Competition of Deformation and Pairing Correlations in N = Z (Stable or Unstable) Nuclei

Eun Ja Ha[1]

in collaboration with

Myung-Ki Cheoun[1] and **Hiroyuki Sagawa**[2]

1) Department of Physics and Origin of Matter and Evolution of Galaxy (OMEG) Institute, Soongsil University, Seoul 156-743, Korea
2) RIKEN and University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan

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4. Effects of the Coulomb and the spin-orbit interaction in a deformed
   mean field on the residual pairing correlations for N=Z nuclei.
5. Summary

❖ Pairing correlation

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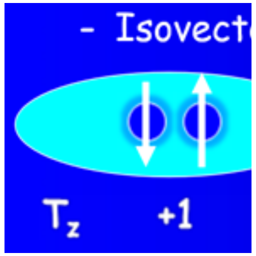
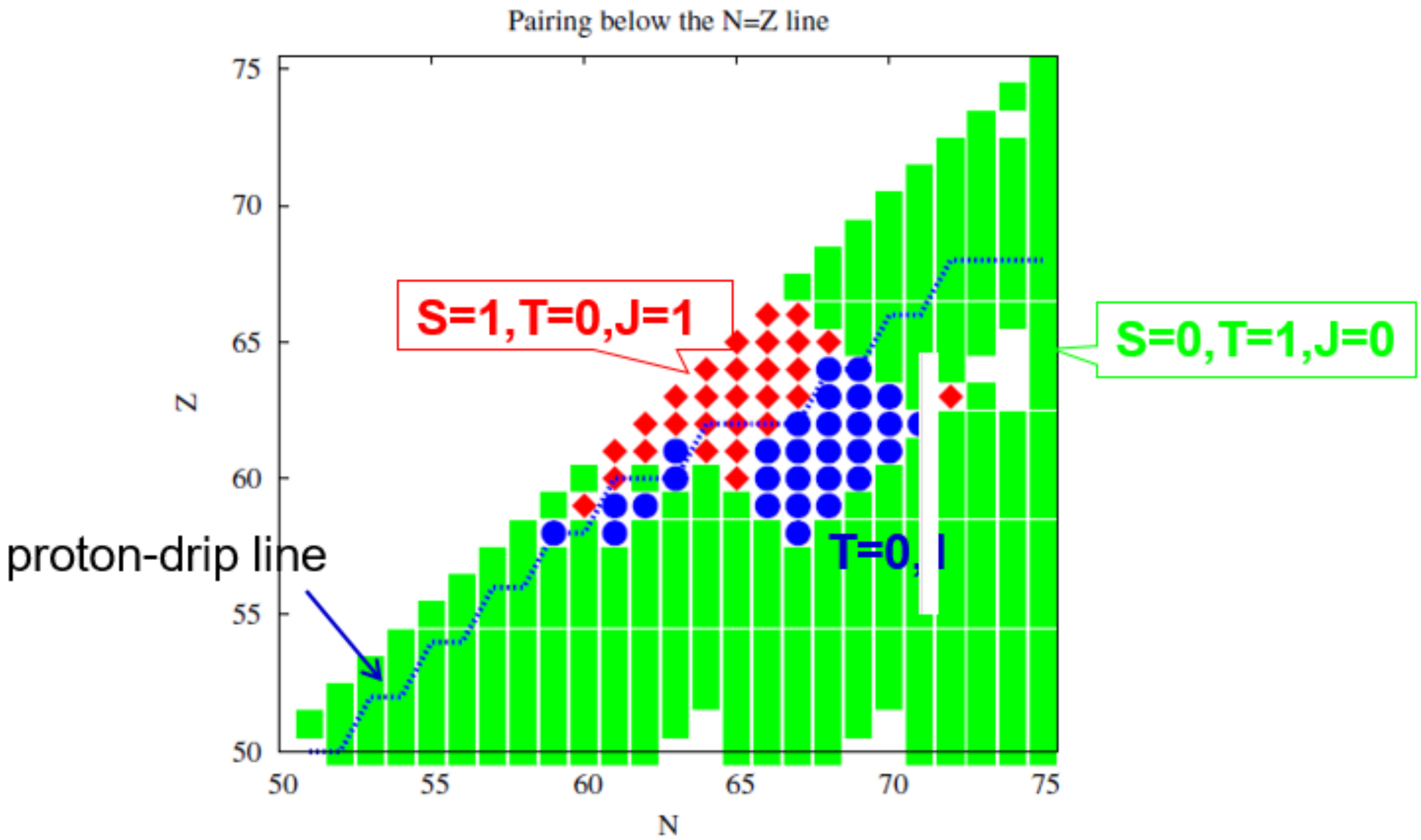
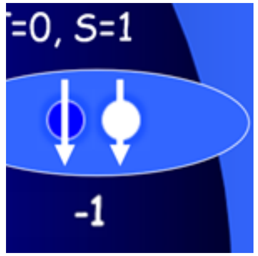
- l

❖ T=1, S … in-







- ➢ Are th
- ➢ For N= … In the … have the maximal spatial overlap, which makes especially T=0 pairing important.
- ➢ There have been many discussions about the coexistence of IS and IV and their competitions.
- ➢ The nuclear structure of N≠Z nuclei, 60< N<70 and 57< Z<64, may also be affected by np correlations. PRL 106, 252502(2011)

- M1 spin transition data shows the IV quenching for the N = Z *sd*-shell nuclei.
  ;T = 0 pairing by the tensor force well-known in deuteron structure may
  become more significant even inside nuclei. PRL 115, 102501(2015)

- In our early papers, the *np* pairing was discussed for GT and double-beta decay using spherical QRPA, which did not include the deformation explicitly and the IS *np* pairing was taken into account by renormalizing the IV *np* pairing.
  M.K. Cheoun *et al*. NPA 561(1993), NPA 564(1993)

- In this work, the **effects of deformation and IS *np* pairing** are taken into account explicitly in the HFB approach.

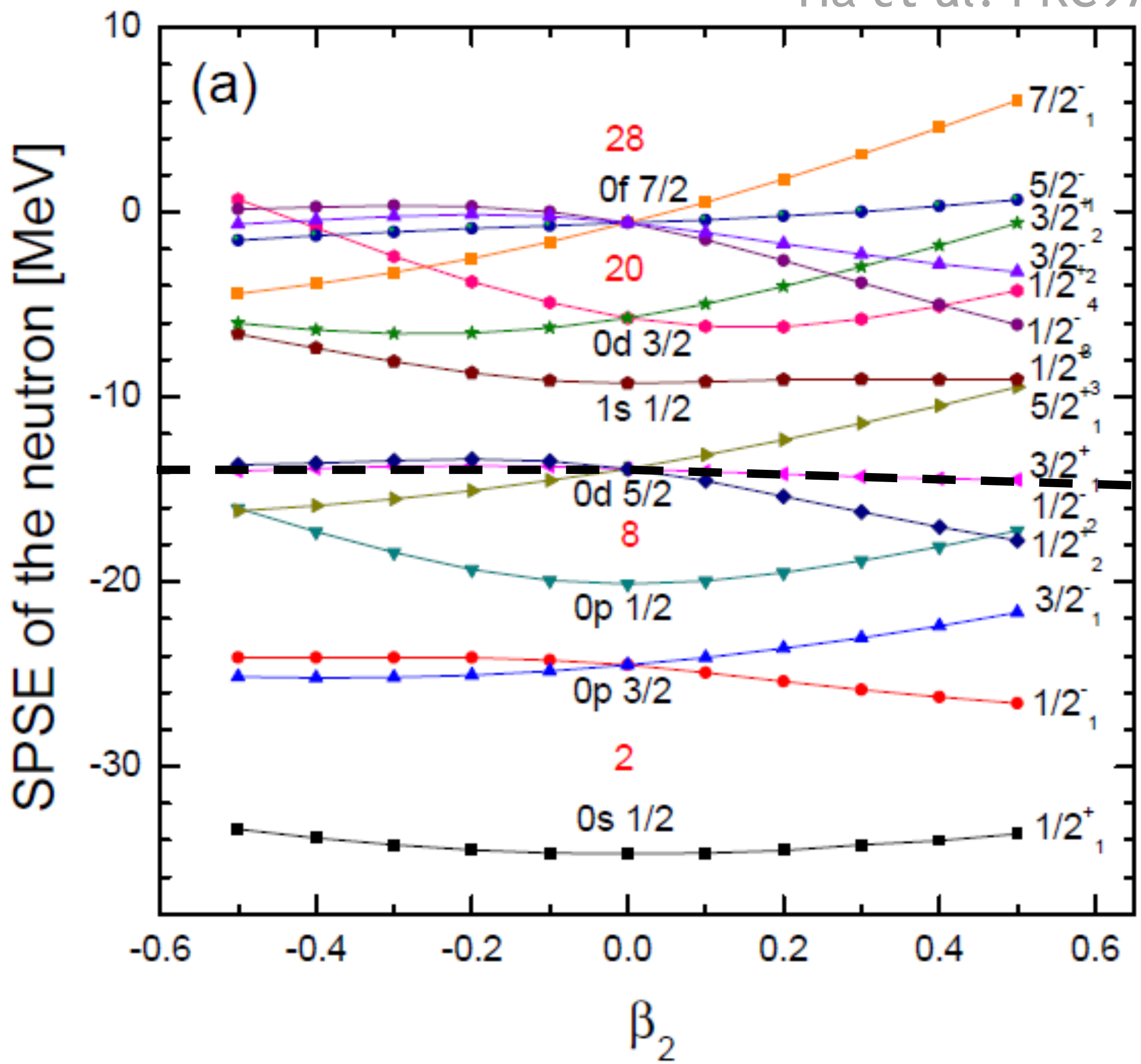
❖ References of our **recent papers**

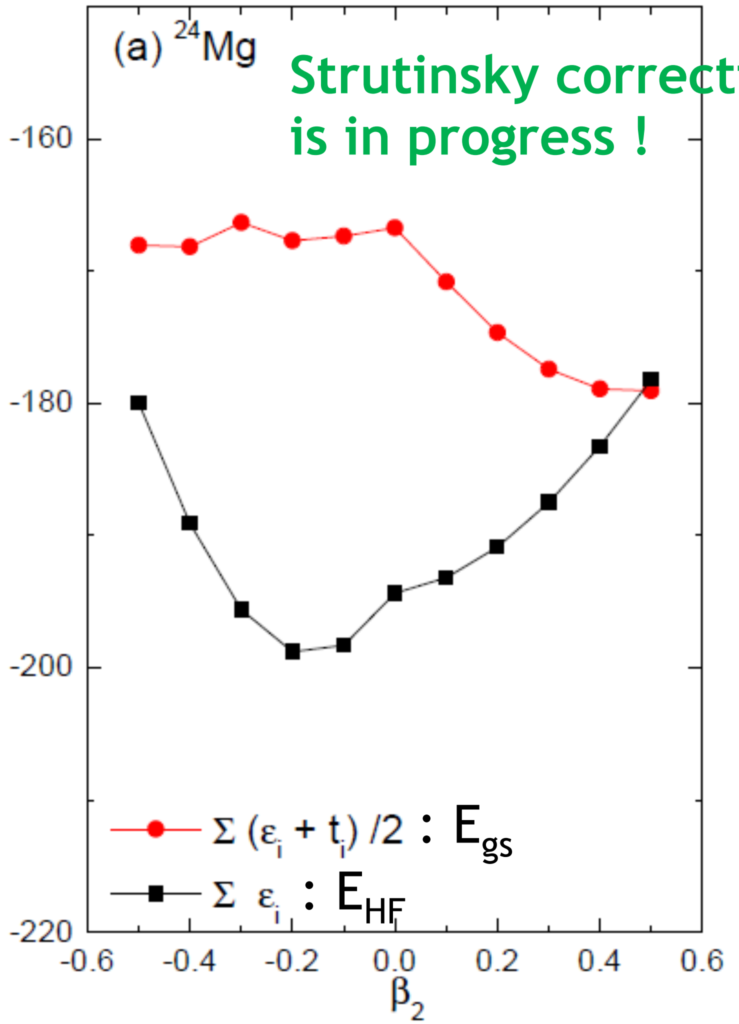
1. Spin singlet and spin triplet pairing correlations on shape evolution in
   sd-shell N=Z nuclei. Ha *et al*. PRC97,024320(2018)
2. Neutron-proton pairing correlations and deformation for N = Z nuclei
   in *pf*-shell by the deformed BCS and HFB approach.
   Ha *et al*. PRC97, 064322(2018)
3. Competition of deformation and neutron-proton pairing in Gamow-Teller
   transitions for ^{56,58}Ni. in preparation.
4. Effects of the Coulomb and the spin-orbit interaction in a deformed mean
   field on the residual pairing correlations for N=Z nuclei.
   Ha *et al*. submitted to PRC.
5. Neutron-proton pairing correlations and deformation for N = Z nuclei in
   sd- and *pf*-shell by deformed BCS and deformed QRPA

Contents

❖ Single particle state E(SPSE) by DWS of ❖ simplest shell model

Ha *et al*. PRC97,024320(2018)





Strutinsky correction is in progress !

➢ In the simplest filling-shell model, we assume that
- no smearing, which means that the occupation provability of nucleon, v², is 1 or 0.
- Fermi energy is located on the each outermost shell (black dotted line)

❖ HF energy and corrected ground state energy

$$E_{HF} = \sum_{i=1}^{A}\left(t_i + \sum_{j=1}^{A} \langle j | V | j \rangle\right)$$

$$= \sum_{i=1}^{A}\left(t_i + \langle i | V_{HF} | i \rangle\right) = \sum_{i=1}^{A} \varepsilon_i$$: Single particle E

$$E_{gs} = \sum_{i=1}^{A} t_i + \frac{1}{2}\sum_{i,j}^{A} \langle j | V | j \rangle$$

$$= \sum_{i=1}^{A}\left(t_i + \frac{1}{2}\langle i | V_{HF} | i \rangle\right) \quad \left(? \langle i | V_{HF} | i \rangle = \varepsilon_i - t_i\right)$$

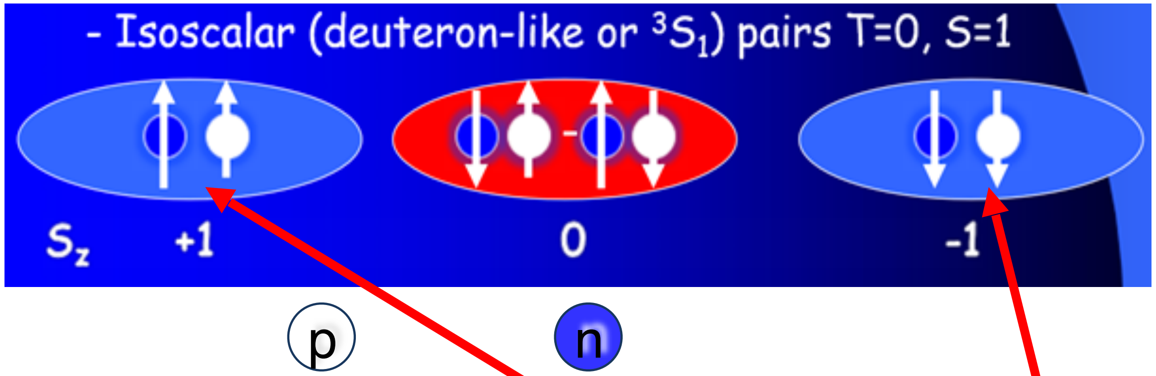
$$= \sum_{i=1}^{A}\left(t_i + \frac{1}{2}(\varepsilon_i - t_i)\right) = \boxed{\sum_{i=1}^{A}\frac{1}{2}(\varepsilon_i + t_i)}$$

$$\therefore E_{gs} \neq \sum_{i=1}^{A} \varepsilon_i$$

We consider the pairing correlations, pp, nn, and np-pairings.

❖ Enhanced T=0 pairing correlation for N=Z nuclei

T=0, S=1 (Isoscalar(IS), spin- triplet)



➢M1 spin transition data shows the IV quenching for the N = Z nuclei in *sd*-shell ; T = 0 pairing becomes more significant.
enhanced T_0 = (T=0) x1.5(IV quenching)x2 (↑↑+↓↓)

Sagawa *et al.*

enhanced T_0 = 3x(T=0)

❖ Shell evolution of ^{24}Mg

$$H = H_0 + H_h$$

$$H_0 = T + V_W \quad (V_c + V_S + V_b)$$

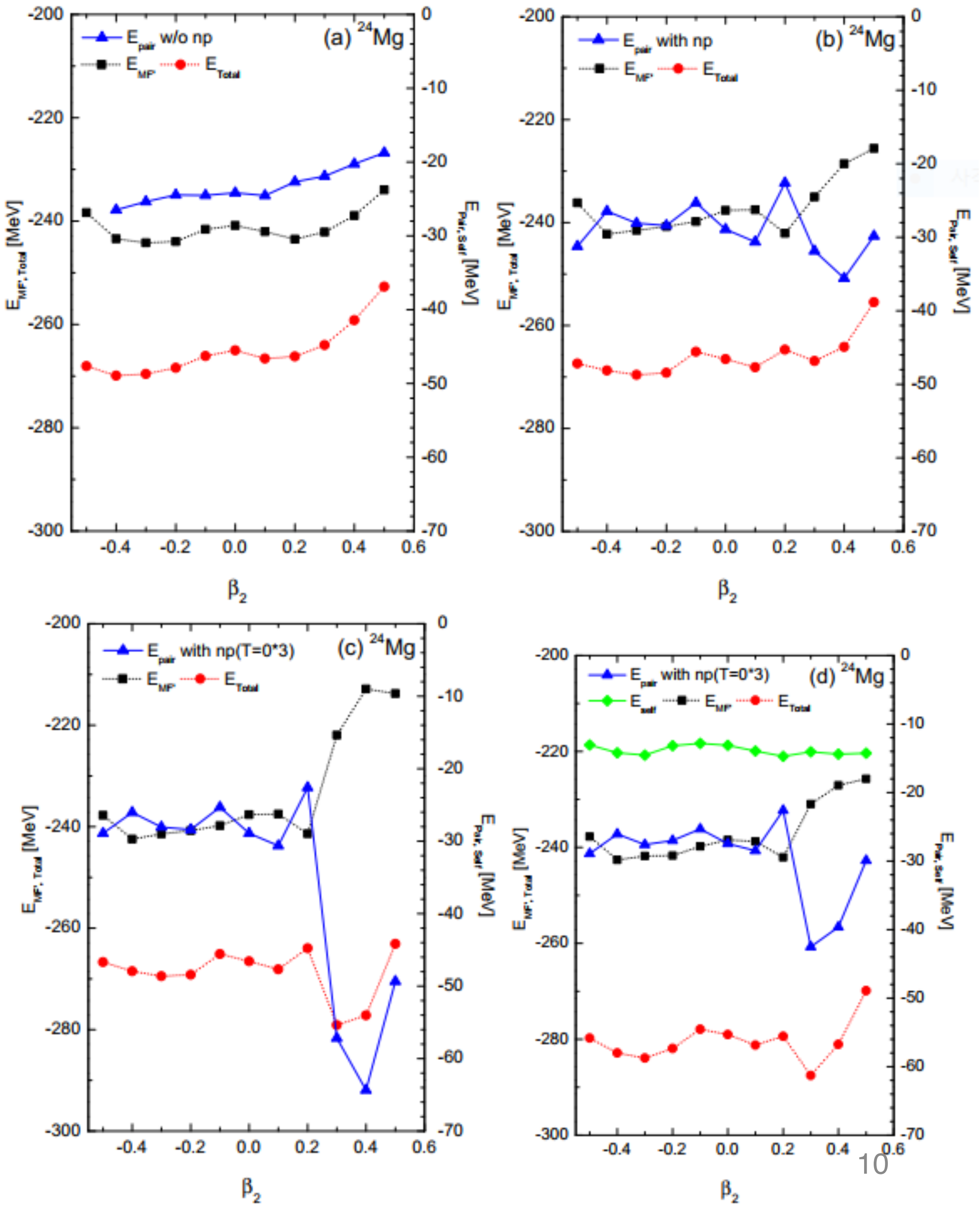
$$E_b = E_M + E_p + E_s$$

(a) without np-pairing
(b) with np-pairing
(c) with enhanced T=0
(d) with enhanced T=0 + self E

