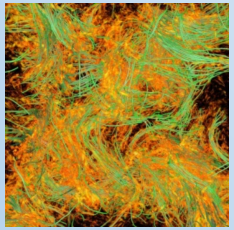
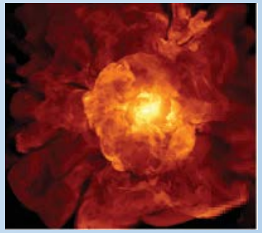
Shaken and Stirred: An Introduction to Shocks and Turbulence in Collisionless Astrophysical Plasmas

Tom Jones (U Minnesota)





Outline

- ✓ Context: Diffuse, "Collisionless", Magnetized Plasmas (but σ <<1) --
  Most of the baryonic matter in the universe

- ✓ Focus on two representative environments: ISM & ICM
  The warm & hot interstellar medium (**ISM**) (Milky Way, but also most galaxies)
  The hot intracluster medium of galaxy clusters (**ICM**)

- ✓ These media are generally highly disturbed
  ---Shocks & turbulence are characteristic (very large scale separations)---

- ✓ On "macro" scales continuum flow dynamics is the standard model paradigm
  HD & MHD

- ✓ Shocks generate turbulence; turbulence generates shocks

Some (Rough) Primitive ISM & ICM Conditions:

Measure:	**Interstellar (ISM): (warm –> hot)**	**Galaxy Clusters (ICM):**
kT (eV)	~1 - 100	~1,000 – 10,000
n_e (cm^{-3})	~ $10^{-3} – 10^{-1}$	~ $10^{-4} – 10^{-2}$
P_g ~ nkT (dyne/cm^2)	~ $10^{-14} – 10^{-13}$	$10^{-13} – 10^{-10}$
B (μG)*	~ 1 - 10	~ 0.1 – 10

*1 μG = 0.1 nT

Some (Rough) ISM & ICM Plasma Lengths

Length	Rough ISM values	Rough ICM values
Coulomb scattering length $\lambda_{coulomb} \sim 0.2$ pc T^2_{keV}/n_e	~ 10^8 km (~1 AU) – 1 pc*	~ 20 pc – 200 kpc
Proton "inertial length" $\lambda_p = c/\omega_{pp} \sim 200$km $n_i^{(-1/2)}$	~ 600 – 6000 km	~2000 – 20,000 km
Debye length $\lambda_D \sim 0.24$ km $(T_{keV}/n_e)^{1/2}$	~ 20 m – 2 km	~ 2 – 100 km
Thermal proton gyro radius $\rho_{gp} \sim 5.6\text{x}10^4$ km $T_{keV}^{(1/2)}/B_{\mu G}$	~ 10^3 – 10^4 km	~ 10^4 – $2\text{x}10^6$ km
Thermal electron gyro radius $\rho_{gp} \sim 1.3\text{x}10^3$ km $T_{keV}^{(1/2)}/B_{\mu G}$	~ 10^2 km	~200 – 50,000 km

← huge

* 1pc ≈ $3\text{x}10^{13}$ km

Some (Rough) ISM & ICM Plasma Velocities

Velocity	Rough ISM Values	Rough ICM Values
Acoustic (sound) velocity* $a = \sqrt{(5P/3\rho)} \sim 500\ T_{keV}^{(1/2)}$ km/s	~ 10 – 100 km/s	~ 500 – 1500 km/s
Alfven velocity $v_A = B/\sqrt{(4\pi\rho)} \sim 2\ B_{\mu G}/n_e^{(1/2)}$ km/s	~ 10 – 100 km/s	~ 20 – 100 km/s

$$^{*}\ a^2 = \gamma \frac{P}{\rho}$$

Some (Rough) ISM & ICM Plasma Frequencies

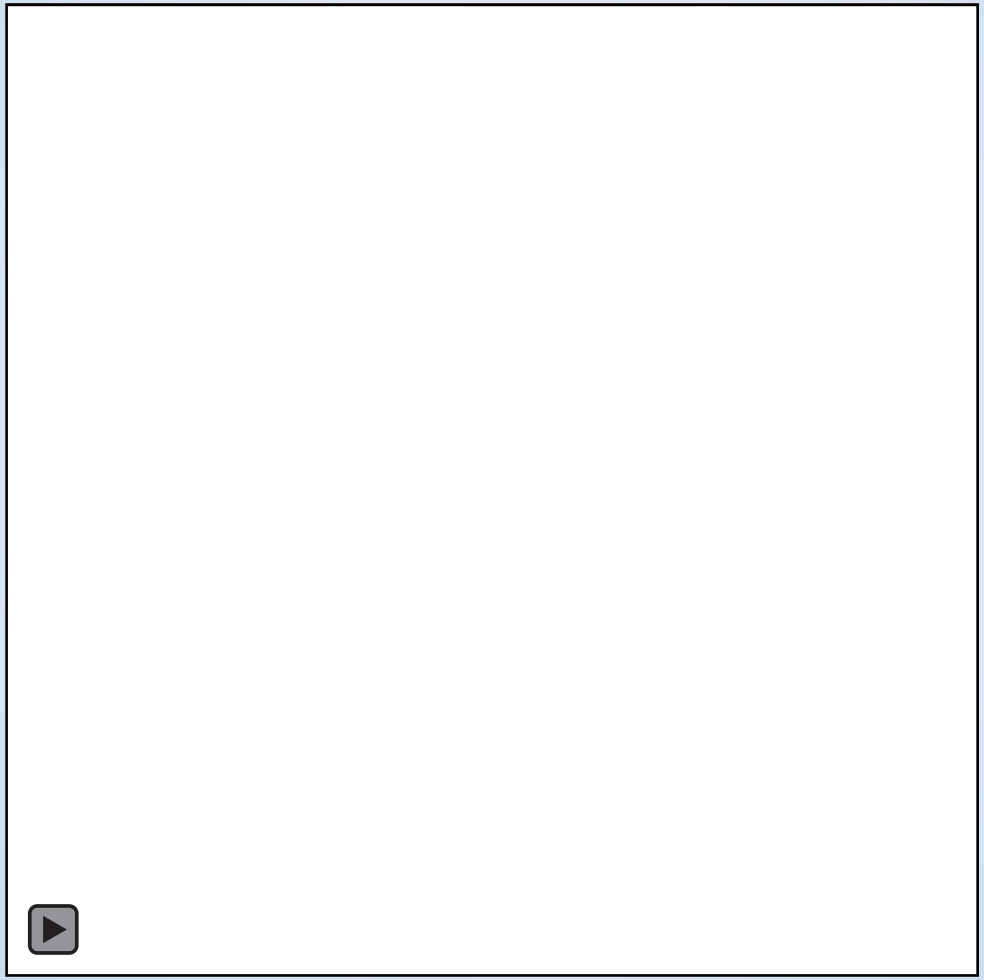
Frequency	Rough ISM Values	Rough ICM Values
Electron plasma frequency $\omega_{pe}/2\pi \approx 10^4\ n^{1/2}{}_e$ Hz	~ 300 – 3000 Hz	100 – 1000 Hz
Electron gryro frequency $\Omega_{ge} \approx 3\ B_{\mu G}$ Hz	~ 3 – 30 Hz	1 – 30 Hz
Electron collision frequency $\nu_{Ce} \approx 3x10^{-9}\ n_e/T^{3/2}{}_{keV}$ Hz	~ 10^{-5} – 10^{-10} Hz	10^{-14} – 10^{-10} Hz

← tiny

Some (Rough) ISM & ICM Dimensionless Plasma Quantities

Quantity	Rough ISM Values	Rough ICM Values	
Plasma "β" $\beta = P/(B^2/8\pi) \approx 8x10^4\ n_e T_{keV}/B_{\mu G}^2$	~ 1 - 10	~ 50 – 10^3	
Debye number $N_d = n_e\lambda_D^3 \sim 10^{13} T_{keV}^{(3/2)} / n_e^{(1/2)}$	~ 10^9 – 10^{13}	~10^{13} – 10^{15}	← huge
Collisionality $R = \nu_{Ce} / \omega_{pe} \sim 10^{-13} n_e^{1/2} / T^{3/2}_{keV}$	~ 10^{-13} - 10^{-9}	~ 10^{-13}	← tiny

Simulation of Evolution of SMC ISM including Supernovae



Hopkins 2015

Illustrating ISM/ICM Collisionless Shocks & Turbulent Flows

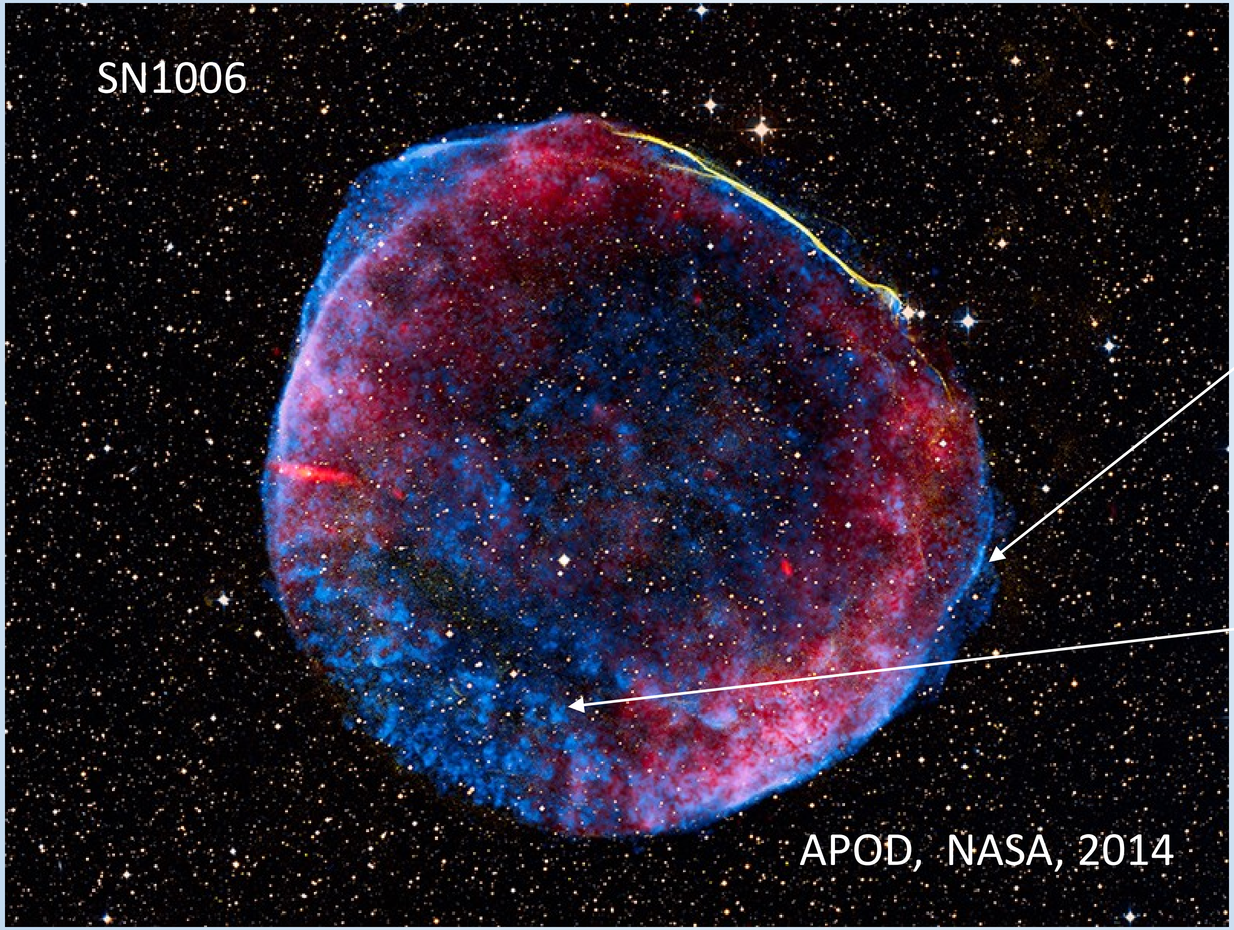
Representative Milky Way ISM Shocks:

Supernova Blast Waves – V_{shock} ~ 100 - 10^4 km/s

M_s ~ few - 1000

$$M_s = \frac{u_{x1}}{a_1}$$

X-ray (blue)
Radio (red)
Optical (yellow)



Shock (blast wave)

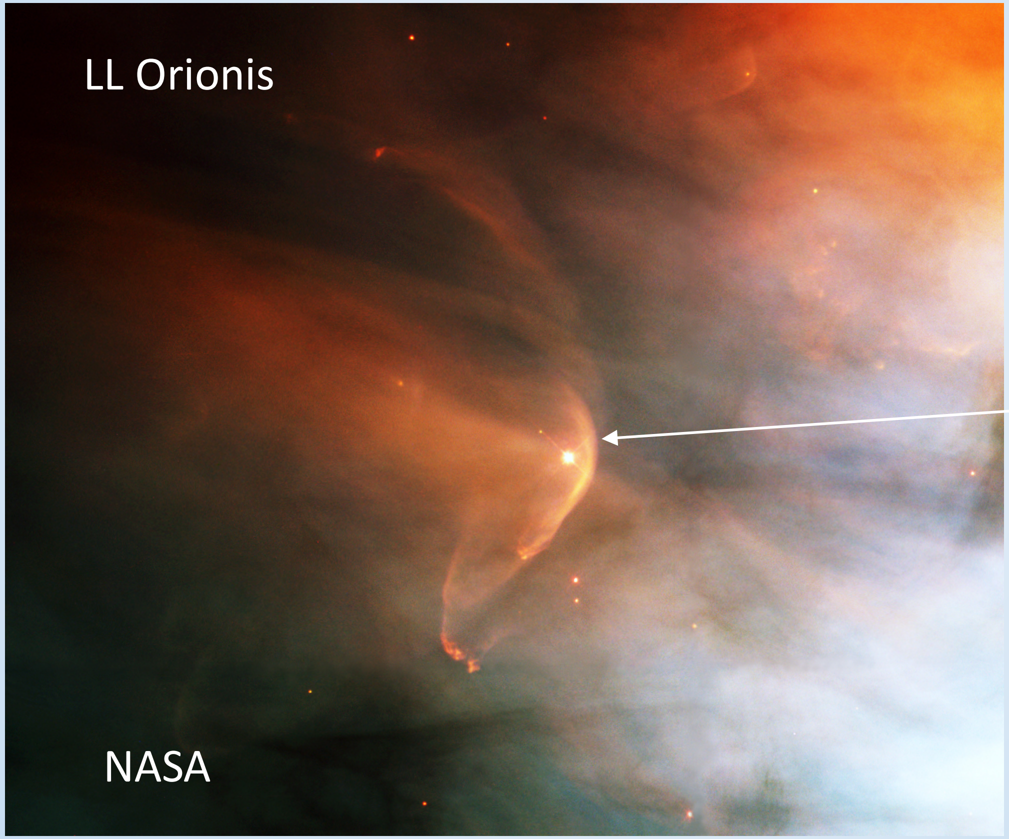
turbulence

Representative Milky Way ISM Shocks:

Stellar Wind Bow Shocks – V_{shock} ~ 100 – 1000 km/s

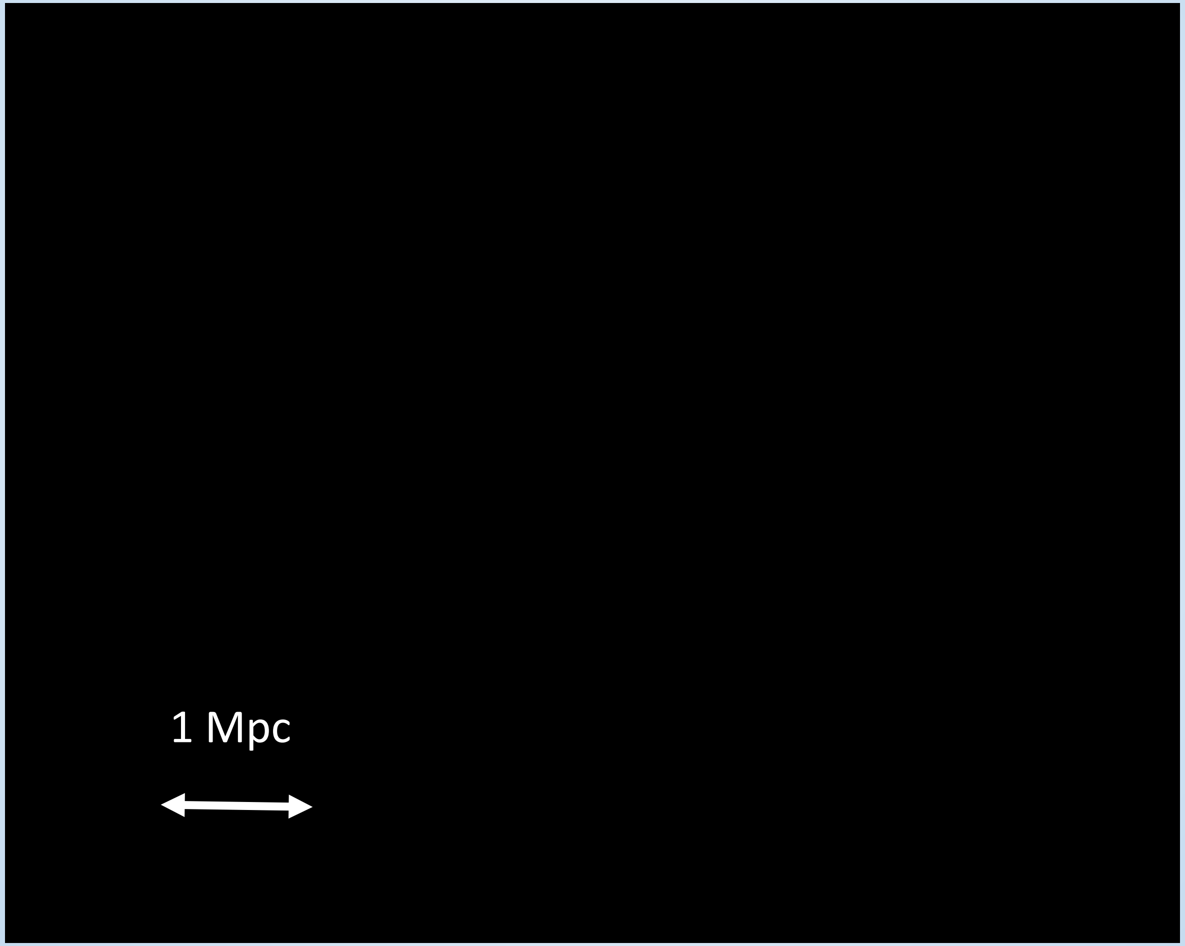
M_s ~ few ~ 10s

$$M_s = \frac{u_{x1}}{a_1}$$



Shock

Simulation of Galaxy Cluster Formation:
3D Rendering of Baryon (Plasma) Density

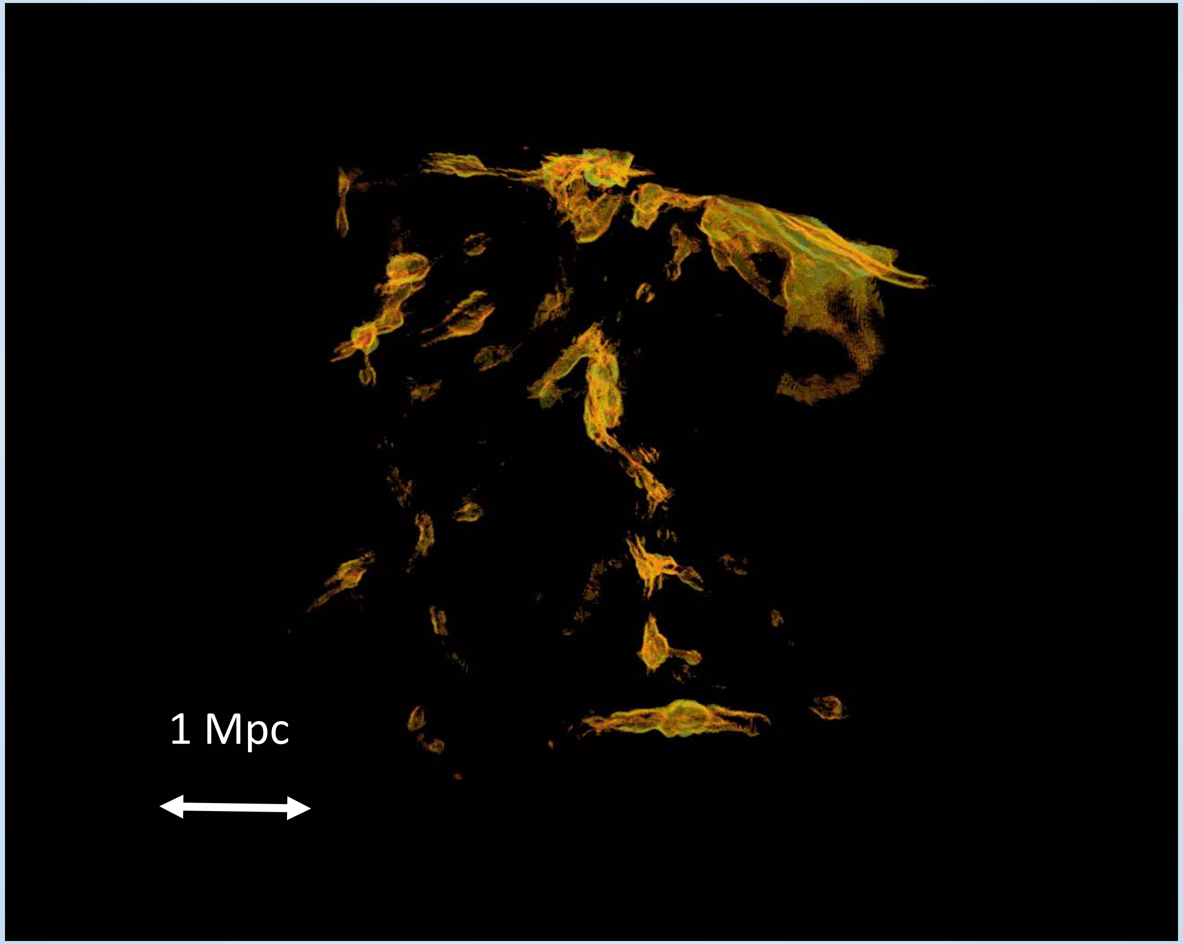


Comoving volume
from z = 10

Vazza + (TJ) 2017

Simulation of Galaxy Cluster Formation
3D Rendering of Shocks – $M_s < 4\text{-}5$
(except "external" shocks -- stronger)

$$M_s = \frac{u_{x1}}{a_1}$$

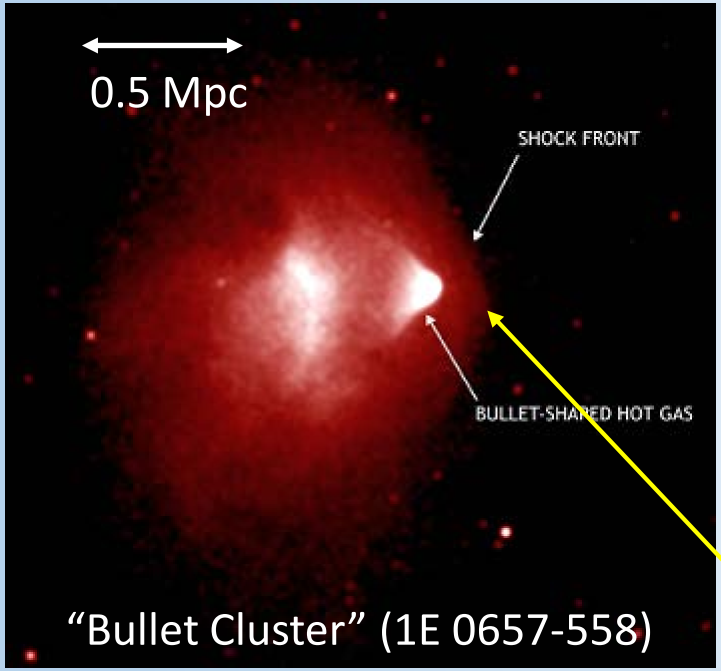


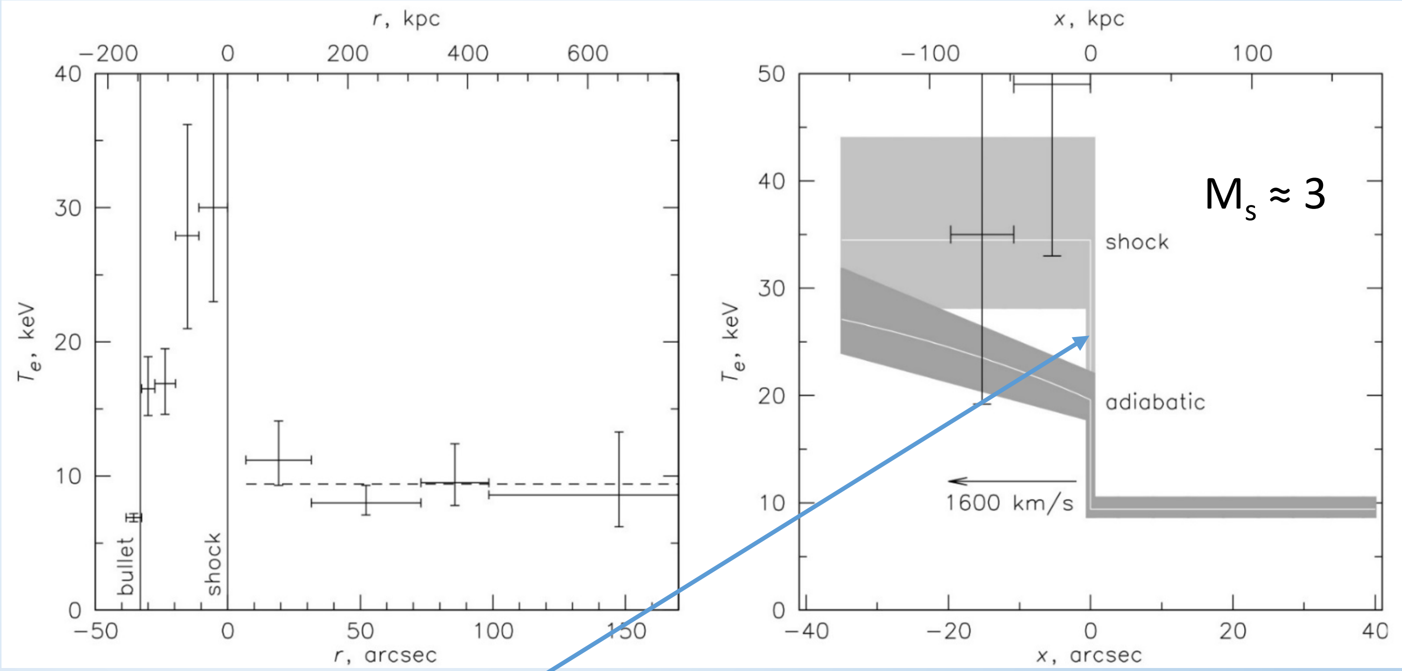
Shock surfaces $M_s > 1.5$
Color code (red => blue
M_s 1.5 => 20)

Vazza + (TJ) 2017

Identifying ICM Shocks:

Cluster Merger Shocks – V_{shock} ~ 1000 – 5000 km/s





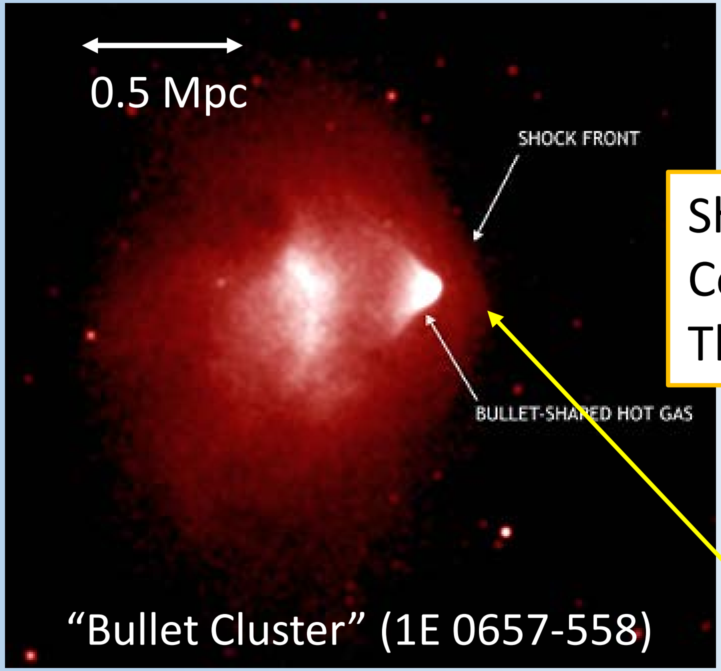
Shock

Markevitch 06

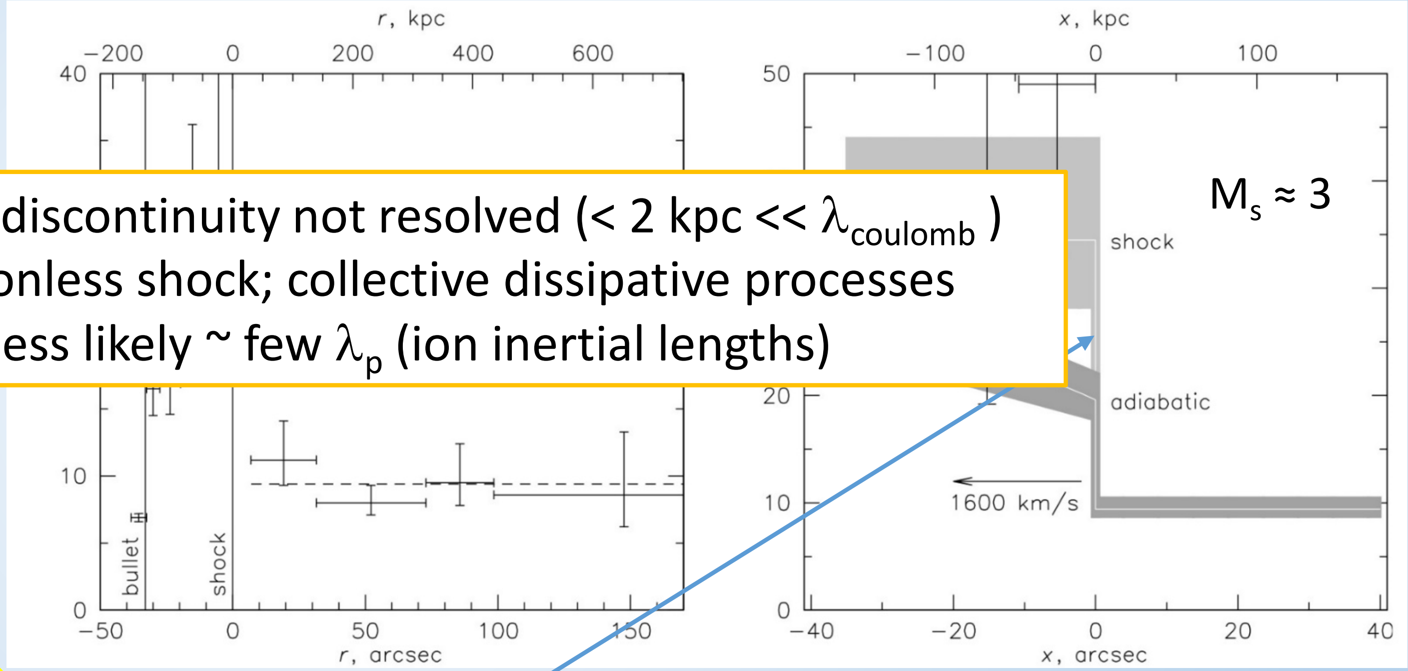
$$M_s = \frac{u_{x1}}{a_1}$$

Identifying ICM Shocks:

Cluster Merger Shocks – V_{shock} ~ 1000 – 5000 km/s



"Bullet Cluster" (1E 0657-558)



Shock discontinuity not resolved (< 2 kpc << $\lambda_{coulomb}$)
Collisionless shock; collective dissipative processes
Thickness likely ~ few λ_p (ion inertial lengths)

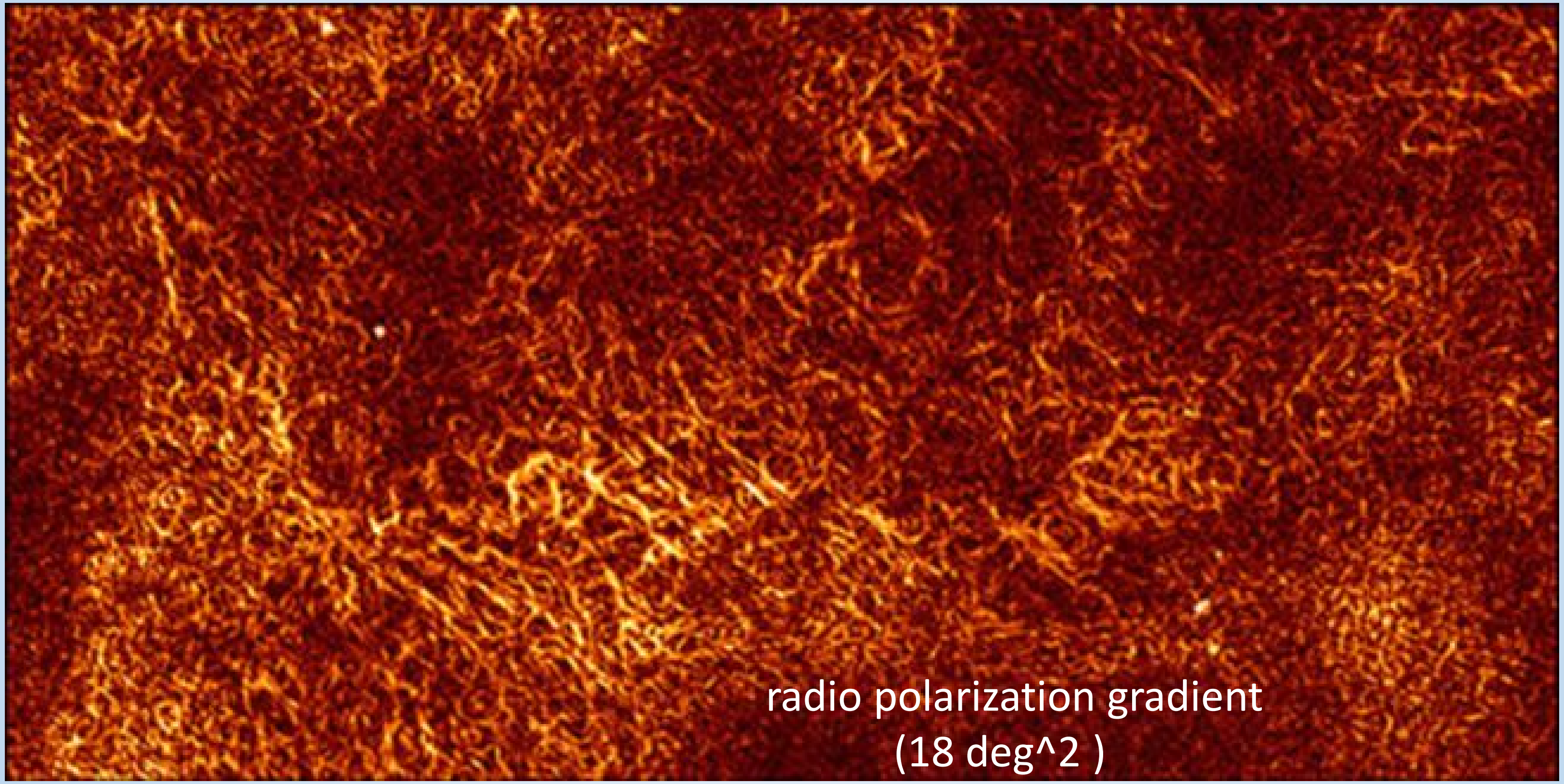
$M_s \approx 3$

Shock

Markevitch 06

$$M_s = \frac{u_{x1}}{a_1}$$

Visualizing Milky Way ISM Turbulence
(“Warm” ISM -- V_{turb} ~ 10 – 20 km/s)

