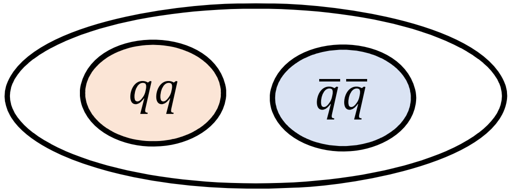
Tetraquark signatures for the two nonets in the light meson system

Hungchong Kim (김흥종)
Korea Aerospace U.

We explore the tetraquark possibility from the two nonets in $J^P = 0^+$.

- Light nonet: $f_0(500), f_0(980), K_0^*(800), a_0(980)$
- Heavy nonet: $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

$qq\bar{q}\bar{q}$ system



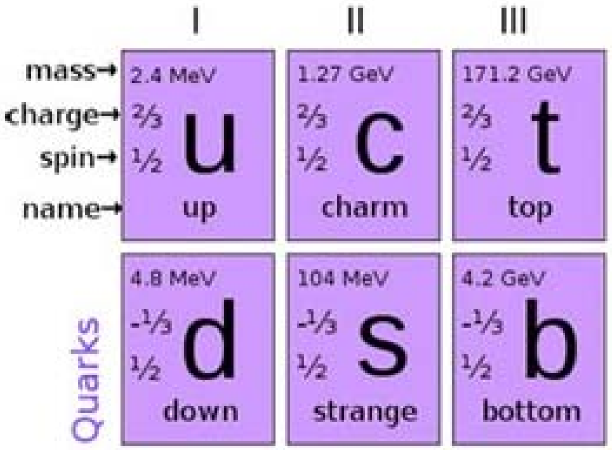
References;
1) EPJC (2017) 77:173, Hungchong Kim, M.K.Cheoun, K.S.Kim,
2) EPJC (2017) 77:435, K.S. Kim, Hungchong Kim,
3) PRD (2018) 97:094005, Hungchong Kim, K.S.Kim, M.K.Cheoun, M.Oka.

2018.10.20, haphy2018, Pohang, Korea

Introduction

To motivate the multiquark study including tetraquarks

- Hadrons are composite particles made out of 6 quarks.



	I	II	III
mass→ charge→ spin→ name→	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top
Quarks	4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom

Figure from Google

⇨ Baryons: $p, n, \Sigma, \Lambda, \Xi, \cdots$
Mesons: $\pi, K, \varrho, \omega, K^*, \cdots$

- Lots of hadrons observed so far!!
  ☞ See the particle listing in the review of Particle Data Group (PDG)
- Certain classification is expected.

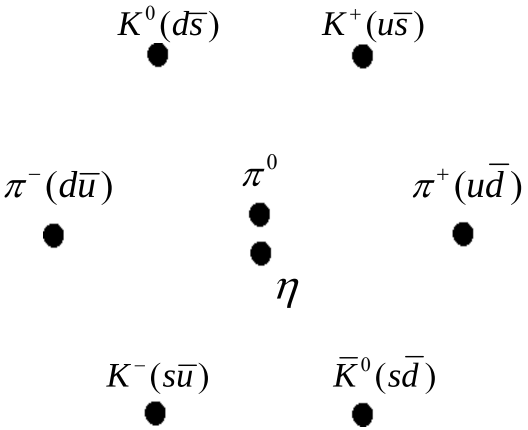
Two things for classification

- Gell-Mann(1964) came out with $\mathrm{SU(3)}_\mathrm{f}$ quark model which classifies hadrons using u, d, s quarks as fundamental rep.
- Color charges bind the quarks inside hadrons, $\mathrm{SU(3)}_\mathrm{c}$
  hadrons are colorless$\Rightarrow 1_c$

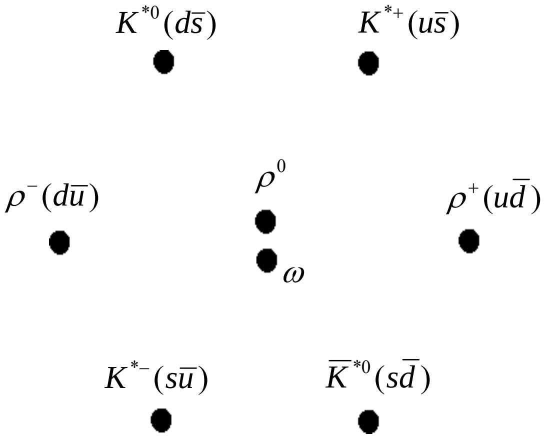
SU(3) quark model

- Hadrons with the same spin and parity form a multiplet (family).
- Members in a multiplet are connected by a SU(3)$_f$ rotation.

Lowest-lying mesons $q\bar{q}$: $3_f \otimes \bar{3}_f = 8_f \oplus 1_f$



$J^P = 0^-$ channel($\mathbf{8}_f$)

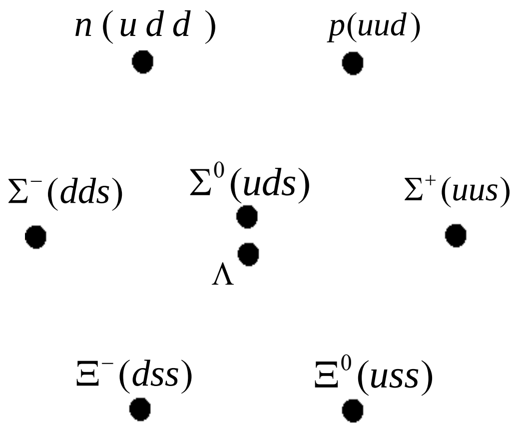


$J^P = 1^-$ channel($\mathbf{8}_f$)

※Flavor singlet members are not shown here.

SU(3) quark model

Lowest-lying baryons qqq: $3_f \otimes 3_f \otimes 3_f = 1_f \oplus 8_f \oplus 8_f \oplus 10_f$

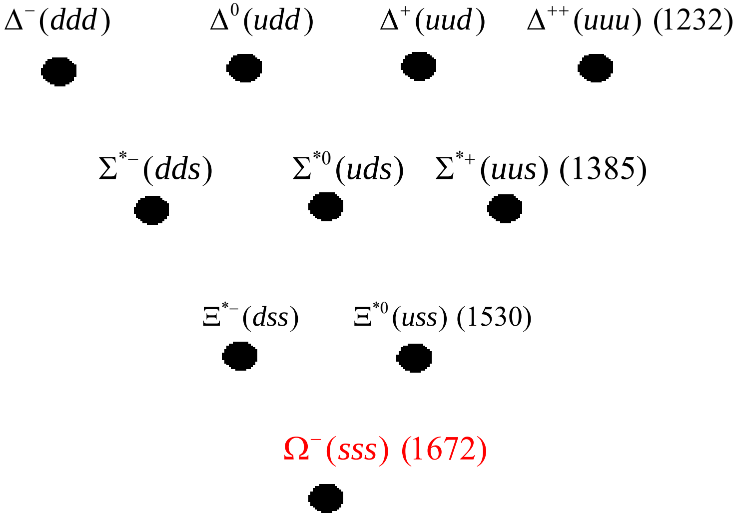

$n\,(u\,d\,d\,)$ $p(uud)$

$\Sigma^-(dds)$ $\Sigma^0(uds)$ $\Sigma^+(uus)$

Λ

$\Xi^-(dss)$ $\Xi^0(uss)$

$J^P = \frac{1}{2}^+$ channel($\mathbf{8}_f$)


$\Delta^-(ddd)$ $\Delta^0(udd)$ $\Delta^+(uud)$ $\Delta^{++}(uuu)$ (1232)

$\Sigma^{*-}(dds)$ $\Sigma^{*0}(uds)$ $\Sigma^{*+}(uus)$ (1385)

$\Xi^{*-}(dss)$ $\Xi^{*0}(uss)$ (1530)

$\Omega^-(sss)$ (1672)

$J^P = \frac{3}{2}^+$ channel($\mathbf{10}_f$)

SU(3) quark model works !

