Seismological applications of sausage waves for inferring the physical parameters in solar *coronal* structures

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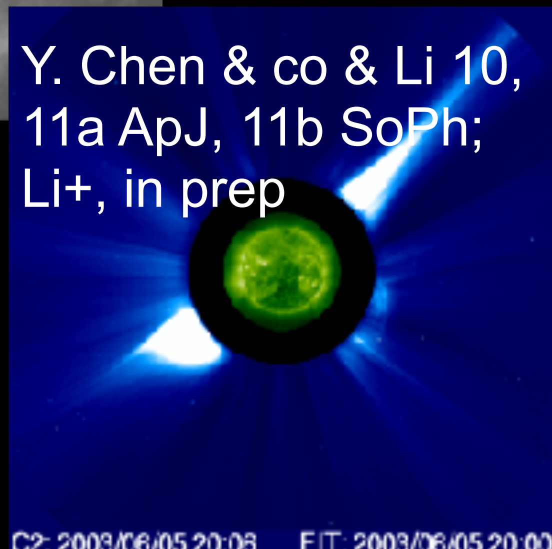
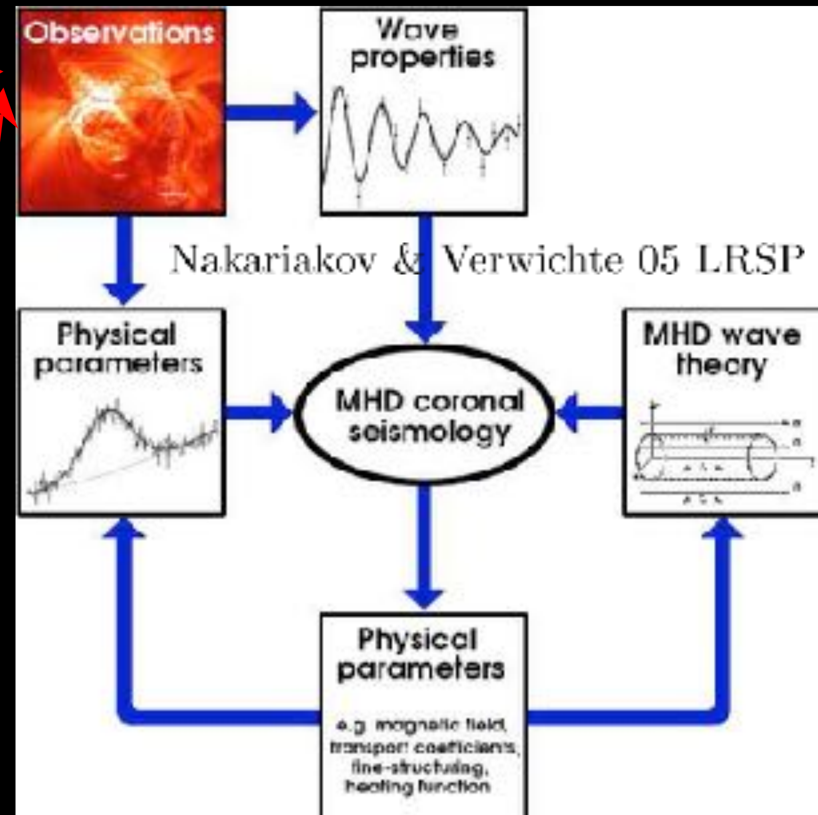
Motivation: the highly structured corona

- Magnetic field
  - shapes the corona & essential in coronal heating
  - hard to measure: emission weak, corona hot [e.g., Cargill 09 SSRv]
- Transverse density structuring
  - heating efficiency, e.g., phase-mixing [Heyvaerts & Priest 83]
  - hard to measure: optically thin; sub-resolution

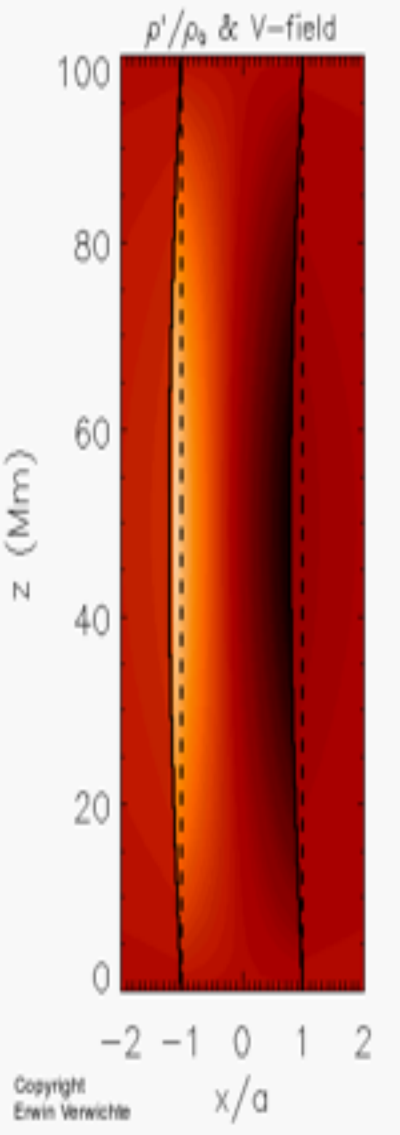
Motivation: solar MHD seismology





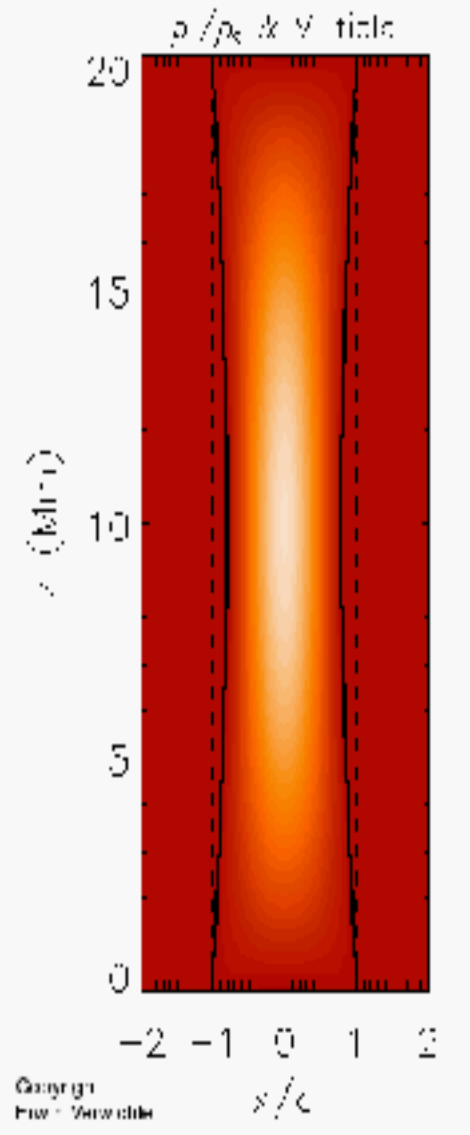
- Key 1 – Observations: waves/oscillations abound, apparent wavelengths: 1-1000Mm, Quasi-Periods: 1 sec – a couple of hours
- Key 2 – Theory: $\mathrm{measureables} = \mathrm{funcs}(\mathrm{measurables}, \mathrm{unknowns})$
  - Idealization of equilibrium structures unavoidable
  - The more realistic this idealization is, the better

Cornerstone of "Local seismology": Standing modes in density-enhanced loops

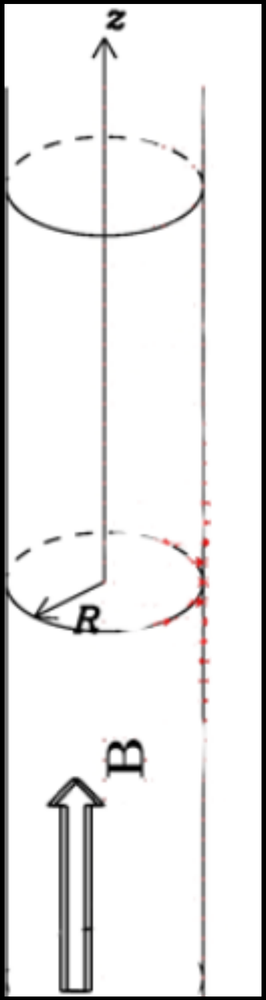


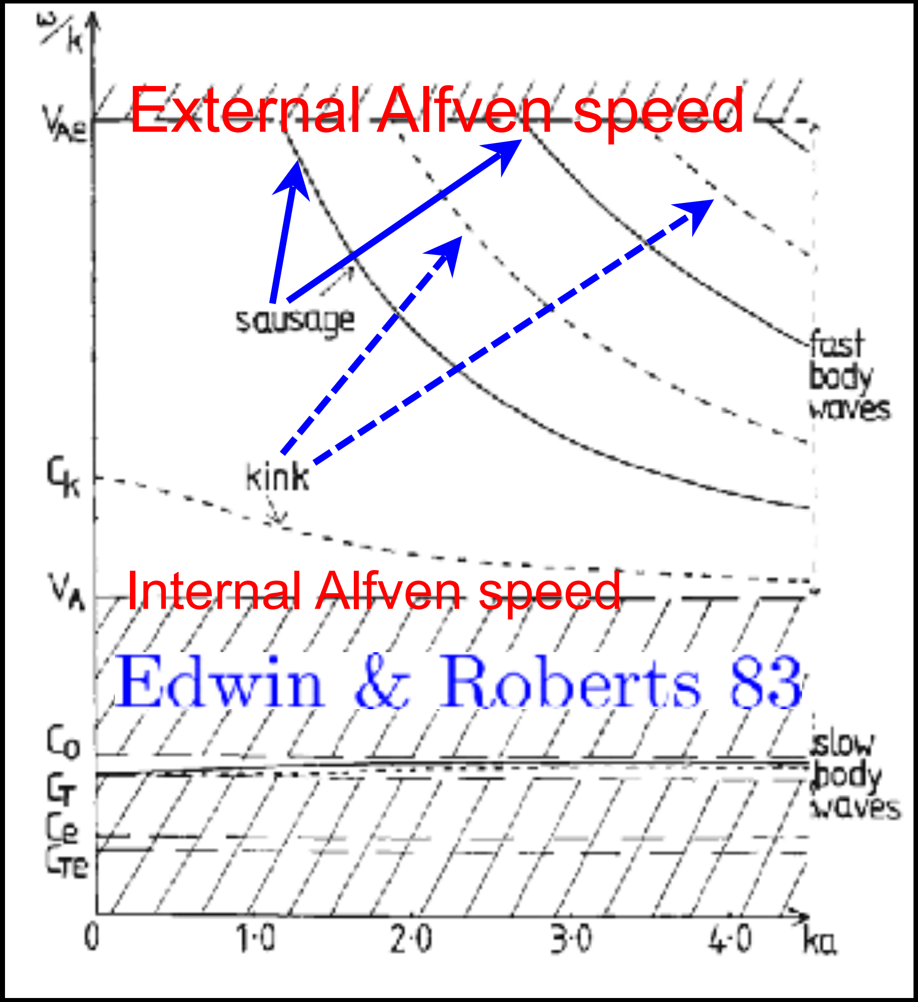
Kink mode	Sausage mode
m=1	m=0
Loop axis displaced	Loop axis not displaced

animations from Nakariakov & Verwichte 05 (LRSP)



Dispersion Diagrams





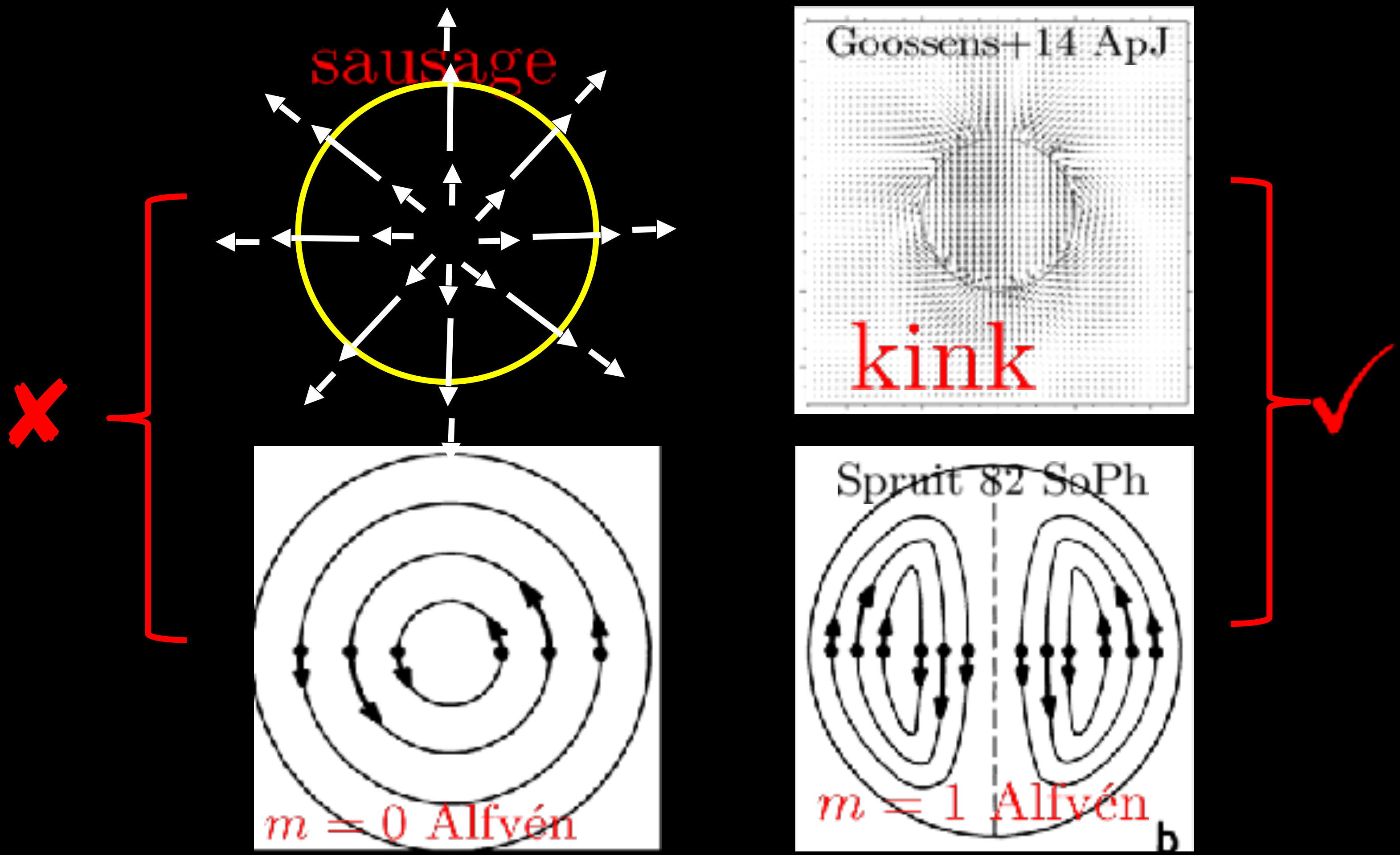
Leaky modes

Trapped modes

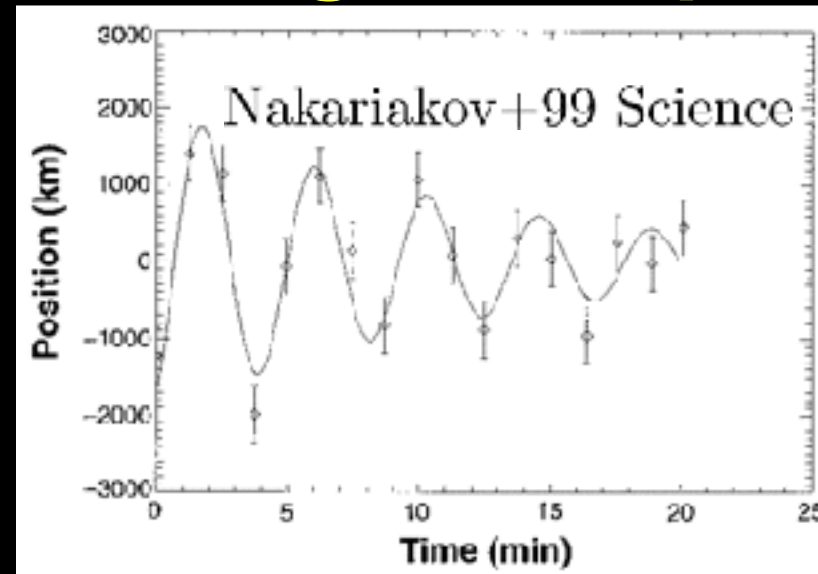
Notes: 1) discontinuous transverse profile 2) an infinite number of solutions 3) kink: longitudinal Alfven; sausage: transverse Alfven

Jargon: resonant coupling

- Two continua result from continuous transverse profiles
  - Cusp $\omega_C = kc_T(r)$ and Alfven $\omega_A = kv_A(r)$
- Resonant (freq. match) coupling between fast and Alfven

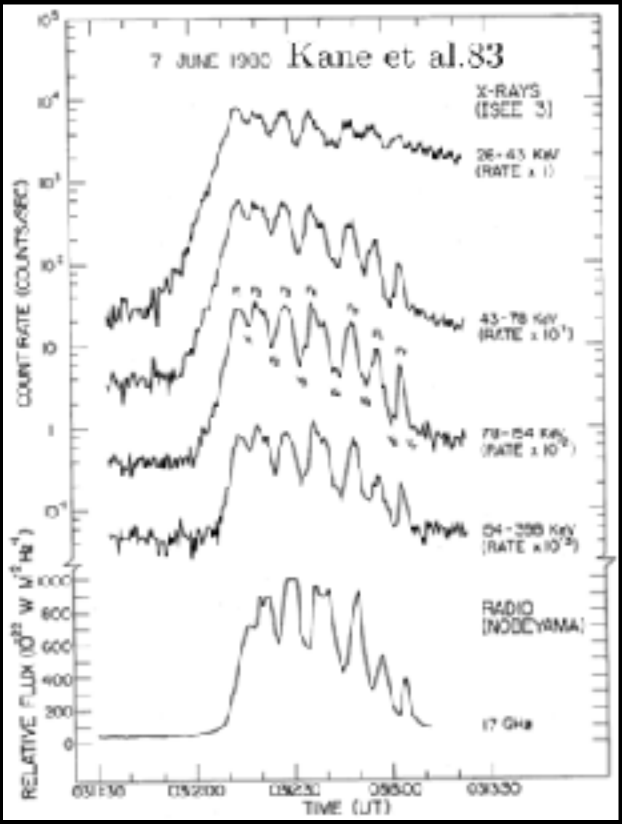


Kink oscillations in active region loops



- Response to low coronal eruptions/ejections [Zimovets & Nakariakov, 15]
  - TRACE[Aschwanden+99, Nakariakov+99, ……] or NOGIS [Hori+07ASPC?]
  - Hinode/EIS [van Doorsselaere+08, Erdelyi & Taroyan 08…]
  - STEREO/EUVI [Verwichte+09, …]
  - SDO/AIA [Aschwanden & Schrijver 11, ...]
- Characteristics
  - Periods: mins to tens of mins, the longer the loop, the large the period
  - tend to be strongly damped over a few cycles

Quasi-Periodic Pulsations in the corona



- Discovered in late 1960’s [Parks & Winckler 69, Frost 69, Rosenberg 70, …].
- Seen in all phases, all passbands, in nearly all (? Inglis+16) flares [Nakariakov+09, Van Doorsselare+16]
- Imaging measurements possible
  - NoRH [e.g.,Asai+01, Nakariakov+03, Kolotkov+15]
  - SDO/AIA [e.g., Su+12, Li, Ning & Co+16, 17]
  - IRIS [e.g., Tian+16; Dennis+17 ?]
- Standing/propagating sausage modes → QPPs with periods ~ secs [reviews by e.g., Aschwanden 87, Aschwanden+04]

