Systematic twist expansion of meson-photon transition form factors in the light-front quark model

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(collaboration with C.-R. Ji and Hui-Young Ryu)

[1] Phys. Rev. D98, 034018 (18) by H. Ryu, C. Ji, HMC
[2] Phys. Rev. D96, 056008 (17) by H. Ryu, C. Ji, HMC
[3] Phys. Rev. D95, 056002 (17) by C. Ji, HMC
[4] Phys. Rev. D98, 014018 (15) by C. Ji, HMC
[5] Phys. Rev. D98, 033001 (14) by C. Ji, HMC

Outline

1. Motivation
2. Why Light-Front?
3. Construction of Self-consistent LFQM
   - Meson DAs (twist-2 and-3) and Transition Form Factors
4. Numerical Results
5. Conclusion

1. Motivation

- **Meson-photon transitions** $P(\pi^0, \eta, \eta') \rightarrow \gamma^* \gamma$ **:**

- Simplest exclusive processes involving the strong interaction
- Significant role for both the low- and high-energy precision tests of the SM

1. Motivation

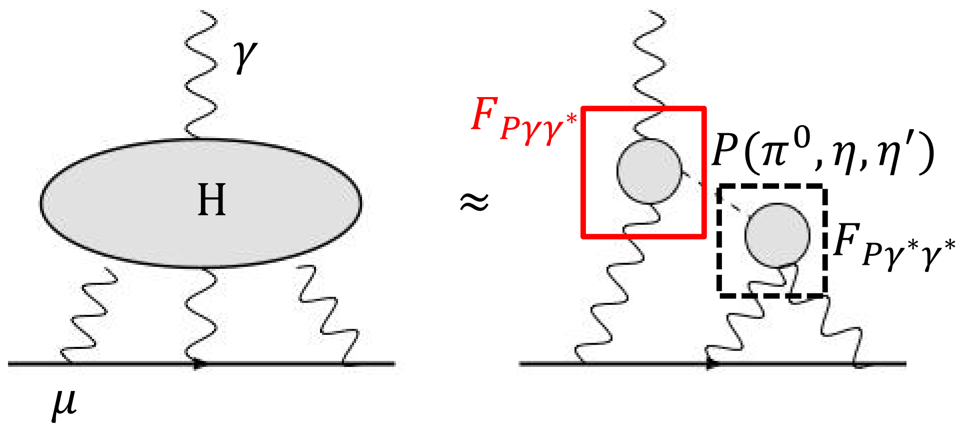
- **Meson-photon transitions** $P(\pi^0, \eta^{(\prime)}, \eta_{c(b)}) \rightarrow \gamma^*\gamma$ **:**

- Simplest exclusive processes involving the strong interaction
- Significant role for both the low- and high-energy precision tests of the SM

1) For the low-energy regime:

The transition form factors(TFFs) enter the prediction of important observables such as $P \rightarrow \ell\bar{\ell}(\ell = e, \mu)$ decays and the Hadronic Light by Light scattering (HLbL) contribution to the muon $(g-2)_\mu$:

e.g.) Pseudoscalar-pole contribution to HLbL.



$a_\mu = (g-2)/2$

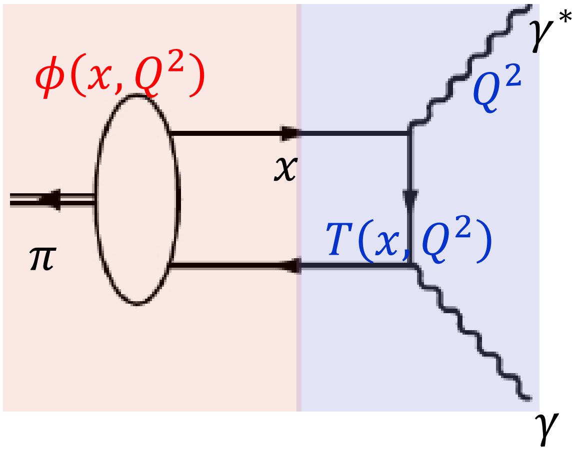
[Exp. –Th.(SM)]($\sim 3\sigma$)
$= (278 \pm 88) \times 10^{-11}$

HLbL
$= (116 \pm 40) \times 10^{-11}$

A. Nyffeler(2016)

2) For the high-energy regime: TFFs can be calculated from pQCD

e.g.) $\pi \to \gamma^* \gamma$ TFF



In theory (pQCD):

$Q^2 F_{\pi\gamma} = f_\pi \sqrt{2} \sim 0.185$ GeV

: Brodsky-Lepage(BL) limit

Calculable in pQCD ;
depends on hard subprocesses

At leading twist:

$$F_{\pi\gamma}(Q^2) = \int T(x,Q^2)\phi(x,Q^2)dx + \cdots$$

ϕ: Nonperturbative (leading twist) meson DA;
Universal and process independent

$$\phi(x,\mu) \propto \int_{|\boldsymbol{k}_\perp|^2 \leq \mu^2} d^2\boldsymbol{k}_\perp \psi(x,\boldsymbol{k}_\perp)$$

$$\xrightarrow{\mu \to \infty} \quad 6x(1-x)\text{: "Asymptotic DA"}$$

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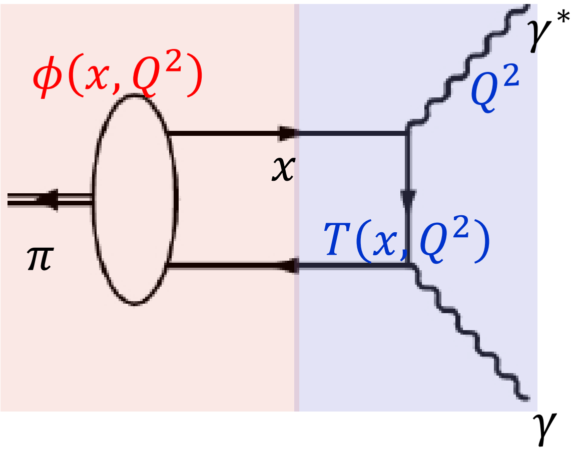

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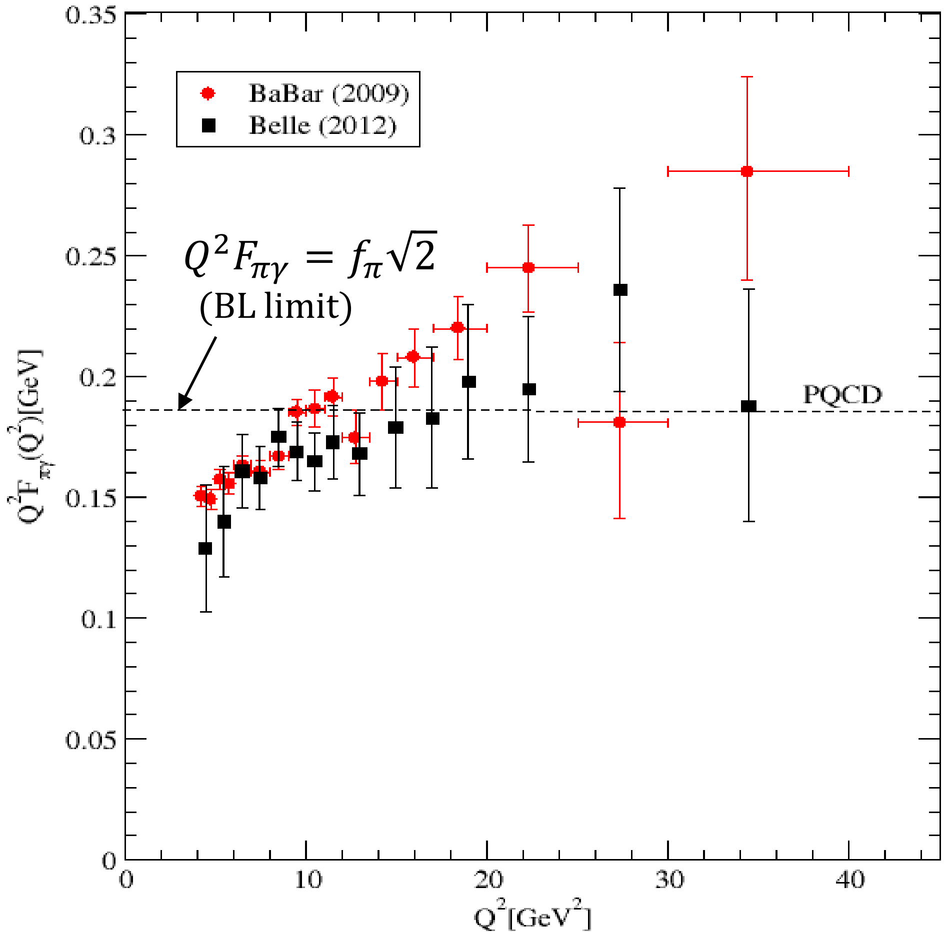
- Higher twist DAs are essential for systematic study of preasymptotic corrections to hard exclusive processes & may contain new information on hadron structure and dynamics of QCD.

- **Experimental status for $F_{\pi\gamma}(Q^2)$** from $e^+e^- \rightarrow e^+e^-\pi^0$



In this talk, I am going to discuss

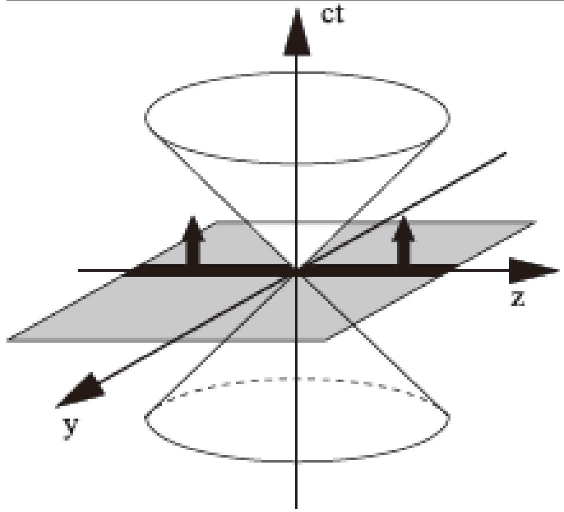
1) Which one (BarBar vs. Belle) is more consistent in our LFQM ?

2) How to explore timelike region as well as spacelike region in our LFQM?

3) How to systematically express $Q^2\ F_{P\gamma}(Q^2)$ in terms of the leading- and higher-twist DAs in our LFQM?

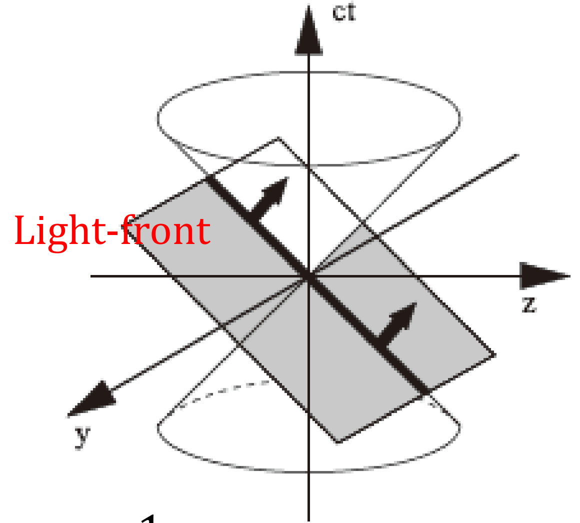
2. Why Light-Front?

Instant form ($x^0 = ct = 0$)



$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

Front form ($x^+ = x^0 + x^3 = 0$)



$$a \cdot b = \frac{1}{2}(a^+ b^- + a^- b^+) - \vec{a} \cdot \vec{b}$$

Hamiltonian	P^0	$P^- = P^0 - P^3$
Momentum	$\boldsymbol{P}_\perp = (P^1,\ P^2)$ P^3	$\boldsymbol{P}_\perp$ $P^+ = P^0 + P^3$
E-P dispersion Relation	$P^0 = \sqrt{M^2 + \vec{P}^2}$	$P^- = \frac{M^2 + \boldsymbol{P}_\perp^2}{P^+}$

Irrational vs. Rational

- Distinguished Features in LFD:

(1) Possess maximum # of interaction free kinematic generators!

LF boost

# of Kinematic generators: 6 $(P^i,\ J^i)$ 7 $(P^+,\ \boldsymbol{P}_\perp, J_3, K_3, K_{1(2)} \pm J_{2(1)})$

Hamiltonian	P^0	$P^- = P^0 - P^3$
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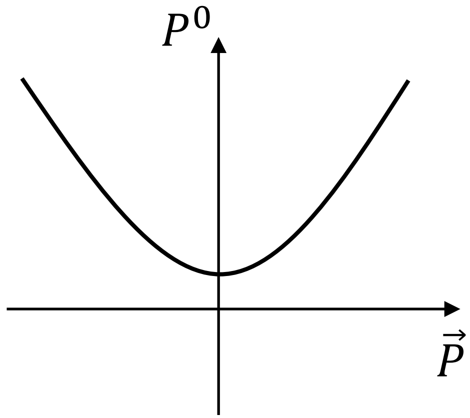
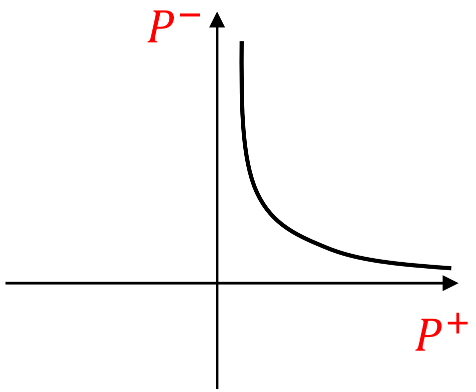
- Distinguished Features in LFD:

(2) Vacuum fluctuations are suppressed!