As far as the lowest-lying resonances are concerned

Mesons: 2 quark states, $\boldsymbol{q\bar{q}}$
Baryons: 3 quark states, $\boldsymbol{qqq}$

Up to three quarks, $q\bar{q}$, qqq are the only possibilities to form the colorless state, 1_c.
※ Hadrons with other combinations like qq, $\bar{q}\bar{q}$, $q\bar{q}\bar{q}$ etc, do not exist because they cannot form 1_c.

But one can make a colorless state, 1_c, with higher number of quarks.
Why not hadrons with 4, 5, 6,… quarks, i.e., **multiquark states** ?.

- No dynamical reason against multiquark states
- Has been great expectation for them!!

Multiquark search

lots of references in the literature

- Jaffe(1977) proposed a diquark-antidiquark model, $qq\bar{q}\bar{q}$, forming a nonet to explain the resonances, $f_0(500), f_0(980), K_0^*(800), a_0(980)$.

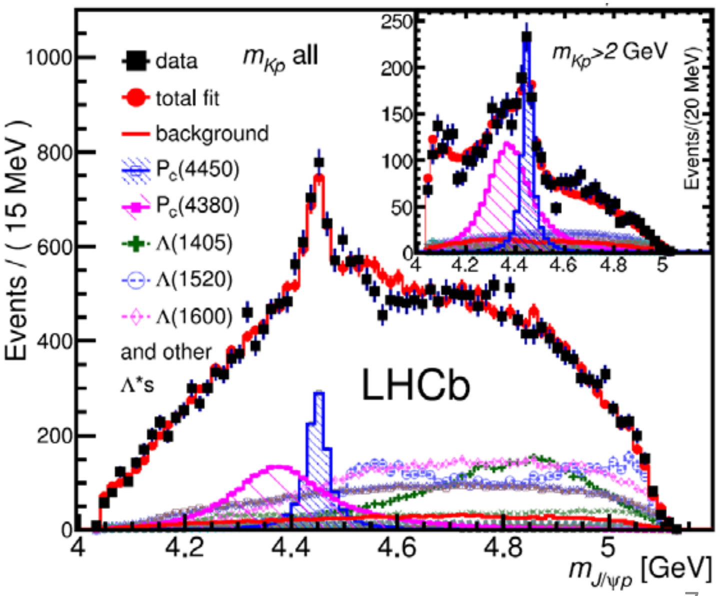
$$qq\bar{q}\bar{q} \in 8_f \oplus 1_f$$

- Recent studies mostly on the states with heavy quarks.
  ☞ For the X,Y,Z spectroscopy, one of promising scenarios is tetraquarks with hidden charm or bottom.
  ☞ LHCb collab. reported the discovery of X(4140), X(4274) which seems to be consistent with $c\bar{c}s\bar{s}$, PRL 118(2017).

- Pentaquark candidates, $P_c(4380)$, $P_c(4450)$ are recently listed in PDG.

PRL 115(2015) 072001

$$\Lambda_b^0 \to P_c^+ K^-, P_c^+ \to J/\psi\ p \quad : uudc\bar{c}$$



Our strategy

- We explore tetraquark possibility in the light meson system.
- In particular, we reexamine the diquark-antidiquark model by Jaffe and motivate tetraquark mixing framework for the resonances in the 0^+ channel.
- Basically we introduce two types of tetraquark and their strong mixing in order to explain two nonets in PDG,
  Light nonet: $f_0(500), f_0(980), K_0^*(800), a_0(980)$
  Heavy nonet: $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

Tetraquark mixing framework

(Our proposal)

A brief review on diquark-antidiquark model

- Well-known model for tetraquark by Jaffe (1977).

- Tetraquarks are constructed by combining diquark(qq) and antidiquark($\bar{q}\bar{q}$), $qq\bar{q}\bar{q}$, $(q = u, d, s)$, while assuming all the quarks are in an S-wave.

- In this construction, the spin-0 diquark with $qq \in J = 0,\ \bar{3}_c, \bar{3}_f,$ is commonly used
  - because this is the most compact object among all possible diquarks.
  - So it can be used as a starting building block for tetraquarks.

〈qq structure [Jaffe, hep-ph/0001123]〉

$Spin$	Color	Flavor	$\langle V_{CS} \rangle$	Type
0	$\bar{3}_c$	$\bar{3}_f$	-2	Attractive
1	6_c	$\bar{3}_f$	$-1/3$	Attractive
1	$\bar{3}_c$	6_f	$2/3$	Repulsive
0	6_c	6_f	1	Repulsive

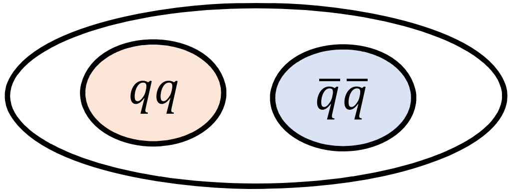
Possible diquarks allowed by Pauli principle.
$\langle V_{CS} \rangle$ is given in a certain unit.

Hyperfine color-spin interaction

$$V_{CS} \propto -\sum_{i \neq j} \lambda_i \cdot \lambda_j J_i \cdot J_j$$

λ_i: Gell-Mann matrix for color
J_i: spin,

$qq\bar{q}\bar{q}$ system



Jaffe model

$qq\bar{q}\bar{q}$ from the spin-0 diquark

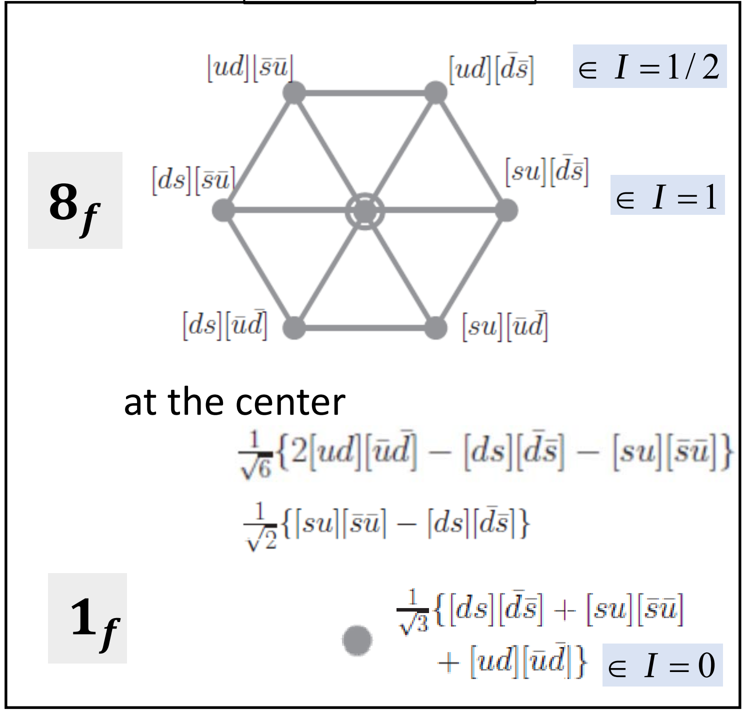
$[qq \in (J = 0, \bar{3}_c, \bar{3}_f)] \otimes [\bar{q}\bar{q} \in (J = 0, 3_c, 3_f)]$