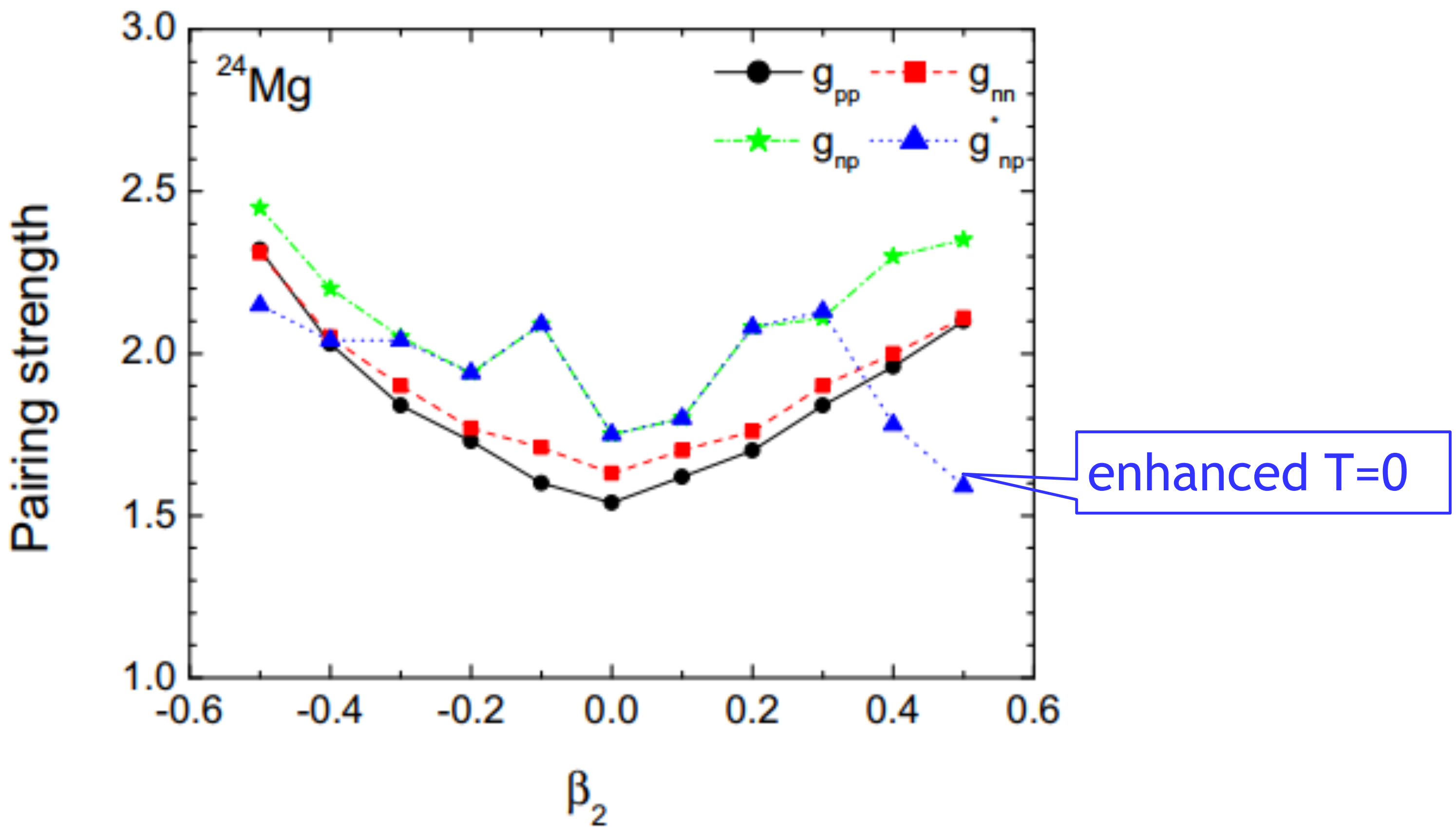
➢ T=0 contribution makes the bounding more stronger due to its attractive property.

➢ **Enhanced IS *np* pairing correlations** may be an indispensable ingredient to understand the prolate deformation.

Nucleus	β_2^{E2} [34]	β_2^{RMF} [35]	β_2^{FRDM}
^{24}Mg	0.605	0.416	0.



❖ Evolution of pairing strength of ^{24}Mg

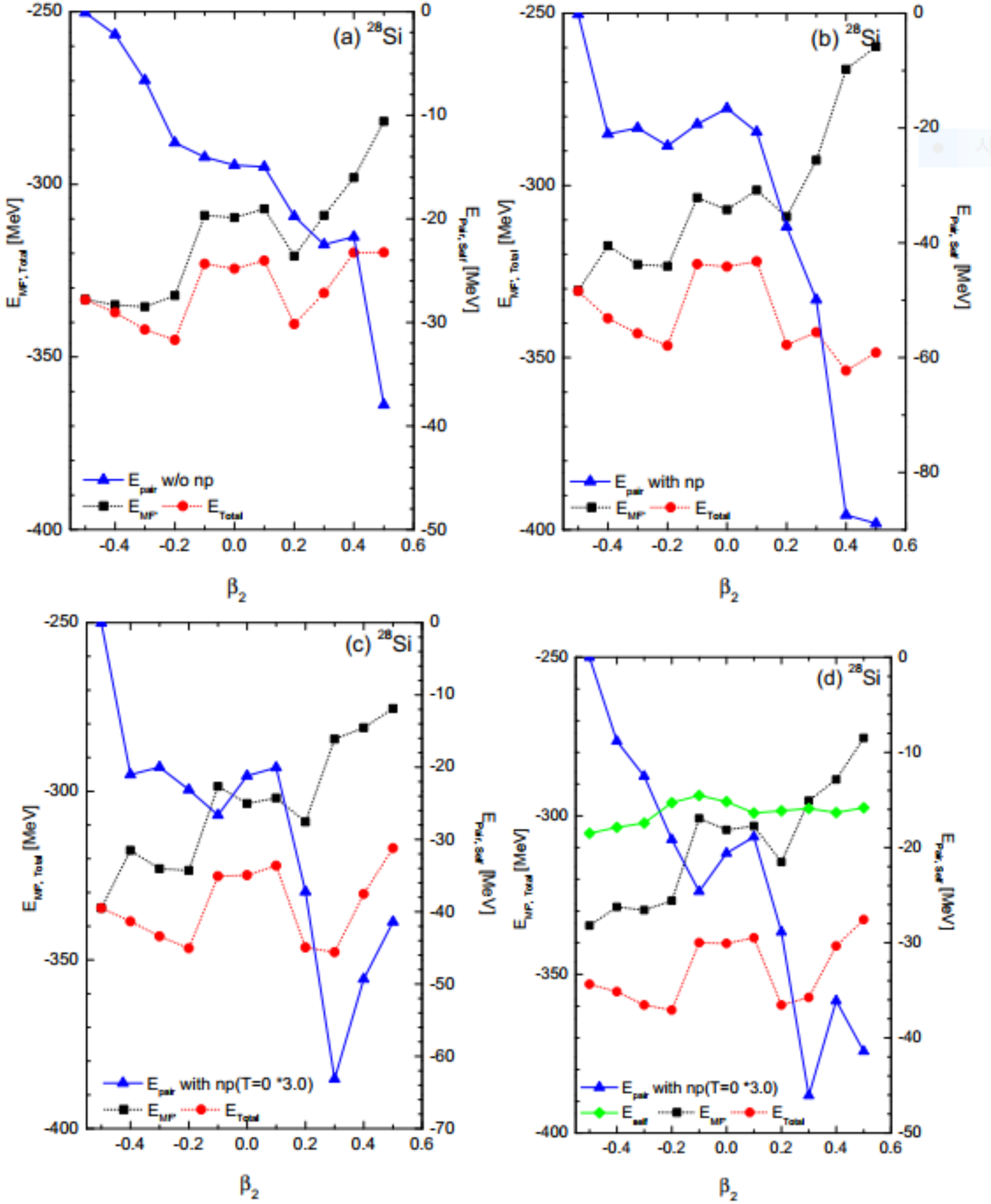


- ➢ g_{np}^* becomes smaller in $|\beta_2| > 0.3$, that is, **the smaller g_{np}^* we have , the larger pairing energy is obtained.**
- ➢ There can be T=0 pairing **(Isoscalar) condensation** in large deformation.
- ➢ There is the coexistence of T=0 and T=1 pairing in $|\beta_2| > 0.3$.

❖ Shell evolution of ^{28}Si

Nucleus	β_2^{E2} [10]	β_2^{RMF} [11]	β_2^{FRDM} [12]
^{24}Mg	0.605	0.416	0.
^{28}Si (prolate)	0.407	x	x
^{28}Si (oblate)	x	− 0.374	− 0.363
^{32}S	0.312	0.186	0.221

- ➢ ^{28}Si can be **oblate deformed** by the strong T = 0 pairing correlations with self energy.

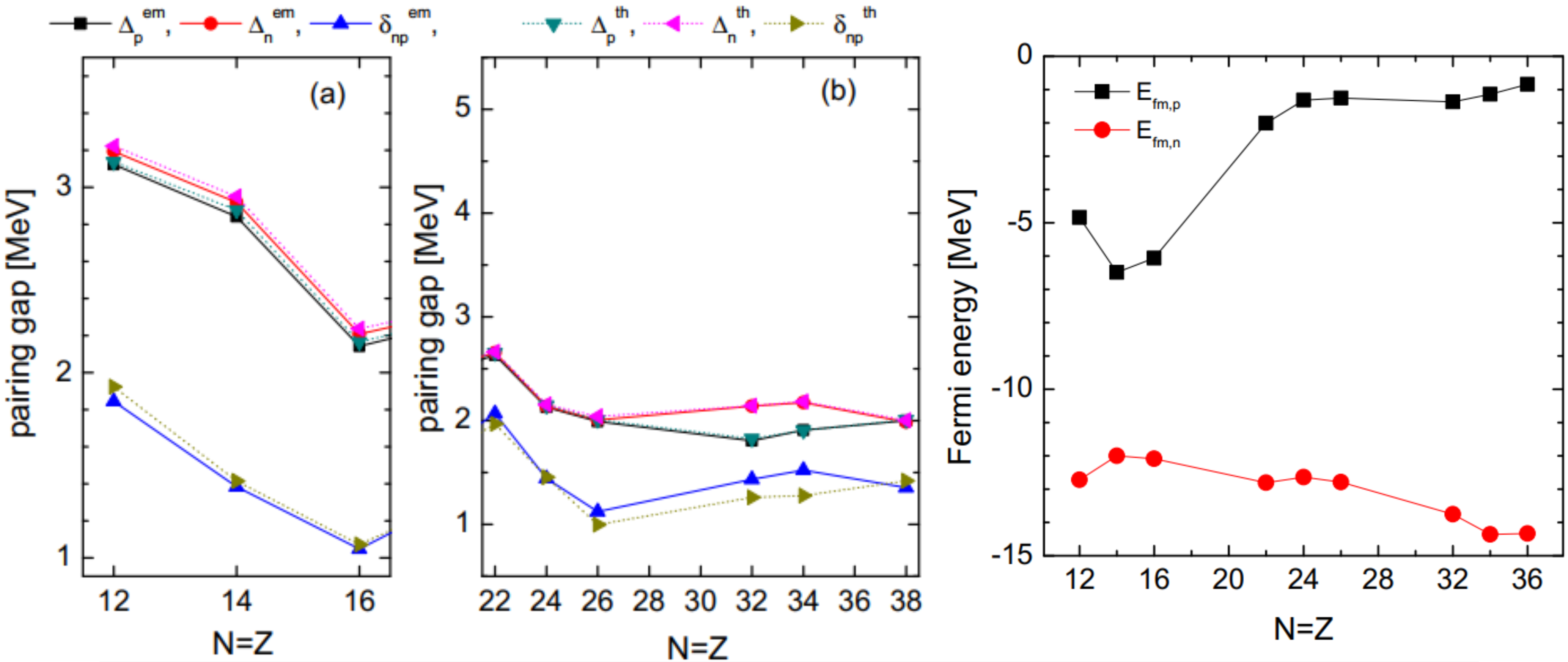


❖In pf-shell N=Z nuclei

Ha *et al*. PRC97, 064322(2018)

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{44}Ti	0.268	0.000	0.011	2.631	2.653	2.068
^{48}Cr	0.368	0.225	0.226	2.128	2.138	1.442
^{52}Fe	0.230	0.186	–0.011	1.991	2.007	1.122
^{64}Ge	0.250	0.217	0.207	1.807	2.141	1.435
^{68}Se	–0.250	–0.285	0.233	1.909	2.174	1.522
^{72}Kr	–0.350	–0.358	–0.366	2.001	1.985	1.353

❖ Pairing gaps & Fermi E evolution in sd- & pf-shell N=Z nuclei

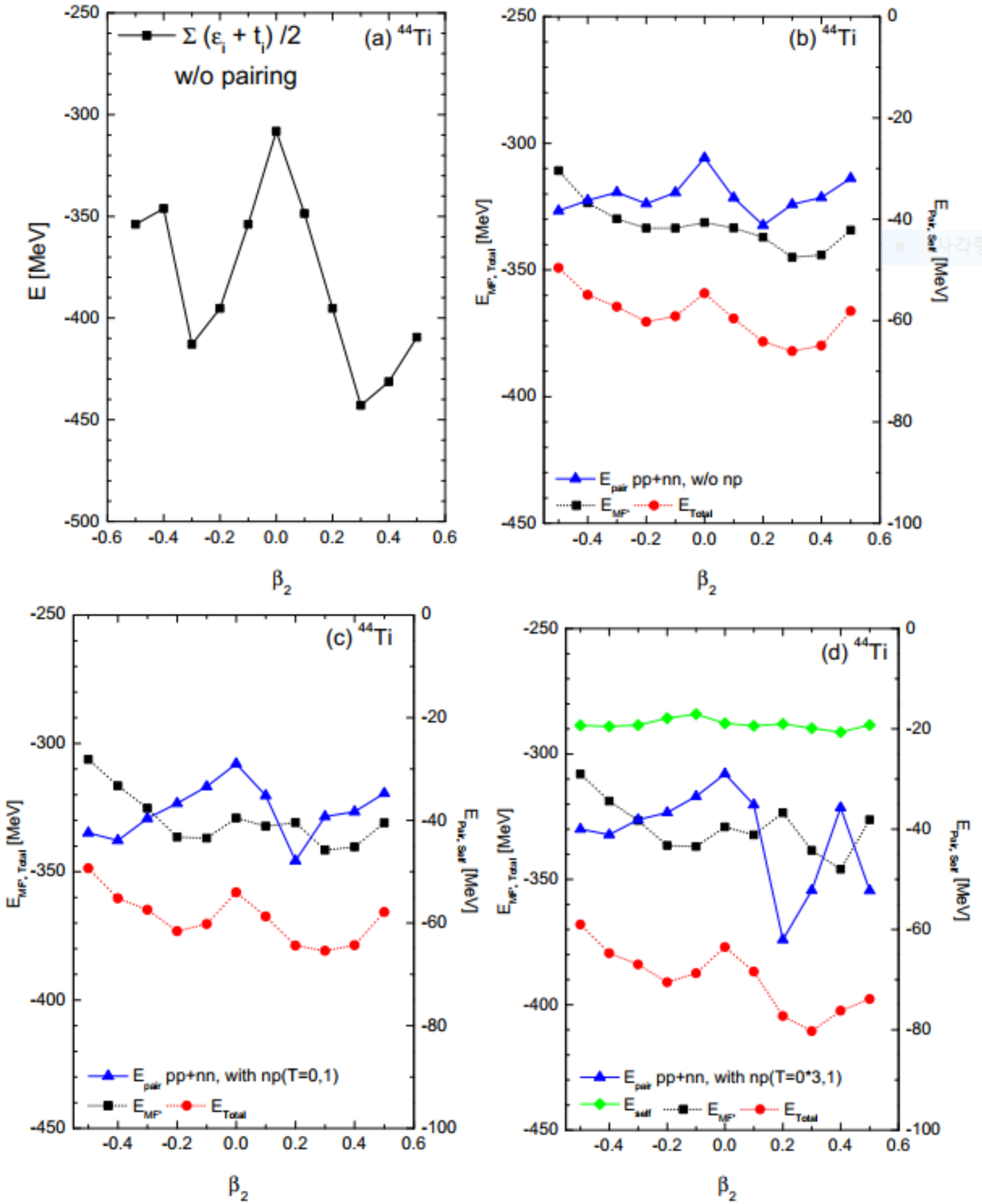


- ➢ Empirical pairing gap by five mass formula.
- ➢ Theoretical pairing gaps are adjusted to reproduce the empirical pairing gaps. Specifically, np-pairing gaps are almost saturated in pf-shell N=Z nuclei.
- ➢ The gap between proton and neutron Fermi E increases as the number of mass increases.