$P^0 = \sqrt{M^2 + \vec{P}^2}$ (Instant Form)	$P^- = \frac{M^2 + P_\perp^2}{P^+}$ (Front form)

Irrational vs. Rational





(While $P^0 \geq 0, \vec{P}$ can be $+$ *or* $-$)

(Both $P^\pm \geq 0$)

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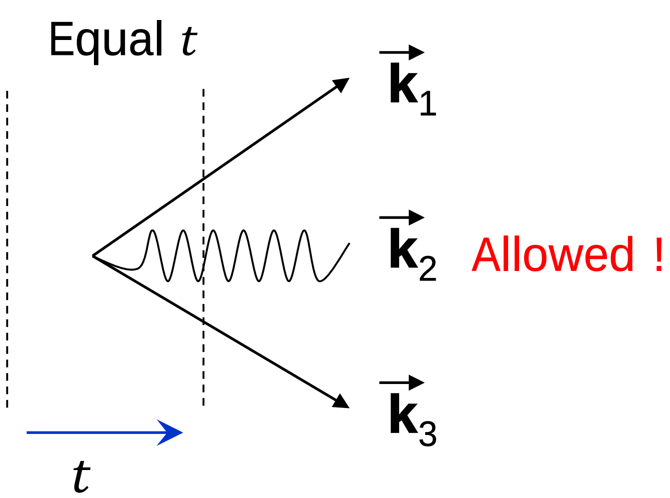

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$

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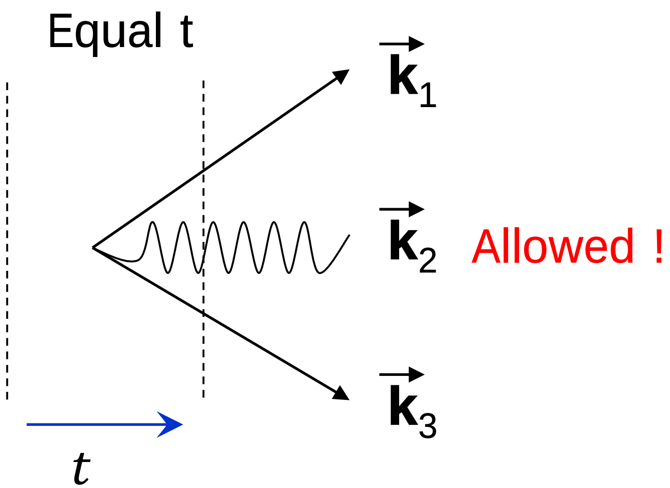
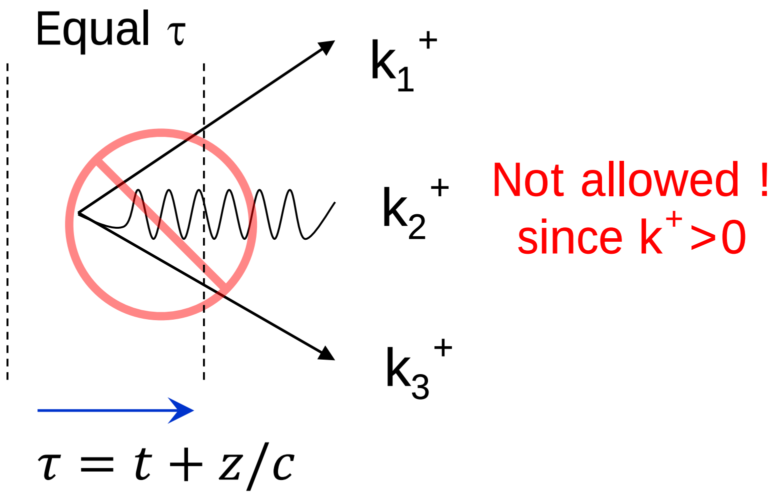
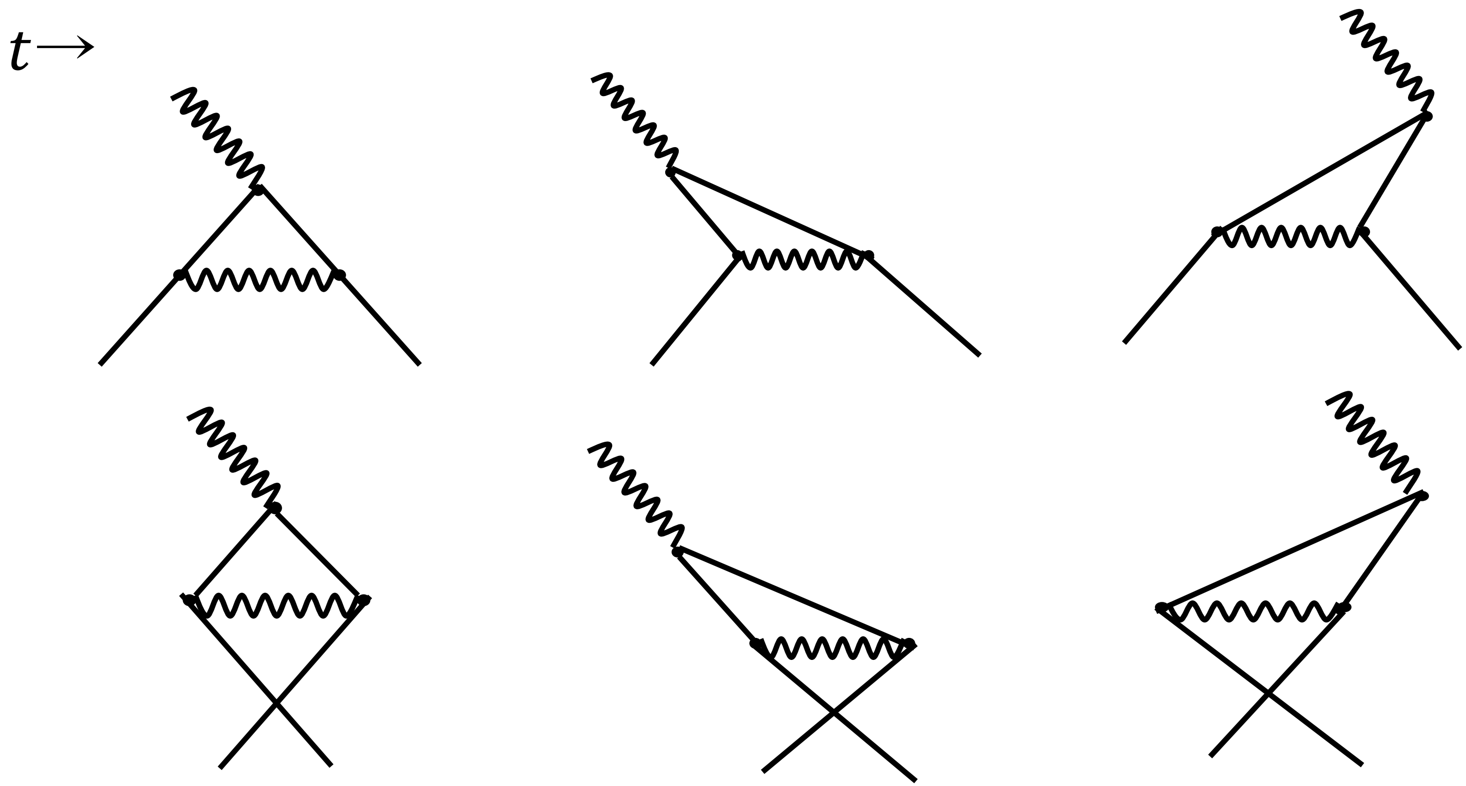




$\vec{k}_1+\vec{k}_2+\vec{k}_3=0$ $k_1^+ + k_2^+ + k_3^+=0$

g-2 calculation

(1) Equal-t formulation:



g-2 calculation

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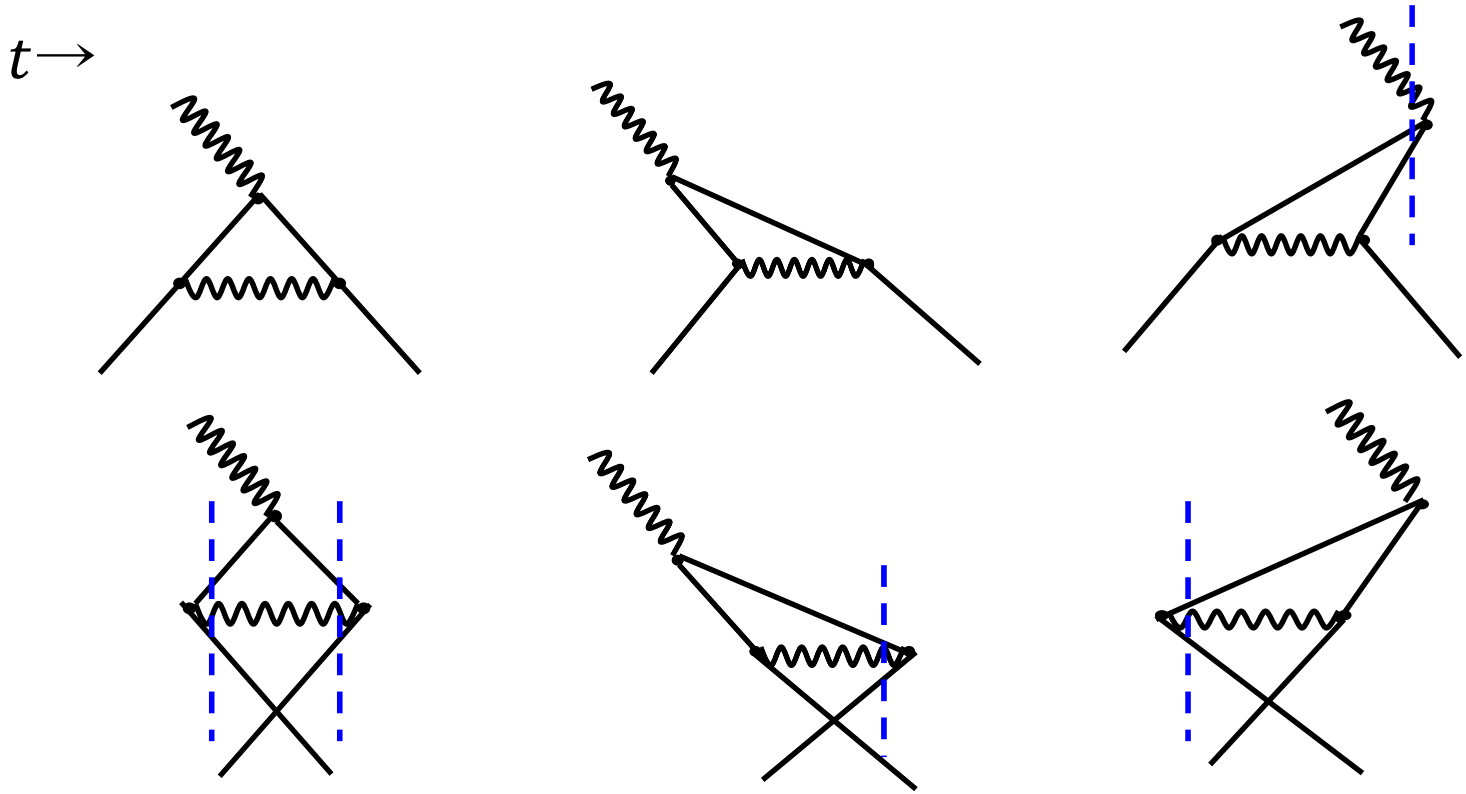
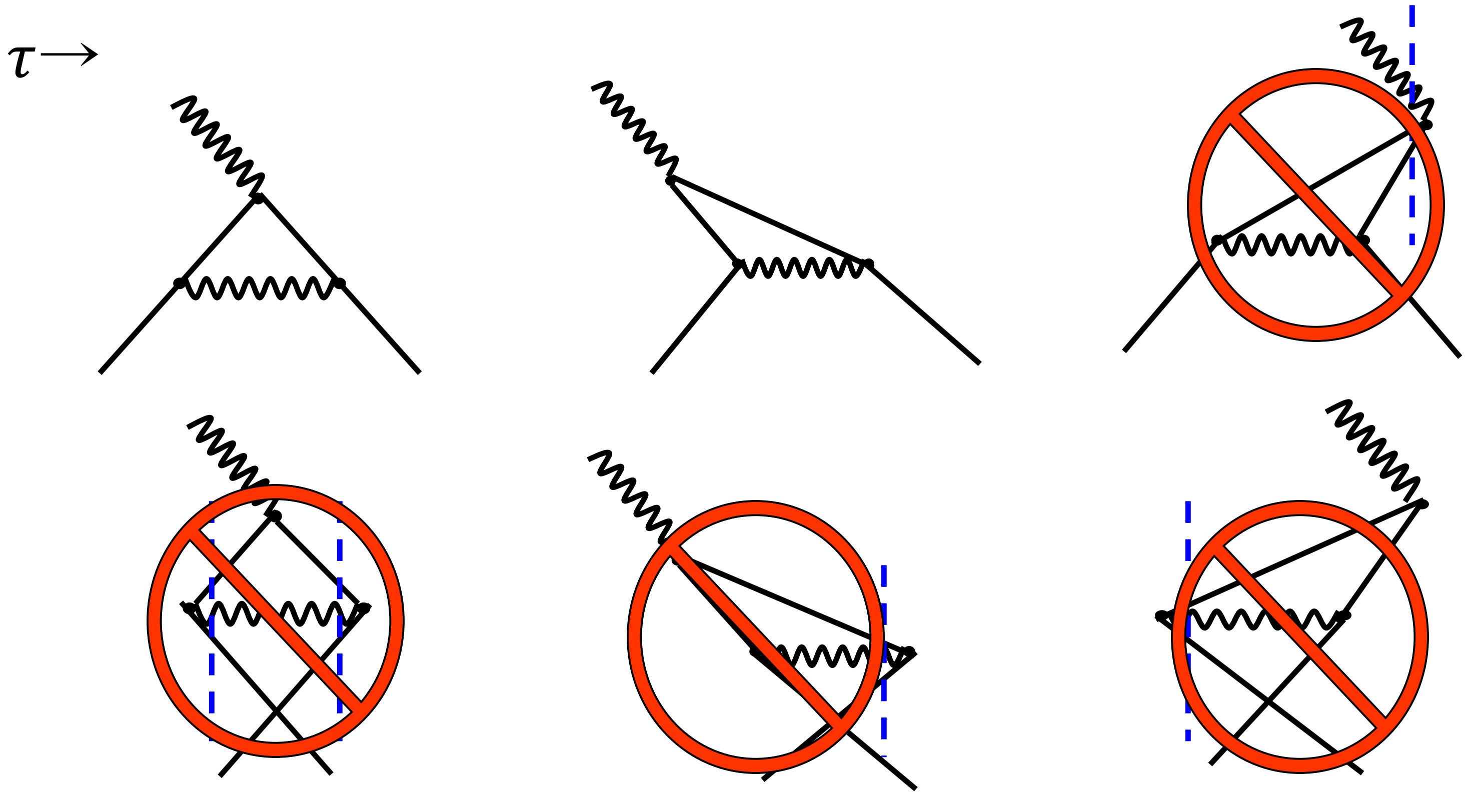