Spin: $[J_{12} = 0] \otimes [J_{34} = 0] = [J = 0] \quad \Longrightarrow \quad |J, J_{12}, J_{34}\rangle = \underline{|000\rangle}$

Color: $\bar{3}_c \otimes 3_c \Rightarrow 1_c$, i.e., $|1_c, \bar{3}_c, 3_c\rangle$, $\quad \frac{1}{\sqrt{12}} \epsilon_{abd} \epsilon^{aef} (q^b q^d)(\bar{q}_e \bar{q}_f)$

Flavor: forming a nonet, $\bar{3}_f \otimes 3_f = 8_f \oplus 1_f$

Flavor nonet



at the center

$\frac{1}{\sqrt{6}}\{2[ud][\bar{u}\bar{d}] - [ds][\bar{d}\bar{s}] - [su][\bar{s}\bar{u}]\}$

$\frac{1}{\sqrt{2}}\{[su][\bar{s}\bar{u}] - [ds][\bar{d}\bar{s}]\}$

$\frac{1}{\sqrt{3}}\{[ds][\bar{d}\bar{s}] + [su][\bar{s}\bar{u}] + [ud][\bar{u}\bar{d}]\}$

Notation: $[ud] = \frac{1}{\sqrt{2}}(ud - du)$, etc.

Characteristics of Jaffe's tetraquarks

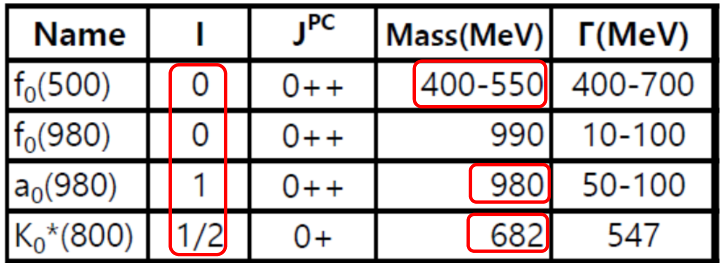
1. Spin and parity are $J^P = 0^+$.
2. Possible isospins are $I = 0, \frac{1}{2}, 1$.
3. The $I_z = 0$ members have $C = +$.
4. The mass ordering among the octet members, $(I = 1) > \left(I = \frac{1}{2}\right) > (I = 0)$, ex) $M([su][\bar{d}\bar{s}]) > M([su][\bar{u}\bar{d}])$.

※ Note, two-quark system ($q\bar{q}$) can form a nonet with the opposite mass ordering.

Possible candidates must be sought from the resonances with $J^{P(C)} = 0^{+(+)}$

Jaffe model

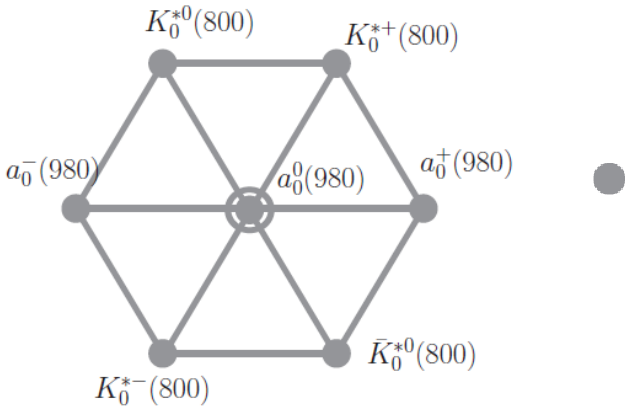
Light nonet (Jaffe's selection)



Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$f_0(500)$	0	0++	400-550	400-700
$f_0(980)$	0	0++	990	10-100
$a_0(980)$	1	0++	980	50-100
K_0*(800)	1/2	0+	682	547

The lowest-lying resonances in $J^{P(C)} = 0^{+(+)}$

- In PDG, the lowest-lying states in $J^P = 0^+$, $f_0(500), f_0(980), K_0^*(800)$, $a_0(980)$, seem to form a nonet ($8_f \oplus 1_f$)
  - A clue for the flavor octet ? Gell-Mann−Okubo mass relation works within $\sim 14\%$, $M^2[a_0(980)] + 3M^2[f_0(500)] \approx 4M^2[K_0^*(800)]$.
- They satisfy the tetraquark characteristics above,
  - the anticipated isospins, $I = 0, \frac{1}{2}, 1$,
  - the mass ordering, $M[a_0(980)] > M[K_0^*(800)] > M[f_0(500)]$.



Light nonet is a strong candidate for the tetraquark
although their masses are rather small to be four-quark states.

Two states in $I = 0$
may be a mixture of: $f_0(500)$, $f_0(980)$

our claim

Another tetraquark in 0^+ can be constructed by the spin-1 diquark

because this spin-1 diquark also forms a bound state even though it is less attractive than the spin-0 diquark.

$\langle qq$ structure$\rangle$

$Spin$	Color	Flavor	$\langle V_{CS}\rangle$
0	$\bar{3}_c$	$\bar{3}_f$	-2
1	6_c	$\bar{3}_f$	$-1/3$
1	$\bar{3}_c$	6_f	$2/3$
0	6_c	6_f	1

$qq\bar{q}\bar{q}$ from the spin-1 diquark in $J^P = 0^+$ channel

Spin: $[J_{12}= 1]\otimes[J_{34} = 1] \Rightarrow [J = 0] \quad \Longrightarrow \ |J,J_{12},J_{34}\rangle = \underline{|011\rangle}$

Color: $6_c\otimes\bar{6}_c \Rightarrow 1_c$, i.e., $|1_c, 6_c, \bar{6}_c\rangle$, $\frac{1}{\sqrt{96}}(q^a q^b + q^b q^a)(\bar{q}_a\bar{q}_b + \bar{q}_b\bar{q}_a)$

Flavor: $\bar{3}_f\otimes 3_f = 8_f\oplus 1_f$ form the same nonet in flavor !

What about $|111\rangle, |211\rangle$?
⇒ see the backup slides

⇒ This 2nd tetraquark also satisfies the tetraquark characteristics above.

- In fact, this 2nd tetraquark is more compact than the one from the spin-0 diquark.
  ⇒ The spin-1 diquark configuration is important as well,
  ⇒ and it cannot be ignored in tetraquark studies.
- But this 2nd type tetraquark requires another nonet to be found in PDG
  ⇒ do we have some candidates ?

Yes ! PDG has another nonet to support our approach.

