Turbulent acceleration in astrophysics
- galaxy clusters & radio galaxies-

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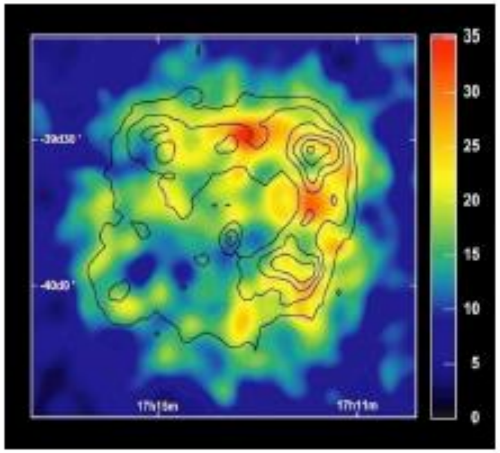
Alexander von Humboldt Stiftung/Foundation

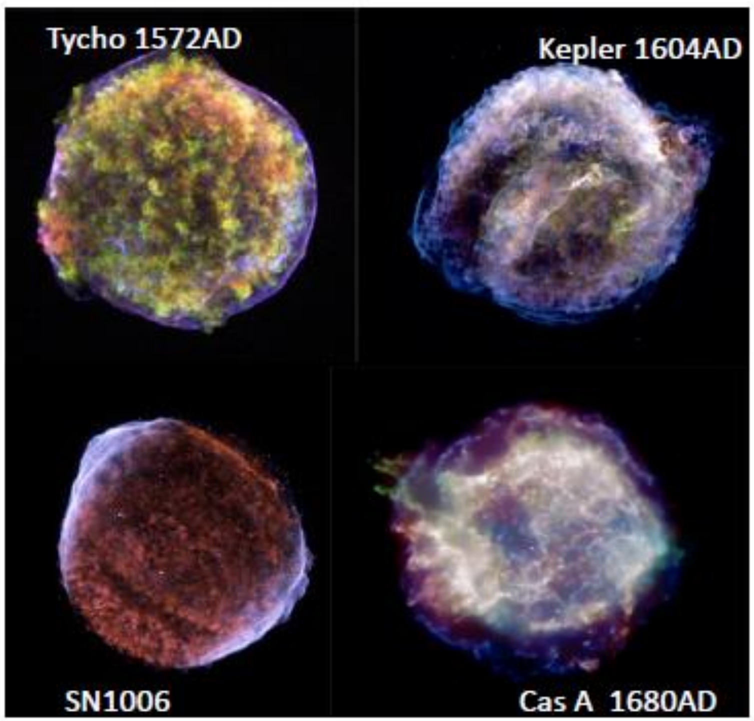
European Commission

Cosmic Accelerators

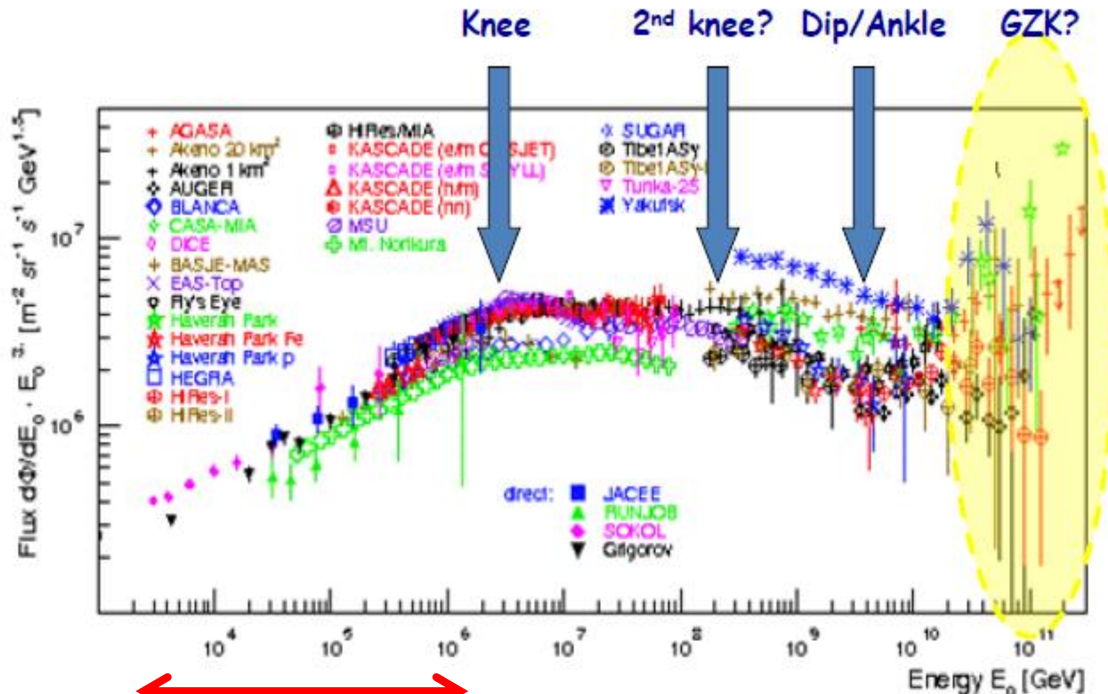
SNR RX J1713.7-3946



Aharonian et al
Nature (2004)



Chandra observations



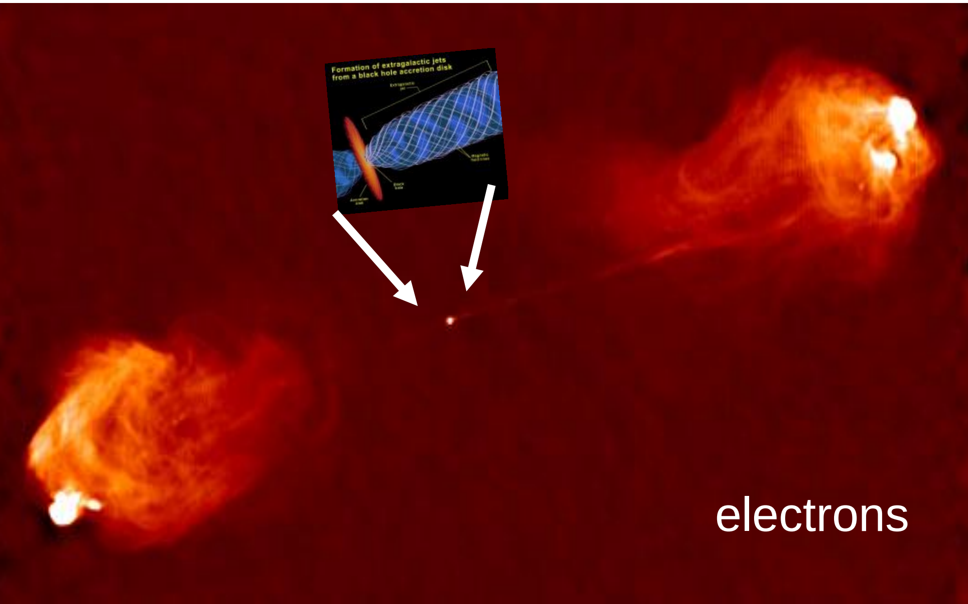
Galactic CRp

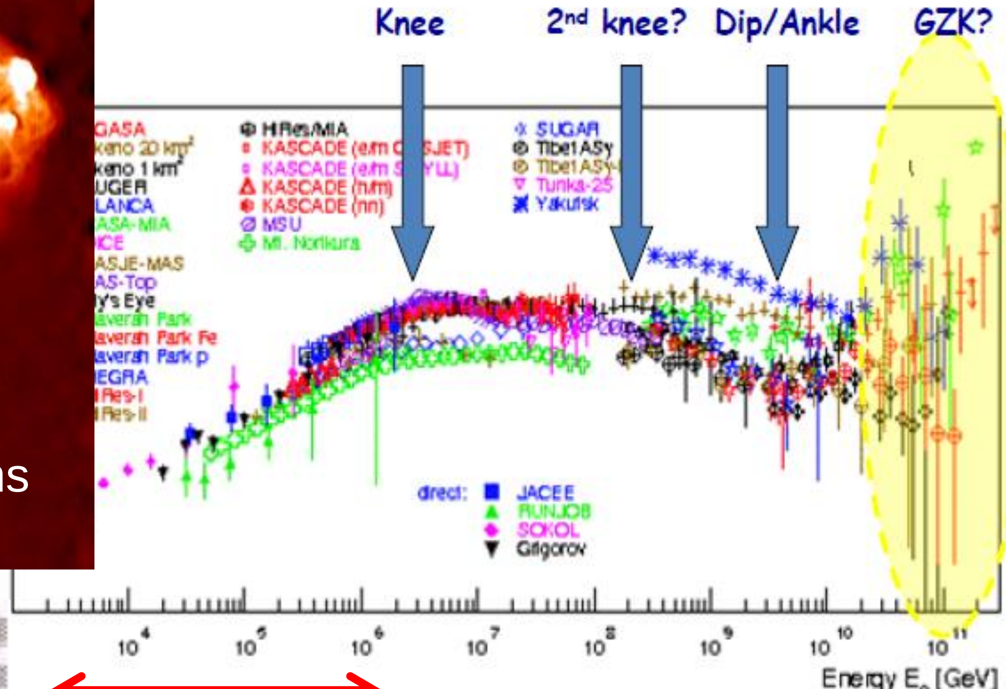
Galactic Fe

Mix of
Galactic &
extragalactic

extragalactic

Cosmic Accelerators





Galactic CRp

Galactic Fe

Mix of
Galactic &
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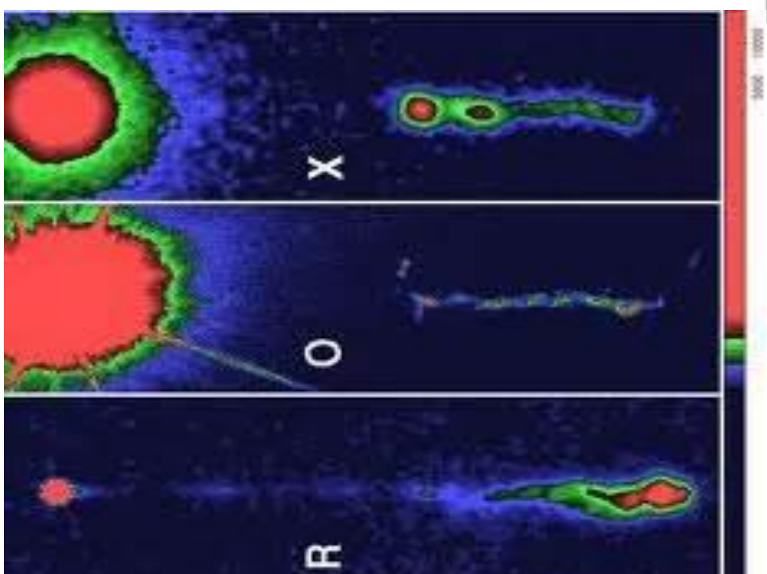
electrons



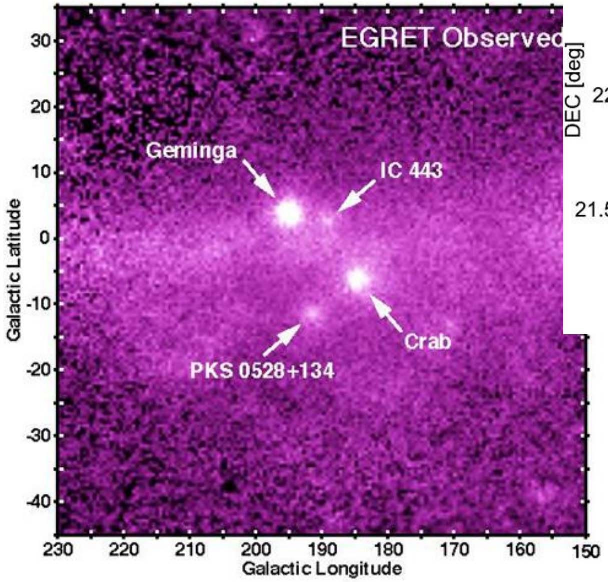
Crab nebula

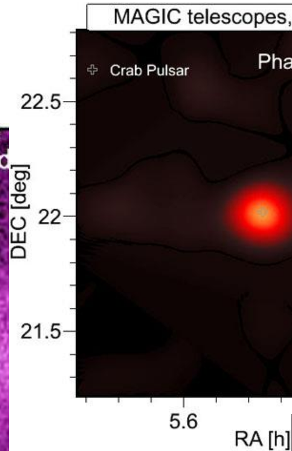
Pulsar WN

$\langle B \rangle_{pc} \approx 100\text{-}300\ \mu G$

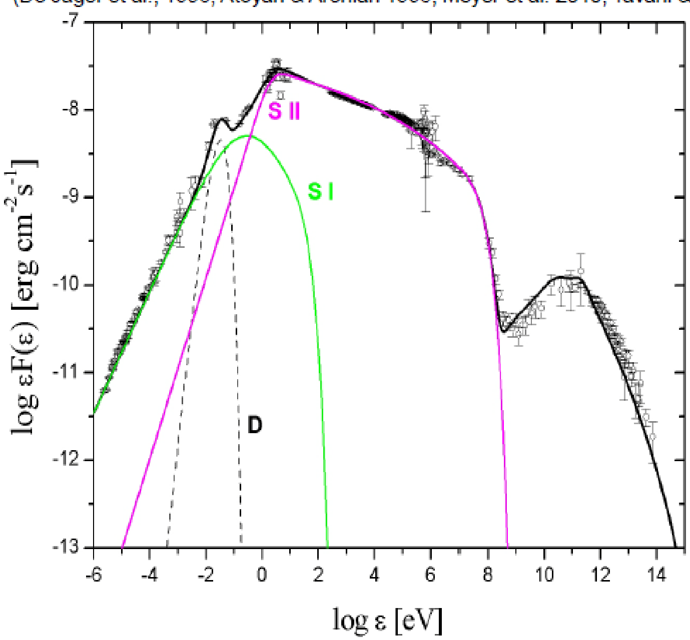
CRe with $E_{max} \approx 10^{14}\text{-}10^{15}$ eV

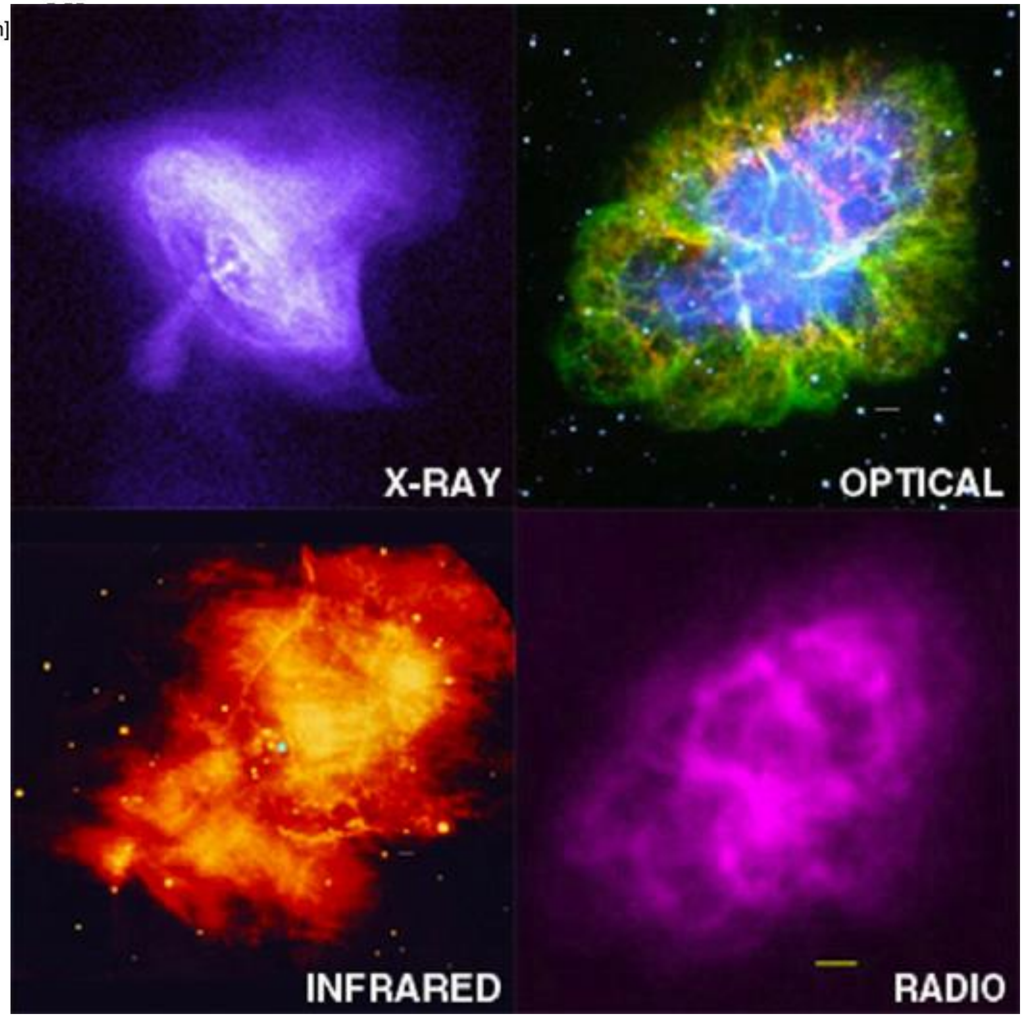
$T_{SYN} \approx 10$ yrs



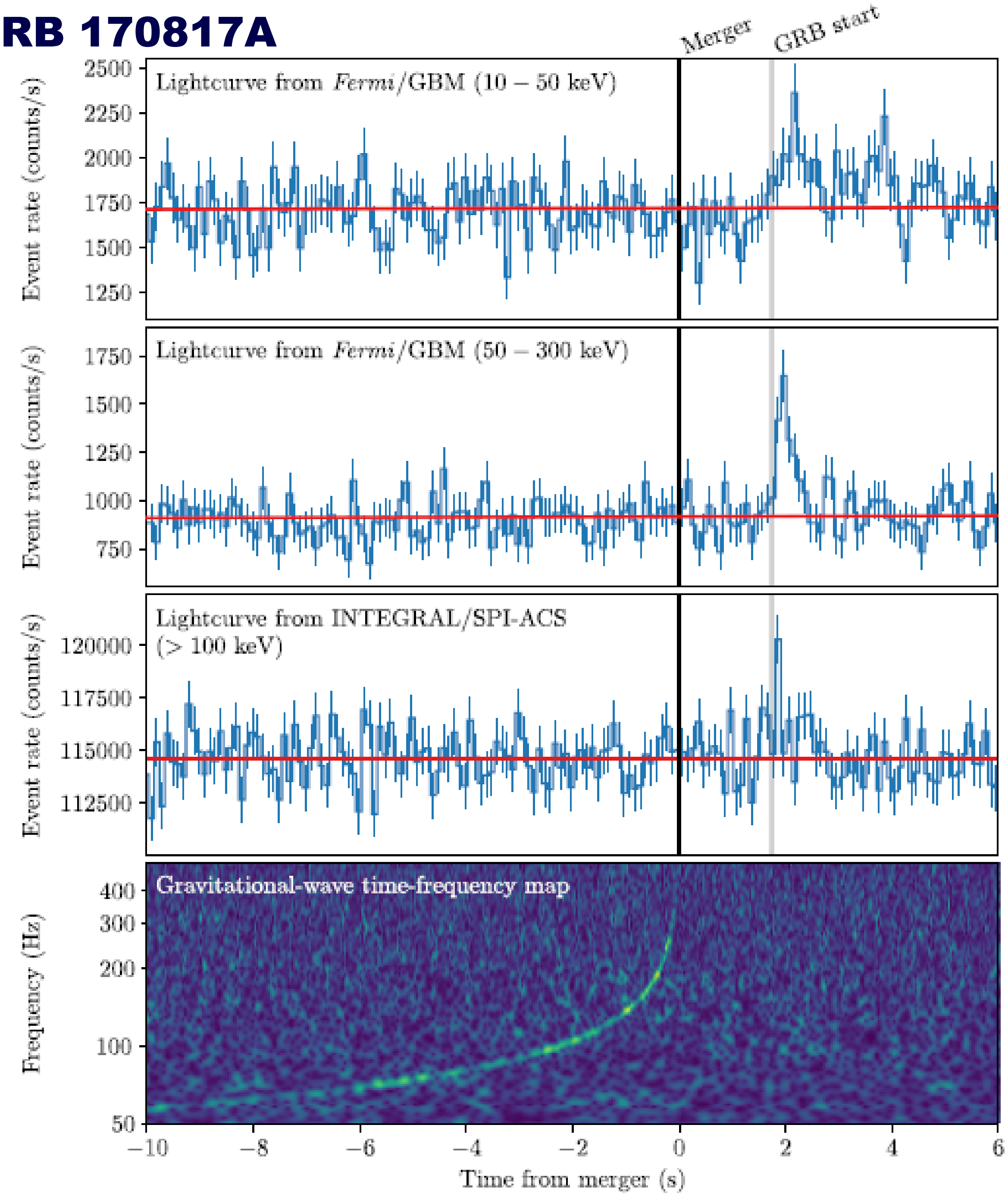


(De Jager et al., 1996, Atoyan & Aronian 1996, Meyer et al. 2010, Tavani & Vittorini, 2012)

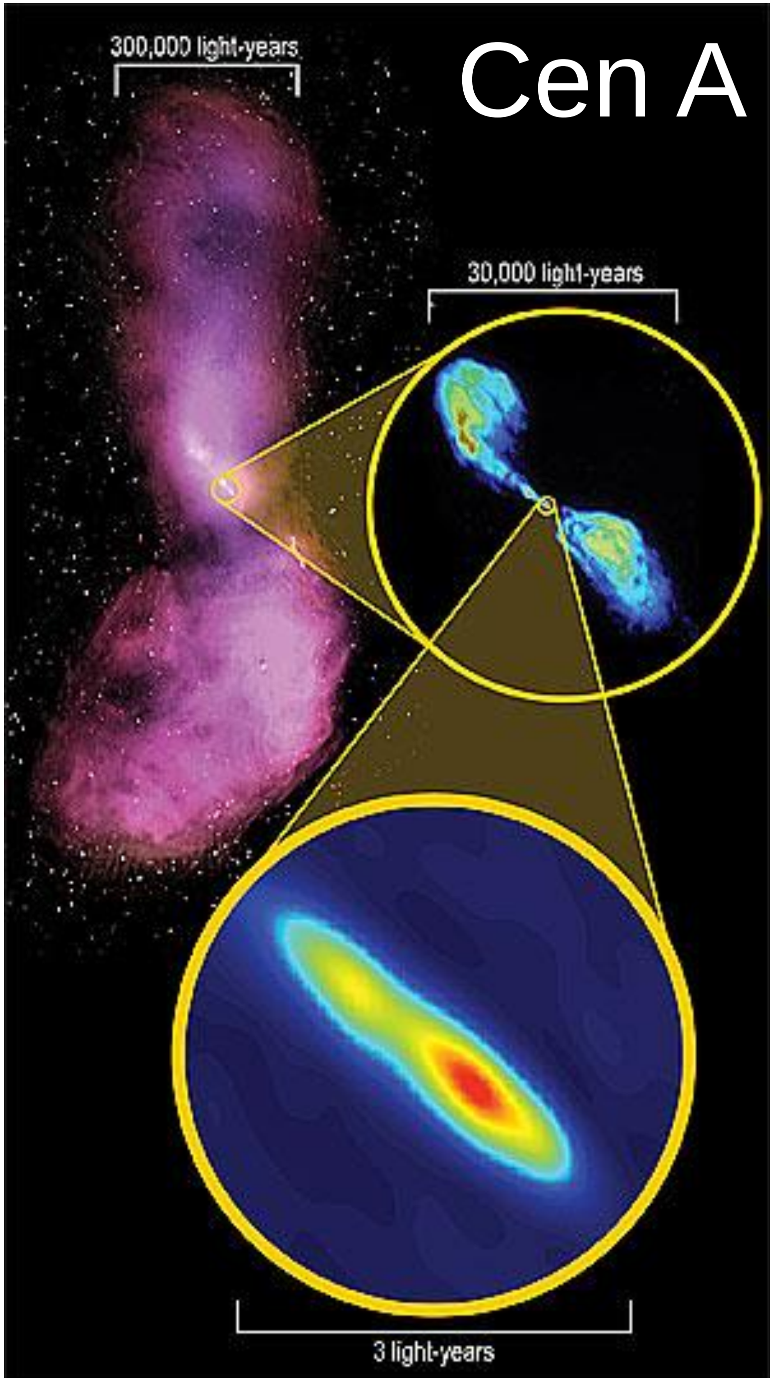


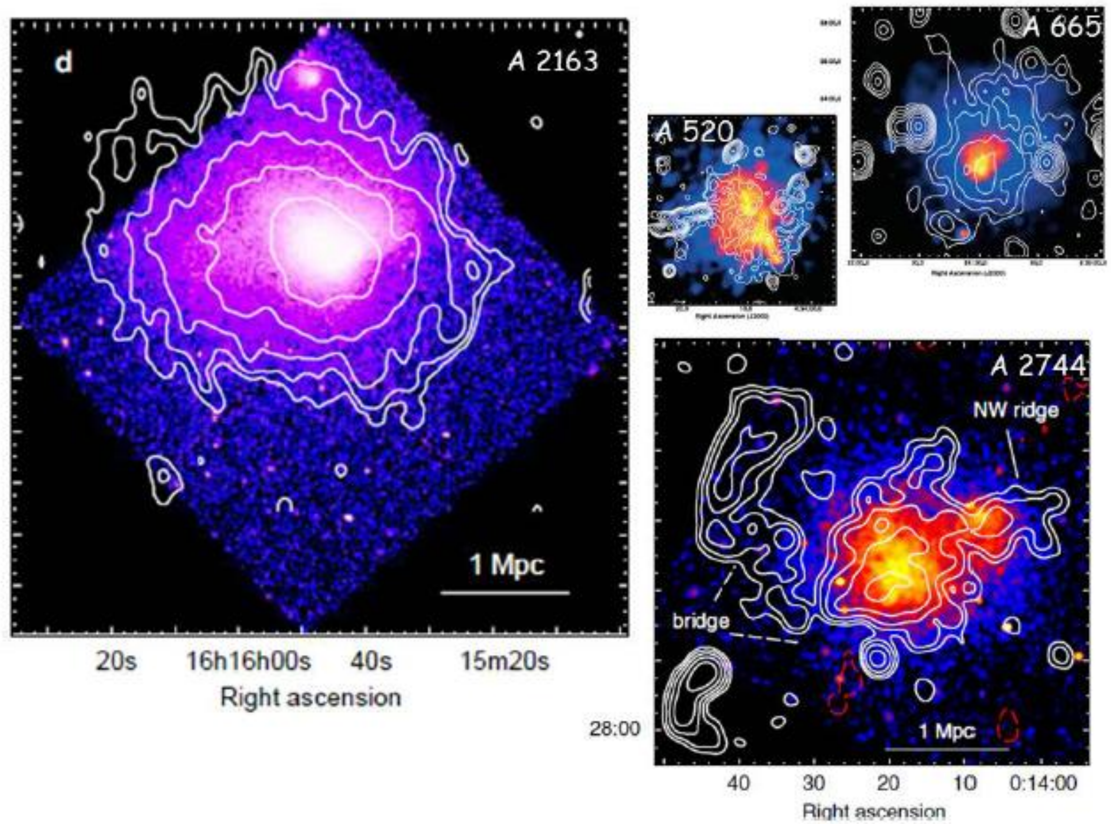


GRB 170817A



Merger
GRB start
Lightcurve from Fermi/GBM (10 – 50 keV)
Lightcurve from Fermi/GBM (50 – 300 keV)
Lightcurve from INTEGRAL/SPI-ACS (> 100 keV)
Gravitational-wave time-frequency map
Event rate (counts/s)
Frequency (Hz)
Time from merger (s)





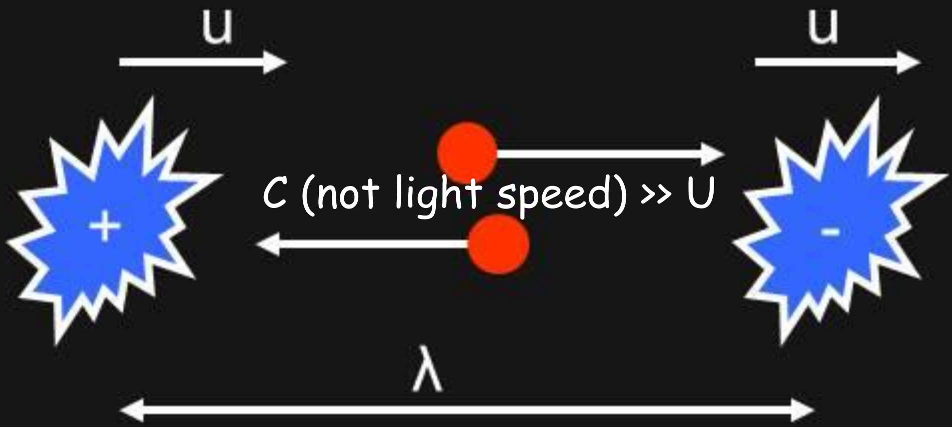
In addition to powerful and fast accelerators there is also evidence for gentle mechanisms operating on long timescales and large volumes

Turbulent (some kind of) acceleration is a natural candidate.