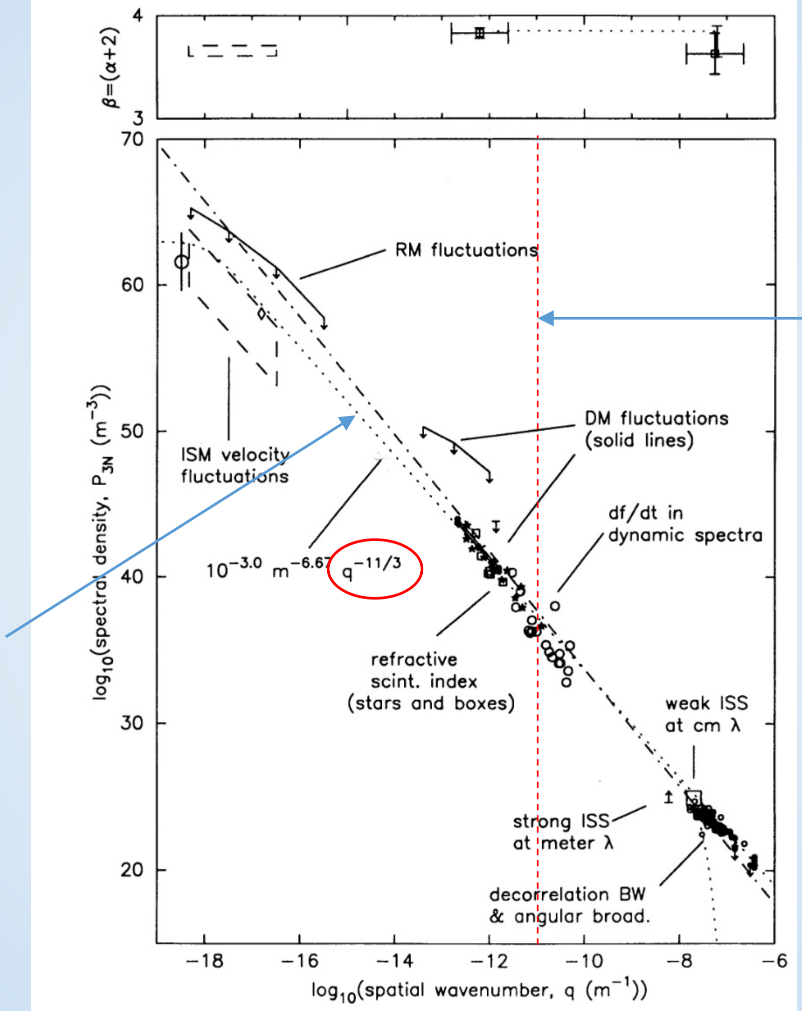


Gaensler + 2011

ISM Turbulence Density Fluctuation PDF
(multiple techniques involved)

$\delta\rho \propto \delta v$

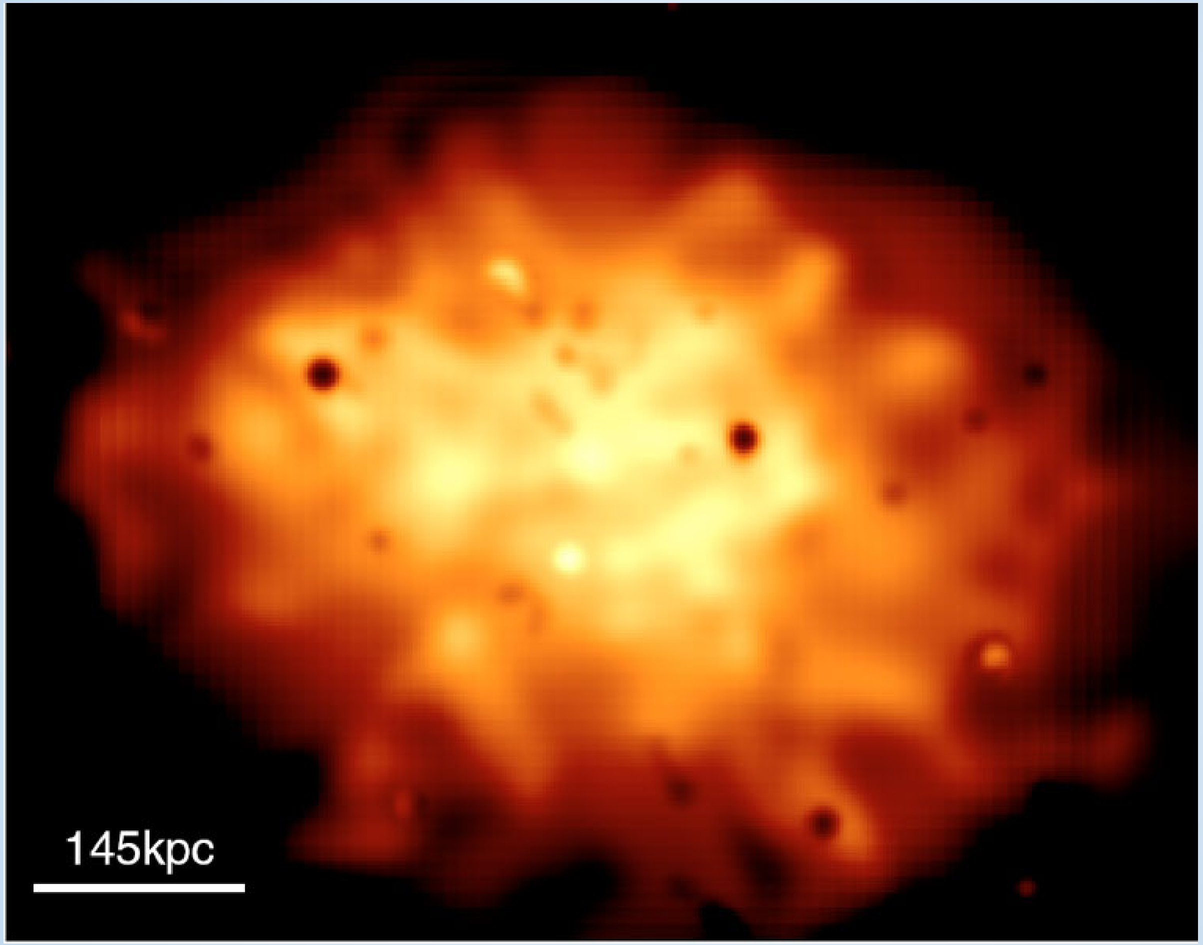
Kolmogorov slope
3D $P(k) \propto k^{-11/3}$
$k^2P(k) \propto k^{-5/3}$



1 AU (10^8 km)

Armstrong+ 95

Visualizing ICM Turbulence: Projected ICM Pressure Distribution –Coma Cluster (X-ray Observations)



Implied

$\delta\rho \propto l^{(1/3)}$

~Kolmogorov

Schuecker + 2005

Kinetic scale, collective, “pseudo” collisions => macro-scale fluid-like behaviors:

Including the magnetic, Lorentz (Maxwell) force (stress), the fluid force equation is

$$\dot{\vec{u}} = \frac{1}{\rho}(-\nabla P + \mu\nabla^2\vec{u} + \frac{1}{c}\vec{j} \times \vec{B})$$

$$\mu = \rho\nu$$

(Electrically neutral, ignore gravity…)

Simpler to start without the Maxwell stress term => Navier Stokes (“hydro”)
(all expressions below are equivalent)

$$\dot{\vec{u}} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\vec{u}$$

$$\frac{\partial\vec{u}}{\partial t} = -\frac{1}{\rho}\nabla P - \nabla(\frac{u^2}{2}) + \vec{u}\times\vec{\omega} + \nu\nabla^2\vec{u}$$

$$\frac{\partial\vec{\omega}}{\partial t} = \frac{1}{\rho^2}\nabla\rho\times\nabla P + \nabla\times(\vec{u}\times\vec{\omega}) + \nu\nabla^2\vec{\omega}$$

$$R = \frac{uL}{\nu} >> 1$$

using $\vec{\omega} = \nabla\times\vec{u}$

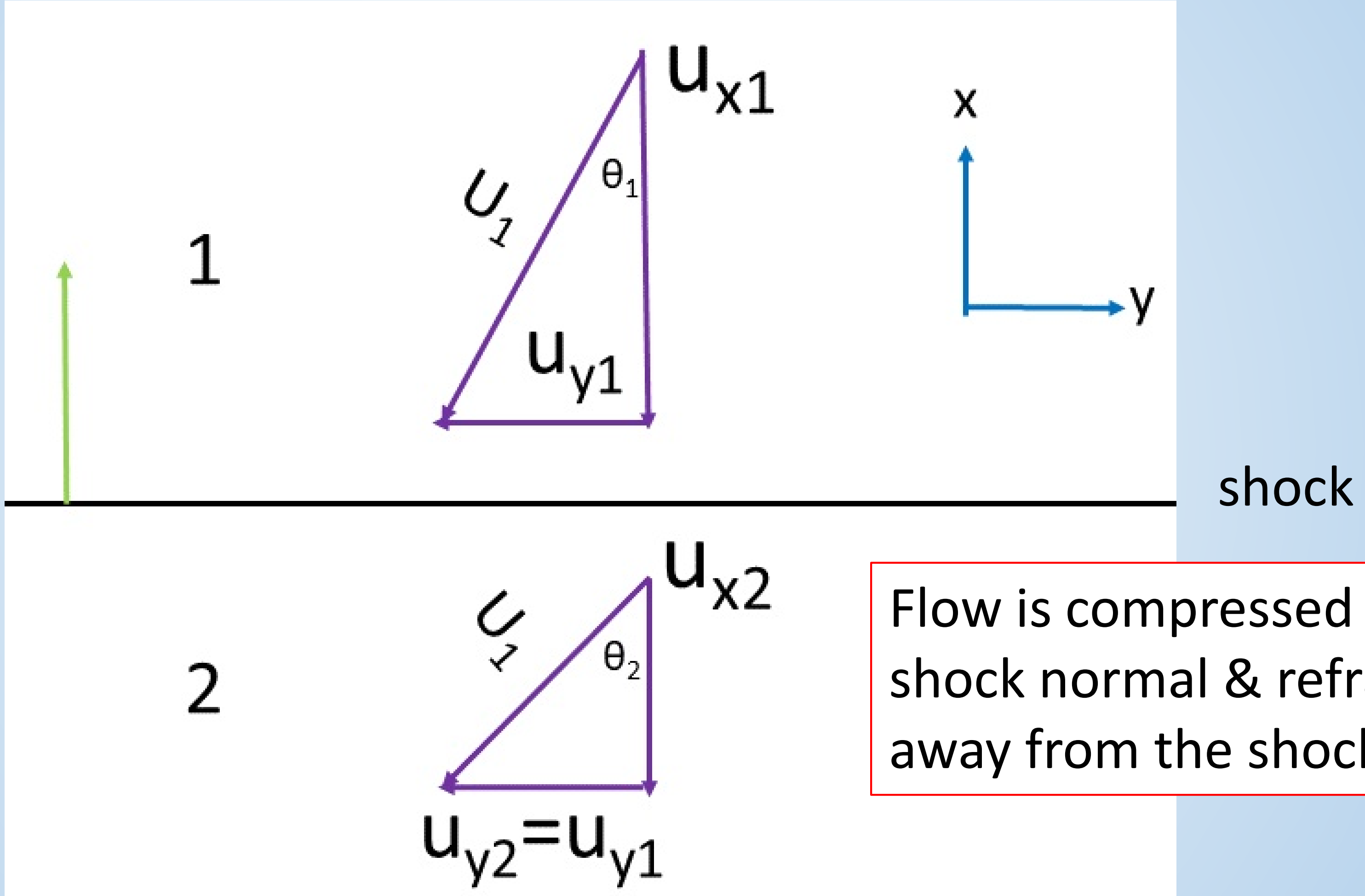
(Vorticity)

Astrophysical Shocks: A Brief Introduction

Begin with Plane, HD shocks: The simplest case

Plane, Steady* Hydro Shock:
(shock in y-z plane with $u_z = 0$)

$$M_s = \frac{u_{x1}}{a_1}$$



Flow is compressed along shock normal & refracted away from the shock normal

*On plasma scales collisionless shocks are not steady – we'll get to that

Uniform flows in regions 1 & 2 with **ω** = 0
Simple jump conditions set by conservation of mass, momentum & energy

$$[A] = A_1 - A_2$$

$$[\rho\ u_x] = 0$$ Mass conservation

$$\left[\rho\ u_x^2 + P\right] = 0$$ x-Momentum conservation

$$[u_y] = 0$$ y-Momentum conservation

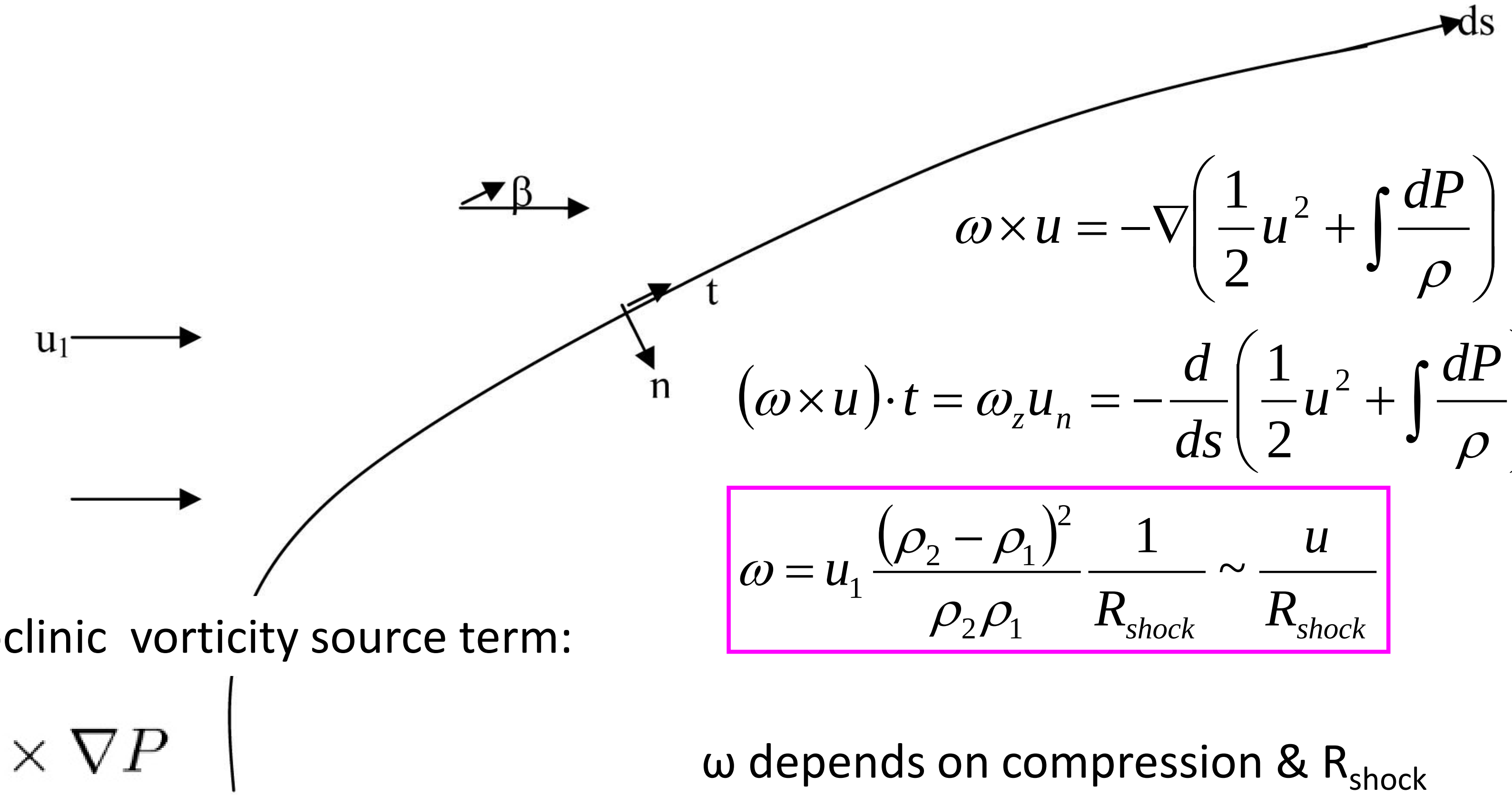
$$\left[\left(\frac{1}{2}\rho\ u^2 + \frac{\gamma\ P}{\gamma - 1}\right)u_x\right] = 0$$ Energy Conservation

$$\frac{\rho_2}{\rho_1} = \frac{u_{x1}}{u_{x2}} = \frac{4\ M_s^2}{M_s^2 + 3} \qquad \frac{P_2}{P_1} = \frac{5}{4}M_s^2 - \frac{1}{4}$$

Rankine-Hugoniot Relations
$\gamma = 5/3$

$$M_{s2}^2 = \left(\frac{u_{x2}}{a_2}\right)^2 = \frac{M_s^2 + 3}{5M_s^2 - 1} < 1$$ Downstream flow is subsonic

If the shock is curved, refraction of incoming uniform flow leads to downstream vorticity (Crocco's Theorem) --source of turbulence



$$\omega \times u = -\nabla\left(\frac{1}{2}u^2 + \int \frac{dP}{\rho}\right)$$

$$(\omega \times u)\cdot t = \omega_z u_n = -\frac{d}{ds}\left(\frac{1}{2}u^2 + \int \frac{dP}{\rho}\right)$$

$$\omega = u_1 \frac{(\rho_2 - \rho_1)^2}{\rho_2 \rho_1} \frac{1}{R_{shock}} \sim \frac{u}{R_{shock}}$$

Also baroclinic vorticity source term:

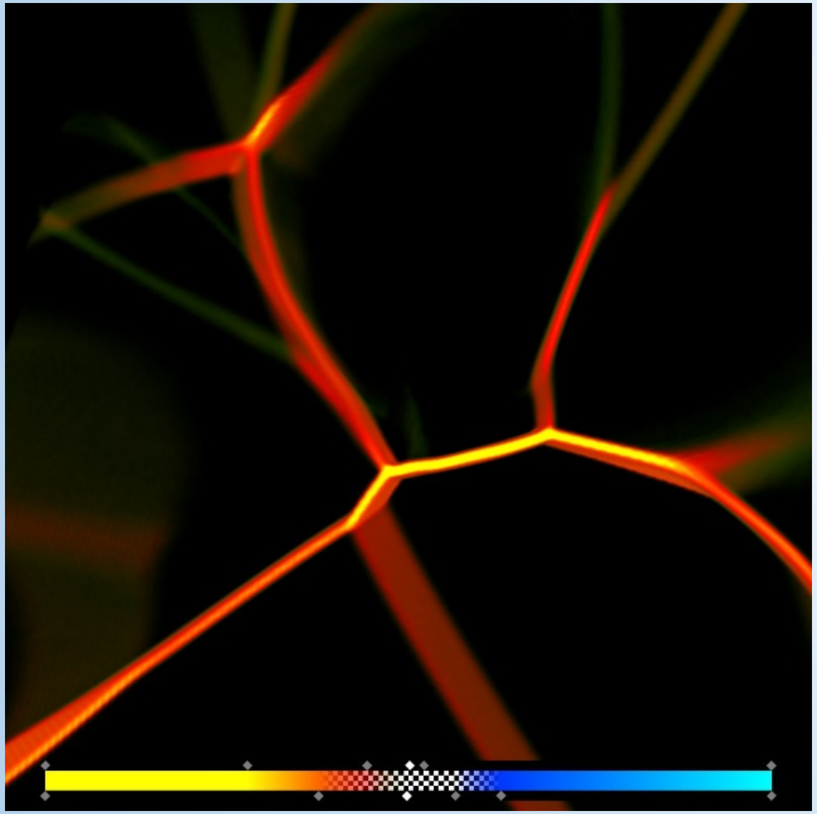
$$\frac{1}{\rho^2}\nabla\rho \times \nabla P$$