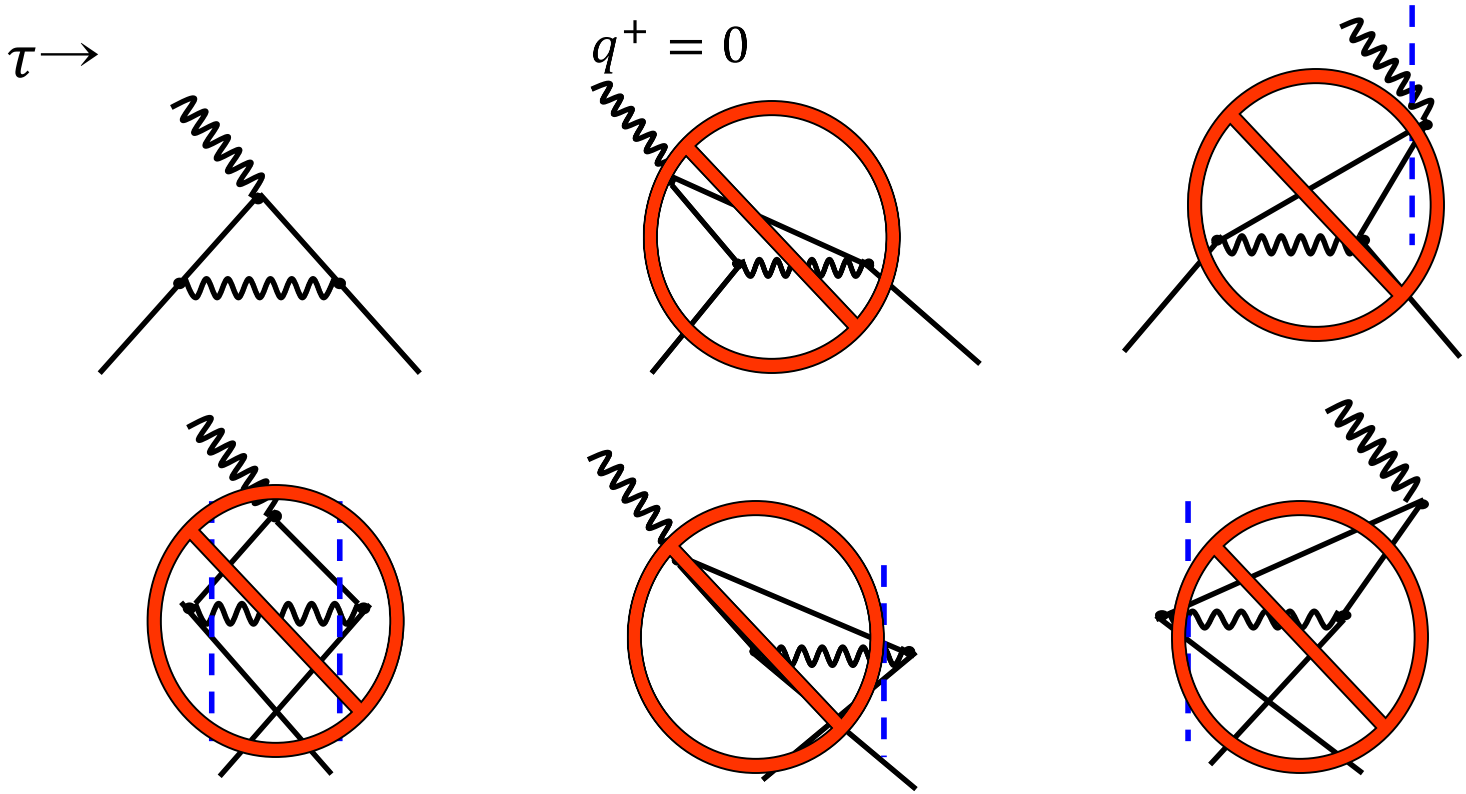
g-2 calculation

(1) Equal light-front τ formulation:



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

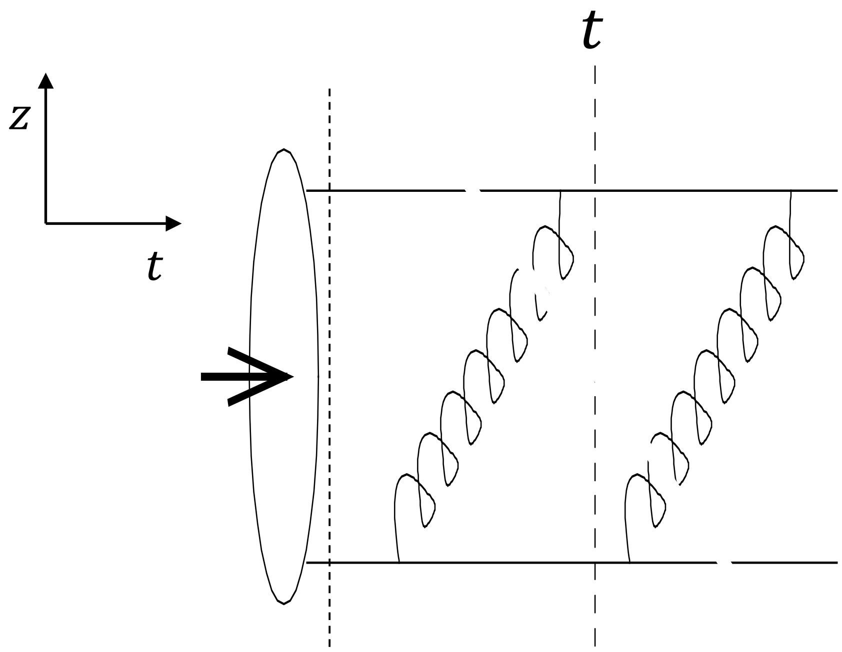
g-2 calculation



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

- Distinguished Features in LFD:

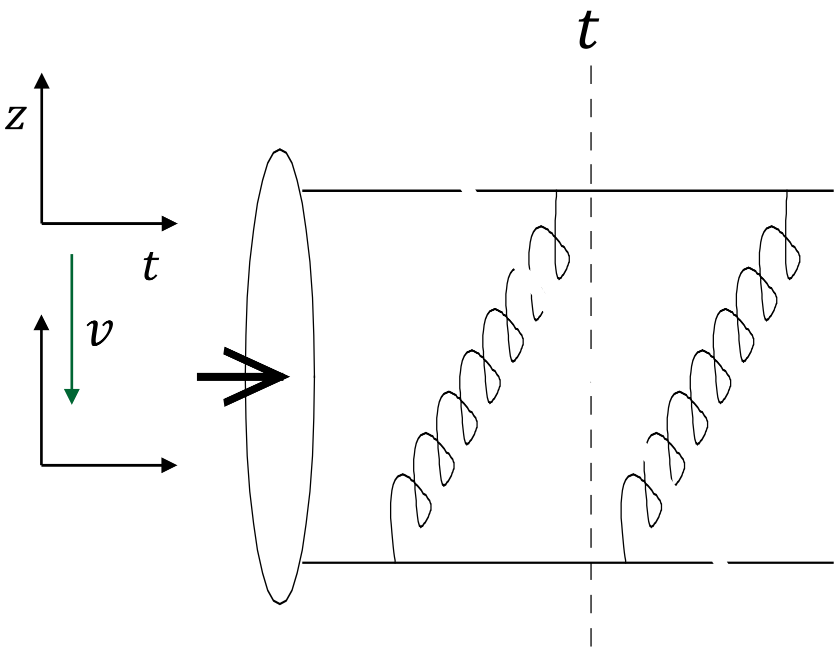
(3) Construct boost invariant LF wave function!



Observe one gluon
at each instant of time!

- Distinguished Features in LFD: Advantages in hadron phenomenology

(1) Construct boost invariant LF wave function!



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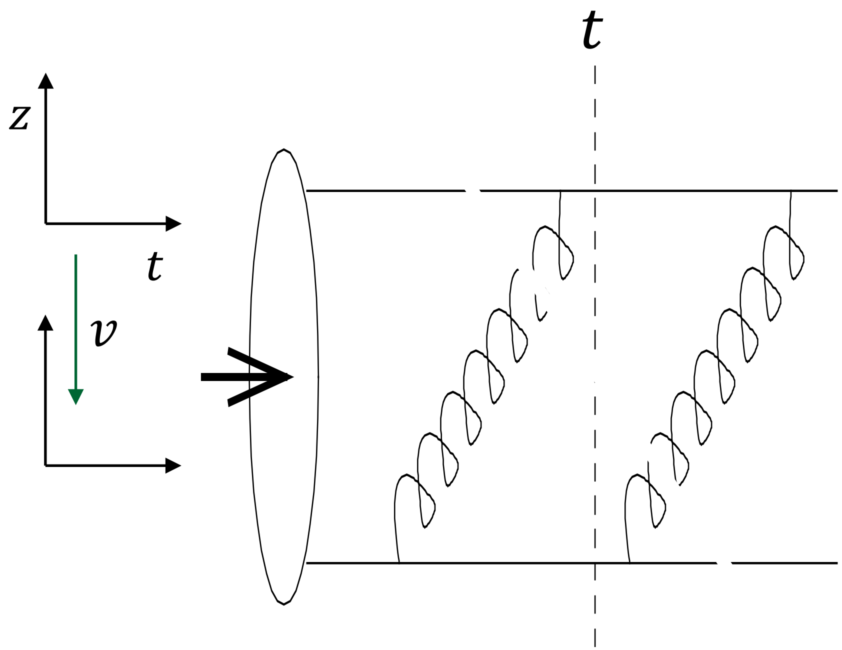

$$z' = \gamma(z + \beta ct)$$
$$ct' = \gamma(ct + \beta z)$$
$$\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

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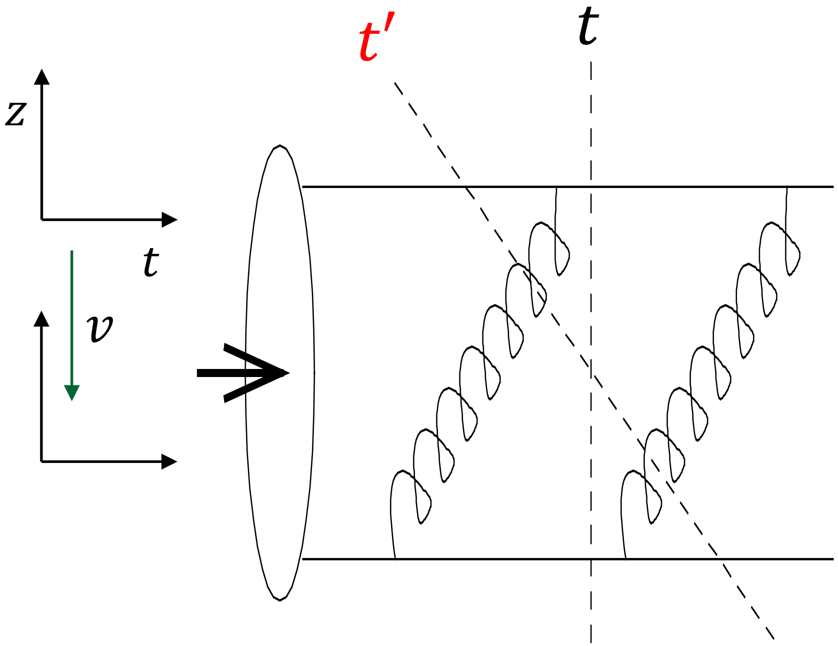