OUTLINE

- Turbulent acceleration: from Fermi to Gyroresonance and diffusion coefficients
- Complications and acceleration models using large-scale turbulence
- Turbulent acceleration in galaxy clusters: Radio Halos, models and observations at low radio frequencies
- Turbulent acceleration in head tail radio galaxies: evidences from observations at low radio frequencies
- APPENDIX: Fokker-Planck equation and coefficients

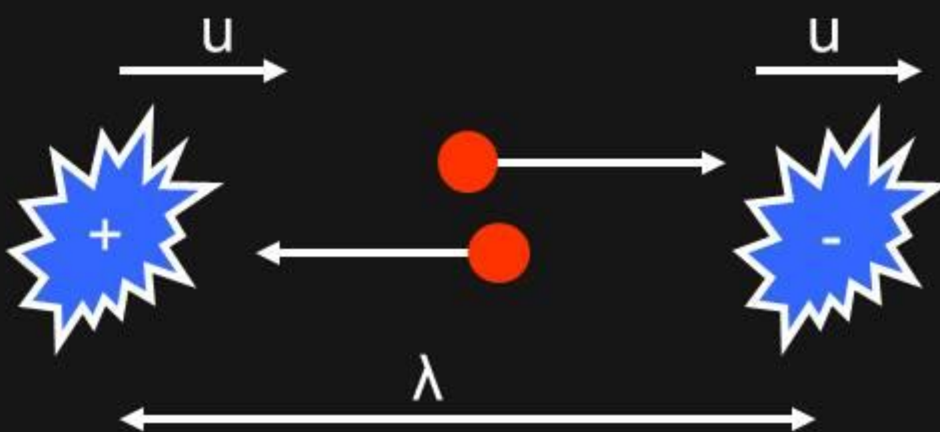
Second order Fermi Mechanisms
(Fermi 1949)



Energy gain per collisions: $\Delta p_{\pm} \approx \pm 2\, p \frac{u}{c}$

Second order Fermi Mechanisms

(Fermi 1949)



Energy gain per collisions: $\Delta p_{\pm} \approx \pm 2 p \frac{u}{c}$

Frequency of collisions: $\nu_{+} = \frac{u + c}{\lambda}$ $\nu_{-} = \frac{c - u}{\lambda}$

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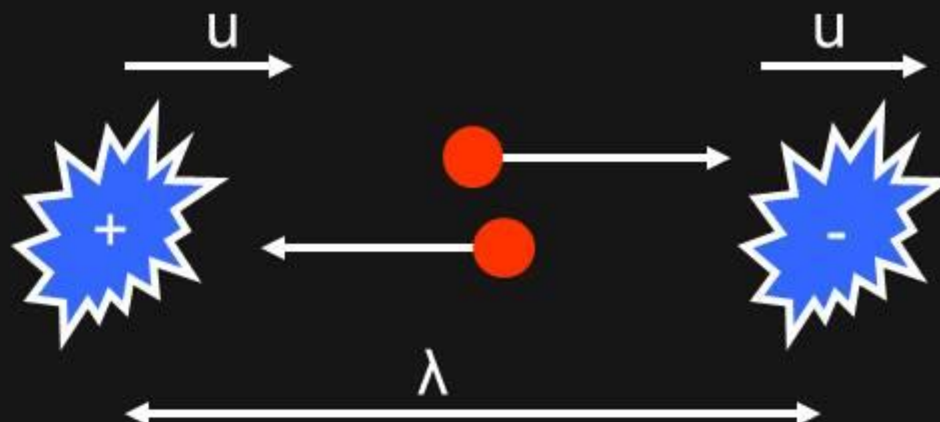

Energy gain per collisions: $\Delta p_{\pm} \approx \pm 2p\frac{u}{c}$

Frequency of collisions: $\nu_{+} = \frac{u+c}{\lambda}$ $\nu_{-} = \frac{c-u}{\lambda}$

$$\left\langle \frac{\Delta p}{\Delta t} \right\rangle = \nu_{+}\Delta p_{+} + \nu_{-}\Delta p_{-} \approx 2p\frac{u^2}{c^2}\frac{c}{\lambda}$$

Second order Fermi Mechanisms

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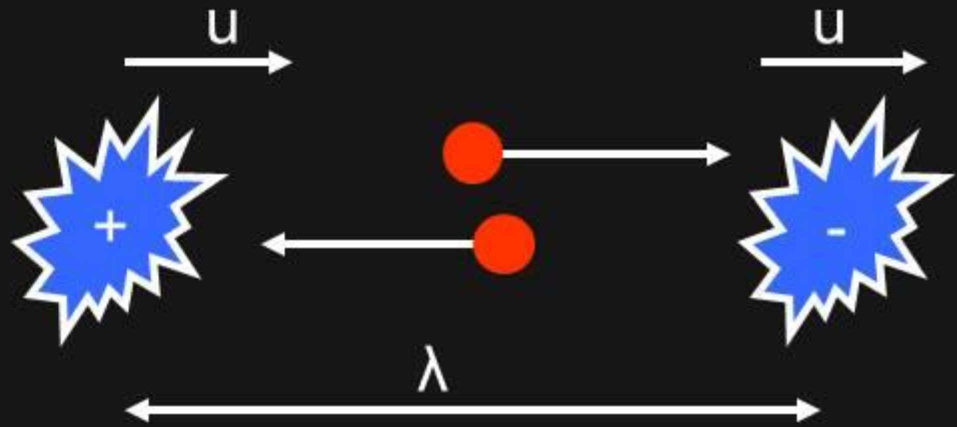

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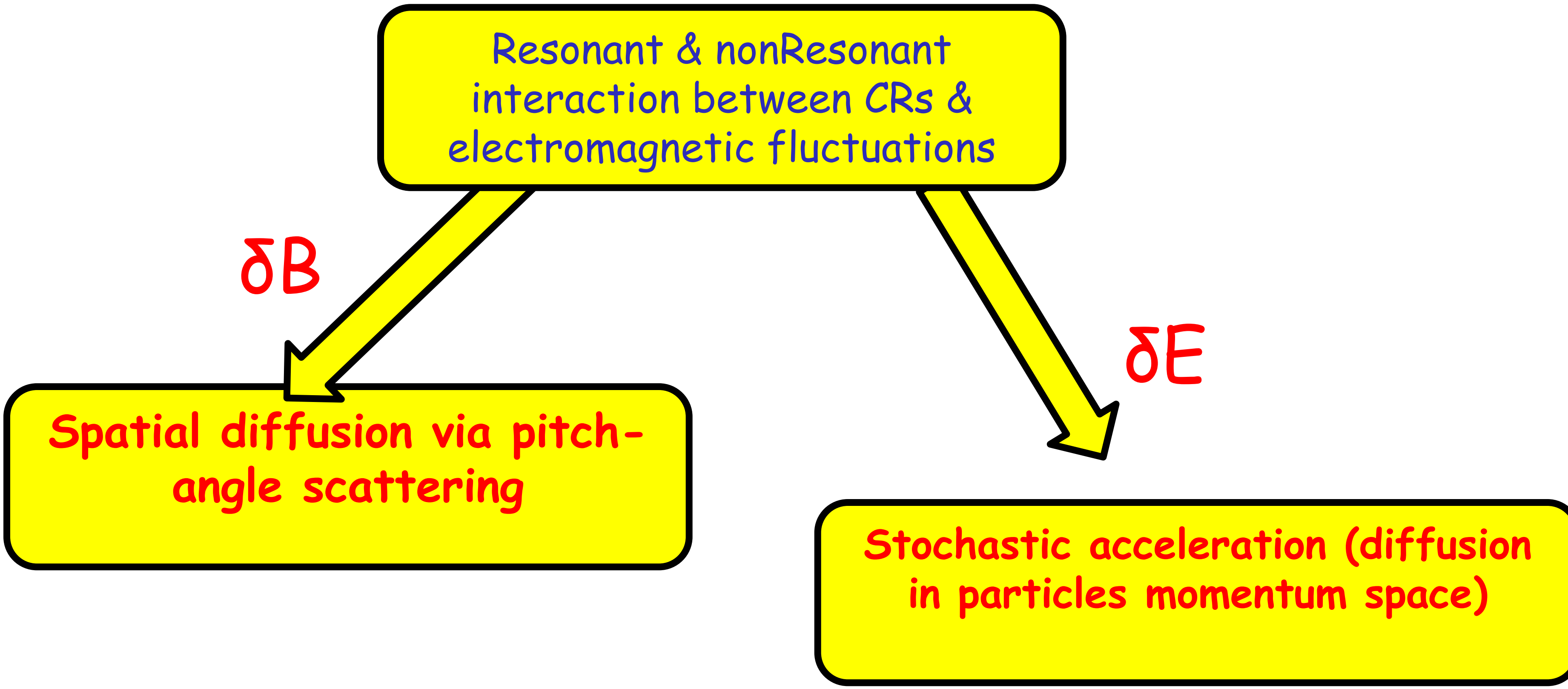
Frequency of collisions: $\nu_+ = \frac{u+c}{\lambda}$ $\nu_- = \frac{c-u}{\lambda}$

$$\left\langle \frac{\Delta p}{\Delta t} \right\rangle = \nu_+ \Delta p_+ + \nu_- \Delta p_- \approx 2 p \frac{u^2}{c^2} \frac{c}{\lambda}$$

Very similar solution in relativistic case

- Particles in a turbulent medium -



- Particles in a turbulent medium -

Resonant & nonResonant interaction between CRs & electromagnetic fluctuations

δB

Spatial diffusion via pitch-angle scattering

$$D_{\mu\mu} \equiv \lim_{t\to\infty} \frac{1}{2t} \langle \Delta\mu(t)\Delta\mu^*(t+\tau)\rangle = \Re \int_0^\infty d\tau \langle \dot{\mu}(t)\dot{\mu}^*(t+\tau)\rangle$$

δE

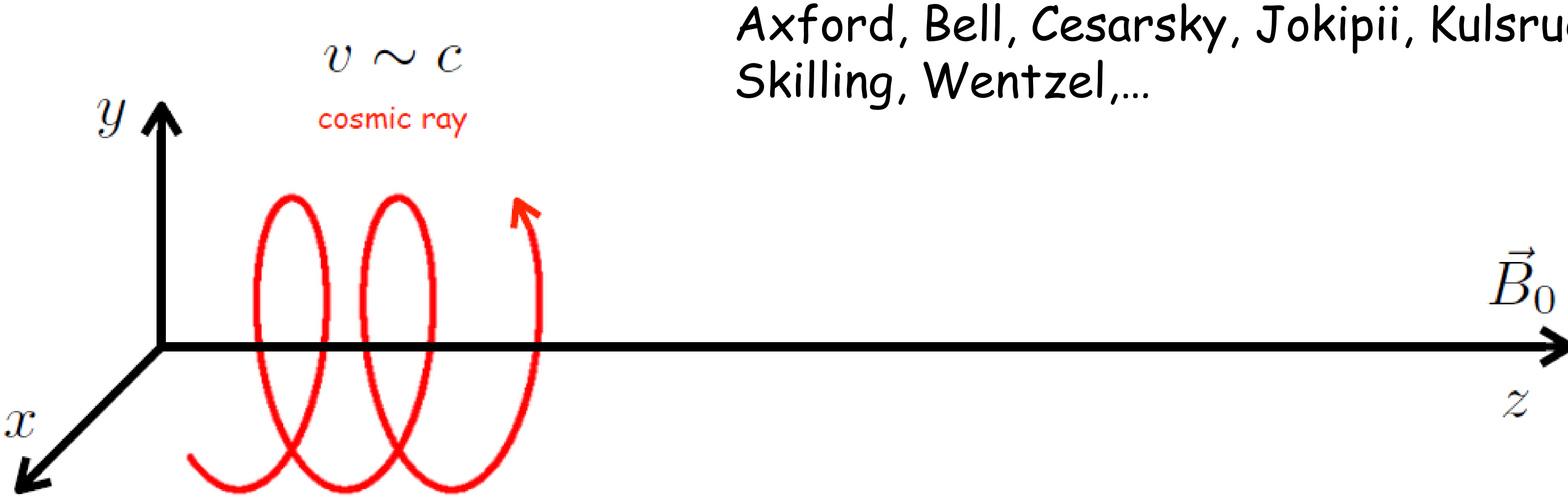
Stochastic acceleration (diffusion in particles momentum space)

$$D_{pp} \equiv \lim_{t\to\infty} \frac{1}{2t} \langle \Delta p(t)\Delta p^*(t+\tau)\rangle = \Re \int_0^\infty d\tau \langle \dot{p}(t)\dot{p}^*(t+\tau)\rangle$$

In the limit of small pitch-angle and momentum changes (ie. δp<<p) the process can be described as a diffusion in the angle and momentum space

Cosmic ray scattering off Alfven waves

Axford, Bell, Cesarsky, Jokipii, Kulsrud, Parker, Skilling, Wentzel,...



angle between particle velocity and B_0

pitch angle -> $\mu = \cos\vartheta$

helical motion

$$\begin{cases} v_{\|} = \mu\, v \\ v_{\perp} = (1-\mu^2)^{1/2} v \end{cases}$$

$$p_z = \text{Constant}$$
$$v_x = V_0 \cos[\Omega\, t]$$
$$v_y = V_0 \sin[\Omega\, t]$$

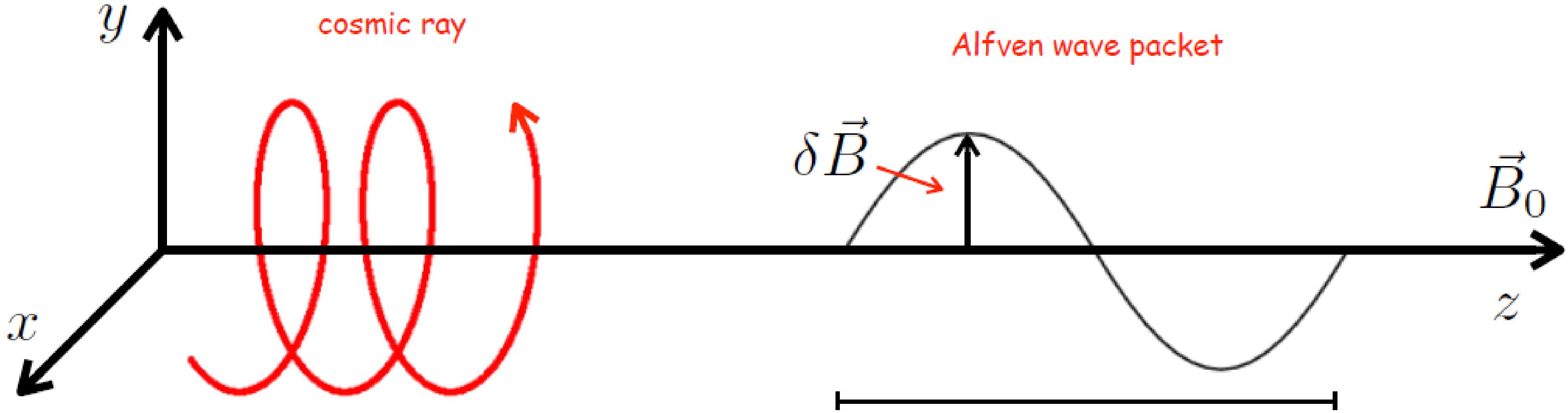
LARMOR FREQUENCY

$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

Cosmic ray scattering off Alfven waves

$v \sim c$ — cosmic ray

$v_A \ll c$ —> wave at rest



$L \approx \lambda = \dfrac{2\pi}{k}$

pitch angle -> $\mu = \cos\vartheta$ ← angle between particle velocity and B_0

helical motion

$$\begin{cases} v_{\parallel} & = & \mu\, v \\ v_{\perp} & = & (1-\mu^2)^{1/2} v \end{cases}$$

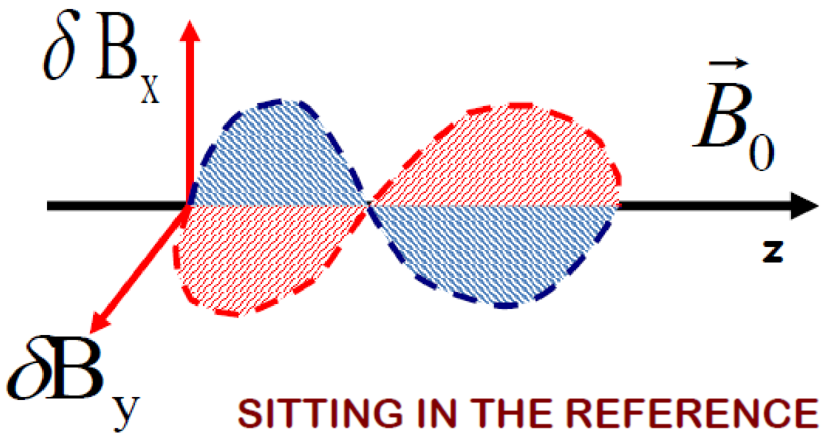
We can sit in the ref frame of the wave

$$\delta\vec{B}_y \sim \delta B \sin(kz)\,\vec{y}$$

$$v_x = v_{\perp} \sin(\Omega\, t + \phi)$$

Ω: gyration frequency; ϕ: arbitrary phase

Cosmic ray scattering off Alfven waves



QLT

$$\delta B << B_0$$

$$\delta\vec{B} \perp \vec{B}_0$$

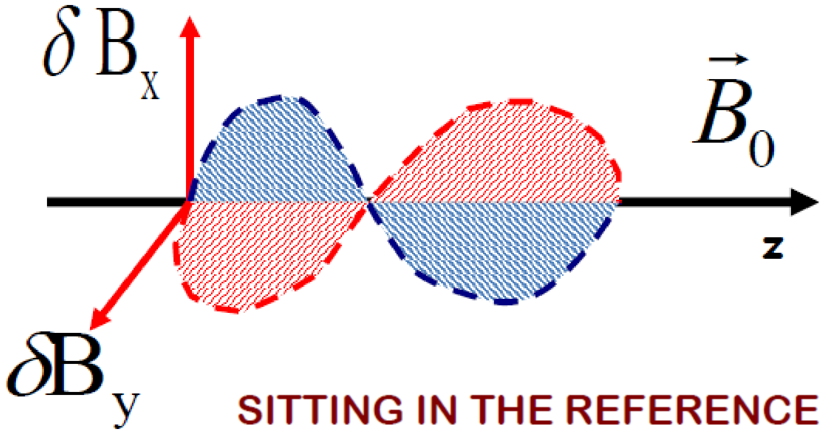
SITTING IN THE REFERENCE FRAME OF THE THE WAVE, THERE IS NO ELECTRIC FIELD...

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_Z = P\mu$

Cosmic ray scattering off Alfven waves



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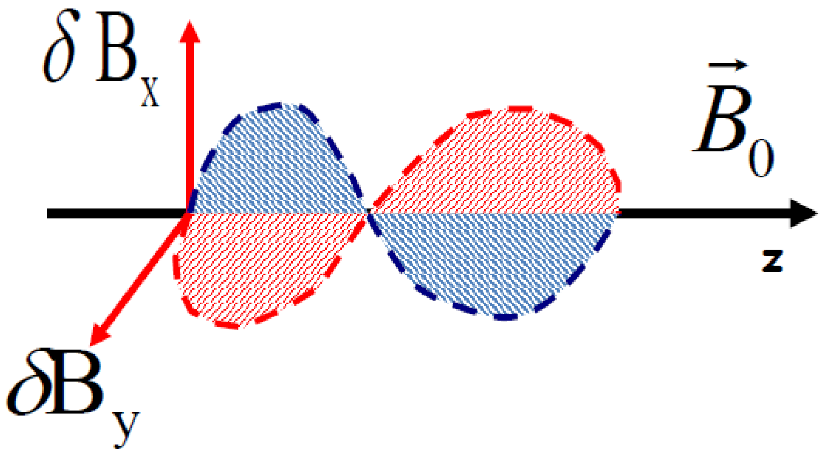
SITTING IN THE REFERENCE FRAME OF THE THE WAVE, THERE IS NO ELECTRIC FIELD...

EQUATION OF MOTION FROM LORENTZ FORCE :

$$\frac{dp_{\parallel}}{dt} = \frac{q}{c} |\vec{v}_{\perp} \times \delta \vec{B}| \qquad p_{\parallel} = p\ \mu$$

Cosmic ray scattering off Alfven waves



$\delta B << B_0$

$\delta \vec{B} \perp \vec{B}_0$

QLT

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2}}{m\gamma c}\left[\cos(\Omega t)\, B_y - \sin(\Omega t)\, B_x\right]$$

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2} B_k}{m\gamma c}\left[\cos(\Omega t)\cos(kz+\psi) + \sin(\Omega t)\sin(kz+\psi)\right]$$

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2} B_k}{m\gamma c}\cos\left[(\Omega - kv\mu)t + \psi\right]$$

Diffusion in pitch angle : diffusion coefficient

$$\frac{d\mu}{dt} = \frac{q(1-\mu^2)^{1/2} B_k}{m\gamma c} \cos\left[(\Omega - kv\mu)t + \psi\right]$$

$$\left\langle \frac{\mathrm{d}\mu}{\mathrm{d}t} \right\rangle = 0$$

$$\Delta\mu\Delta\mu = \frac{q^2(1-\mu^2)B_k^2}{m^2\gamma^2c^2} \int dt \int dt' \cos\left[(\Omega - kv\mu)t + \psi\right] \cos\left[(\Omega - kv\mu)t' + \psi\right]$$

$$< \frac{\Delta\mu\Delta\mu}{\Delta t} >_\psi = \frac{q^2(1-\mu^2)\pi B_k^2}{m^2\gamma^2c^2} \frac{1}{v\mu} \delta\left(k - \frac{\Omega}{v\mu}\right)$$

CORRECTION FOR LAB FRAME

Many waves with spectrum $W_k = B_k^2/4\pi$

$$\overbrace{< \frac{\Delta\mu\Delta\mu}{\Delta t} >}^{D_{\mu\mu}} = \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2} \frac{1}{v\mu} 4\pi \int dk \frac{B_k^2}{4\pi} \delta\left(k - \frac{\Omega}{v\mu}\right)$$

Gyroresonance

$$\delta(k_\parallel v_\parallel - \omega + n\Omega)$$

Diffusion in pitch angle : timescale and spatial diff

Gyroresonance

$$\delta(k_{\|}v_{\|} - \omega + n\Omega)$$

$$D_{\mu\mu} = \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2}\frac{1}{v\mu}4\pi\int dk\frac{B_k^2}{4\pi}\delta(k - \frac{\Omega}{v\mu})$$

$$\tau scatt \approx \frac{\mu^2}{D_{\mu\mu}}$$

iotropisation time

particles lose memory of the initial pitch angle

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transport equation

$$\frac{\partial f}{\partial t} + \mu v\frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[(1-\mu^2)D_\vartheta\frac{\partial f}{\partial \mu}\right]$$

particles move along B
with velocity μv

$D_{\mu\mu}$

Diffusion in pitch angle : timescale and spatial diff

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$$\delta(k_{\|} v_{\|} - \omega + n\Omega)$$

$$D_{\mu\mu} = \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2}\frac{1}{v\mu}4\pi\int dk\frac{B_k^2}{4\pi}\delta\left(k-\frac{\Omega}{v\mu}\right);$$

$$\tau scatt \approx \frac{\mu^2}{D_{\mu\mu}}$$

iotropisation time

particles lose memory of the initial pitch angle

mean free path → λ ; particle velocity → v

$$D \sim \lambda v \sim (\tau_s v) v \sim \frac{v^2}{D_{\mu\mu}}$$

$$D = \frac{v^2}{8}\int_{-1}^{1} d\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}}$$

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Resonant & nonResonant interaction between CRs & electromagnetic fluctuations

δB

δE

Spatial diffusion via pitch-angle scattering

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Momentum diffusion : coefficient

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$$\tau scatt \approx \frac{\mu^2}{D_{\mu\mu}}$$

iotropisation time

particles are isotropized in the rest frame of the wave

Now we come back to to the Lab frame where there is an Electric field associated with the moving waves

[long MHD waves ($\omega < \Omega/\beta_{pl}$)] $(\frac{\delta B}{\delta E})^2 \approx (\frac{c}{V_A})^2$

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[long MHD waves ($\omega < \Omega/\beta_{pl}$)] $\left(\frac{\delta B}{\delta E}\right)^2 \approx \left(\frac{c}{V_A}\right)^2$