❖ Shell evolution of ^{44}Ti

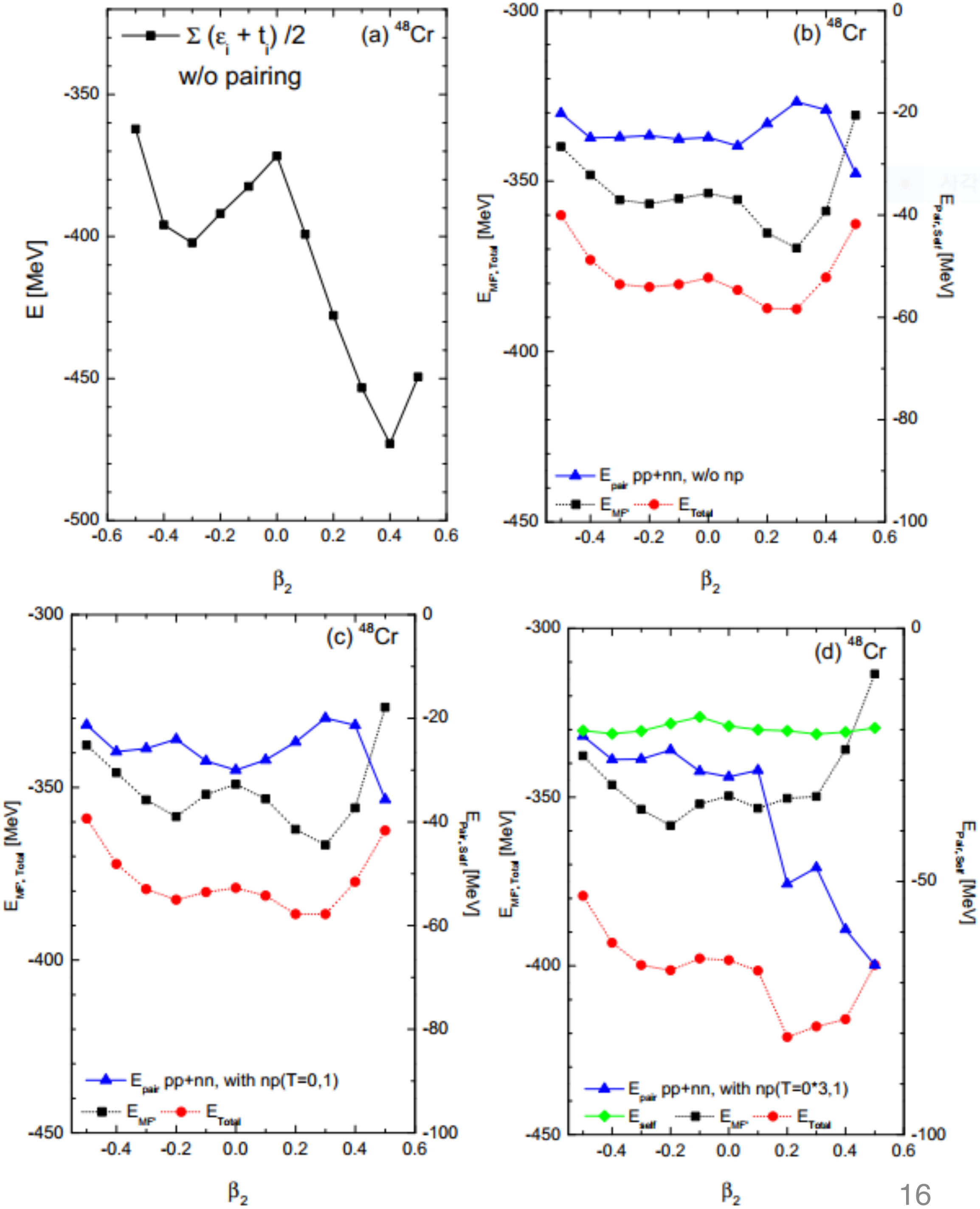
Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366

The enhanced T=0 force makes
the bounding more stronger.



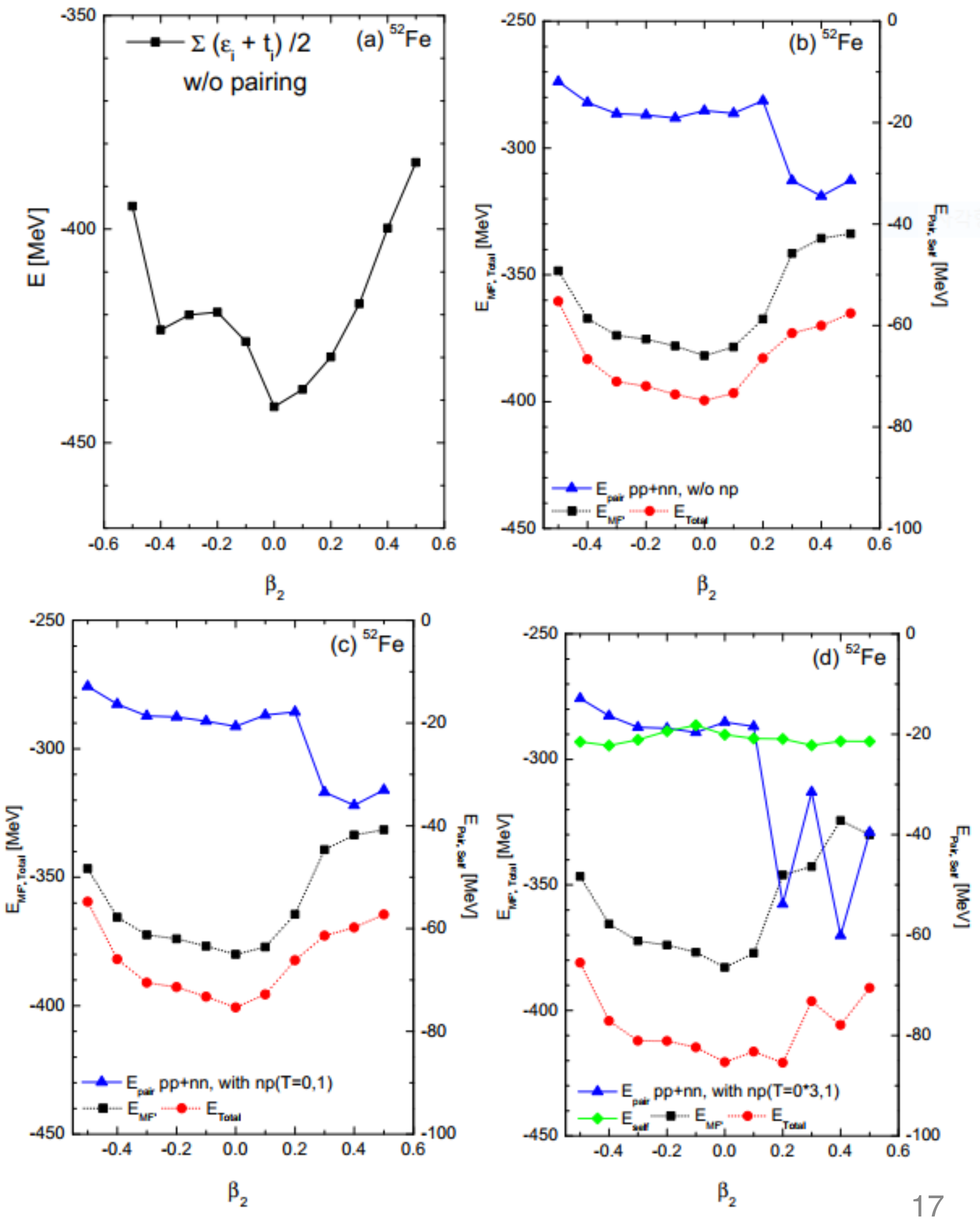
❖ Shell evolution of ^{48}Cr

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



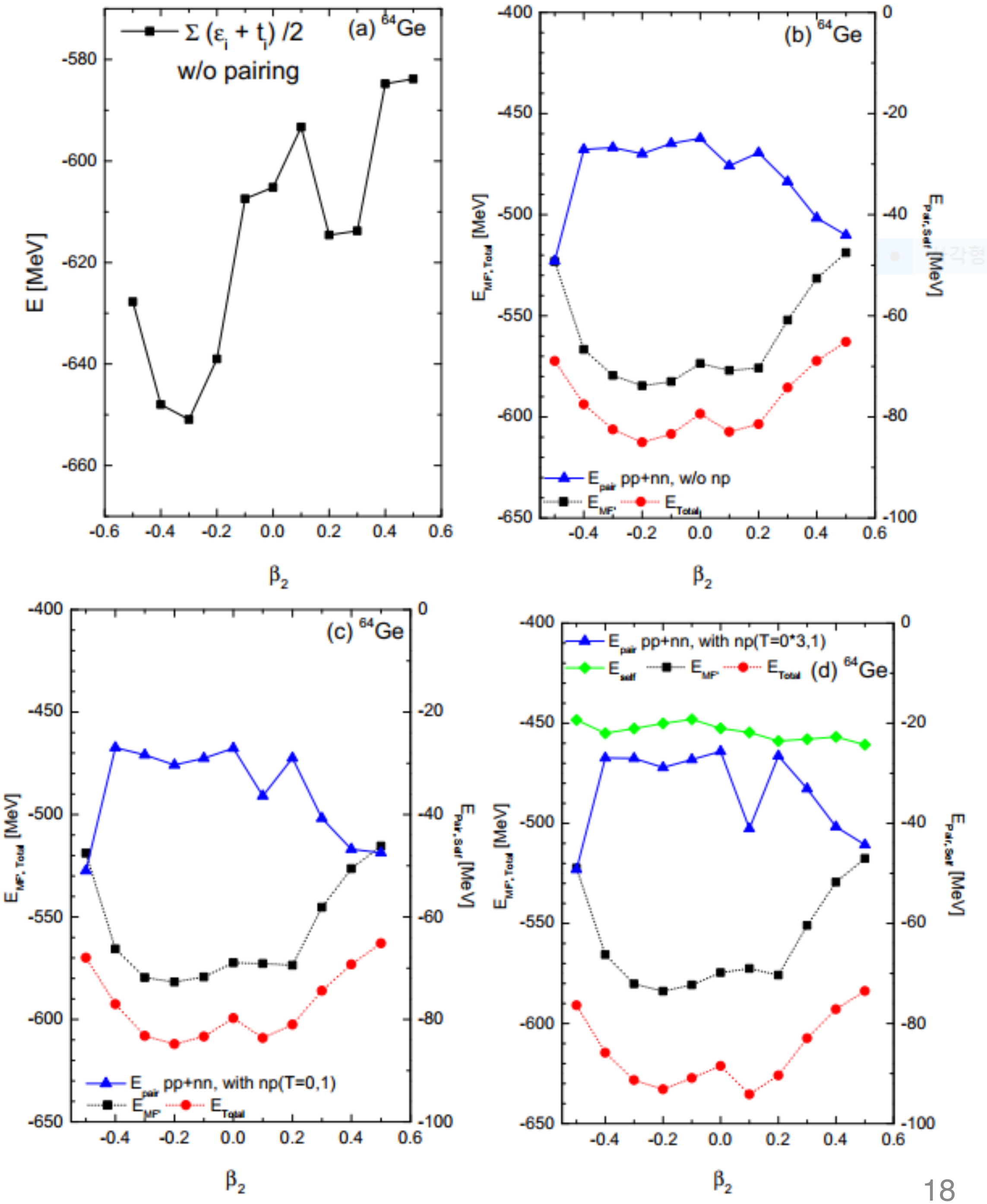
❖ Shell evolution of ^{52}Fe

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	–0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	–0.250	–0.285	0.233
^{72}Kr	–0.350	–0.358	–0.366



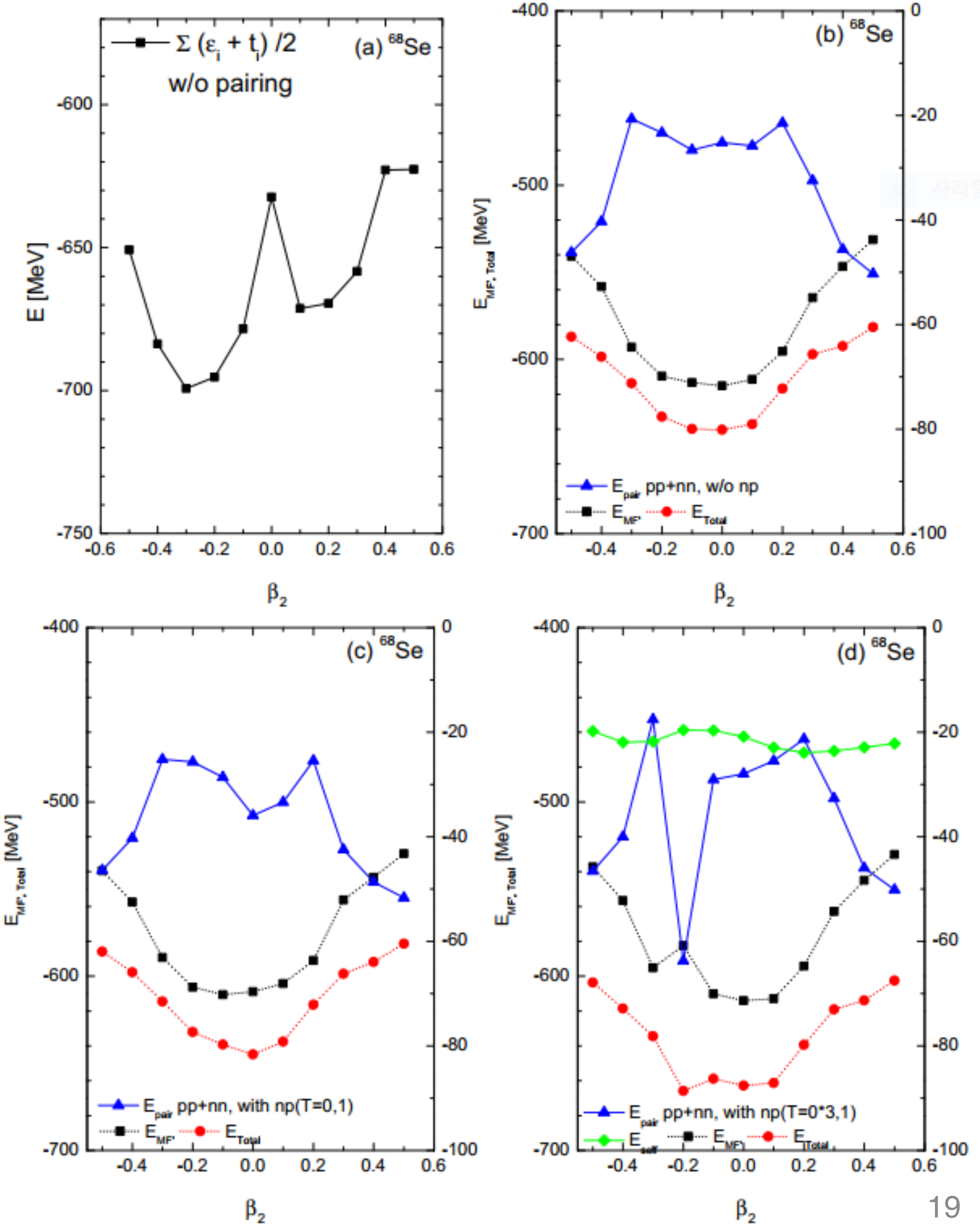
❖ Shell evolution of ^{64}Ge

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



❖ Shell evolution of ^{68}Se

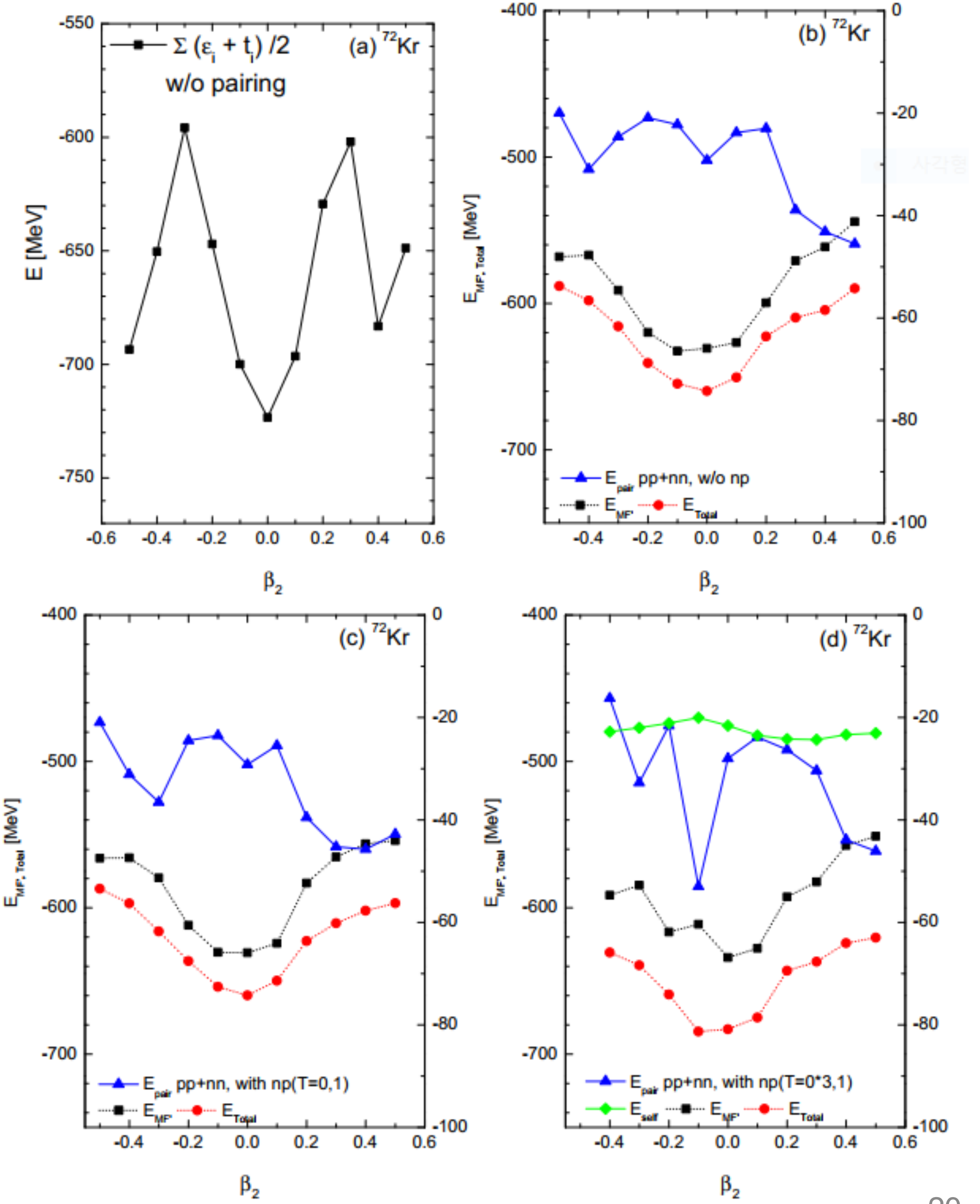
Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