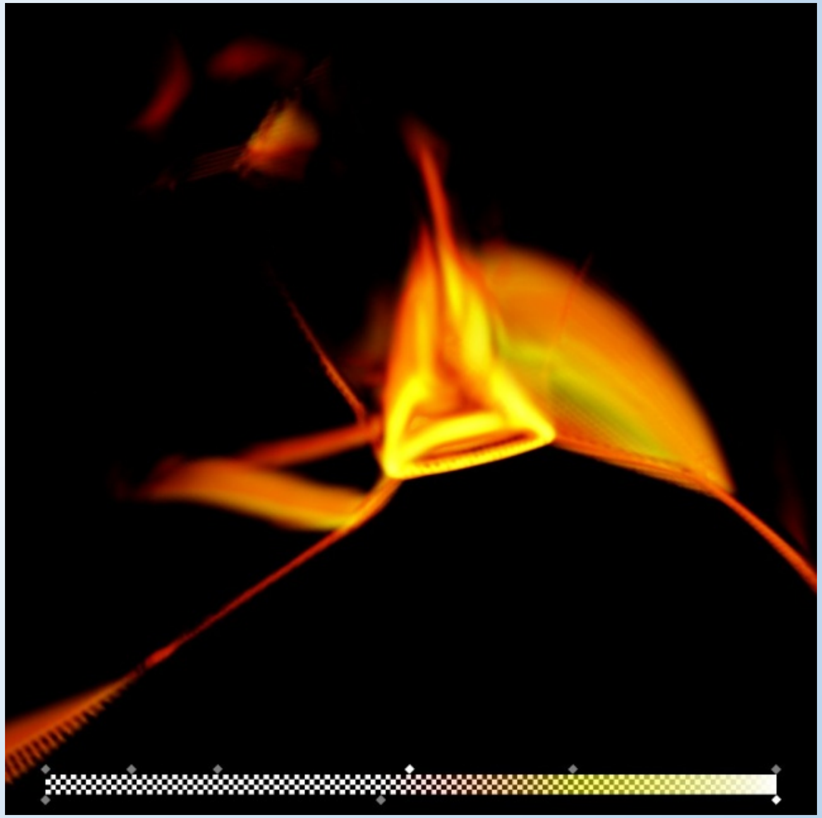
ω depends on compression & R_{shock}

Illustration of Vorticity Coming from Shock Intersections

3D CFD Simulation

Intersecting shocks => Vorticity





Porter, Jones & Ryu 2015

MHD

(Anisotropic) Maxwell stress along with & magnetic induction

$$\frac{1}{c}\vec{j}\times\vec{B} = -\nabla(P_B) + \frac{1}{4\pi}(\vec{B}\cdot\nabla)\vec{B}$$

$$\nabla\cdot\vec{B} = 0$$

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{u}\times\vec{B})$$

using $\vec{j} = \frac{c}{4\pi}\nabla\times\vec{B}$ (so) $-\frac{1}{c}\frac{\partial E}{\partial t}\to 0$ and $P_B = \frac{B^2}{8\pi}$

=> Three wave families: (fast, intermediate & slow)

mixed compress/shear

$$c_{f,s}^2 = \frac{a^2+v_A^2}{2} \pm \frac{1}{2}\sqrt{(a^2-v_A^2)^2+4a^2v_{Ay}^2}$$

"shear"

$$c_i^2 = v_{Ax}^2$$

Note: $\frac{\delta B_y}{B_y} = \frac{c\ \delta u_x}{c^2-v_{Ax}^2} = \frac{c^2}{c^2-v_{Ax}^2}\frac{\delta\rho}{\rho}$

If $\beta \sim (a/v_A)^2 \gg 1$

$$c_f^2 \approx a^2 + v_{Ay}^2$$

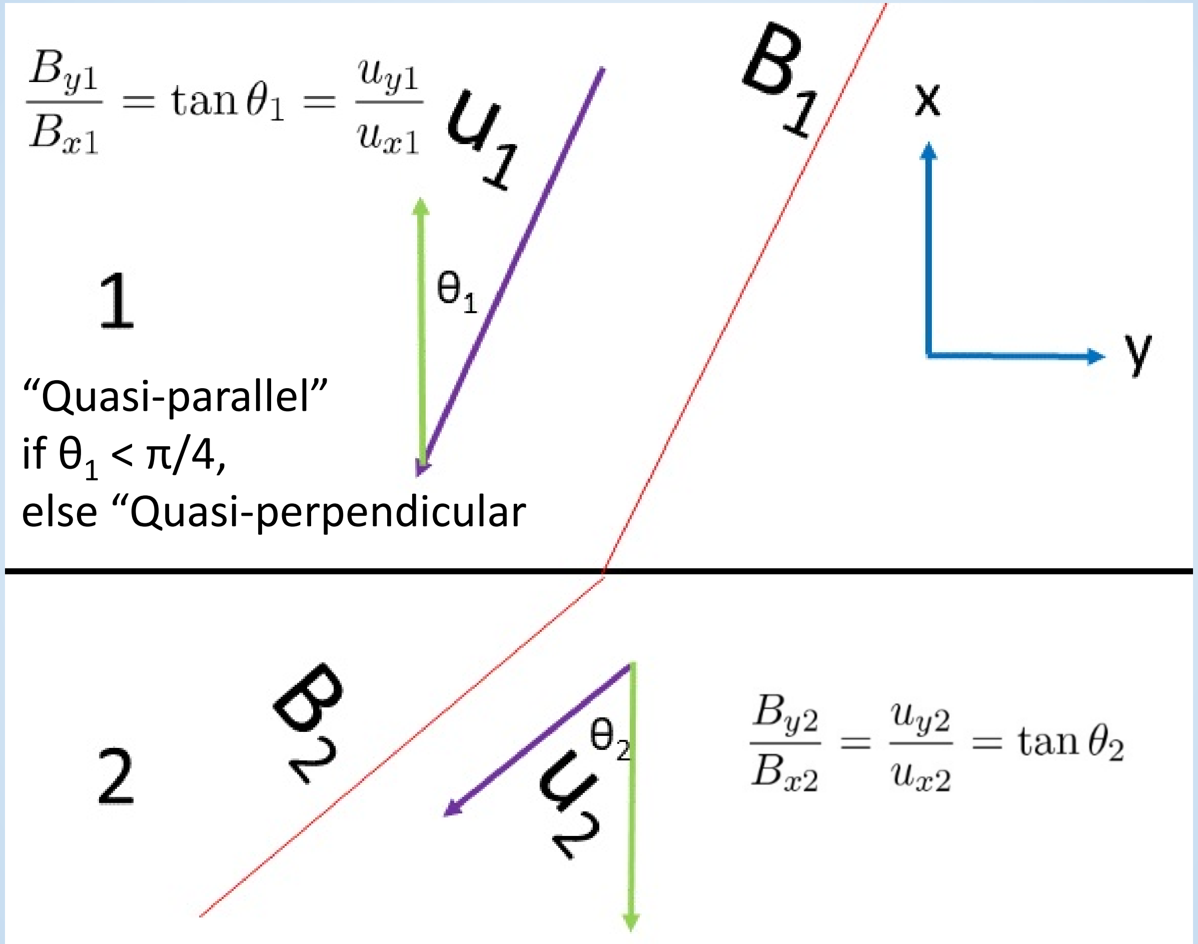
$$c_s^2 \approx v_{Ax}^2 - \frac{5}{6\beta}v_{Ay}^2$$

Sign of $\delta B_y/B_y$ opposite for f & s waves

Planar MHD Shock (Fast Mode)

(If $B_{y1} \neq 0$, boost along y into "de Hoffmann-Teller Frame": **u ∥ B so E = 0**)

$$\vec{E} = \frac{\vec{u}}{c} \times \vec{B}$$



shock

MHD Shock Jump Conditions

$$[\rho\ u_x] = 0$$ Mass conservation (unchanged)

$$\left[\rho\ u_x^2 + P + \frac{B_y^2}{8\pi}\right] = 0$$ x-Momentum conservation

$$\left[\rho\ u_x\ u_y - \frac{B_x B_y}{4\pi}\right] = 0$$ y-Momentum conservation

$$\left[(\frac{1}{2}\rho\ u^2 + \frac{\gamma\ P}{\gamma - 1} + \frac{B_y^2}{4\pi})u_x - \frac{B_x B_y}{4\pi}u_y\right] = 0$$ Energy conservation

Modified by Maxwell stresses

$$[B_x] = 0$$ Magnetic flux conservation

$$[B_y\ u_x - B_x\ u_y] = 0$$ Magnetic induction

Our Astrophysical Shocks of Interest are Collisionless:

Short Version of the Story:

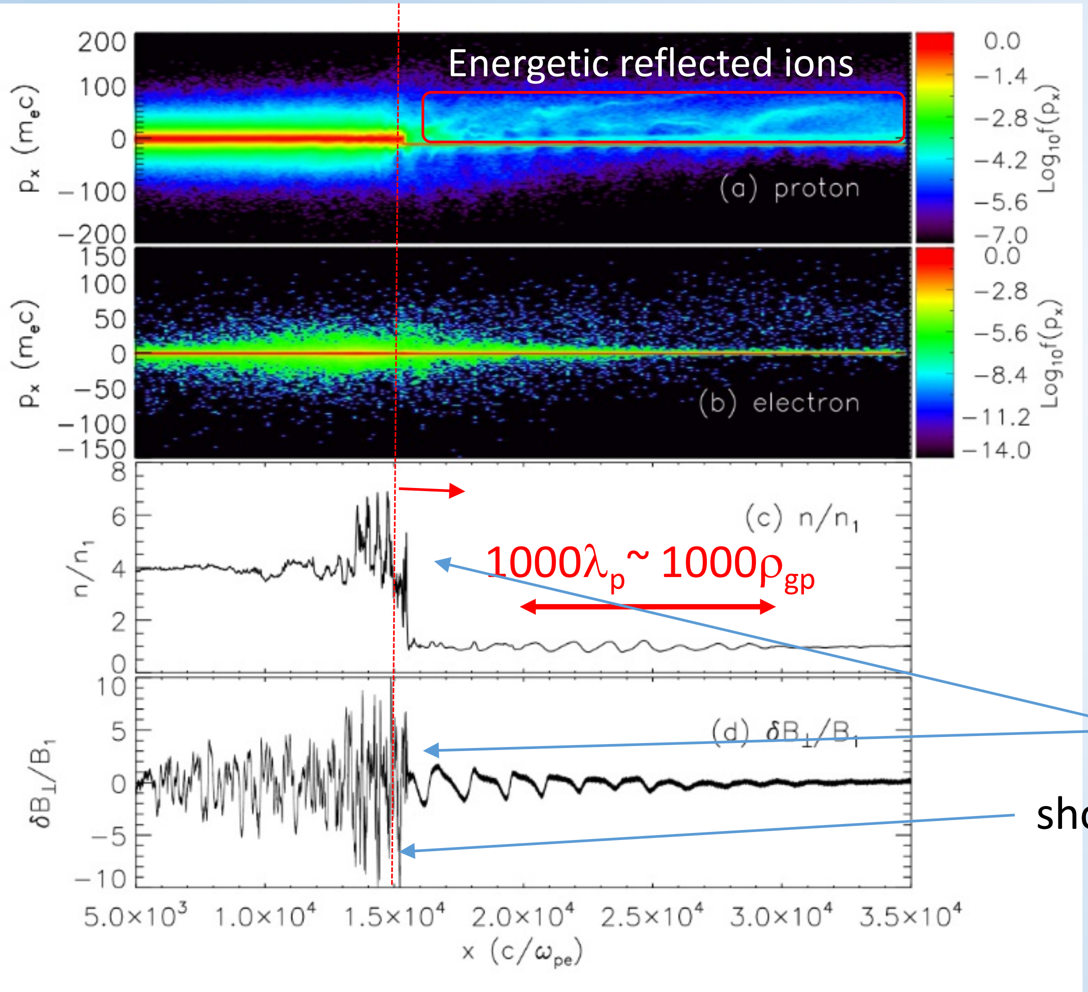
Plasma instabilities (e.g., firehose, mirror & current)
on scales ~ proton gyro and proton inertial lengths
generate waves, reflect & scatter ions (some resonantly
& some non-resonantly) in the “shock foot” ,
dissipate kinetic energy and accelerate (energize) ions.
Note: electrons & ions depend on different instabilities-
coupling is not simple – more to come

Details depend on strength & obliquity of B_1 M_A & M_s (β)

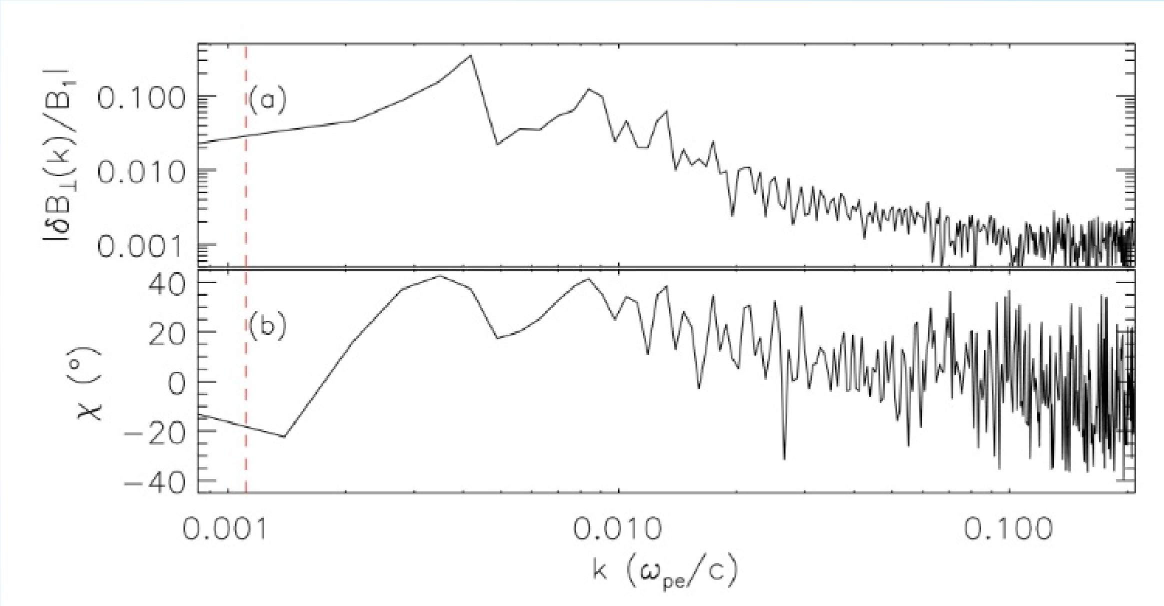
Often modeled through “Particle in Cell” (PIC) & “hybrid” (particle ions, electron fluid)

Illustrative 1D PIC Simulation:
Quasi-parallel shock, β≈1/√2
M_s = 40, M_A = 20, θ_1 = 30°
u_s = 0.1c m_p/m_e = 100

Particle & field distributions



Magnetic fluctuation spectrum (foot)



Note overshoots

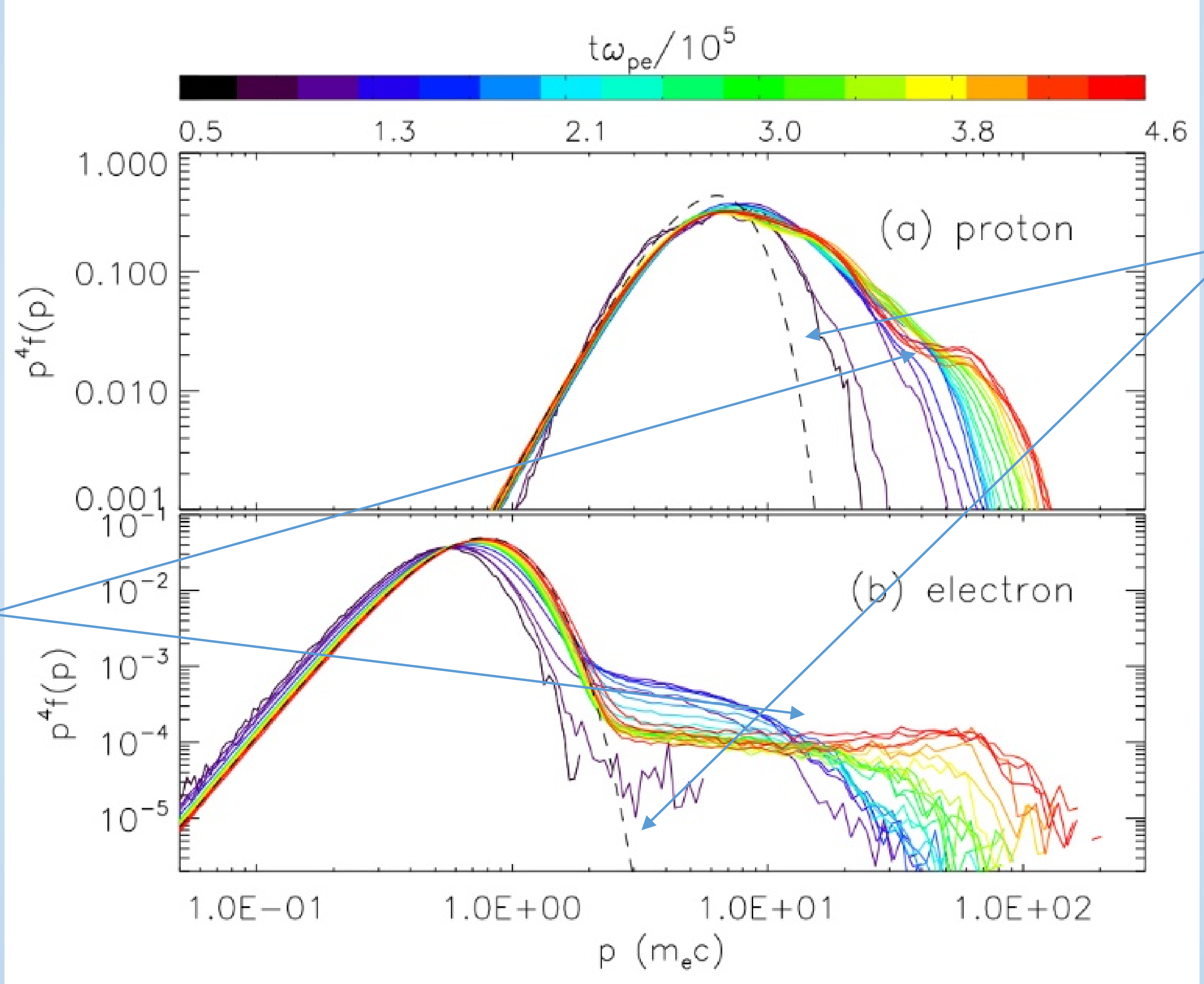
shock

Park+2015

Downstream particle spectra over time
(Previous PIC Simulation)