In the same timescale particles change momentum

$$\Delta p \sim p\frac{v_A}{c}$$

$$D_{pp} = \frac{2\pi^2 e^2 v_A^2}{c^3}\int_{k_{\min}(p)}^{k_{\max}} W_A(k)\frac{1}{k}\left[1-\left(\frac{v_A}{c}+\frac{\Omega m_e}{pk}\right)^2\right]dk$$

$$k_{\min} = \frac{\Omega m}{p}\frac{1}{(1 \pm v_A/v)}$$

$$\tau_{acc} \approx \frac{p^2}{D_{pp}}$$

$$\tau_{acc} \sim \tau scatt\left(\frac{c}{V_{ph}}\right)^2$$

Evolution of particles interacting with Alfven waves

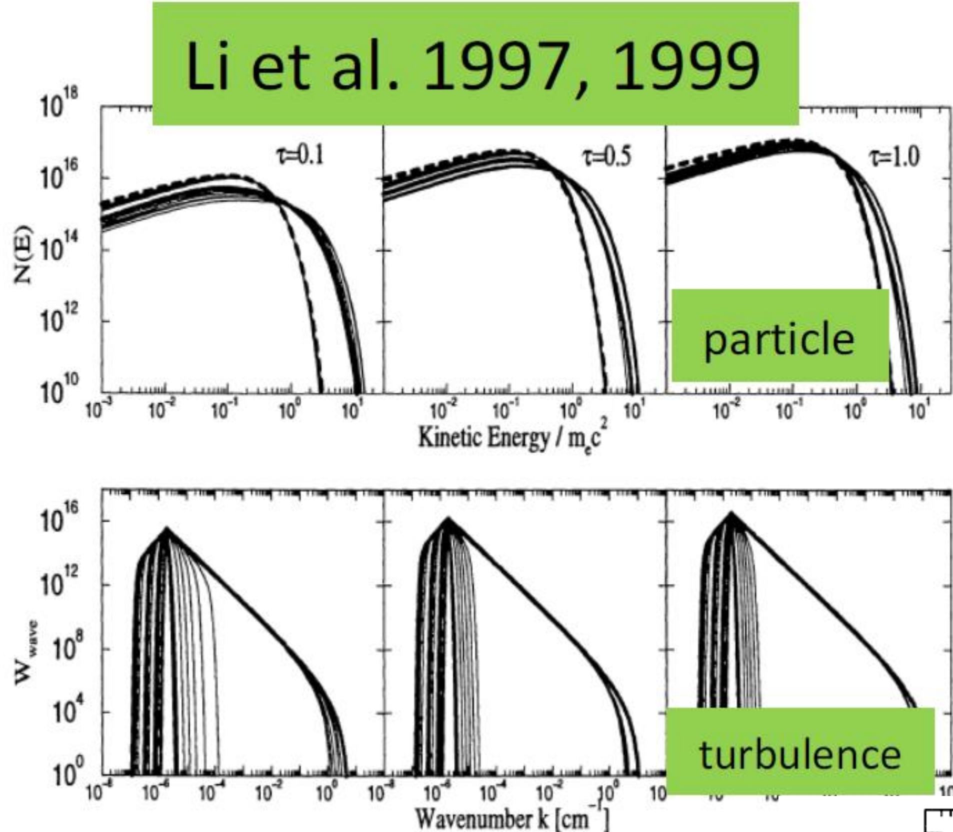
$$D_{\mu\mu} = \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2}\frac{1}{v\mu}4\pi\int dk\frac{B_k^2}{4\pi}\delta\left(k-\frac{\Omega}{v\mu}\right)$$

$$D(p) = \frac{2\pi^2e^2v_A^2}{c^3}\int_{k_{min}(p)}^{k_{max}} W_A(k)\frac{1}{k}\left[1-\left(\frac{v_A}{c}+\frac{\Omega m_e}{pk}\right)^2\right]dk$$

Fokker-Planck Equation

$$\frac{\partial f}{\partial t}+\mu v\frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left[\left(D_{\mu\mu}\frac{\partial}{\partial\mu}+D_{\mu p}\frac{\partial}{\partial p}\right)f(p,\mu,t)\right]+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\times\left(D_{\mu p}\frac{\partial}{\partial\mu}+D_{pp}\frac{\partial}{\partial p}\right)f(p,\mu,t)\right]$$

Spectrum of particles



Electrons/Positrons

Q_e: secondaries from CRp-p collisions

$$\frac{\partial N_e(p,t)}{\partial t} = \frac{\partial}{\partial p}\left(N_e(p,t)\left[\left(\frac{dp}{dt}\right)_{rad} + \left(\frac{dp}{dt}\right)_i - \frac{2}{p}D_{pp}\right]\right) + \frac{\partial}{\partial p}\left(D_{pp}\frac{\partial N_e(p,t)}{\partial p}\right) + Q_e(p,t)$$

losses + sys acceleration — p-diffusion

Protons

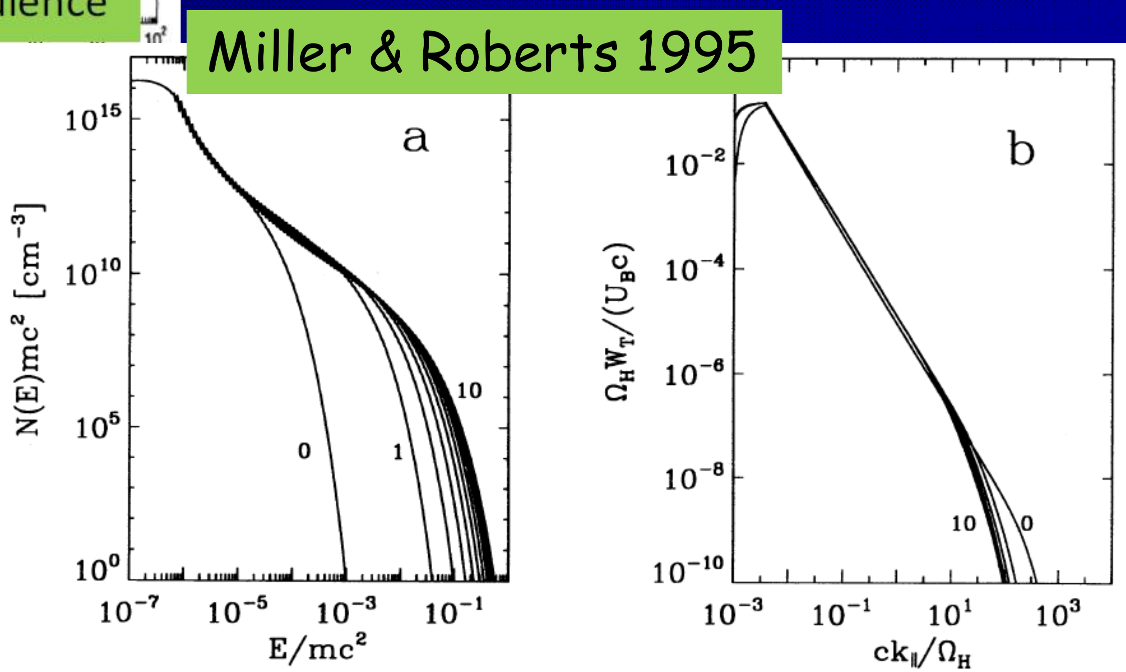
$$\frac{\partial N_p(p,t)}{\partial t} = \frac{\partial}{\partial p}\left(N_p(p,t)\left[\left(\frac{dp}{dt}\right)_i - \frac{2}{p}D_{pp}\right]\right) + \frac{\partial}{\partial p}\left(D_{pp}\frac{\partial N_p(p,t)}{\partial p}\right) + Q_p(p,t)$$

losses + sys acceleration — p-diffusion — injection

Turb. Modes

$$\frac{\partial \mathcal{W}(k,t)}{\partial t} = \frac{\partial}{\partial k}\left(k^2 D_{kk}\frac{\partial}{\partial k}\left(\frac{\mathcal{W}(k,t)}{k^2}\right)\right) - \sum_i \Gamma_i(k,t)\mathcal{W}(k,t) + I(k,t)$$

mode coupling — collisionless dampings — injection



In general the spectrum of accelerated particles in turbulent acceleration is NOT a simple power-law.
This affectes the properties of (nonthermal) radiation spectra

- Curved spectra
- Steep spectra

OUTLINE

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Complication : scale anisotropy

neglecting factors of order unity:

Gyroresonance

$\delta(k_{\|} v_{\|} - \omega + n\Omega)$ ⇨ $k v_z \approx \Omega = \dfrac{v_\perp}{R_L}$ ⇨ $R_L \approx \dfrac{1}{k}$

Galaxy : $R_{res} \approx$ Mm , $R_A \approx$ 1 pc … 10 dex

Galaxy Clusters : $R_{res} \approx$ Mm , $R_A \approx$ 1 kpc … 13 dex

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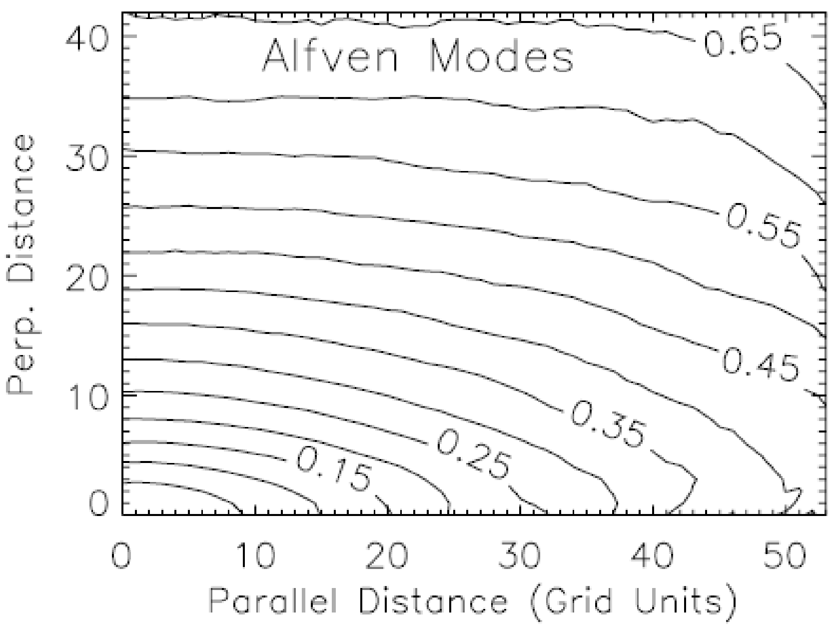
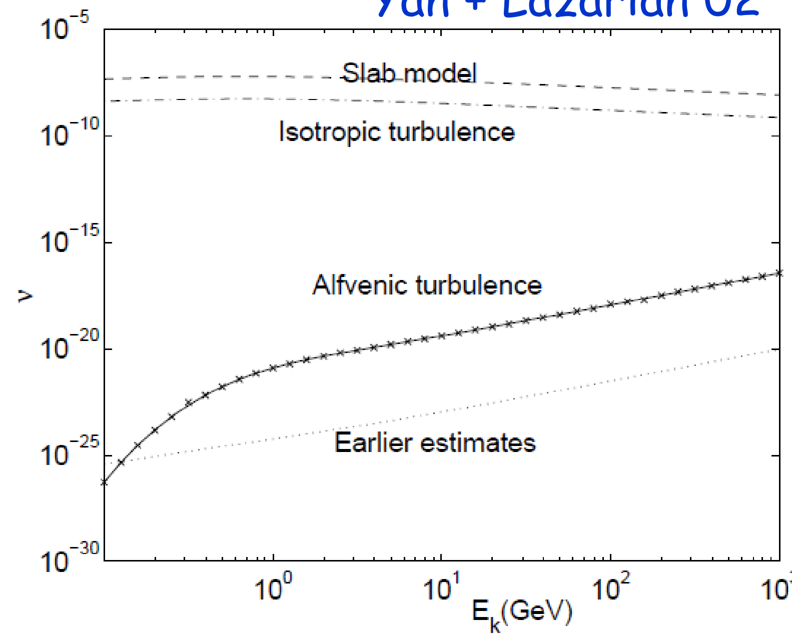
Goldreich & Sridhar 95,
Maron & Goldreich 01, .. Cho & Lazarian 03...



Yan + Lazarian 02



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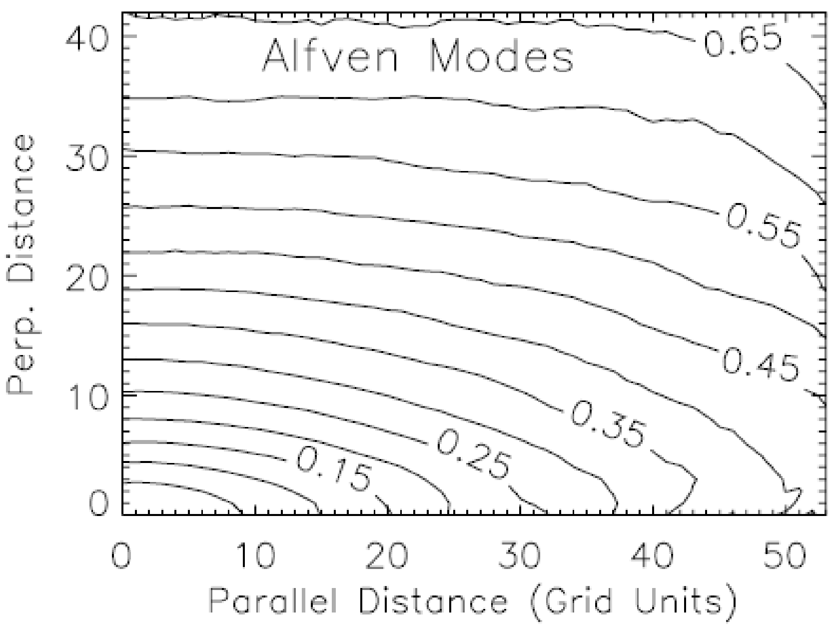
Goldreich & Sridhar 95,
Maron & Goldreich 01, .. Cho & Lazarian 03…



Possible solution :

Waves must be generated at small, quasi-resonant scales
Self-generated waves, instabilities :

- Streaming instability (Kulsrud, … ..Blasi, Amato, Lazarian, Yan, Wiener, Zweibel, Oh)
- Many other instabilities….

Fermi-like mechanisms using large scale turbulence

Transit-Time-Damping (TTD, magnetic Landau Damping)

[Fisk 76, Schliskeiser+Miller 98,..Yan+Lazarian 04, Brunetti+Lazarian 07..]

Coupling between particles magnetic moment and magnetic field gradients

$$\omega - k_{//} v_{//} = 0$$

Magnetic Pumping

[Swann 33, .. Melrose 80, ..]

Particles compressions and rarefactions in B (betatron), coupled with an effective scattering mechanism .

$$D_{pp} \approx \alpha_B \frac{\omega_{\Delta B}^2}{v_{SC}} \left(\frac{\Delta B}{B} \right)^2 p^2$$

Compression by large-scale trubulence

[Ptuskin 88, .. Cho+Lazarian 06, … Brunetti 16..]

$$\frac{\partial p}{\partial t} = -\frac{\nabla \mathbf{u}}{3} p$$

Particles diffusing through turbulence (acoustic) in a medium experience stochastic compressions/rarefactions and are statistically accelerated at a rate depending on spatial diffusion coefficient and turbulence

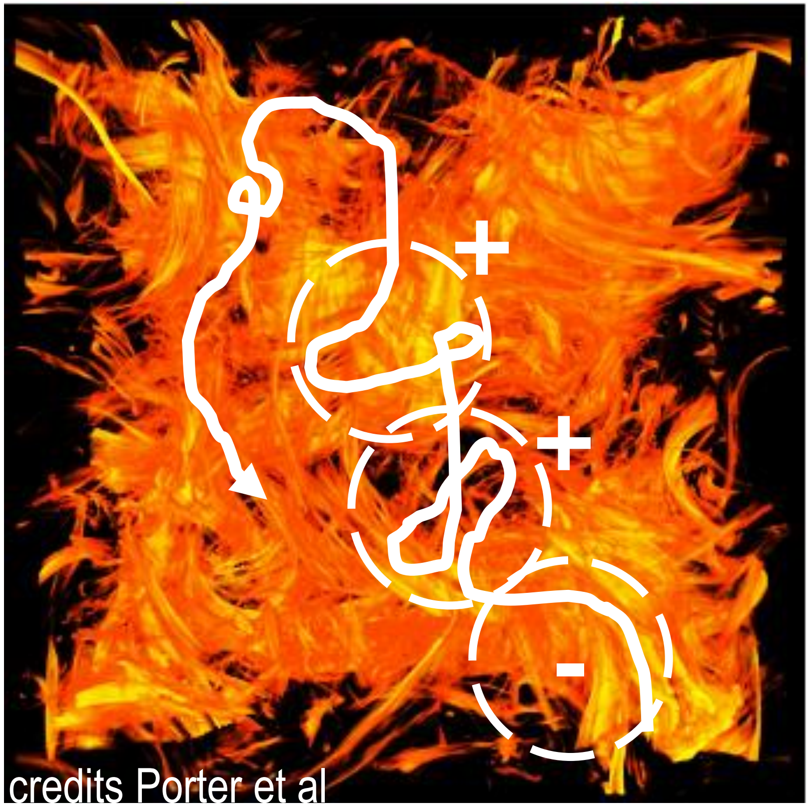
$$D_{pp} = \frac{2}{9} p^2 D \int_k \frac{dy y^2 \mathcal{K}(y)}{c_s^2 + y^2 D^2}$$

Fermi-like mechanisms using large scale turbulence

Reacceleration in super-Alfvenic turbulent reconnection

[Brunetti & Lazarian 16, Xu & Zhang 18, ..]

Particles diffusing in super-Alfvenic turbulence experience cycles of positive and negative acceleration via interaction with collapsing (in reconnection regions) and expanding (in dynamo regions) magnetic field lines.



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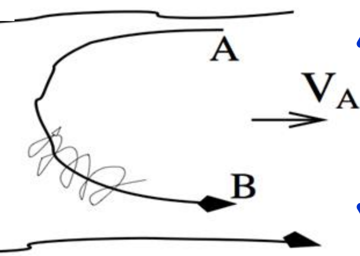
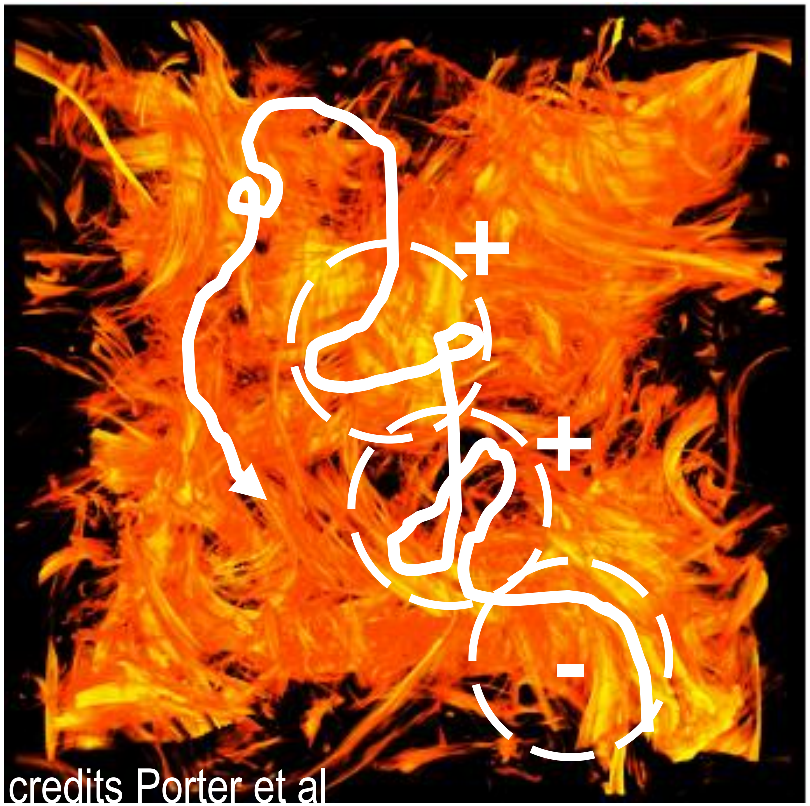
V_A (LV99, deGouveia+Lazarian 05)

(1) $$\frac{dp}{dt} \sim \phi \frac{V_A}{\lambda_{mfp}} p$$



$\approx l_A = L_0 M^{-3}$

V_A



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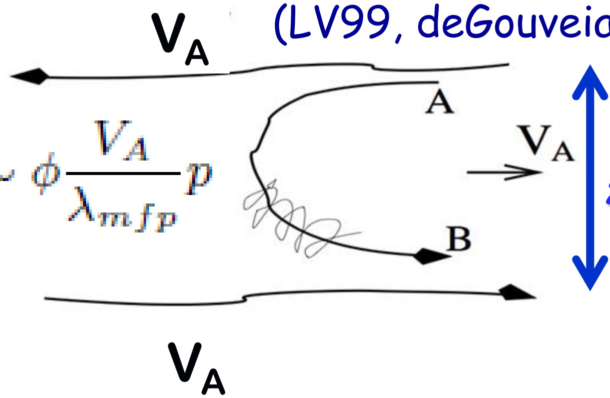

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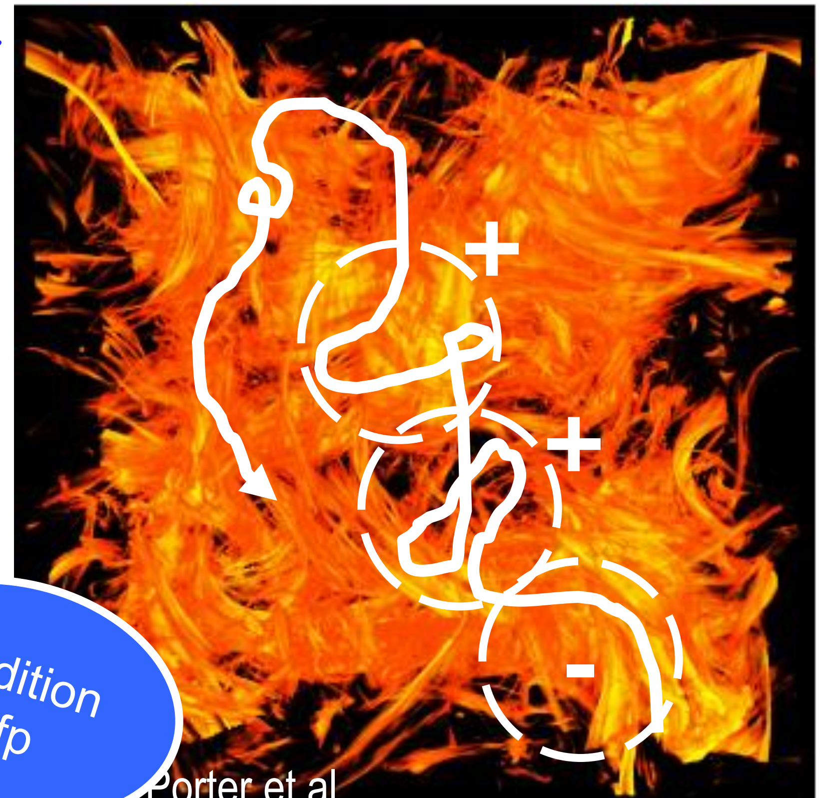

Condition on mfp

(2) particles diffuse faster than eddy turnover time $\approx l_A / V_A$

Fermi-like mechanisms using large scale turbulence

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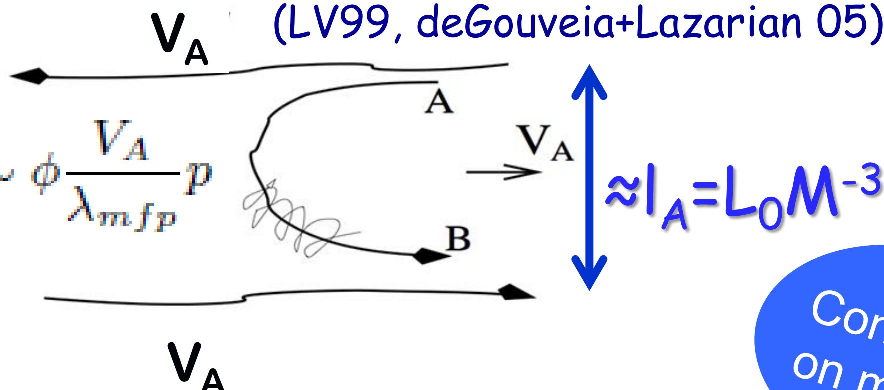

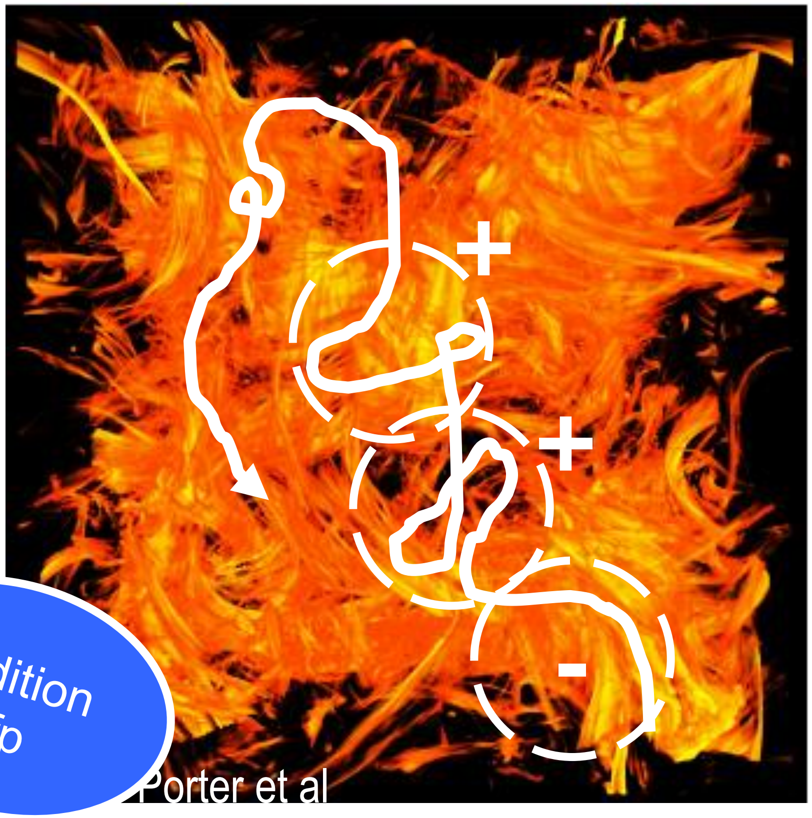

Condition on mfp

(2) **particles diffuse faster than eddy turnover time $\approx l_A/V_A$**

(3) **Case :** $p >> \Delta p$

$\Delta p = p\phi \frac{V_A}{\lambda_{mfp}}$ X $\frac{l_A^2}{D}$ } per cycle

Fermi-like mechanisms using large scale turbulence

Reacceleration in super-Alfvenic turbulent reconnection

[Brunetti & Lazarian 16, Xu & Zhang 18, ..]