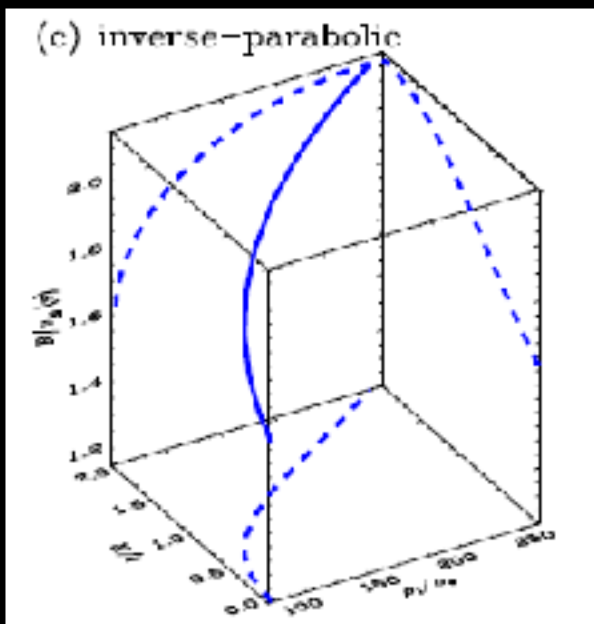
Aim of this talk

- Aim: develop inversion schemes for sausage modes to infer transverse Alfven time (eventually B) & transverse density structuring
- Key: develop theories that account for continuous structuring
- Words of caution
  - Cold MHD (plasma beta =0, slow modes absent; non-zero beta examined by S.X. Chen+16 ApJ, 18 ApJ)
  - Structures on which we impose perturbations
    - ➢ Straight: magnetic twist neglected
    - ➢ In Magnetostatic equilibrium ($\partial/\partial t = 0$, $\boldsymbol{v}= 0$; flow effects see Li+13 ApJ, 14 AA)
    - ➢ Structured only in the transverse direction (uniform in axial direction)
- Despite all these words of caution, we still need to start from scratch (very few theories accounted for continuous structuring, Nakariakov & co 12 for tubes, Nakariakov & Roberts & co 88, 95)

Seismology with sausage oscillations

$$P_{\rm saus} = \frac{R}{v_{\rm Ai}} F_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right)$$

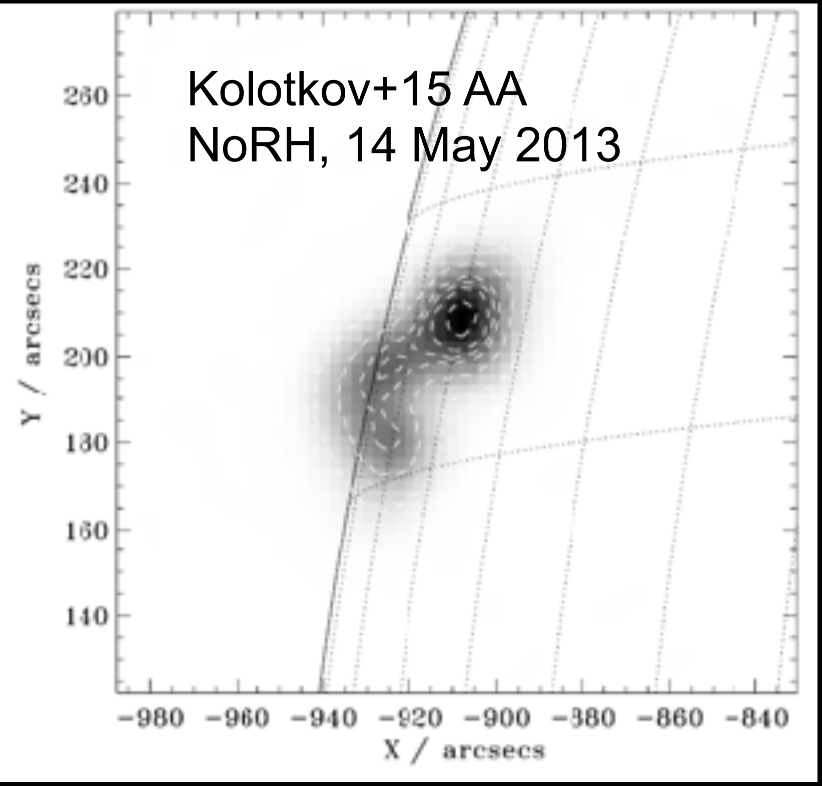
$$\frac{\tau_{\rm saus}}{P_{\rm saus}} = G_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right)$$

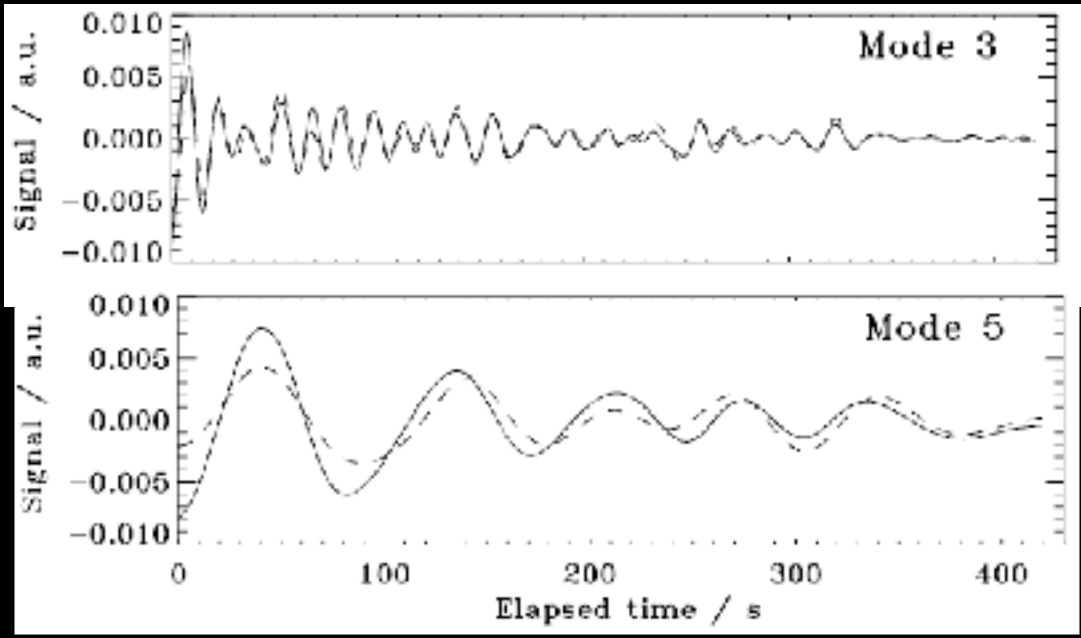


- ➢ $R/v_{\rm Ai}$: max/min = 1.8
- ➢ den. contrast: max/min = 2.9
- ➢ Layer width l/R: [0, 2] possible

- key for finding *F* and *G* [Chen, Li, +15, ApJ 812 22]: no resonant coupling between fast sausage and *m*=0 Alfven
- Problem under-determined
  - Unknowns: R/v_Ai; L/R, l/R, rho_i/rho_e; density formulation in the Transition Layer
  - For spatially unresolved measurements, the best one can do

Can do better with Spatially resolved multi-mode QPPs





- Geometrical parameters known
  - L=4.e4 km & r_e = 4.e3 km
- Two modes identified
  - saus: P = 15 s & tau = 90 s
  - kink: P = 100 s & tau = 250 s

Spatially resolved multi-mode QPPs

$$P_{\mathrm{saus}} = \frac{R}{v_{\mathrm{Ai}}} F_{\mathrm{saus}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}} \right)$$

$$\frac{\tau_{\mathrm{saus}}}{P_{\mathrm{saus}}} = G_{\mathrm{saus}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}} \right)$$

Analytical DR [Chen+15ApJ]

Lateral leakage

$$\boxed{P_{\mathrm{kink}} = \frac{L}{v_{\mathrm{Ai}}} F_{\mathrm{kink}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}} \right)}$$

$$\tau_{\mathrm{kink}} = \frac{L}{v_{\mathrm{Ai}}} H_{\mathrm{kink}} \left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}} \right)$$

Linear resistive MHD [Terradas+06, Guo, Chen, Li+16]

Resonant Absorption

- 3 Unknowns: v_{Ai}, l/R, $\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$
- 4 Eqs ? We use 3, the remaining one as a safety check

spatially resolved multi-mode QPPs

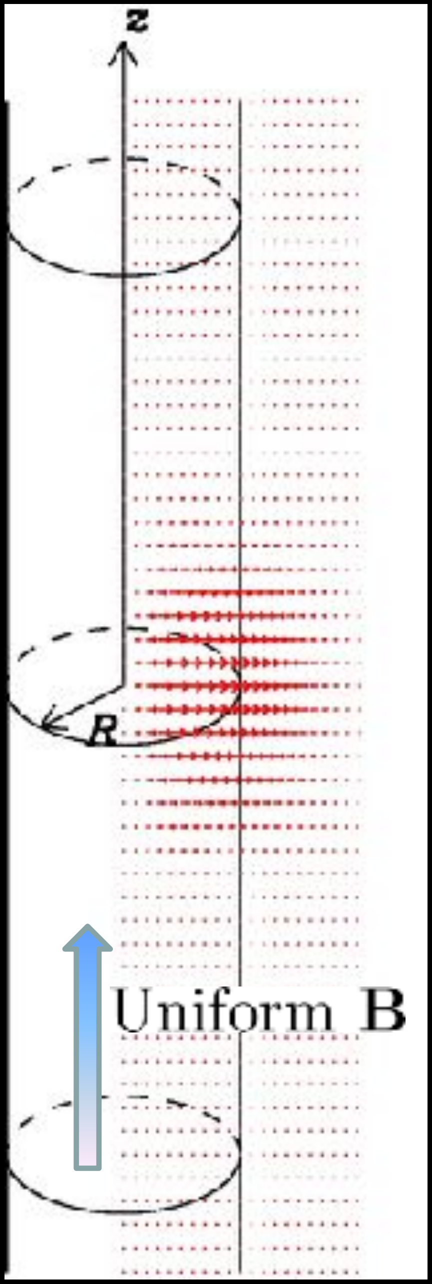
Guo, Chen, Li+16 SoPh

profile	l/R	ρ_i/ρ_e	v_{Ai} (km/s)	$P_{kink,theory}$ (s)
linear	0.167	28.5	653.8	91.5
parabolic	0.240	28.4	657.7	89.2
inverse-parabolic	0.277	31.1	593.7	102.5
sine	0.284	29.9	620.5	95.9

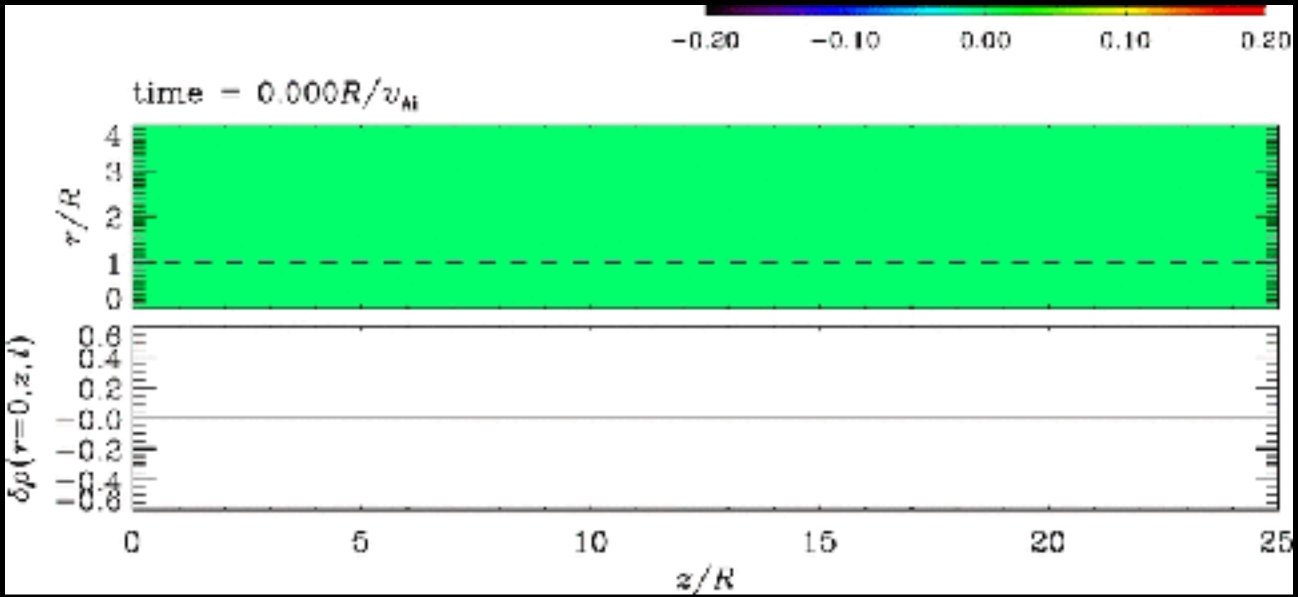
measured kink period = 100 sec

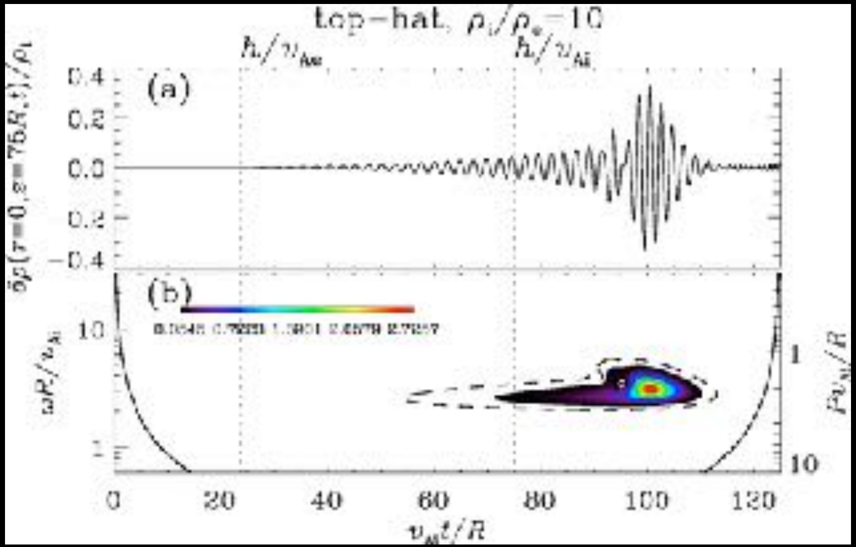
- possible to tell the details on transverse structuring, at least in principle
- key: the more observables, the better

Sausage wave trains in cold tubes



Linear MHD simulations by self-developed code[Yu, Li+16, 17]

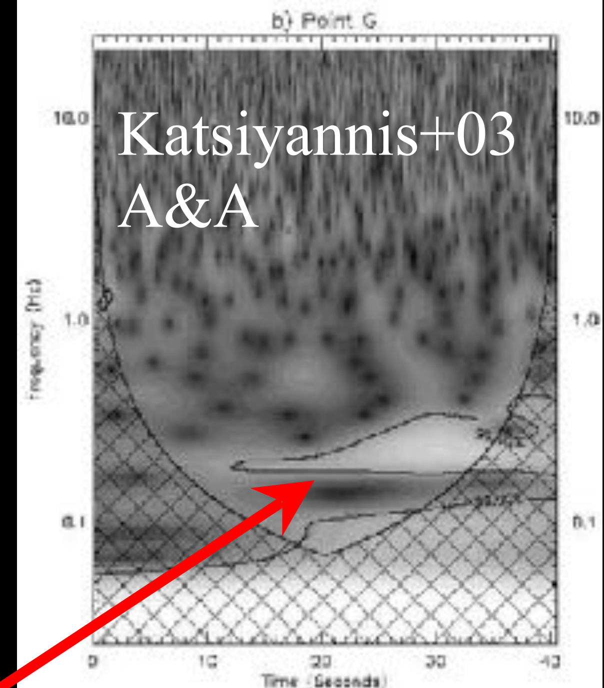


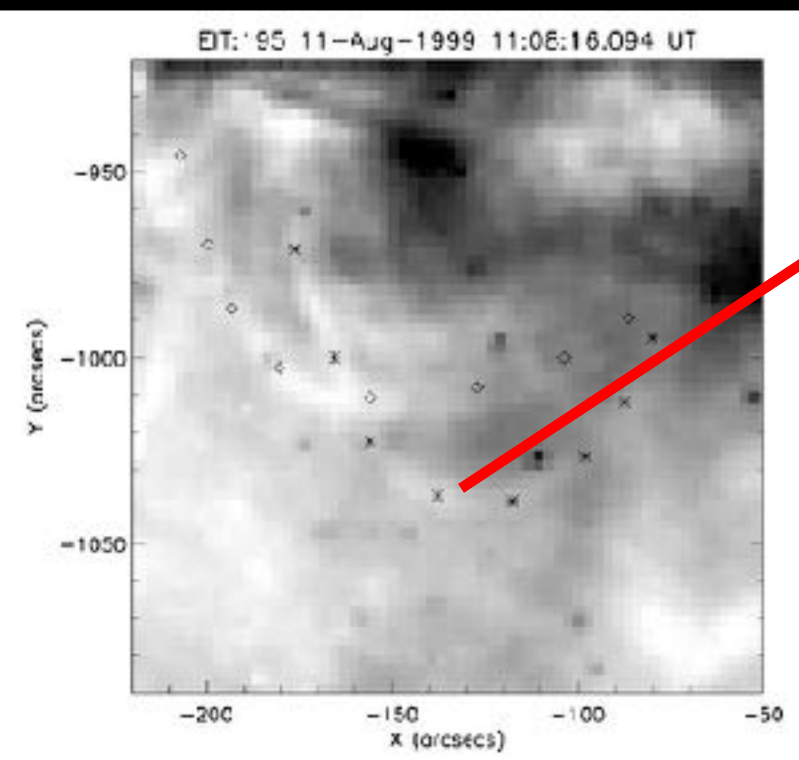


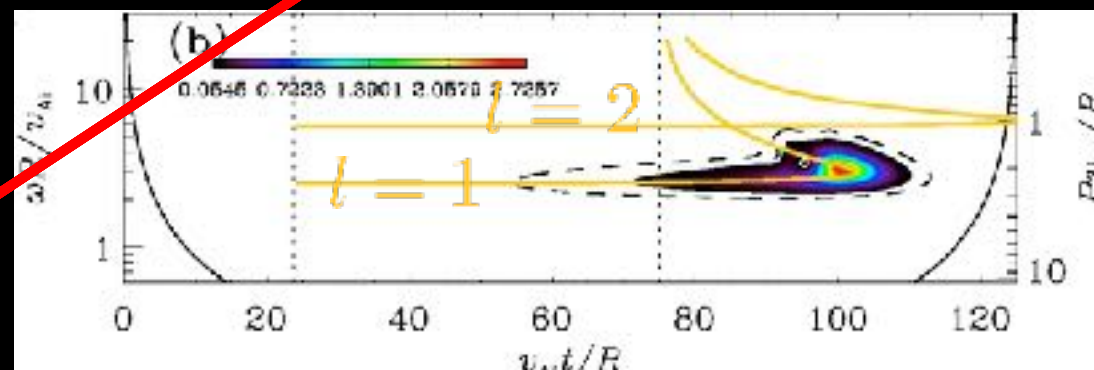
Similar simulations
Slabs [Murawski+91,92, Selwa+07, Nakariakov+04]
Tubes [Shestov+15]

"crazy tadpole"-like Morlet spectra

- WL measurements [Williams+01, 02; Katsiyannis+03, Rudawy+04, …, Samanta+16]
- dm-fiber bursts [e.g., Karlicky's group]