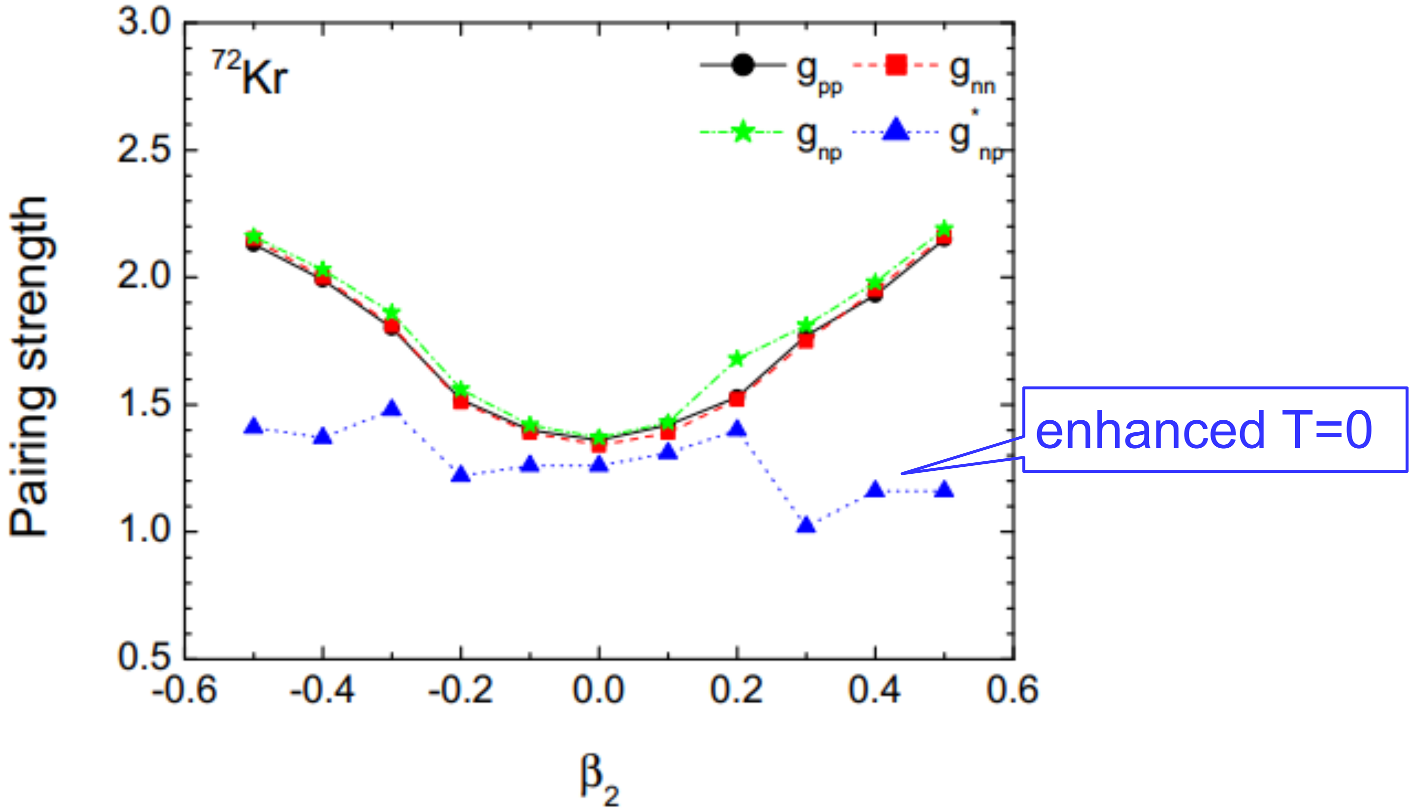
Even the oblate deformation can be explained by the unlike-pairing correlations !

❖ Shell evolution of ^{72}Kr

Nucleus	β_2^{E2} [9]	β_2^{RMF} [10]	β_2^{FRDM} [11]
^{44}Ti	0.268	0.000	0.011
^{48}Cr	0.368	0.225	0.226
^{52}Fe	0.230	0.186	−0.011
^{64}Ge	0.250	0.217	0.207
^{68}Se	−0.250	−0.285	0.233
^{72}Kr	−0.350	−0.358	−0.366



❖ Evolution of pairing strength of ^{72}Kr

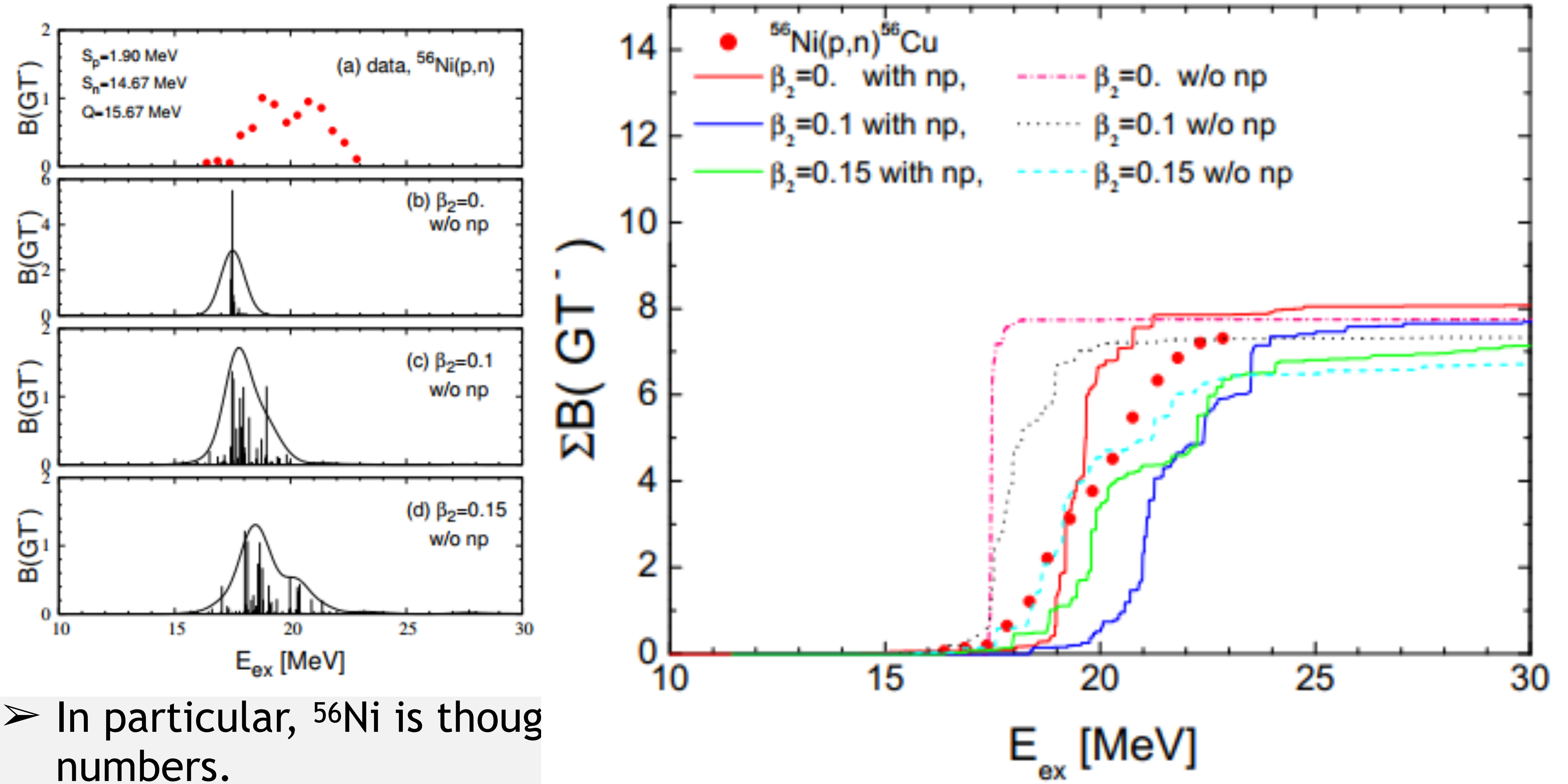


➢ There is also the **coexistence of T=0 and T=1** pairing at large deformation similarly to *sd*-shell N=Z nuclei.

Contents

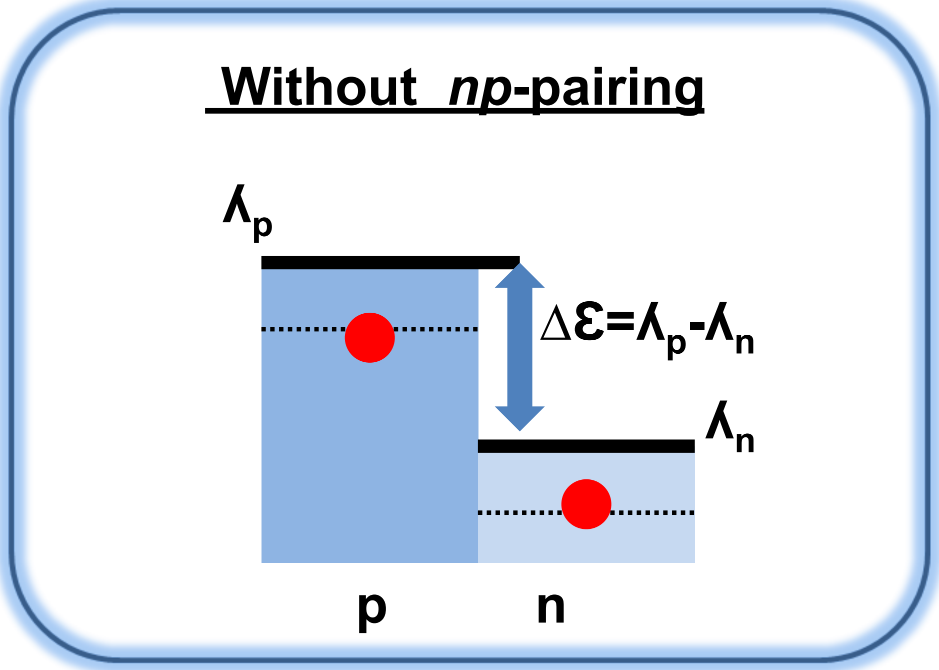
❖ Gamow-Teller strength for ^{56}Ni

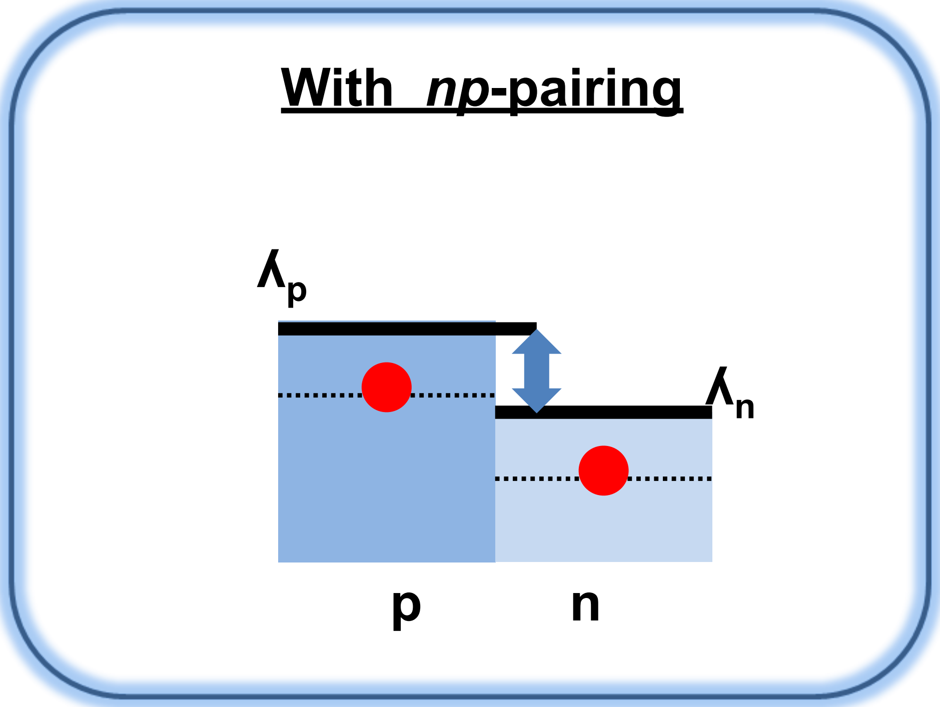
Ha *et al.* accepted to PRC



- ➢ In particular, ^{56}Ni is thoug numbers.
- ➢ If we take α-cluster model for ^{56}Ni, the ground state may be slightly deformed. PRC 84, 024302(2011)
- ➢ The *np* pairing effects turn out to be able to properly explain the GT strength although the deformation is also another important property. **The high-lying GT peak in the two peaks stems from the repulsive np pairing through the reduction of Fermi energies of protons and neutrons**

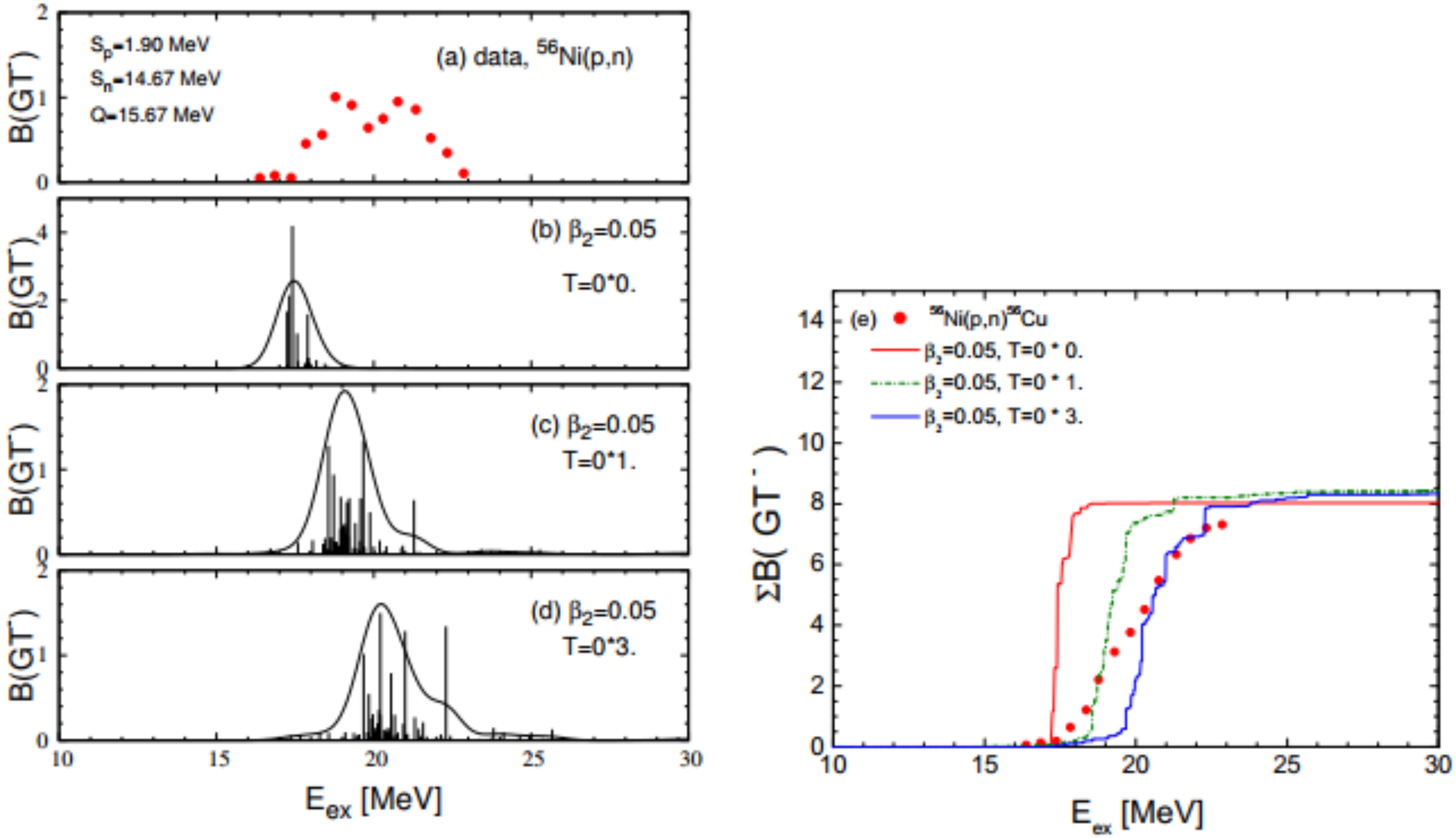
❖ proton and neutron Fermi E





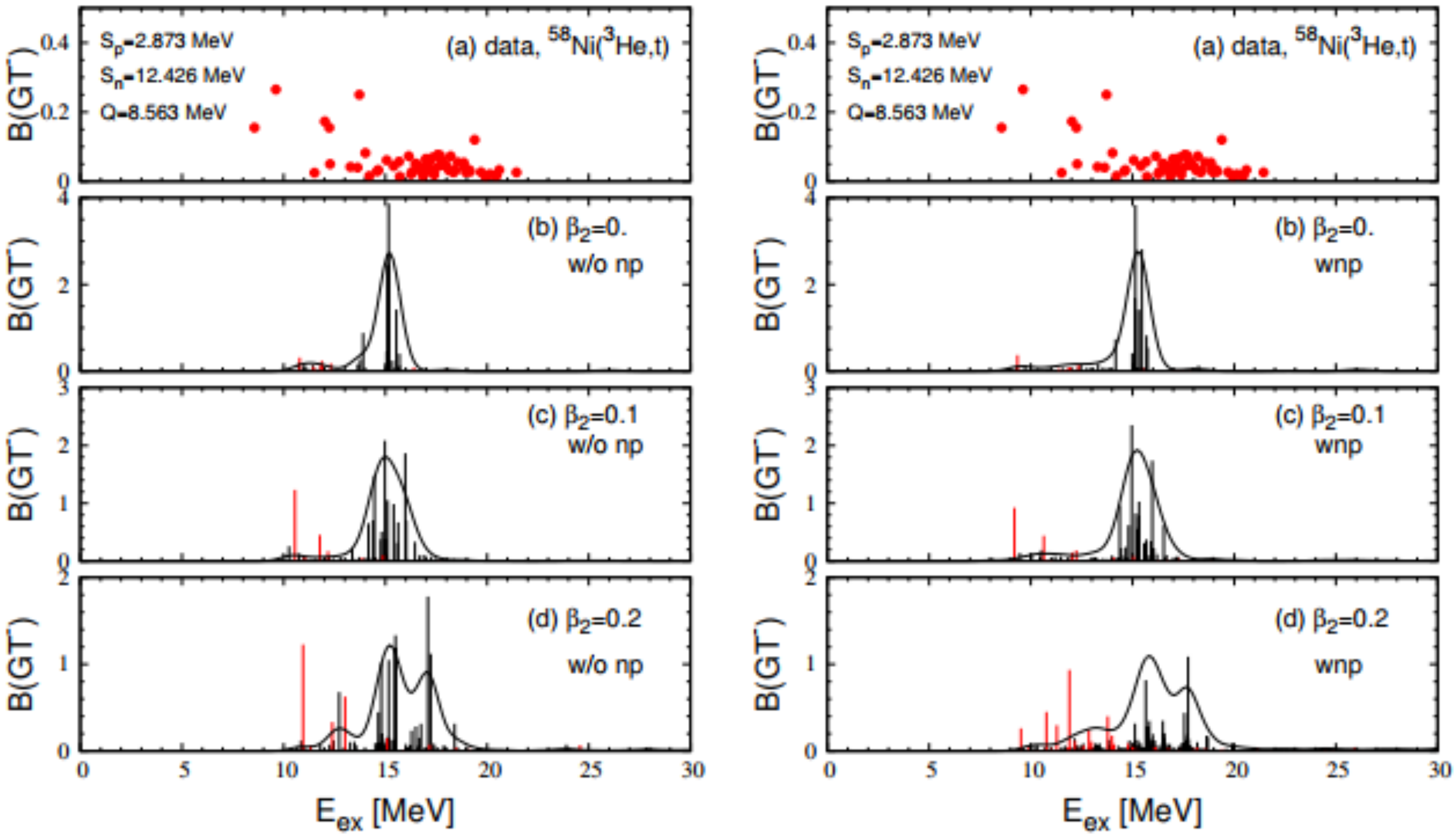
The np pairing makes the Fermi energy difference small, which can induce the GT transition more effectively and give rise to the high-lying GT states !!!