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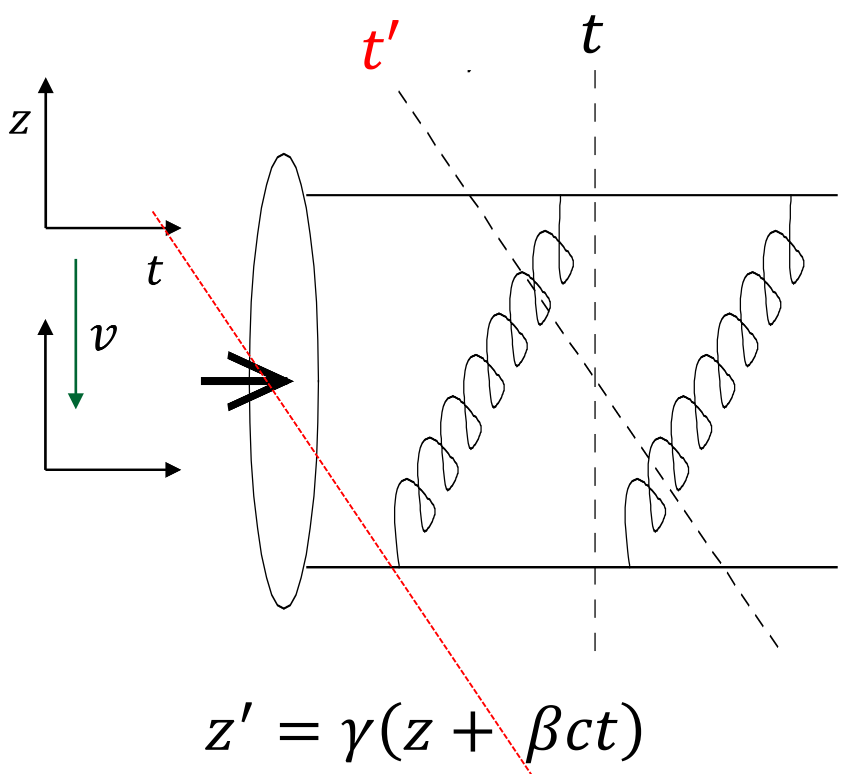

Observe two gluons in boosted frame!

→ Wave function is not boost invariant in equal t-quantization.

$$z' = \gamma(z + \beta ct)$$
$$ct' = \gamma(ct + \beta z)$$
$$\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- Distinguished Features in LFD: Advantages in hadron phenomenology

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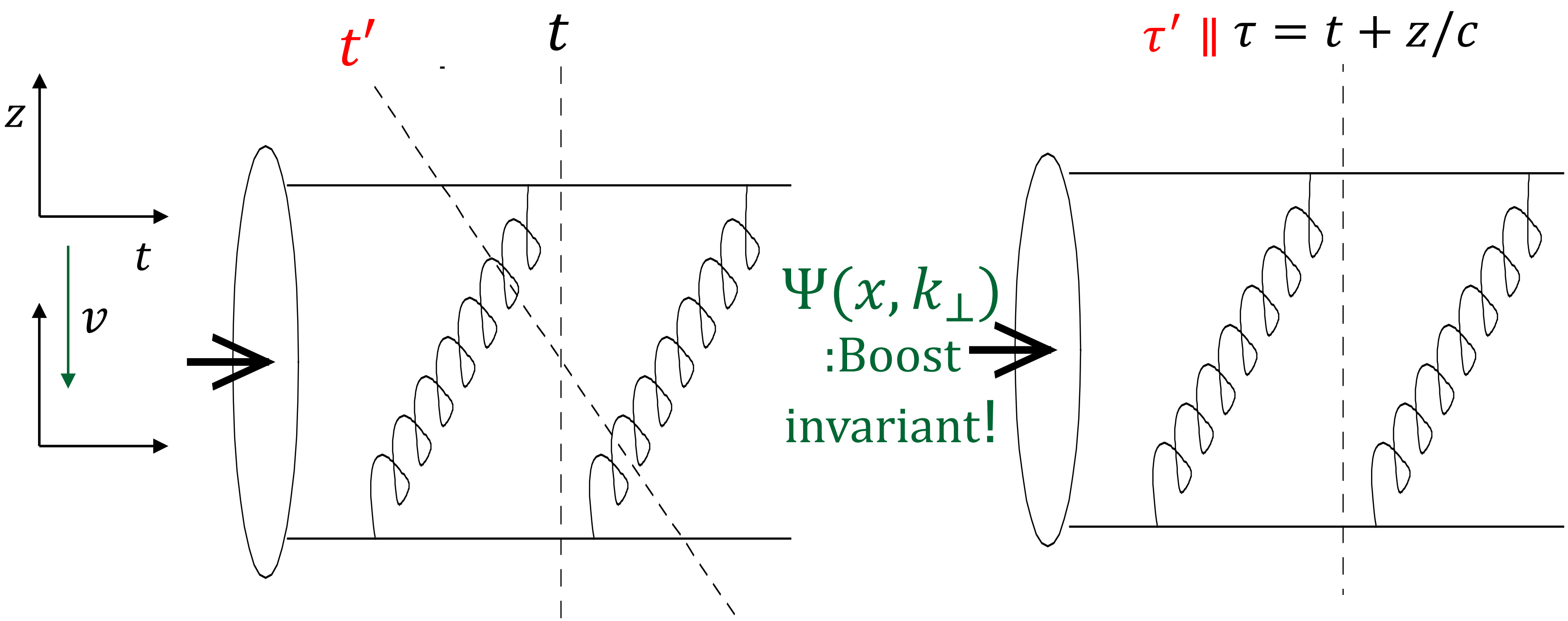

$$z' = \gamma(z + \beta ct)$$
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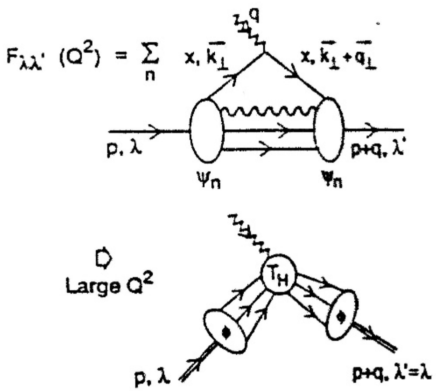
$$\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\tau' = e^{\phi}\tau$$
$$\gamma = \cosh\phi$$
$$\beta\gamma = \sinh\phi$$

Applications to Hadron Phenomenology

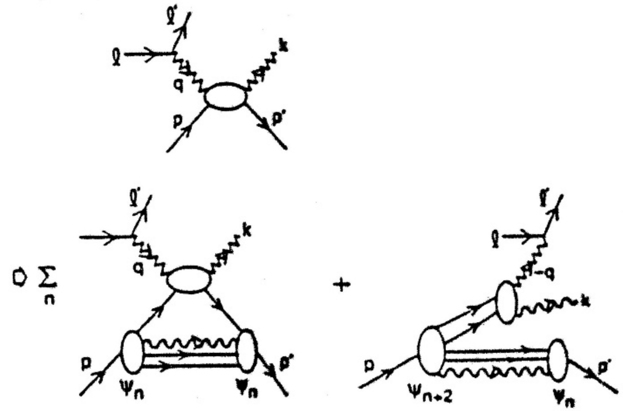
Form Factors l p →l' p'

<p' λ'|J$^+$(0)|p λ>

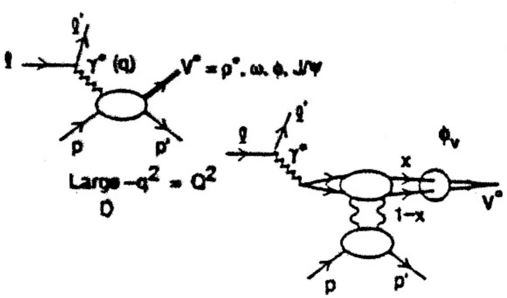


Virtual Compton γ* p→γ' p'

Large − q^2 = Q^2 <p'λ'|J^μ(z)J^ν(0)|pλ>

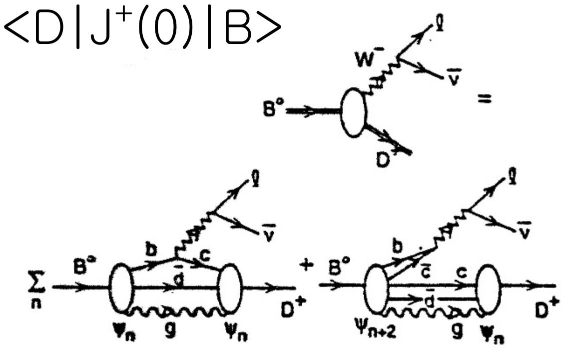


Vector Meson Leptoproduction γ*p→V*p'

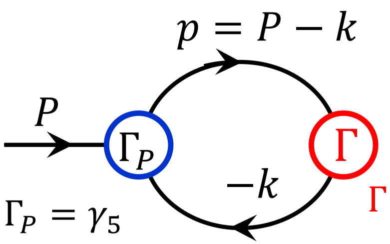


Weak Decay

<D|J$^+$(0)|B>



Twist-2 and-3 DAs of a Pseudoscalar Meson (in LC gauge)



$\Gamma = \gamma^\mu \gamma_5, \gamma_5, \sigma^{\mu\nu}\gamma_5$

Twist-2:

$$\langle 0|\bar{q}(z)\boldsymbol{\gamma^\mu\gamma_5} q(-z)|M(P)\rangle = i f_M P^\mu \int_0^1 dx\ e^{i(2x-1)P\cdot z}\, \phi^A_{2;M}(x)$$

Twist-3:

$$\langle 0|\bar{q}(z) i\boldsymbol{\gamma_5} q(-z)|M(P)\rangle = f_M \mu_M \int_0^1 dx\ e^{i(2x-1)P\cdot z}\phi^P_{3;M}(x)$$

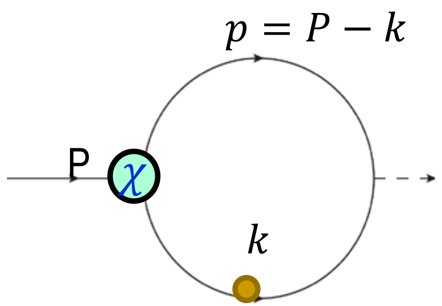
$$\langle 0|\bar{q}(z)\boldsymbol{\sigma_{\alpha\beta}\gamma_5} q(-z)|M(P)\rangle = -\frac{i}{3} f_M \mu_M (P_\alpha z_\beta - P_\beta z_\alpha)\int_0^1 dx\ e^{i(2x-1)P\cdot z}\phi^\sigma_{3;M}(x)$$

- Normalization constant μ_M results from quark condensate via

$$\mu_\pi = \frac{M^2}{m_q + m_{\bar{q}}} = -2\frac{\langle \bar{q}q\rangle}{f_\pi^2}$$: Gell-Mann-Oakes-Renner relation

3. Construction of the Self-Consistent LFQM

LFBS model:



$$\int d^4k\, H(p^2, k^2)(\cdots)$$

$$\int dk^- \Downarrow \quad e.g)\ H = \frac{g}{(p^2 - \Lambda^2 + i\epsilon)^n}$$

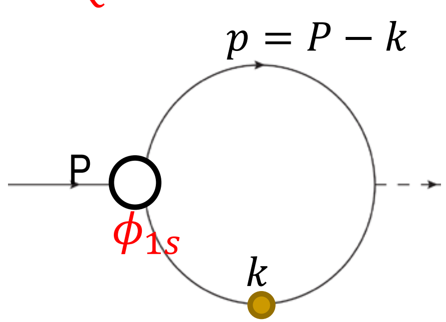
$$\int [d^3\vec{k}]\, \chi(x, k_\perp)(\cdots)$$

$\Downarrow$

$$\sqrt{2N_c}\frac{\chi}{1-x} = \frac{\phi_{1s}}{\sqrt{m^2 + \boldsymbol{k}_\perp^{(\prime)2}}}$$
; Mathcing Condition

PRD89,033011(14);
PRD91,014018(15)

LFQM:



Gaussian W.F.: $\phi_{1s} \propto \exp(-\frac{\vec{k}^2}{2\beta^2})$

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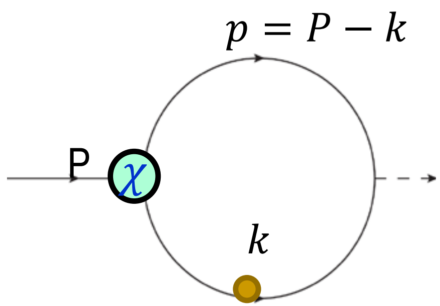

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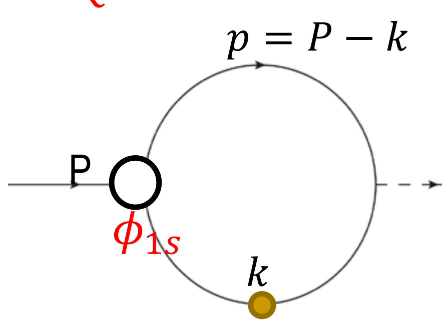


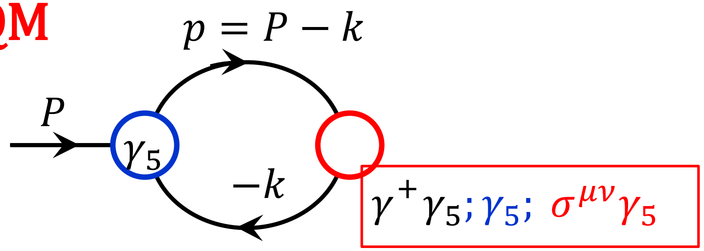
$$\Psi_{(\uparrow\downarrow-\downarrow\uparrow)} = \boxed{\frac{1}{\sqrt{2}}(\mathcal{R}_{\uparrow\downarrow} - \mathcal{R}_{\downarrow\uparrow})}\,\phi_{1s}(x, \boldsymbol{k}_\perp) = \boxed{\frac{m}{\sqrt{m^2 + \boldsymbol{k}_\perp^2}}}\,\phi_{1s}$$

Spin-orbit w.f.