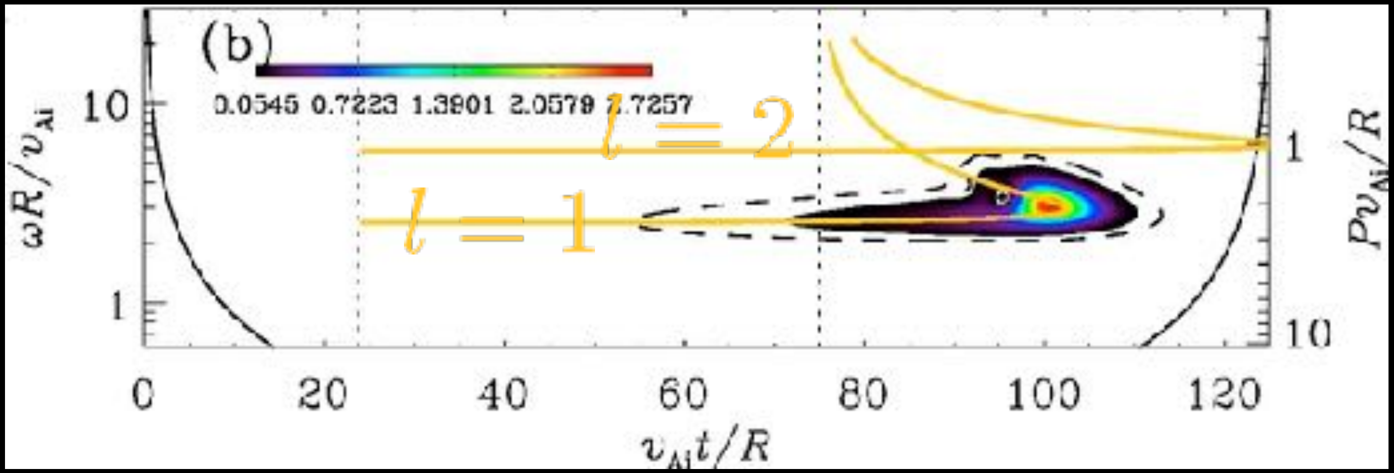




$$\omega - h/v_{\mathrm{gr}}$$

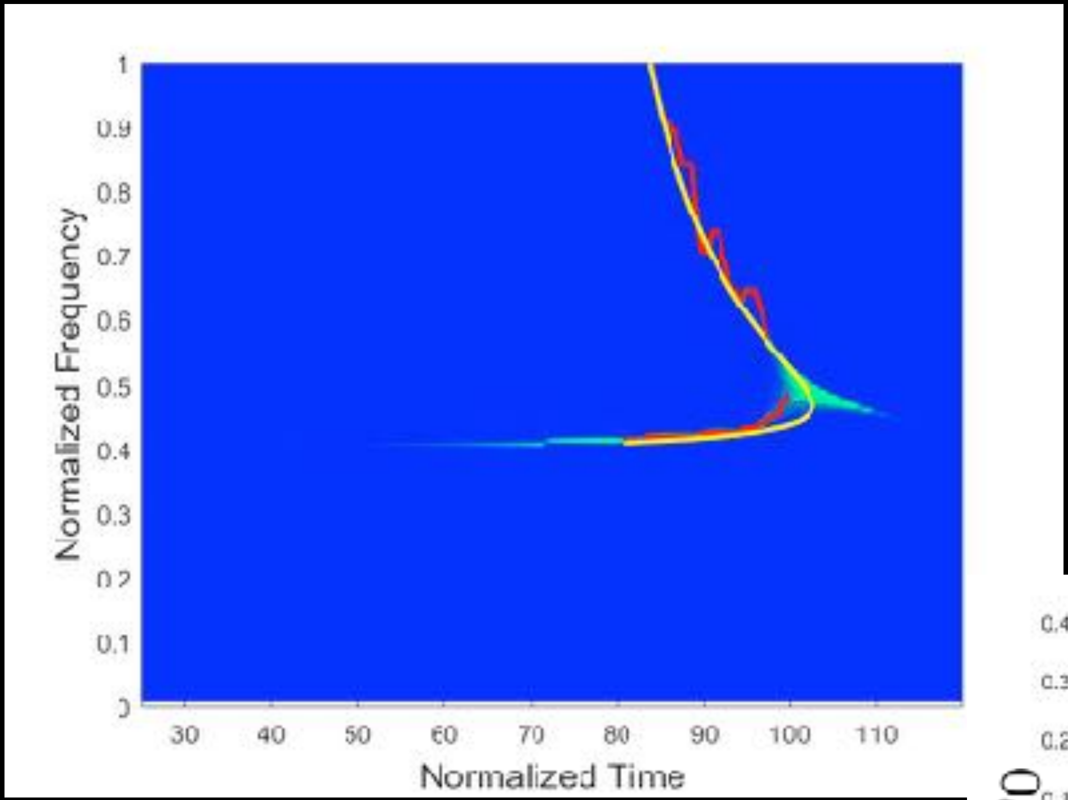
New: Freq - wavepacket arrival time [Yu+16, 17, Li+18a]

Quantitative inversion

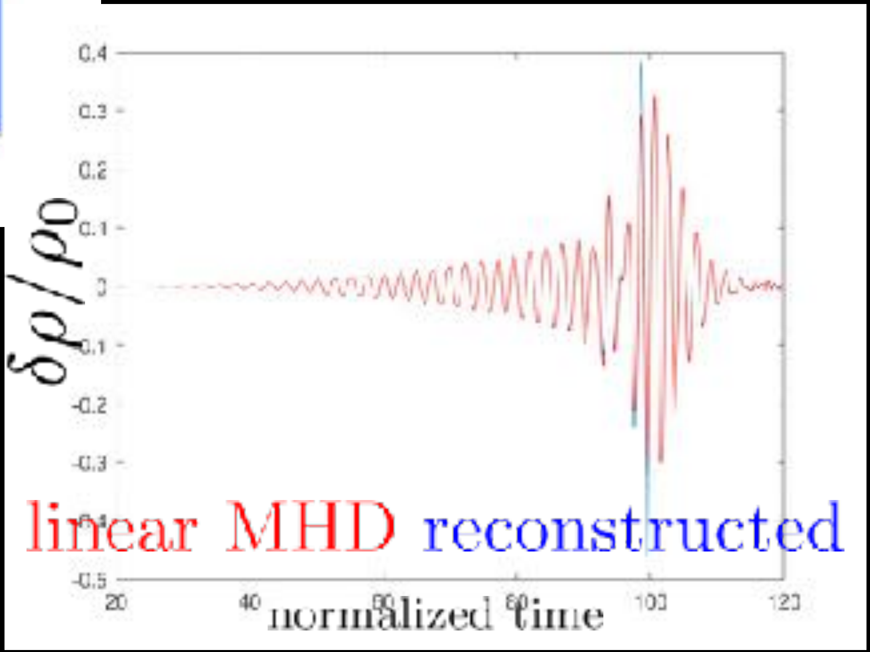


- Key: group speed curves shape Morlet spectra
- Why difficult? Not easy to extract instantaneous freq.
- First attempt with wavelet synchrosqueezed transform (WSST, Li+18c in prep, so far applied to computed data only)
  - WSST assumes that signal $= \sum_k a_k(t) \sin[2\pi f_k(t)t] + \text{noise}$
  - WSST uses Continuous Wavelet Transform (CWT) as input, but reduces energy smearing by frequency reassignment [Daubechies+11]
  - WSST implemented in Matlab after R2017b
  - Safety check possible: reconstruction allowed

Illustration of the idea [Li+18c in prep]

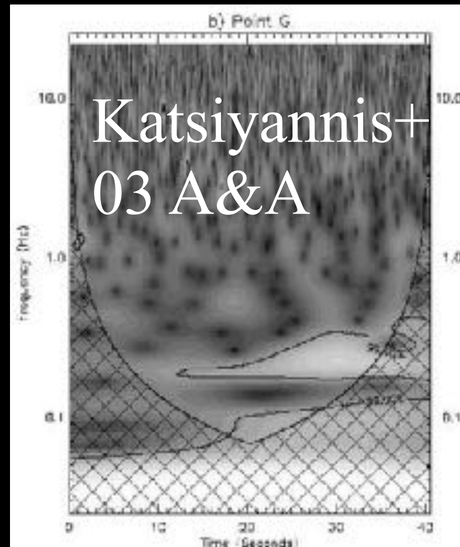


- Color map: WSST spectrum from density time series
- Red: ridge ~ instantaneous freq.
- Yellow: wavepacket arrival time - freq



Inversion scheme [Li+18c in prep]

Input: intensity series (say, 5303) cadence < secs + time of wave source (say, flare)



Linear MHD eigen-value problem solver

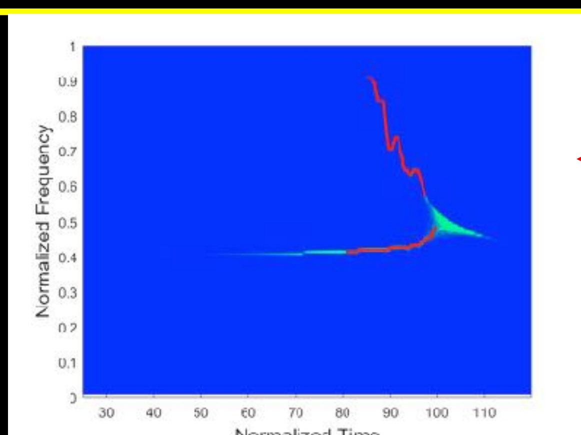
guess 2: den profile func $(\rho_i/\rho_e, \mu)$

$$\frac{v_{\rm gr}}{v_{\rm Ai}} - \frac{fR}{v_{\rm Ai}}$$

guess 1: $R/v_{\rm Ai}$

guess 3: h/R

WSST & ridge extraction



iteration

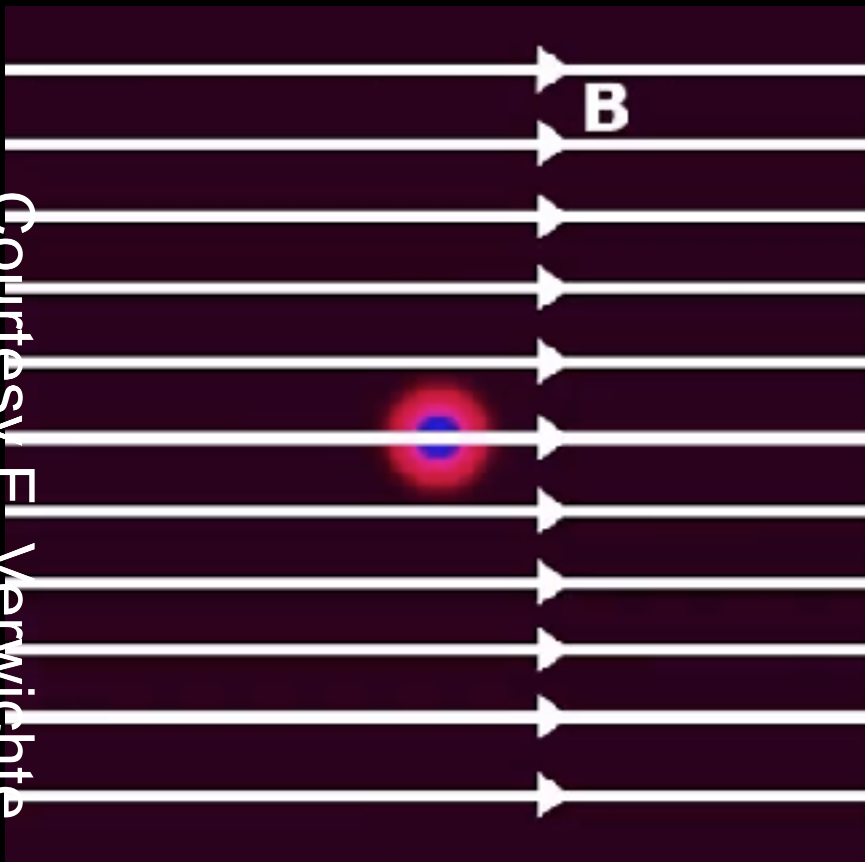
$$\frac{h}{v_{\rm gr}}\frac{v_{\rm Ai}}{R} - \frac{fR}{v_{\rm Ai}}$$

Summary

- Fast sausage waves in coronal tubes
  - axisymmetric, strong compressibility & strong dispersion
- Standing sausage modes
  - Multi-mode measurements are very helpful
- Impulsively generated sausage wavetrains
  - Useful for interpreting oscillatory behavior associated with flares, rapid ones (quasi-period~secs) in particular
  - Morlet spectra shaped by group speed curves of trapped modes
  - Can help probe sub-resolution transverse density structuring

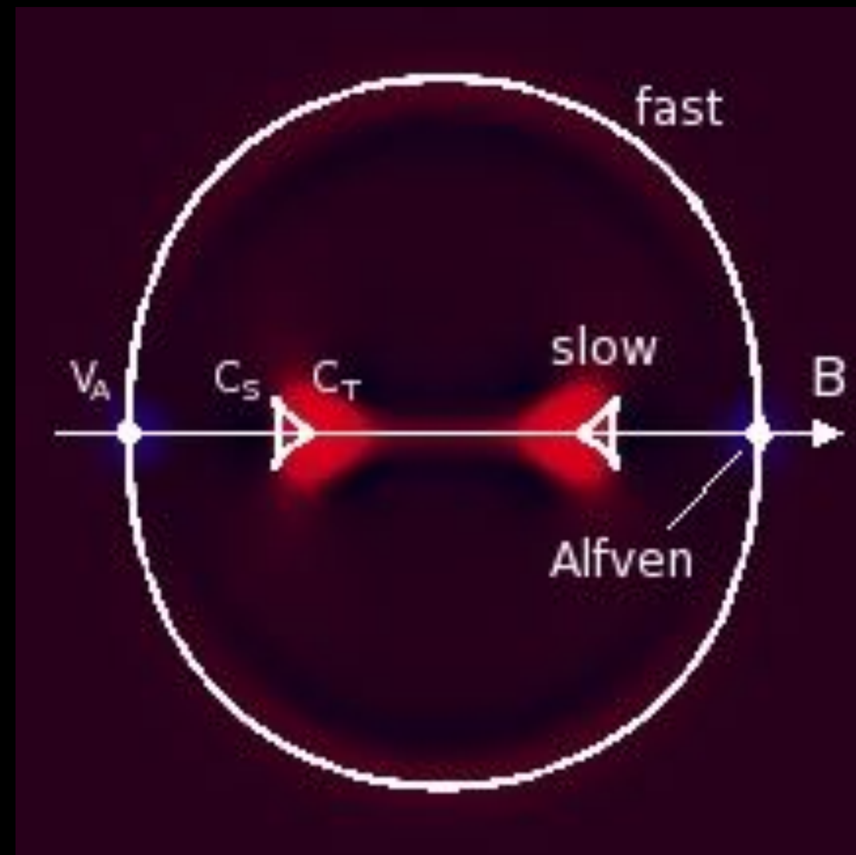
BKUP SLIDES

Response of uniform medium to isotropic pressure pulse



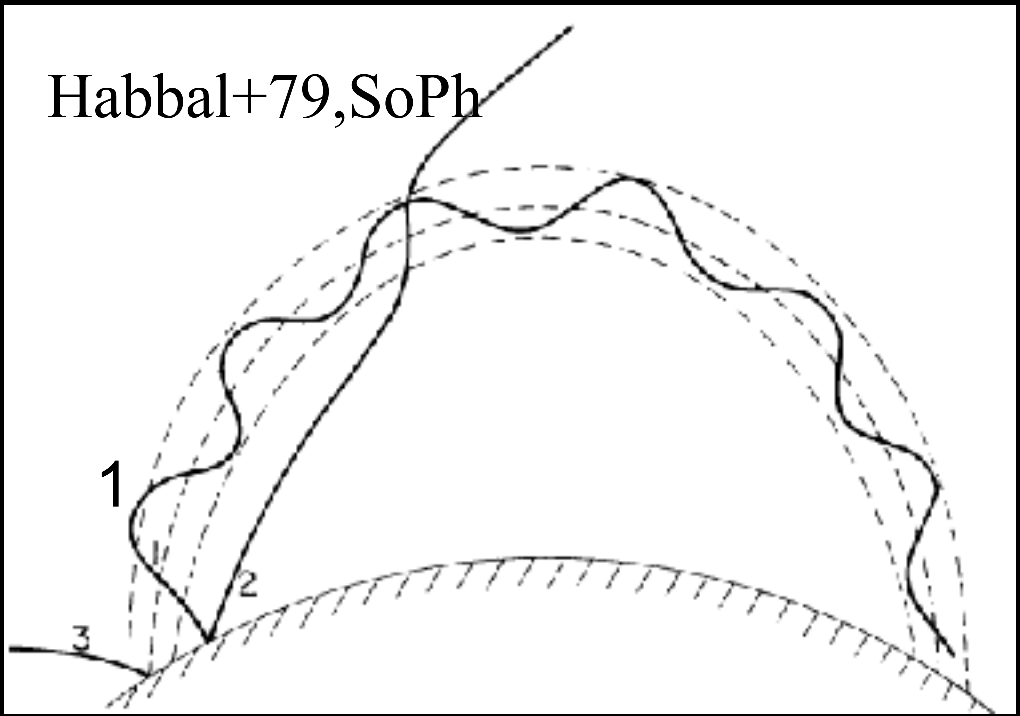
Courtesy E. Verwichte

time-dependent, 2D simulation



group speed diagram

Wave trapping from perspective of ray theory



Equilibrium: an isothermal, gravitationally stratified, corona with a density enhanced “loop”

$$\mathbf{v}_g = \frac{d\mathbf{x}}{dt} = \frac{\partial \omega}{\partial \mathbf{k}} = v_A \hat{\mathbf{k}},$$

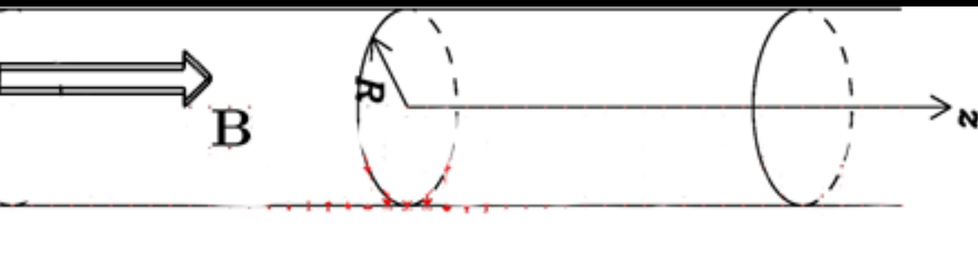
$$\frac{d\mathbf{k}}{dt} = -\frac{\partial \omega}{\partial \mathbf{x}} = -k \frac{\partial v_A}{\partial \mathbf{x}}$$

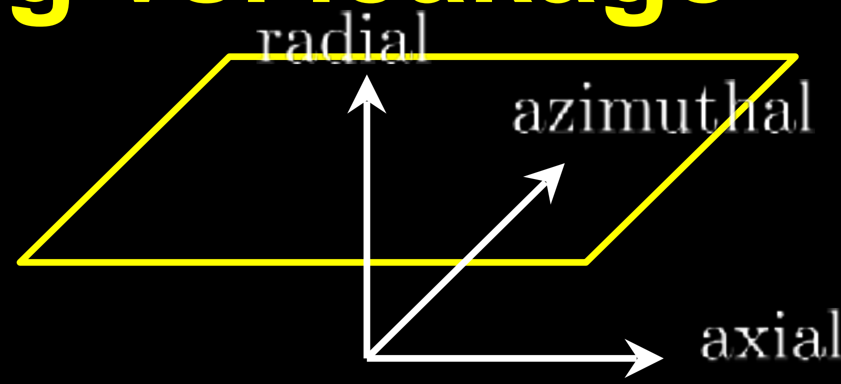
ray 3: originates from inside the loop but does not get trapped
ray 2: originates from outside the loop, not trapped
ray 1: originates from outside the loop, trapped