❖ IS *np* pairing effects on B(GT)



➢ The shift of the GT strength distributions by the enhanced T=0 *np* pairing is mainly attributed to the IS coupling condensation. Even with the small deformation, the second peak appears by the T=0 pairing.

❖ Gamow-Teller strength and IAR for ^{58}Ni (N:



- ➢ The *np* pairing makes the IAR(isobaric analogue resonance) concentrated around 12 MeV, which is consistent with the results in PRC 69(2004) at $\beta_2 = 0.2$.
- ➢ The deformation effect turned out to be more important rather than the *np* pairing correlations since the *np* pairing effects become the smaller with the increase of *N* - *Z* number. Some spurious states peculiar to QRPA lead to small distribution of IAR state.

Contents

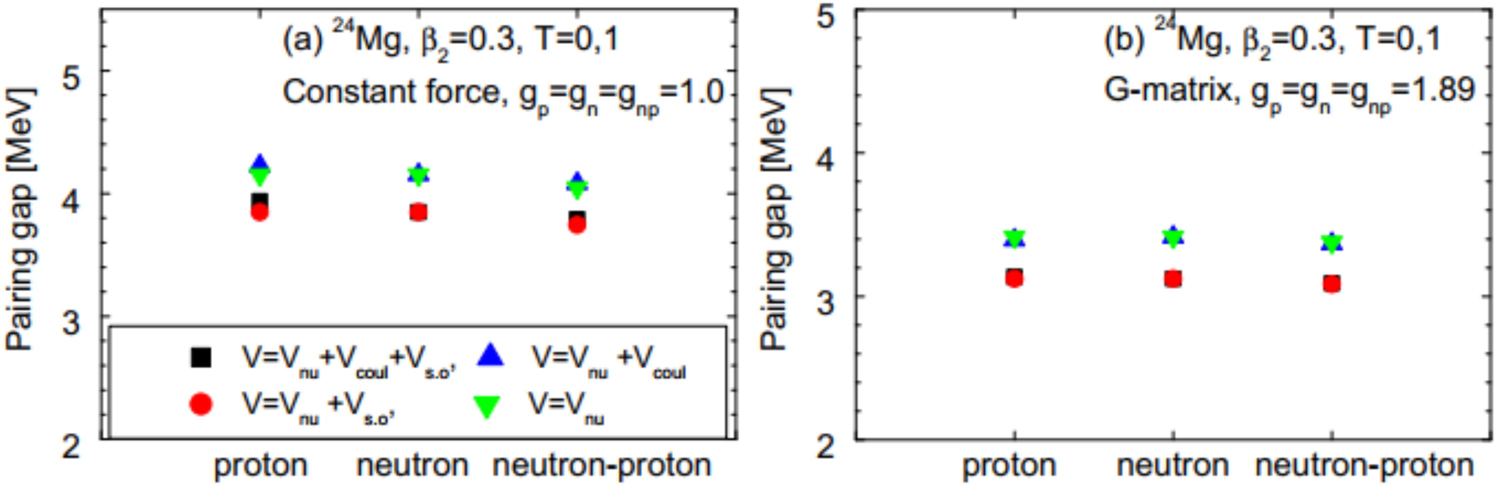
We investigated how the Coulomb and SO interaction as well as the deformation affect on the residual pairing correlations.

- Constant PME(pairing matrix element): the pairing under the Wigner spin-isospin SU(4) symmetry in the absence of the Coulomb and the spin-orbit interaction.
- Brueckner G-Matrix PME : state dependent taken into account shell structure effects, the realistic description of ground state.

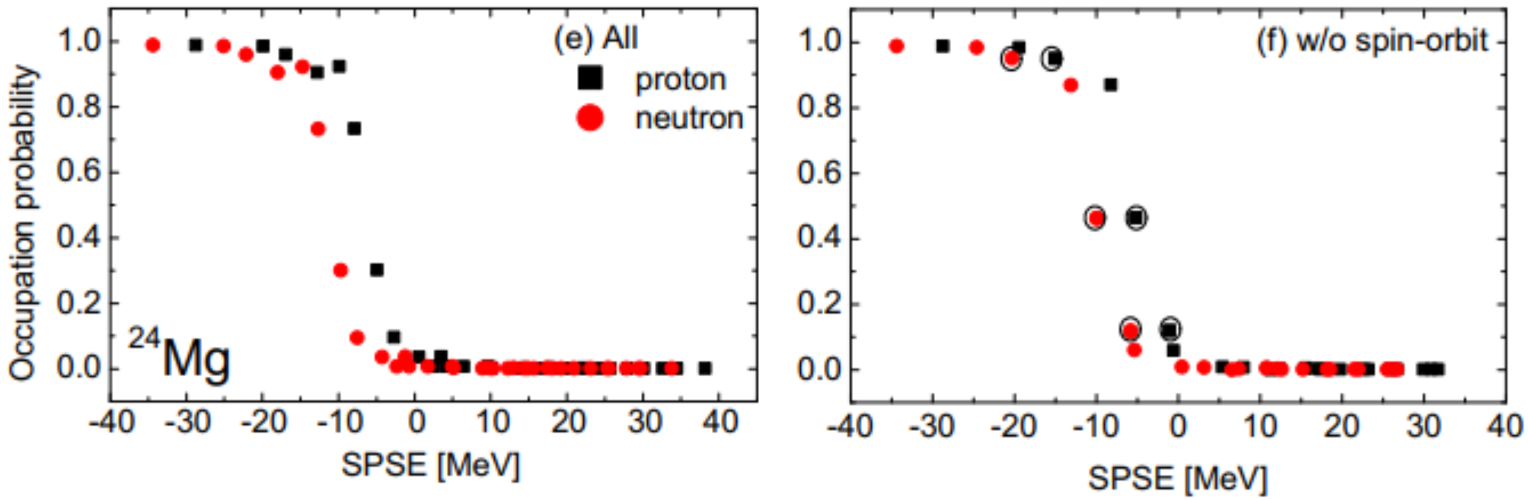
❖ Pairing gaps of pp, nn, and np for *sd*-shell N=Z ^{24}Mg

Ha *et al*. submitted to PRC

$$H = H_0 + H_h, \quad H_0 = T + (V_n + V_{s.o} + V_c)$$

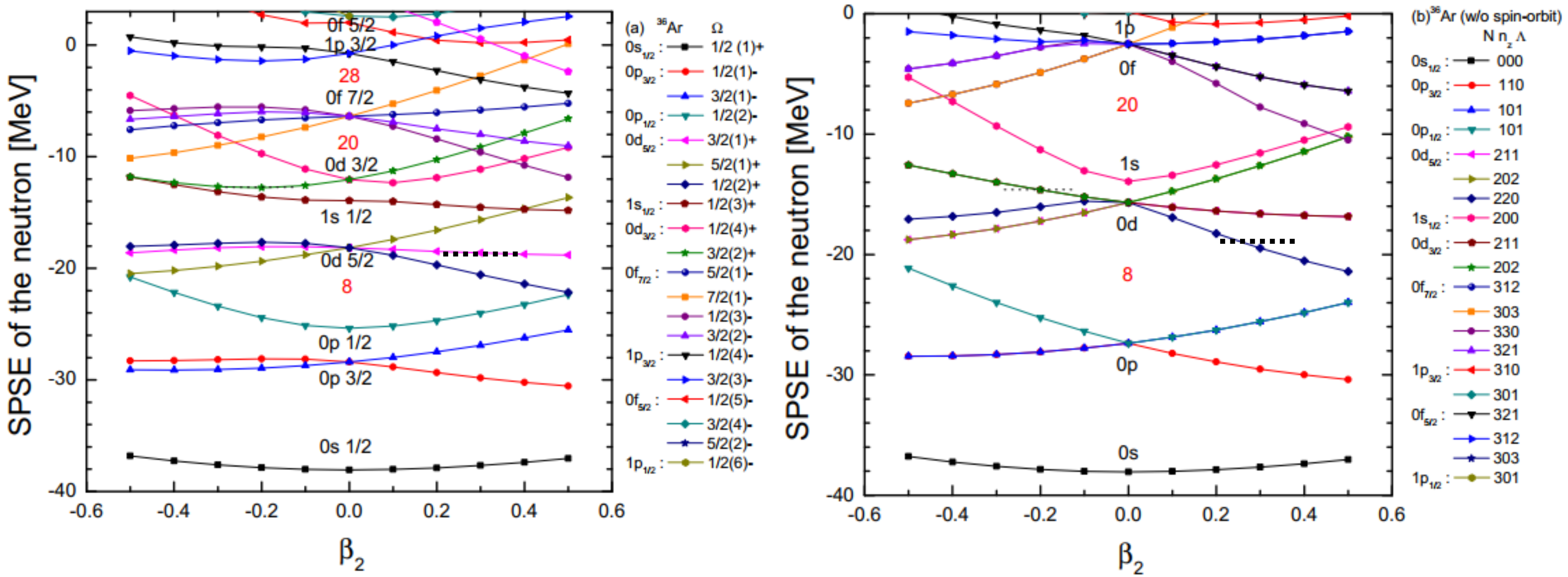


- ➢ The charge independence symmetry is approximately conserved for ^{24}Mg.
- ➢ Coulomb force does not affect the pairing gaps !!



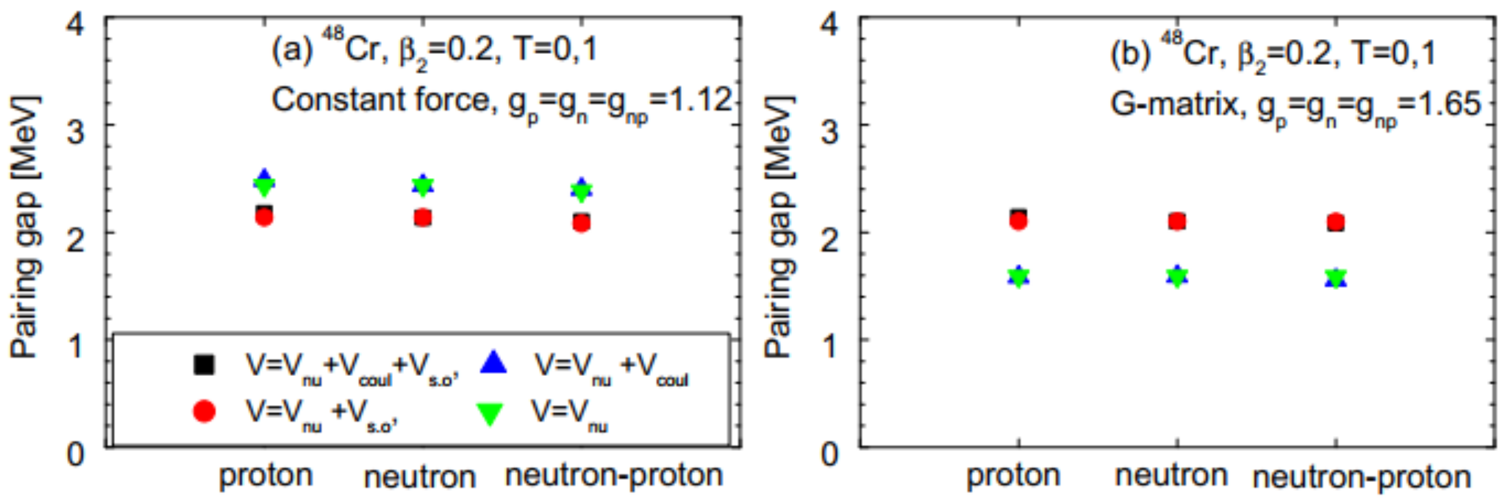
- ➢ The smearing of the Fermi surface increases when there is no SO force, which increases the pairing gap.

❖ Shell evolution with(without) spin-orbit(SO) force in ^{24}Mg

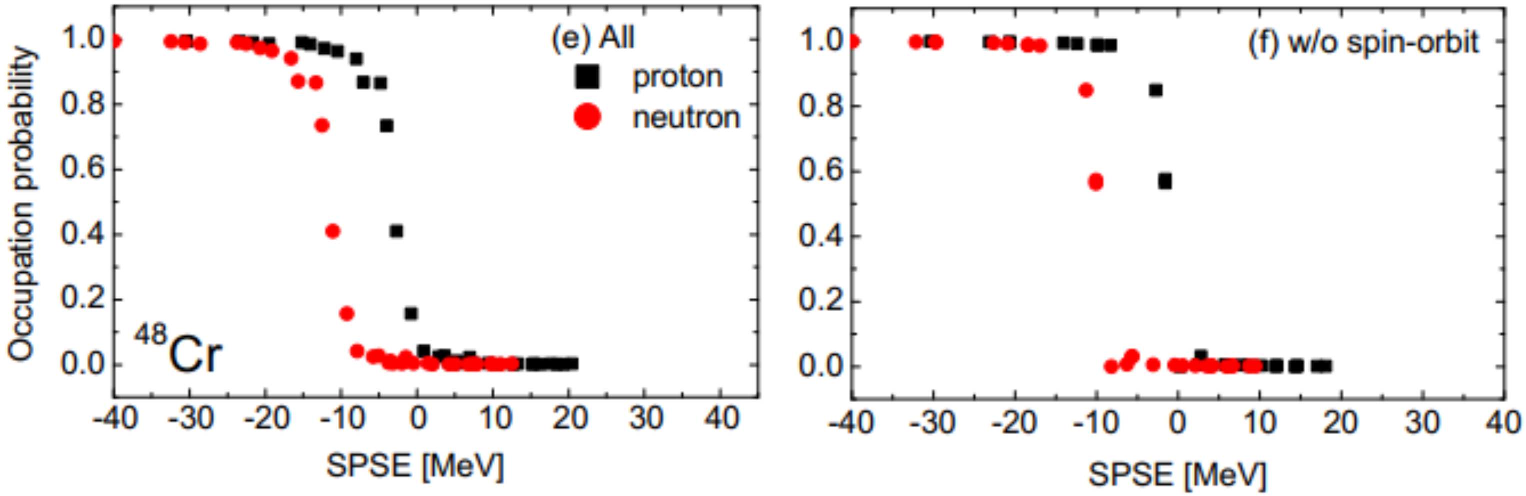


➢ Why does the pairing gaps grow without the SO?
: Many degenerate states appear w/o the SO force, $0d_{3/2}$+ $0d_{5/2}$ → 0d, which keep more
particles in the smearing region and makes more smearing and larger pairing gaps.

❖ in *pf*-shell N=Z nuclei ^{48}Cr

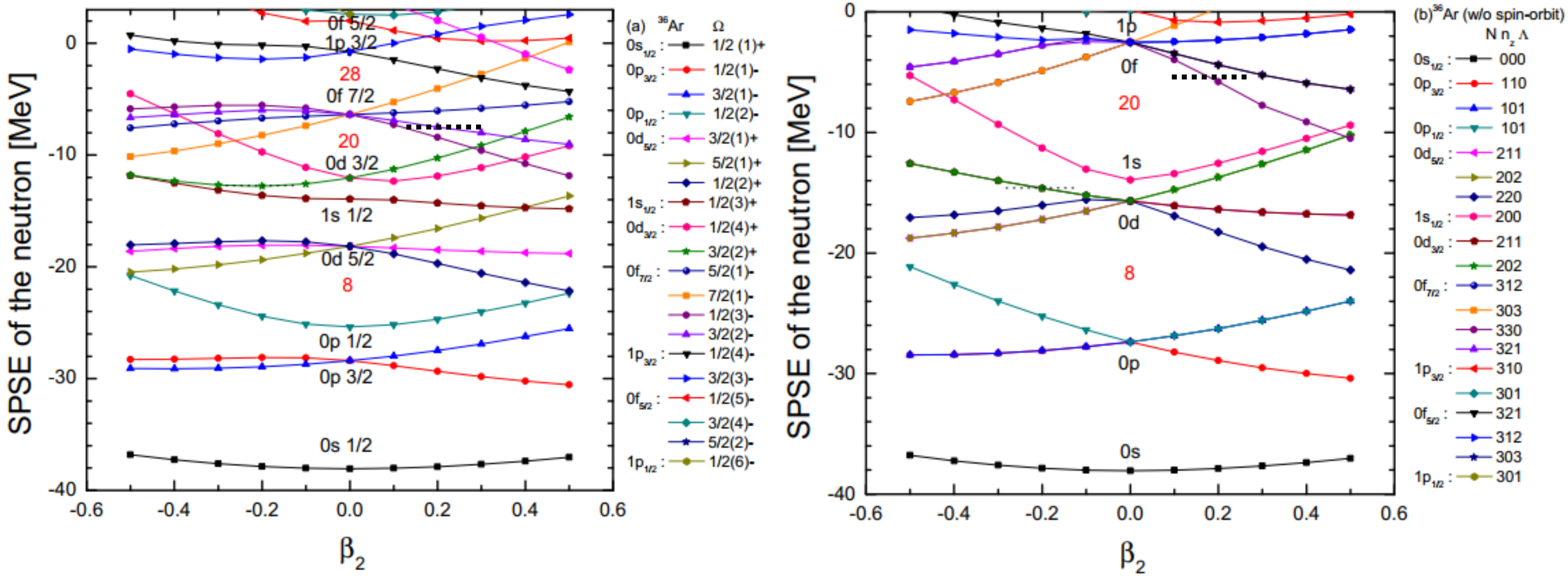


- ➢ The charge independence symmetry is approximately conserved for ^{48}Cr.
- ➢ Coulomb force also does not affect the pairing gaps !!



- ➢ The SO force increases the smearing at the Fermi surface, which increases the pairing gap.