Normalization: $1 = \int \frac{dx\, d^2\boldsymbol{k}_\perp}{16\pi^3} |\phi_{1s}(x, \boldsymbol{k}_\perp)|^2$

1) Meson DAs in LFQM

Pseudoscalar Meson:



$$M_0^2 = \frac{m^2 + \boldsymbol{k}_\perp^2}{x(1-x)}$$

Twist 2 DA $of\ \pi$:

$$\phi_{2;\pi}^A(x) = \frac{2\sqrt{2N_c}}{f_\pi} \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} \Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x, \boldsymbol{k}_\perp)$$

Pseudoscalar Twist 3 DA $of\ \pi$:

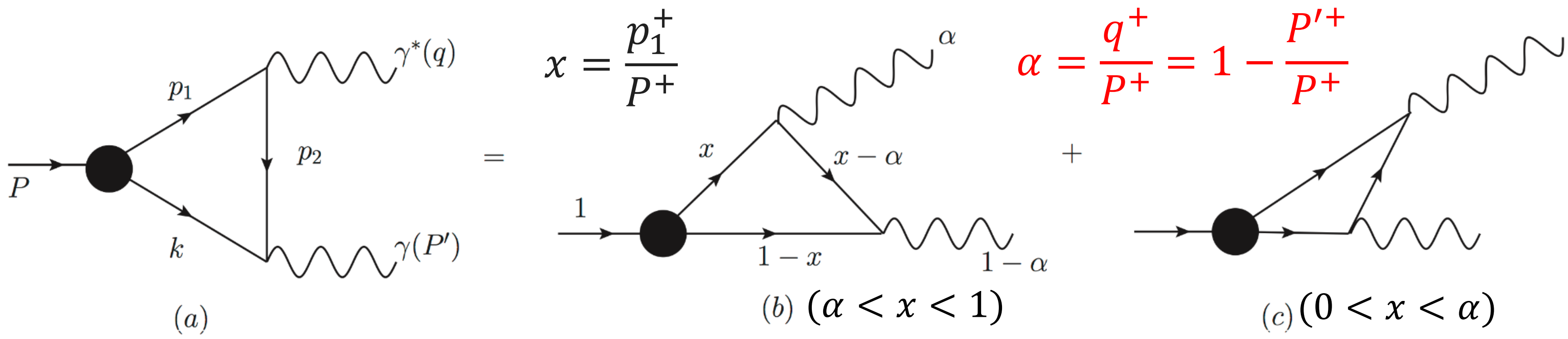
$$\phi_{3;\pi}^P(x) = -\frac{f_\pi}{\langle \bar{q}q \rangle} \frac{\sqrt{2N_c}}{2} \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} \frac{M_0^2}{m} \Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x, \boldsymbol{k}_\perp)$$

Pseudotensor Twist 3 DA $of\ \pi$:

$$\phi_{3;\pi}^\sigma(x) = \frac{3f_\pi}{\langle \bar{q}q \rangle} \sqrt{2N_c} \int \frac{d^2\boldsymbol{k}_\perp}{16\pi^3} \int_0^x \frac{(1-2x')dx'}{x'(1-x')} \frac{(m^2+\boldsymbol{k}_\perp^2)}{m} \Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x', \boldsymbol{k}_\perp)$$

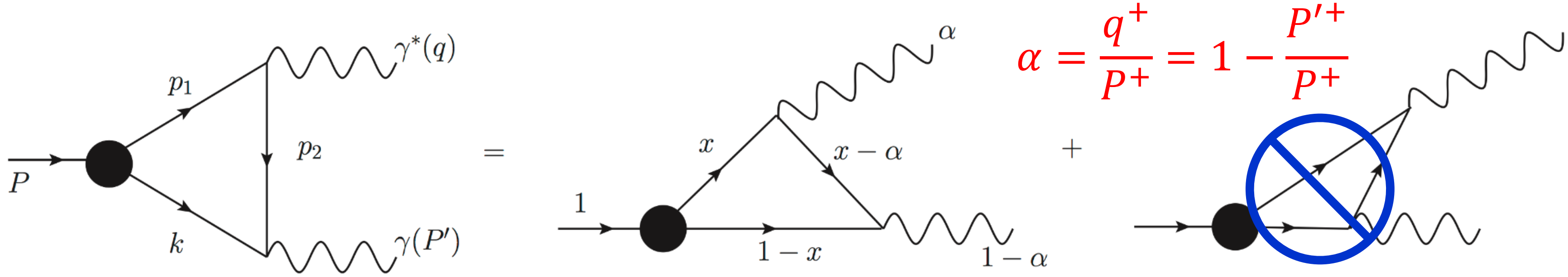
2) $F_{P\gamma}(Q^2)$ for $P \to \gamma^*\gamma$ Manifestly Covariant Model

$$\Gamma^\mu = \langle \gamma(P-q)|J^\mu_{em}|P(P)\rangle = ie^2 F_{P\gamma}(Q^2)\epsilon^{\mu\nu\varrho\sigma}P_\nu\epsilon_\rho q_\sigma$$



1. Equivalence between Covariant Calculation and Light-Front Calculation

Covariant Calculation	LF Calculations in different reference frames
Diagram (a) =	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)



(a) (b) $(\alpha < x < 1)$ (c) $(0 < x < \alpha)$

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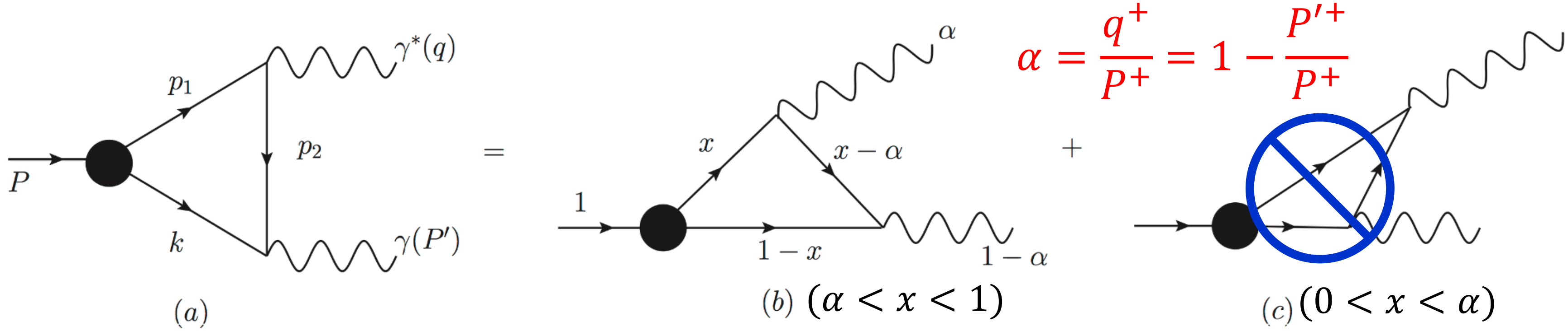

$$\alpha = \frac{q^+}{P^+} = 1 - \frac{P'^+}{P^+}$$

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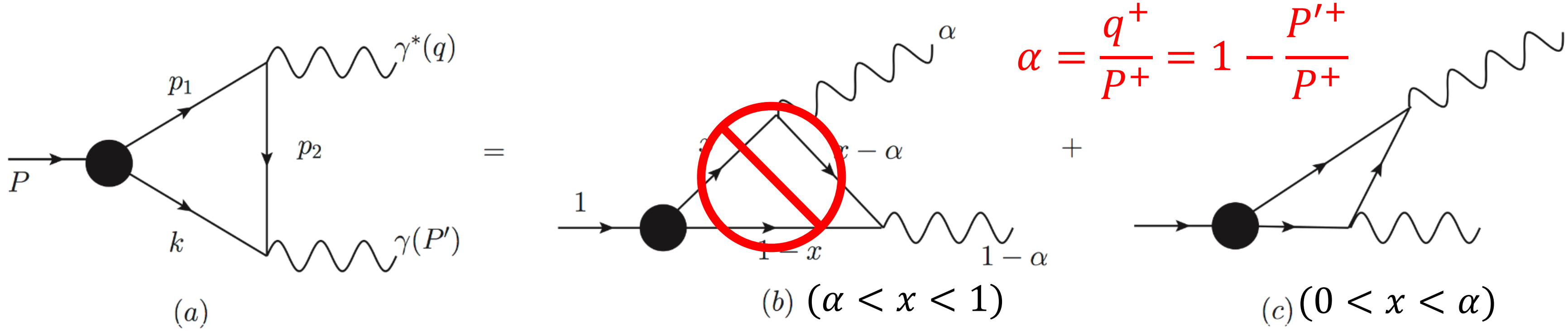
$$[F_{\pi\gamma}]^{\text{SLF}}_{\alpha\to 0} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{x(1-x)}\int d^2\boldsymbol{k}_\perp \frac{m_Q}{M'^2_0}\Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x,\boldsymbol{k}_\perp)$$

$$M'^2_0 = \frac{m^2 + \boldsymbol{k}'^2_\perp}{x(1-x)}$$

$$\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp + (1-x)\boldsymbol{q}_\perp$$

$$Q^2 = \boldsymbol{q}^2_\perp = -q^2$$

Popular reference frame but not effective in calculating timelike region $(q^2 > 0)$



$$\alpha = \frac{q^+}{P^+} = 1 - \frac{P'^+}{P^+}$$

1. Equivalence between Covariant Calculation and Light-Front Calculation

Covariant Calculation	LF Calculations in different reference frames
Diagram (a)=	(b) + (c) for $0 < \alpha < 1$ ($q^+ \neq 0$)
	(b) for $\alpha = 0$ ($q^+ = 0$) : defined in $q^2 < 0$
	(c) for $\alpha = 1$ ($q^+ \neq 0$) : defined in $q^2 > 0$
$F_{(a)}^{Cov}(q^2) = [F_{(b)}^{LF} + F_{(c)}^{LF}]_{0<\alpha<1} = [F_{(b)}^{LF}]_{\alpha=0} = [F_{(c)}^{LF}]_{\alpha=1}$	

$$[F_{\pi\gamma}]^{\mathrm{SLF}}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{\Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x,\boldsymbol{k}_\perp)}{M_0^2 - q^2}$$

Our new findings!