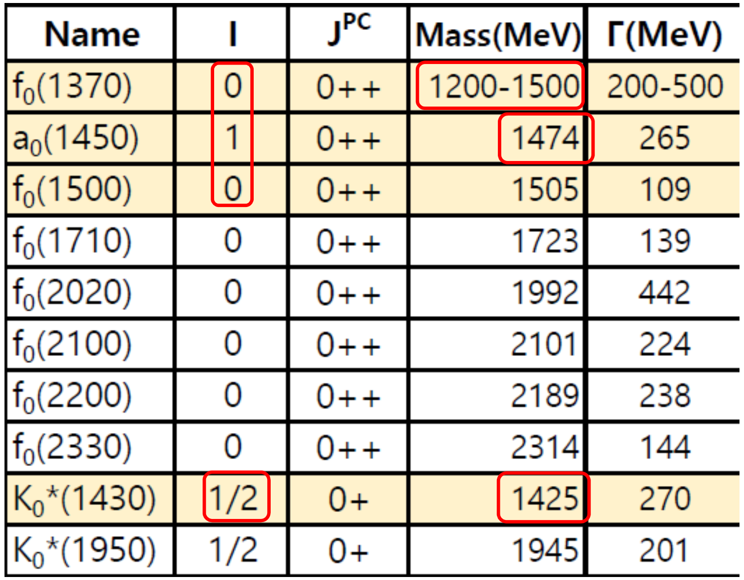
our observation

Heavy nonet (our selection)

- A similar nonet can be selected from higher resonances in $J^P = 0^+$, $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$
  - GMO relation within $\sim 6\%$, $M^2[a_0(1450)] + 3M^2[f_0(1370)] \approx 4M^2[K_0^*(1430)]$
- They have the anticipated isospins, $I = 0, \frac{1}{2}, 1$.
- Their mass ordering, though marginal, still holds here,

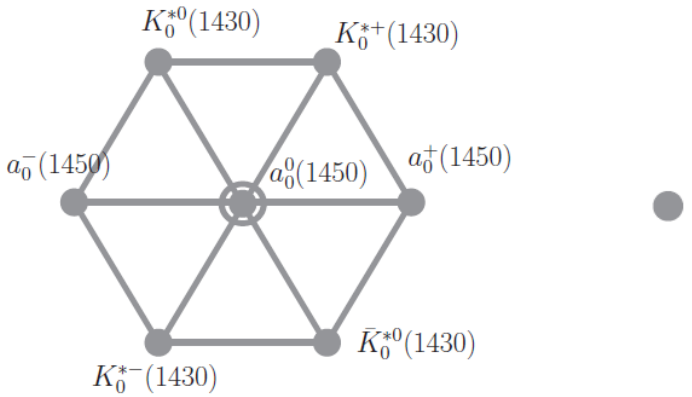
$M[a_0(1450)] > M[K_0^*(1430)]$ with $\Delta M \sim 50$ MeV, $M[K_0^*(1430)] \gtrsim M[f_0(1370)]$.

The 'marginal' ordering can be explained partially by our hyperfine masses (more later!).



Name	I	J^{PC}	Mass(MeV)	Γ(MeV)
$f_0(1370)$	0	0++	1200-1500	200-500
$a_0(1450)$	1	0++	1474	265
$f_0(1500)$	0	0++	1505	109
$f_0(1710)$	0	0++	1723	139
$f_0(2020)$	0	0++	1992	442
$f_0(2100)$	0	0++	2101	224
$f_0(2200)$	0	0++	2189	238
$f_0(2330)$	0	0++	2314	144
$K_0^*(1430)$	1/2	0+	1425	270
$K_0^*(1950)$	1/2	0+	1945	201

$J^{P(C)} = 0^{+(+)}$ with higher masses



Two states in $I = 0$
may be a mixture of $f_0(1370), f_0(1500)$

Heavy nonet could be the 2nd candidate for the tetraquark !

half summary

We have two tetraquark types in $J^P = 0^+$,

- differed by the spin and color configuration,

  $|000\rangle_{\bar{3}_c,3_c} \Longrightarrow |000\rangle \qquad |011\rangle_{6_c,\bar{6}_c} \Longrightarrow |011\rangle.$

- Both form the same nonet in flavor ($8_f \oplus 1_f$).

PDG also has two nonets in $J^P = 0^+$,

- which satisfy GMO relation, the mass ordering expected from the tetraquarks.

Light nonet (Jaffe's selection)

The lowest-lying in 0^+,
$f_0(500), f_0(980), K_0^*(800), a_0(980)$

Heavy nonet (additional selection by us)

From higher resonances in 0^+,
$f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$

The huge mass gap between the two $\gtrsim 500$ MeV

How to match the two sets ?

Two tetraquark types $\Longleftrightarrow$ Two nonets in PDG

A crucial observation is that

- the two tetraquarks, $|000\rangle$, $|011\rangle$, mix through the hyperfine color-spin interaction !

$$V_{CS} \propto \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

λ_i: Gell-Mann matrix for color,
J_i: spin,
m_i: constituent quark mass

- The mixing terms are nonzero, $\langle 011|V_{CS}|000\rangle \neq 0$.
- $\langle V_{CS}\rangle$ forms a 2x2 matrix in the bases, $|000\rangle, |011\rangle$, constituting the hyperfine mass matrix.

Our main claim is that

- physical resonances, the two nonets in PDG, can be identified by the eigenstates that diagonalize the 2x2 matrix,
  i.e., the two nonets in PDG must be superposition of $|000\rangle, |011\rangle$.
- In fact, the mixing is found to be strong so it can explain the large mass gap between the two nonets (later!).

This is our tetraquark mixing framework for the two nonets in $J^P = 0^+$.

☞ We look for its phenomenological signatures from experimental observables such as masses or decay properties !

skip

Other approaches for the two nonets

other approaches

The two-quark picture($q\bar{q}$) with $\ell = 1$

- can make nonets also with $J^P = 0^+$.
- Does this picture explain the two nonets in PDG ?   My answer is `No’

$$q\bar{q}: (S = 0,1) \otimes (\ell = 1) \Longrightarrow J = 0,1,2$$

Total J	Configuration	# of confs.
$J = 0$	$(S = 1, \ell = 1)$	one
$J = 1$	$(S = 0, \ell = 1), (S = 1, \ell = 1)$	two
$J = 2$	$(S = 1, \ell = 1)$	one

- This picture has only one configuration in $J^P = 0^+$, not enough to explain the two nonets in $J^P = 0^+$.

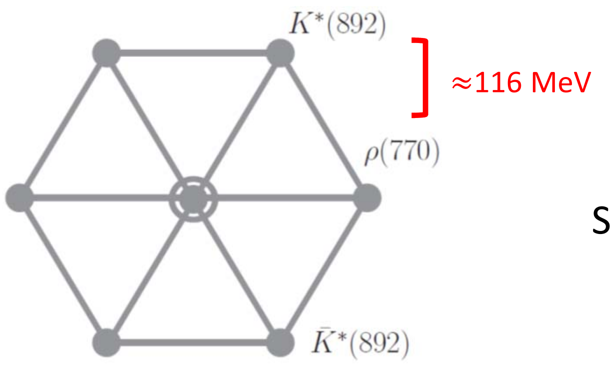
Alternatively, one may view

- the heavy nonet as the $q\bar{q}$ with $\ell = 1$ while maintaining the $qq\bar{q}\bar{q}$ picture for light nonet.

☞ This is not realistic for the heavy nonet.