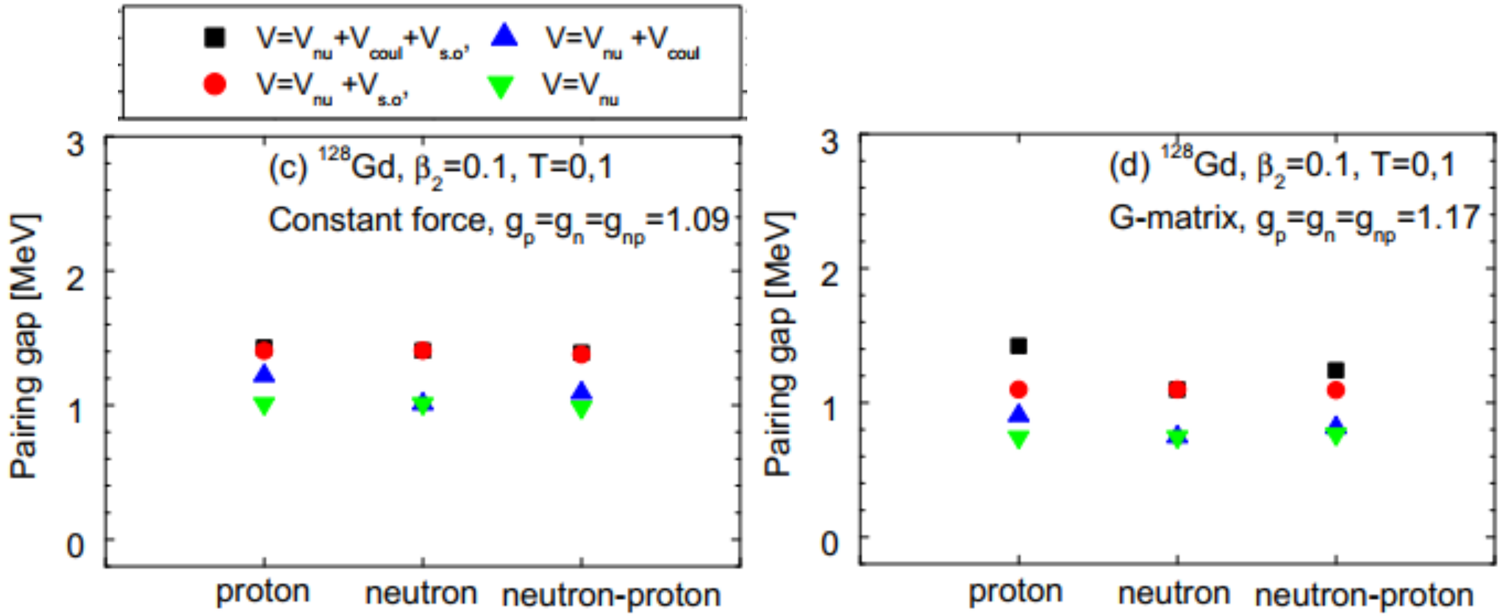
❖ Shell evolution with(without) spin-orbit(SO) force in ^{48}Cr



➢ Why does the pairing gaps grow with the SO?

: By the SO force, the occupation probability by the $3/2_2^+$ and $1/2_4^+$ in $0d_{3/2}$ shell increase and makes more smearing and larger pairing gaps.

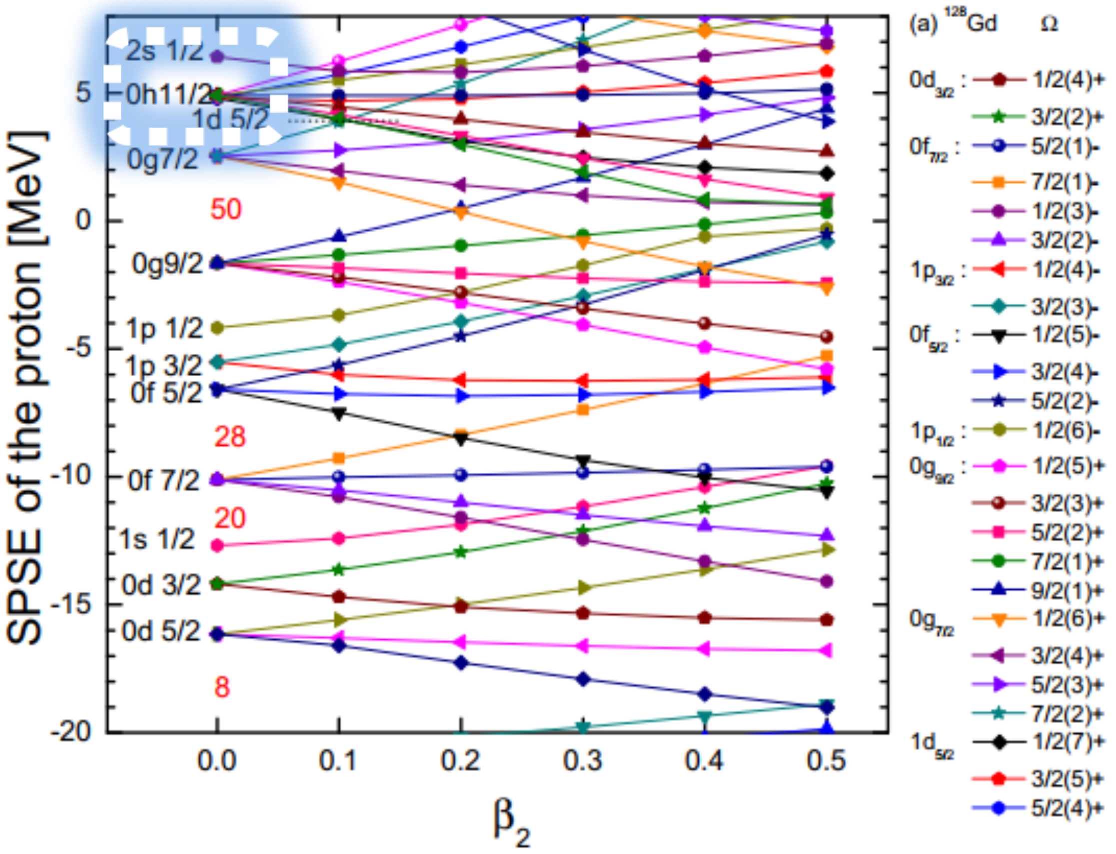
❖ in *sdgh*-shell N=Z nuclei ^{128}Gd



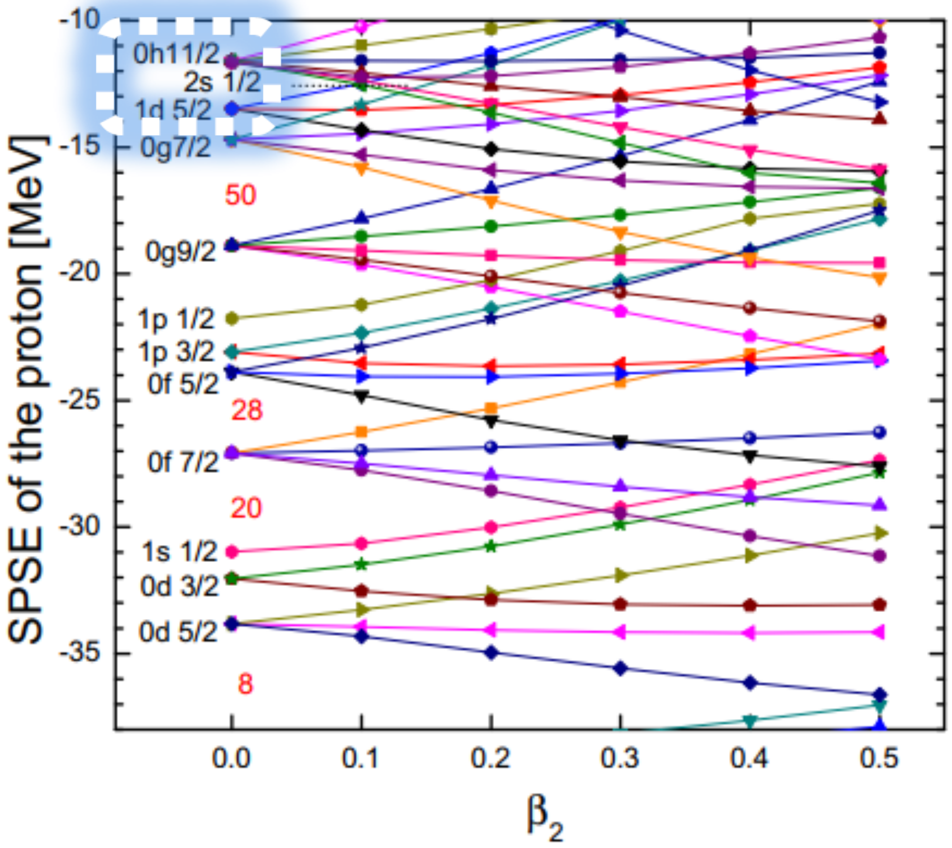
- ➢ The CF does not affect the pairing gaps at least *sd*-shell nuclei but the CF increases the pairing gap with G-Mat PME for heavy nucleus.
- ➢ The SO force also increase the pairing gaps for two PME.
- ➢ For heavy nuclei, the CF can be more important rather than the SO force.

❖ Reordering of SPSE in ^{128}Gd by the Coulomb force

- **With Coulomb force**



- **Without Coulomb force**

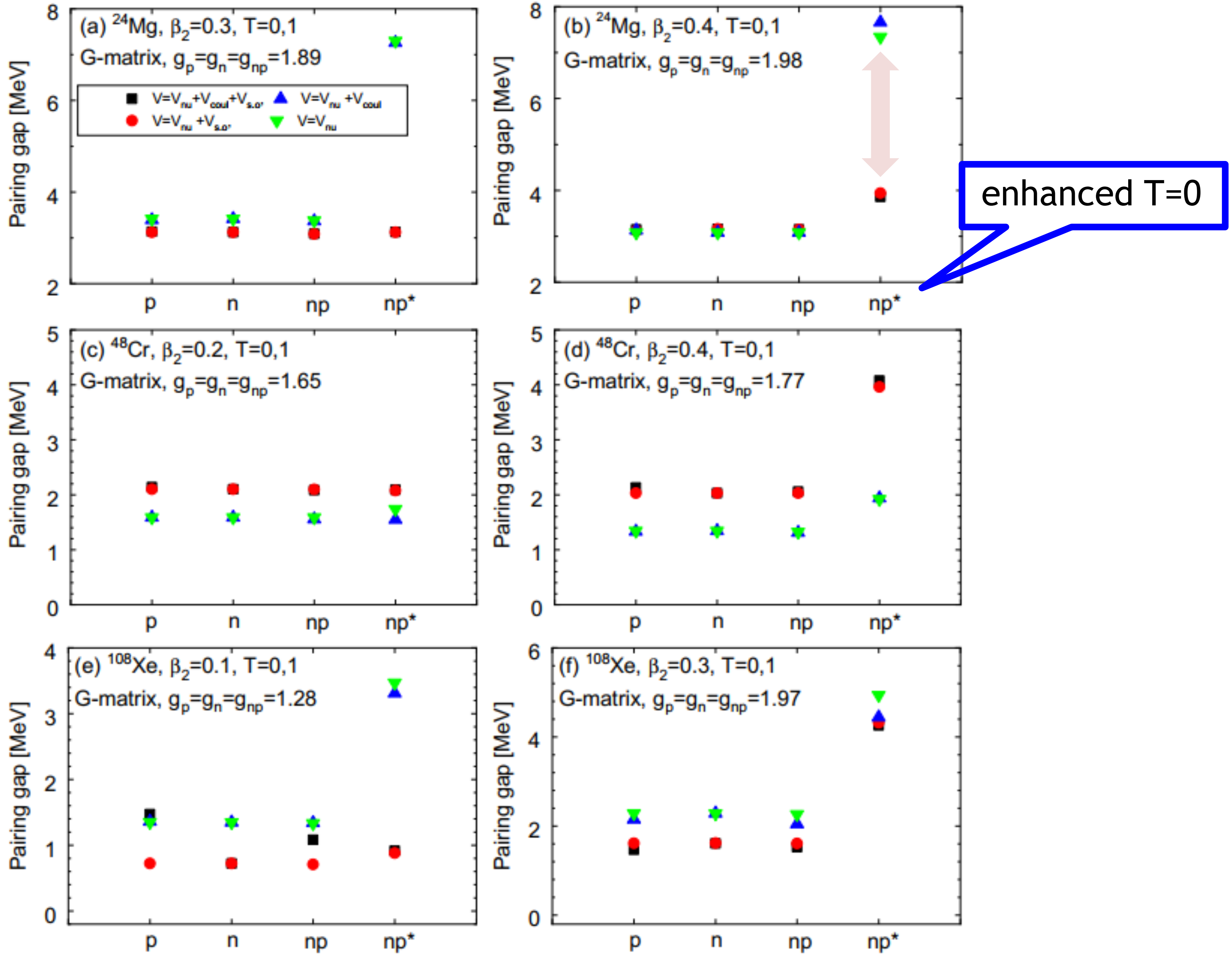


➢ The reordering of *sdgh*-shell near to Fermi surface by the Coulomb force(CF) affect the pairing gaps.

➢ $0h_{11/2}$ and $1d_{5/2}$ are almost overlapped by CF, which increase the occupation probability.

➢ The large smearing by the CF makes a large pairing gap

❖ Competition of isoscalar(IS) and isovector(IV) pairing in N=Z nuclei

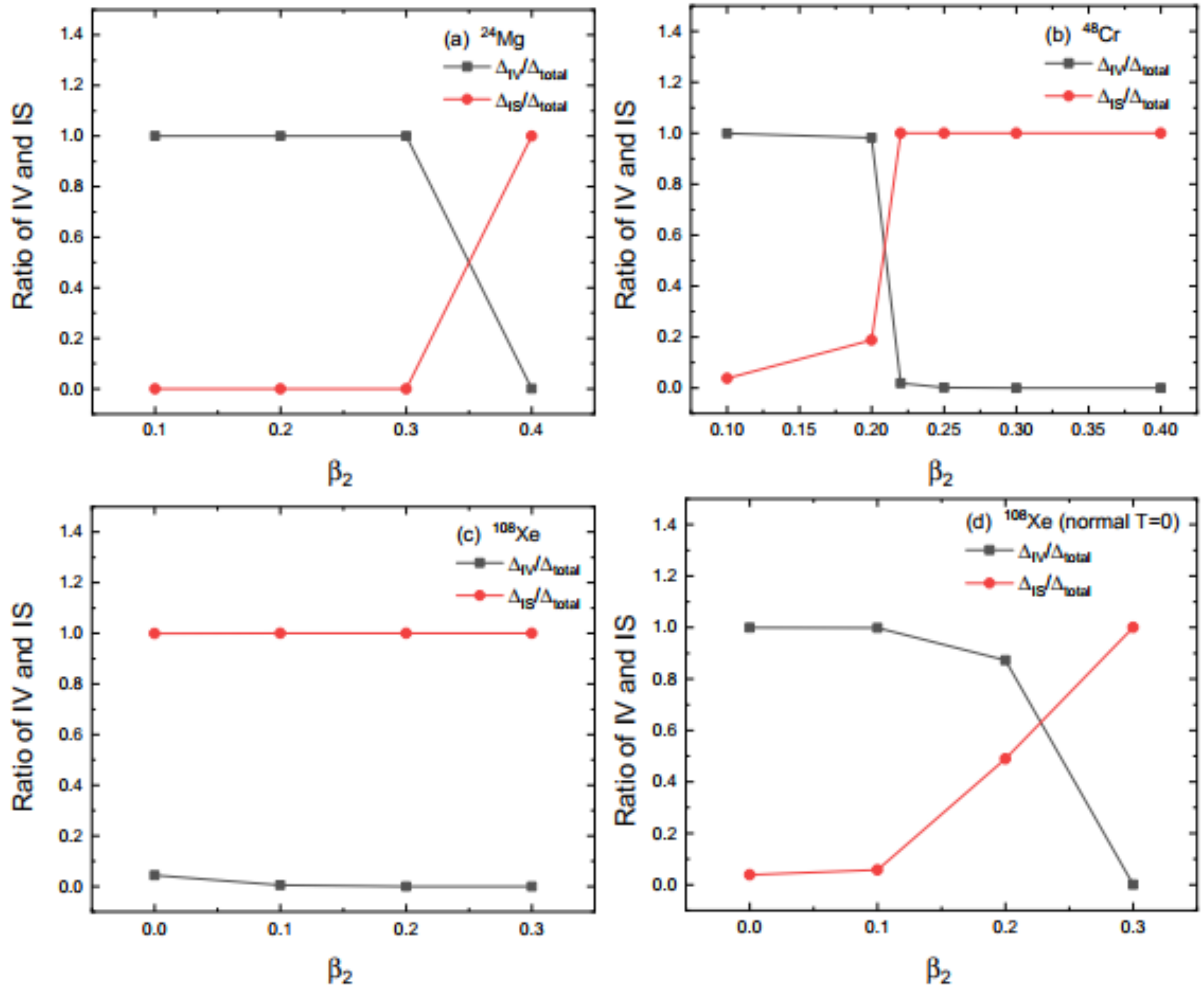


➢ For the enhanced T=0, the np pairing gap by the SO switch-off is large apart from (c) and (f).

❖ Ratio of IV and IS *np*-pairing

a)~c): enhanced T=0

d)): normal T=0



- ➢ **IS condensation by the enhanced T=0 *np* pairing** may happen in deformed ^{24}Mg and ^{48}Cr.
- ➢ rapid phase transition from IV to IS component in the *np* pairing. **But it may happen slower in heavy nuclei, which may mean the coexistence of IV and IS in some deformation region.**
- ➢ For heavy nuclei, ^{108}Xe, T=0 *np* pairing is dominant by the enhanced T=0 and the

Summary

1. We find a coexistence of two types of superconductivities (T=0 and T=1) at the
 $|\beta_2| > 0.3$ region in ^{24}Mg .
2. The enhanced IS *np* pairing interaction is shown to play important roles of shape
 deformations.
3. The IS condensation by the enhanced $T = 0$ pairing may happen not only in *sd*-shell, but also in *pf*-shell nuclei.
4. The IS condensation part plays a vital role to explain the GT strength distribution of ^{56}Ni nucleus.
5. The Coulomb force and the SO force are shown to change the smearing by change of ordering of SPS.
6. The state-dependent Brueckner G-PME takes into account shell structure effects
 on the residual interaction and enables us to do realistic description of ground
 states of the $N = Z$ nuclei.
7. IS condensation due to the enhanced T=0 *np* pairing may happen in a deformed
 ^{24}Mg and ^{48}Cr.
8. For heavy N=Z nuclei, the transition from IV to IS component may happen more

Thanks for your attention !!

Back-up files

❖ How to include the deformation?

<u>Deformed Woods-Saxon(WS) potential</u>

(cylindrical WS, Damgaard *et al* 1969)

$$V(\boxed{?}) = \frac{-V_0}{1+\exp\left(\boxed{?}a\right)}, \qquad V_{so} = -\lambda(\boxed{?}2mc)^2 \,\mathrm{grad}\, V(\boxed{?})(\boxed{?}\sigma \times \boxed{?}p)$$

$$\boxed{?}(u, v; \beta_2, \beta_4) = CS(u,v)/\left|\nabla_{u,v}S(u,v)\right|, \qquad z = Cu,\ \rho = Cv$$

distance function

surface function

β_2 : quadrupole deformation parameter

β_4 : hexadecapole deformation parameter

➢We can determine these two parameters by taking values giving the minimum ground state energy.

