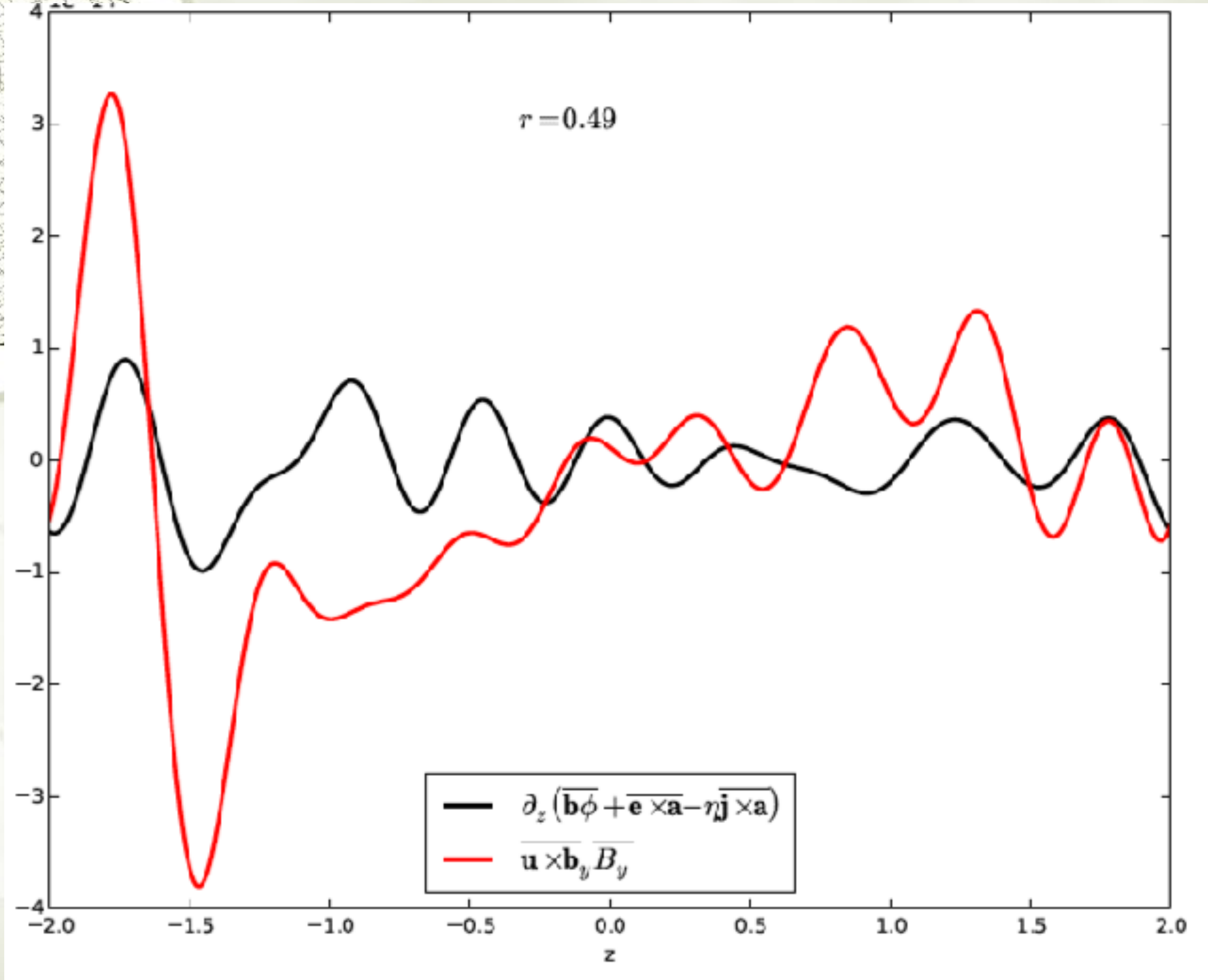
- This same mechanism can be applied to accretion disks.
- For accretion disks the MRI drives both the turbulence and the dynamo with a correlation time similar to the inverse of the shear (or the rotation rate).
- Eddies can be very anisotropic with long azimuthal wavelengths and (sometimes) very large vertical wavenumbers.

Implications for the MRI?

- Magnetic helicity flux will be vertical.
- If the vertical wavenumber is very large (as in unstratified simulations) then the dynamo will be weak and easily suppressed.
- For stratified simulations, where the vertical wavelength is not small, saturation will be at an Alfven speed which is a fraction of the disk thickness divided by the correlation time.



Current and Magnetic Helicity Link
1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
$r = 0.95$
$k_f^2 = 3104.41$
$k_f^2\ \overline{\mathbf{a}\cdot\mathbf{b}}$
$\overline{\mathbf{j}\cdot\mathbf{b}}$
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
z



r = 0.49
$\partial_z \left(\overline{\mathbf{b}\phi} + \overline{\mathbf{e}\times\mathbf{a}} - \eta\overline{\mathbf{j}\times\mathbf{a}} \right)$
$\overline{\mathbf{u}\times\mathbf{b}}_y \overline{B}_y$
z
4
3
2
1
0
−1
−2
−3
−4
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

A problem for grid-based simulations

- Spectral codes conserve global magnetic helicity reasonably well.
- Simulations with disk stratification do significantly worse (stars, galaxies, accretion disks)
- The size of convective cells varies enormously in stars, adequate resolution is required for the smallest cells that contribute to the dynamo.

A second problem for understanding disks

- A vertical field will affect the disk dynamics and may replace the dynamo or make it more efficient (vertical bending is suppressed).
- Magnetic buoyancy and radial drag will combine to concentrate external poloidal field at small radii, helping drive winds and jets. (Jafari and Vishniac 2018)
- This implies that a global solution for the disk magnetosphere is critical for understanding observations.

How do galactic magnetic fields grow?

- At early times the small scale dynamo pushes the disordered magnetic field towards equipartition (linear growth).
- As the magnetic nears equipartition with the turbulence the magnetic helicity current starts up, and the turbulence begins to generate long wavelength modes of the magnetic field. At this point they are decoupled from each other.

The Growth of Galactic Magnetic Fields (continued)

- Eventually the large scale fields will be strong enough to drive the inverse cascade, i.e.

$$2\vec{B}\bullet\left\langle \vec{v}\times\vec{b}\right\rangle \sim -\nabla\bullet\vec{j}_n$$

- At this point the accumulated magnetic helicity will be rapidly transferred to the large scale field, resulting in super-exponential growth.

The Growth of Galactic Magnetic Fields (finale)

- After a short time we achieve a steady balance between div(j) and the dynamo effect.
- The field grows linearly with a steadily decreasing growth rate until turbulent mixing and buoyant losses balance the growth rate.
- The whole process takes a few galactic rotation periods and gives a typical Alfven speed ~ disk height x rotation

Summary

- Ignoring kinetic helicity and considering a dynamo driven by the magnetic helicity flux explains how stellar magnetic fields scale with rotation.
- This model avoids the difficulties posed by “alpha suppression”, which plague dynamos driven by kinetic helicity.
- The requirement that computer simulations conserve magnetic helicity on eddy scales over dynamo growth time scales is unexpectedly difficult. Fixing this might require an explicit local conservation of magnetic helicity.

And Also…

- It also explains the rapid growth of large scale magnetic fields in galaxies, and the MRI dynamo.

There is no large scale kinematic dynamo in the limit of small resisitivity.