Quasi-parallel shock, β≈1/√2
$M_s = 40$, $M_A = 20$, $\theta_1 = 30^o$

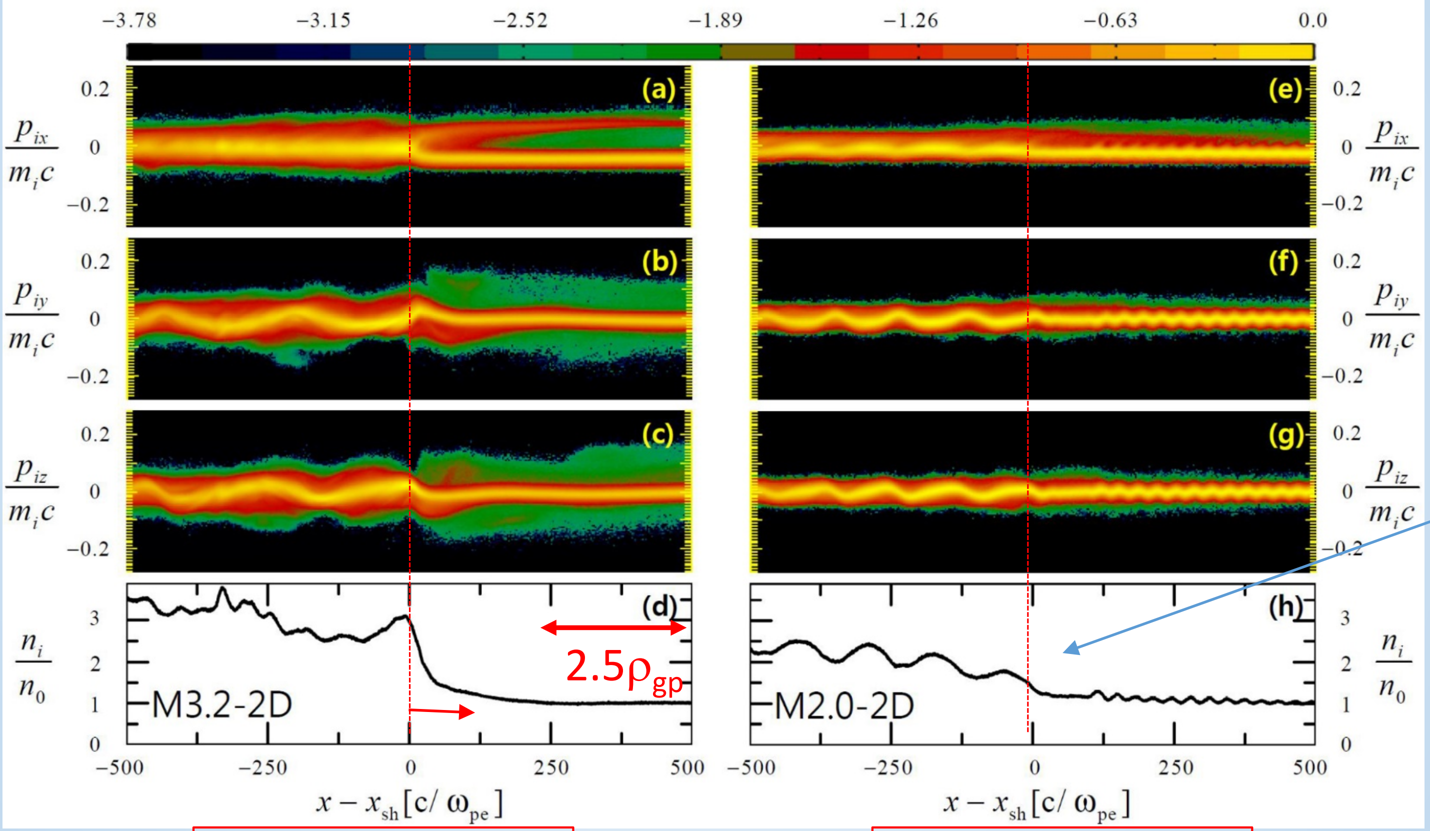
Particles reflected at shock gain energy through cross-B drifts & repeated scatterings into upstream & downstream flows "Diffusive Shock" Fermi I acceleration



Maxwellian

Park+2015

Illustrative 2D PIC Simulations:
Weak (low M_s) Quasi-parallel shocks, β≈100
$\theta_1 = 13^o$ $m_p/m_e = 100$



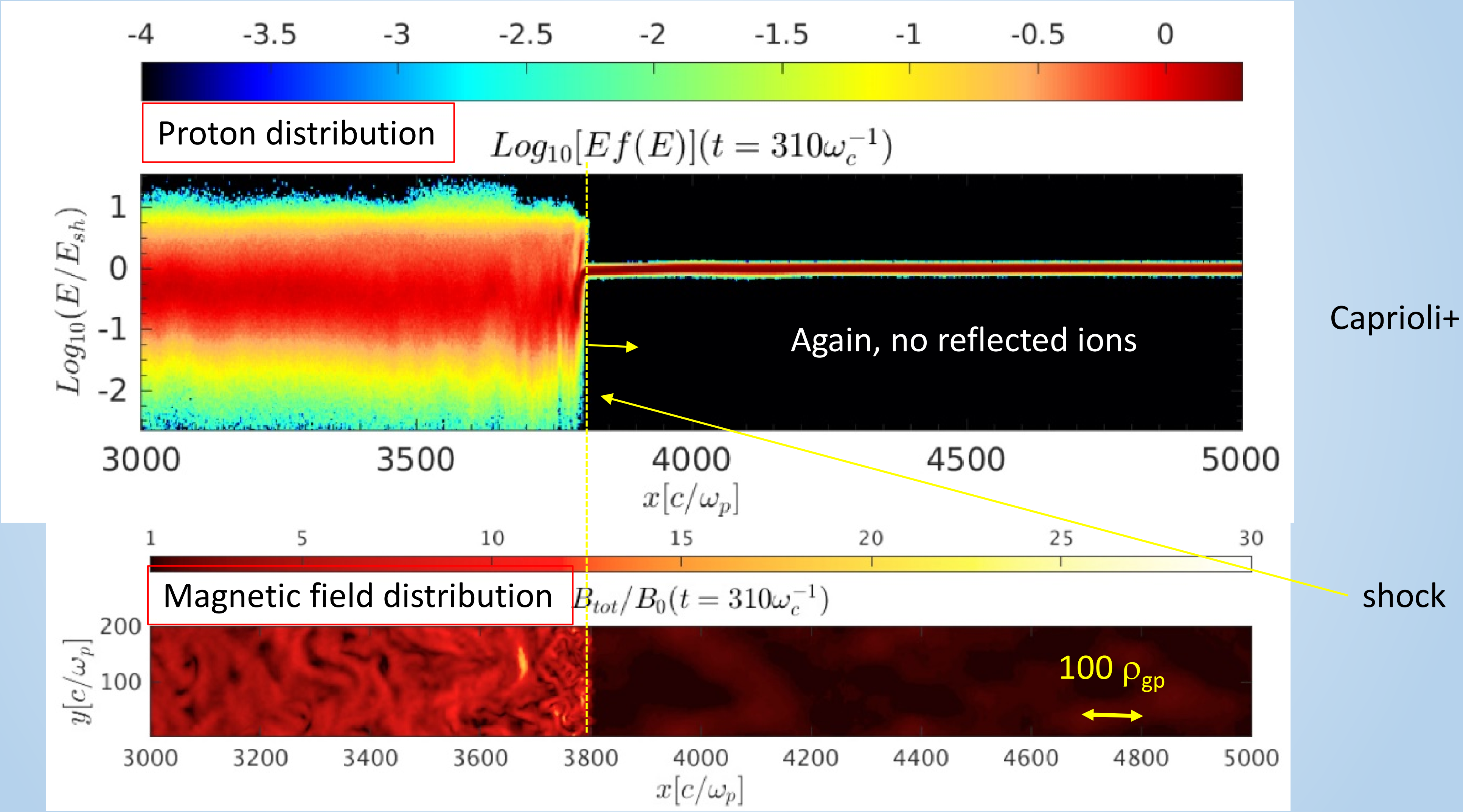
"Subcritical" shock
(ions not reflected)

Ha+2018

$M_s = 3.2$, $M_A = 29$
$u_s = 0.027c$

$M_s = 2.0$, $M_A = 18$
$u_s = 0.052c$

Illustrative Hybrid Simulation
Quasi-perpendicular shock, β≈1
$M_s = M_A = 30$, $\theta_1 = 80^o$



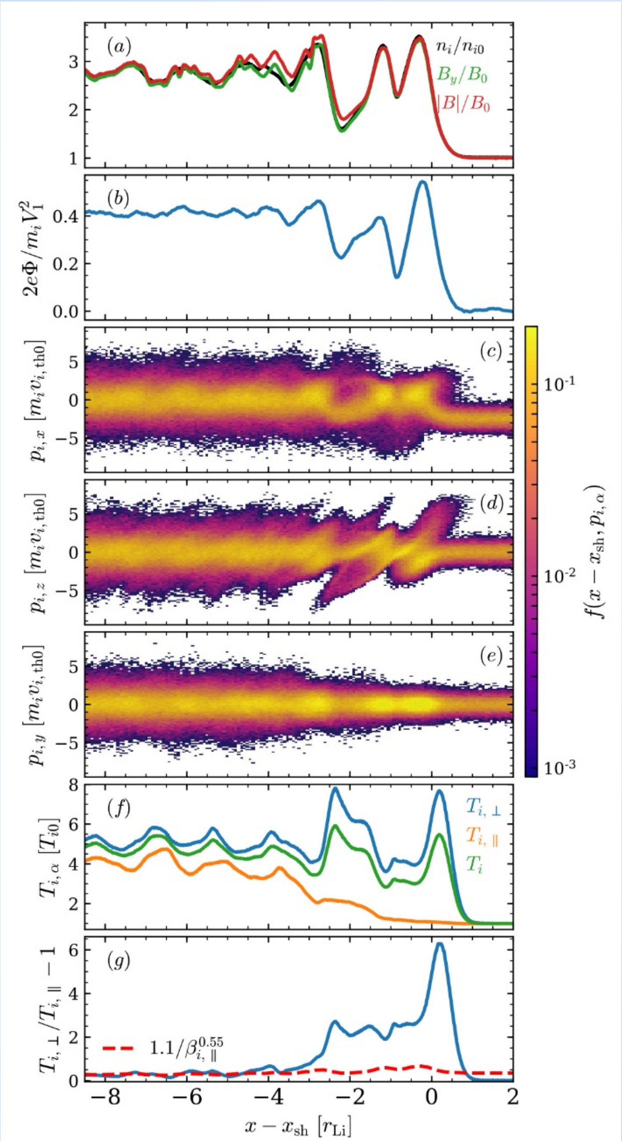
Caprioli+ 2018

How are thermal electrons heated in collisionless shocks?

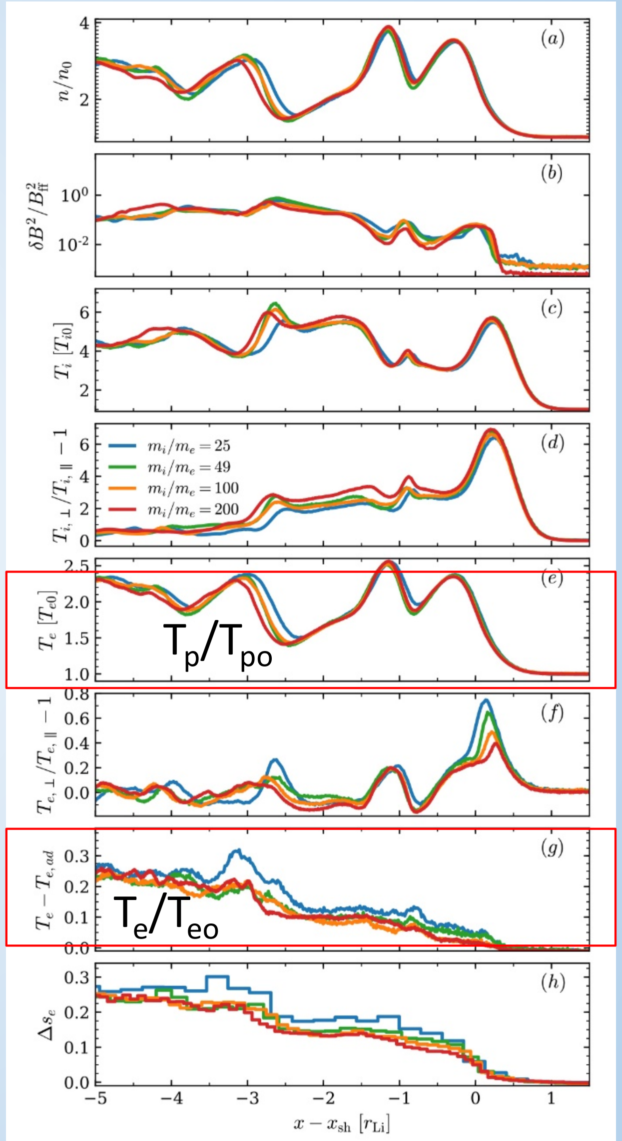
Guo + 2017
2D PIC Sims
(m_p/m_e = 49)
M_s = 3
Perpendicular shock
β = 16

Electron heating depends on:
magnetic field amplification
→ anisotropic electron velocity
→ resonant “Whistler instability”
to energize electrons

Shock Structure (n_p, B, ϕ, f_p, T_p)

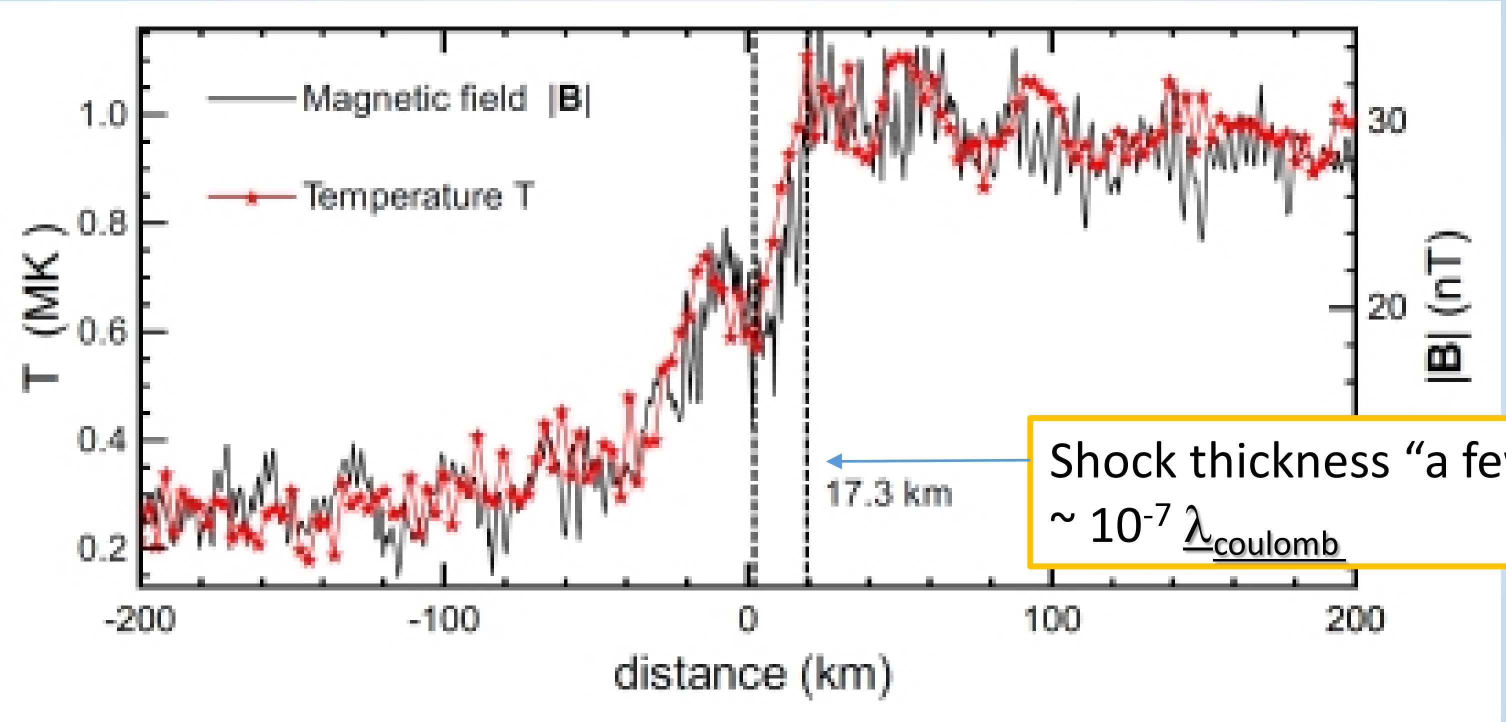


Shock Structure, n_p, B, T_p, T_e)