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• The virtue of $\alpha = 1$ frame

$$[F_{\pi\gamma}]^{\mathrm{SLF}}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{\Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x,\boldsymbol{k}_\perp)}{M_0^2 - q^2}$$

Systematic twist expansion is possible!

$$\frac{1}{M_0^2 - q^2} = \frac{1}{M_0^2 + Q^2} = \frac{1}{Q^2} - \frac{M_0^2}{Q^4} + \cdots$$

$$Q^2\, F_{\pi\gamma}(q^2) = \frac{f_\pi}{3\sqrt{2}}\int_0^1 \frac{dx}{(1-x)}\left[2\,\phi^A_{2;\pi}(x) - 4\frac{m_Q}{Q^2}\mu_\pi \phi^P_{3;\pi}(x) + \,O\left(\frac{1}{Q^{2n}}\right)\right]$$

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At sufficiently high Q^2

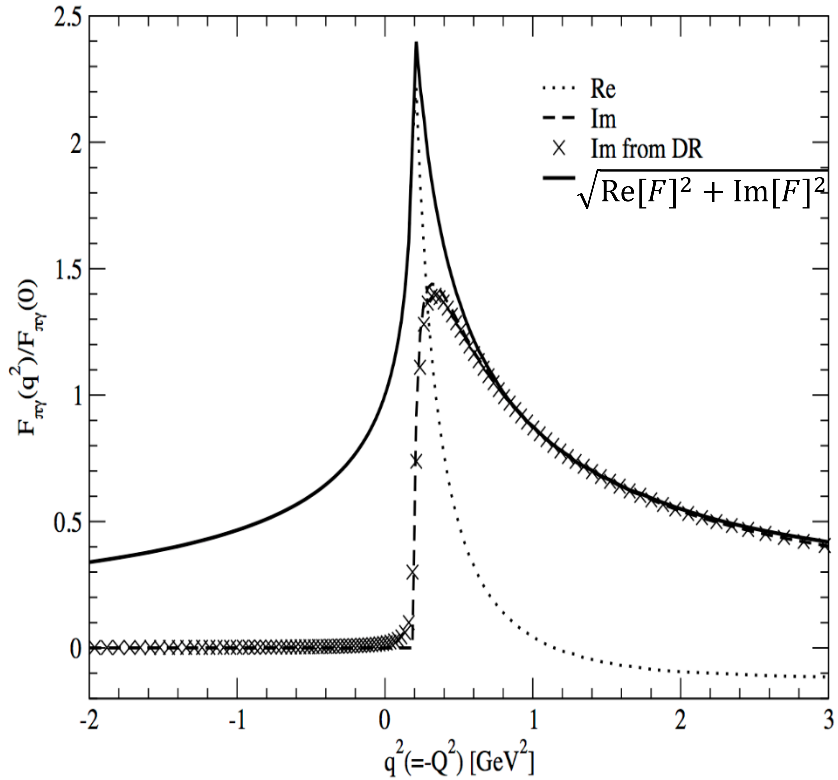
$$Q^2\, F_{\pi\gamma}(q^2) \approx \frac{f_\pi}{3\sqrt{2}}\int_0^1 \frac{dx}{(1-x)}\left[2\,\phi^A_{2;\pi}(x)\right] = f_\pi\sqrt{2} \sim 0.185\ \mathrm{GeV}$$

$$if\ \phi^A_{2;\pi}(x) = \phi_{as}(x) = 6x(1-x)$$

Dispersion Relation(DR) for
$\boldsymbol{F(q^2) = \mathrm{Re}\, F(q^2) + i\mathrm{Im}\, F(q^2)}$:

$$\mathrm{Re}\, F(q^2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, F(q'^2)}{q'^2 - q^2} dq'^2$$

$$\mathrm{Im}\, F(q^2) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Re}\, F(q'^2)}{q'^2 - q^2} dq'^2$$



Our LFQM explicitly satisfies DR.

For the $(\boldsymbol{\eta}, \boldsymbol{\eta}') \to \boldsymbol{\gamma\gamma^*}$ transitions:

Use $\eta - \eta'$ mixing scheme in the quark-flavor basis

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \qquad \eta_q = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \eta_s = s\bar{s}$$

Transition form factor $\boldsymbol{F_{P\gamma}}$ mixing scheme for $\boldsymbol{P \to \gamma\gamma^*}$ $(\boldsymbol{P = \pi^0, \eta, \eta'})$

$$F_{\pi\gamma}(q^2) = \frac{(e_u^2 - e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}}$$

$$F_{\eta\gamma}(q^2) = \cos\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}} - \sin\phi \, e_s^2 \, I_{\rm tot}^{m_s}$$

$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\rm tot}^{m_{u(d)}} + \cos\phi \, e_s^2 \, I_{\rm tot}^{m_s}$$

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Quadratic(linear) Gell-Mann-Okubo mass formula: $\phi = [44.7^o, 31.7^o]$

Transition form factor $\boldsymbol{F_{P\gamma}}$ mixing scheme for $\boldsymbol{P \to \gamma\gamma^*}$ $(\boldsymbol{P = \pi^0, \eta, \eta'})$

$$F_{\pi\gamma}(q^2) = \frac{(e_u^2 - e_d^2)}{\sqrt{2}} I_{\text{tot}}^{m_{u(d)}}$$

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$$F_{\eta'\gamma}(q^2) = \sin\phi \frac{(e_u^2 + e_d^2)}{\sqrt{2}} I_{\text{tot}}^{m_{u(d)}} + \cos\phi\, e_s^2\, I_{\text{tot}}^{m_s}$$

We shall use $\phi = (37 \pm 5)^o$

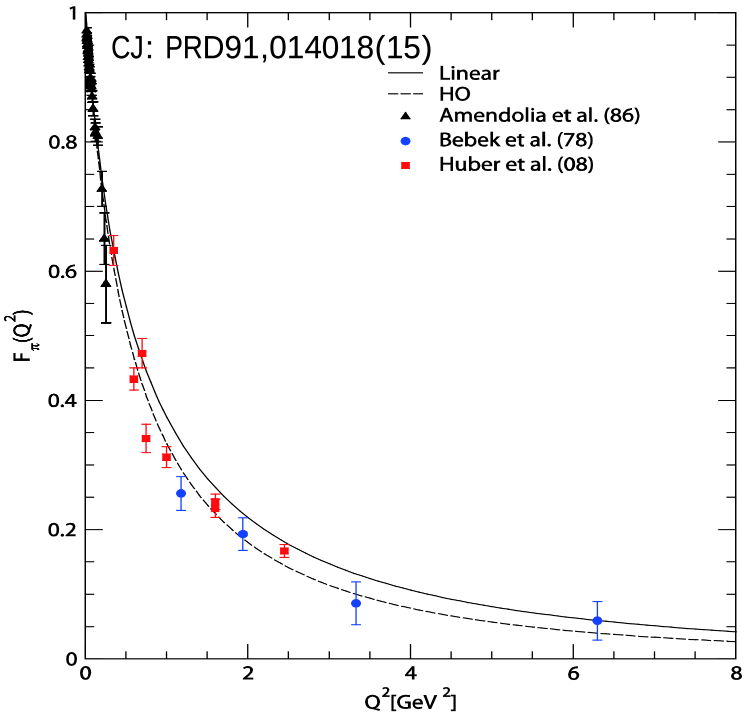
4. Numerical Results

(in unit of GeV)

Model	m_q	m_s	m_c	β_{qq}	β_{ss}	β_{cc}
	0.22	0.45	1.30	0.3659	0.4128	0.6509

CJ: PRD59, 074015(99); PLB460, 461(99)

1) Pion E&M form factor



2) Pion Charge Radius

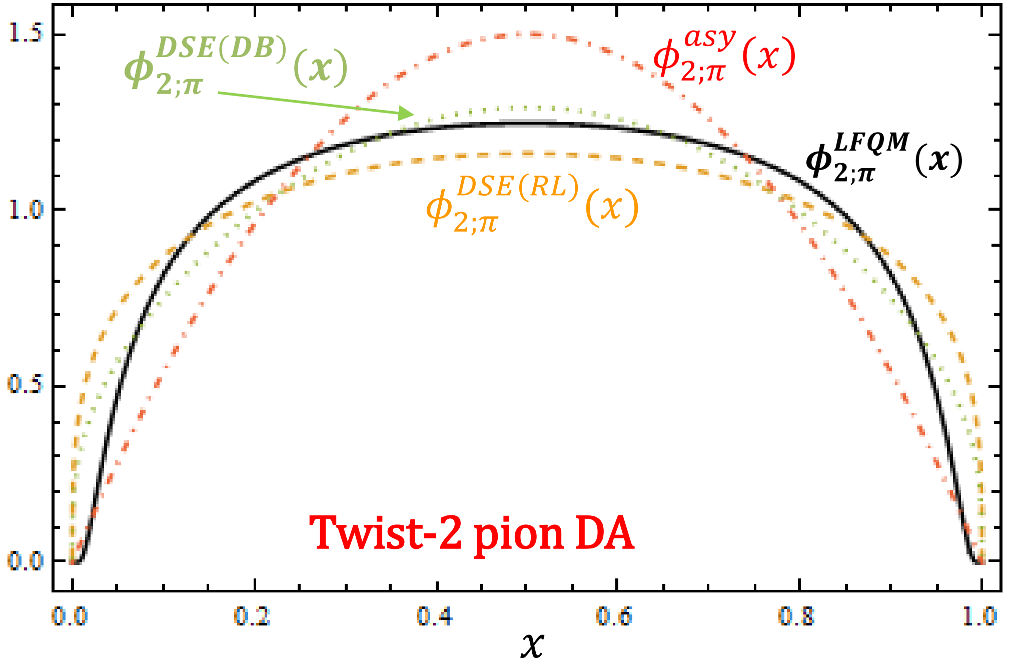
	LFQM	Exp.
$\langle r_\pi^2 \rangle^{1/2}$ [fm]	0.652	0.672(8)

3) Decay Constants

	LFQM	Exp.
f_π [MeV]	130	130.41(23)
f_0	$1.16 f_\pi$	$1.17 f_\pi$[1] $1.25 f_\pi$[2]
f_8	$1.32 f_\pi$	$1.26 f_\pi$[1] $1.28 f_\pi$[2]

[1]Feldmann,Kroll, Stech, PRD58,114006(98)

[2] Leutwyler, Nucl. Phys. B (Proc. Suppl.) 64,223(98)



$$\langle \xi^n \rangle = \int_0^1 dx\, \xi^n \phi_{2;\pi}(x)$$

where $\xi = x - (1 - x)$

$\langle \xi^2 \rangle$; measure of the width of the DA

$$\boldsymbol{\langle \xi^2 \rangle_{\pi}^{LFQM} = 0.24}$$

$\langle \xi^2 \rangle_{\pi}^{RL(DB)} = 0.28\ (0.25)$ [Chang et al. 13]

$\langle \xi^2 \rangle_{\pi}^{asy} = 0.20\ (\phi_{2;\pi}^{asy} = 6\mathrm{x}(1 - \mathrm{x}))$

$\langle \xi^2 \rangle_{\pi}^{LAT} = 0.27 \pm 0.04$ [Braun et al. 06]

$\langle \xi^2 \rangle_{\pi}^{flat} = 1/3 (\phi_{2;\pi}^{flat} = 1)$

$\langle \xi^2 \rangle_{\pi}^{Ads/QCD} = 0.25 (\phi_{2;\pi}^{Ads/QCD} = \frac{8}{\pi}\sqrt{x(1-x)})$

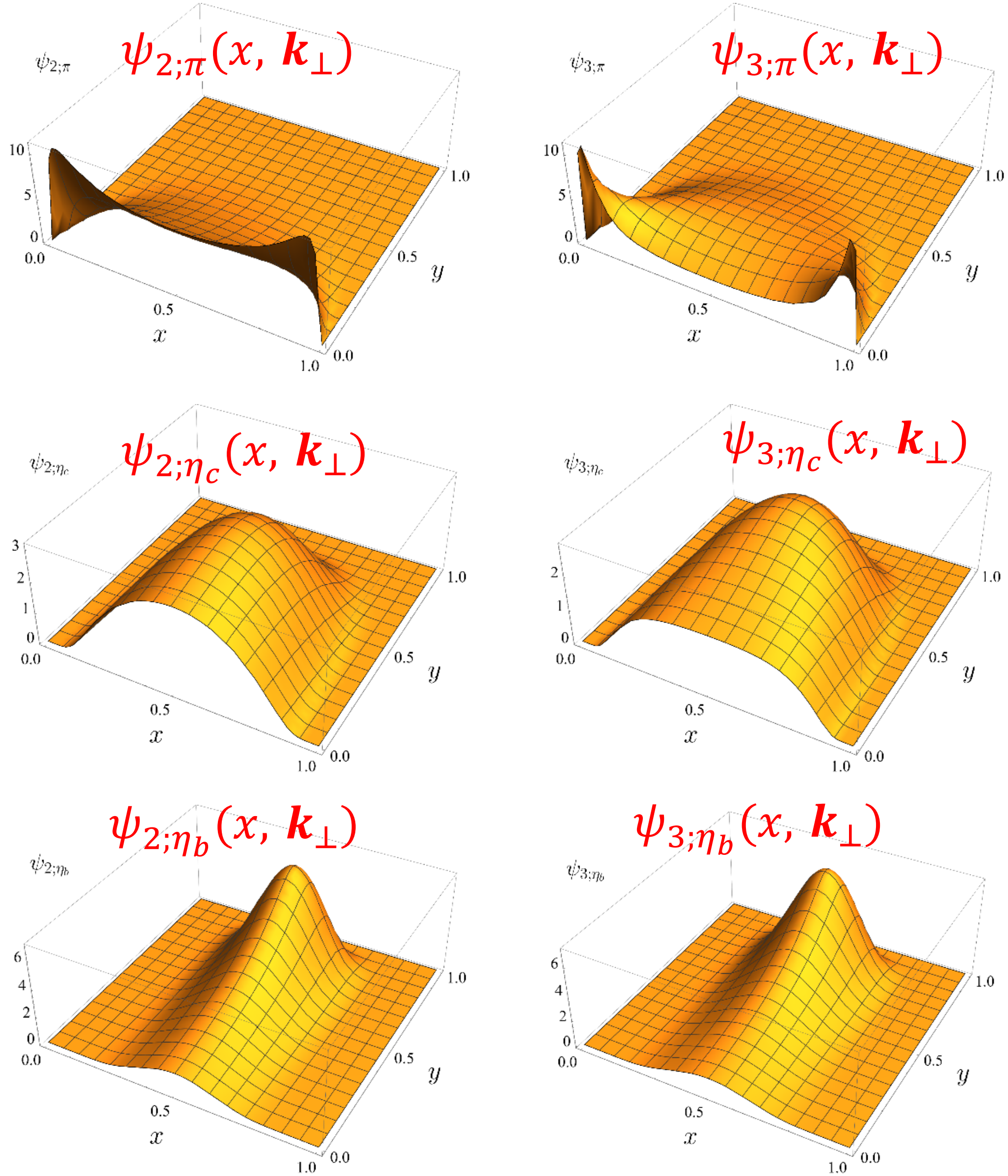
[Brodsky et al. 11]