Jargon 1: trapping vs. leakage

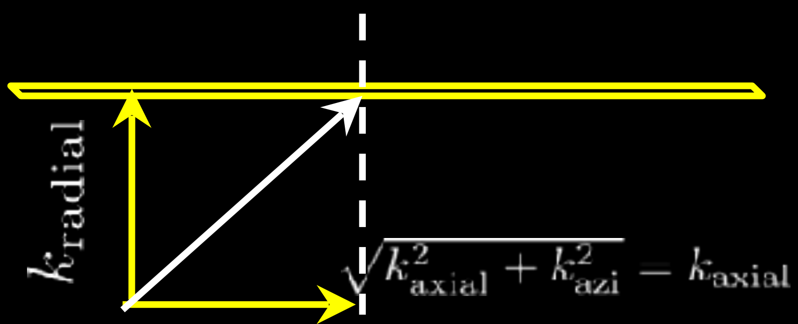
$$\rho_{\text{internal}} > \rho_{\text{external}}$$

$$\rightarrow v_{\text{fast,in}} < v_{\text{fast,ext}}$$



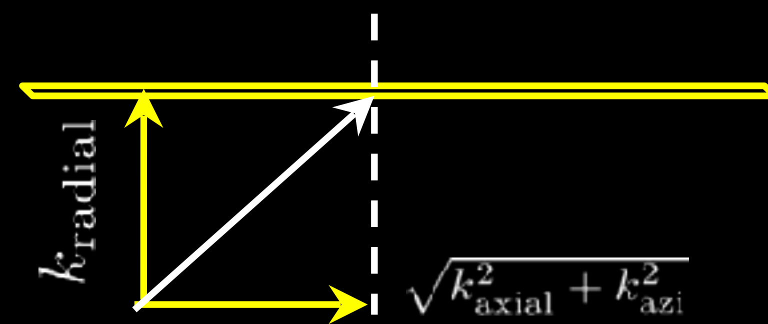


sausage m=0, k_azimuthal=0



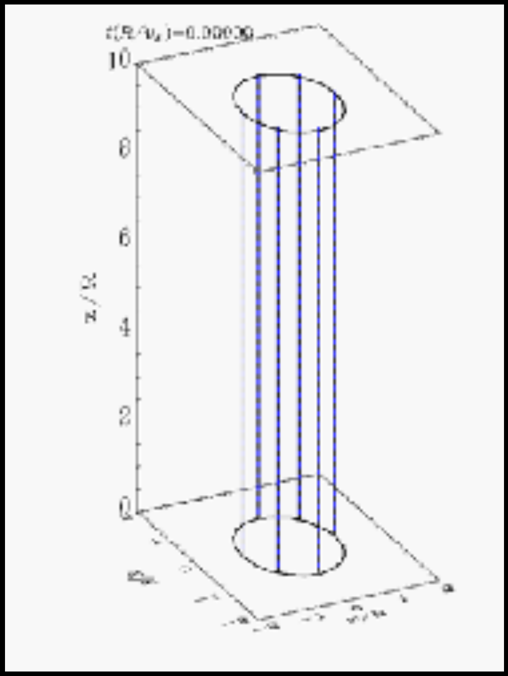
trapped only when k_axial>k_crit

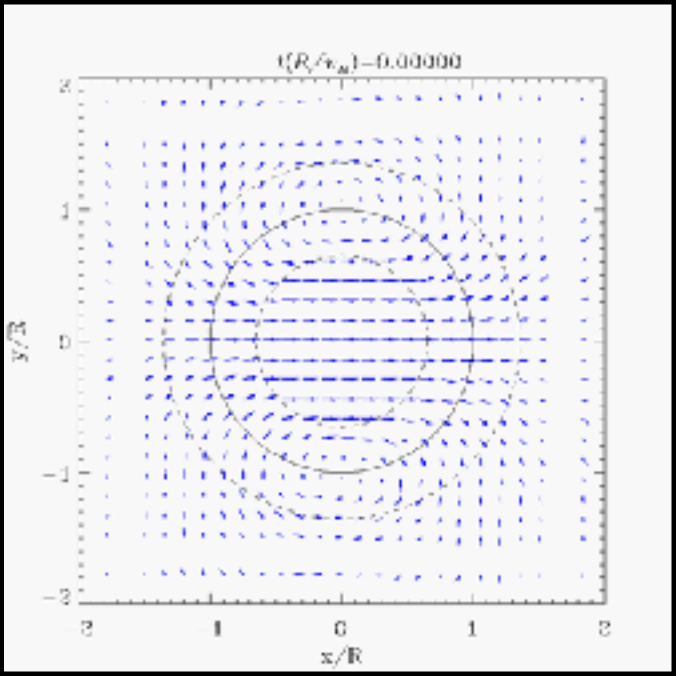
kink m=1, k_azimuthal=1/*R*



may be trapped for arbitrary k_axial

Damping of standing kink modes





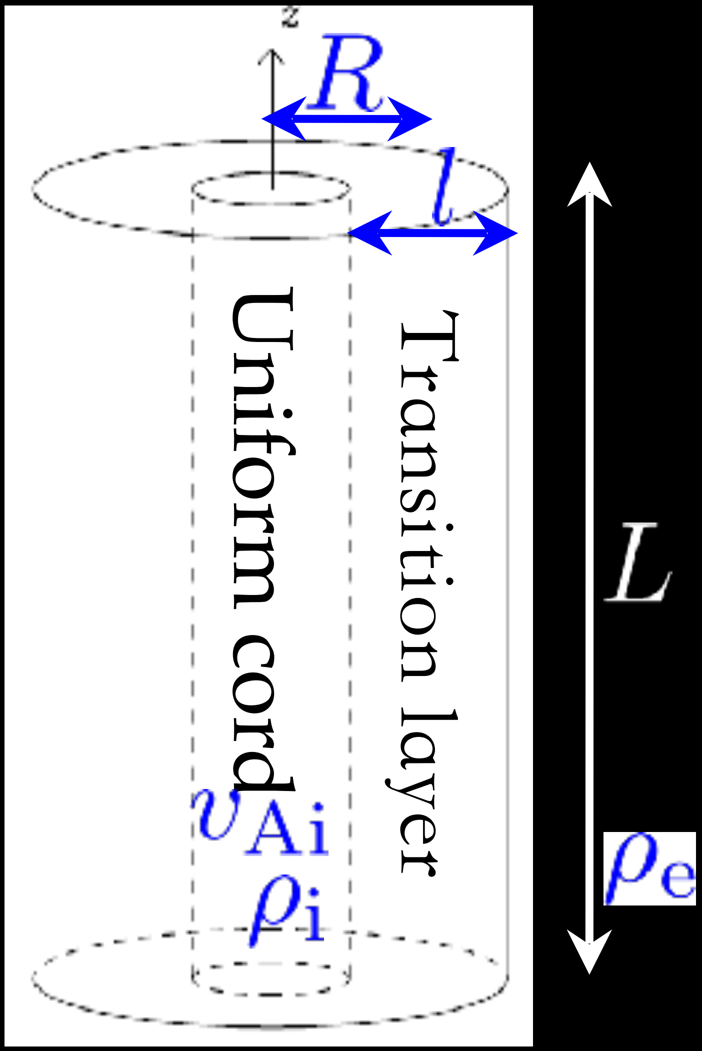
$$\frac{L}{R} = 10, \frac{\rho_\mathrm{i}}{\rho_\mathrm{e}} = 5, \text{linear:} \frac{l}{R} = 0.7$$

3D MHD simulation with PLUTO (by M.-G. Guo)

V field in transverse cut at loop apex

- Resonant absorption = resonant coupling between kink & m=1 Alfven [see review by Goossens+11 SSRv]
  - idea originated in fusion community in 1970’s [Tataronis, Grosmann, L. Chen, Hasegawa], introduced to solar by Ionson 78 [also Wentzel 79, Hollweg 88….]
  - occurs only for continuous transverse density profile → damping rate useful for inferring density lengthscale!
  - Theory in seismological context initiated by Hollweg & Yang 88, Sakurai+Goossens+Hollweg 90, 91, 92] for thin boundaries; for arbitrary layer thickness by Soler+13

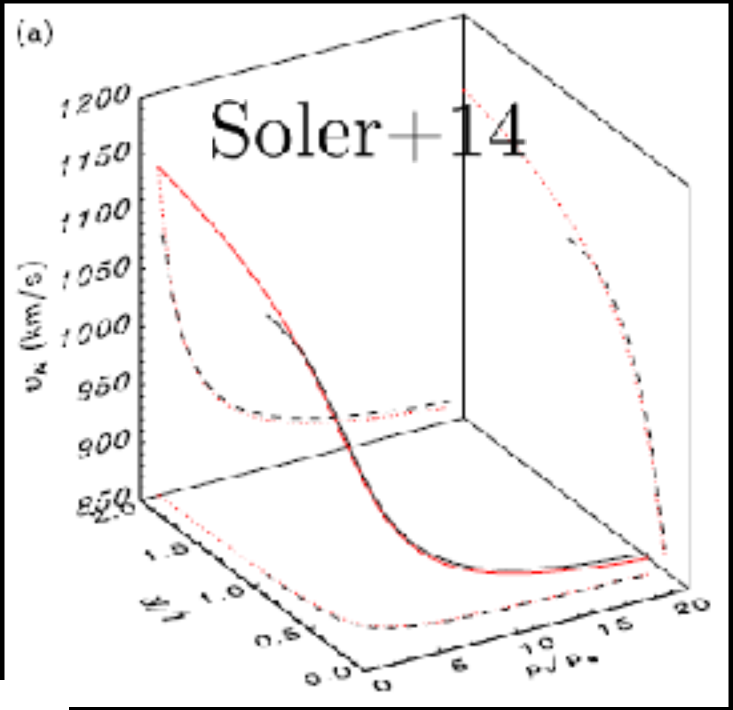
Seismology with kink oscillations



$$P_{\mathrm{kink}} = \frac{L}{v_{\mathrm{Ai}}} F_{\mathrm{kink}}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}\right)$$

$$\tau_{\mathrm{kink}} = \frac{L}{v_{\mathrm{Ai}}} H_{\mathrm{kink}}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}\right)$$

Resonant Absorption

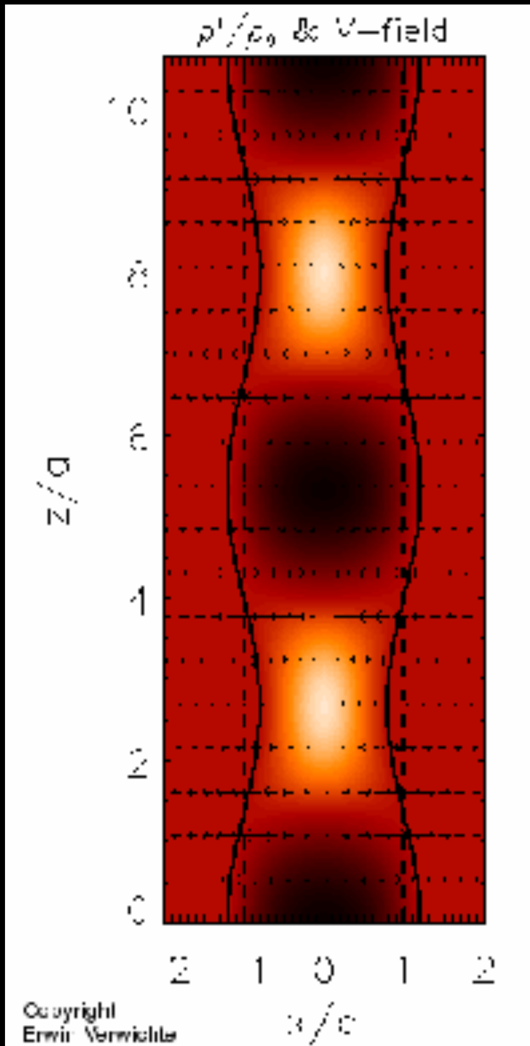


Uniform external

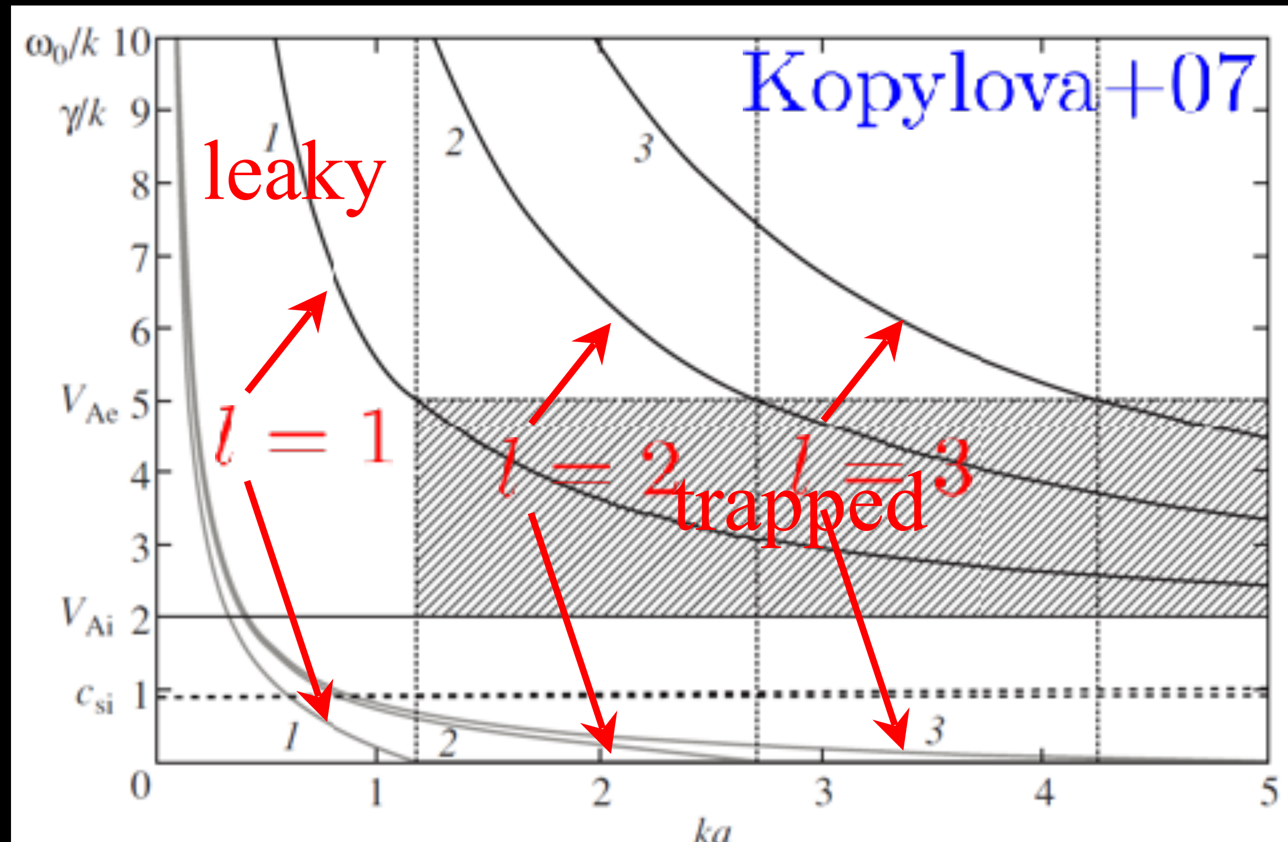
Oscillatory behavior attributable to sausage modes

- Rapid oscillatory behavior in radio bursts
  - Wiggles in Zebra-patterns in type IV bursts: seconds [e.g., S. Yu+13, 16; Kaneda+15, 18, …]
  - dm-fiber bursts: secs to tens of secs [e.g., Fu+90, Zhao+90; … Karlicky & co, 09a, b AA, 11SoPh, 12a, b AA, …, 13AA, 14 ApJ, …]
- Rapid oscillatory behavior (quasi-periods~ seconds) in WL measurements almost exclusively found during total eclipses
  - 1980 Hyderabad: 5303; 0.5-2 sec(?) [e.g.,Pasachoff & Landman 84]
  - 1983 Indonesian: 5303; 0.5-4 sec(?) [e.g.,Pasachoff & Ladd 87]
  - ……
  - 1999 Bulgaria: 5303; 6 sec [Williams+01, 02; Katsiyannis+03]
  - 2010 Chile: 5303 & 6374; 6-25 sec [Samanta+16]
  - ……

Fast sausage waves in coronal tubes:jargons



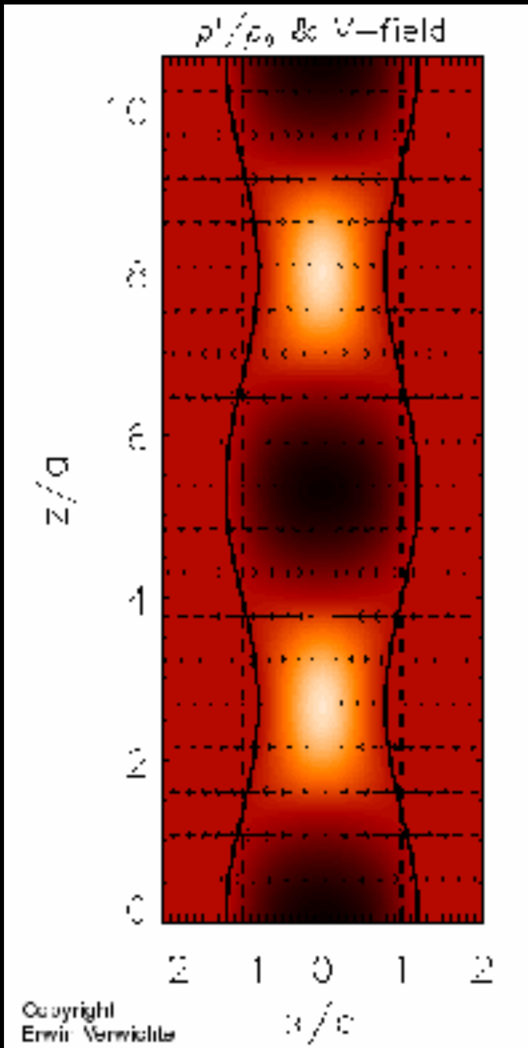
Stationary Propataging waves [Nakariakov & Verwichte 05 LRSP]

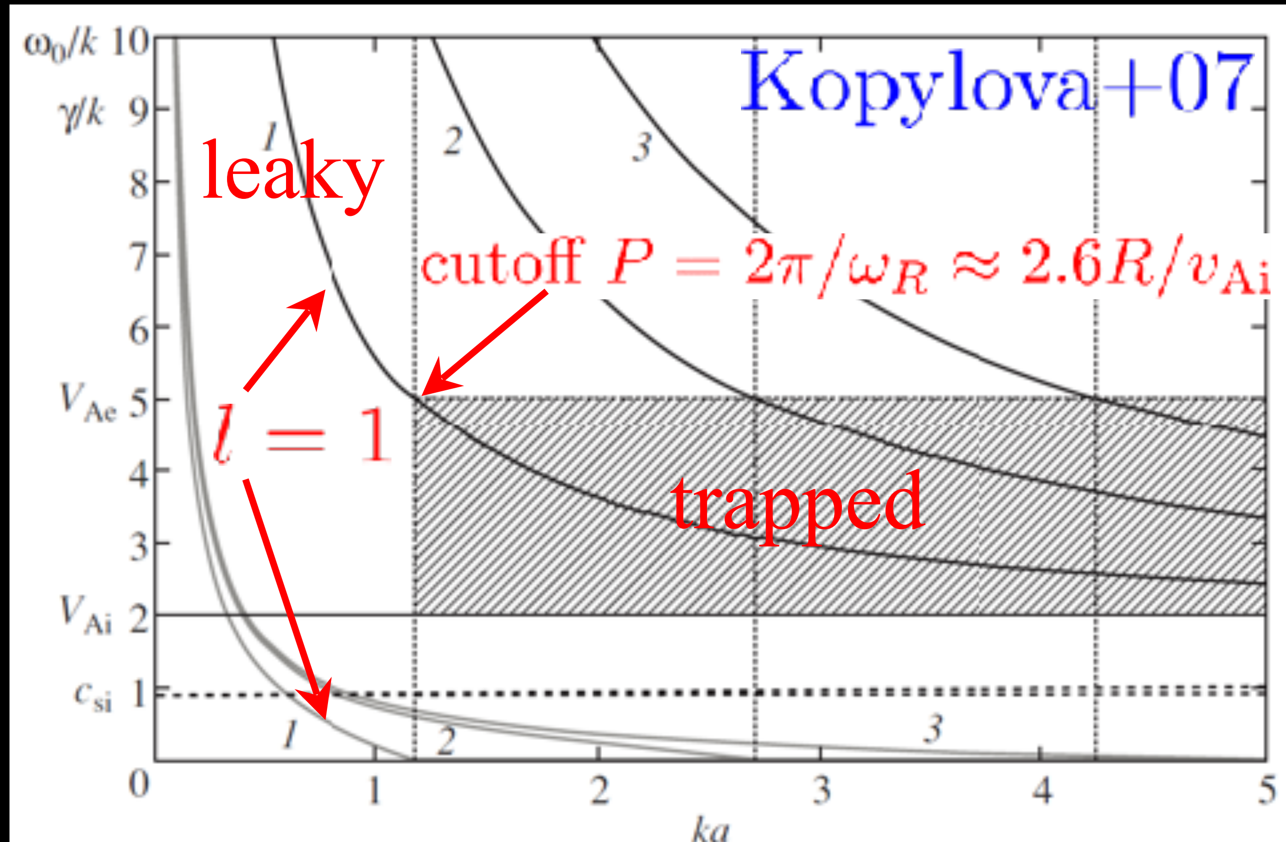


Axial phase speed: **real part (thick)** +imaginary (thin) [Rosenberg 70; Zaitsev & Stepanov 75; Spruit 82; Cally 86; …]

Transverse Structuring: Discontinuous

Fast sausage waves in coronal tubes:jargons





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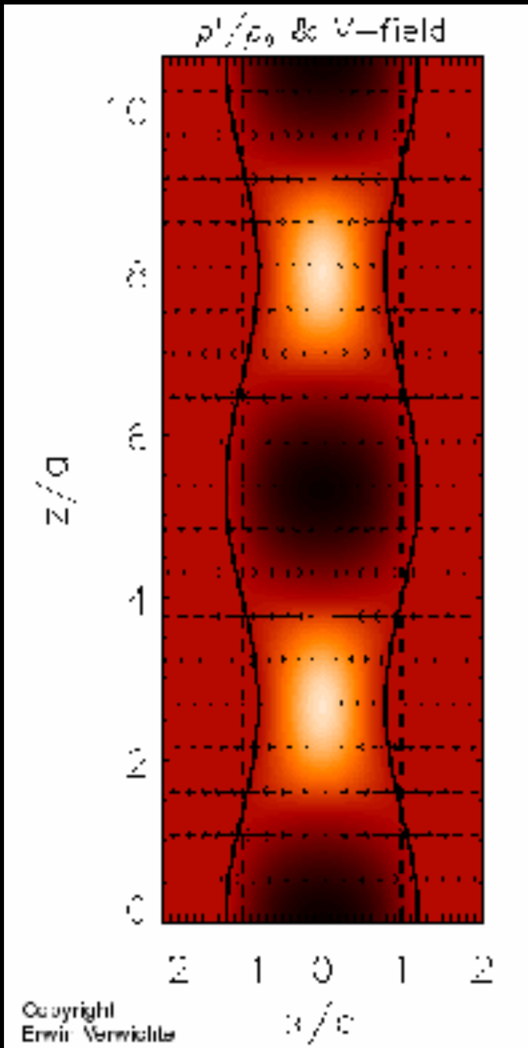