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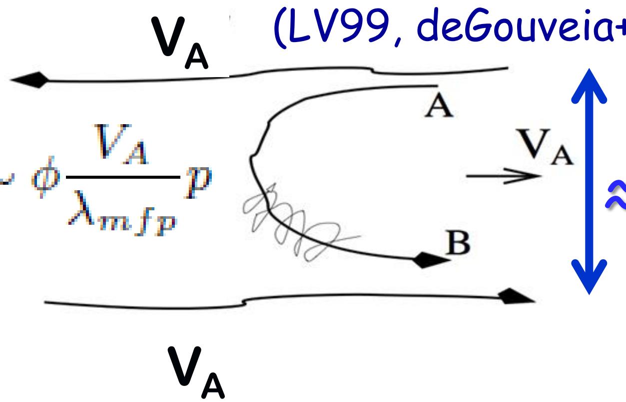

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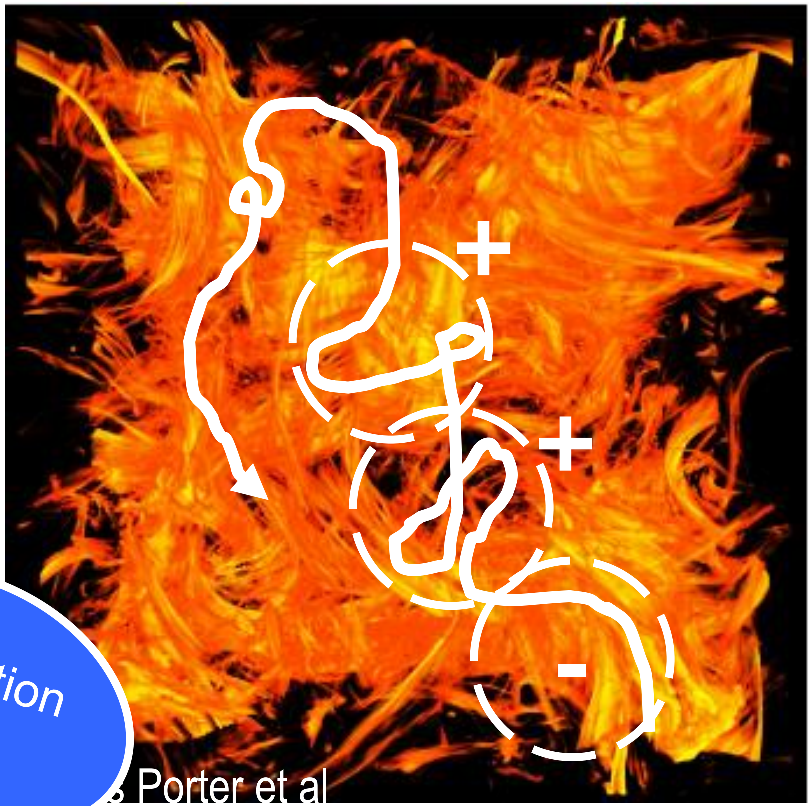


Condition on mfp

(2) particles diffuse faster than eddy turnover time $\approx l_A / V_A$

(3) Case : $p >> \Delta p$

$$D_{pp} = \langle \frac{\Delta p \Delta p}{2 \Delta t} \rangle \sim 3 \sqrt{\frac{5}{6}} \frac{c_s^2}{c} \frac{\sqrt{\beta_{pl}}}{L_o} M_t^3 \psi^{-3} p^2$$

where $\lambda_{mfp} = \psi l_A$

- Particles in a turbulent medium -

Resonant & nonResonant interaction between CRs & electromagnetic fluctuations

δB

Spatial diffusion via pitch-angle scattering

$$D_{\mu\mu} \equiv \lim_{t\to\infty} \frac{1}{2t} \langle \Delta\mu(t)\Delta\mu^*(t+\tau)\rangle = \Re \int_0^\infty d\tau \langle \dot{\mu}(t)\dot{\mu}^*(t+\tau)\rangle$$

δE

Stochastic acceleration (diffusion in particles momentum space)

$$D_{pp} \equiv \lim_{t\to\infty} \frac{1}{2t} \langle \Delta p(t)\Delta p^*(t+\tau)\rangle = \Re \int_0^\infty d\tau \langle \dot{p}(t)\dot{p}^*(t+\tau)\rangle$$

In the limit of small pitch-angle and momentum changes (ie. δp<<p) the process can be described as a diffusion in the angle and momentum space

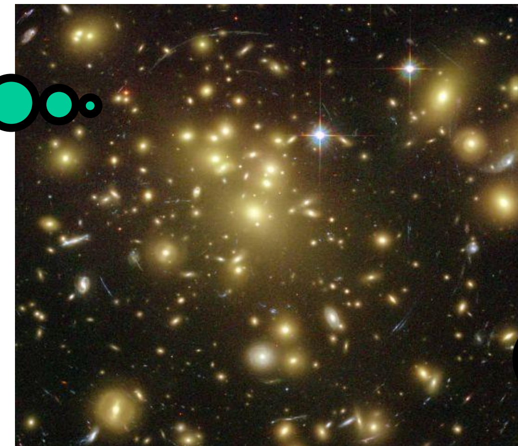
OUTLINE

- Turbulent acceleration: from Fermi to Gyroresonance and diffusion coefficients
- Complications and acceleration models using large-scale turbulence
- Turbulent acceleration in galaxy clusters: Radio Halos, models and observations at low radio frequencies
- Turbulent acceleration in head tail radio galaxies: evidences from observations at low radio frequencies
- APPENDIX: Fokker-Planck equation and coefficients

Clusters of galaxies:

the largest gravitational structures in the Universe ($M \approx 10^{14}$-$10^{15} M_{sun}$, $R_V \approx$ 2-3 Mpc)

≈30-300 galaxies



Galaxy cluster matter :

Barions ❑ **10%** of stars in galaxies

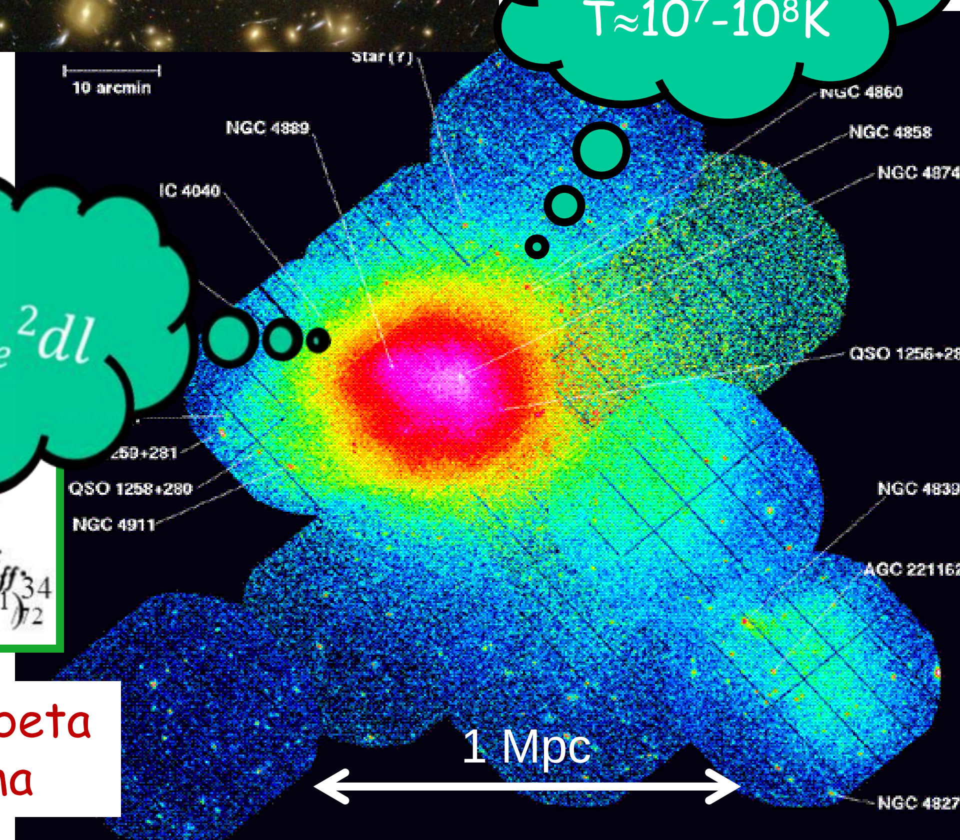
❑ **15-20% of** hot diffuse gas

Dark Matter **70%**

$n \approx 10^{-3} cm^{-3}$

$T \approx 10^7$-$10^8 K$

$\sim \int n_e^2 dl$

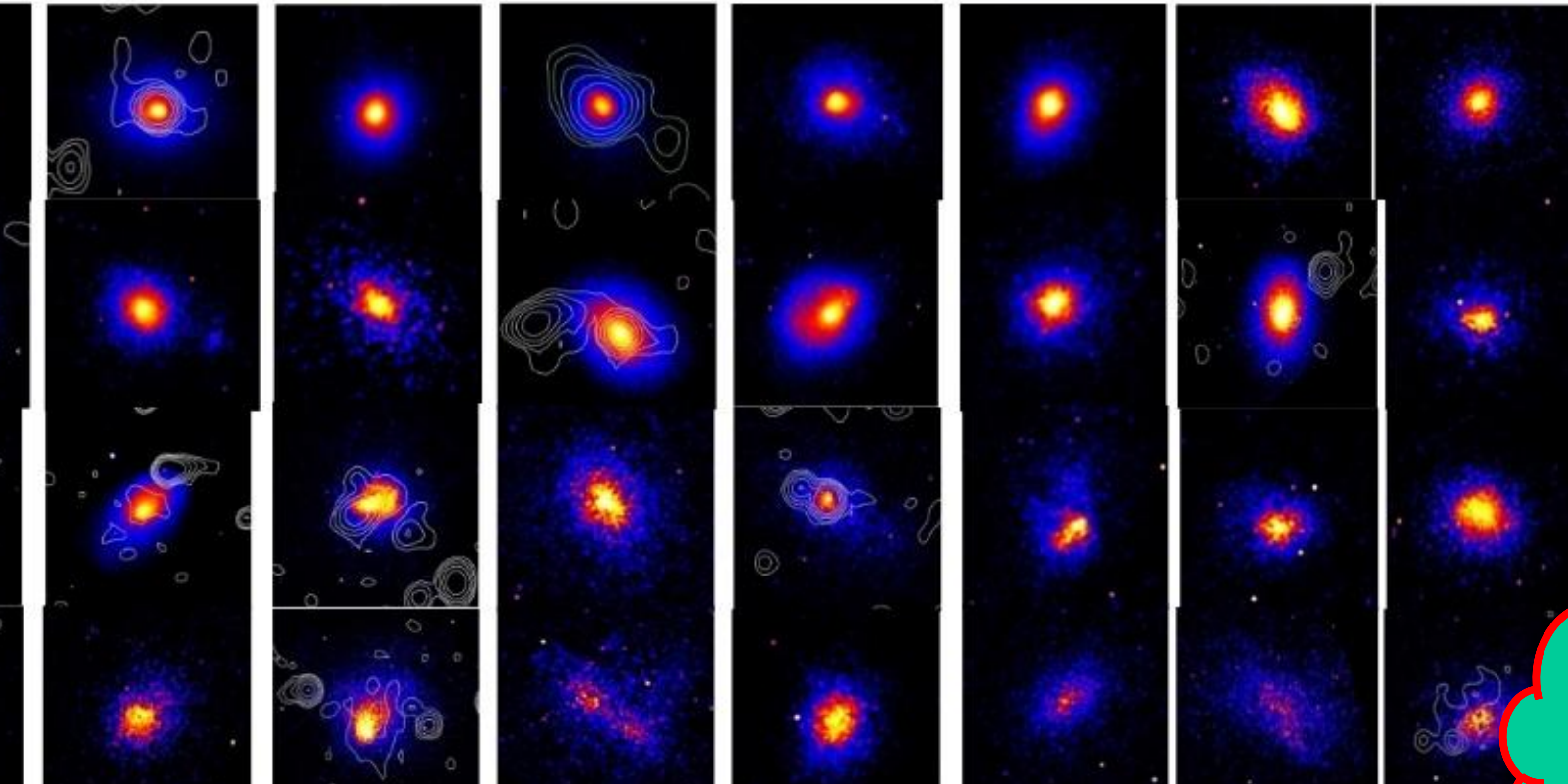


Emissivity per unit frequency (~ X-ray spectrum)

$$\varepsilon_\nu^{ff} \equiv \frac{dW}{dV\,dt\,d\nu} = 6.8\times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}$$

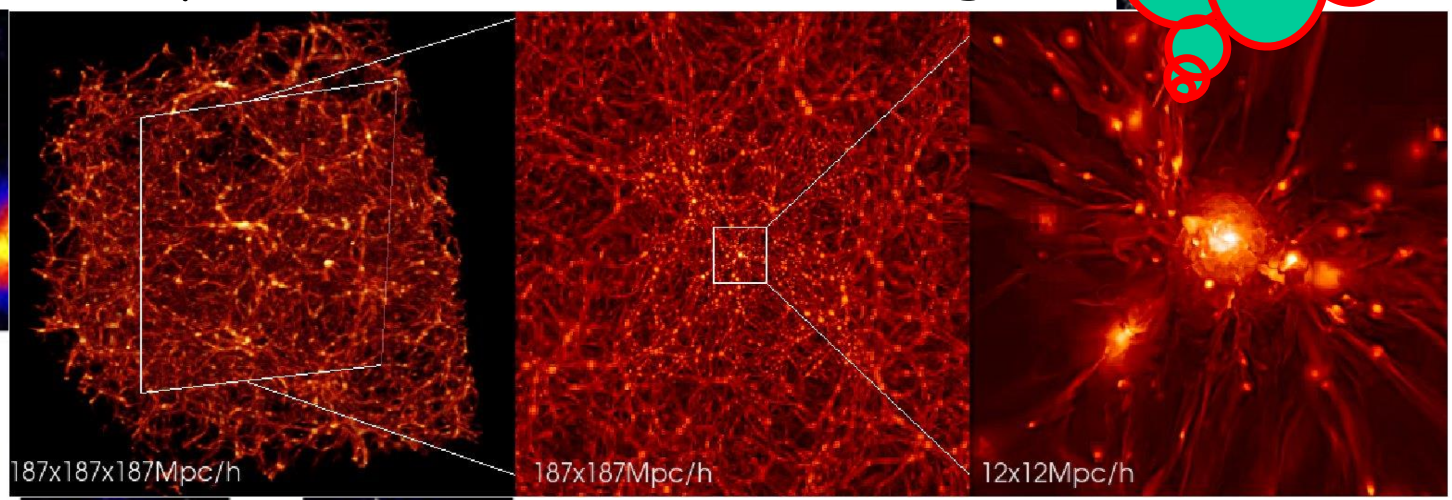
(erg s^{-1} cm^{-3} Hz^{-1})

The intra-cluster-medium (ICM) is a high beta (from Faraday RM) weakly collisional plasma



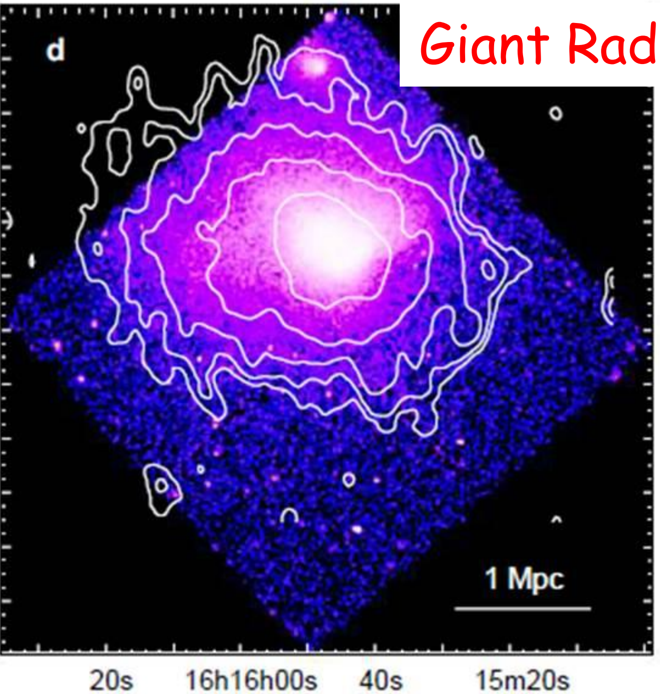
Galaxy clusters formation : mergers

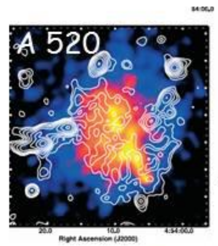
Mergers dissipate 10^{63-64} erg in 1 Gyr

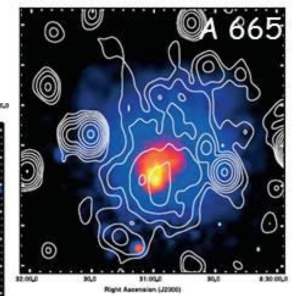


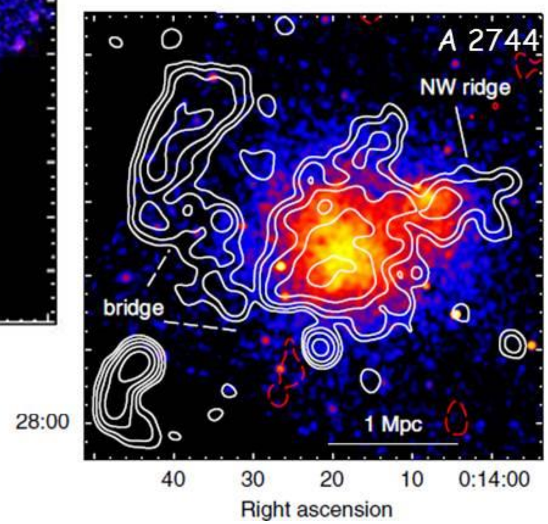
Cluster-scale radio emission

Giant Radio Halos









- Steep spectrum sources ($\alpha > 1$, $F(\nu) \approx K \nu^{-\alpha}$)
- Low + smooth brightness and unpolarised (< 5%)

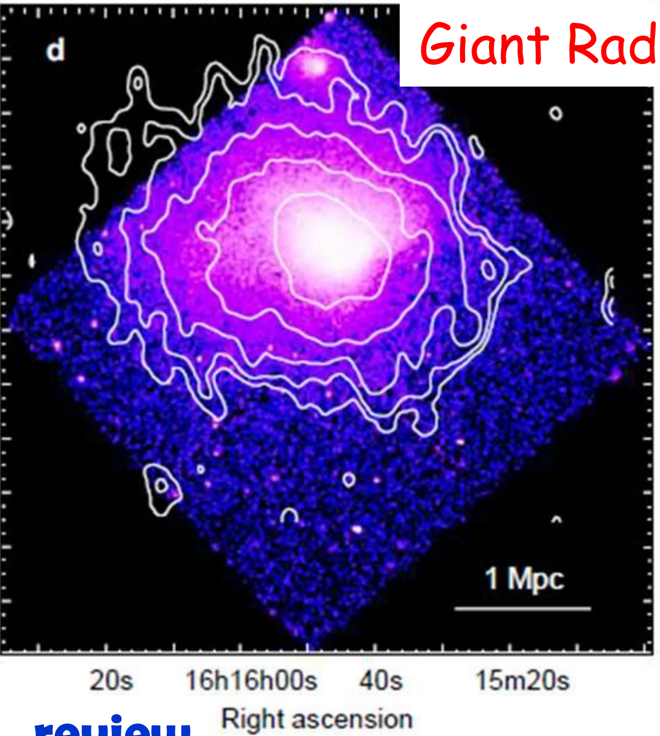
review Brunetti+Jones 14

Syn frequency

$$\nu_{SYN} \simeq 4.6\gamma^2 B_{\mu G}(1+z)^{-1}$$

GeV+ electrons (protons?) and μG B distributed on Mpc-scales...

Cluster-scale radio emission



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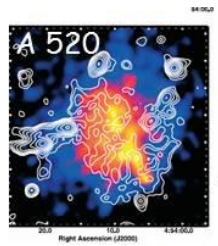

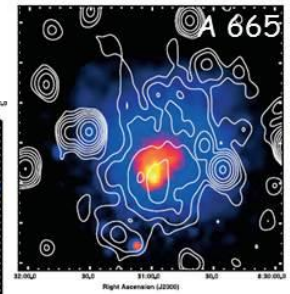

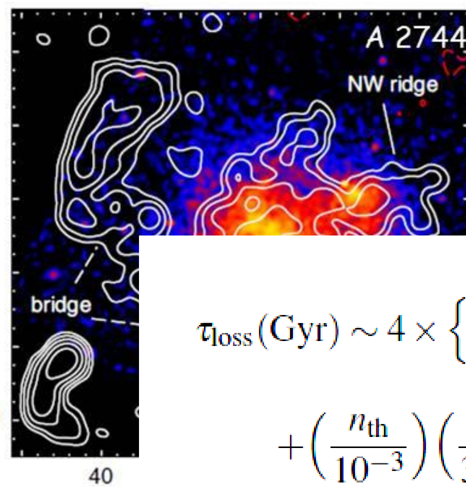

review Brunetti+Jones 14

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Electrons time-scale

$$\tau_{\rm loss}({\rm Gyr}) \sim 4\times\left\{\frac{1}{3}\left(\frac{\gamma}{300}\right)\left[\left(\frac{B_{\mu G}}{3.2}\right)^2\frac{\sin^2\theta}{2/3}+(1+z)^4\right] + \left(\frac{n_{\rm th}}{10^{-3}}\right)\left(\frac{\gamma}{300}\right)^{-1}\left[1.2+\frac{1}{75}\ln\left(\frac{\gamma/300}{n_{\rm th}/10^{-3}}\right)\right]\right\}^{-1} \approx 100\ {\rm Myr}$$

Syn frequency

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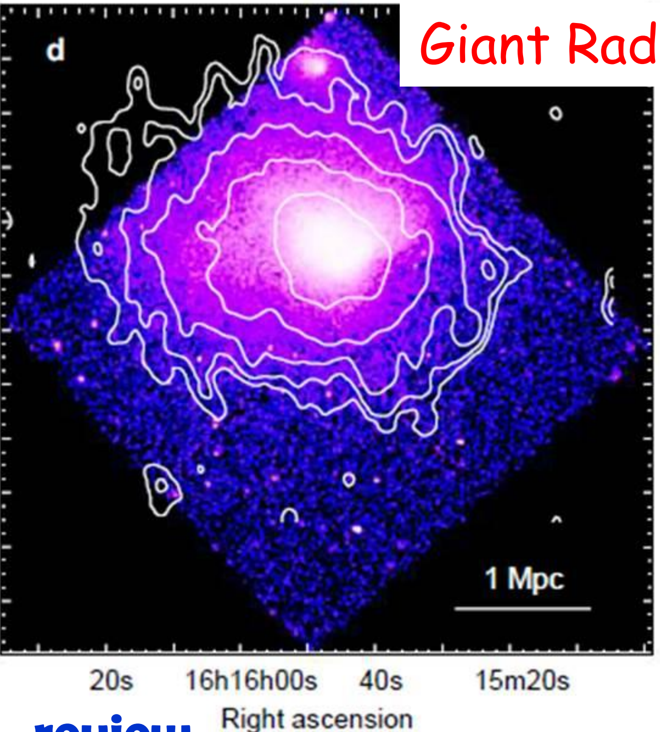
GeV+ electrons (protons?) and μG B distributed on Mpc-scales…

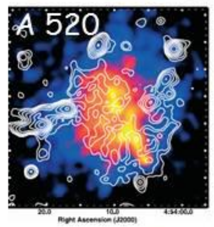
Diffusion timescale

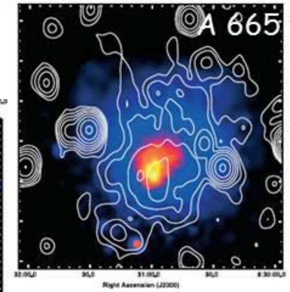
$$\tau_{diff} \approx \frac{1}{4}\frac{L^2}{D} \approx 10\ {\rm Gyr}$$

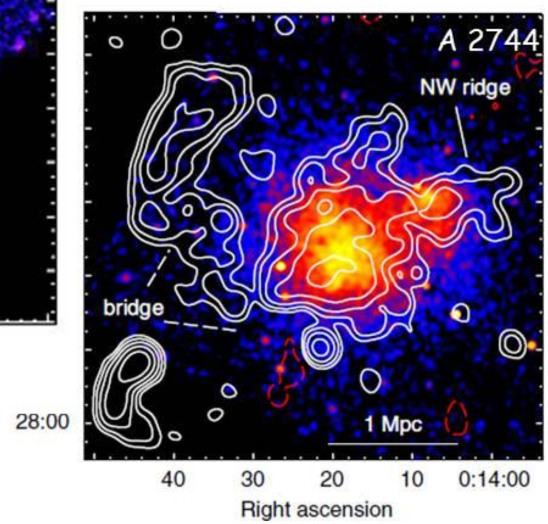
Cluster-scale radio emission

Giant Radio Halos









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- In situ acceleration TURBULENCE ?