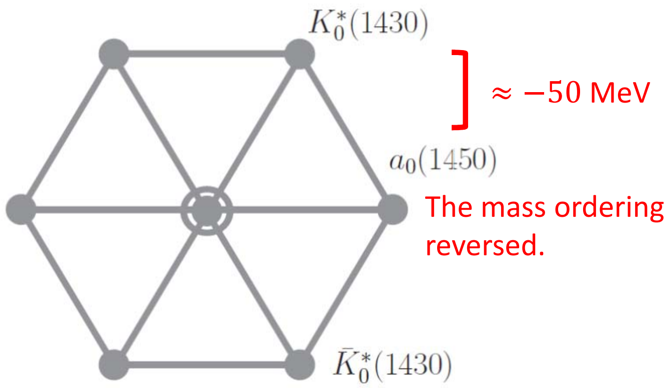
- The heavy nonet, if viewed as the $q\bar{q}$ with $\ell = 1$, must have the configuration $(S = 1$, vector nonet$) \otimes (\ell = 1) \Rightarrow J = 0$

  $\Rightarrow$ orbital excitations of the vector mesons, ρ, ω, K^*, ϕ.



$\Rightarrow$

Spin-Orbit (SO)
$(\ell = 1)$



The mass ordering is reversed.

- In this picture, SO makes the heavy nonet ‘heavier’ than the vector nonet.
- To reproduce the reversed gap (≈ -50 MeV), SO must have strong isospin dependence, strong enough to flip the mass ordering established by the quark masses.

  ☞ This picture seems not realistic !

Mixture of $q\bar{q}$, $qq\bar{q}\bar{q}$

other approaches

One may view the two nonets as a mixture of $q\bar{q}$ $(\ell = 1)$, and the four-quark state ($qq\bar{q}\bar{q}$) ?   Black et.al, PRD 59(1999)

- Black et.al introduce the effective fields corresponding to $q\bar{q}$ and $qq\bar{q}\bar{q}$ nonets, and make SU(3) invariant Lagrangian among them.
- As pointed by Maiani et.al. EPJC50(2007), the required mixing seems too large given the fact that the very different configurations are involved.

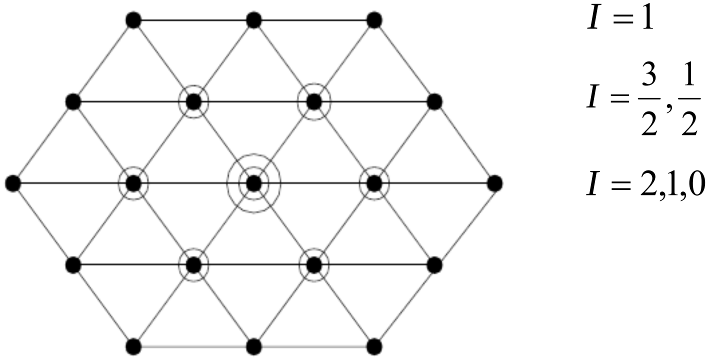
- In particular, $q\bar{q}(\ell = 1)$, $qq\bar{q}\bar{q}$ do not mix under the color-spin interaction ! $\langle q\bar{q}|qq\bar{q}\bar{q}\rangle = 0, \langle q\bar{q}|V_{CS}|qq\bar{q}\bar{q}\rangle = 0.$
- That is, hard to imagine such a mixing term from well-known quark-quark interactions.

other approaches

One may view the two nonets as meson-meson bound states.

- Since mesons are colorless, this model suggests shallow bound states.
  $\Rightarrow$ Expect to have narrow mass gaps between the resonances and the two-meson masses,
  ex) $f_0(980){\sim}K\bar{K}$ since $M[f_0(980)]{\sim}2M[K]$.

- Since the lowest-lying mesons form a nonet in flavor, the flavor structure of the meson-meson states would be much diverse including 27-plet
  $$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$$
  $\Rightarrow$ PDG does not support this picture. (ex. no 0^+ resonances with $I = 2$.)



$\mathbf{27}_f$

But note Jido et.al(PRL2006)

testing ground

Now returning to tetraquark mixing framework, its testing ground is the two nonets.

Isospin	Light nonet	Heavy nonet
$I=1$	$a_0(980)$	$a_0(1450)$
$I=1/2$	$K_0^*(800)$	$K_0^*(1430)$
$I=0$	$f_0(500)$	$f_0(1370)$
	$f_0(980)$	$f_0(1500)$

$\Leftarrow$ close to the 8_f member

$\Leftarrow$ close to the 1_f member

Flavor mixing on isoscalars

- The $I=0$ members are subject to additional flavor mixing between $|\mathbf{8}_f\rangle_{I=0}$, $|\mathbf{1}_f\rangle_{I=0}$, known as the OZI rule.
- Depending on how the flavor mixing is implemented, we consider three cases (Hungchong Kim et.al., PRD2018),
  - – SU(3)$_f$ Symmetric Case, SSC (no flavor mixing)
  - – Ideal Mixing Case, IMC
  - – Realistic Case with Fitting, RCF

According to our mixing scheme

- first we need to calculate the hyperfine masses, $\langle V_{CS} \rangle$, w.r.t. $|000\rangle, |011\rangle$ in each isospin channel. Then its diagonalization generates the physical states.

Hyperfine masses, $\langle V_{CS} \rangle$

Color-spin interaction for four-quark system

hyperfine masses

$$V_{CS} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

for all the pairs among 4 quarks

$$= v_0 \left[\lambda_1 \cdot \lambda_2 \frac{J_1 \cdot J_2}{m_1 m_2} + \lambda_3 \cdot \lambda_4 \frac{J_3 \cdot J_4}{m_3 m_4} + \lambda_1 \cdot \lambda_3 \frac{J_1 \cdot J_3}{m_1 m_3} + \lambda_1 \cdot \lambda_4 \frac{J_1 \cdot J_4}{m_1 m_4} + \lambda_2 \cdot \lambda_3 \frac{J_2 \cdot J_3}{m_2 m_3} + \lambda_2 \cdot \lambda_4 \frac{J_2 \cdot J_4}{m_2 m_4} \right]$$

Master formulas for $\langle V_{CS} \rangle$

$v_0 = (-192.9\ \mathrm{MeV})^3$ from the mass splitting, $D_2^*(2463) - D_0^*(2318)$

$\langle J, J_{12}, J_{34} \vert V \vert J, J_{12}, J_{34} \rangle$	Corresponding formulas for one specific flavor combination, $q_1 q_2 \bar{q}^3 \bar{q}^4$
$\langle 000 \vert V_{CS} \vert 000 \rangle$	$2v_0 \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} \right]$
$\langle 011 \vert V_{CS} \vert 011 \rangle$	$\frac{v_0}{3} \left[\frac{1}{m_1 m_2} + \frac{1}{m_3 m_4} + \frac{5}{m_1 m_3} + \frac{5}{m_1 m_4} + \frac{5}{m_2 m_3} + \frac{5}{m_2 m_4} \right]$
mixing, $\langle 000 \vert V_{CS} \vert 011 \rangle$	$\underline{\sqrt{\frac{3}{2}} v_0 \left[\frac{1}{m_1 m_3} + \frac{1}{m_1 m_4} + \frac{1}{m_2 m_3} + \frac{1}{m_2 m_4} \right]} \neq 0$

$\Longleftarrow$ only diquark and antidiquark pairs contribute

$\Longleftarrow$ all the pairs contribute

$\Longleftarrow$ only quark-antiquark pairs contribute

Ex) For the $I = 1$ member, a_0^+

Since its flavor is $[su][\bar{d}\bar{s}]=\frac{1}{2}(su - us)\left(\bar{d}\bar{s} - \bar{s}\bar{d}\right)$, we sum over all the flavor combinations

$$\langle V_{CS} \rangle = \frac{1}{4}\left[\langle V_{CS} \rangle_{su\bar{d}\bar{s}} + \langle V_{CS} \rangle_{su\bar{s}\bar{d}} + \langle V_{CS} \rangle_{us\bar{d}\bar{s}} + \langle V_{CS} \rangle_{us\bar{s}\bar{d}} \right].$$

hyperfine masses

Hyperfine mass matrix in the $I = 1$ channel corresponding to $a_0(980)$, $a_0(1450)$.