➢ To exploit G-matrix elements, which is calculated on the spherical basis, deformed bases are **expanded in terms of the spherical bases.**

$$|\alpha\Omega_\alpha >= \sum_a B_a^\alpha \ |a\Omega_\alpha >,$$

Deformed SPS

Sph. HO w. f.

❖ In sd-shell N=Z nuclei , Q_{exp} of ^{28}Si is different from ^{24}Mg and ^{32}S

Nucleus	β_2^{E2} [10]	β_2^{RMF} [11]	β_2^{FRDM} [12]	$Q_{exp.}$ [14, 15]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{24}Mg	0.605	0.416	0.	$-0.29 \sim -0.07$	3.123	3.193	1.844
^{28}Si (prolate)	0.407	x	x	x	2.841[a]	2.917 [a]	1.384[a]
^{28}Si (oblate)	x	-0.374	-0.363	$0.16 \sim 0.18$	2.841[a]	2.917[a]	1.384[a]
^{32}S	0.312	0.186	0.221	$-0.12 \sim -0.18$	2.141	2.207	1.047

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E2\uparrow)}{e^2}\right]^{1/2} \qquad (R_0 = 1.2A^{1/3})$$

in the rotational model, $Q_{J^\pi} = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_0$.

for 2^+, $Q_{2+} = -2/7\, Q_0$

Q_{2+} : **experimental quadrupole moment**

Q_0 : **intrinsic quadrupole moment**

➢ ^{28}Si is not heavy. Where does it come from ?

❖ Pairing correlation

BCS

J=0

T=1 j m j -m

deformed BCS

K=0

Ω -Ω

(J=0,1, 2, 3 •••)

T=0 J=1, 3, 5, •••

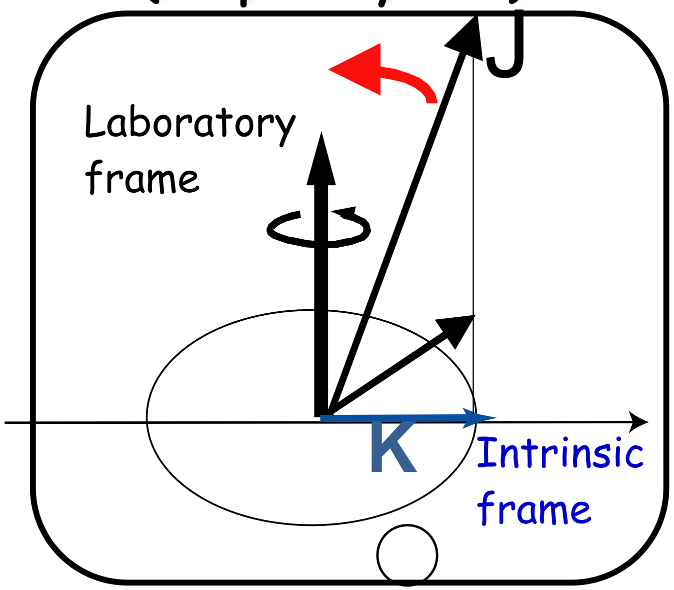
T=1 J=0, 2, 4, •••

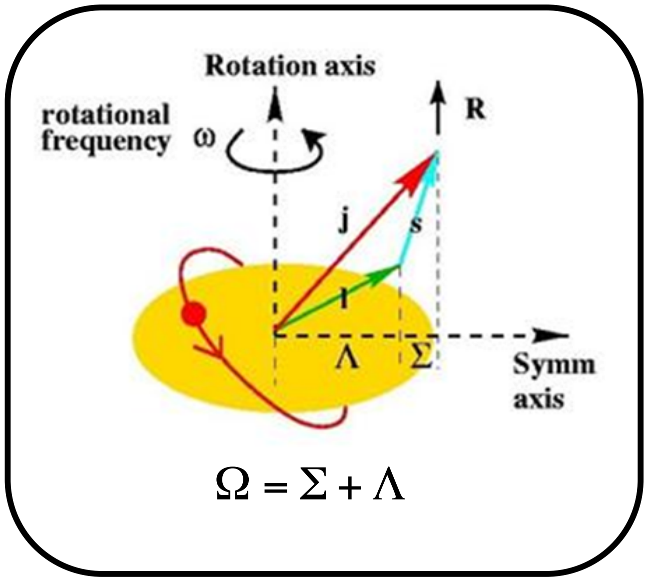
Ω= ½

j ≥ Ω

j	Ω
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{1}{2}$
$\frac{5}{2}$	$\frac{1}{2}$
$\frac{7}{2}$	$\frac{1}{2}$
?	

(Coupled system)

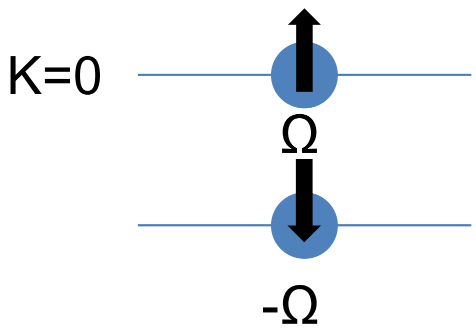




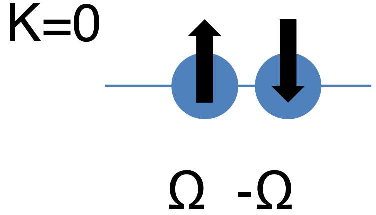
➢ Since the deformed SPS are expanded in terms of the spherical SP bases the different total angular momenta of the SP basis states would be mixed.

❖ Pairing correlation

deformed HFB



deformed BCS



BCS

$$\Delta_{p\bar{p}_\alpha} = \Delta_{\alpha p \bar{\alpha} p} = -\sum_{J,c} g_{\rm pp} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} G(\underline{aacc}, J, T=1)(u^*_{1p_c} v_{1p_c} + u^*_{2p_c} v_{2p_c})$$

$$\Delta_{p\bar{n}_\alpha} = \Delta_{\alpha p \bar{\alpha} n} = -\sum_{J,c} g_{\rm np} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} [G(aacc, J, T=1) Re(u^*_{1n_c} v_{1p_c} + u^*_{2n_c} v_{2p_c})$$

$$+ \; iG(aacc, J, T=0) Im(u^*_{1n_c} v_{1p_c} + u^*_{2n_c} v_{2p_c})] \;,$$

HFB

$$\Delta_{p\bar{p}_\alpha} = \Delta_{\alpha p \bar{\alpha} p} = -\sum_{J,c,d} g_{\rm pp} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\delta} c} G(\overline{aacd}, J, T=1)(u^*_{1p_c} v_{1p_d} + u^*_{2p_c} v_{2p_d})$$

❖ Self energy in BCS

$$H_0 = \sum_{b}^{A} 2\left[v_b^2\left(\eta_b + \tfrac{1}{2}\mu_b\right) - \tfrac{1}{2}u_b v_b \Delta_b\right]$$

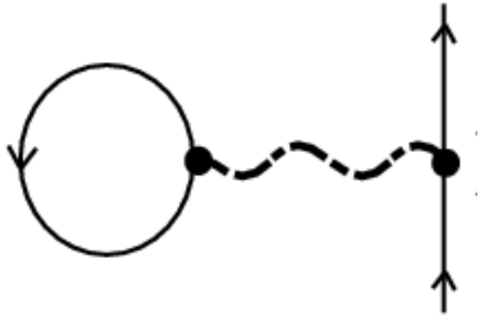
$\eta_b \rightarrow E_{mean}$, $\mu_b \rightarrow E_{self}$, $u_b v_b \Delta_b \rightarrow E_{pair}$

BCS eq.

$$\eta_b \equiv \varepsilon_b - \lambda - \mu_b$$

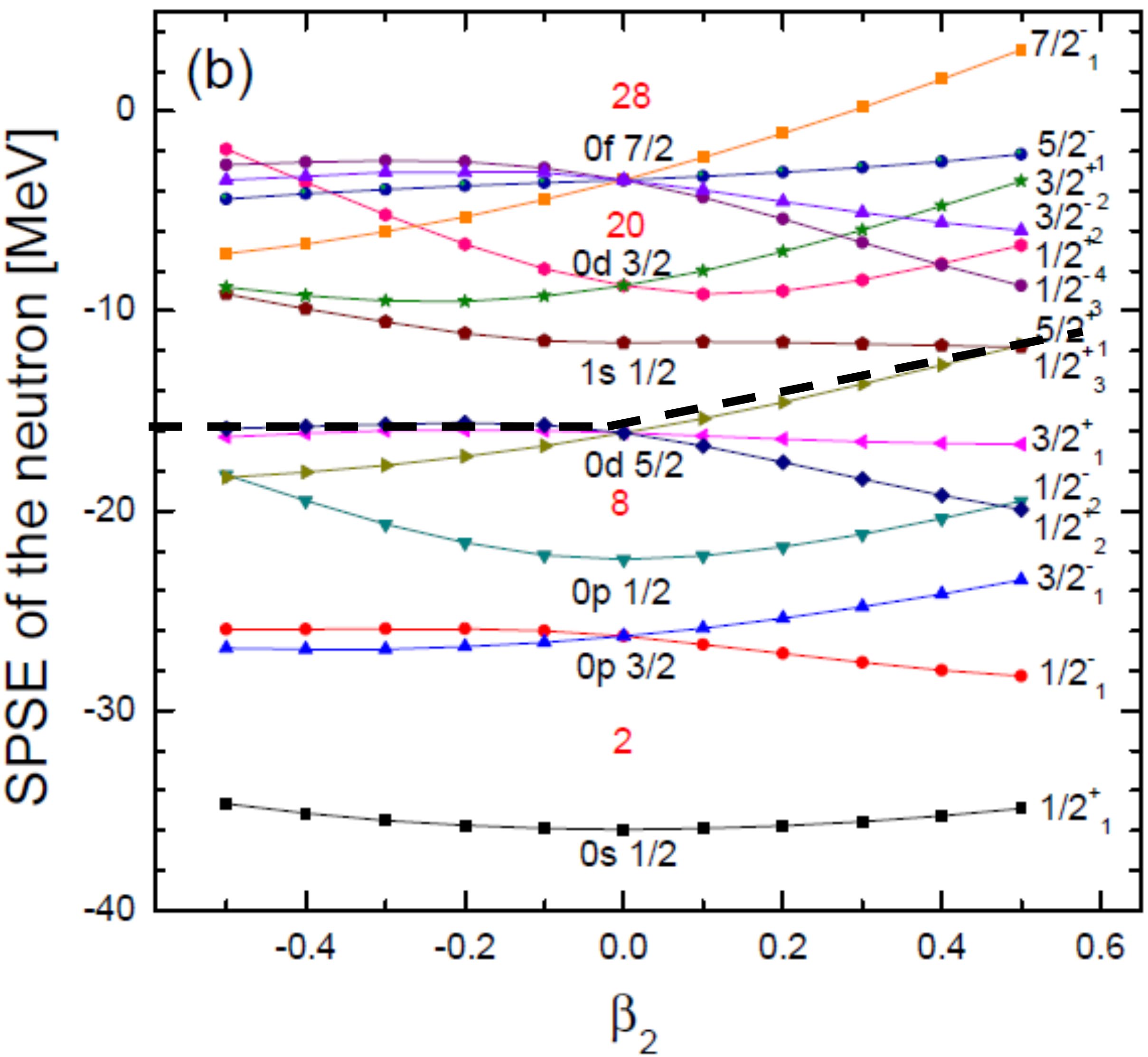
$$\mu_b = -\tfrac{1}{2}\sum_{a,J} v_a^2 \hat{J}^2 \langle ab:J|V|ab:J\rangle \quad \text{: self energy}$$

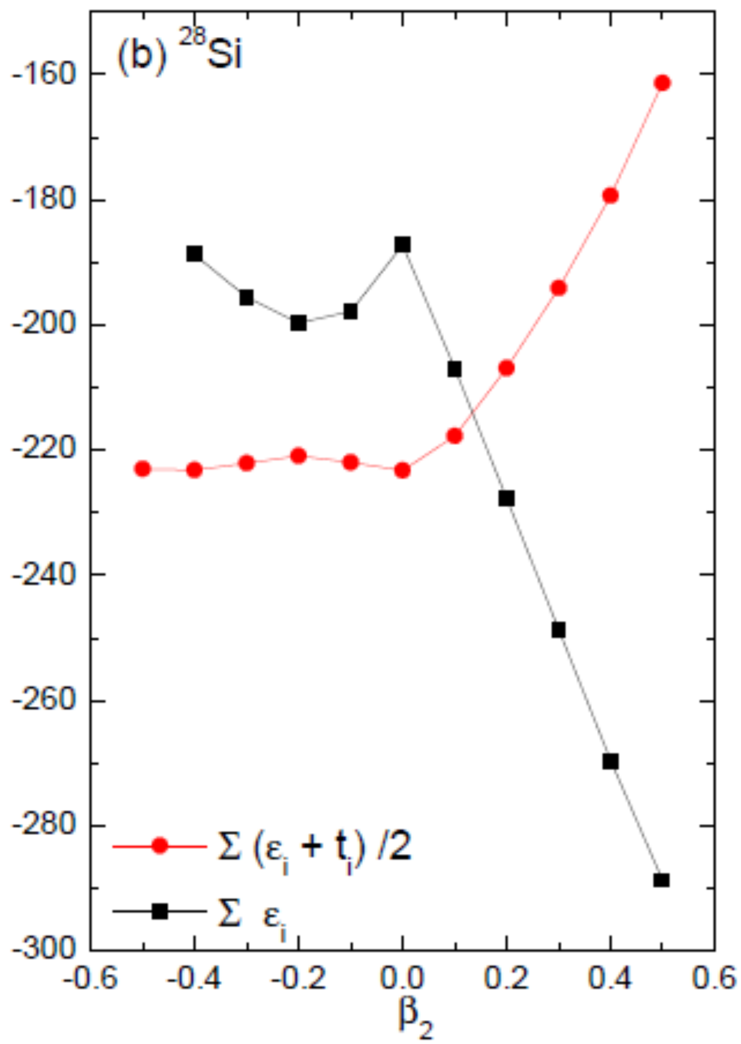
$$\Delta_b = -\sum_{a} u_a v_a \langle aa;0|V|bb:0\rangle \quad \text{: pairing gap}$$



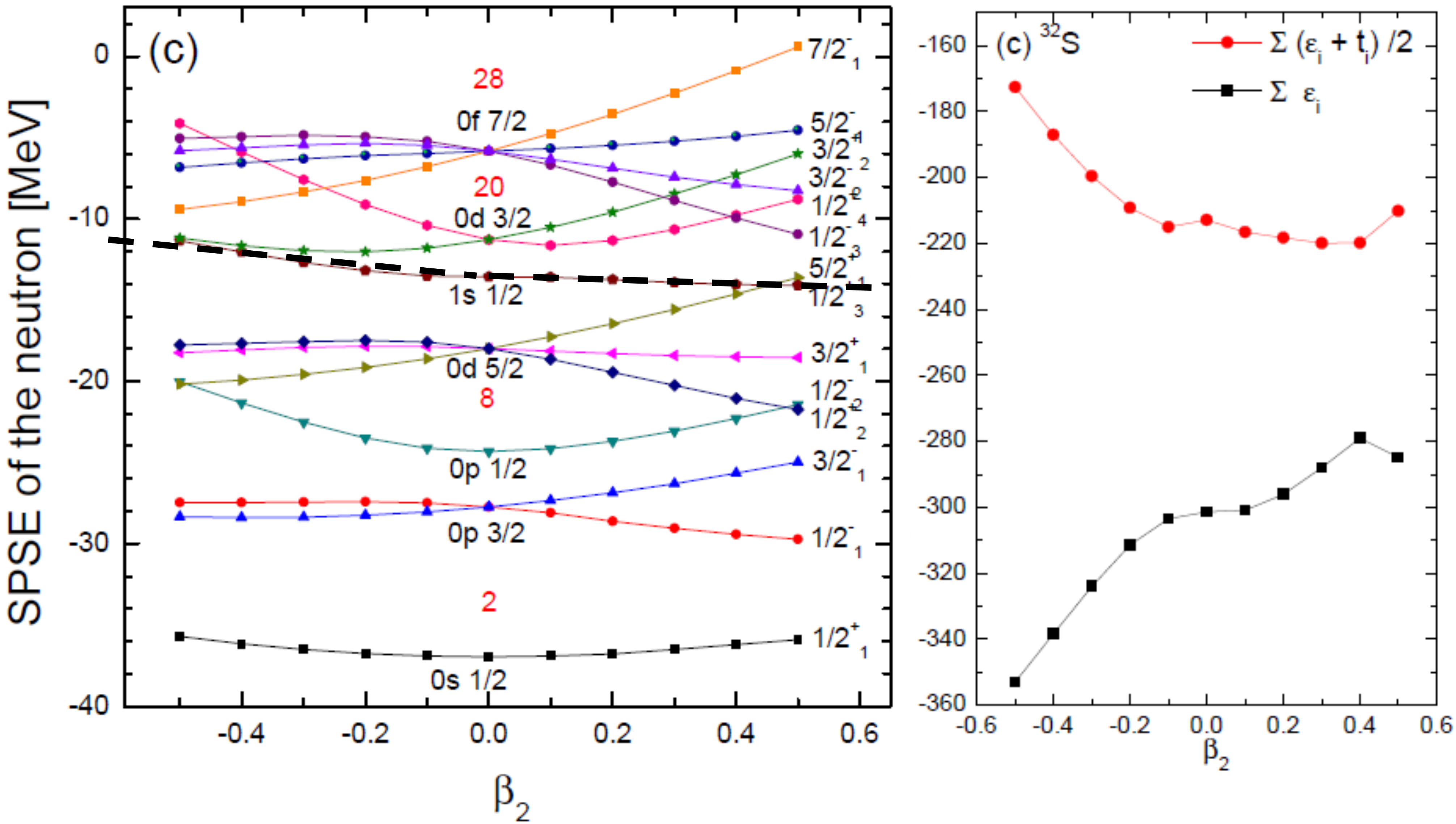
➢ The self energy term was usually neglected in BCS eq. because it results from particle-hole correlations beyond the BCS and affects a renormalization of the single particle energy.