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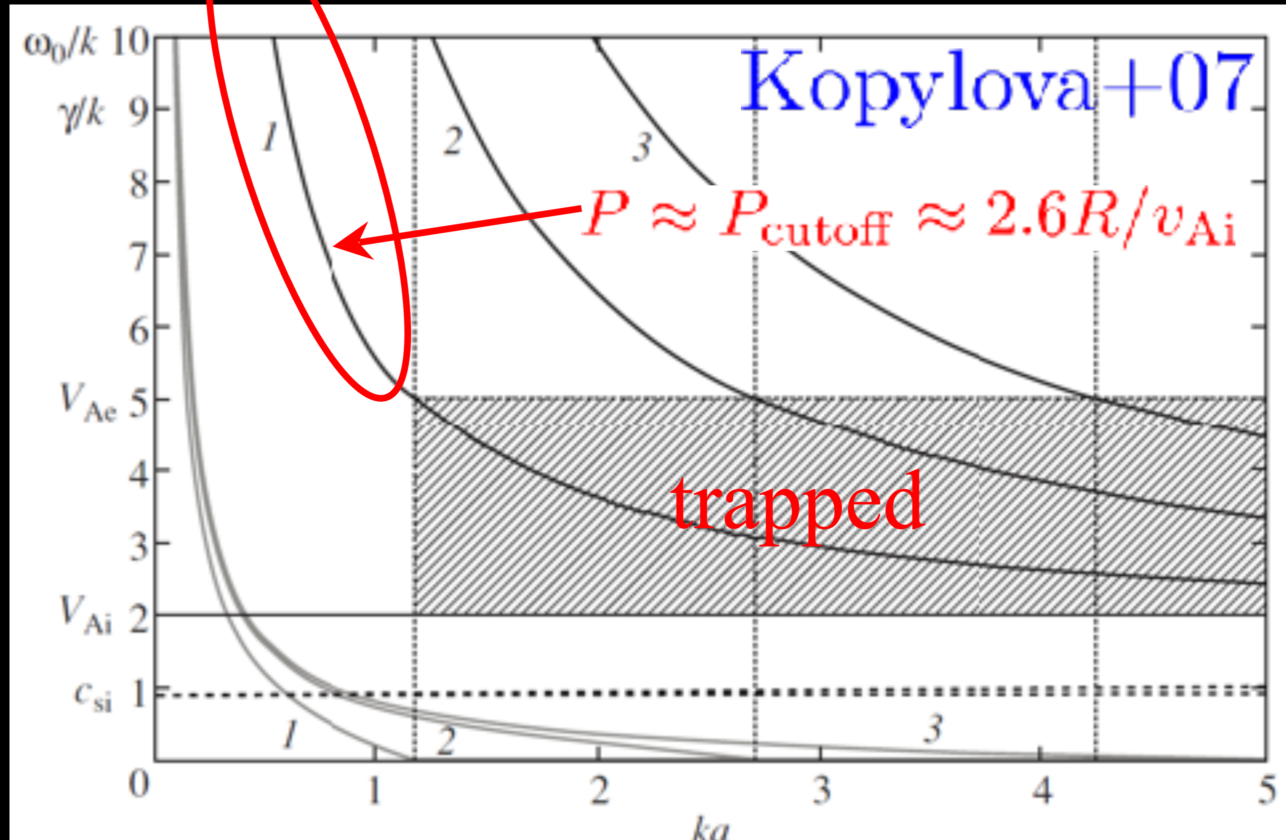
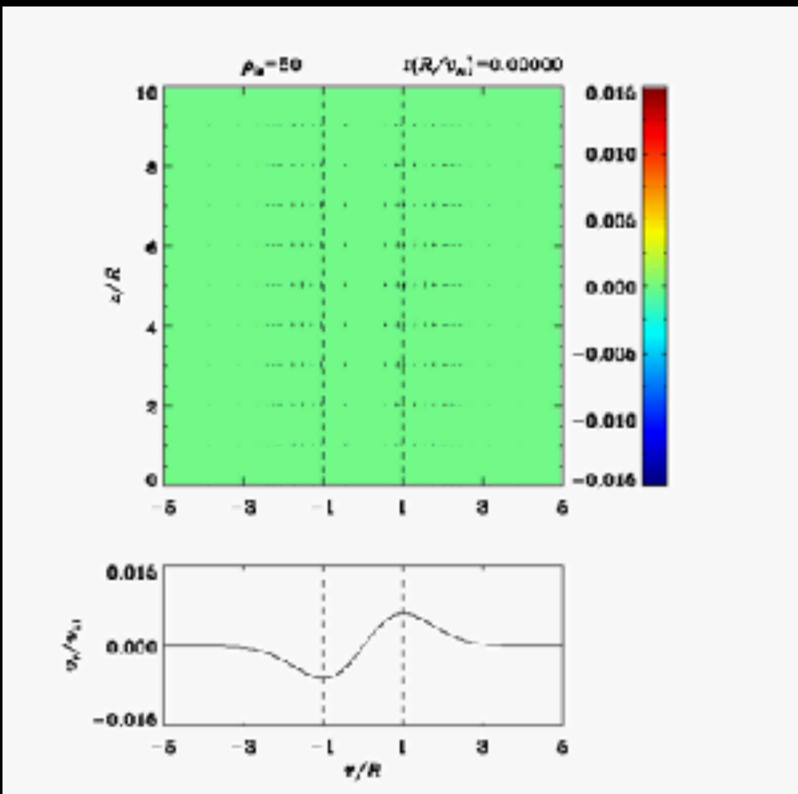


Axial phase speed: **real part (thick)** +imaginary (thin) [Rosenberg 70; Zaitsev & Stepanov 75; Spruit 82; Cally 86; …]

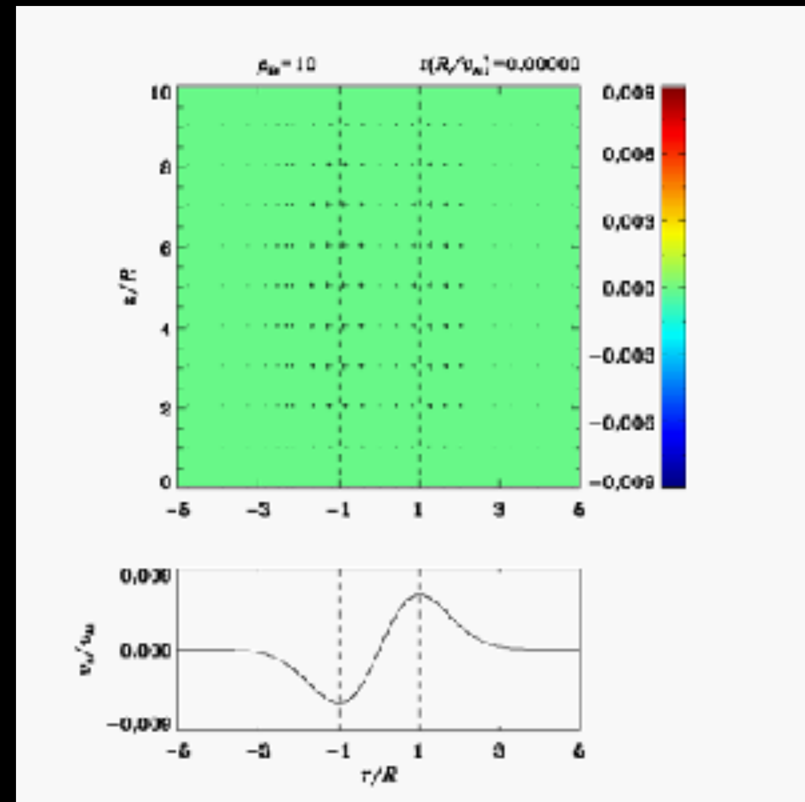
Transverse Structuring: Discontinuous

Two regimes of sausage modes

trapped



leaky

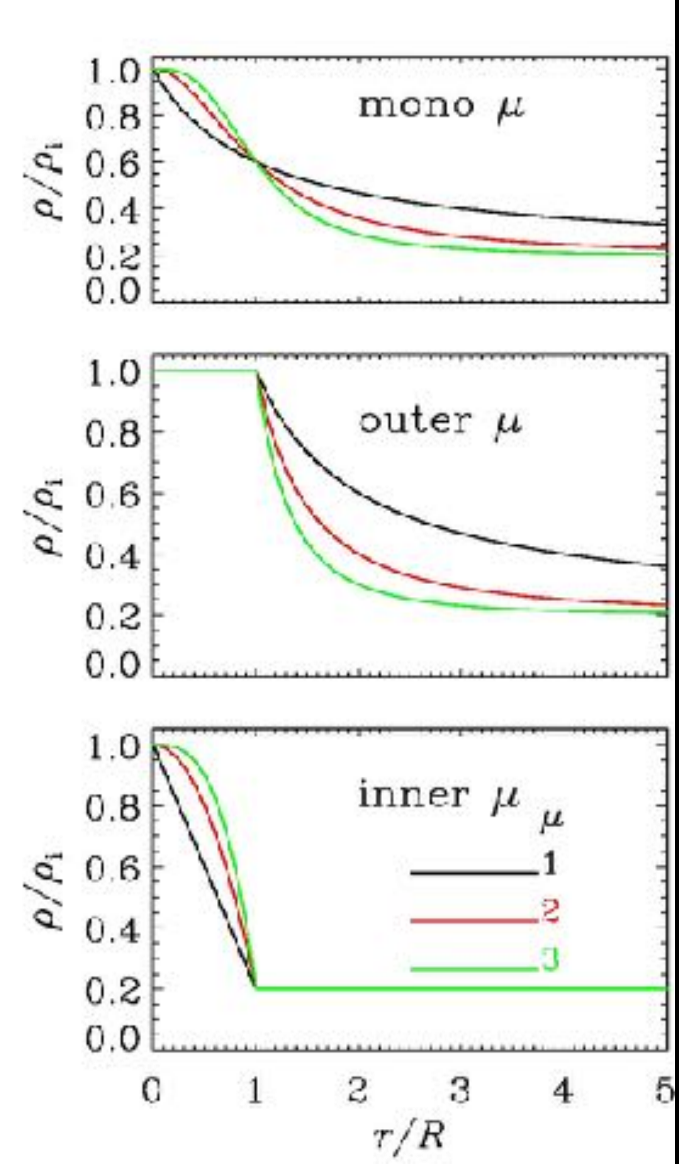


Upper: density variation & *v* field

Lower: transverse (radial) profile of transverse v

simulations with PLUTO by M.Z. Guo

New: Continuous Transverse Structuring



$$\rho(r) = \rho_c + (\rho_i - \rho_c) f(r)$$

$$f(r) = \frac{1}{1 + (r/R)^\mu}$$

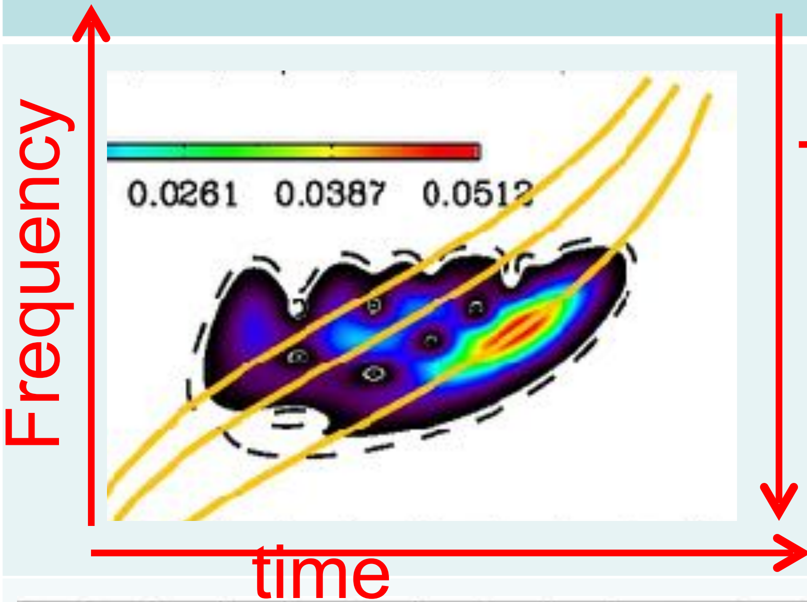
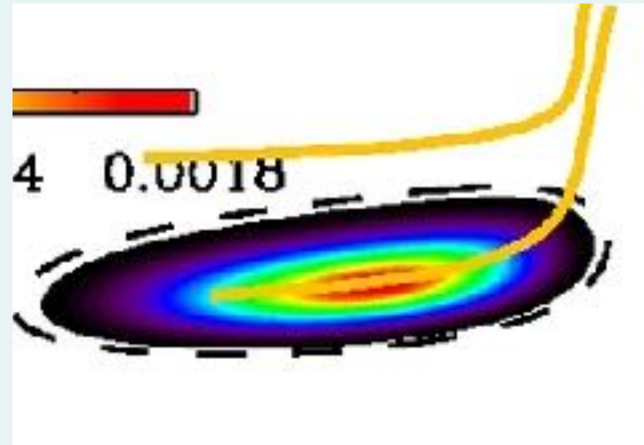
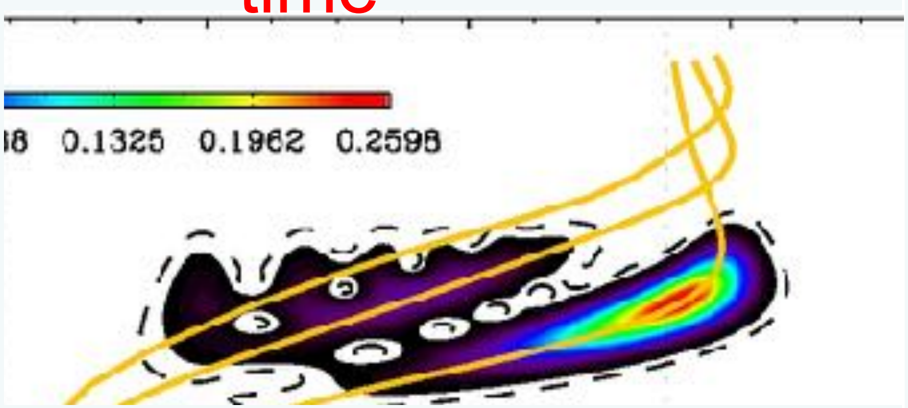
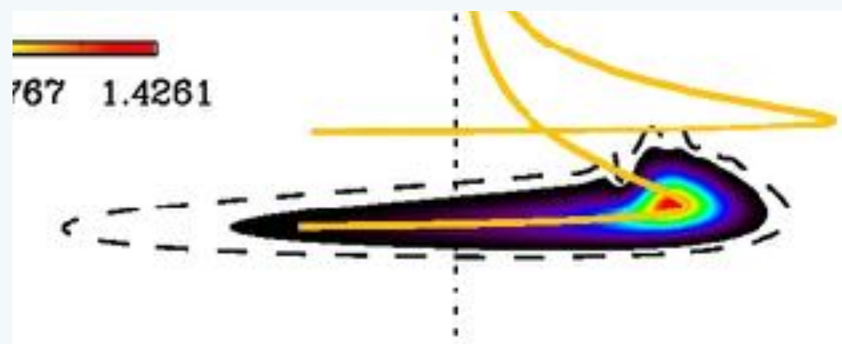
Yu+17 ApJ, Li+18a ApJ, 18b in prep.

$$f(r) = \begin{cases} 1, & 0 \le r \le R \\ (r/R)^{-\mu}, & r \ge R. \end{cases}$$

$$f(r) = \begin{cases} 1 - \left(\frac{r}{R}\right)^\mu, & 0 \le r \le R, \\ 0, & r \ge R. \end{cases}$$

$$\left\{ \begin{array}{l} \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\tilde{\xi}}{\mathrm{d}r}\right) + \left(\frac{\omega^2}{v_\mathrm{A}^2} - k^2 - \frac{1}{r^2}\right)\tilde{\xi} = 0 \\ \tilde{\xi}(r = 0) = 0 \\ \tilde{\xi}(r \to \infty) \to 0 \end{array} \right.$$

Morlet spectra from linear MHD simulations

	Cutoff-free	With cutoffs
monotonical		
non-nonotonical		

Monotonicity of group speed curves

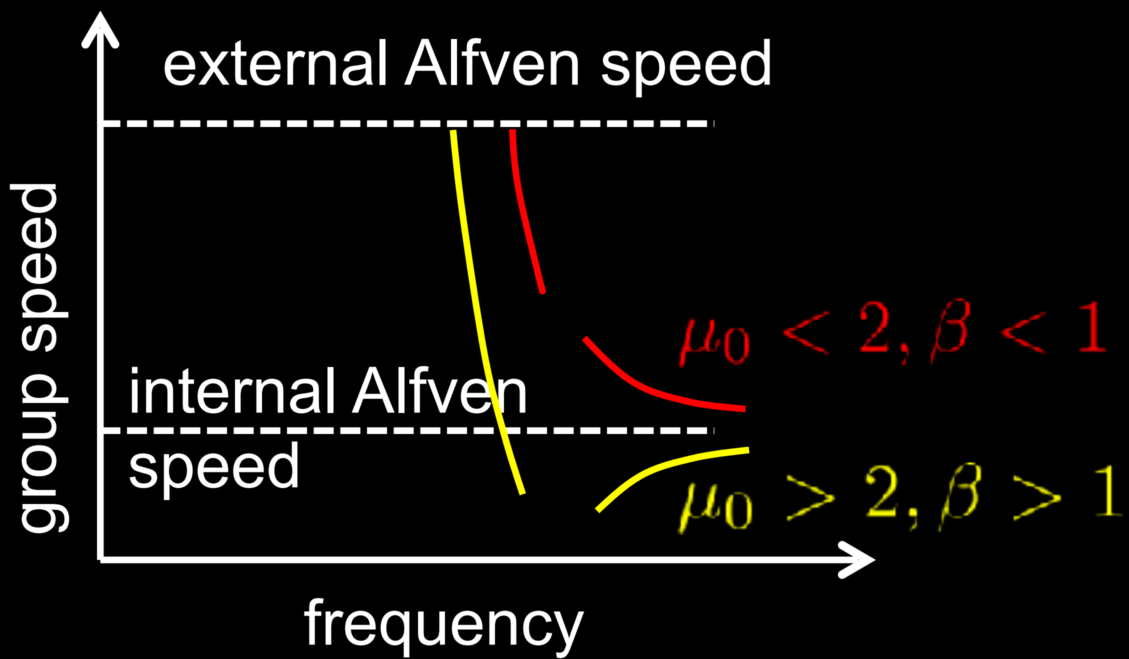
Numerical solutions to eigen-value problem
[Yu, Li+16 ApJ 833, 51; 17 ApJ 836, 1]

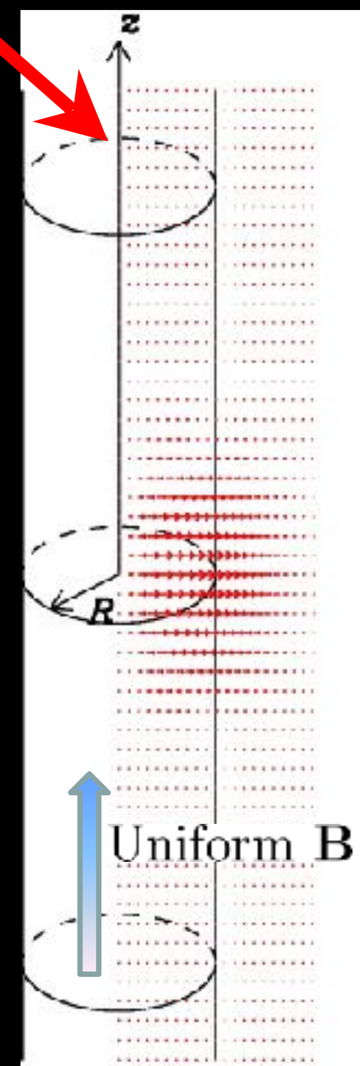
$$\rho(r) = \rho_e + (\rho_i - \rho_e) f(r)$$

$$f(r) \approx 1 - (r/R)^{\mu_0}$$

$$\xrightarrow[\text{Numerical}]{kR \gg 1} \quad \frac{v_{gr}^2}{v_{Ai}^2} \approx 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^{\beta}$$

$$c_l = h_l \left(1 - \frac{\rho_e}{\rho_i}\right)^{1/\mu_0} \qquad \beta = \frac{2\mu_0}{\mu_0 + 2}$$





Monotonicity of group speed curves

Slab [Li+18a ApJ 855, 53, mu_0=1 or 2; Li+18b in prep., arbitrary mu_0 ≠ 2]

$$\xrightarrow[\text{WKB}]{kR\gg 1} \boxed{\frac{v_{\rm gr}^2}{v_{\rm Ai}^2} \approx 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^{\beta} \qquad c_l = h_l\left(1-\frac{\rho_{\rm e}}{\rho_{\rm i}}\right)^{1/\mu_0} \qquad \beta = \frac{2\mu_0}{\mu_0+2}}$$