- Diagonalization leads to the physical hyperfine masses

$$\begin{array}{c|cc} \langle V_{CS}\rangle & |000\rangle & |011\rangle \\ \hline |000\rangle & \underline{-173.9} & \underline{-222.3} \\ |011\rangle & -222.3 & \underline{-331.5} \end{array} \quad \rightarrow \quad \begin{array}{c|cc} \langle V_{CS}\rangle & |0_A^{a_0}\rangle & |0_B^{a_0}\rangle \\ \hline |0_A^{a_0}\rangle & -16.8 & 0.0 \\ |0_B^{a_0}\rangle & 0.0 & -488.5 \end{array}$$

and eigenstates corresponding to $a_0(980), a_0(1450)$,

$$|0_A^{a_0}\rangle = -0.817|000\rangle + 0.577|011\rangle \quad \Rightarrow |a_0(1450)\rangle$$

$$|0_B^{a_0}\rangle = 0.577|000\rangle + \underline{0.817|011}\rangle \quad \Rightarrow |a_0(980)\rangle\,.$$

This identification follows from $\langle 0_A^{a_0}|V_{CS}|0_A^{a_0}\rangle > \langle 0_B^{a_0}|V_{CS}|0_B^{a_0}\rangle$

As advertised,

- $|011\rangle$ is found to be more compact, $\langle 000|V_{CS}|000\rangle > \langle 011|V_{CS}|011\rangle$.
  ☞ $a_0(980)$ has more probability to stay in $|011\rangle$ rather than in $|000\rangle$!!
  ☞ We emphasize that $|011\rangle$ must be considered in tetraquark studies.

- The **strong mixing** causes **large separation** in hyperfine masses [$\Delta\langle V_{CS}\rangle \approx 500$ MeV].
  ☞ This can explain the large mass gap (500 MeV or so) between the two nonets.

mixing parameters

Similar results can be obtained in the other isospin channels.

- The mixing formulas for the two nonets are collectively represented by

$$\begin{aligned} |\text{Heavy nonet}\rangle &= -\alpha|000\rangle + \beta|011\rangle \\ |\text{Light nonet}\rangle &= \beta|000\rangle + \alpha|011\rangle \end{aligned}$$

Hyperfine masses ($\Delta\langle V_{CS}\rangle \approx 500$ MeV)

Isospin	α	β	Light	$\langle V_{CS}\rangle$	Heavy	$\langle V_{CS}\rangle$
$I = 1$	0.8167	0.5770	$a_0(980)$	-488.5	$a_0(1450)$	-16.8
$I = 1/2$	0.8130	0.5822	$K_0^*(800)$	-592.7	$K_0^*(1430)$	-26.9
$I = 0$ (RCF)	0.8136	0.5814	$f_0(500)$	-667.5	$f_0(1370)$	-29.2
$I = 0$ (RCF)	0.8157	0.5784	$f_0(980)$	$\underline{-535.1}$	$f_0(1500)$	$\underline{-20.1}$

close to the 8_f member

mixing parameters ($\alpha > \beta$)

Immediate signatures to support the tetraquark mixing framework

signature 1

Hyperfine masses explain partially the marginal mass ordering in the heavy nonet !

Isospin	Light	$\langle V_{CS} \rangle$	Heavy	$\langle V_{CS} \rangle$
$I = 1$	$a_0(980)$	-488.5	$a_0(1450)$	-16.8
$I = 1/2$	$K_0^*(800)$	-592.7	$K_0^*(1430)$	-26.9
$I = 0$ (RCF)	$f_0(500)$	-667.5	$f_0(1370)$	-29.2
$I = 0$ (RCF)	$f_0(980)$	$\underline{-535.1}$	$f_0(1500)$	$\underline{-20.1}$

~100

~10

close to the 8_f member

- For the octet members, our hyperfine masses are ordered, $\langle V_{CS} \rangle_{I=1} > \langle V_{CS} \rangle_{I=1/2} > \langle V_{CS} \rangle_{I=0}$, the same as the masses, $M[a_0] > M[K_0^*] > M[f_0]$.
  ☞ $\langle V_{CS} \rangle$ is also responsible for the mass ordering.

- But the $\langle V_{CS} \rangle$ splitting becomes narrower for the heavy nonet,
  ~ 100 MeV for light nonet,
  ~ 10 MeV or less for heavy nonet.

signature 2

Mass splitting formula works very well for our tetraquarks.

$$\Delta M_H \approx \Delta\langle V_{CS}\rangle$$

$$V_{CS} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

The mass difference between hadrons with the same flavor content can be approximated by their hyperfine mass splitting (we understand why).

For $I = 1$

Heavy nonet	Light nonet	ΔM_{exp} (MeV)	$\Delta\langle V_{CS}\rangle$ (MeV)		
			SSC	IMC	RCF
$a_0(1450)$	$a_0(980)$	**494**	**471.7**	-	-

☞Our mixing scheme works very well !

equal when $m_u = m_d$

For $I = 0, 1/2$

M_{exp} is broad or not fixed well

$f_0(1500)$	$f_0(980)$	**515**	**541.7**	471.7	515
$f_0(1370)$	$f_0(500)$	875	611.7	681.7	638.3
$K_0^*(1430)$	$K_0^*(800)$	743	565.8	-	-

- The $I = 0$ results do not depend much on how the flavor mixing is implemented.
- For the last two lines, precise agreement is not anticipated as the participating resonances are either broad or their masses are poorly known.

The strong mixing qualitatively generates the huge gap (500 MeV or so) between the two nonets.

signature 3

The mixing parameters, α, β, support flavor nonet structure.

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

Isospin	Light	Heavy	α	β
$I = 1$	$a_0(980)$	$a_0(1450)$	0.8167	0.5770
$I = 1/2$	$K_0^*(800)$	$K_0^*(1430)$	0.8130	0.5822
$I = 0$ (RCF)	$f_0(500)$	$f_0(1370)$	0.8136	0.5814
$I = 0$ (RCF)	$f_0(980)$	$f_0(1500)$	0.8157	0.5784

close to the 8_f member

- α, β, are determined separately in each isospin through diagonalization.
- But they are almost the same independent of isospins,

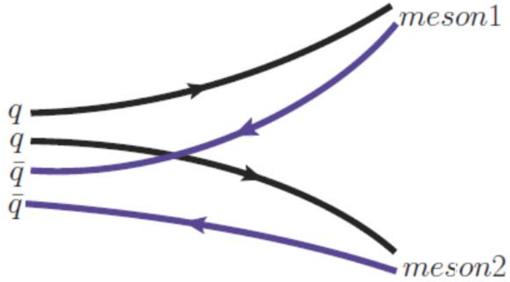
  $$\alpha \approx \sqrt{\frac{2}{3}} = 0.8165, \quad \beta \approx \sqrt{\frac{1}{3}} = 0.5773.$$
- The same α, β indicate that |Heavy nonet>, |Light nonet> form the SU(3) flavor nonet consistently with the two nonets in PDG.