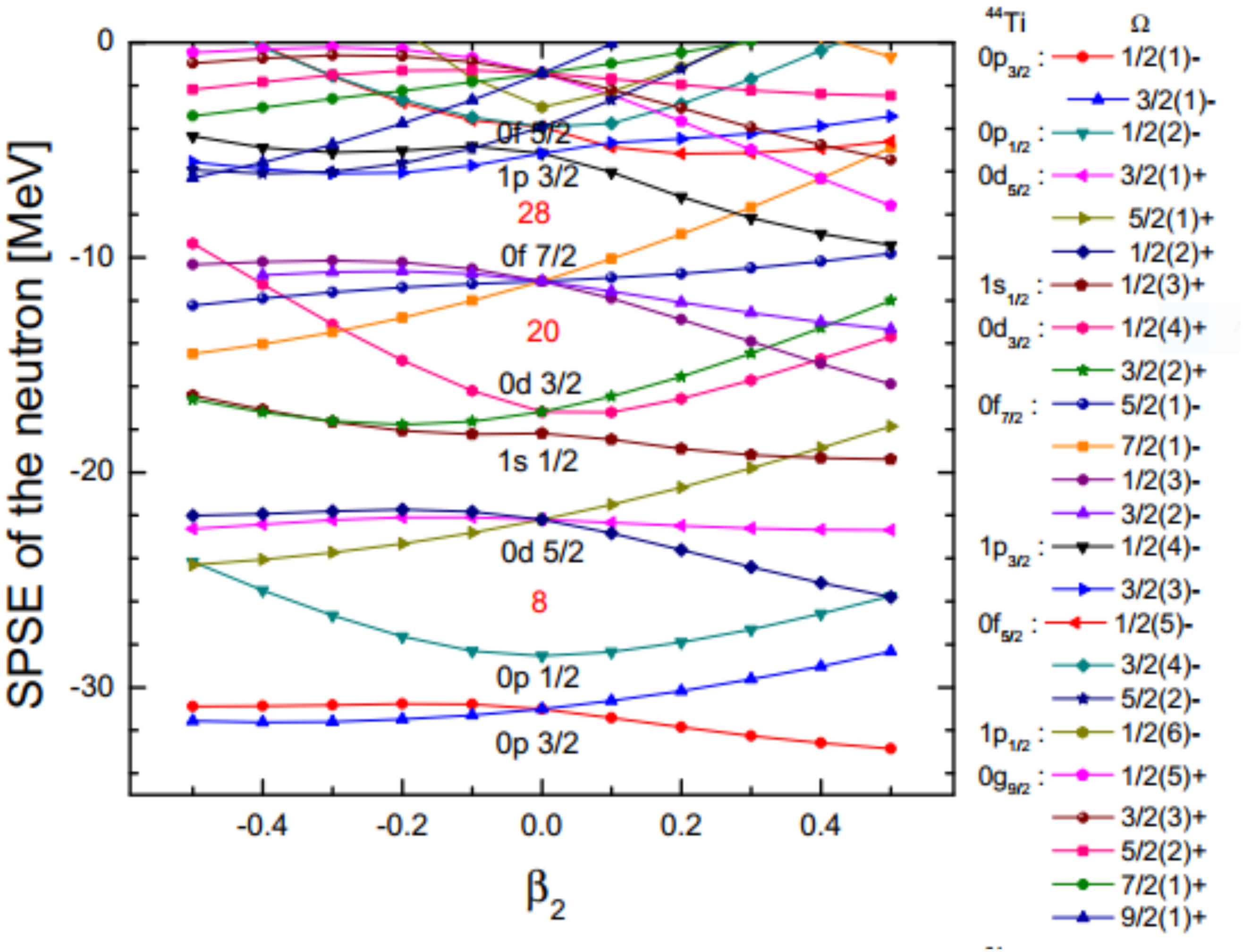
❖ Shell evolution & the simplest shell model of ^{28}Si





❖ Shell evolution & the simplest shell model of ^{32}S





SPSE of the neutron [MeV]
0
-10
-20
-30
-0.4
-0.2
0.0
0.2
0.4
β_2
0f 5/2
1p 3/2
28
0f 7/2
20
0d 3/2
1s 1/2
0d 5/2
8
0p 1/2
0p 3/2
^{44}Ti
Ω
$0p_{3/2}$: 1/2(1)-
3/2(1)-
$0p_{1/2}$: 1/2(2)-
$0d_{5/2}$: 3/2(1)+
5/2(1)+
1/2(2)+
$1s_{1/2}$: 1/2(3)+
$0d_{3/2}$: 1/2(4)+
3/2(2)+
$0f_{7/2}$: 5/2(1)-
7/2(1)-
1/2(3)-
3/2(2)-
$1p_{3/2}$: 1/2(4)-
3/2(3)-
$0f_{5/2}$: 1/2(5)-
3/2(4)-
5/2(2)-
$1p_{1/2}$: 1/2(6)-
$0g_{9/2}$: 1/2(5)+
3/2(3)+
5/2(2)+
7/2(1)+
9/2(1)+

❖ Parameter set of Deformed Woods-Saxon

Table 1
Set of parameter values defined by the program according to the input value of the ICHOIC variable. The symbols P (N) refer to the protons (neutrons). The λ values in the case of the Chepurnov parametrisation are defined by $\lambda = 23.8\ (1+2*(N-Z)/A)$. Blomqv.–Wahlb. stands for Blomqvist and Wahlborn. The values of r_0 and a are in fermi, V_0 in MeV, κ and λ dimensionless

Parametrisation	λ (P)	λ (N)	r_{0-so} (P)	r_{0-so} (N)	r_0 (P)	r_0 (N)	κ	V_0	a	ICHOIC
Blomqv.–Wahlb.	32.0	32.0	1.270	1.270	1.270	1.270	0.67	51.0	0.67	0
Rost	17.8	31.5	0.932	1.280	1.275	1.347	0.86	49.6	0.70	1
Chepurnov	calc.		1.240	1.240	1.240	1.240	0.63	53.3	0.63	2
"optimal"	A-dependent				1.275	1.347	0.86	49.6	0.70	3
"universal"	36.0	35.0	1.20	1.310	1.275	1.347	0.86	49.6	0.70	4
"input"				parameters read from input						5
def.-dependent INCREA = 1	deformation-dependent (only for $\beta_2 > 0.325$)				depend on ICHOIC					0–5

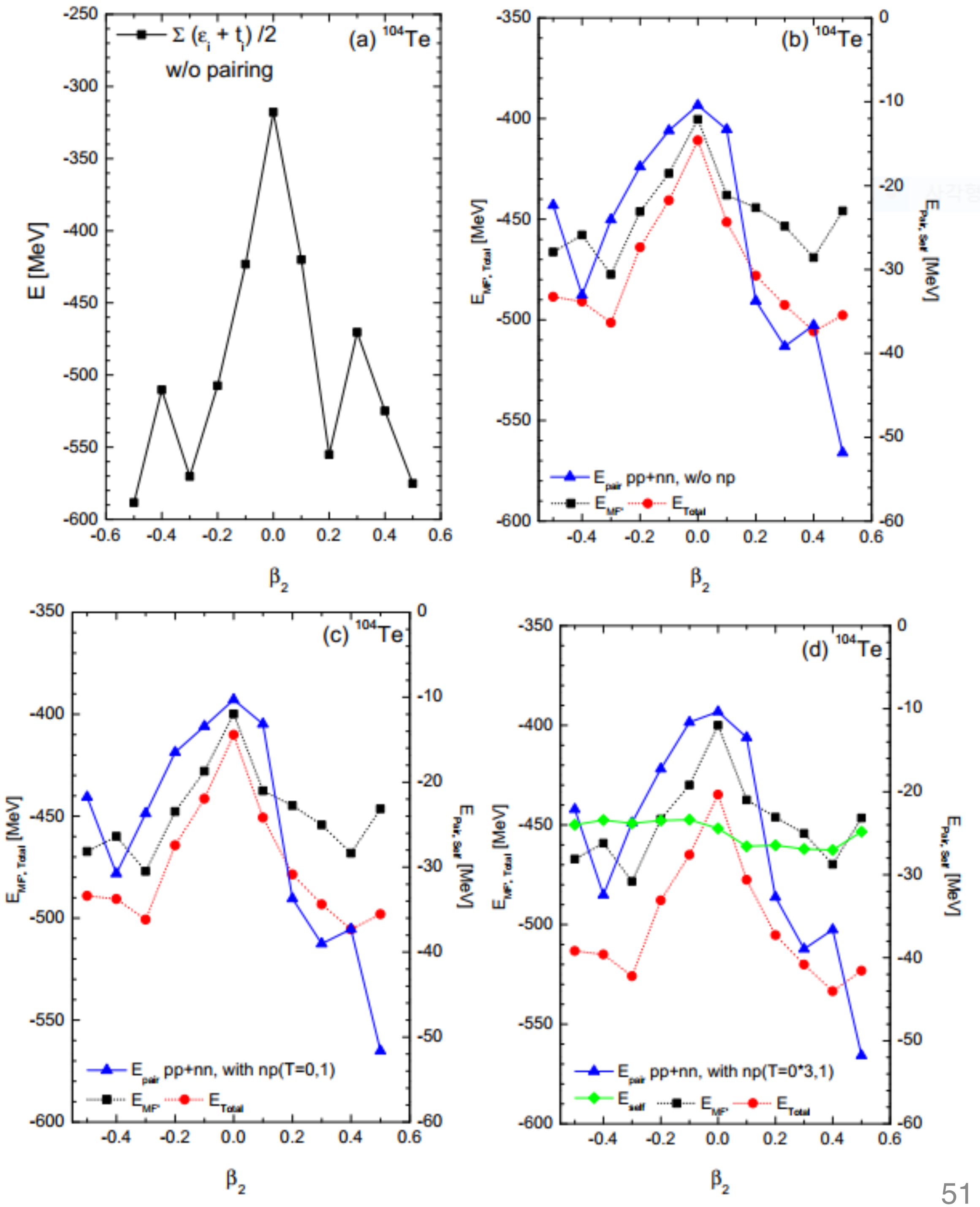
❖ In gd-shell N=Z nuclei

Nucleus	β_2^{RMF} [10]	β_2^{FRDM} [11]	β_2^{KTUY} [10]	Δ_p^{emp}	Δ_n^{emp}	δ_{np}^{emp}
^{104}Te	–	–0.011	0.039	1.520	1.548	0.665
^{116}Ce	0.285	0.282	0.145	1.452	1.530	0.697
^{128}Gd	0.350	0.341	0.194	1.415	1.393	0.592

❖ Used parameters in this work.

* $N_{max} = 0$ (…)
* $N_{max} = 5$ (…)
* … : … .
* … : … − … … …
* g_p (g_p) = 0.9 (1.5) … − … (… − …) … .

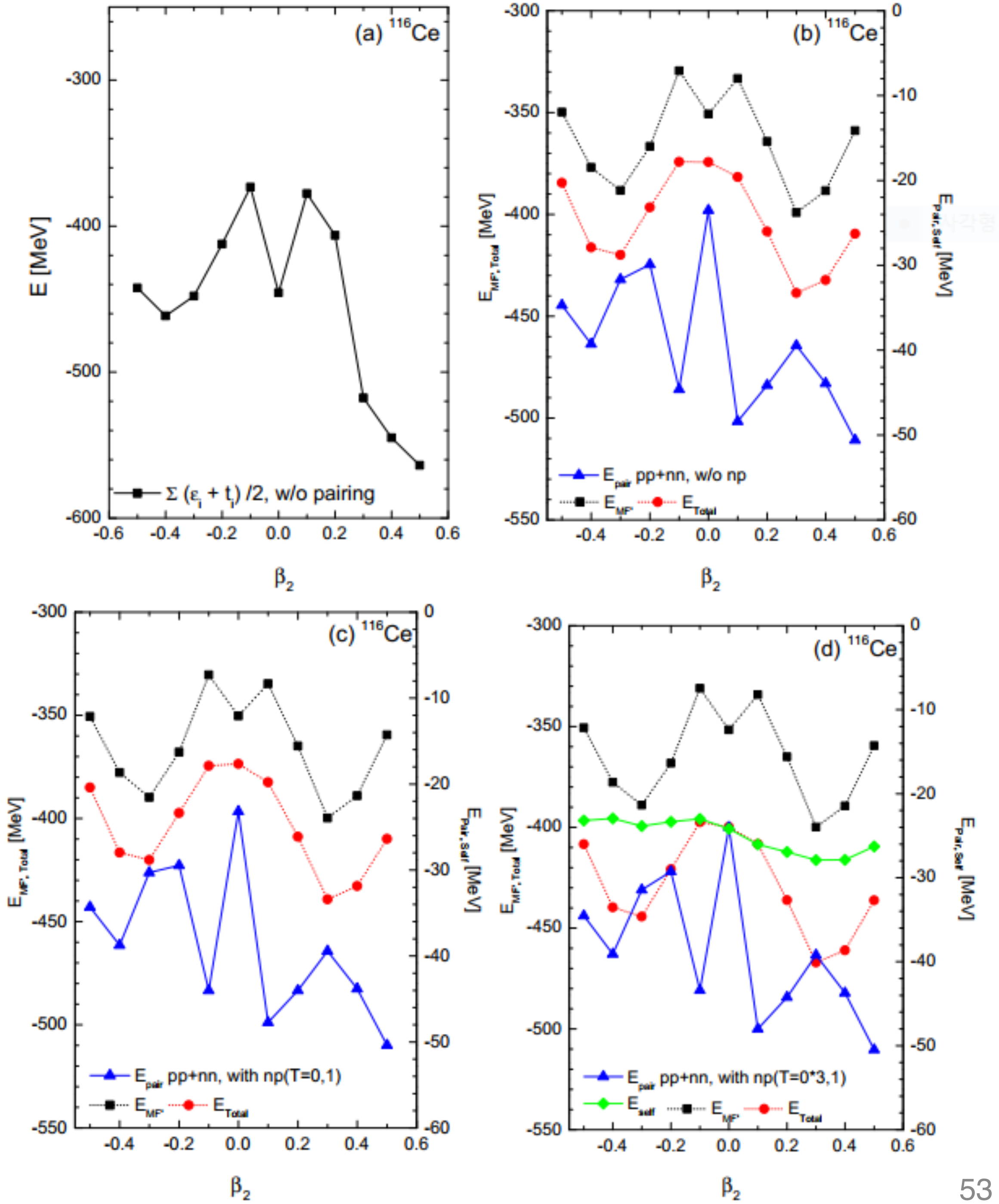
❖ ground state E of 104Te



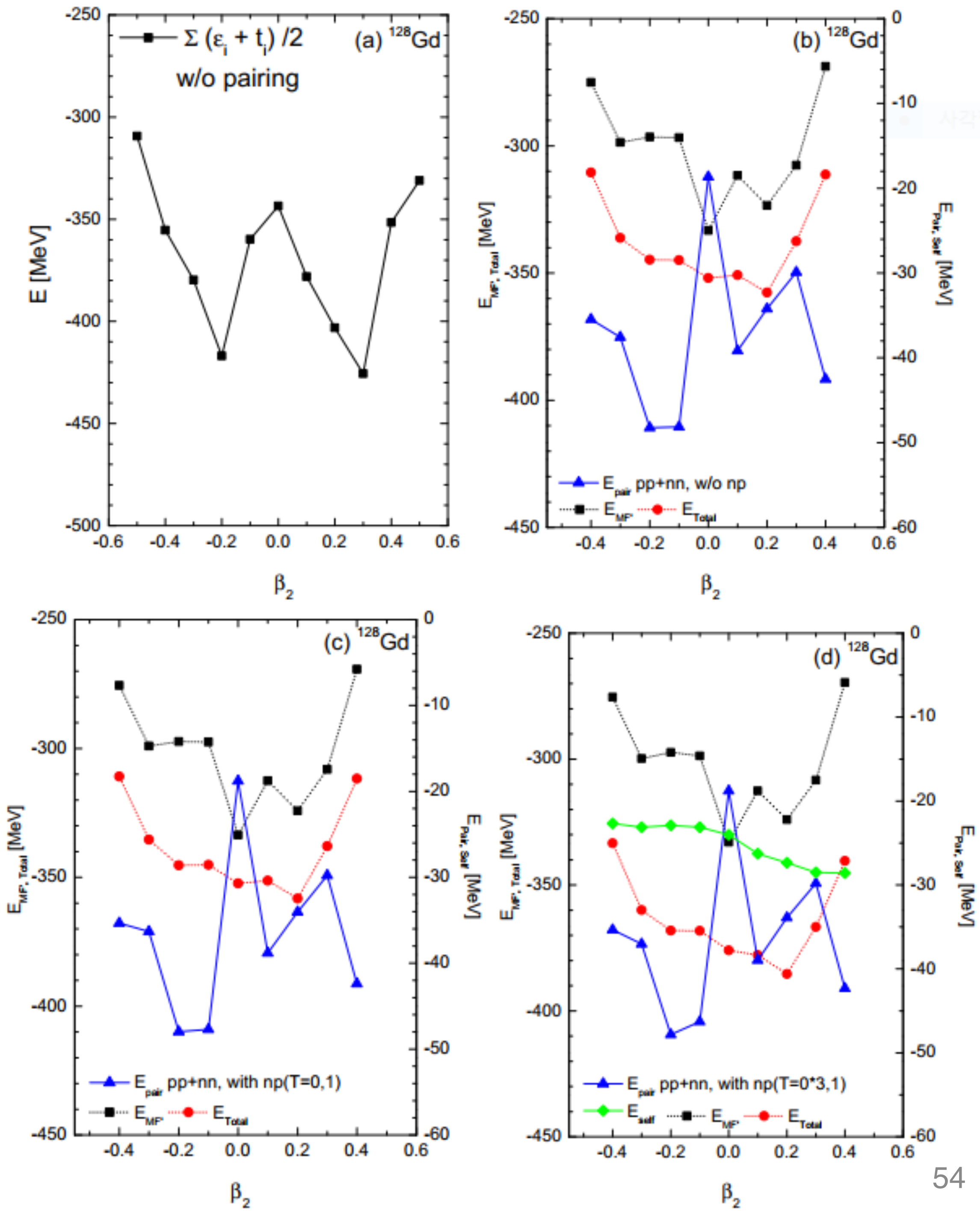
deformed Hartree Fock Bogoliubov (DHFB) transformation,

$$\begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ a_{\bar{1}} \\ a_{\bar{2}} \end{pmatrix}_\alpha = \begin{pmatrix} u_{1p} & u_{1n} & v_{1p} & v_{1n} \\ u_{2p} & u_{2n} & v_{2p} & v_{2n} \\ -v_{1p} & -v_{1n} & u_{1p} & u_{1n} \\ -v_{2p} & -v_{2n} & u_{2p} & u_{2n} \end{pmatrix}_\alpha \begin{pmatrix} c_p^\dagger \\ c_n^\dagger \\ c_{\bar{p}} \\ c_{\bar{n}} \end{pmatrix}_\alpha$$

❖ ground state E of 116Ce



❖ ground state E of 128Gd



❖ Two-body interaction

Realistic two body interaction inside nuclei was taken by Brueckner g-matrix, which is a solution of the Bethe-Salpeter Eq., derived from the Bonn-CD potential for nucleon-nucleon interaction in free space.

$$g(\omega)_{ab,cd} = V_{ab,cd} + V_{ab,cd} \frac{Q_p}{\omega - H_0} g(\omega)_{ab,cd}$$

a,b,c,d : single particle states from the Woods-Saxon potential.
$V_{ab,cd}$: phenomenological nucleon-nucleon potential in free space.