--A Quick Reality Check--
Earth's Bow Shock Profile

$\lambda_{coulomb}$ ~ 1 AU



Shock thickness "a few ion inertial lengths"
~ 10^{-7} $\lambda_{coulomb}$

M_s ~ 3.5

Schwartz + 11

Astrophysical Turbulence: A Brief Introduction

On macro scales >> λ_p again apply fluid model
(compressible, MHD)

Extract "fluctuating" fields (here velocity)

$$\vec{u} = <\vec{u}> + \vec{u}_{turb}$$

$$<\vec{u}_{turb}> = 0$$

Apply "Hodge-Helmholtz (HH) Decomposition" to isolate compressive & solenoidal motions

"sonic waves" "shear waves"

$$\vec{u}_{turb} = -\nabla\phi + \nabla \times \vec{a} = \vec{u}_c + \vec{u}_s$$

So,

$$\nabla \times \vec{u}_c = 0$$

$$\nabla \cdot \vec{u}_s = 0$$

Vorticity then relates to the solenoidal component

$$\vec{\omega} = \nabla \times \vec{u}_{turb} = \nabla \times \vec{u}_s$$

(Note also)

$$|\omega| \sim \frac{\ell}{\delta u_s} \sim \frac{1}{\tau_{eddy}}$$

So, vorticity measures eddy turn over rate

Simplest path to HH Decomposition: Fourier Transforms

$$\vec{u} = -\nabla\phi + \nabla \times \vec{a} = \vec{u}_c + \vec{u}_s$$

$$\vec{U}_k = i\vec{k}\Phi + i\vec{k} \times \vec{A} = \vec{U}_{kc} + \vec{U}_{ks}$$

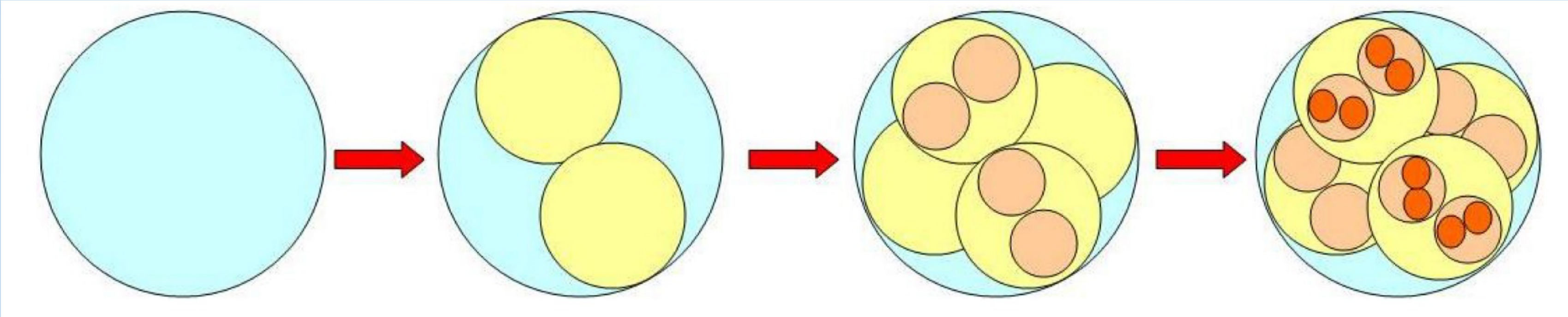
$$\vec{U}_{kc} = \vec{k}\left(\vec{k} \cdot \vec{U}_k\right)/|k|^2$$

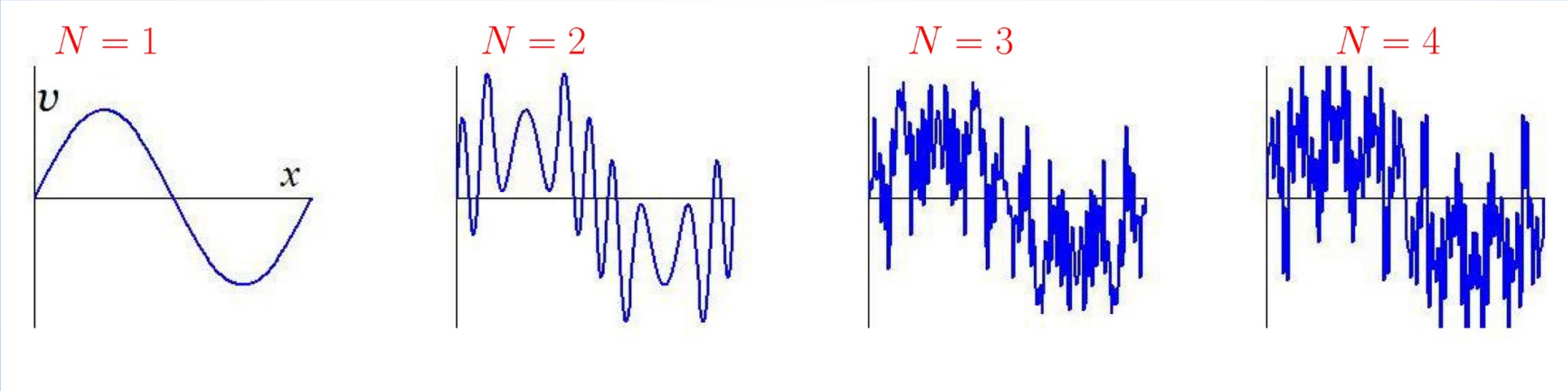
$$\vec{U}_{ks} - -\vec{k} \times \left(\vec{k} \times \vec{U}_k\right)/|k|^2$$

MHD a bit more complicated; 3 modes (anisotropic stresses)

Development of Turbulence:

Fluctuating motions "driven" on large scales will "cascade" down to dissipative scales





Shukurov 2015

Typically the Solenoidal Turbulent Motions Dominate Compression: (but in compressible media, there will always be both)

Fluctuating motions "driven" on large scales will "cascade" down to dissipative scales

In steady state HD turbulence leads to so-called Kolmogorov (or K41) kinetic spectrum

Cascading energy flux

$$\frac{(\delta u_\ell)^2}{\tau_{eddy}} \sim \frac{(\delta u_\ell)^3}{\ell} = \text{const}$$

(within "inertial" range)
between driving and dissipation scales

$$\delta u_\ell^2 \propto \ell^{2/3} \propto E_s(\ell) \propto \int_{k_{min}\sim 1/\ell}^{\infty} E_{sk} dk$$

$$E_{sk} \sim U_{sk}^2 \propto k^{-5/3}$$

(within "inertial" range)
between driving and dissipation scales

Evolution of vorticity: curl of Navier Stokes Eqn.

Recall: vorticity measures an 'inverse eddy time'

$$\vec{\omega} = \nabla \times \vec{u} \sim \frac{u_\ell}{\ell} \sim \frac{1}{t_{eddy,\ell}}$$

In K41 turbulence: $\omega_\ell \propto \ell^{-2/3}$

From Navier-Stokes:

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{u} \times \vec{\omega}) + \frac{1}{\rho} \nabla \ln(\rho) \times \nabla P - \nu \nabla^2 \vec{\omega},$$

or

$$\frac{\partial \vec{\omega}}{\partial t} = -\nabla \cdot (\vec{u}\vec{\omega}) + (\vec{\omega} \cdot \nabla)\vec{u} + \frac{1}{\rho} \nabla \ln(\rho) \times \nabla P - \nu \nabla^2 \vec{\omega},$$

Conservative

Stretch Amplification

Baroclinic Source Term

Dissipation (numerical here)

Easier to Work with Scalar Enstrophy [ε = (1/2)ω^2]

Conservative

Sources

Amplification

$$\frac{\partial \omega^2}{\partial t} = F_{adv} + F_{stretch} + F_{comp} + F_{baroc} + F_{diss},$$

$$F_{adv} = -\nabla \cdot \vec{u}\omega^2 = -(\omega^2 \nabla \cdot \vec{u} + \vec{u} \cdot \nabla \omega^2),$$

$$F_{stretch} = -2\vec{\omega} \cdot (\vec{\omega} \cdot \nabla)\vec{u}$$

$$F_{comp} = -\omega^2 \nabla \cdot \vec{u} = \omega^2 \frac{d\rho}{dt},$$

$$\begin{aligned} F_{baroc} &= 2\frac{\vec{\omega}}{\rho} \cdot \nabla \ln(\rho) \times \nabla P \\ &= 2\mathcal{R}\, \vec{\omega} \cdot \nabla \ln(\rho) \times \nabla T \\ &= 2\rho^{2/3}\vec{\omega} \cdot \nabla \ln(\rho) \times \nabla S, \end{aligned}$$

where $\mathcal{R} = k_B/(\mu m_H)$

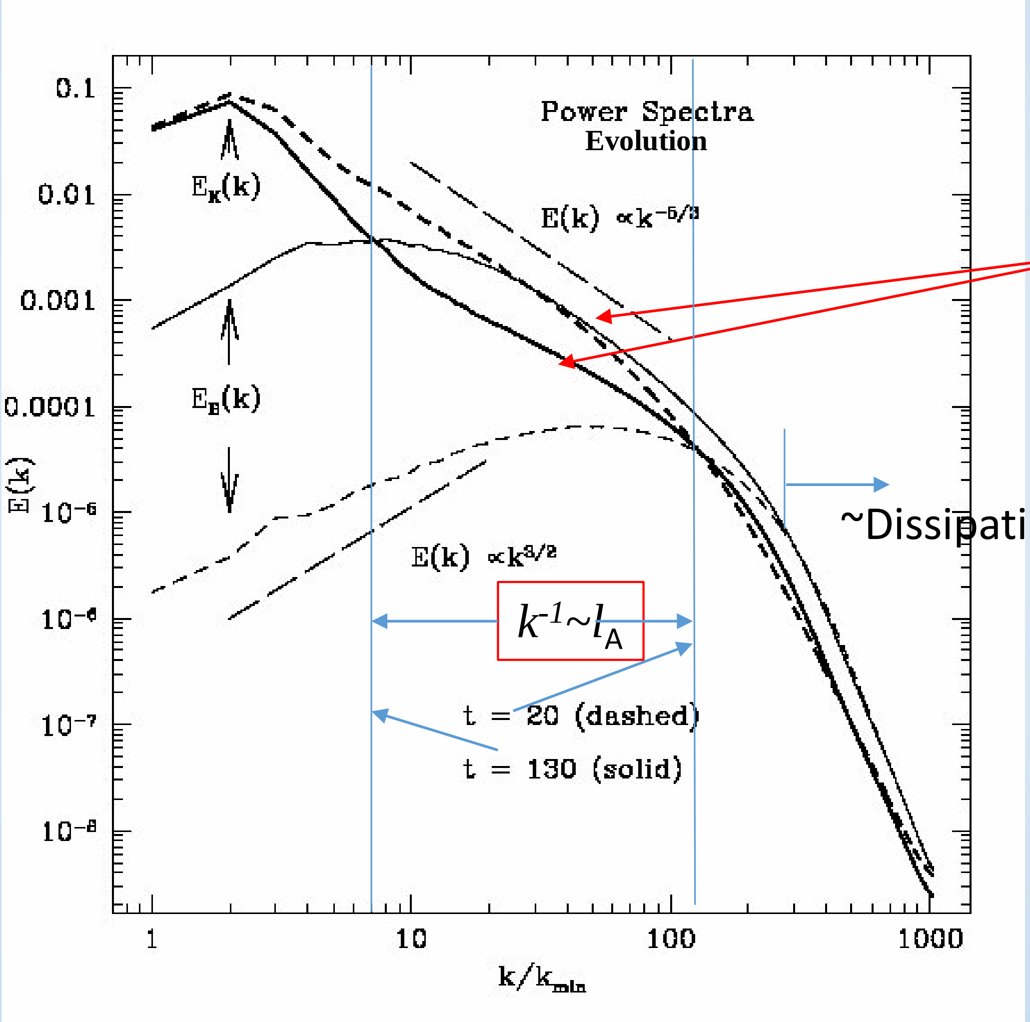
Typically, F_{baroc} is dominant source term; $F_{stretch}$ dominates amplification

$$F_{diss} = -2\nu\vec{\omega} \cdot \nabla^2 \vec{\omega}$$

Compressible Fluid Turbulence Properties Depend on Forcing: Component Proportions & Spectral Slopes in a Simulation

Solenoidal Forcing
$\nabla\cdot\delta v = 0$, $\nabla\times\delta v \neq 0$

$M_s \sim ½$
MHD with
$\beta_0 = 10^6$
$\tau_{eddy} \sim 10$



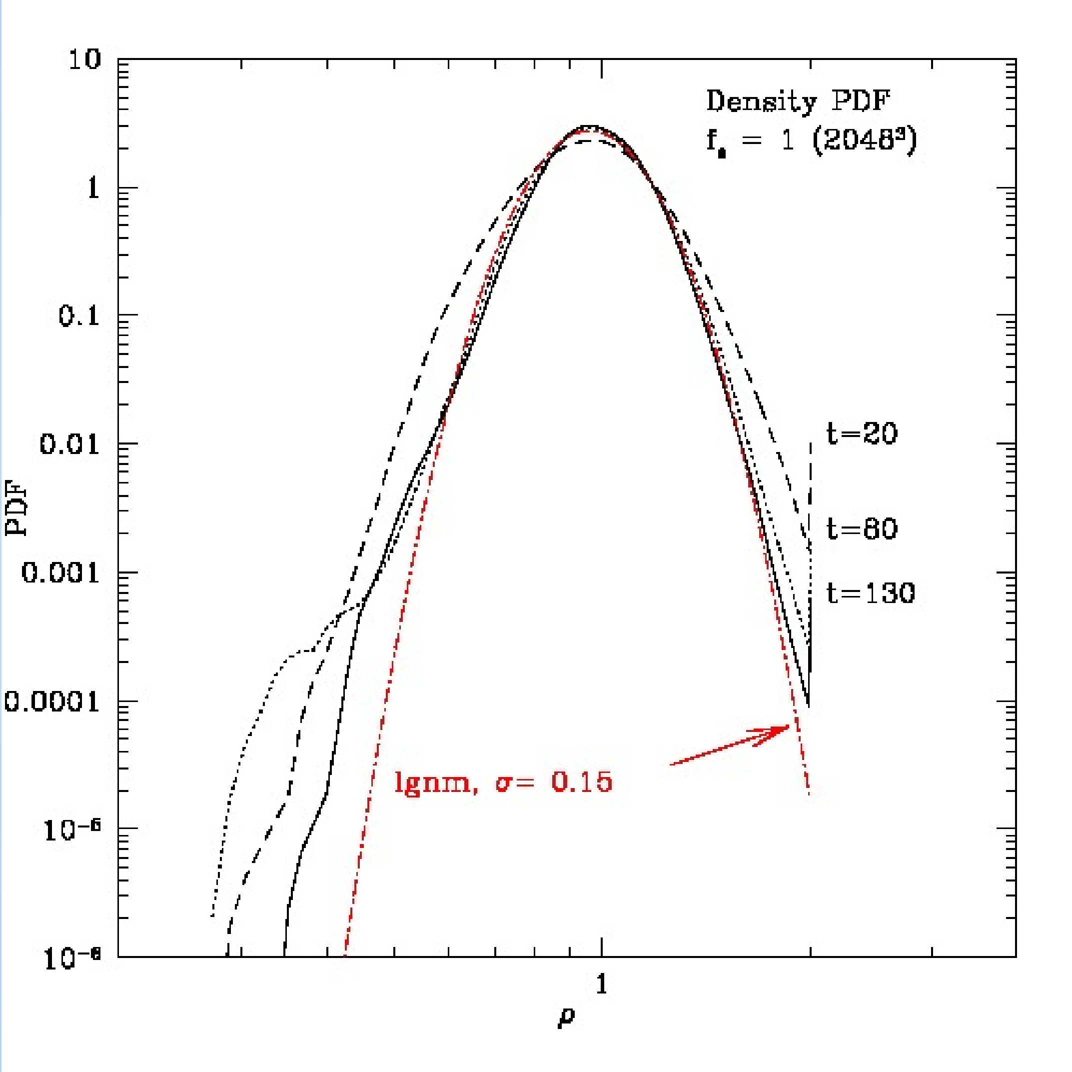
Solenoidal Modes
(B-field extracts energy @ late times)

(Compressive Modes < 10%)

~Dissipation scales

Porter, Jones & Ryu 2015

There are density fluctuations even for pure solenoidal driving



Simulation data are in black

Lognormal distribution with $\sigma = 0.15$ in red

Agrees with semi-analytic models (e.g., Molina + 12, Konstandin + '12)
$\sigma^2 = \ln(1 + b^2M^2)$
using their $b \approx 1/3$ ($f_s = 1$) and $M = 0.5$