Other signatures from fall-apart modes

These may provide exclusive signatures for tetraquark mixing framework.

Tetraquarks can decay through the fall-apart mechanism

- because of two-meson channels existing in the tetraquark wave functions.



$qq\bar{q}\bar{q}$ fall-apart decay

One can see two-meson channels

- by rearranging the tetraquarks into quark-antiquark pairs in color space.

$$\overset{(24)}{} \quad q_1 q_2 \bar{q}^3 \bar{q}^4 \Longrightarrow [(8_c)_{13} \otimes (8_c)_{24}]_{1_c} \oplus \boxed{[(1_c)_{13} \otimes (1_c)_{24}]_{1_c}} \quad \underset{(13)}{}$$

two-meson channels

- When $(1_c)_{13}$, $(1_c)_{24}$ are in the spin-0 state,
  ☞we have PS-PS mode, PP
- When $(1_c)_{13}$, $(1_c)_{24}$ are in the spin-1 state,
  ☞we have V-V mode, VV

Coefficients of two-meson channels represent relative strengths of fall-apart decay.
Interesting signatures can be seen in the relative strengths !

Spin and color factors in the two-meson modes

relative strengths

$$PP = |0,0\rangle_{13}|0,0\rangle_{24}$$

$$VV = \frac{1}{\sqrt{3}}\Big[|1,1\rangle_{13}|1,-1\rangle_{24} - |1,0\rangle_{13}|1,0\rangle_{24} + |1,-1\rangle_{13}|1,1\rangle_{24}\Big] \,,$$

	$\vert 000\rangle$	$\vert 011\rangle$
Spin factors	$\frac{1}{2}PP + \frac{\sqrt{3}}{2}VV$	$\frac{\sqrt{3}}{2}PP - \frac{1}{2}VV$
Color factors	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$
Spin-color for PP	$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{2}}$
Spin-color for VV	$\frac{1}{2}$	$-\frac{1}{\sqrt{6}}$

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$

Heavy nonet	Light nonet
$-\frac{\alpha}{2\sqrt{3}} + \frac{\beta}{\sqrt{2}}$	$\frac{\beta}{2\sqrt{3}} + \frac{\alpha}{\sqrt{2}}$
$-\frac{\alpha}{2} - \frac{\beta}{\sqrt{6}}$	$\frac{\beta}{2} - \frac{\alpha}{\sqrt{6}}$

※By construction, the flavor factors are the same for $|000\rangle, |011\rangle$.

Due to the relative sign differences, we find that

- PP modes: suppressed in heavy nonet but enhanced in the light nonet.
- VV modes: enhanced in heavy nonet but suppressed in the light nonet.

☞Interesting signatures from the tetraquark mixing framework !

In this talk, we discuss only the PP modes from the isovector ($I = 1$) resonances, $a_0(980)$, $a_0(1450)$ and compare them with experiment data.

- The main features are maintained for the $I = 0, 1/2$ cases but we do not consider the other channels because expt. data are rather limited for them.

- Also, we do not discuss the VV modes because most of them are not accessible experimentally by their kinematics.

Fall-apart strength of $a_0(980)$, $a_0(1450)$ into two PS mesons

- PP modes in flavor: $[su][\bar{d}\bar{s}] \Longrightarrow (s\bar{d})(u\bar{s}) - (s\bar{s})(u\bar{d})$

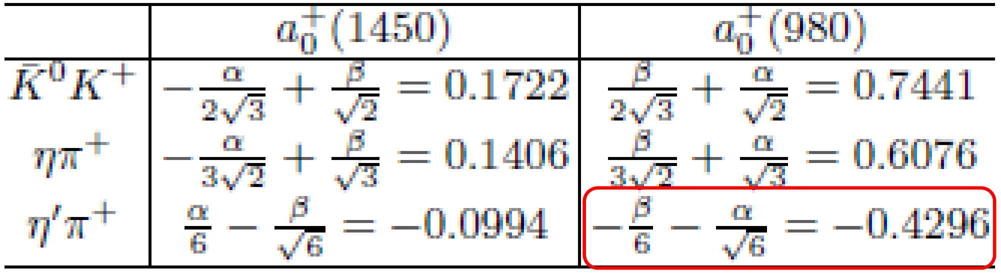
$$\Longrightarrow \bar{K}^0K^+ + \sqrt{\tfrac{2}{3}}\eta\pi^+ - \frac{1}{\sqrt{3}}\eta'\pi^+$$

- By combining with the spin-color factors

$-\frac{\alpha}{2\sqrt{3}} + \frac{\beta}{\sqrt{2}}$ for the heavy nonet

$\frac{\beta}{2\sqrt{3}} + \frac{\alpha}{\sqrt{2}}$ for the light nonet

Coupling strength of the fall-apart modes into two PS mesons up to an overall constant



	$a_0^+(1450)$	$a_0^+(980)$
$\bar{K}^0K^+$	$-\frac{\alpha}{2\sqrt{3}} + \frac{\beta}{\sqrt{2}} = 0.1722$	$\frac{\beta}{2\sqrt{3}} + \frac{\alpha}{\sqrt{2}} = 0.7441$
$\eta\pi^+$	$-\frac{\alpha}{3\sqrt{2}} + \frac{\beta}{\sqrt{3}} = 0.1406$	$\frac{\beta}{3\sqrt{2}} + \frac{\alpha}{\sqrt{3}} = 0.6076$
$\eta'\pi^+$	$\frac{\alpha}{6} - \frac{\beta}{\sqrt{6}} = -0.0994$	$\boxed{-\frac{\beta}{6} - \frac{\alpha}{\sqrt{6}} = -0.4296}$

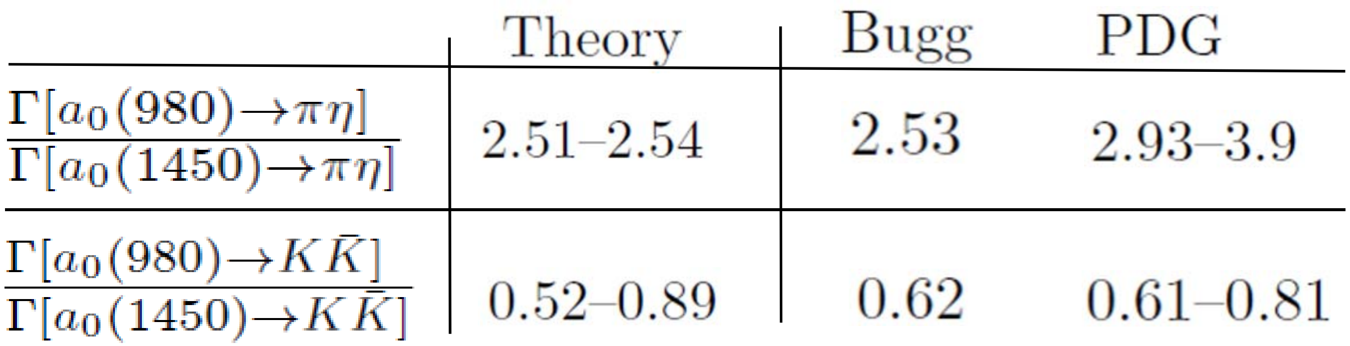
kinematically not allowed

- The relative enhancement factor is about ‘four’ !

partial width ratios

This signature can be tested effectively from the following ratios !