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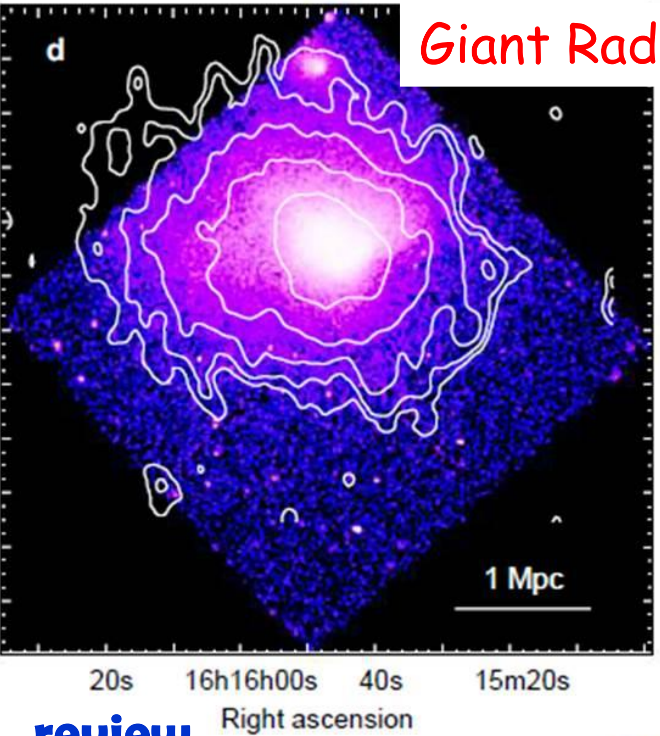

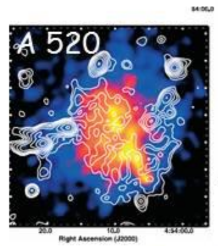

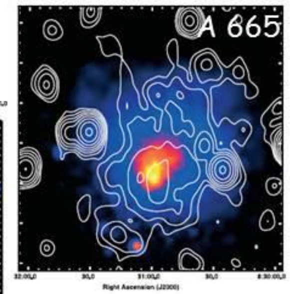

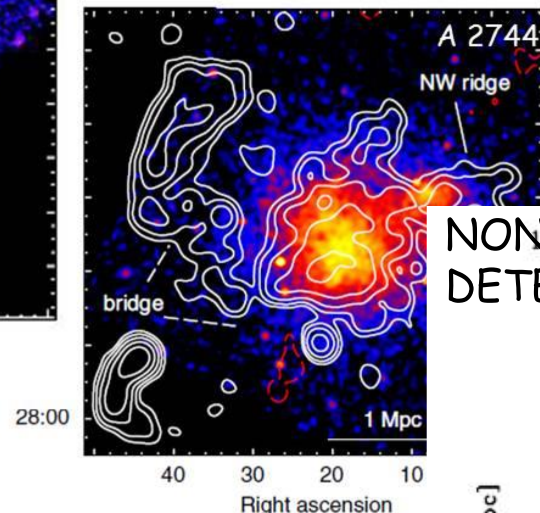

review

Brunetti+Jones 14

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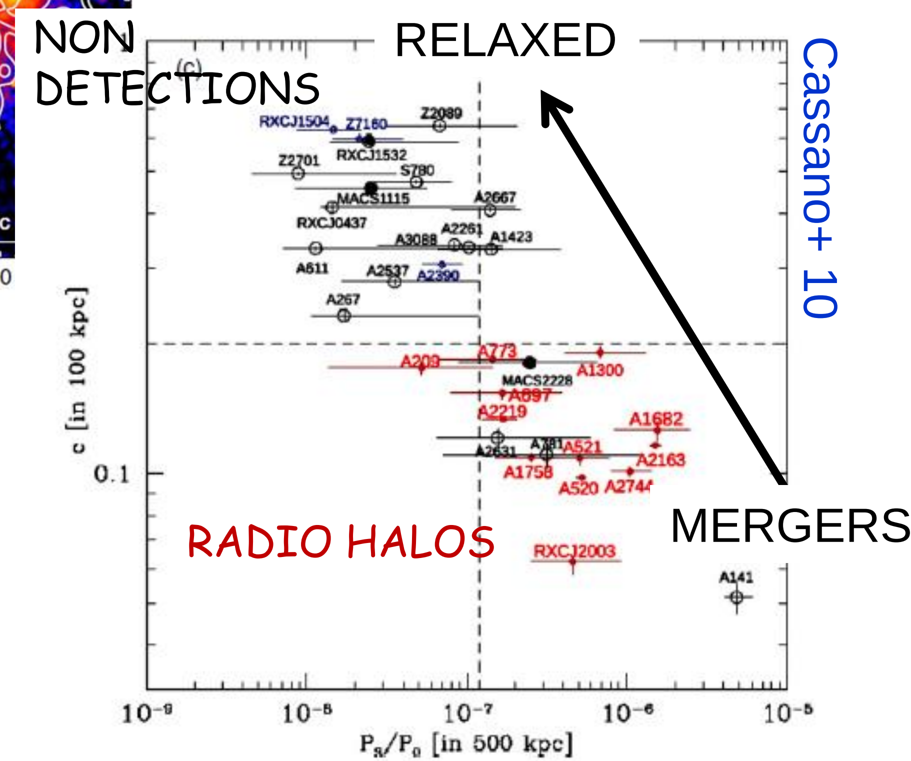
- In situ acceleration TURBULENCE ?
- Connection with MERGERS



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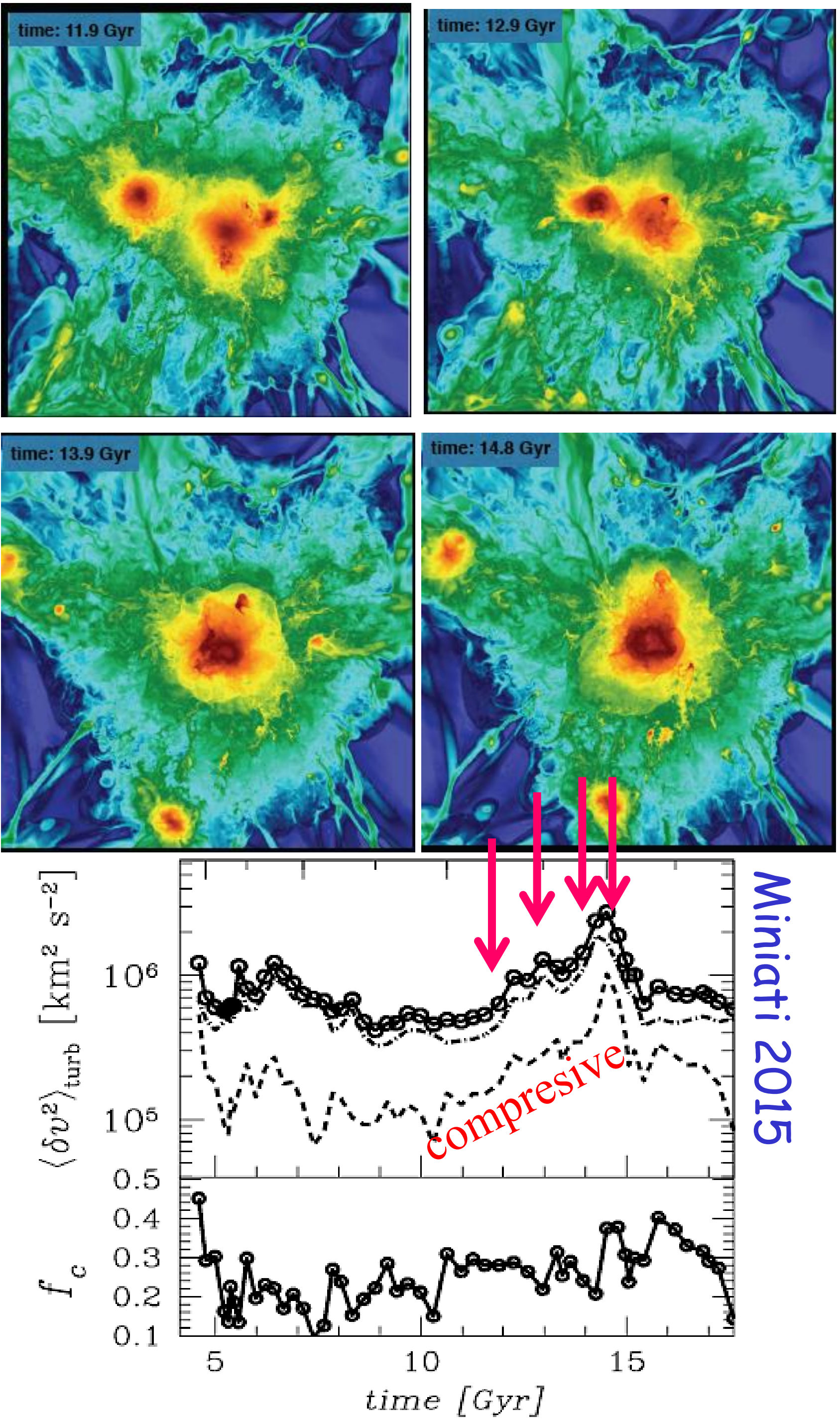
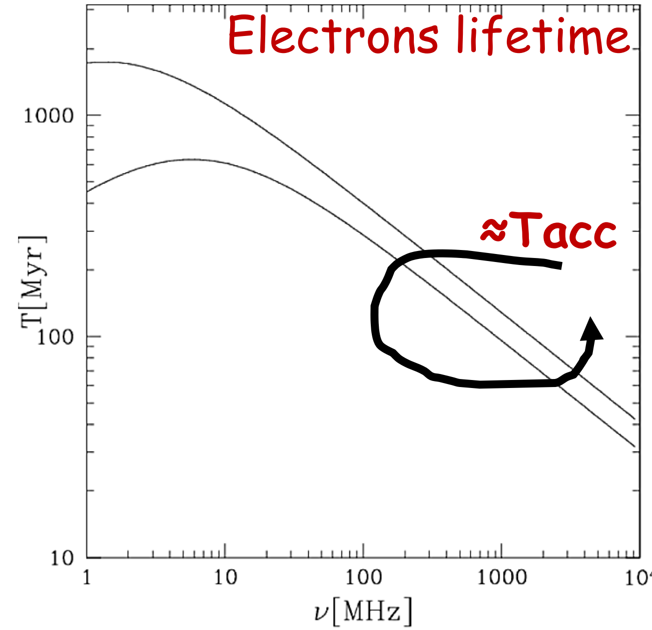
TURBULENT ACCELERATION





- ❑ Tm ≈ 1-3 Gyr
- ❑ Tt ≈ L/δV ≈ 200-500 Myr
- ❑ Tescape > Gyr

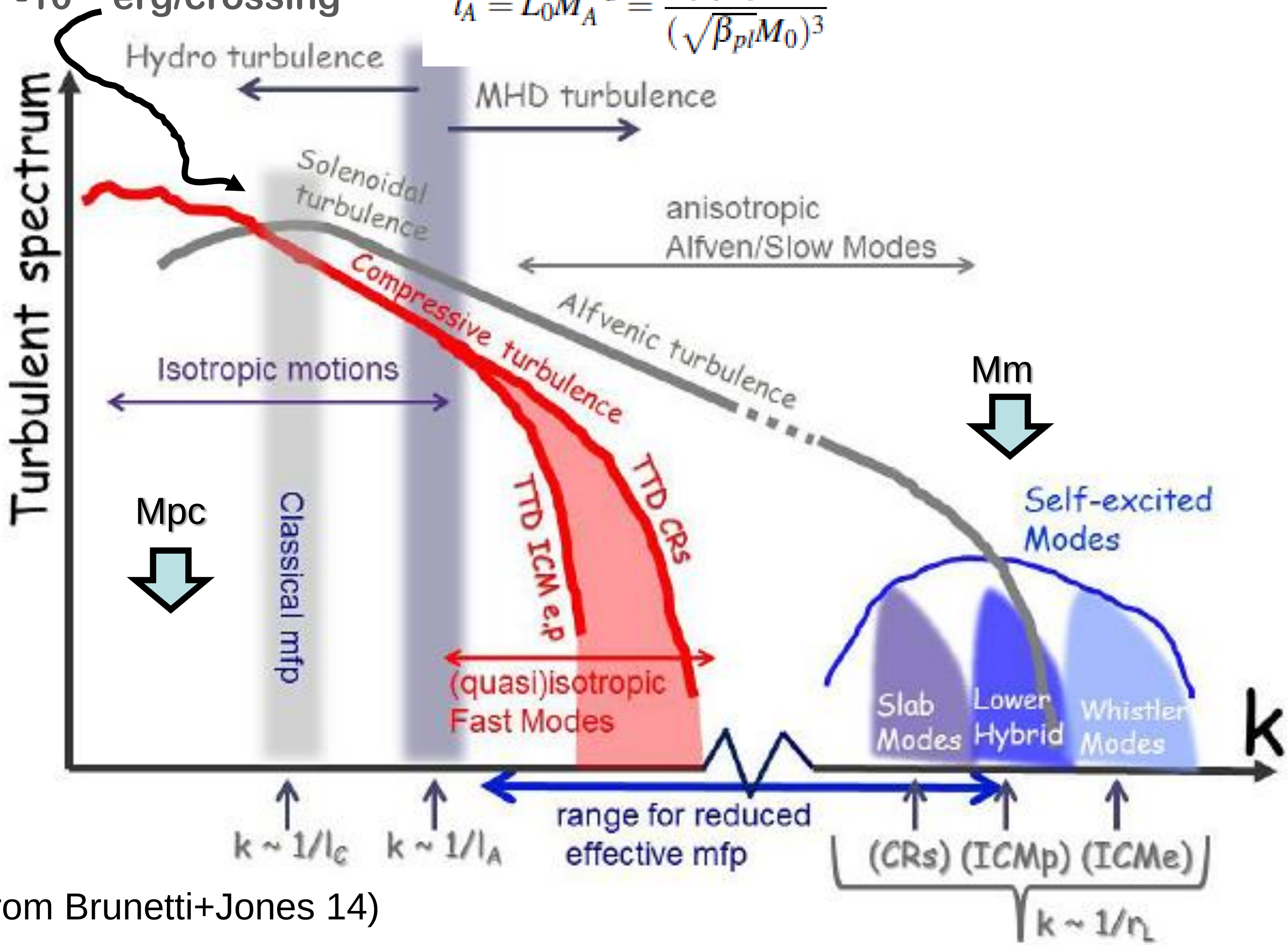
Tacc < Tt < Tes ≈ Tm

- Energy is transported from Mpc to Mm scales into non-thermal particles.
- This requires a hierarchy of complex mechanisms and plasma/kinetic effects !

TURBULENCE IN THE ICM

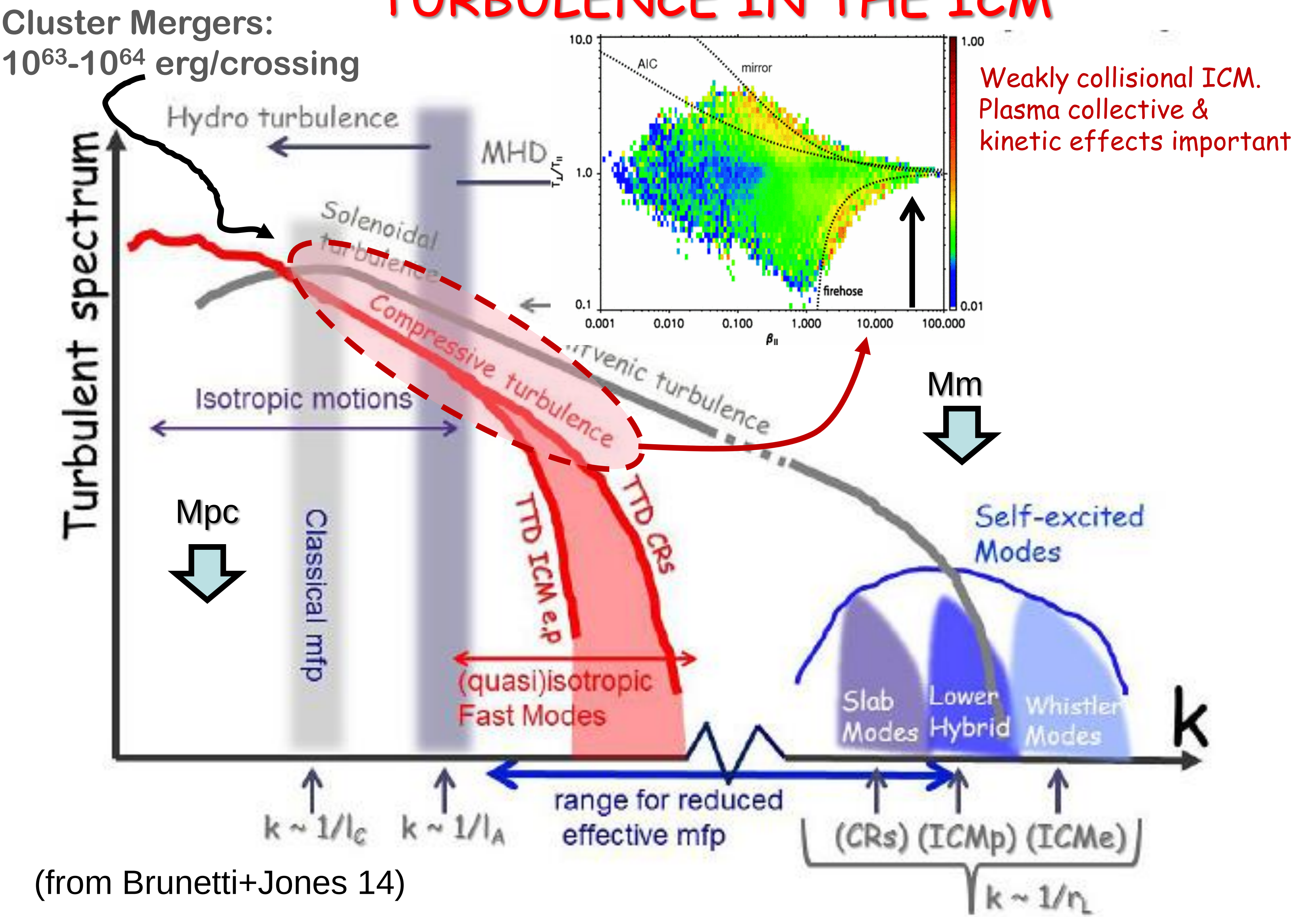
Cluster Mergers:
10^{63}-10^{64} erg/crossing

$$l_A = L_0 M_A^{-3} = \frac{(6/5)^3 L_0}{(\sqrt{\beta_{pl}} M_0)^3}$$



(from Brunetti+Jones 14)

TURBULENCE IN THE ICM



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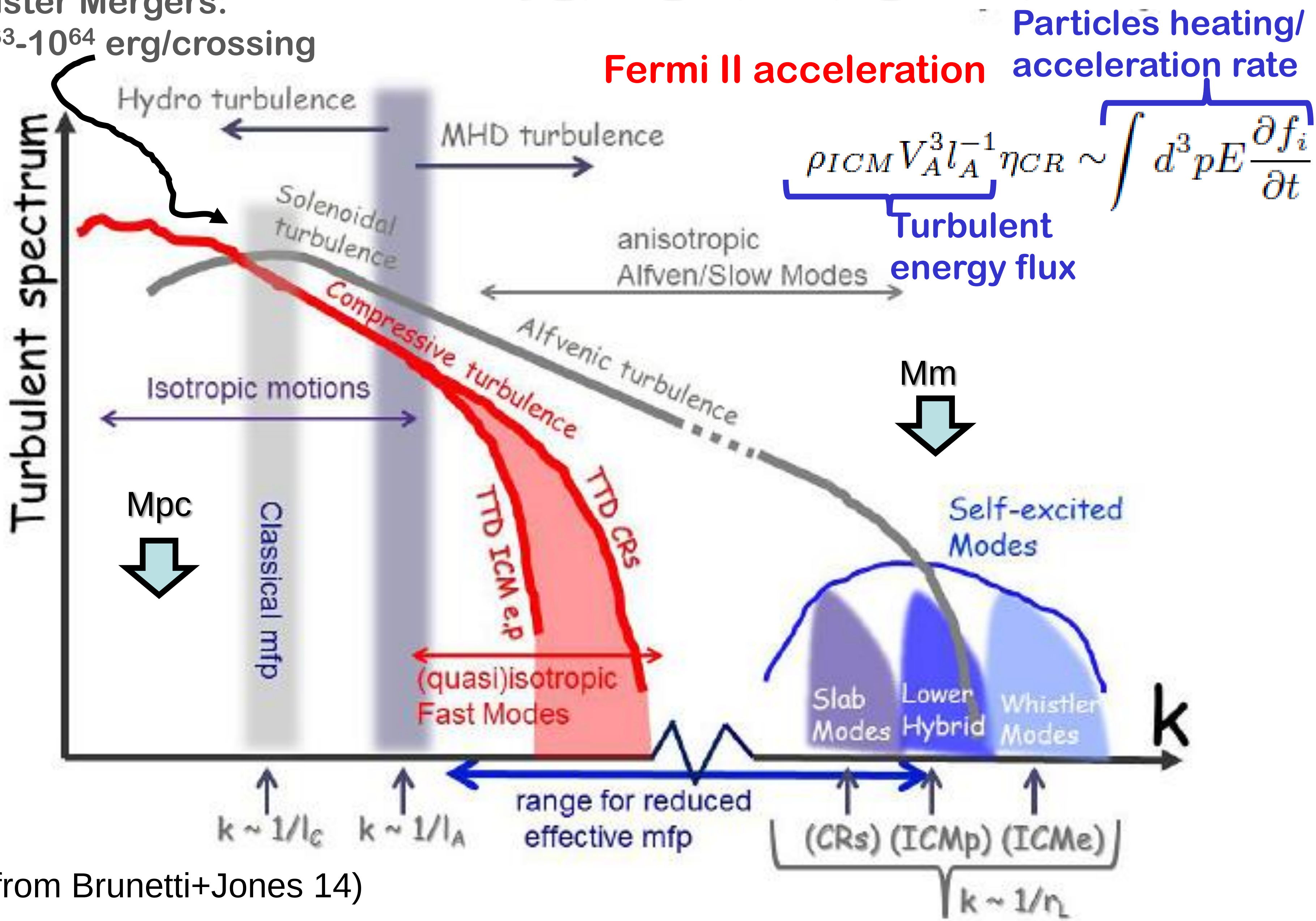
Fermi II acceleration



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TURBULENT ACCELERATION : example

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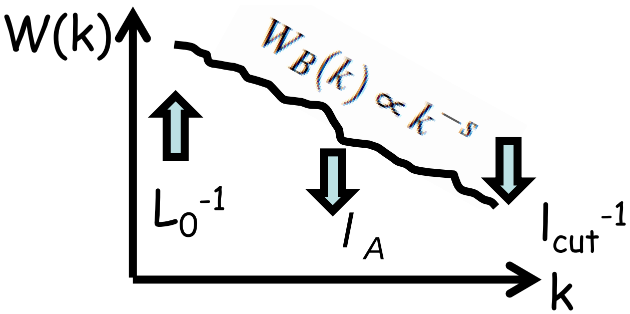

Turbulent energy flux

Particles heating/ acceleration rate

$$\rho_{ICM} V_A^3 l_A^{-1} \eta_{CR} \sim \int d^3p E \frac{\partial f_i}{\partial t} = \int d^3k W(\vec{k}) \Gamma(\vec{k})$$

Efficiency depends on turbulent acceleration model and on plasma conditions (Brunetti+Lazarian 07)

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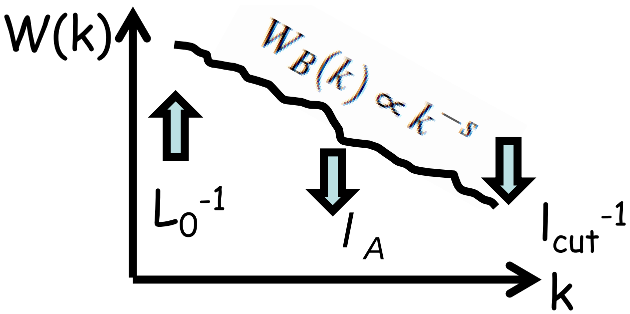

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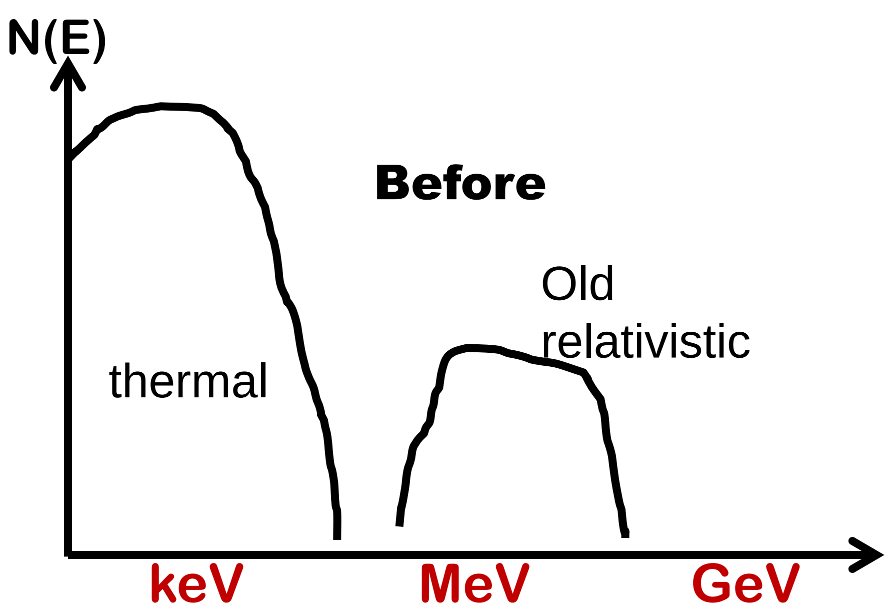

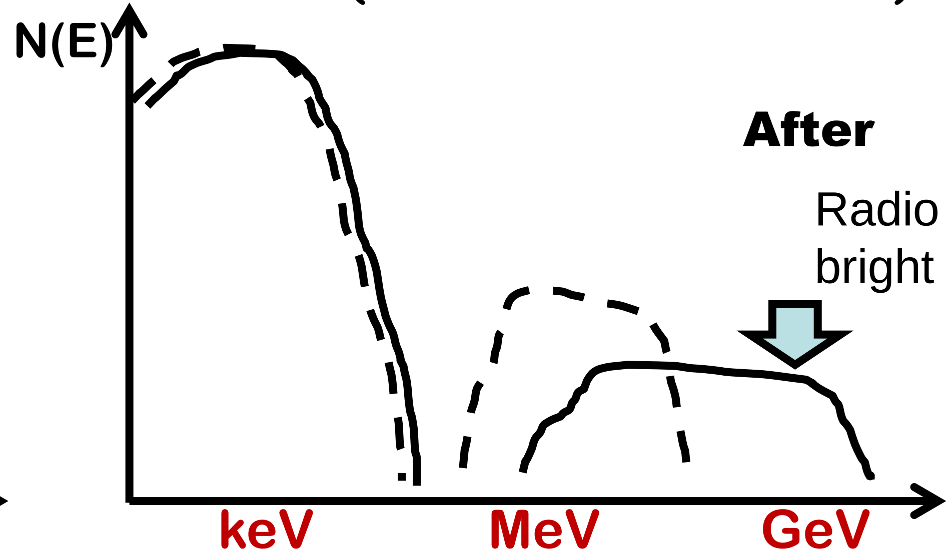

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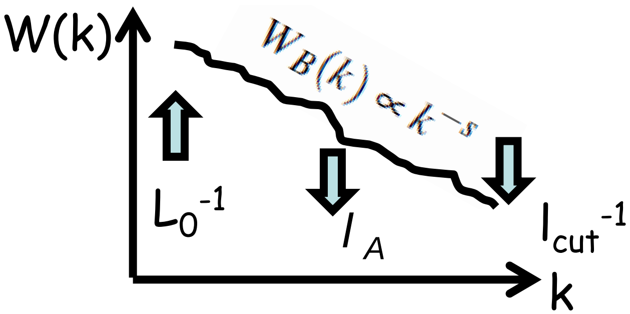

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Efficiency depends on turbulent acceleration model and on plasma conditions (Brunetti+Lazarian 07)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathrm{p}} \left[D_{\mathrm{pp}} \frac{\partial f}{\partial \mathrm{p}} \right]$$

$$\Gamma = -\mathrm{i} \left(\frac{E_i^* K_{ij}^a E_j}{16\pi W} \right)_{\omega_\mathrm{i}=0} \omega_\mathrm{r},$$

$$W(k, \omega) = \frac{1}{16\pi} \left[B_{k\,i}^* B_{k\,i} + E_{k\,i}^* \frac{\partial}{\partial \omega} \left(\omega K_{ij}^\mathrm{h} \right) E_{k\,j} \right]_{\omega_\mathrm{i}=0},$$