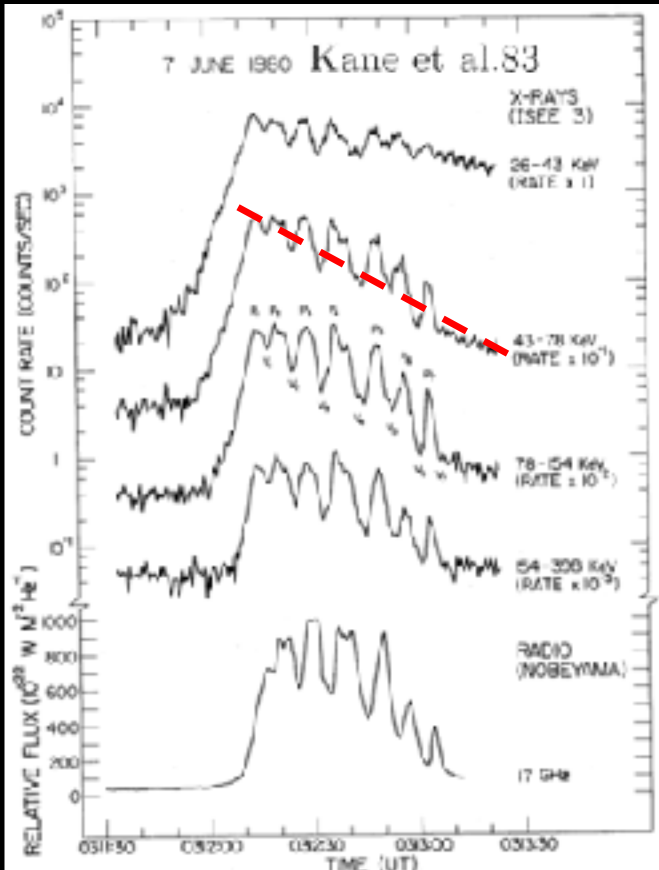
$$h_l = \left(2l - \frac{1}{2}\right)\sqrt{\pi}\,\frac{\Gamma\left(\frac{1}{\mu_0}+\frac{3}{2}\right)}{\Gamma\left(\frac{1}{\mu_0}+1\right)}$$

accurate to several % when $\mu_0 \lesssim 4-10$

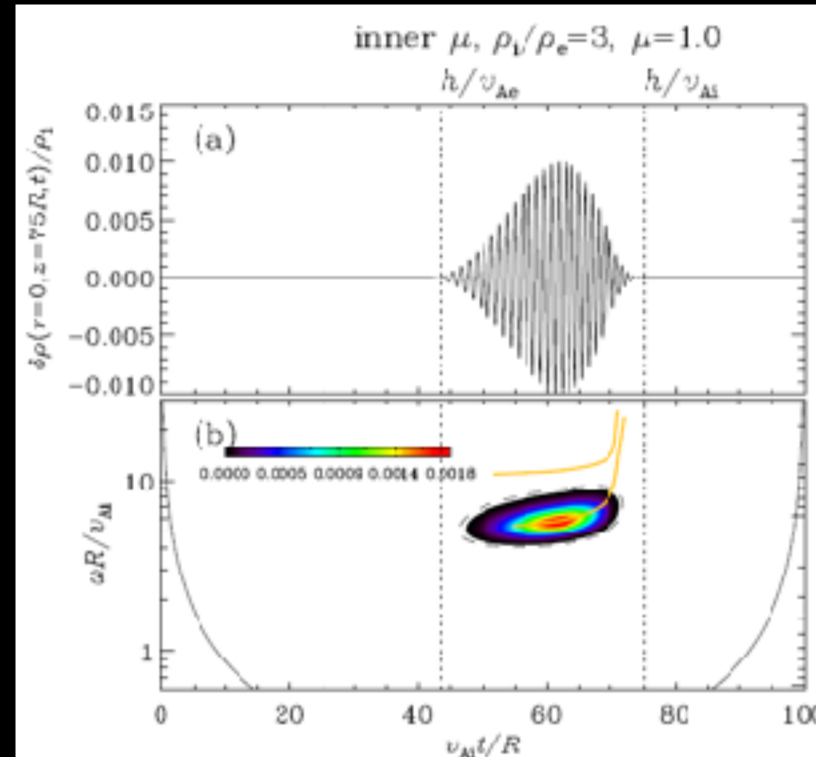
Cylinder [Yu+17 ApJ, mu_0 = 2; Lopin & Nagorny 15 ApJ, mu_0 ≠ 2]

$$\xrightarrow[\text{WKB}]{\rho_{\rm i}/\rho_{\rm e}\gg 1} \boxed{\frac{v_{\rm gr}^2}{v_{\rm Ai}^2} \overset{kR\gg 1}{\approx} 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^{\beta} \quad \beta = \begin{cases} \frac{2\mu_0}{\mu_0+2}, & \mu_0 > 2 \\ \frac{\mu_0}{\mu_0+2}\ ?, & \mu_0 < 2 \end{cases}}$$

Qualitative inversion



?



- Density profile of this flaring loop
  - sufficiently flat at large *r* → cutoff freq.
  - sufficiently steep at loop axis → monotonic group speed curve

frontiers
Research Topics

Magnetohydrodynamic Waves in the Solar Atmosphere: Heating and Seismology

Frontiers in Astronomy and Space Sciences, Frontiers in Physics

- Scope: any original research that addresses 1) how the solar atmospheric waves can be used for seismology, 2) how important waves are for heating the solar atmosphere
- Web: https://www.frontiersin.org/research-topics/8315/magnetohydrodynamic-waves-in-the-solar-atmosphere-heating-and-seismology

Submission Deadlines

23 September 2018	Abstract
18 January 2019	Manuscript

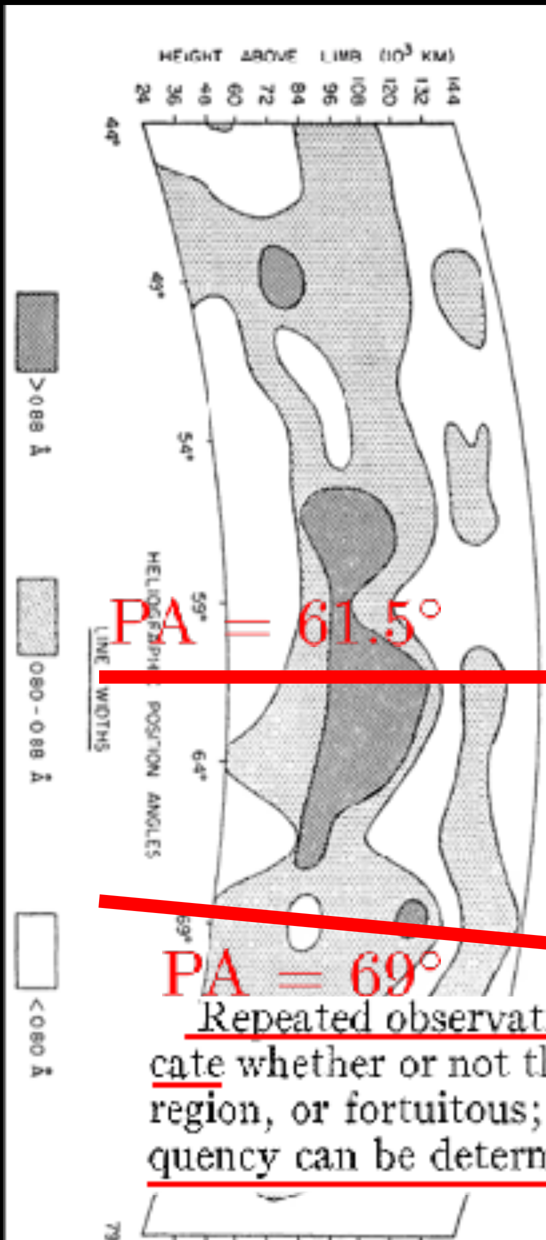
Participating Journals

Manuscripts can be submitted to this Research Topic via the following journals:

Frontiers in
Astronomy and Space Sciences
Stellar and Solar Physics

Frontiers in
Physics
Stellar and Solar Physics

Earliest (?) attempt in SMS



- Cowling's textbook "Magnetohydrodynamics" published in 1957
- Billings' attempt using oscillatory patterns in 5303 linewidth [1959 ApJ, 130, 215]

wave of the type described above. A typical half-width fluctuation is about 0.10 A. This corresponds to a velocity of 5.7 km/sec. Then if the amplitude of the magnetic fluctuations is related to the velocity amplitudes by

$$\frac{h}{H_0} = \frac{v}{a}, \tag{1}$$

where H_0 is the intensity of the existent magnetic field, h the amplitude of the field fluctuation, v the plasma velocity, and

$$a = \frac{H_0}{\sqrt{(4\pi\rho)}}, \tag{2}$$

the velocity of the magnetohydrodynamic wave,

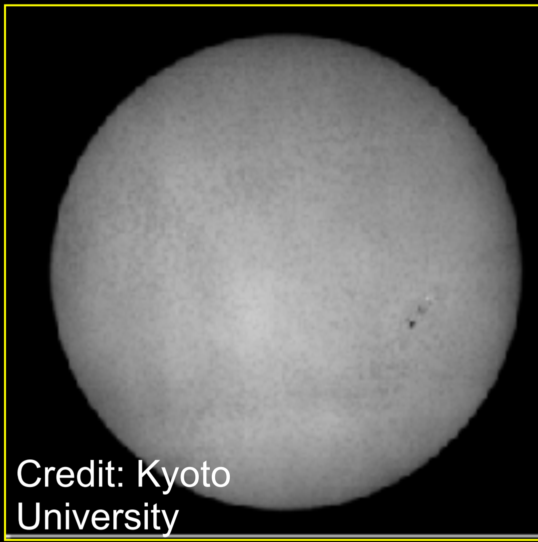
$$h = v\sqrt{(4\pi\rho)} \tag{3}$$

independent of H_0. For $\rho = 1.66 \times 10^{-16}$ gm/cm^3, corresponding to 10^8 protons/cm^3, the

$\rightarrow h \lesssim 0.05\mathrm{G}$

Repeated observations of this type, made over a period of several hours, should indicate whether or not the wavelike distribution of line widths is characteristic of a coronal region, or fortuitous; whether the waves are progressive or standing; and, if their frequency can be determined, the intensity of the magnetic field, H_0.

Earliest (?) attempt in SMS



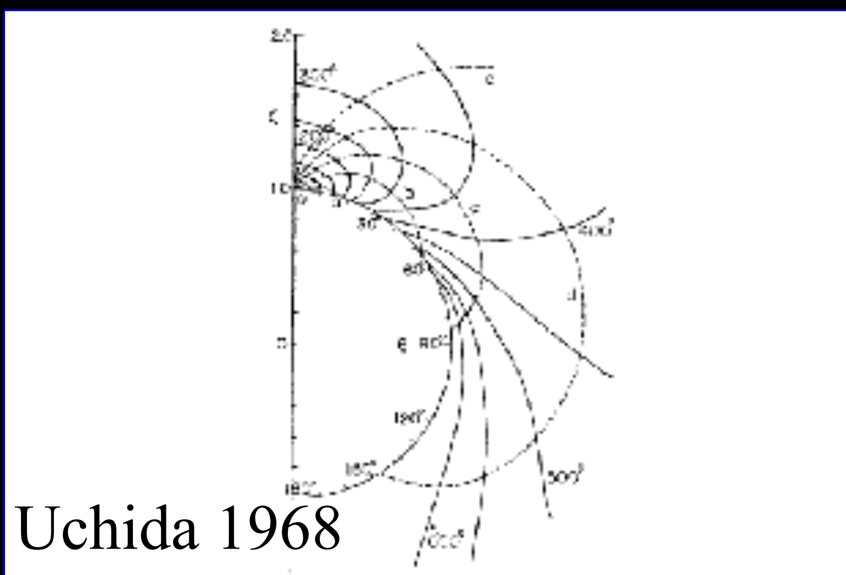
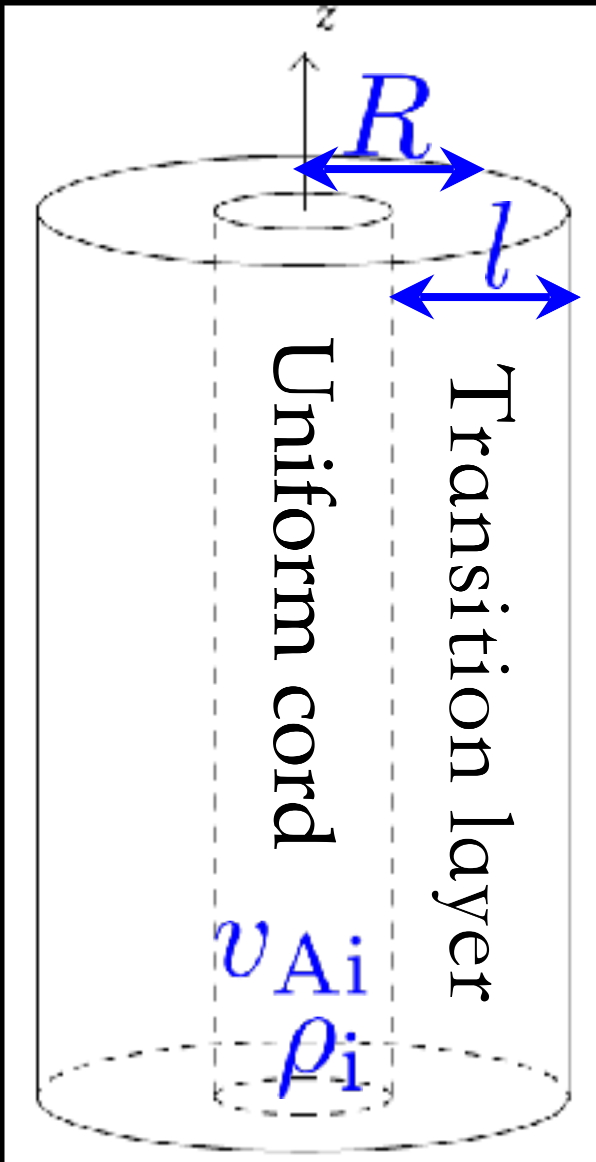
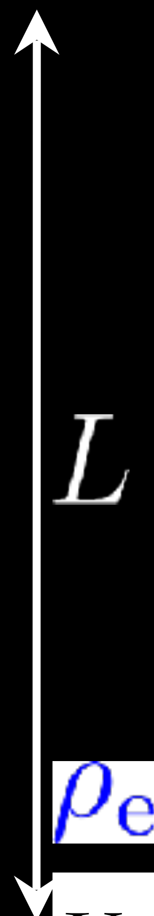


Fig. 4. Representative paths of wave-packets (dashed lines) and the shape of wavefront expanding in time (solid lines) from a source located at $\zeta = 1$, and $\theta = 0$ in a model corona with [illegible]. The wavefronts are labeled with the time lapse after the explosion, and the [illegible] values for the paths a, b, c, d, and e are [illegible], respectively.

- Moreton waves [Moreton+61]
  - Big-flare-associated
  - propagate with 500 – 2000 km/s out to 5e5 km
- Uchida's suggestion[68,70]
  - interface between coronal shock front and the chromosphere
  - the coronal shock/fast wave, if observed, useful to **deduce coronal B**
  - Pioneering "global seismology" [Warmuth 15 LRSP, P.F. Chen 16]

A couple of definitions

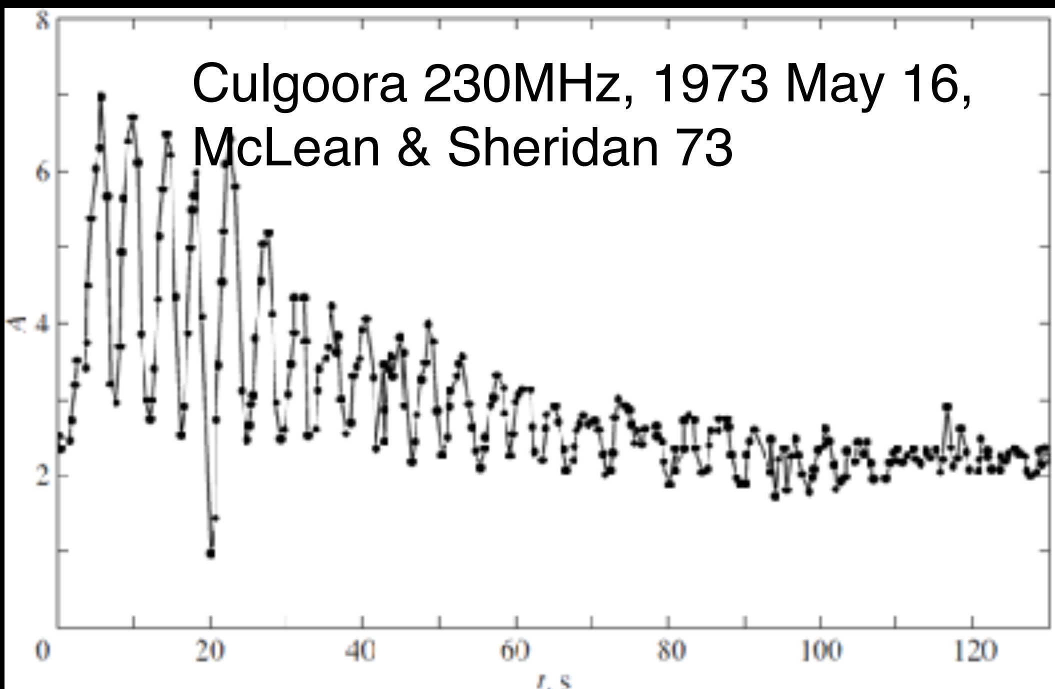




- Geometrical
  - *L:* Loop length
  - *R:* mean radius
  - *l: transition layer width*
- Physical
  - v_{Ai}: internal Alfven speed
  - $\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$:density contrast

Uniform external medium

spatially unresolved QPPs

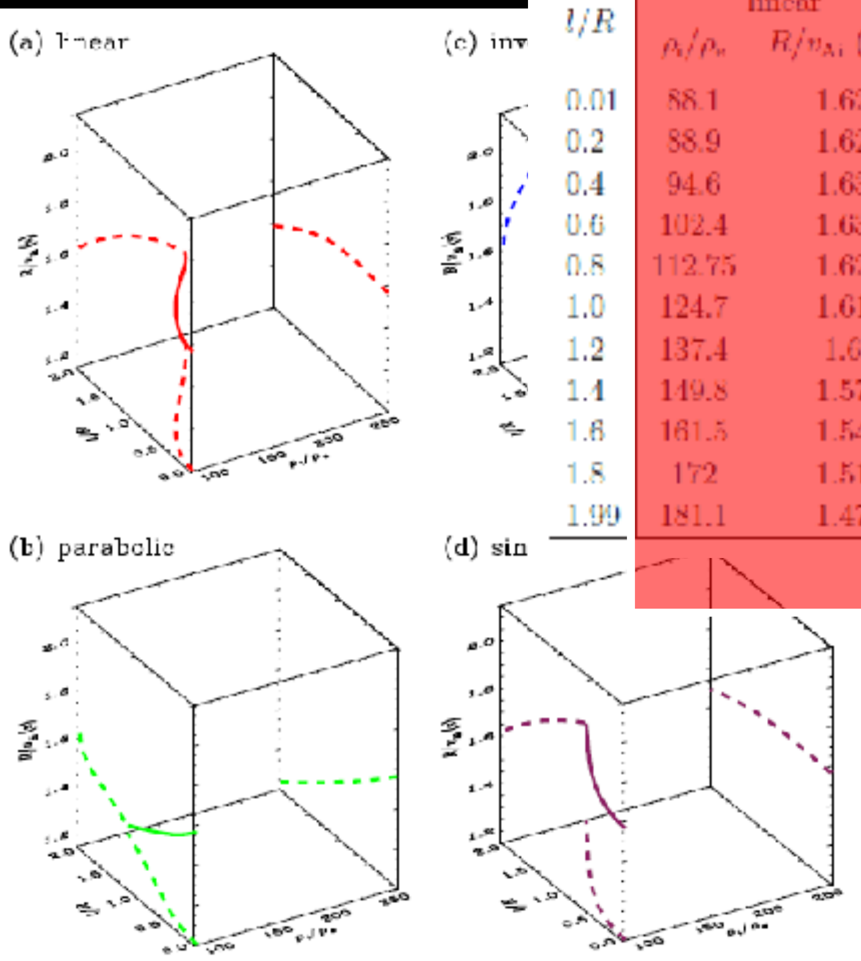


- P = 4.3 s, tau/P=10
- Attributing attenuation to leakage, the best one can do is

$$P_{\rm saus} = \frac{R}{v_{\rm Ai}} F_{\rm saus}\left(\cancel{\frac{L}{R}}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right),$$

$$\frac{\tau_{\rm saus}}{P_{\rm saus}} = G_{\rm saus}\left(\cancel{\frac{L}{R}}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right).$$

Spatially unresolved QPPs



l/R	linear ρ_i/ρ_e	linear R/v_{Ai} (sec)	parabolic ρ_i/ρ_e	parabolic R/v_{Ai} (sec)	inverse-parabolic ρ_i/ρ_e	inverse-parabolic R/v_{Ai} (sec)	sinc ρ_i/ρ_e	sinc R/v_{Ai} (sec)
0.01	88.1	1.62	88.2	1.62	88.1	1.62	88.1	1.62
0.2	88.9	1.62	89.2	1.57	89.3	1.68	89.1	1.62
0.4	94.6	1.63	92	1.52	93.2	1.74	91.9	1.62
0.6	102.4	1.63	95.8	1.48	00.4	1.81	96.4	1.63
0.8	112.75	1.62	100.4	1.43	11.5	1.88	102.4	1.63
1.0	124.7	1.61	105.4	1.39	27.5	1.95	109.8	1.62
1.2	137.4	1.6	110.5	1.35	48.4	2.02	118.2	1.62
1.4	149.8	1.57	115.4	1.3	73.2	2.07	127.4	1.61
1.6	161.5	1.54	120	1.26	00.1	2.1	136.9	1.6
1.8	172	1.51	124.4	1.22	26.8	2.12	146.4	1.58
1.99	181.1	1.47	128.2	1.18	51.4	2.13	155.4	1.56

- R/v_{Ai}: max/min = 1.8
- den. contrast: max/min = 2.9
- l/R: [0, 2] possible

Finite gas pressure?