Based on expt. analysis



	Theory	Bugg	PDG
$\frac{\Gamma[a_0(980)\to\pi\eta]}{\Gamma[a_0(1450)\to\pi\eta]}$	2.51–2.54	2.53	2.93–3.9
$\frac{\Gamma[a_0(980)\to K\bar{K}]}{\Gamma[a_0(1450)\to K\bar{K}]}$	0.52–0.89	0.62	0.61–0.81

Bugg: PRD78,074023(2008)

- The ratios eliminate the dependence on the overall constant.
- Note also that the partial widths depend on kinematical factors as well as the coupling strengths.

The agreement is quite good !

- Only disagreement is in the 1st ratio in comparison with the PDG ratio but both results still point toward the enhancement and suppression of the couplings.

Summary

- We have proposed a tetraquark mixing framework for the two nonets,
  Light nonet: $f_0(500), f_0(980), K_0^*(800), a_0(980)$
  Heavy nonet: $f_0(1370), f_0(1500), K_0^*(1430), a_0(1450)$
  Clues for the tetraquark nonets: Quantum numbers (J^{PC}, I), GMO relation, the mass ordering
- Two types of tetraquark, $|000\rangle, |011\rangle$, are found to mix strongly through the color-spin interaction.
- Tetraquark mixing framework suggests that their mixture, which diagonalizes the hyperfine masses, can generate the two nonets in PDG..

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle$$
$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle$$
$$\alpha \approx \sqrt{2/3}, \beta \approx \sqrt{1/3}$$

- Signatures to support our tetrquark mixing framework.
  - Hyperfine mass splitting agrees with the mass splitting between the two nonets (500 MeV or so).
  - Hyperfine masses partially explain the marginal mass ordering seen in the heavy nonet.
  - The mixing parameters: almost the same showing that our mixing formulas generate the flavor nonets consistently with the two nonets in PDG.
  - Fall-apart modes: we find that
    - PP modes: suppressed in heavy nonet but enhanced in the light nonet.
    - VV modes: enhanced in heavy nonet but suppressed in the light nonet.

- Signature from the PP modes has been tested relatively well through the ratios of partial widths, $a_0(980), a_0(1450) \Longrightarrow K\bar{K}, \eta\pi$.

Our work provides a new view on tetraquarks, especially how they are realized in the actual spectrum, i.e., through ``mixing framework''.

Back up slides

digression

One question

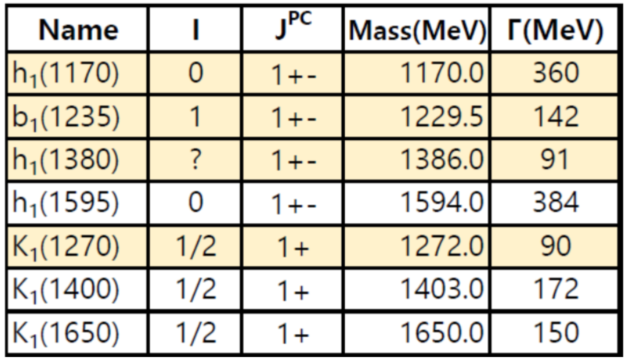
- The spin-1 diquark scenario requires additional nonets to be found in $J^P = 1^{+-}, 2^{++}$ corresponding to the configurations

$$|111\rangle_{6_c,\bar{6}_c} \qquad |211\rangle_{6_c,\bar{6}_c}$$

※ One can prove that C-parity is negative for $J = 1$, positive for $J = 2$.

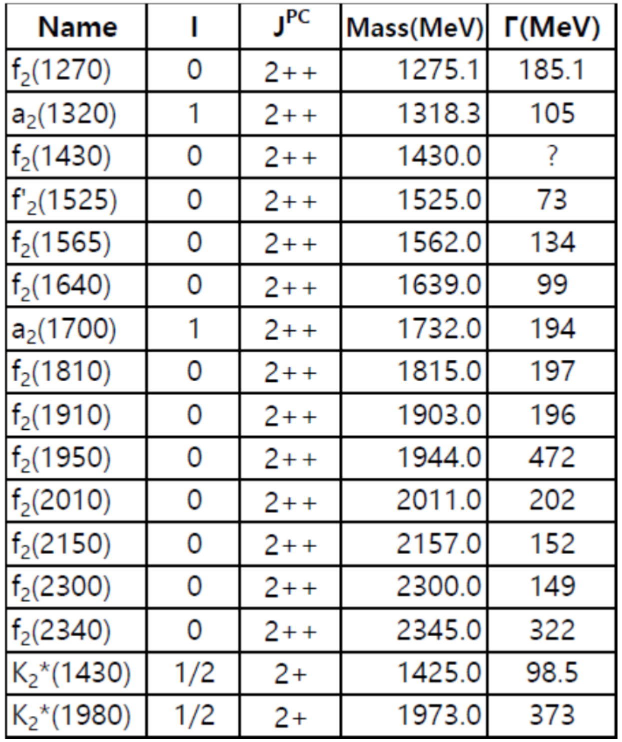
Are there such nonets in PDG ? My answer is ‘Maybe’.

- There are lots of resonances to choose but the candidate selection is not definite.



Name	I	J^PC	Mass(MeV)	Γ(MeV)
h_1(1170)	0	1+-	1170.0	360
b_1(1235)	1	1+-	1229.5	142
h_1(1380)	?	1+-	1386.0	91
h_1(1595)	0	1+-	1594.0	384
K_1(1270)	1/2	1+	1272.0	90
K_1(1400)	1/2	1+	1403.0	172
K_1(1650)	1/2	1+	1650.0	150

$J^{P(C)} = 1^{+(-)}$ resonances



Name	I	J^PC	Mass(MeV)	Γ(MeV)
f_2(1270)	0	2++	1275.1	185.1
a_2(1320)	1	2++	1318.3	105
f_2(1430)	0	2++	1430.0	?
f'_2(1525)	0	2++	1525.0	73
f_2(1565)	0	2++	1562.0	134
f_2(1640)	0	2++	1639.0	99
a_2(1700)	1	2++	1732.0	194
f_2(1810)	0	2++	1815.0	197
f_2(1910)	0	2++	1903.0	196
f_2(1950)	0	2++	1944.0	472
f_2(2010)	0	2++	2011.0	202
f_2(2150)	0	2++	2157.0	152
f_2(2300)	0	2++	2300.0	149
f_2(2340)	0	2++	2345.0	322
K_2*(1430)	1/2	2+	1425.0	98.5
K_2*(1980)	1/2	2+	1973.0	373

$J^{P(C)} = 2^{+(+)}$ resonances

- Highlighted members can be selected but with some ambiguity,
  - unknown isospin of $h_1(1380)$,
  - the mass ordering, slightly violated, $M[b_1(1235)] < M[K_1(1270)]$

digression

The selection is ambiguous

- maybe due to further mixings with additional tetraquarks constructed by other diquarks, and possible contamination from two-quark component with $\ell = 1$.
- This ambiguity does not mean that $|111\rangle, |211\rangle$ do not exist.
  ⇒ It simply says that the candidates do not stand out in a well-separated entity.
  ⇒ It does not rule out our mixing framework in the 0^+ channel.