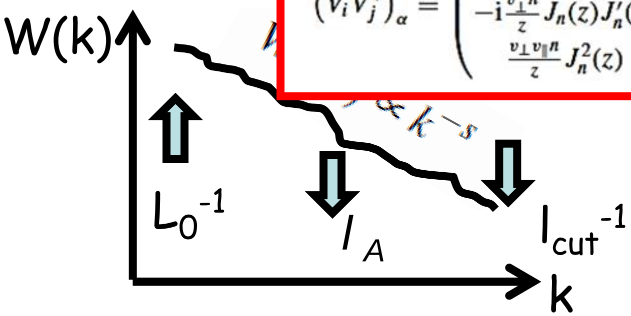
TU[R...]

$$K_{ij} = \delta_{ij} + 2\pi \sum_\alpha m_\alpha \left(\frac{\omega_{p,\alpha}}{\omega}\right)^2 \int\int dp_\perp p_\perp dp_\parallel \left[\frac{v_\parallel}{v_\perp}\left(v_\perp \frac{\partial}{\partial p_\parallel} - v_\parallel \frac{\partial}{\partial p_\perp}\right)\hat{f}_\alpha(p) b_i b_j + \sum_{n=-\infty}^{\infty} \frac{(V_i V_j^*)_\alpha}{\omega - n\Omega_\alpha - k_\parallel v_\parallel} \times \left(\frac{\omega - k_\parallel v_\parallel}{v_\perp}\frac{\partial}{\partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel}\right)\hat{f}_\alpha(p)\right], \quad (16)$$

where $\omega_{p,\alpha} = \sqrt{4\pi N_\alpha e_\alpha^2/m_\alpha}$ is the plasma frequency for the species α, $b_i = (B_o/|B_o|)_i$ is the unit vector along the magnetic field, $\Omega_\alpha = (e_\alpha B_o/m_\alpha c)/\gamma$ is the Larmor frequency of particles α,

$$(V_i V_j^*)_\alpha = \begin{pmatrix} \left(\frac{v_\perp n}{z}\right)^2 J_n^2(z) & i\frac{v_\perp^2 n}{z} J_n(z)J_n'(z) & \frac{v_\perp v_\parallel n}{z} J_n^2(z) \\ -i\frac{v_\perp^2 n}{z} J_n(z)J_n'(z) & v_\perp^2 (J_n'(z))^2 & -i v_\perp v_\parallel J_n(z) J_n'(z) \\ \frac{v_\perp v_\parallel n}{z} J_n^2(z) & i v_\perp v_\parallel J_n(z) J_n'(z) & v_\parallel^2 J_n^2(z) \end{pmatrix}_\alpha \qquad z_\alpha = k_\perp p_\perp / m_\alpha \Omega_o^\alpha \quad (17)$$

$$l_A = L_0 M_A^{-3} = \ldots$$



$$\rho_{ICM} V_A^3 l_A^{-1} \eta_{CR} \sim \int d^3p E \frac{\partial f_i}{\partial t} = \int d^3k W(\vec{k}) \Gamma(\vec{k})$$

Efficiency depends on turbulent acceleration model and on plasma conditions (Brunetti+Lazarian 07)

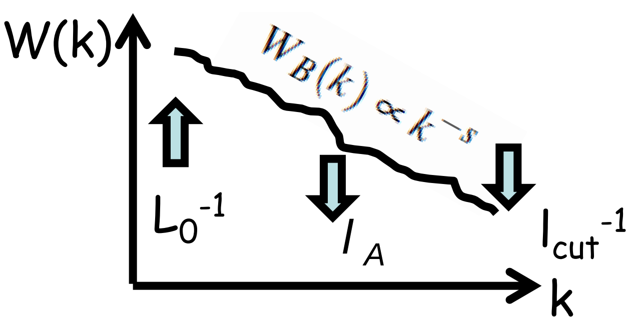
$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathbf{p}}\left[D_{\mathbf{pp}} \frac{\partial f}{\partial \mathbf{p}}\right]$$

$$\Gamma = -i\left(\frac{E_i^* K_{ij}^a E_j}{16\pi W}\right)_{\omega_i = 0} \omega_r,$$

$$W(k, \omega) = \frac{1}{16\pi}\left[B_{k\,i}^* B_{k\,i} + E_{k\,i}^* \frac{\partial}{\partial \omega}(\omega K_{ij}^h) E_{k\,j}\right]_{\omega_i = 0},$$

TURBULENT ACCELERATION : example

$$l_A = L_0 M_A^{-3} = \frac{(6/5)^3 L_0}{(\sqrt{\beta_{pl}} M_0)^3}$$



Turbulent energy flux

Particles heating/ acceleration rate

$$\rho_{ICM} V_A^3 l_A^{-1} \eta_{CR} \sim \int d^3p E \frac{\partial f_i}{\partial t} = \int d^3k W(\vec{k}) \Gamma(\vec{k})$$

TTD with Magnetosonic/fast Modes

$\omega - k_{//} v_{//} = 0$

$$\tau_{acc} \approx \frac{p^2}{D_{pp}}$$

fraction of energy available at small scales

$$D_{pp} = \mathscr{A}(s,..) \frac{\delta B^2}{B_0^2} \left(\frac{L_0}{l_{cut}} \right)^{1-s} \left(\frac{c_s^2}{c l_{cut}} p^2 \right) \approx \mathscr{A} \frac{\delta V_{l_{cut}}{}^2}{c l_{cut}} p^2$$

l_{cut} is the dominant scale for acceleration

Classical Fermi II

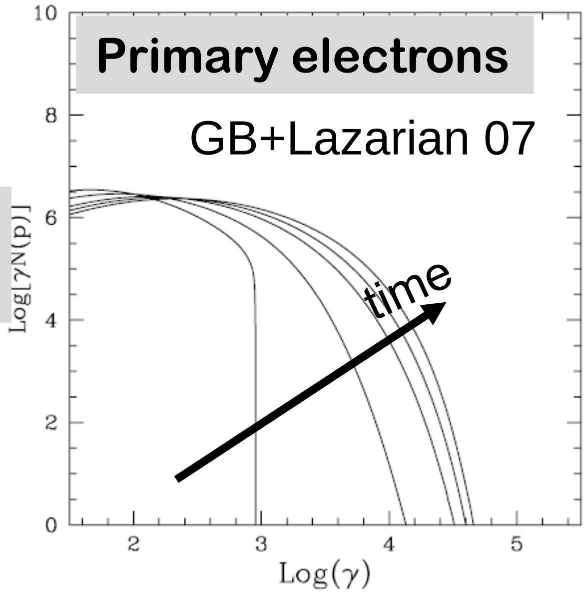
$T_{acc} \approx 10–1000$ Myr

TTD with Magnetosonic/fast Modes

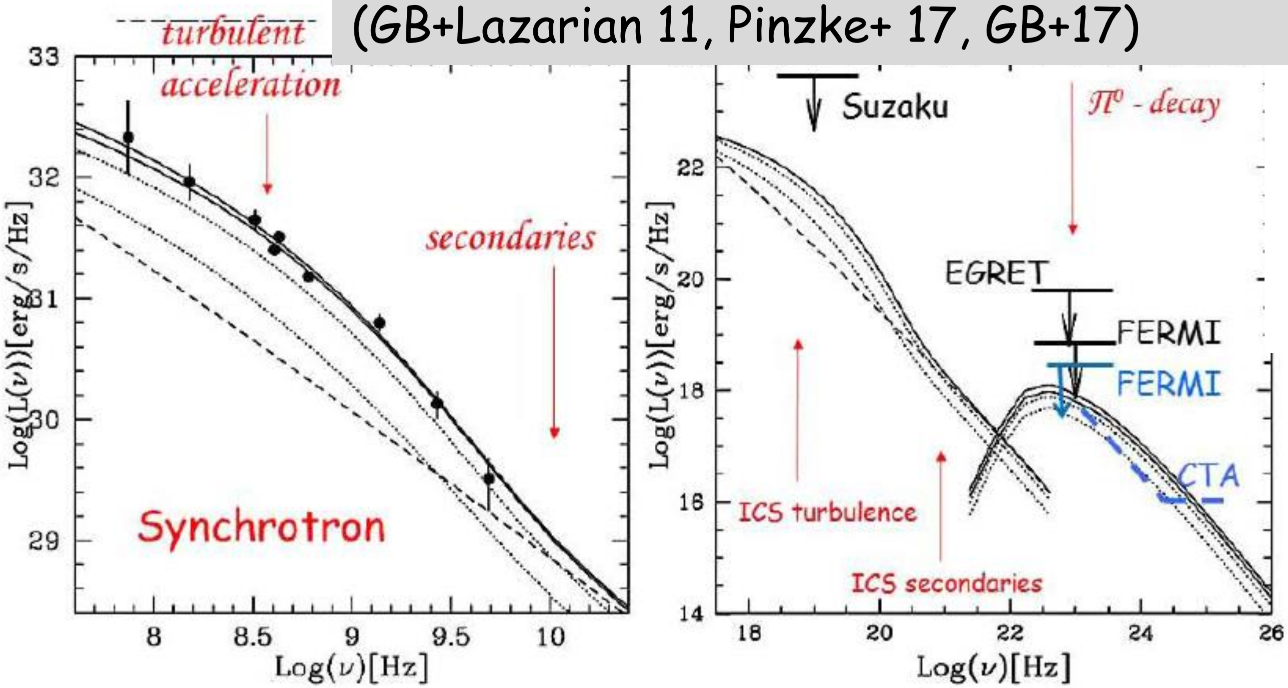
$\omega - k_{//} v_{//} = 0$

$$\frac{\partial f(p,t)}{\partial t} + (\mathbf{V}\cdot\nabla)f - \nabla\cdot\{\mathbf{n}D(\mathbf{n}\cdot\nabla)f\} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\mathscr{D}_{pp}\frac{\partial f}{\partial p} - p^2\left|\frac{dp}{dt}\right|_{loss} f\right) + Q(p,t)$$

$$D_{pp} = \mathscr{A}(s,..)\frac{\delta B^2}{B_0^2}\left(\frac{L_0}{l_{cut}}\right)^{1-s}\left(\frac{c_s^2}{c\, l_{cut}}p^2\right)$$



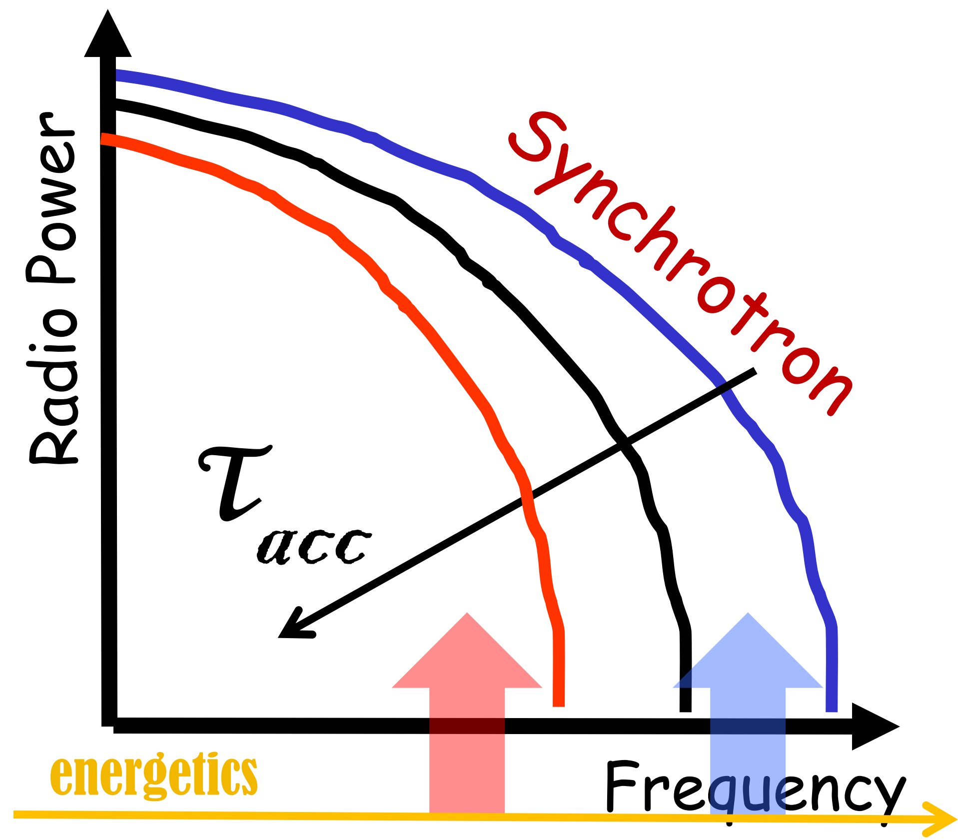
SED from primary+secondary electrons
(GB+Lazarian 11, Pinzke+ 17, GB+17)



TURBULENT ACCELERATION IS A GENTLE MECHANISM

$T_{acc} \approx p^2/D_{pp} \approx$10-1000 Myr

Reacceleration time is of the same order of magnitude of the energy-losses time-scale of the electrons emitting Syn radiation in the radio band !



Max Syn frequency

$$\nu_s/\mathrm{GHz} \sim (\tau_{acc}/400\mathrm{Myr})^{-2}$$

[assuming rough equipartition between Syn and inverse Compton losses]

Radio Halos predicted to be a mix of different populations including with very steep spectrum sources «invisible» at classical frequencies.
(Cassano, Brunetti, Setti 06
Brunetti + al 2008 Nature 455,944)

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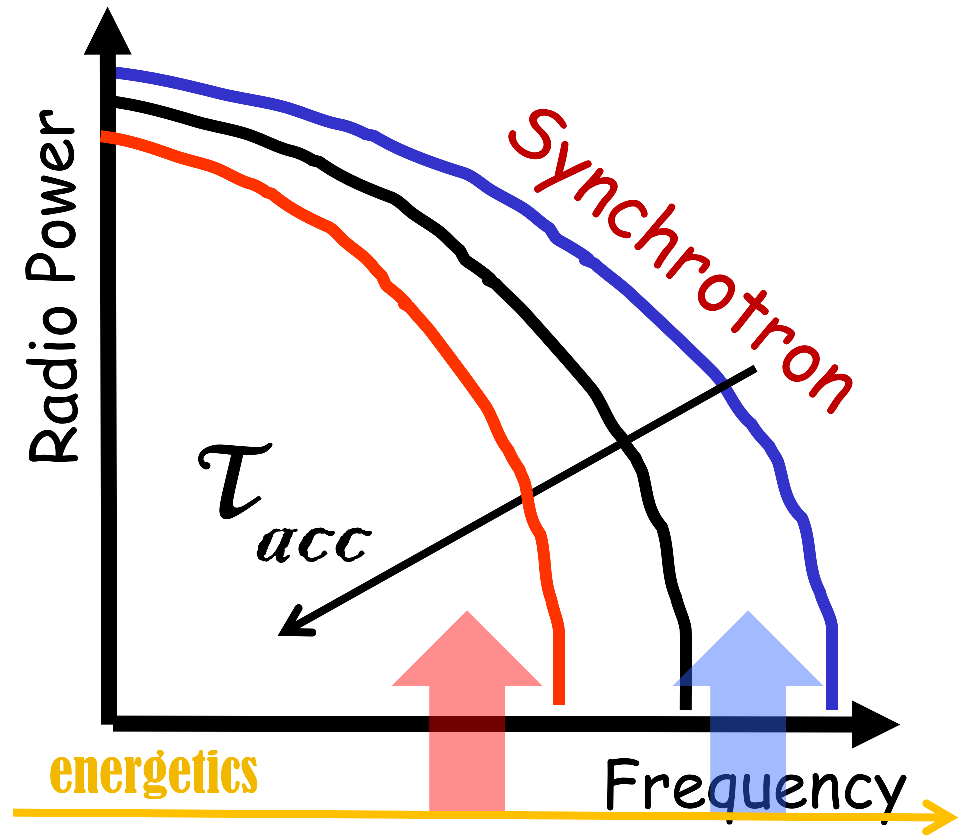

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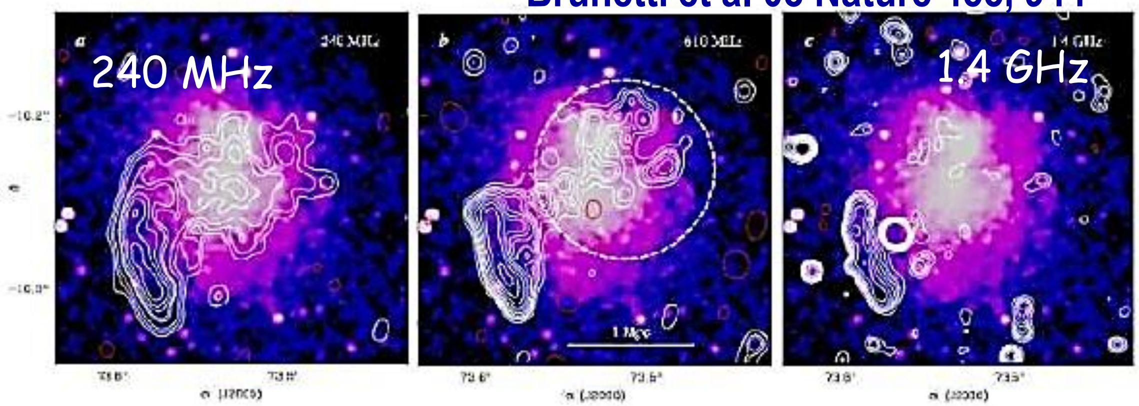
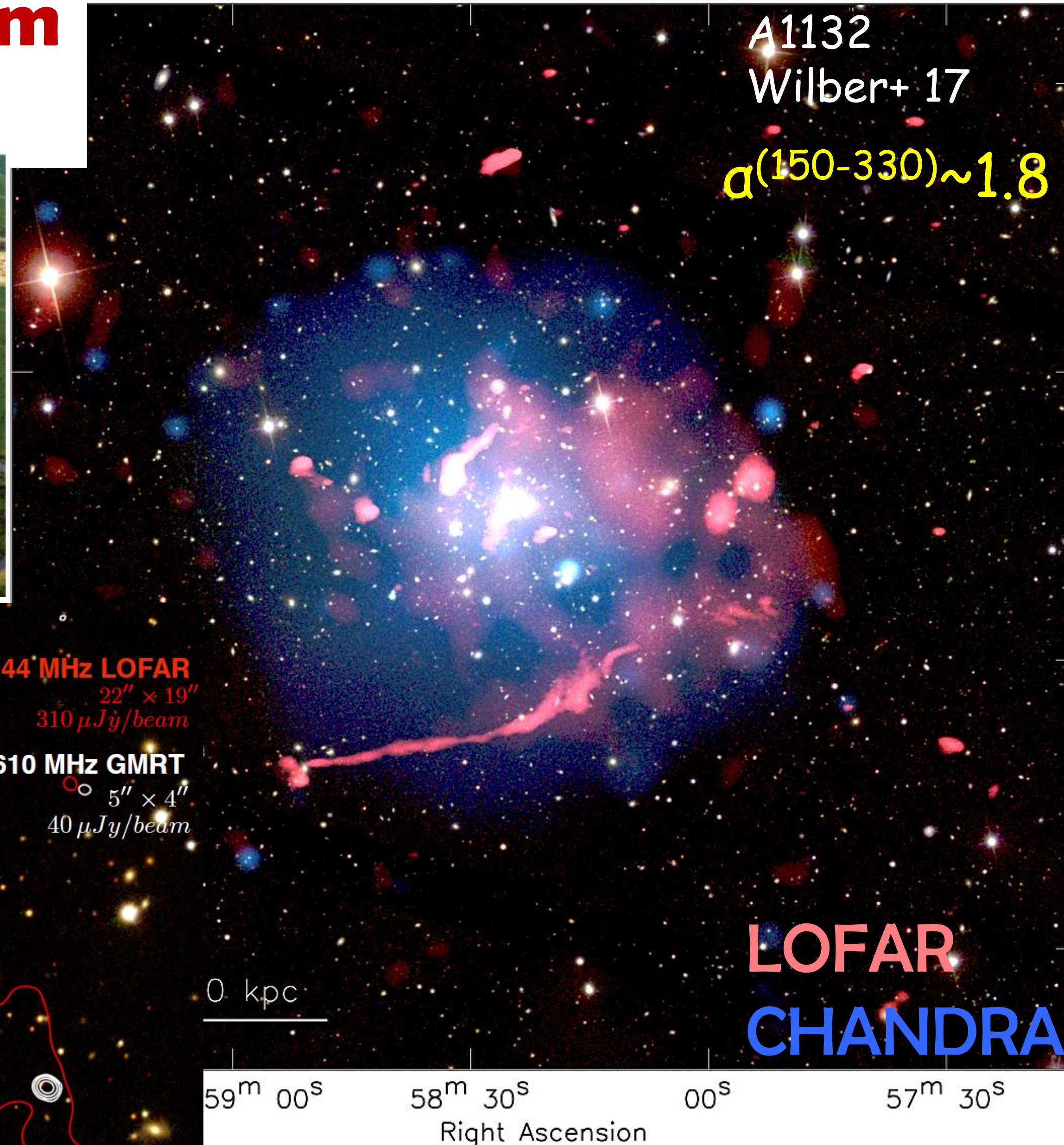


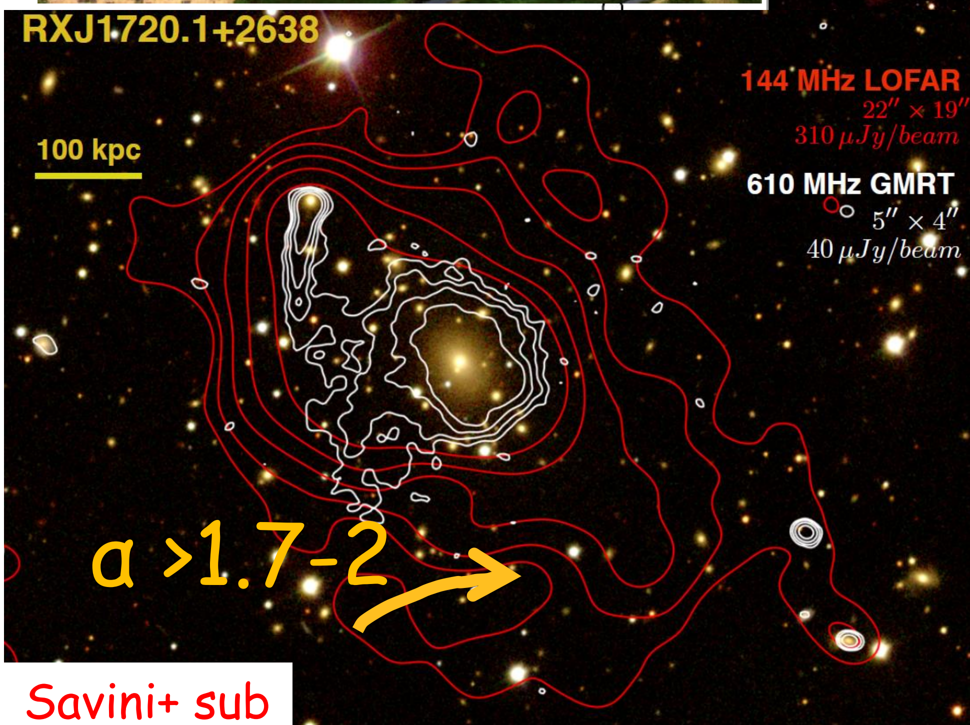
The largest radio telescope on the way to the SKA : 25000 antennae

LOFAR

\# 10-80 MHz
\# 120-240 MHz

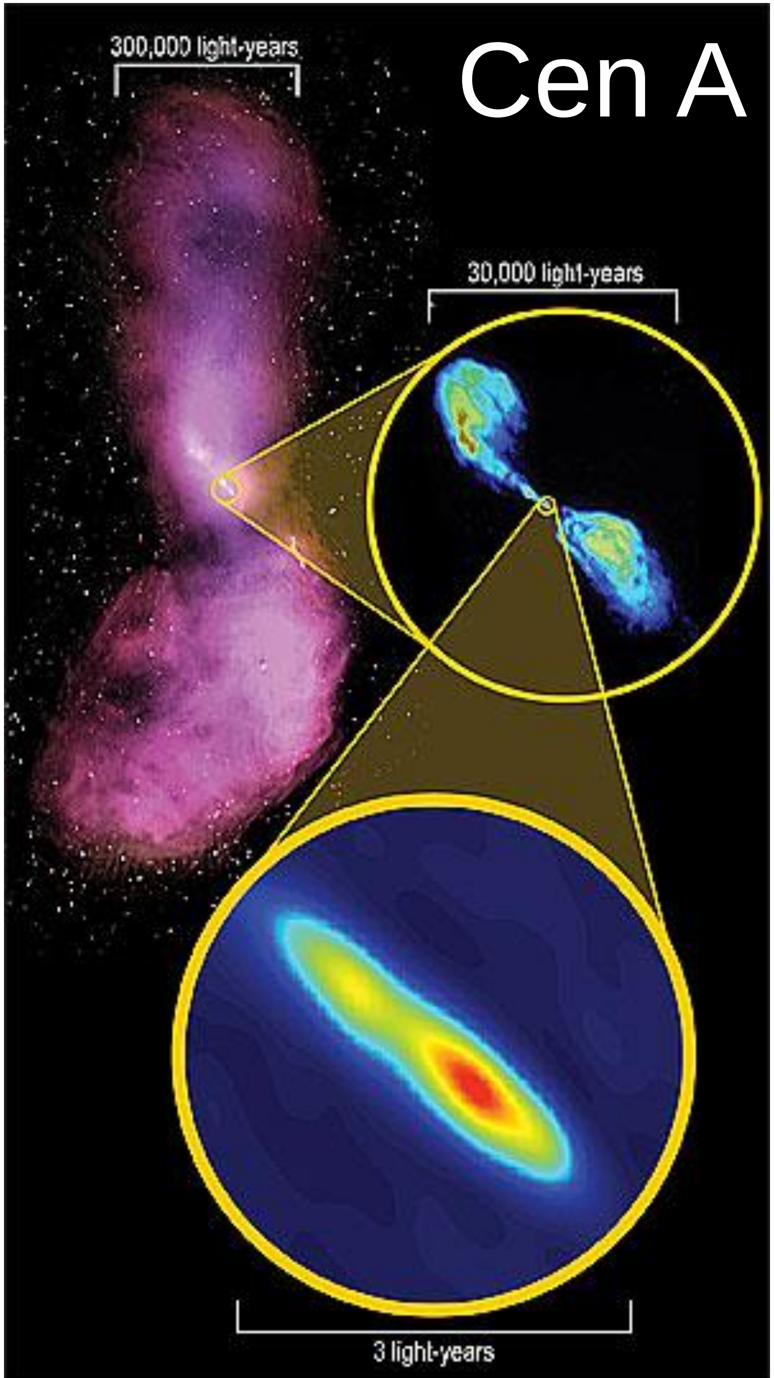
Ultra-steep spectrum radio halos





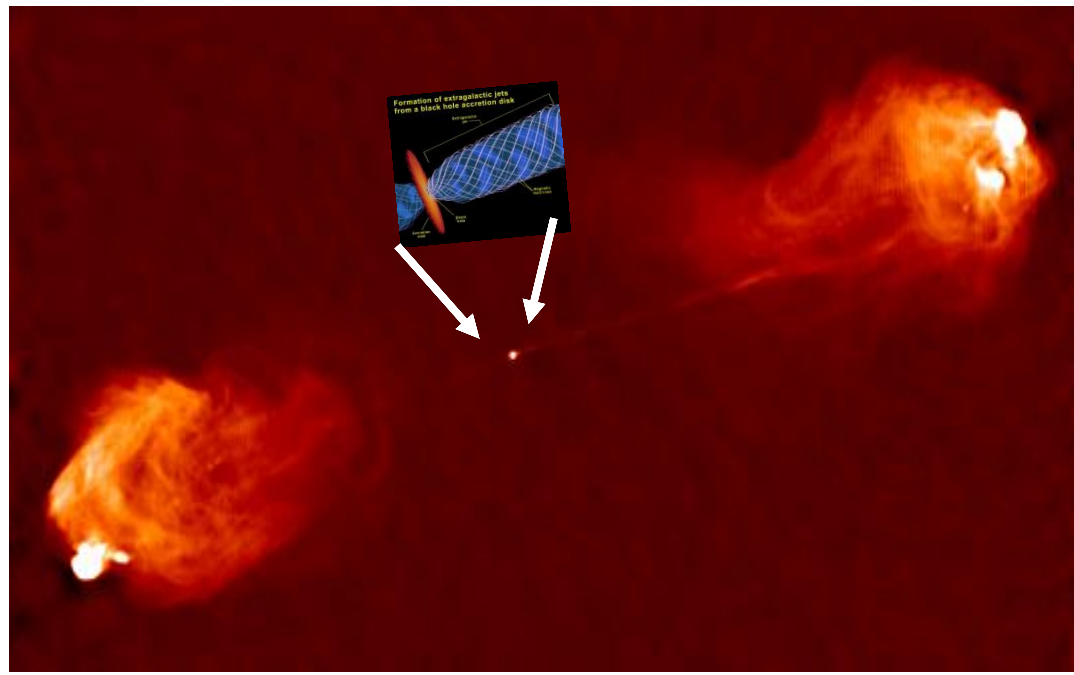
OUTLINE

- ❑ Turbulent acceleration: from Fermi to Gyroresonance and diffusion coefficients
- ❑ Complications and acceleration models using large-scale turbulence
- ❑ Turbulent acceleration in galaxy clusters: Radio Halos, models and observations at low radio frequencies
- ❑ Turbulent acceleration in head tail radio galaxies: evidences from observations at low radio frequencies
- ❑ APPENDIX: Fokker-Planck equation and coefficients