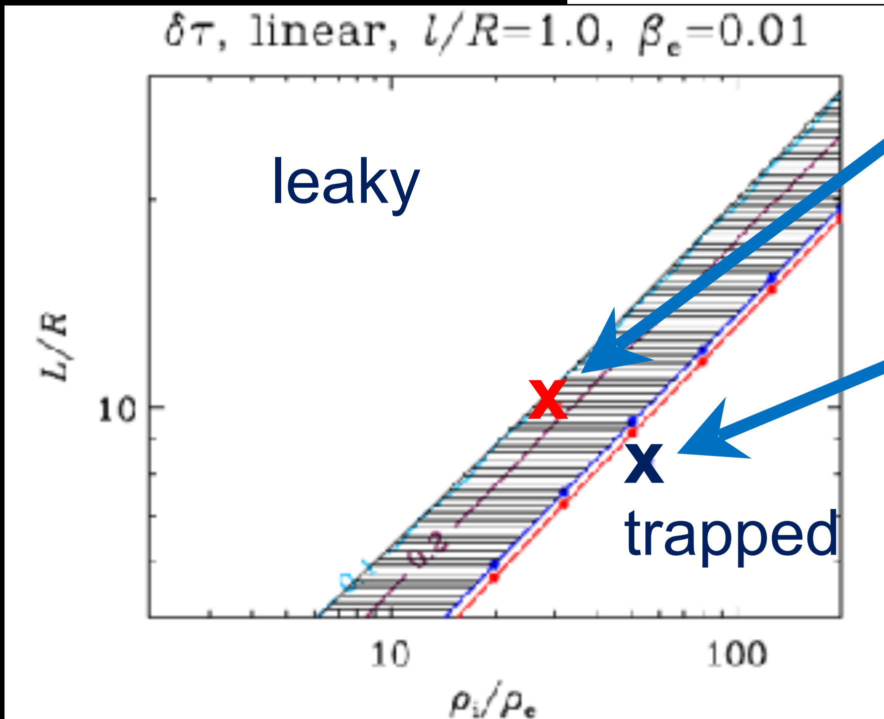
- Interior-Transition layer-Exterior for both density & temperature (Chen, Li+16 ApJ 833, 114)
- Solution method essentially the same as cold MHD case

$$\Longrightarrow \quad \frac{\omega R}{v_{\mathrm{Ai}}} = \mathcal{G}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}, \beta_{\mathrm{i}}, \beta_{\mathrm{e}}\right)$$

or better as ?

$$\frac{\omega R}{v_{\mathrm{fi}}} = \mathcal{H}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}, \beta_{\mathrm{i}}, \beta_{\mathrm{e}}\right) \qquad v_{\mathrm{fi}} = \sqrt{v_{\mathrm{Ai}}^2 + c_{\mathrm{si}}^2}$$

$$\frac{\omega R}{v_{\mathrm{fi}}} = \mathcal{H}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}, \beta_{\mathrm{i}}, \beta_{\mathrm{e}}\right)$$



NoRH (Kolotkov+15)

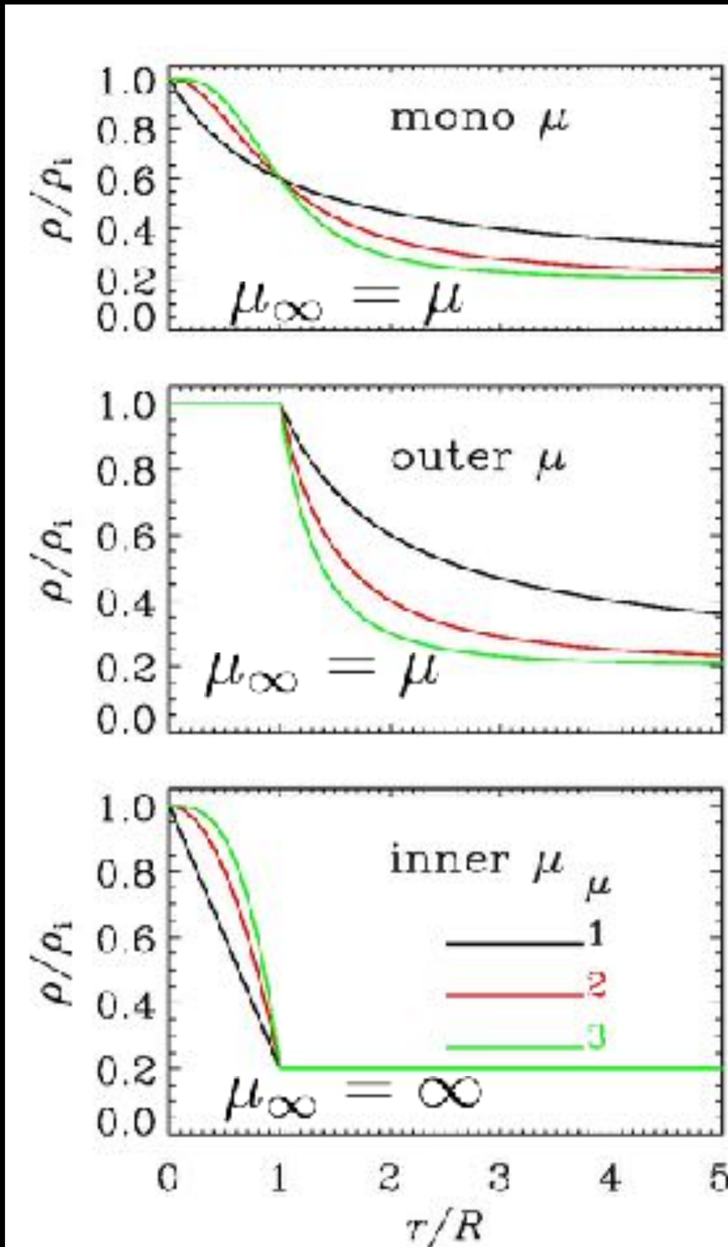
NoRH (Nakariakov+03);

IRIS (Tian+ 16)

$$\delta\tau = \max \left| \frac{\tau^{\beta\neq0}(\beta_{\mathrm{i}} \in [0, 1])}{\tau^{\mathrm{cold}}} - 1 \right|$$

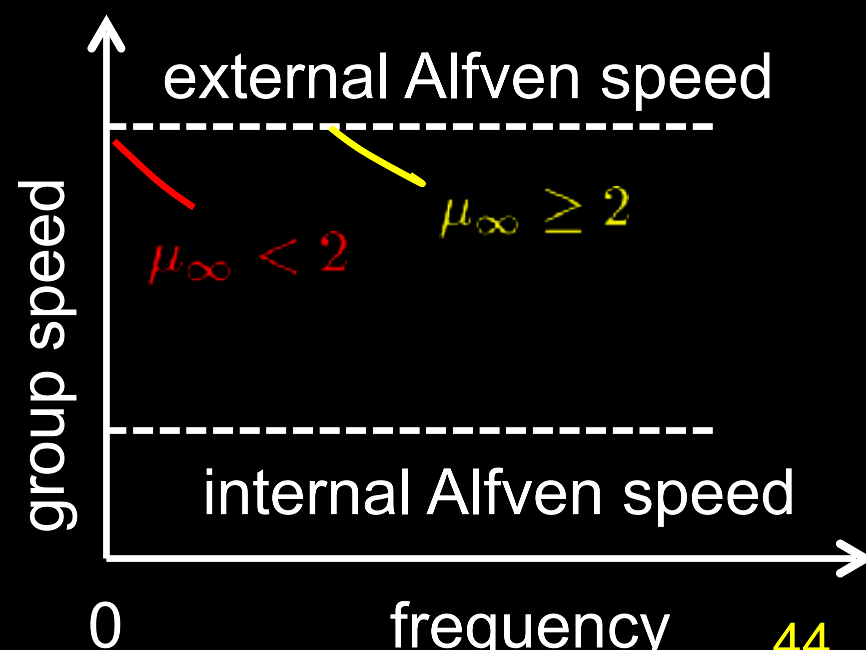
- P in R/v_{fi} not sensitive to internal beta → cold MHD results can be used to invert period, but the derived v_{Ai} actually means v_{fi}
- Cold MHD results can be used to invert damping time if mode is deep in the leaky regime

Existence of cutoff frequency



$$f(r) \approx (r/R)^{-\mu_\infty}$$

Oscillation theorem [Lopin & Nagorny 15; Yu+17]



Spatially resolved QPPs ?

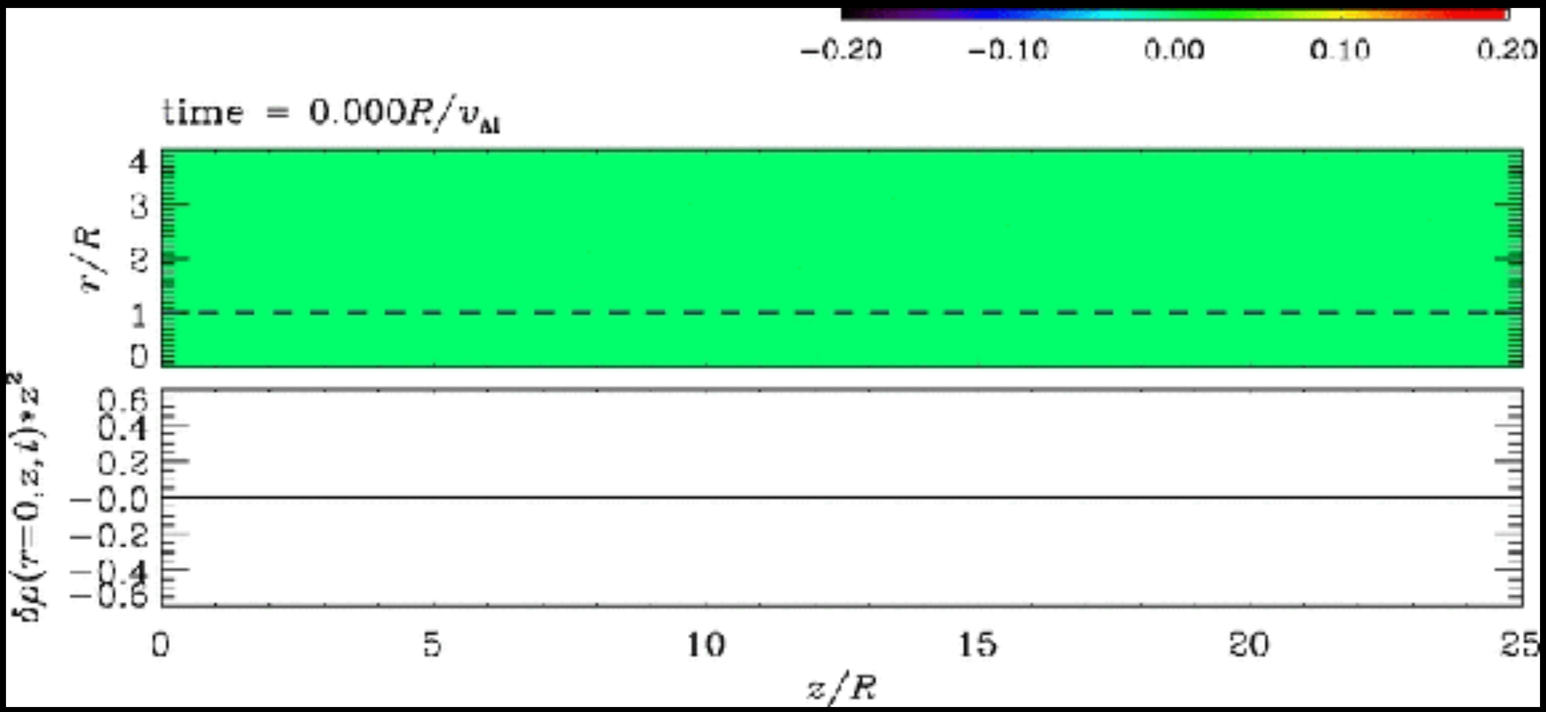
$$P_{\rm saus} = \frac{R}{v_{\rm Ai}} F_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right),$$

$$\frac{\tau_{\rm saus}}{P_{\rm saus}} = G_{\rm saus}\left(\frac{L}{R}, \frac{l}{R}, \frac{\rho_{\rm i}}{\rho_{\rm e}}\right).$$

- Counting knowns and unknowns
  - knowns: L, R; P, τ
  - unknowns: $v_{\rm Ai}$, l/R, $\rho_{\rm i}/\rho_{\rm e}$
- Problem remains under-determined

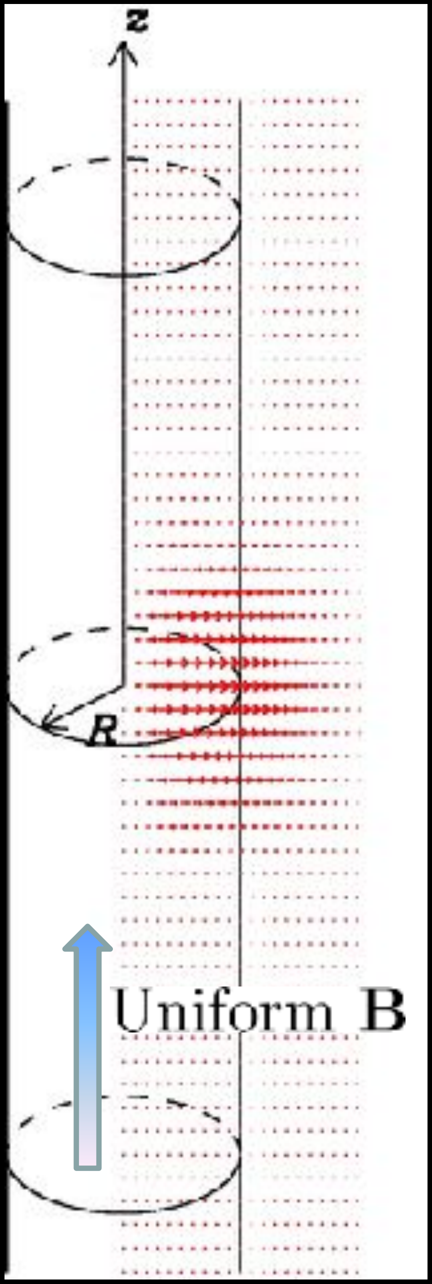
The uniform case

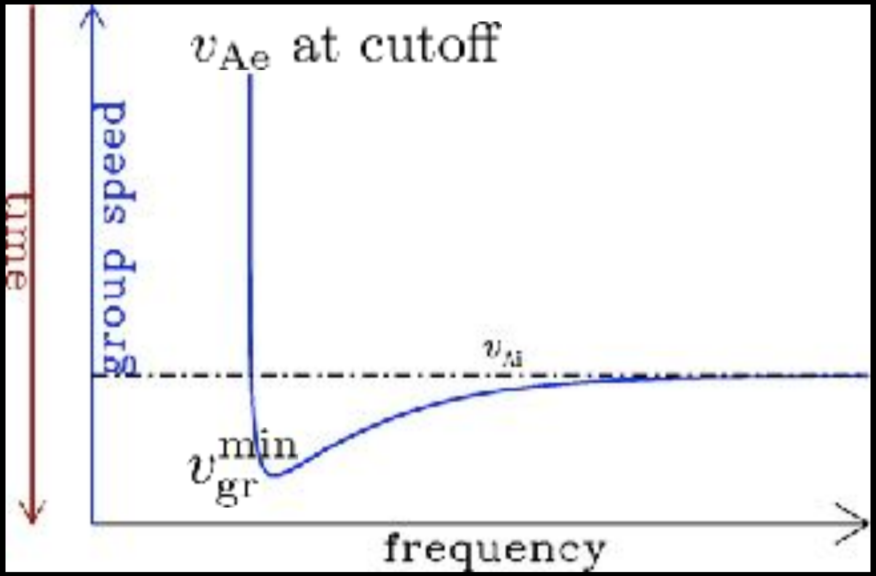
- No density structuring. No dispersion. Pulse does not change its shape



Upper: density perturbation in an r-z plane. Here r-transverse, z-axial direction
Lower: density perturbation sampled at the tube axis

Sausage wave trains in cold tubes

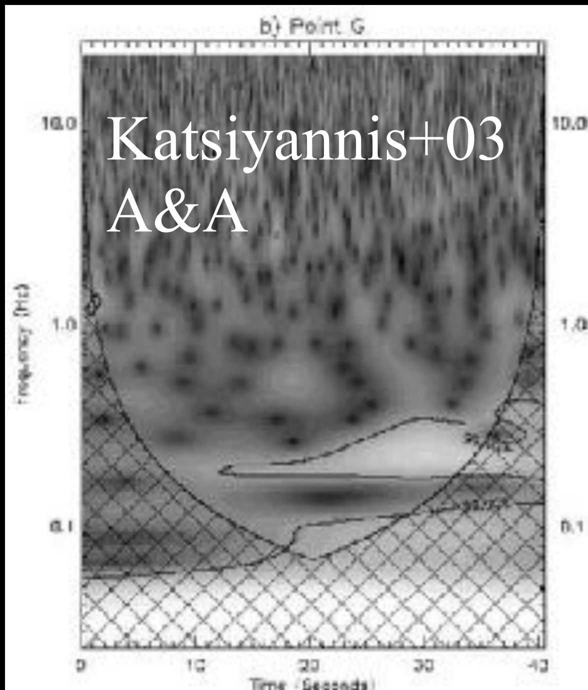




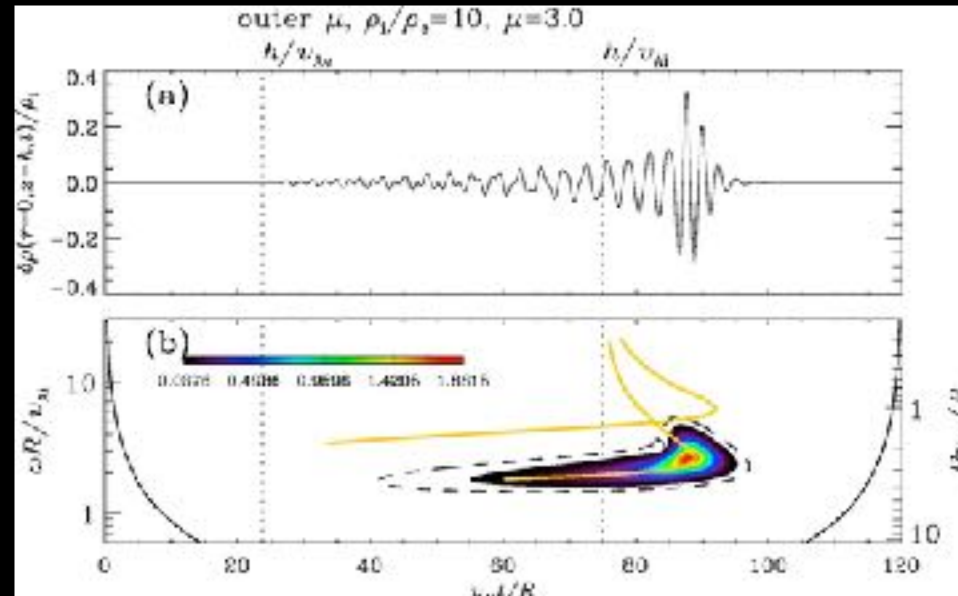
Roberts+83, 84, Edwin & Roberts 86, 88; analogy with explosive sound waves in shallow water [Pekeris 48]

- Effects of a simple step forward toward reality: continuous transverse profile?
- Qualitative difference arises, details of transverse structuring can be probed

Implications: WL observations



?