$\delta\rho$ comes from compressive motions

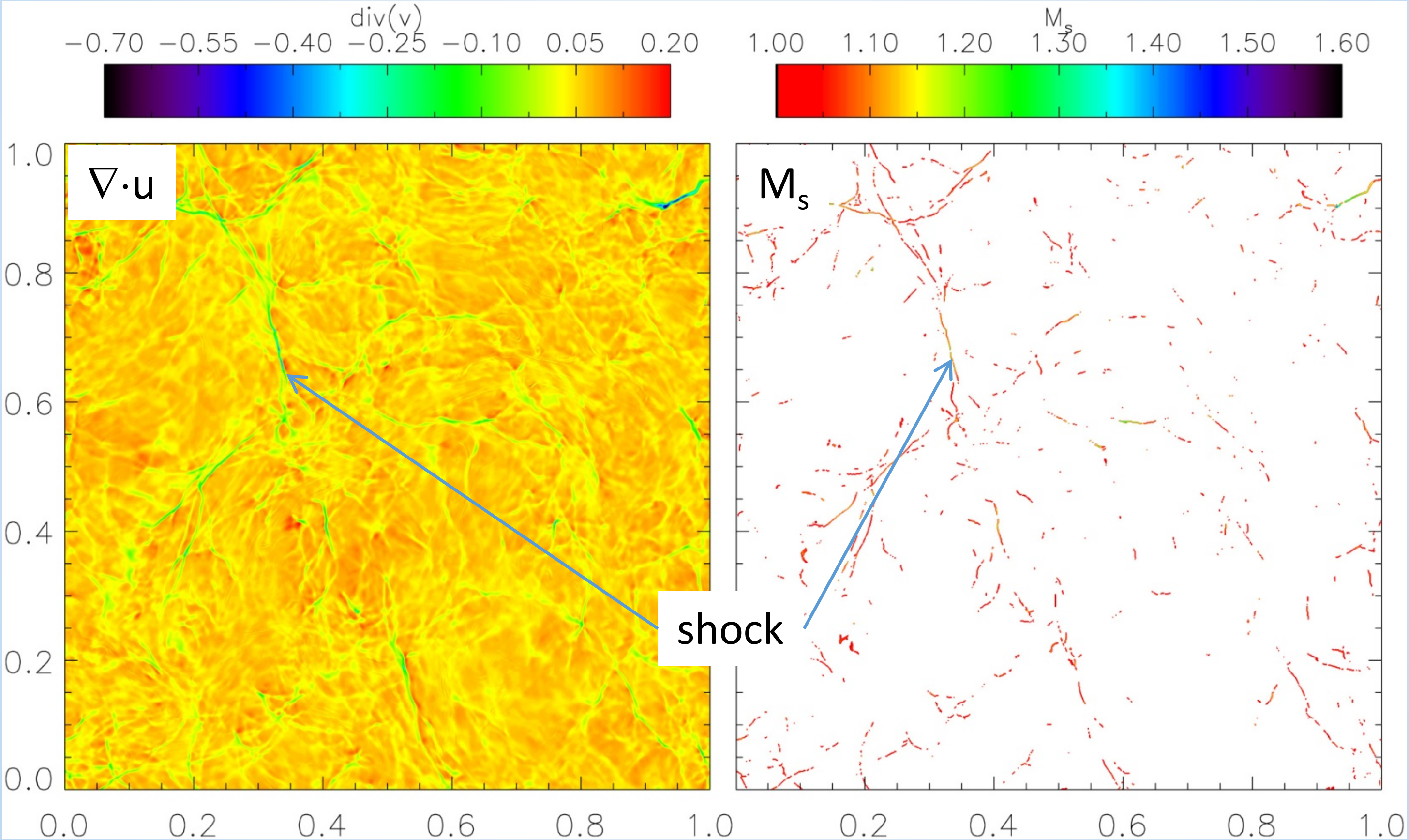
Using $\sigma^2 \sim (\delta\rho/\rho)^2 \sim (\delta u/c)^2$ for sound

Estimate: $E_c \sim (1/2)\,\sigma^2 P$,

$E_c/E_k \sim \sigma^2/M^2 \sim 0.09$ (0.07 measured)

Porter, Jones & Ryu 2015

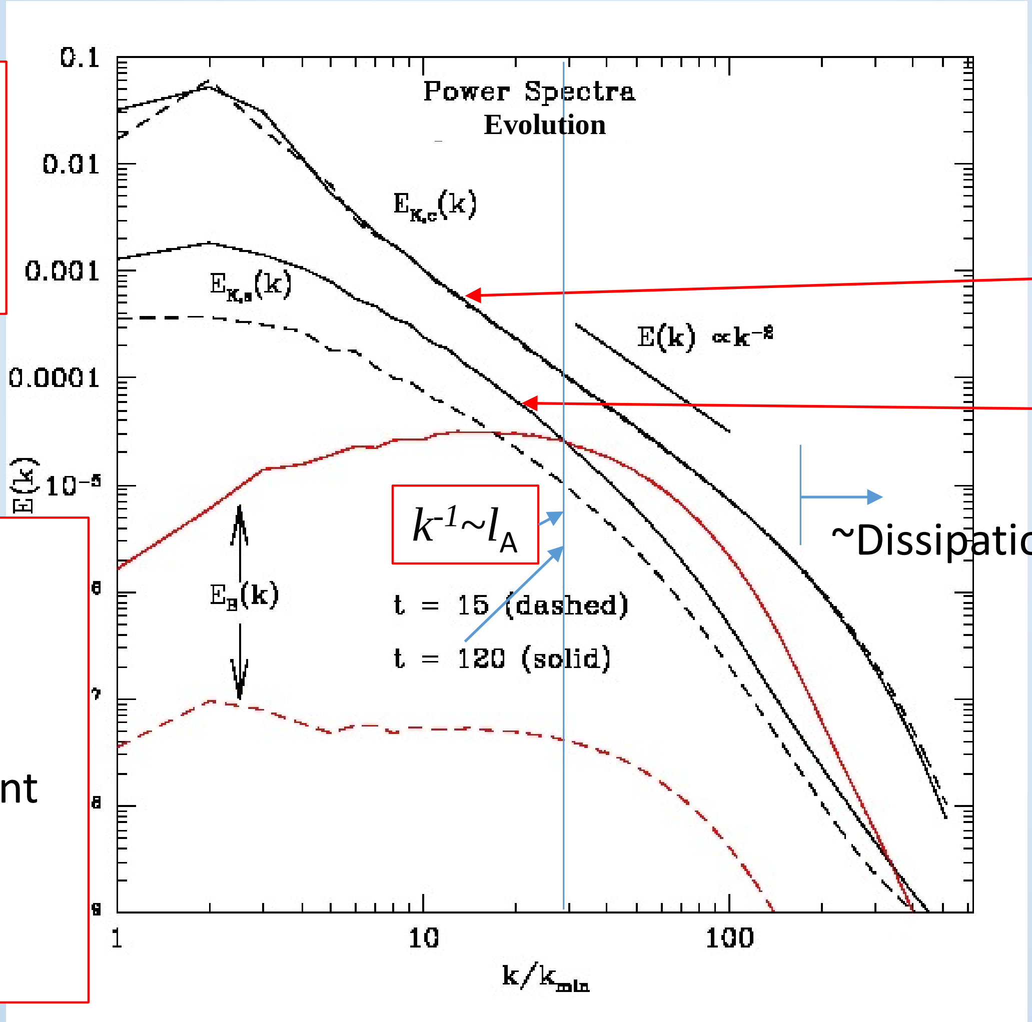
There actually are shocks even with $u_{RMS} \approx 1/2$ a (2D slice at t ≈ 100 ~ 10 τ_{eddy})



Porter, Jones & Ryu 2015

Compressible Fluid Turbulence Properties Depend on Forcing: Component Proportions & Spectral Slopes

Compressive Forcing
$\nabla\times\delta v=0$, $\nabla\cdot\delta v \neq 0$,
MHD



Compressive Modes

Solenoidal Modes
(< 10%)

~Dissipation scales

"Real World" Turbulence Forcing includes both Solenoidal and Compressive Forcing:

⇒ Variable Compressive Component
⇒ Variable Steepness
⇒ Intermittent Distribution

Porter, Jones & Ryu 2015

Our Media are magnetized:
MHD – Maxwell stresses included

$$\dot{\vec{u}} = \frac{1}{\rho}(-\nabla P + \mu\nabla^2\vec{u} + \frac{1}{c}\vec{j}\times\vec{B})$$

$$\frac{1}{c}\vec{j}\times\vec{B} = -\nabla(P_B) + \frac{1}{4\pi}(\vec{B}\cdot\nabla)\vec{B}$$

Maxwell stresses important when: $\frac{\delta u^2}{\ell} \lesssim \frac{v_A^2}{\ell}$

Balance for $l \sim l_A$

$$P_B = \frac{B^2}{8\pi}$$

Stresses are anisotropic: => anisotropic turbulent cascade for shear mode (intermediate)

Simplest MHD cascade picture:
Cross field plasma motions inhibited—
⇒"critical balance" timescale condition
for perpendicular & parallel shear waves
(Goldreich & Sridhar 1995)

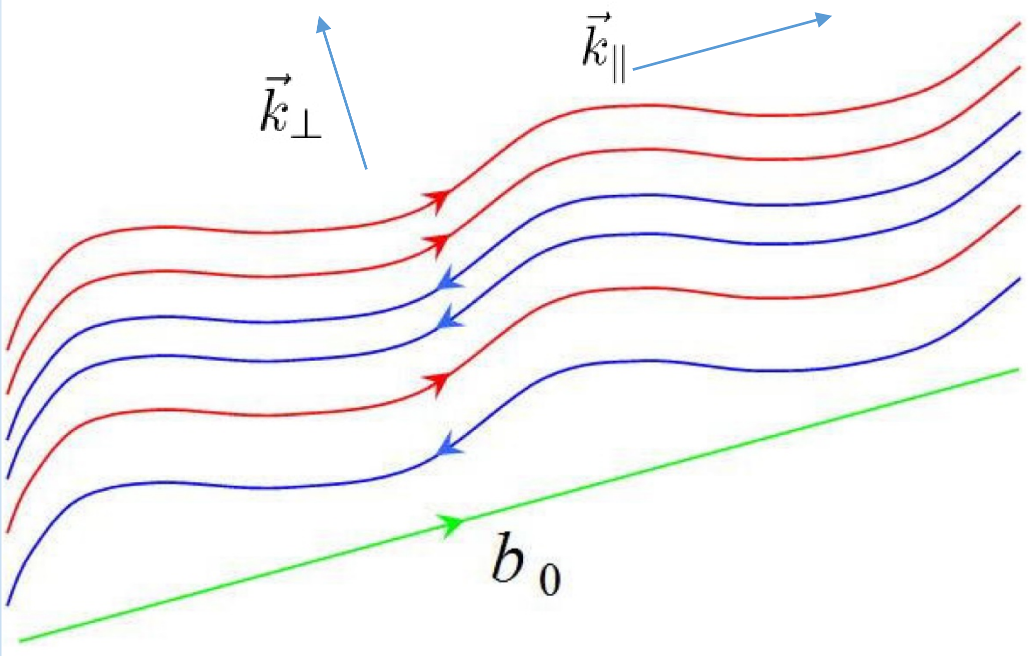
$k_\perp \delta u_\perp \approx k_{||} v_A$ + $\delta u_\perp \propto \ell_\perp^{1/3} \sim k_\perp^{-1/3}$ => $k_{||} \propto k_\perp^{2/3}$

⇓

$E_{k_\perp} \propto k_\perp^{-5/3}$

$E_{k_{||}} \propto k_{||}^{-5/2}$

Over-simplified structures. Not really coherent



Shukurov 2015

An initially weak magnetic field can be amplified by "small scale" dynamo:

The Magnetic Induction Equation using Generalized Ohm's Law

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + \eta \nabla^2 B - \frac{1}{e n_e^2} \nabla n_e \times \nabla P_e$$

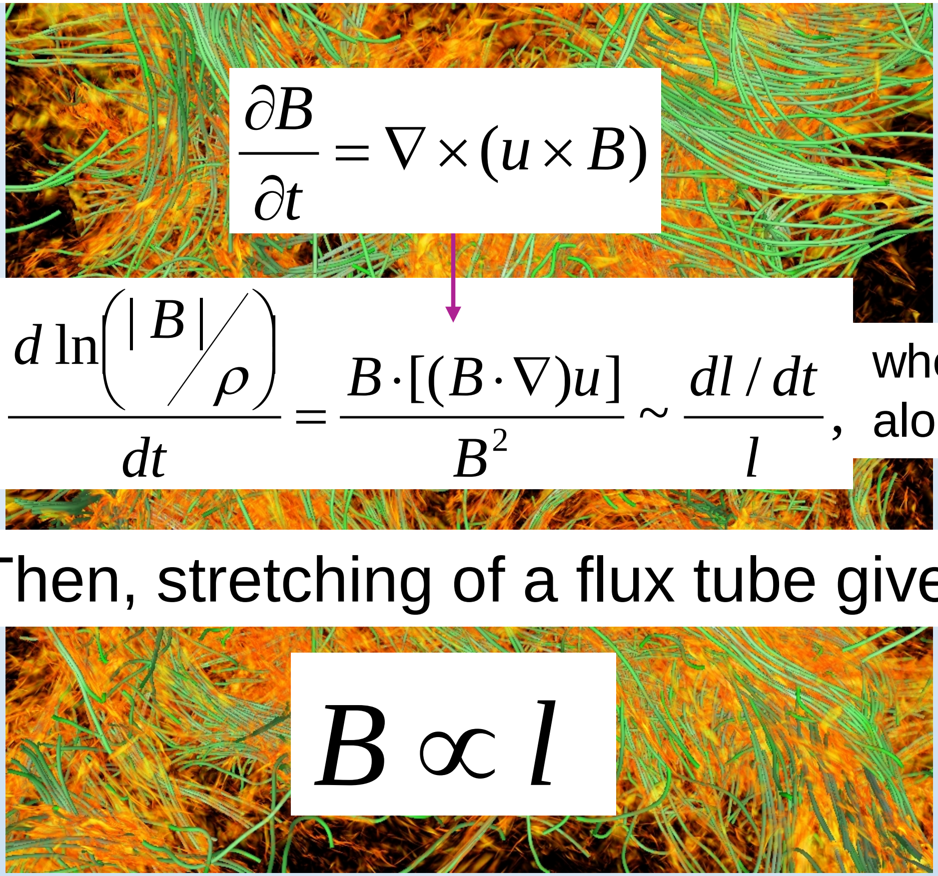
η is resistivity

Mathematical structure same as the Vorticity Equation:

- Source Term (Biermann Battery) when $\nabla n_e \times \nabla P_e \neq 0$
  (e.g., at curved shocks)
- Field intensity, B(t) $\propto$ l(t) --stretch and fold amplification)
- Dissipation, diffusion measure, $R_M = uL/\eta = P_r R_e$
  Where $P_r = \nu/\eta$ is the Prandtl number

Essential Turbulent Dynamo Concepts:

1) In ideal MHD (sans resistivity) B obeys



$$\frac{\partial B}{\partial t} = \nabla \times (u \times B)$$

$$\frac{d \ln\left(|B| / \rho \right)}{dt} = \frac{B \cdot [(B \cdot \nabla) u]}{B^2} \sim \frac{dl/dt}{l},$$

where l measures length along a flux tube. ρ is mass density

Then, stretching of a flux tube gives*:

$$B \propto l$$

* Just like vorticity, e.g., making a tornado by stretching a weak vortex

With "Small, but Finite" Dissipation, "Stretch, Twist & Fold" Cycle
⇒ 2) Exponential Field Amplification on Smallest Solenoidal Turbulent Scales

A. Brandenburg, K. Subramanian / Physics Reports 417 (2005) 1–209

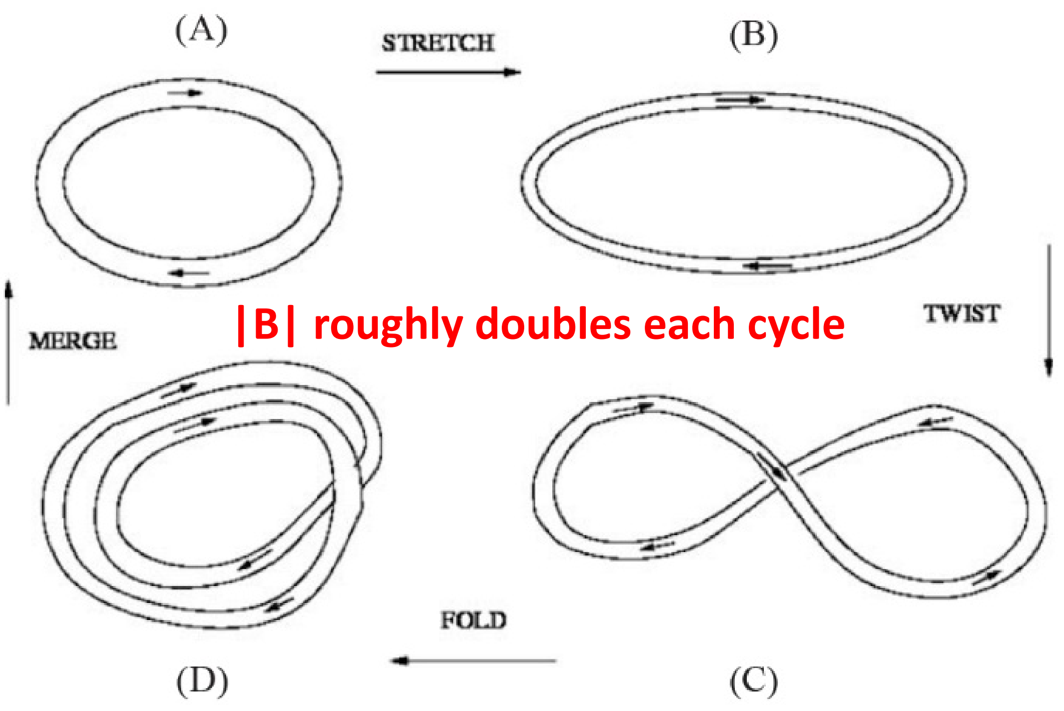


Fig. 4.6. A schematic illustration of the stretch-twist-fold-merge dynamo.

$$t_{grow} \propto \frac{\ell}{u_\ell} \propto \ell_{min}^{2/3} \quad \text{for}$$

$$u_\ell \propto \ell^{1/3} \quad \text{(Kolmogorov)},$$

Essential conditions:

1)Small viscous scale, $\ell_{min} << L$ ($\ell_{min}\, u_{\ell min} \sim \nu$)
High Reynolds ($R = U_L L/\nu >> 100$) flow

2)Resistive and viscous scales similar
[Prandtl ($Pr = \nu/\eta) \sim 1$]
Magnetic diffusion time not less than viscous dissipation time, $t_{visc} \sim (\ell_{min})^2/\nu$

3) Eventually, magnetic tension resists turbulent motions

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \frac{1}{4\pi\ \rho}(\vec{B} \cdot \nabla \vec{B}$$

when balanced on a scale ℓ

$$\rightarrow \frac{u_\ell^2}{\ell} \sim \frac{1}{4\pi\ \rho} \frac{B_\ell^2}{\ell},$$

$$u_\ell \sim v_A(\ell), \quad \text{where } v_A = B/\sqrt{4\pi\rho},$$

Saturation on this scale (smallest scales first)