Radio Galaxies

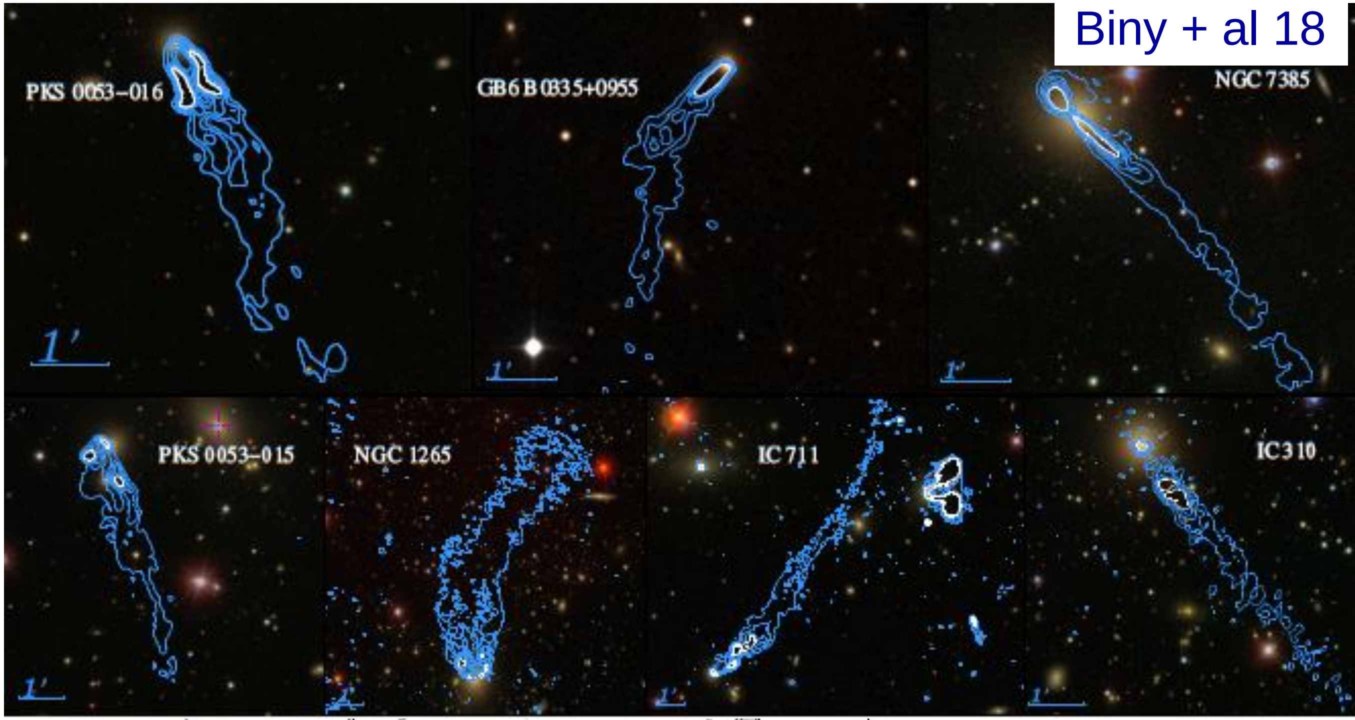
- Energy is extracted from a supermassive BH
- Relativistic jet : kinetic + Poynting flux
- Particle acceleration : shocks , reconnection
- Hot spots : shocks
- Backflow : reacceleration ? Mixing ?

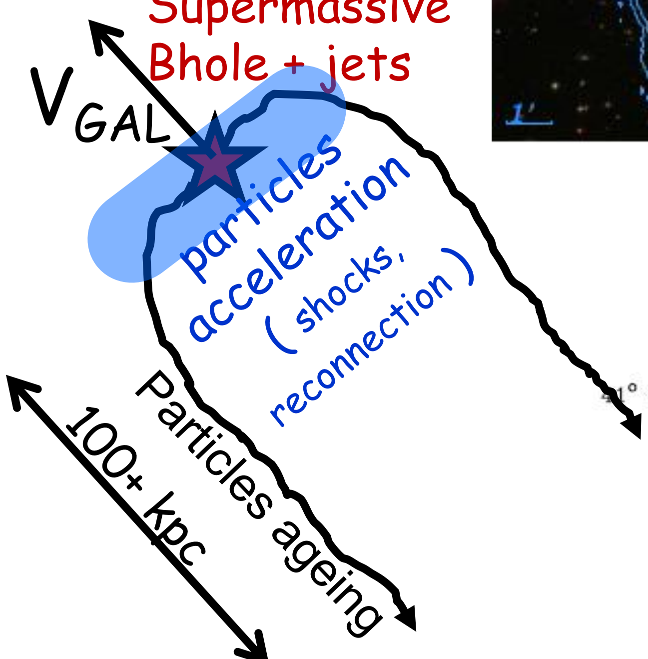


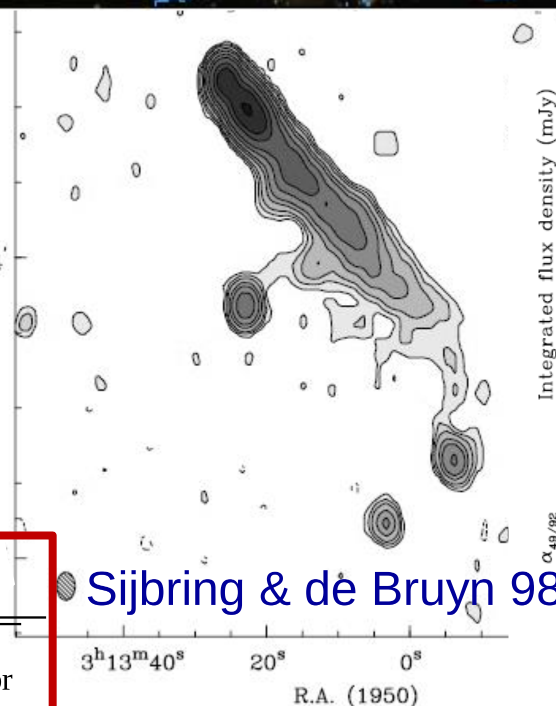
Head Tail Radio Galaxies

- In galaxy clusters
- Probes of jet/cocoon ICM interaction

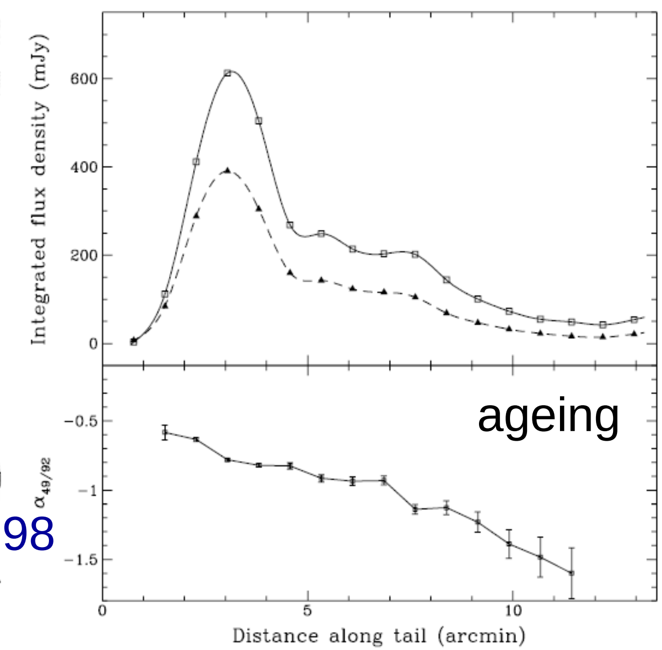
$$R \approx L_{GAL} \frac{\rho_j}{\rho_{ICM}} \left(\frac{V_j}{V_{GAL}}\right)^2$$







Sijbring & de Bruyn 98



$$L/V_{GAL} \approx \tau_{syn} \propto \frac{\sqrt{B}}{B^2 + B_{IC}^{\ 2}} \frac{1}{\sqrt{(1+z)\nu_{br}}}$$

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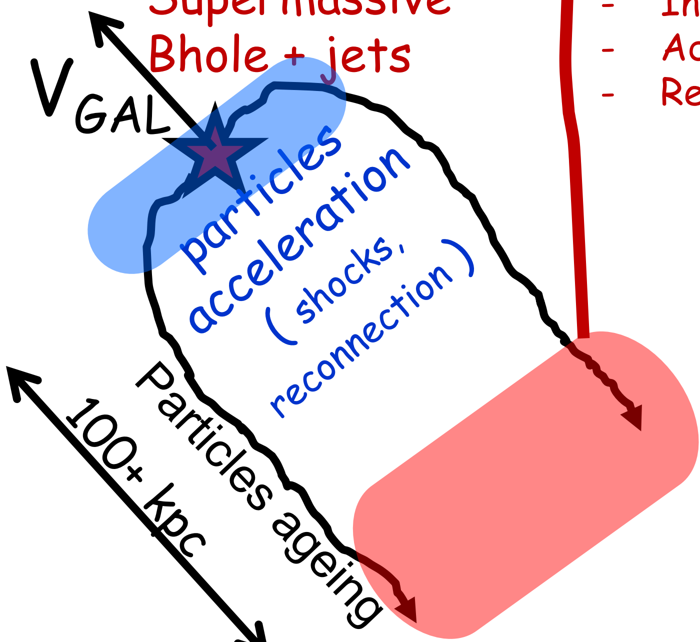
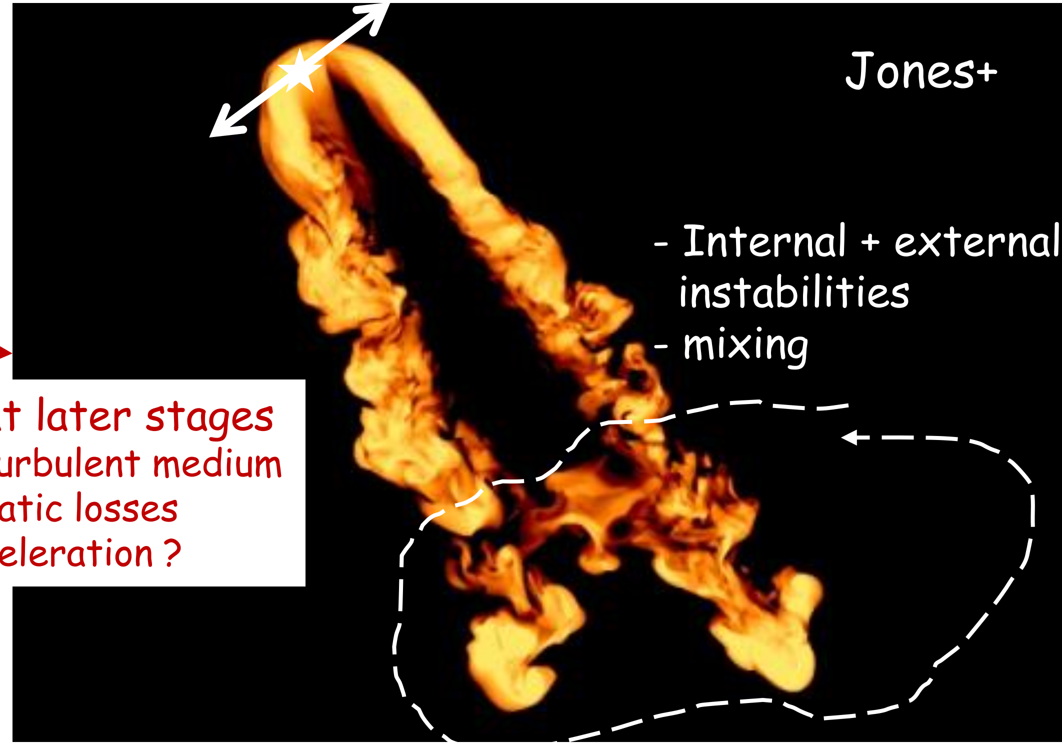
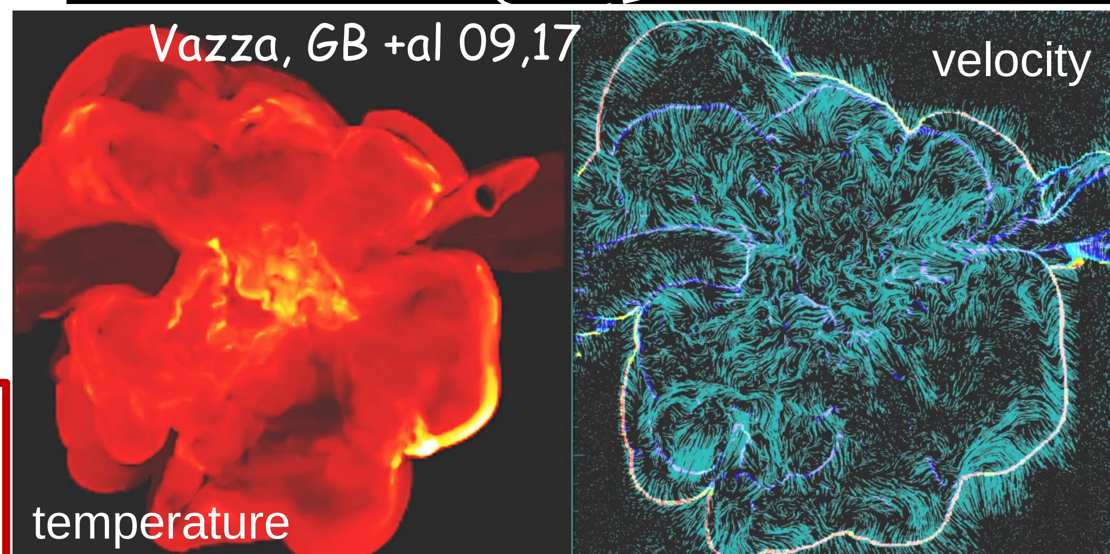
Supermassive Bhole + jets



$$L/V_{GAL} \approx \tau_{syn} \propto \frac{\sqrt{B}}{B^2 + B_{IC}^{\;2}} \frac{1}{\sqrt{(1+z)\nu_{br}}}$$

Mixing at later stages
- In a turbulent medium
- Adiabatic losses
- Reacceleration ?





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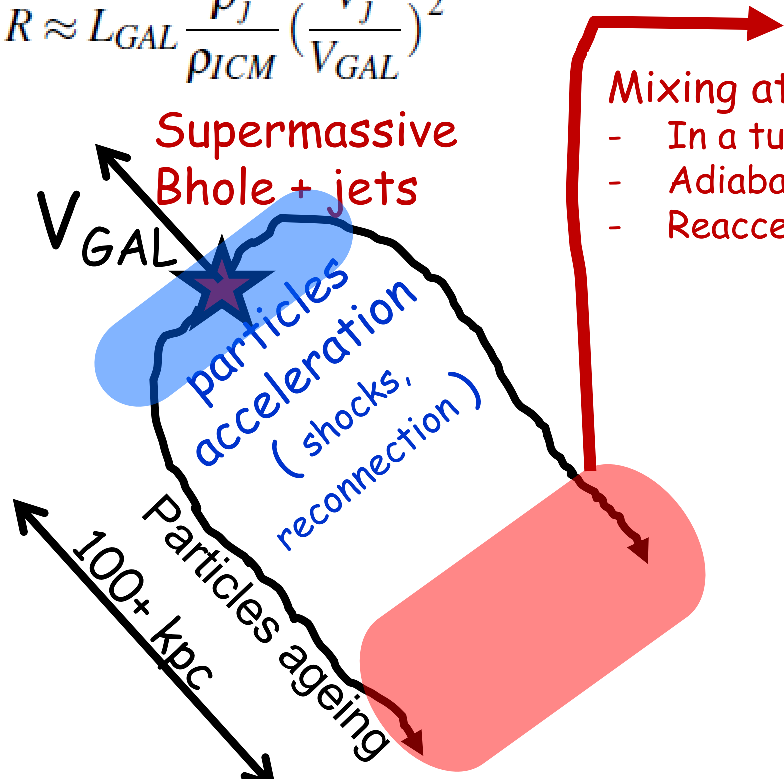

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$$L/V_{GAL} \approx \tau_{syn} \propto \frac{\sqrt{B}}{B^2 + B_{IC}^{\ 2}} \frac{1}{\sqrt{(1+z)\nu_{br}}}$$

How long does it take ??

$$(\delta x)^2 \approx \; x \delta V_x \Delta t \approx x \left(\frac{x}{L_0}\right)^{\frac{1}{3}} \delta V_0 \Delta t$$

$$\Delta t \approx \frac{l}{\delta V_0} \left(\frac{L_0}{l}\right)^{\frac{1}{3}} \approx \text{100 Myr}$$

Richardson diffusion

Head Tail Radio Galaxies

Where (frequency) we have to observe ??

- In galaxy clusters
- Probes of jet/cocoon ICM interaction

$$R \approx L_{GAL} \frac{\rho_j}{\rho_{ICM}} \left(\frac{V_j}{V_{GAL}}\right)^2$$

Mixing at later stages
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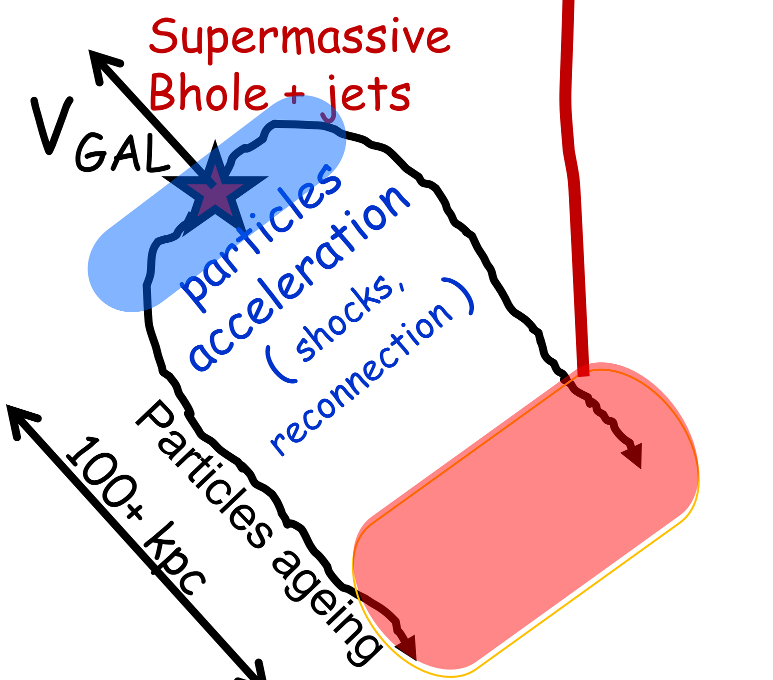

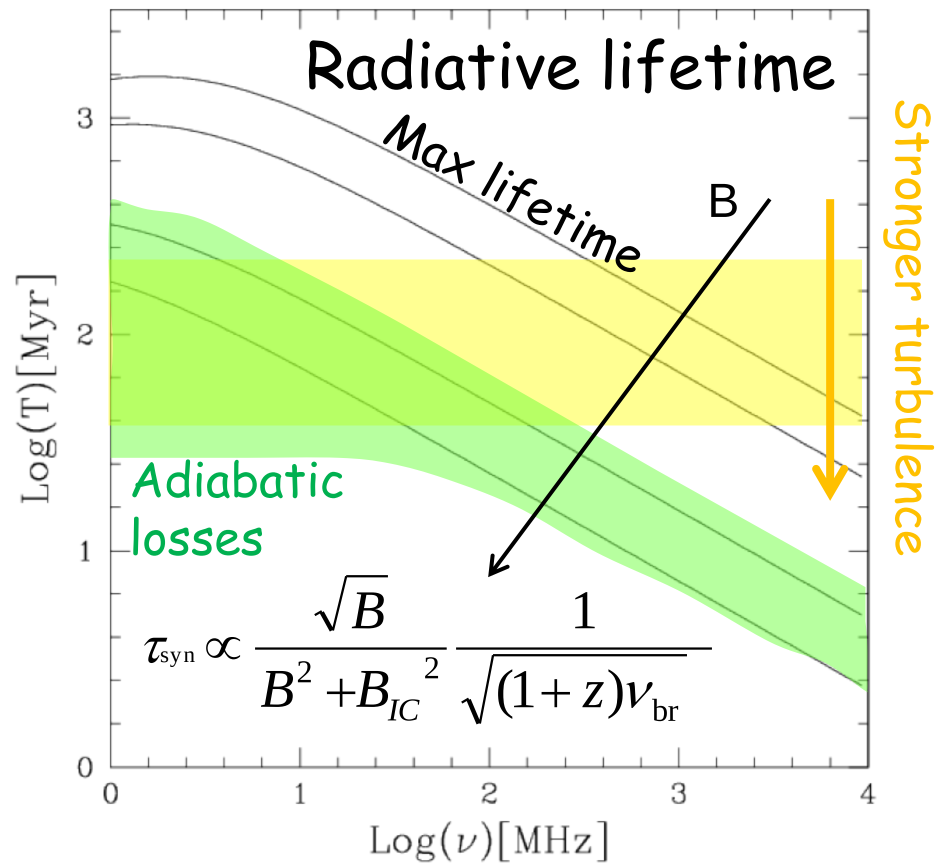

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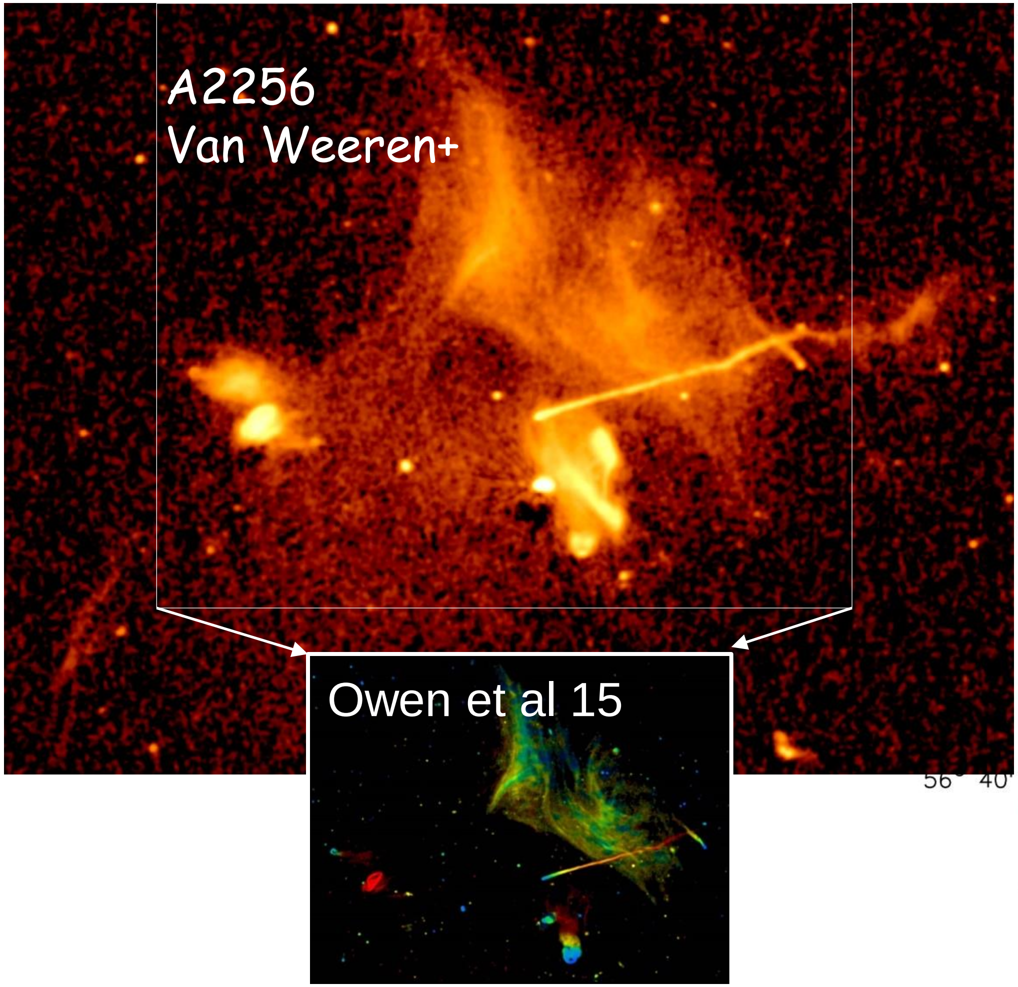
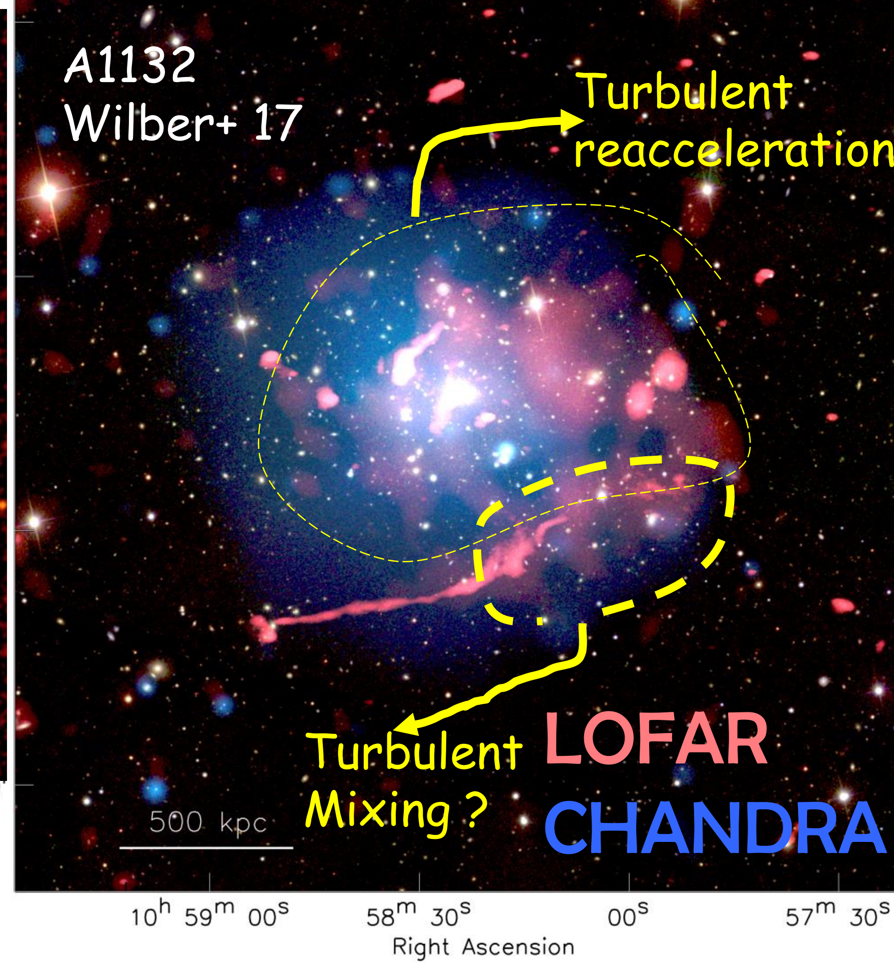