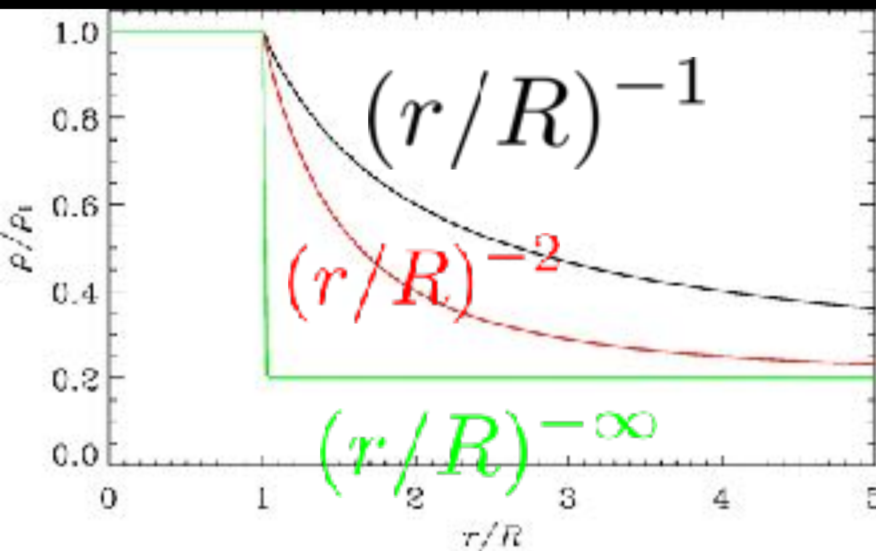


- Density profile of this active region loop
  - sufficiently flat at large r → cutoff freq.
  - sufficiently flat at small r → non-monotonic group speed curve
- Quasi-periods → transverse Alfven time

Mathematical reasons: cylindrical

$$f(r) \approx 1 - (r/R)^{\bar{\mu}=\infty}$$

Yu+17 ApJ



$$nR\frac{J_0(nR)}{J_1(nR)} = mR \quad \nu \mid \frac{1}{2} \quad \frac{W_{\nu-1,1}(2mR)}{W_{\nu,1}(2mR)}$$

$$\boxed{(nR)\frac{J_0(nR)}{J_1(nR)} = 1 - \nu - (mR)\frac{K_{\nu-1}(mR)}{K_\nu(mR)}}$$

$$\boxed{n\frac{J_0(nR)}{J_1(nR)} = -m\frac{K_0(mR)}{K_1(mR)}}$$

$$\frac{v_{\rm gr}^2}{v_{\rm Ai}^2} \approx 1 - \frac{j_{1,l}^2}{(kR)^2}$$

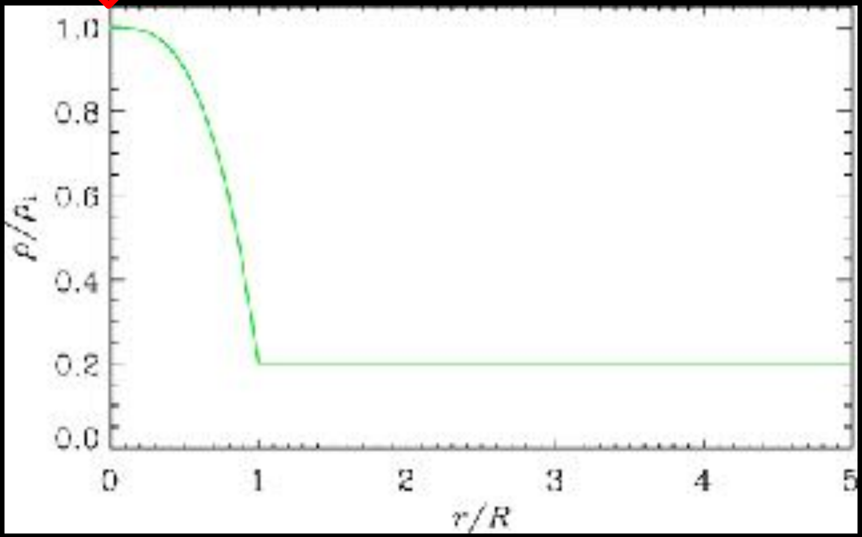
?

$$\frac{v_{\rm gr}^2}{v_{\rm Ai}^2} \approx 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^\beta .$$

$$\beta = \frac{2\bar{\mu}}{\bar{\mu}+2} = 2$$

Mathematical reasons: cylindrical

$$f(r) = 1 - (r/R)^{\bar{\mu}=2}$$



$$-(mR)\frac{K_0(mR)}{K_1(mR)} \qquad 2 - p + \alpha p \frac{M(\alpha \mid 1, 3, p)}{M(\alpha, 2, p)}$$

Yu, Li+16 ApJ
extending Pneuman 65

$$\frac{v_{\rm gr}}{v_{\rm Ai}} \approx 1 - \frac{2l^2(1 - \rho_{\rm e}/\rho_{\rm i})}{(kR)^2}$$

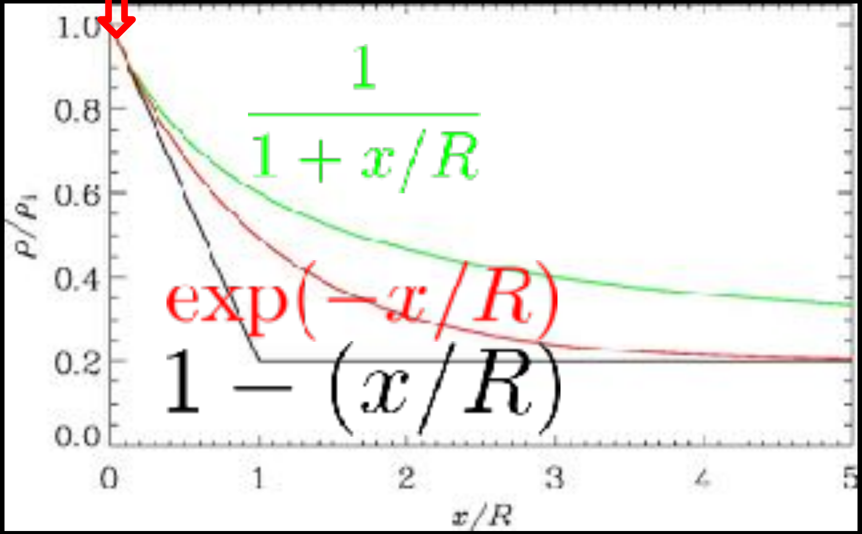
?

$$\frac{v_{\rm gr}^2}{v_{\rm Ai}^2} \approx 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^{\beta}.$$

$$\beta = \frac{2\bar{\mu}}{\bar{\mu}+2} = 1$$

Mathematical reasons: slab

$$f(r) \approx 1 - (x/R)^{\bar{\mu}=1}$$



Li+ 18 ApJ

$$\frac{\mathrm{Bi}(X_0)\mathrm{Ai}'(X_1) - \mathrm{Ai}(X_0)\mathrm{Bi}'(X_1)}{\mathrm{Bi}(X_0)\mathrm{Ai}(X_1) - \mathrm{Ai}(X_0)\mathrm{Bi}(X_1)} = -\frac{\bar{m}}{D^{1/3}}$$

$$\boxed{J_{2\bar{m}}(2\sqrt{\bar{D}}) = 0}$$

$$\boxed{W_{\nu,1/2}(2\bar{m}) = 0}$$

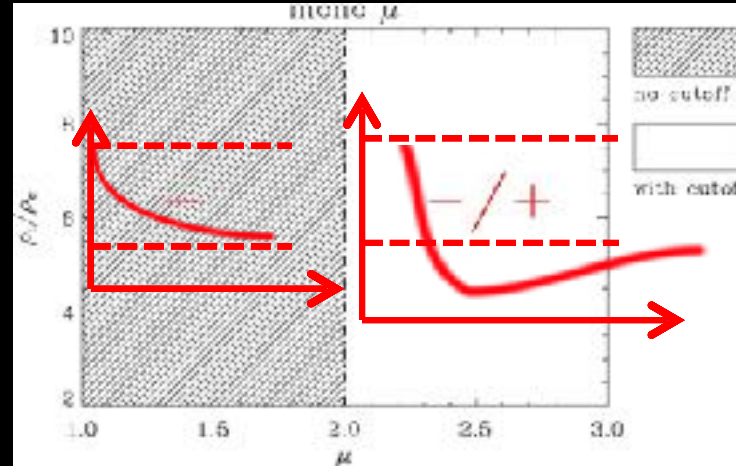
$$\frac{v_{\mathrm{gr}}^2}{v_{\mathrm{Ai}}^2} \approx 1 + \frac{1}{3}\left[\frac{3(4l-1)\pi(1-\rho_e/\rho_i)}{8kR}\right]^{2/3}$$

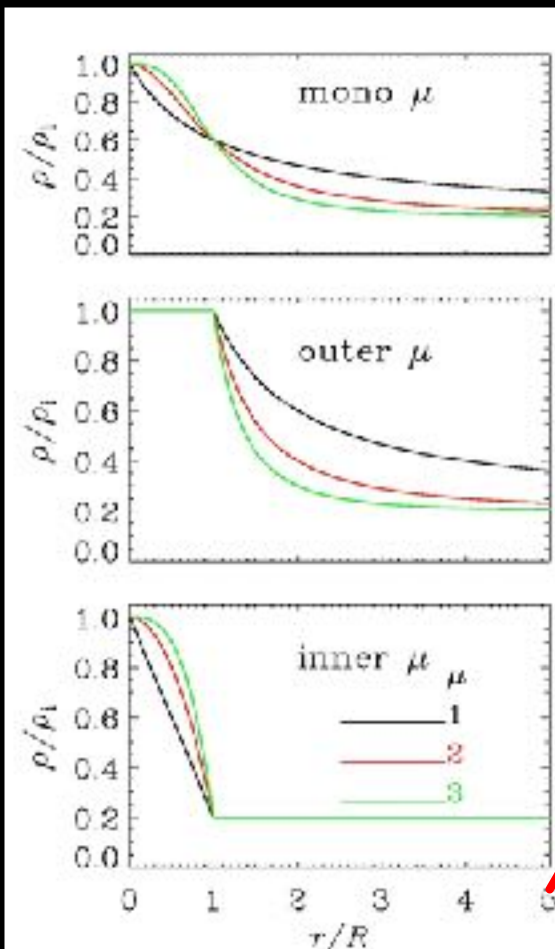
?

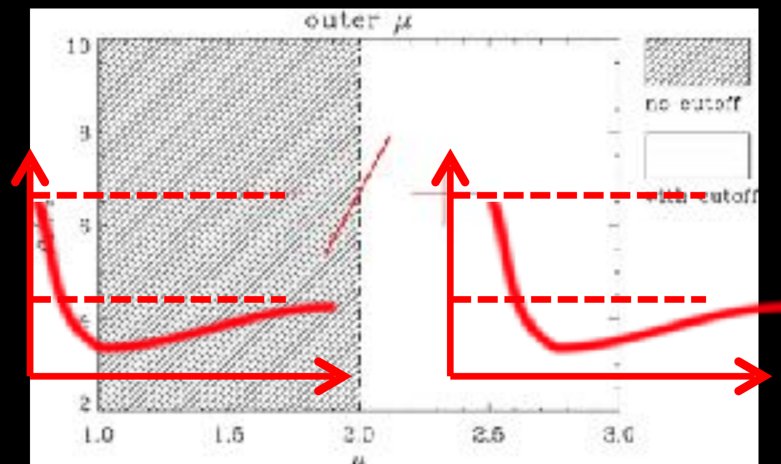
$$\frac{v_{\mathrm{gr}}^2}{v_{\mathrm{Ai}}^2} \approx 1 + (1-\beta)\left(\frac{c_l}{kR}\right)^{\beta}.$$

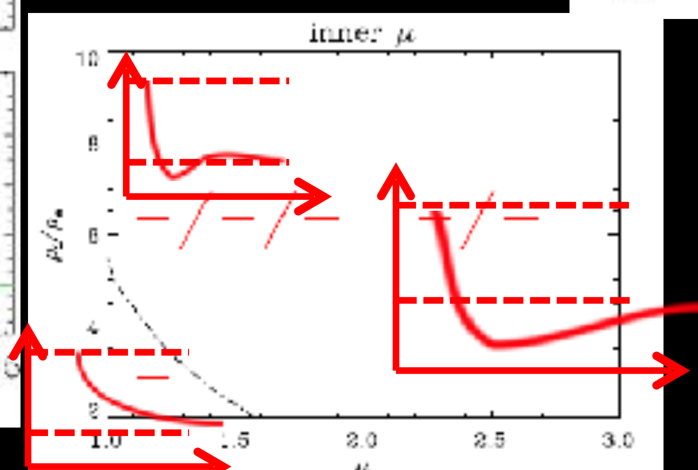
$$\beta = \frac{2\bar{\mu}}{\bar{\mu}+2} = \frac{2}{3}$$

Group speed curves



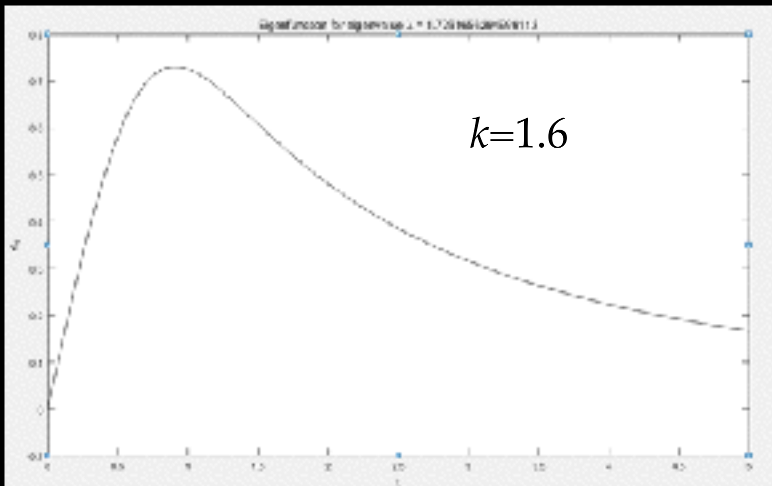


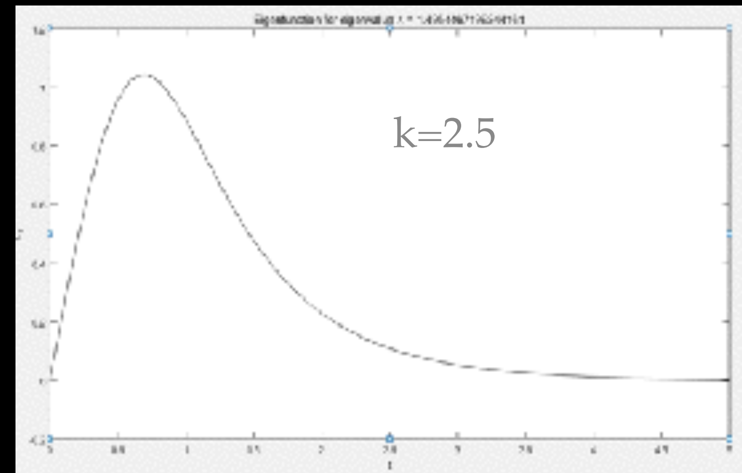


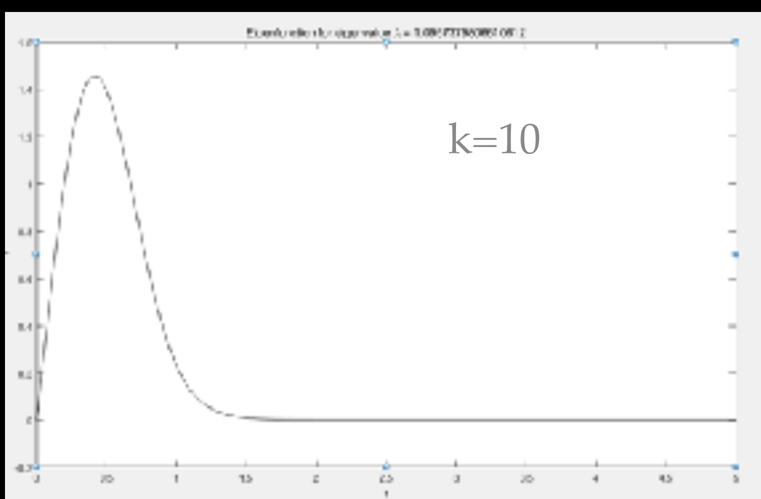


Eigen-functions as k ↗

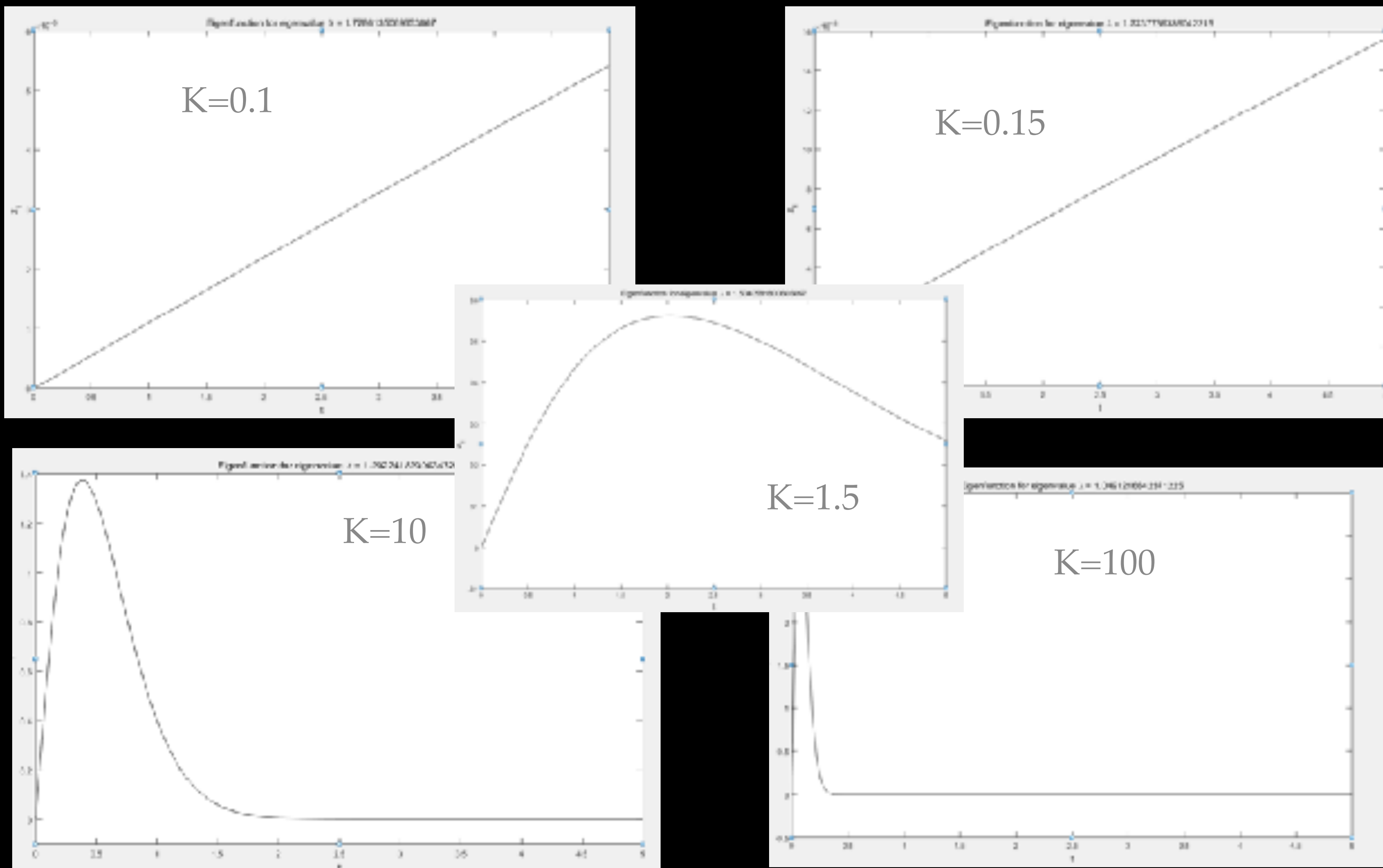
mono $\mu : \rho_{\mathrm{i}}/\rho_{\mathrm{e}} = 3, \mu = 4.5$







- Eigen-functions


K=0.1
K=0.15
K=1.5
K=10
K=100