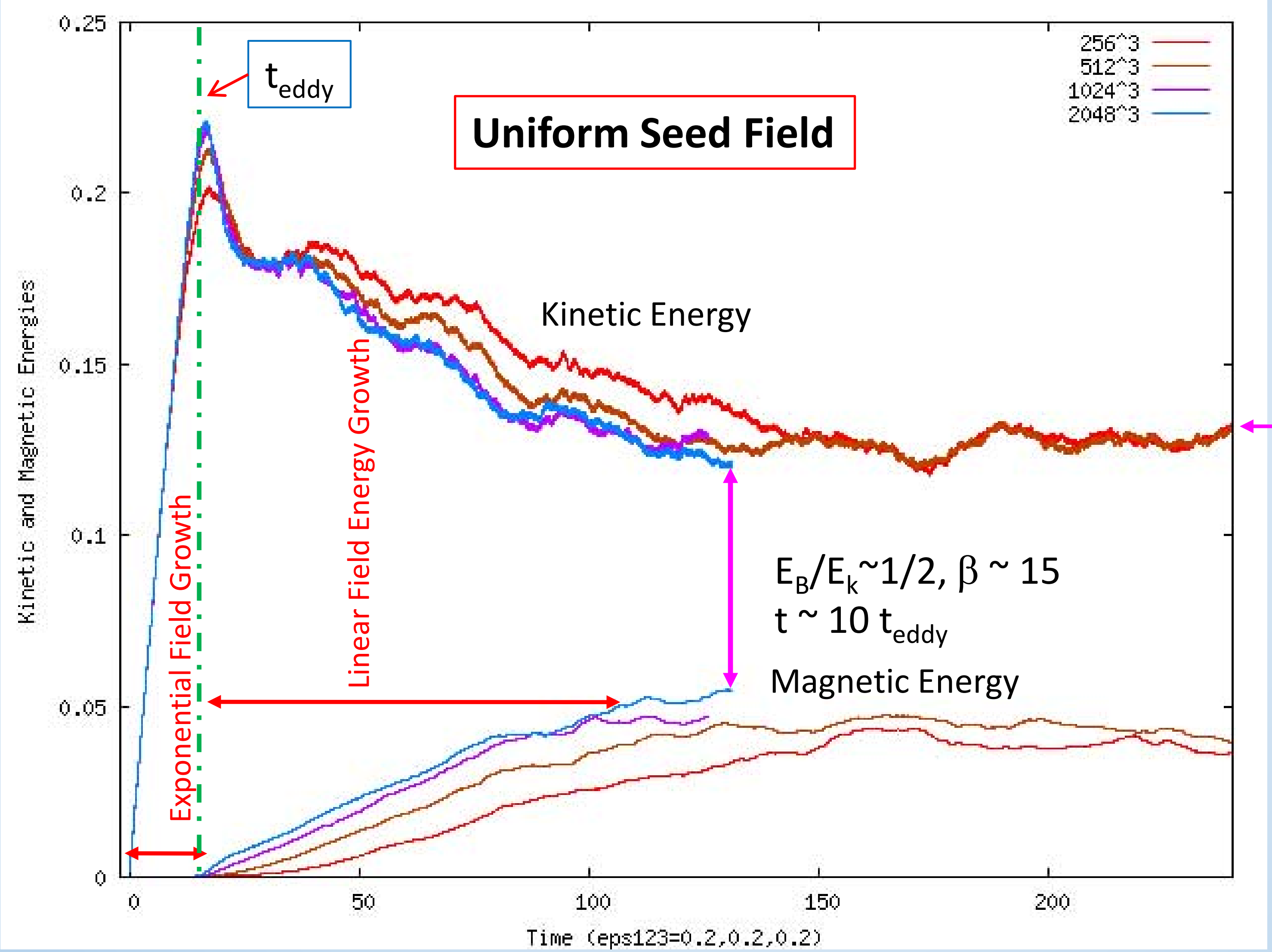
$$u_\ell \propto \ell^{1/3} \quad \text{(Kolmogorov)},$$
$$u_\ell^2 \sim B_\ell^2 \ \sim \ell^{2/3}$$

Energy magnetic increases with scale

$$t_{sat} \propto \frac{\ell}{u_\ell} \propto \ell^{2/3},$$
$$\frac{dB_\ell^2}{dt} \sim \ \text{const}$$

Total magnetic energy increases as saturation scale, so, linear rate with time
(energy extraction only a few % of dissipation)

Energy Evolution from Previous (Solenoidal) Turbulence Simulation

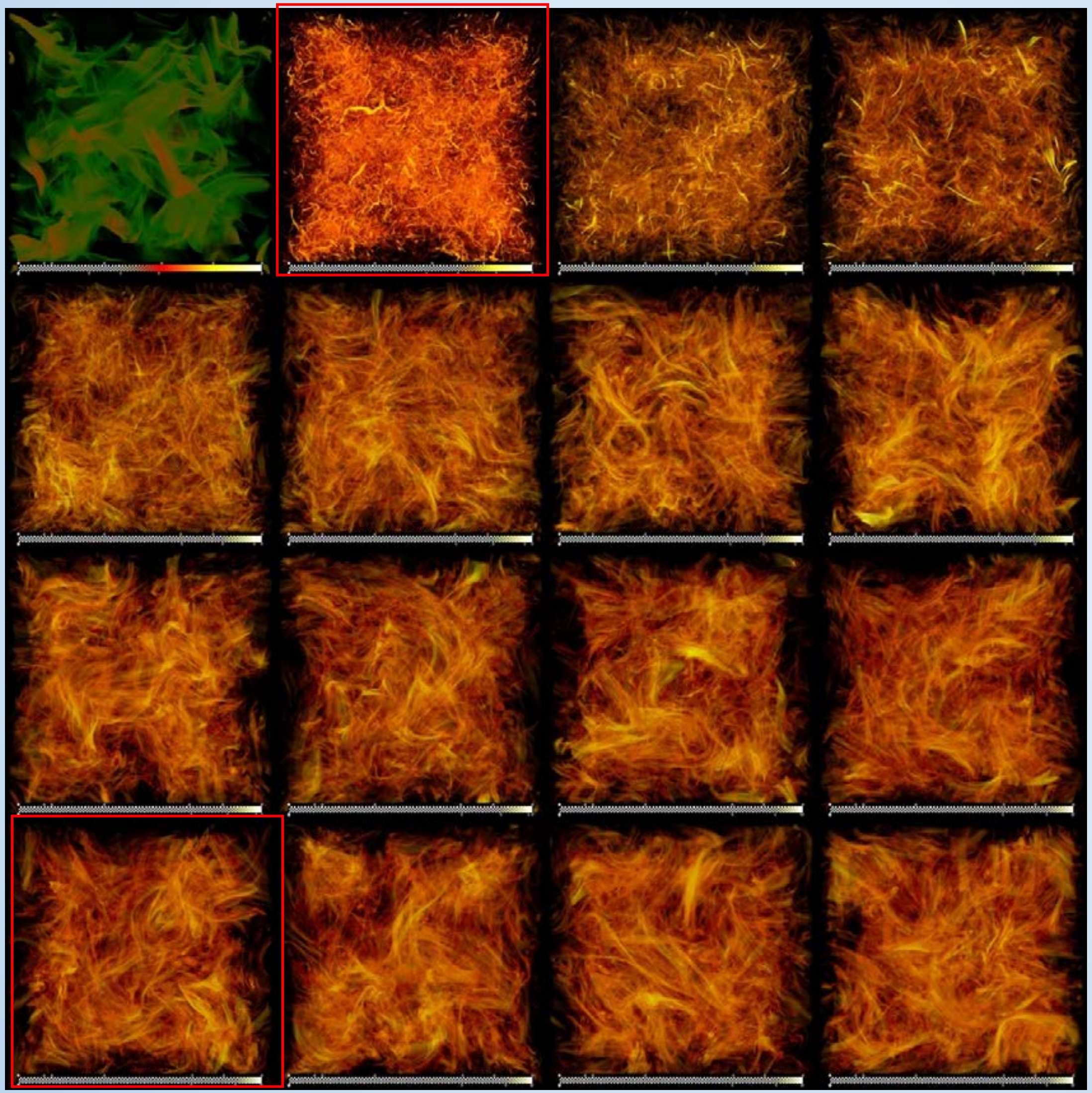


u_{rms}~0.5 c_s
P_{turb} ~ 0.08 P

Porter, Jones & Ryu 2015

Rendering of Magnetic Field Distribution Evolution (Same Simulation)

time →



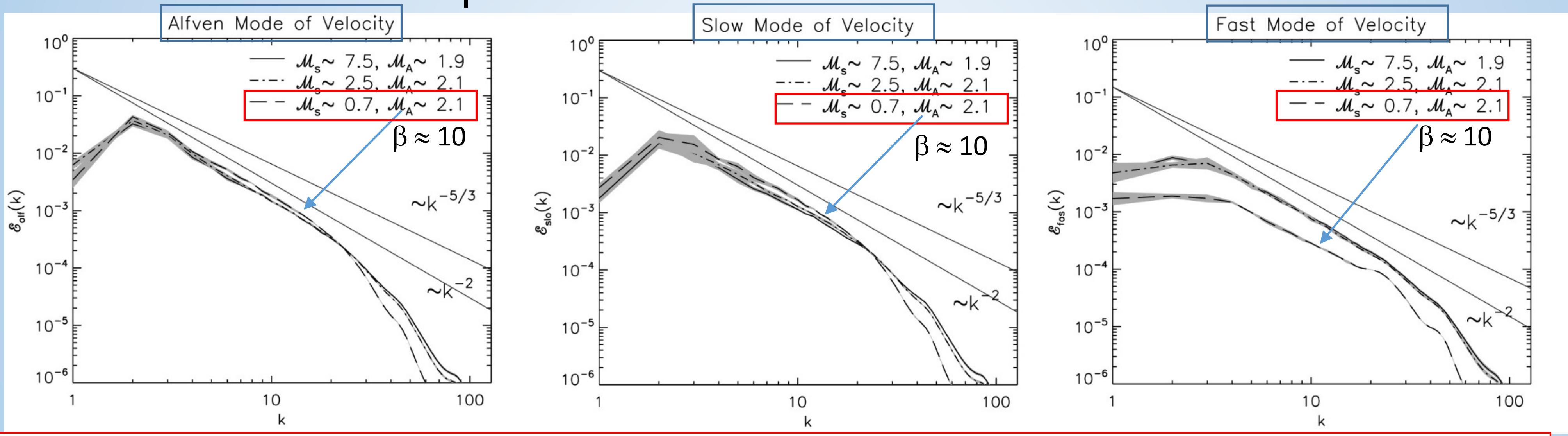
$t=10, 20, ..., 160$
$t \approx 0.7, 1.3, ..., 10\ t_{eddy}$

Log B
volume rendered

1024^3 box

Porter, Jones & Ryu 2015

Compressible MHD Domain Turbulence: Velocity Power Spectra By MHD Mode- For $l < l_A$ ($\delta u < v_A$) in the MHD Domain Solenoidal => Alfven Mode Compressive => Fast & Slow Modes



Fast mode = magnetosonic => $\delta\rho$ correlates with δB (pressure fluctuations enhanced); $c \approx a$ in high β
Slow mode => $\delta\rho$ anti-correlates with δB (little or no total pressure fluctuation); $c < v_A$ in high-β

Kowal & Lazarian 2010

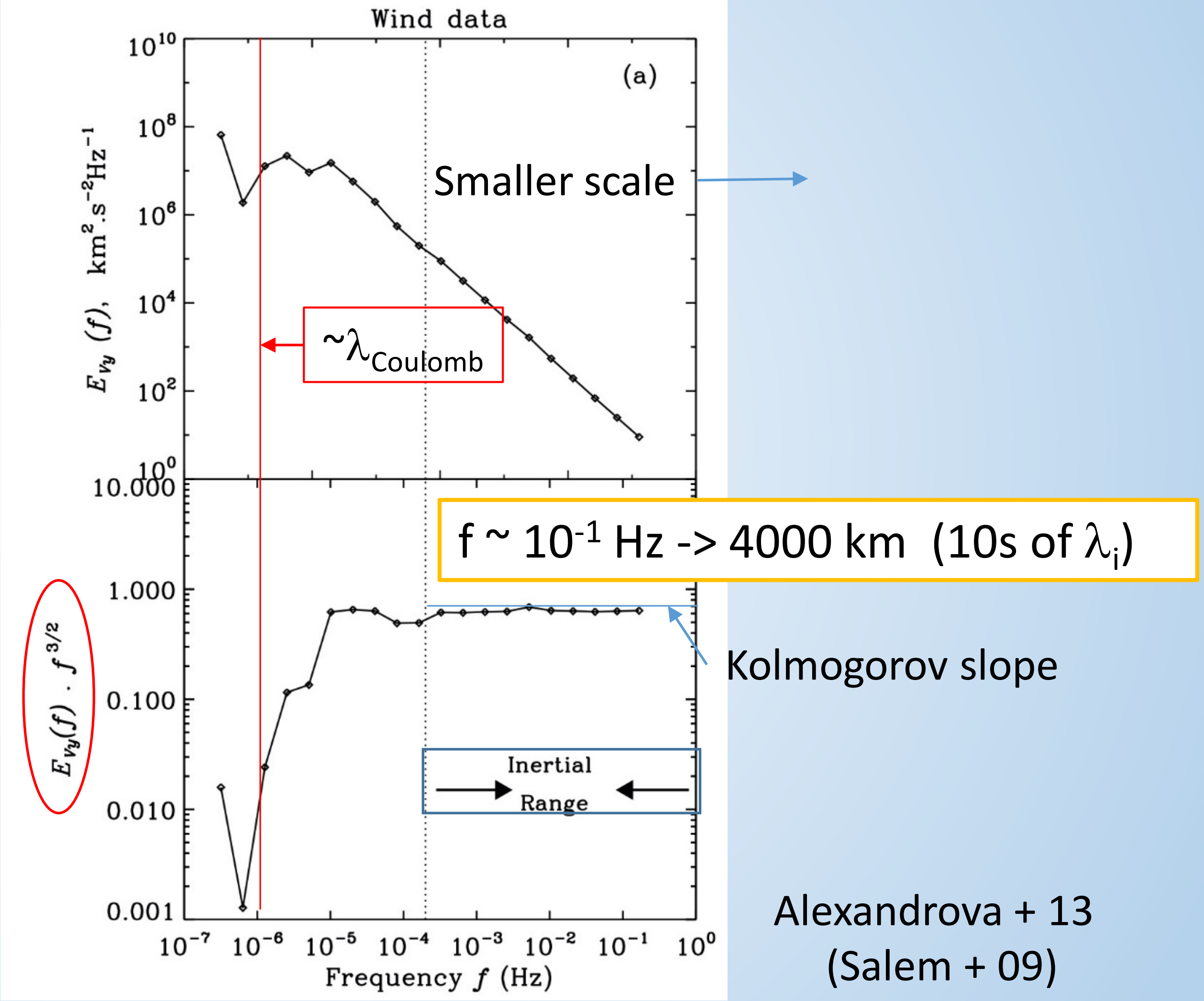
As for shocks:
the physics of the turbulence cascade on scales approaching ρ_{gp} and/or λ_p governed by collective, kinetic interactions.

Illustration sub-scattering-scale plasma dynamics: Solar Wind inertial turbulence extends to ion inertial scales --Kinetic (Velocity) Power Spectrum--

$$\ell \sim \frac{V_w}{f} \sim 400 \text{ km} \frac{1}{f} \sim 3 \times 10^{-6} \text{ AU} \frac{1}{f}$$

Inertial range l ~ 1000 km – .01 AU

IPM
$0.1 < \beta < 10$

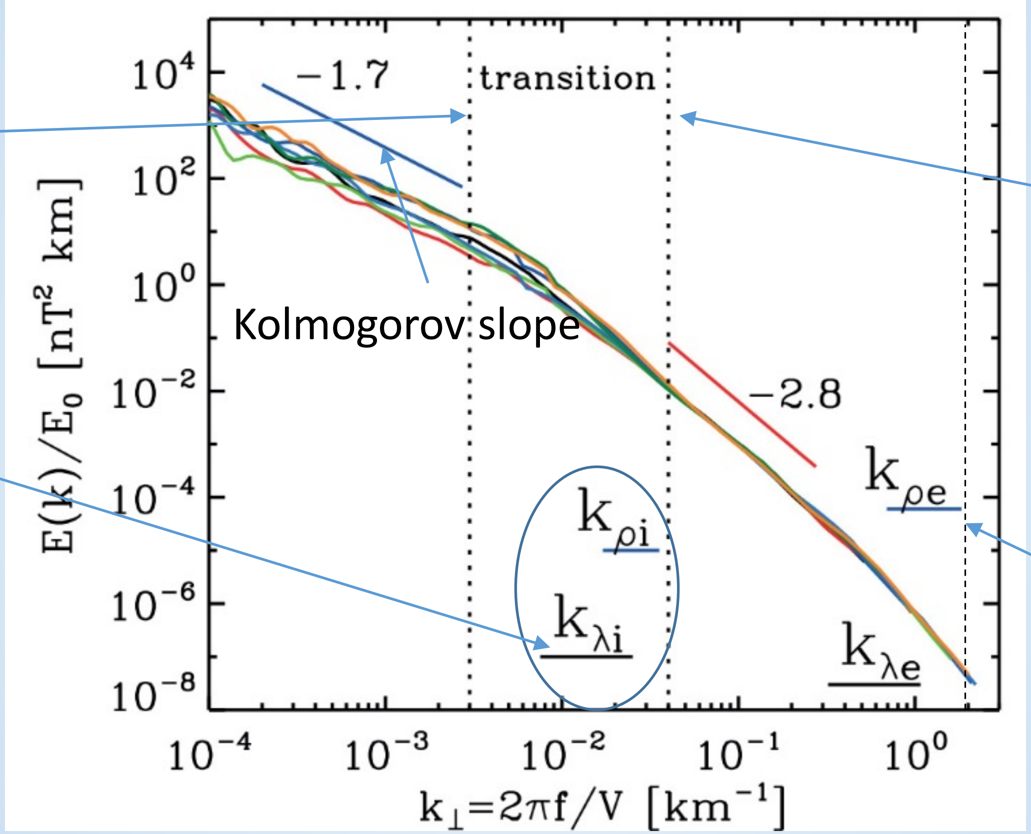


Alexandrova + 13
(Salem + 09)

--Solar Wind Magnetic Field Turbulent Spectrum--

l = 2000 km

Steepens below *l* ~ 10 λ_i
(λ_i = c/ω_{pi} ,Ion inertial length)



Proton Gyro-radius
l ~150 km

Electron Gyro-radius
l ~35 km

Alexandrova + 13

IPM
0.1 < β < 10

Energy Cascades to kinetic scales, but detailed dissipation outcomes require detailed microphysics

Summary & Conclusion

- Diffuse plasmas are the dominant form of baryonic matter in the universe
- These media are mostly almost collisionless and weakly to moderately magnetized
- Plasma-scale, kinetic instabilities predominantly control particle behaviors
- On “macro” length and time scales the behaviors of the media are essentially fluid-like
- On “kinetic” scales plasma processes are critically important, especially to particle distributions
- These media are generally “agitated” in response to local energy sources; scale separations very large: shocks and turbulence are ubiquitous.

감사합니다!