$\langle \xi^2 \rangle_{\pi}^{delta} = 0\ (\phi_{2;\pi}^{delta} = \delta(x - \frac{1}{2}))$

Transverse momentum Dependent DA(TMDA):

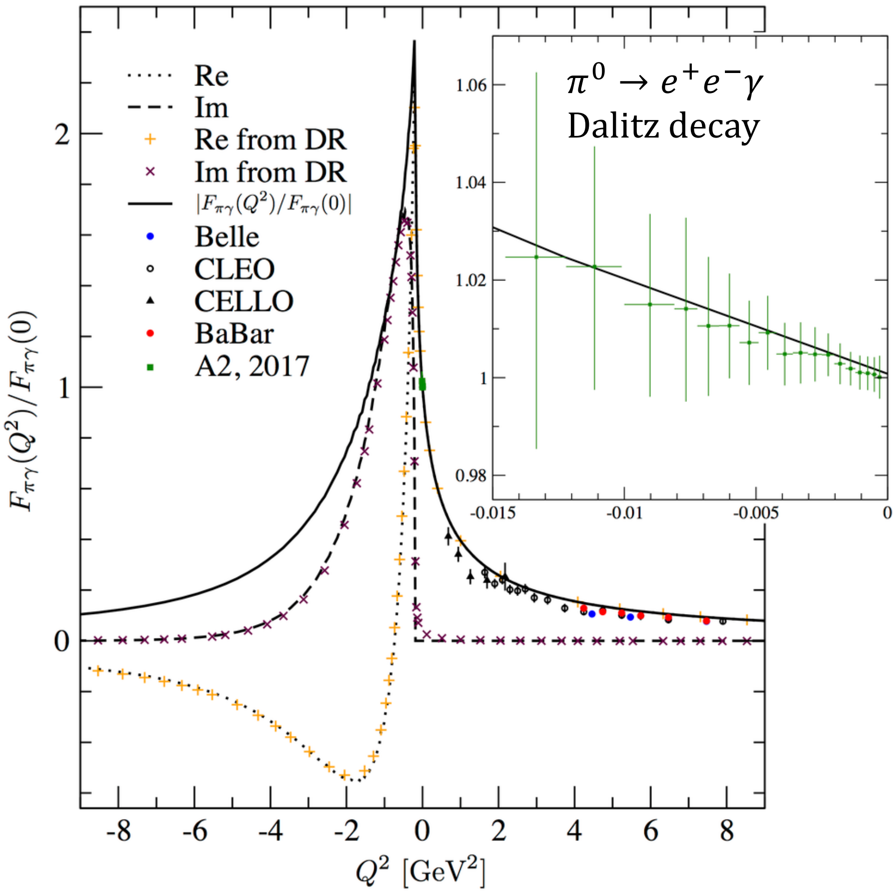
$\psi_{n;M}(x,\, \boldsymbol{k}_\perp)$

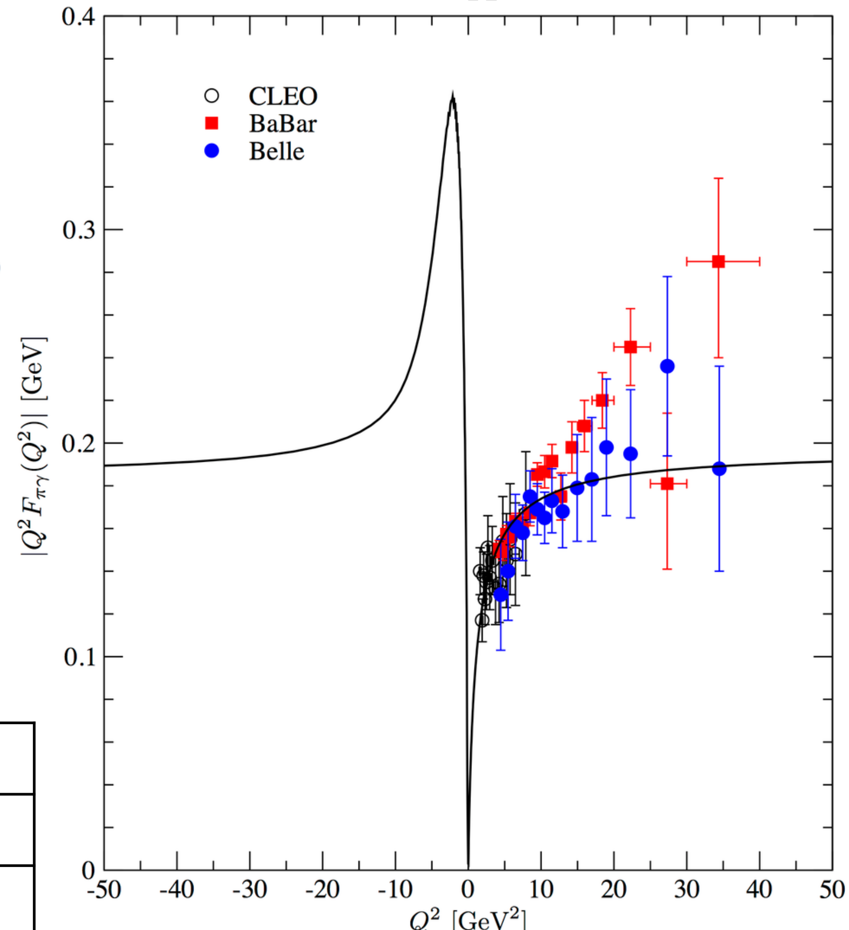
$$\phi_{n;M}(x)$$
$$= \int d^2\boldsymbol{k}_\perp\ \psi_{n;M}(x,\, \boldsymbol{k}_\perp)$$
$$= \int_0^1 dy\, \psi_{n;M}(x, y)$$



Results for $F_{\pi\gamma}(q^2)$

$$F(m_{ll} = q) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}} \approx 1 + a_\pi \frac{m_{ll}^2}{m_\pi^2}$$

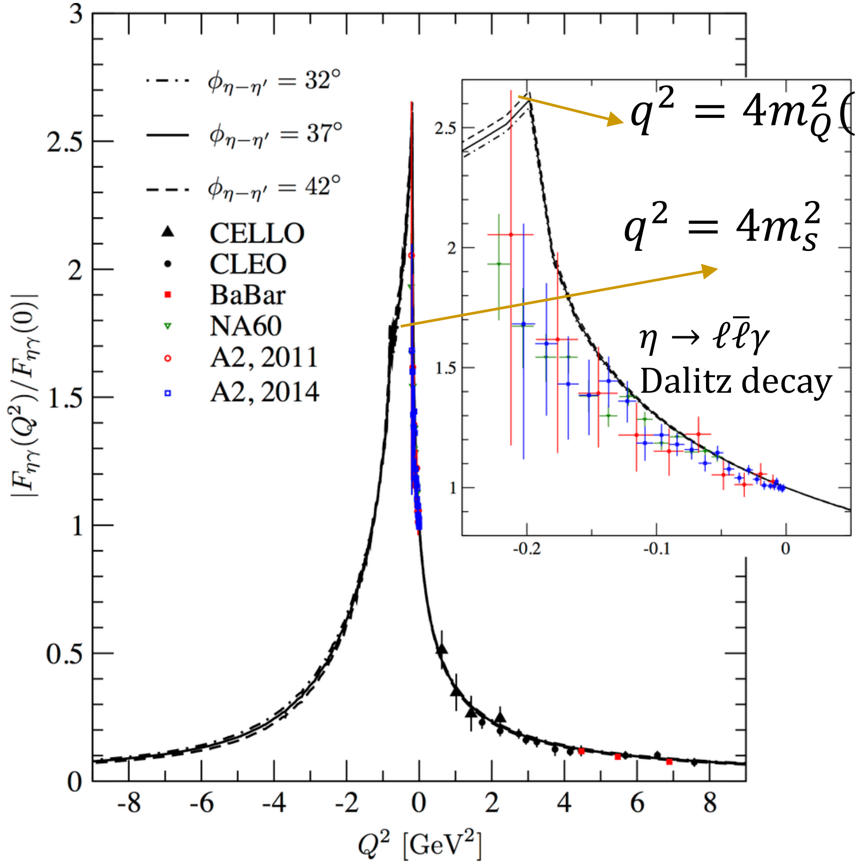




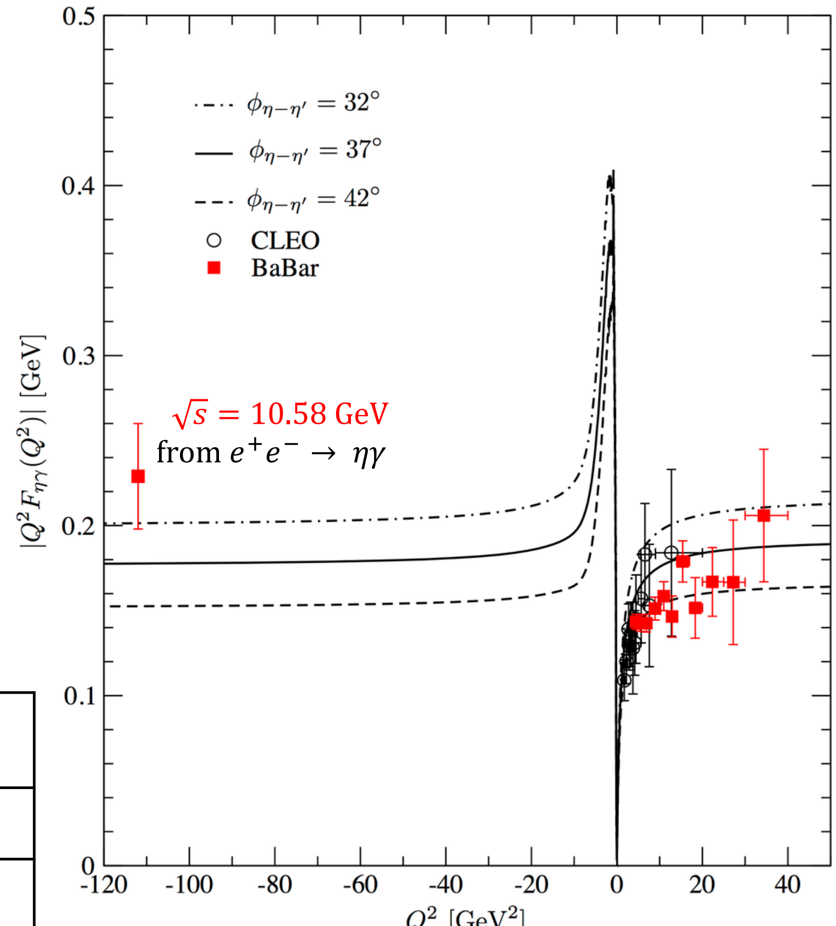
Slope parameter a_π:

Ours	0.0355
A2 at MAMI(16)	0.030 ± 0.010
World average(PDG)	0.032 ± 0.004

Results for $F_{\eta\gamma}(q^2)$



$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$

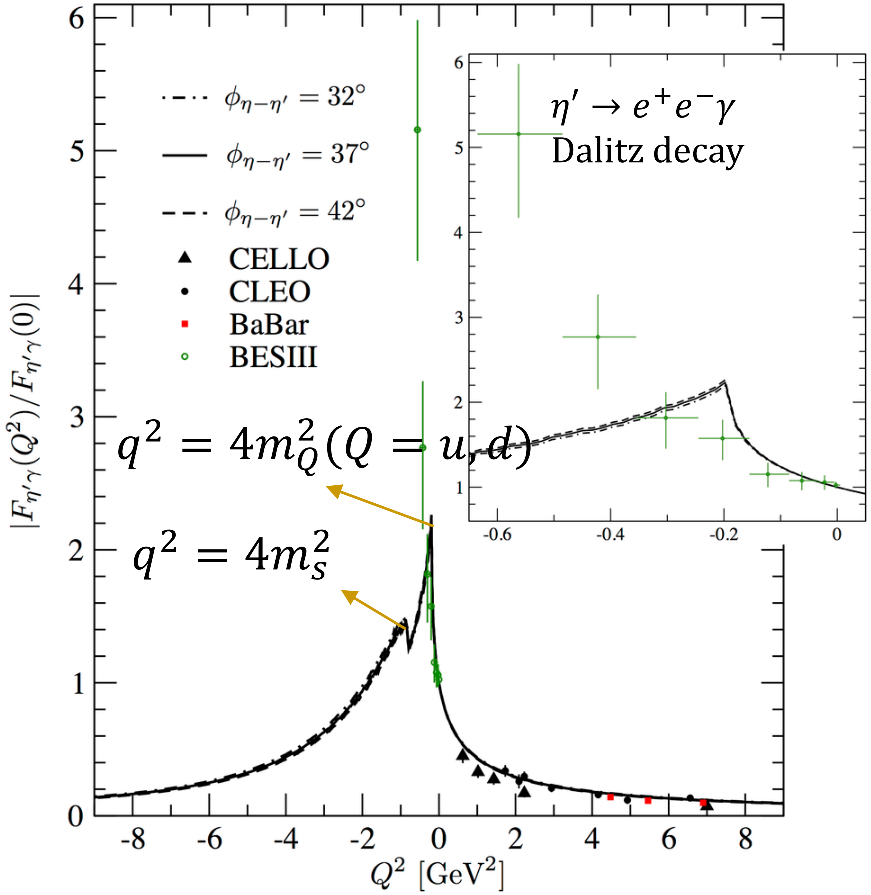


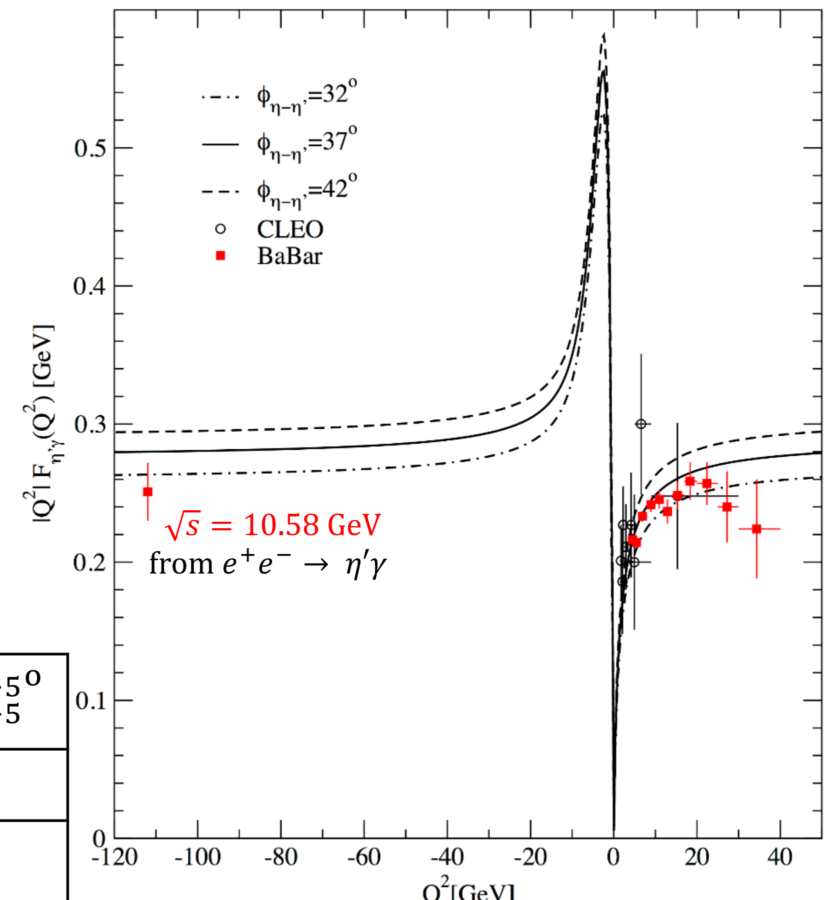
Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$2.112^{-0.031}_{+0.038}$ for $\phi = 37^{-5}_{+5}{}^{\circ}$
A2 at MAMI	$1.95 \pm 0.15 \pm 0.10$
NA2 at CERN	$1.95 \pm 0.17 \pm 0.05$

Results for $F_{\eta'\gamma}(q^2)$

$$F(m_{ll}) = \frac{1}{1 - \frac{m_{ll}^2}{\Lambda^2}}$$





Slope parameter Λ^{-2} [GeV^{-2}]:

Ours	$1.732^{-0.035}_{+0.031}$ for $\phi = 37^{-5}_{+5}{}^{o}$
BESIII (2015)	1.60 ± 0.25
Lepton-Col.(1979)	1.7 ± 0.4

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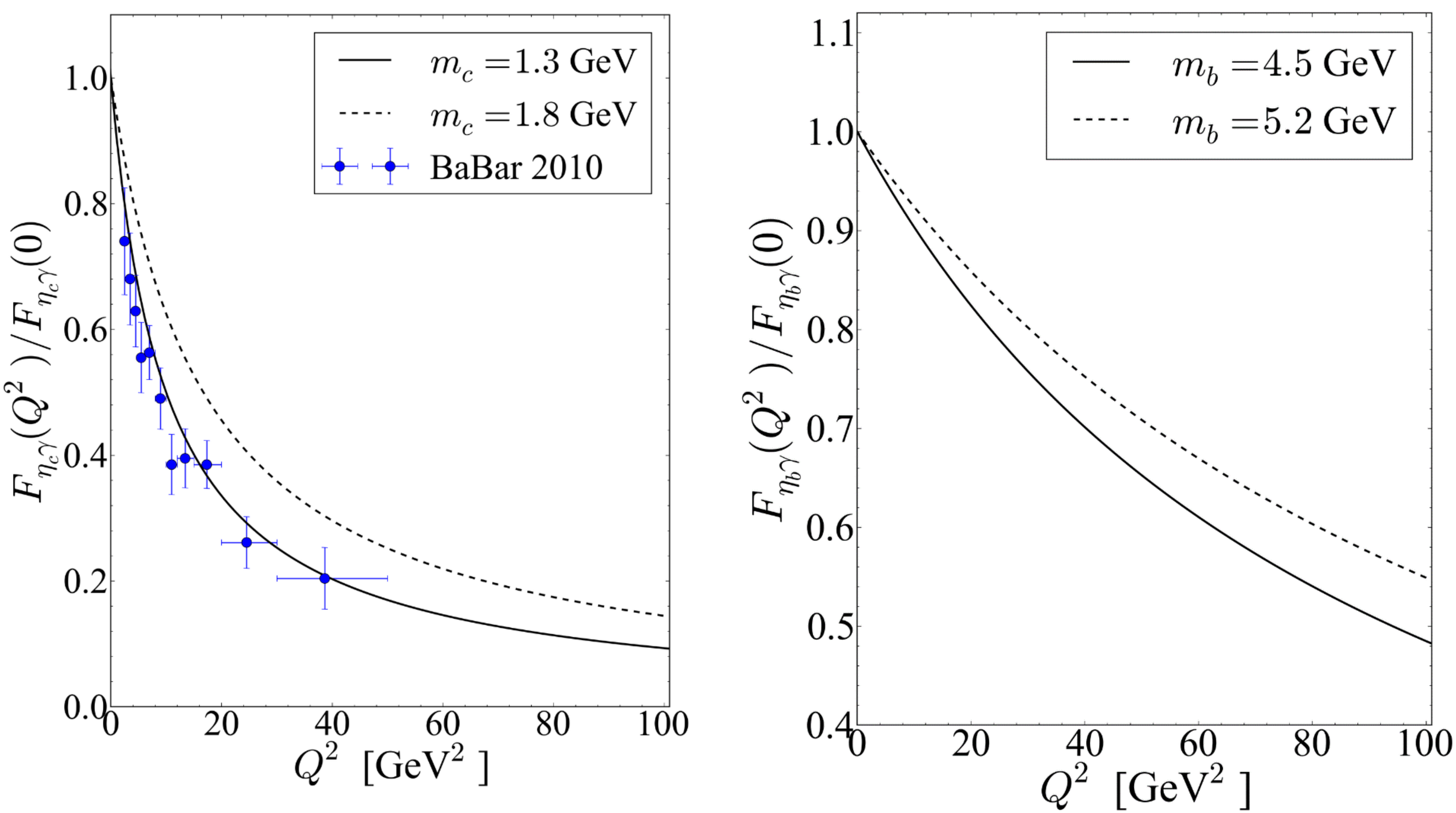

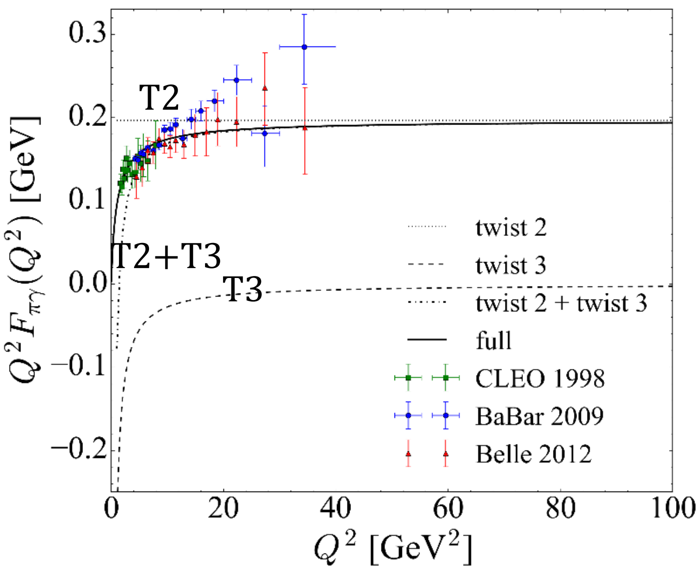

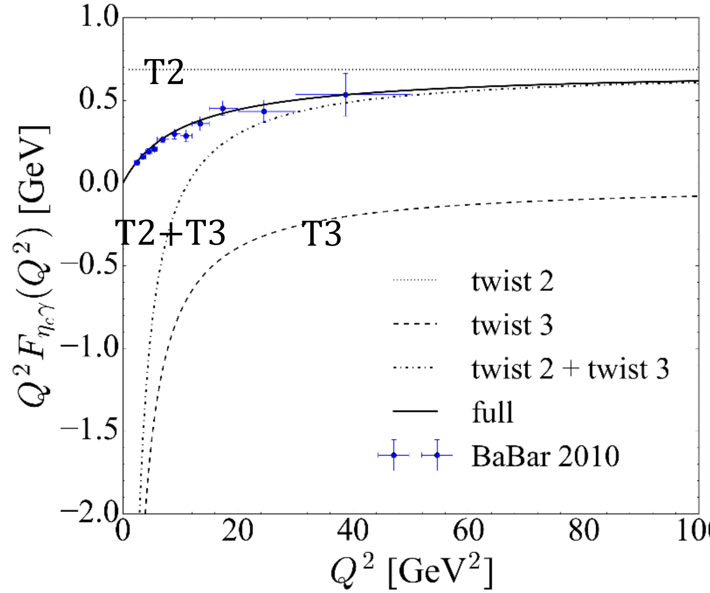

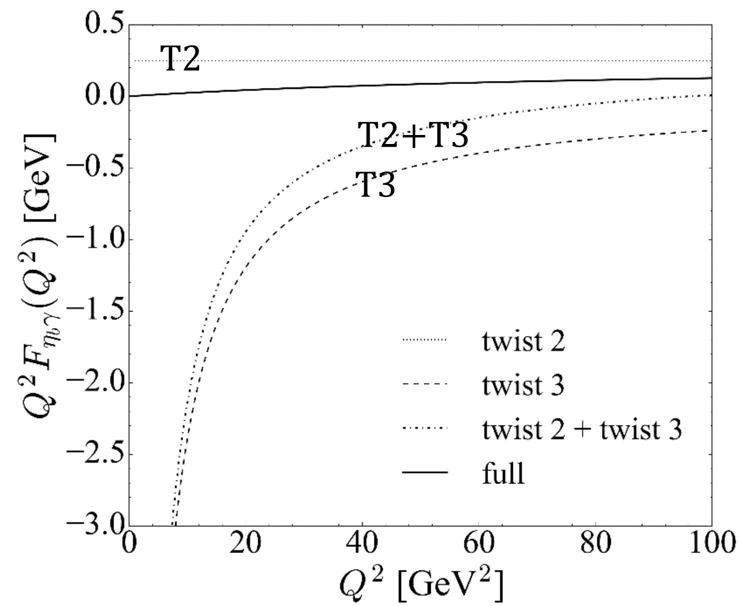

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5. Conclusion

- We investigate $(\pi^0, \eta^{(\prime)}, \eta_{c(b)}) \to \gamma\gamma^*$ transitions both for the spacelike and timelike regions using the LFQM.
  - We present the new direct method to explore the timelike region and show the agreement with the result from the DR.

$$[F_{\pi\gamma}]^{SLF}_{\alpha\to 1} = \frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\boldsymbol{k}_\perp \frac{\Psi_{(\uparrow\downarrow-\downarrow\uparrow)}(x,\boldsymbol{k}_\perp)}{M_0^2 - q^2}$$

  - Our results for $Q^2 F_{\pi\gamma}(Q^2)$ are consistent with the PQCD prediction showing a scaling behavior for both timelike and spacelike regions.
  - Our LFQM provides a straightforward systematic twist expansion of TFFs.

$$Q^2\, F_{\pi\gamma}(q^2) = \frac{f_\pi}{3\sqrt{2}}\int_0^1 \frac{dx}{(1-x)}\left[2\,\phi^A_{2;\pi}(x) - 4\frac{m_Q}{Q^2}\mu_\pi \phi^P_{3;\pi}(x) + \, O\left(\frac{1}{Q^{2n}}\right)\right]$$

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