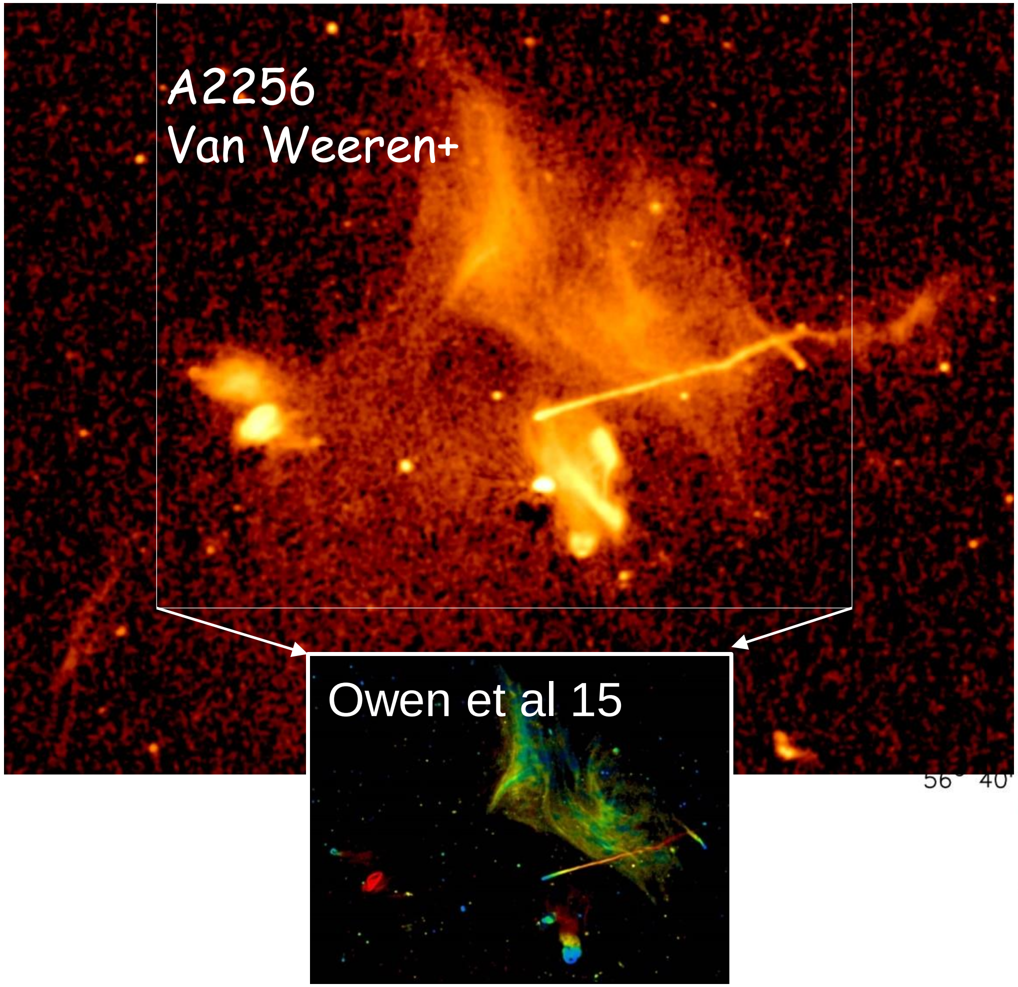
We start to see late stages and mixing !

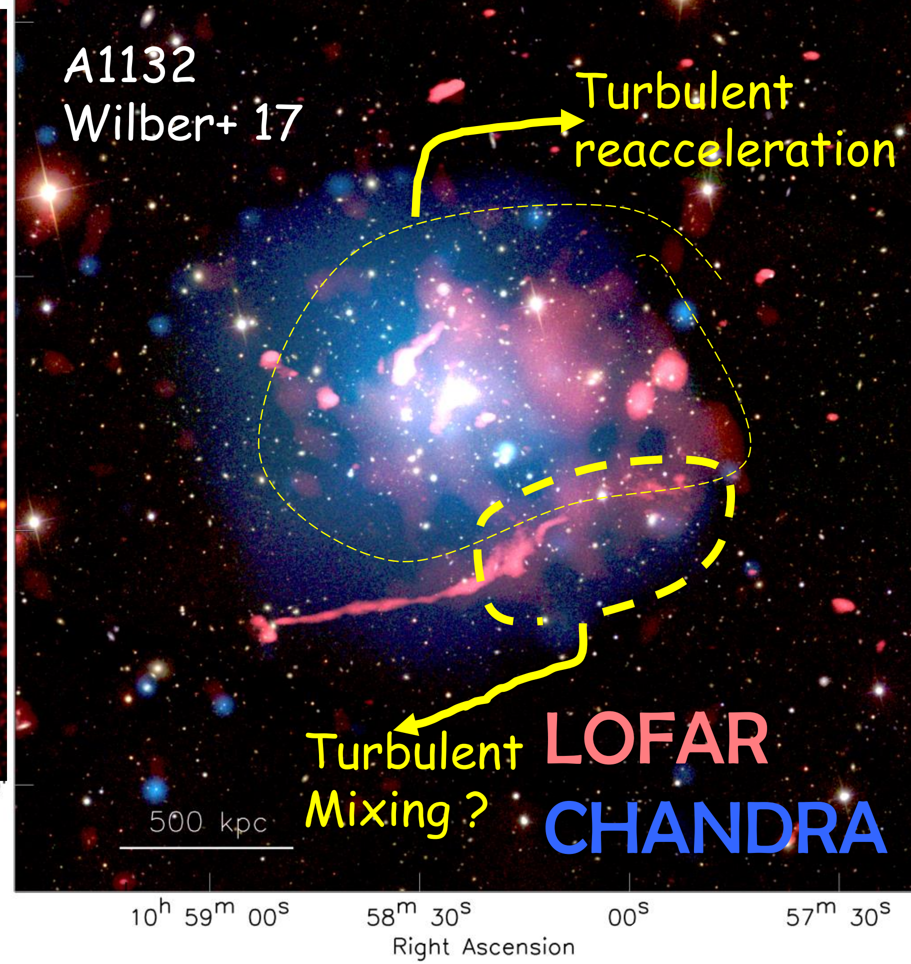
Surprises !

- Head tails are too long…

$$L/V_{GAL} > \tau_{syn} \propto \frac{\sqrt{B}}{B^2 + B_{IC}^{\,2}} \frac{1}{\sqrt{(1+z)\nu_{br}}}$$

- Electrons do not cool fast in the mixing phase





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$$L/V_{GAL} > \tau_{syn} \propto \frac{\sqrt{B}}{B^2 + B_{IC}^2} \frac{1}{\sqrt{(1+z)\nu_{br}}}$$

Are we looking at reacceleration ??

SCIENCE ADVANCES | RESEARCH ARTICLE

2017

PHYSICAL SCIENCES

Gentle reenergization of electrons in merging galaxy clusters

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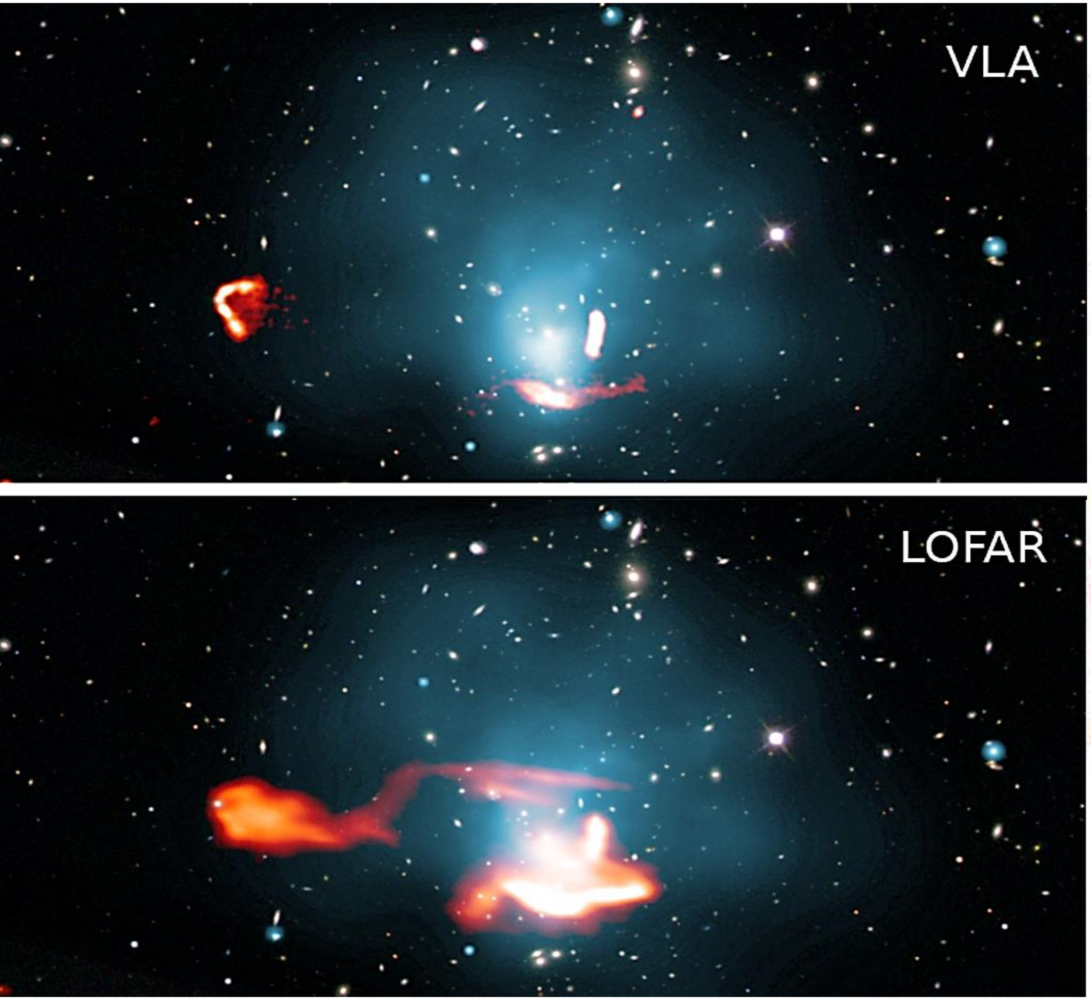
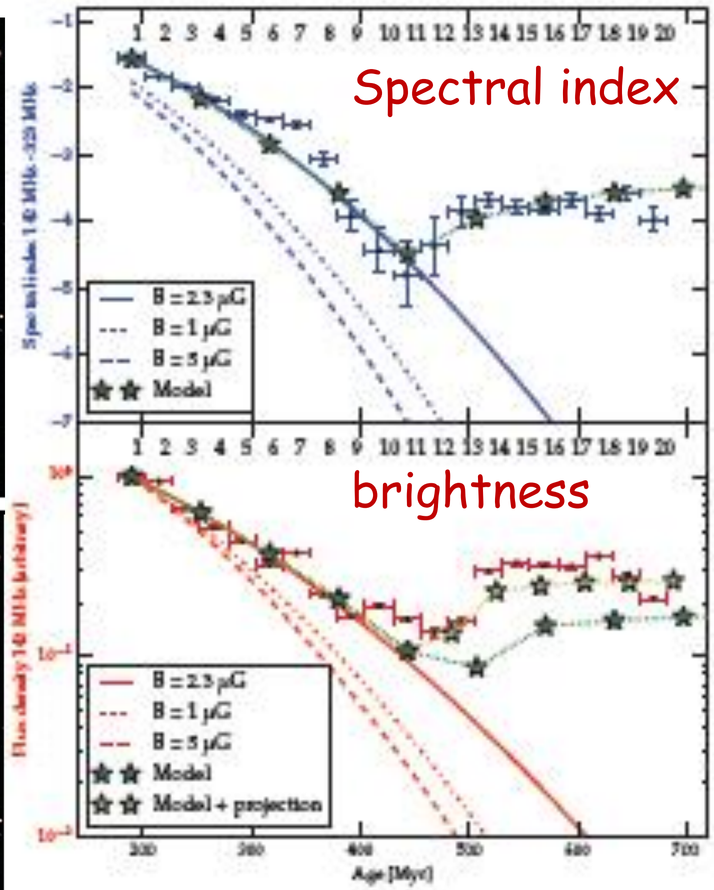
Marcus Brüggen,[2] Torsten A. Enßlin,[4] Reinout J. van Weeren,[1,5]
Annalisa Bonafede,[2,3] Huub J. A. Röttgering[1]

BIG SURPRISE





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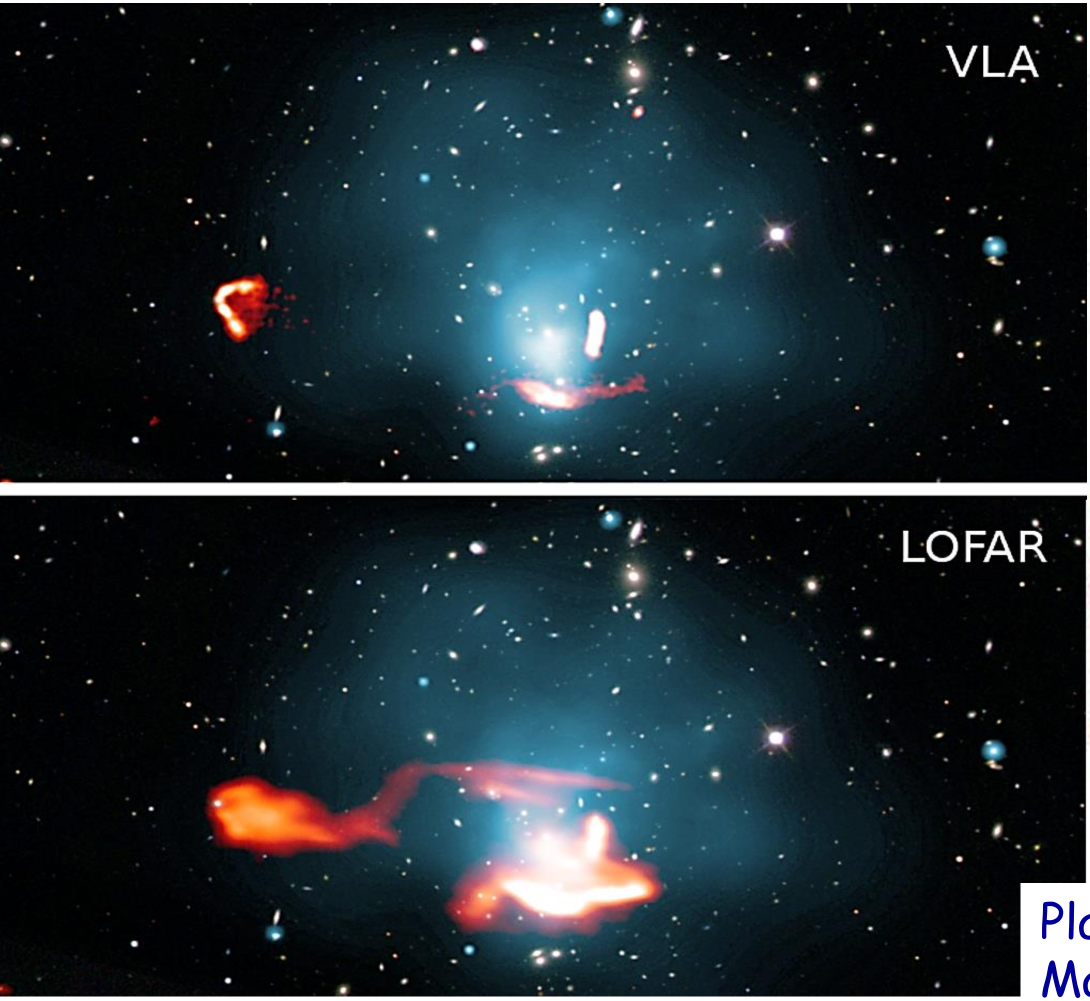
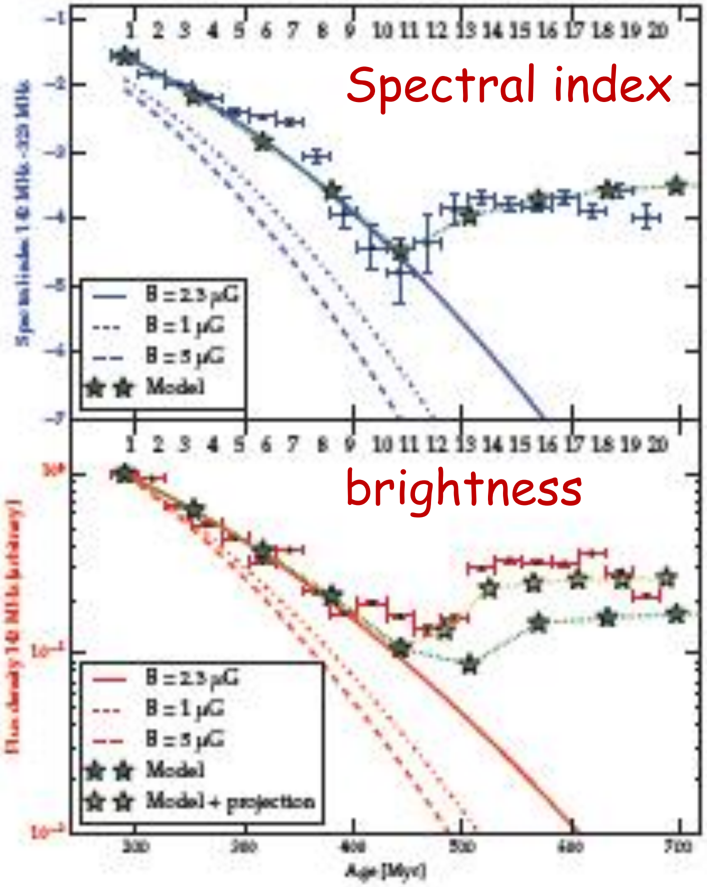
adding physics

$$\frac{\partial N_e(p,t)}{\partial t} = \frac{\partial}{\partial p}\left[N_e(p,t)\left(\left|\frac{dp}{dt}\right|_r - \frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{pp})\right)\right] + \frac{\partial^2}{\partial p^2}\left[D_{pp} N_e(p,t)\right]$$

$$\tau_{acc} = p\left(\langle\frac{dp}{dt}\rangle\right)^{-1} = p^3\left(\frac{\partial p^2 D_{pp}}{\partial p}\right)^{-1} \sim 500\text{-}700 \text{ Myr}$$





Plasma effects & microturbulence

Magnetic pumping ??

Revision of life-cycle of rel plasma !

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Evolution of particles interacting with Alfven waves

$$D_{\mu\mu} = \frac{q^2(1-\mu^2)\pi}{m^2\gamma^2c^2}\frac{1}{v\mu}4\pi\int dk\frac{B_k^2}{4\pi}\delta(k-\frac{\Omega}{v\mu})$$

$$D(p) = \frac{2\pi^2e^2v_A^2}{c^3}\int_{k_{\min}(p)}^{k_{\max}} W_A(k)\frac{1}{k}\left[1-\left(\frac{v_A}{c}+\frac{\Omega m_e}{pk}\right)^2\right]dk$$

Fokker-Planck Equation

$$\frac{\partial f}{\partial t}+\mu v\frac{\partial f}{\partial z}=\frac{\partial}{\partial\mu}\left[\left(D_{\mu\mu}\frac{\partial}{\partial\mu}+D_{\mu p}\frac{\partial}{\partial p}\right)f(p,\mu,t)\right]+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\times\left(D_{\mu p}\frac{\partial}{\partial\mu}+D_{pp}\frac{\partial}{\partial p}\right)f(p,\mu,t)\right]$$

Fokker-Planck equations

particle distribution in phase-space

$$f(\mathbf{x},\mathbf{p},t)\,dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

The aim is to derive an equation describing the 6D+1 evolution of f(..) subject to stochastic changes in **p**

Fokker-Planck equations

particle distribution in phase-space

$$f(\mathbf{x},\mathbf{p},t)\,dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

Probability density that particles change momentum by **Δp** in a time Δt.
Assume this is a Markov process…

$$\mathscr{P}(\mathbf{p},\Delta\mathbf{p}) \iff \int d\Delta\mathbf{p}\,\mathscr{P}(\mathbf{p},\Delta\mathbf{p}) = 1$$

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$$\Downarrow$$

$$f(\mathbf{p},\mathbf{x}+\mathbf{V}\Delta t,t+\Delta t) = \int d\Delta\mathbf{p}\,f(\mathbf{p}-\Delta\mathbf{p},\mathbf{x},t)\,\mathscr{P}(\mathbf{p}-\Delta\mathbf{p},\Delta\mathbf{p})$$

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make a Taylor expansion

$$f(\mathbf{p}-\Delta\mathbf{p},\mathbf{x},t) = f(\mathbf{p},\mathbf{x},t) - \frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\frac{\partial^2 f}{\partial \mathbf{p}^2} + \ldots$$

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$$f + \left[\frac{\partial f}{\partial t} + (\mathbf{V}\cdot\nabla)f\right]\Delta t =$$

$$= \int d\Delta\mathbf{p}\left(f - \frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\frac{\partial^2 f}{\partial \mathbf{p}^2} + \ldots\right)\left(\mathscr{P} - \frac{\partial \mathscr{P}}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\frac{\partial^2 \mathscr{P}}{\partial \mathbf{p}^2} + \ldots\right)$$

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The aim is to derive an equation describing the 6D+1 evolution of f(..) subject to stochastic changes in **p**

$$f + \left[\frac{\partial f}{\partial t} + (\mathbf{V}\cdot\nabla)f\right]\Delta t =$$

$$= \int d\Delta\mathbf{p}\left(f - \frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\frac{\partial^2 f}{\partial \mathbf{p}^2} + \ldots\right)\left(\mathscr{P} - \frac{\partial \mathscr{P}}{\partial \mathbf{p}}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\frac{\partial^2 \mathscr{P}}{\partial \mathbf{p}^2} + \ldots\right)$$

some tedious algebra…

$$\frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\,f\frac{\partial^2 \mathscr{P}}{\partial \mathbf{p}^2} + \frac{\partial \mathscr{P}}{\partial \mathbf{p}}\frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p}\Delta\mathbf{p} + \frac{1}{2}\Delta\mathbf{p}\Delta\mathbf{p}\,\mathscr{P}\frac{\partial^2 f}{\partial \mathbf{p}^2} = \frac{1}{2}\frac{\partial^2(\mathscr{P}f)}{\partial \mathbf{p}^2}$$

$$\mathscr{P}\frac{\partial f}{\partial \mathbf{p}}\Delta\mathbf{p} + f\frac{\partial \mathscr{P}}{\partial \mathbf{p}}\Delta\mathbf{p} = \frac{\partial(\mathscr{P}f)}{\partial \mathbf{p}}\Delta\mathbf{p}$$

……………………

Fokker-Planck equations

particle distribution in phase-space

$$f(\mathbf{x},\mathbf{p},t)\,dx\,dy\,dz\,dp_x\,dp_y\,dp_z$$

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definitions :

$$\langle\frac{\Delta\mathbf{p}\Delta\mathbf{p}}{\Delta t}\rangle = \frac{1}{\Delta t}\int d\Delta\mathbf{p}\,\mathscr{P}(\mathbf{p},\Delta\mathbf{p})\Delta\mathbf{p}\Delta\mathbf{p}$$

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Relation between FP-coefficients

$$\frac{\partial}{\partial\mathbf{p}}\langle\frac{\Delta\mathbf{p}}{\Delta t}\rangle = \frac{1}{2}\frac{\partial}{\partial\mathbf{p}}(\frac{\partial}{\partial\mathbf{p}}\langle\frac{\Delta\mathbf{p}\Delta\mathbf{p}}{\Delta t}\rangle)$$

Consequence of detailed balance :

$$\mathscr{P}(\mathbf{p},-\Delta\mathbf{p}) = \mathscr{P}(\mathbf{p}-\Delta\mathbf{p},\Delta\mathbf{p})$$

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$$\frac{\partial f}{\partial t} + (\mathbf{V}\cdot\nabla)f = \frac{\partial}{\partial \mathbf{p}}\left[D_{\mathbf{pp}}\frac{\partial f}{\partial \mathbf{p}}\right]$$

$$D_{pp} \equiv \lim_{t\to\infty}\frac{1}{2t}\langle \Delta p(t)\Delta p^*(t+\tau)\rangle = \Re\int_0^\infty d\tau \langle \dot{p}(t)\dot{p}^*(t+\tau)\rangle$$