

# Effects of isospin asymmetry on nuclear stopping

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Kyungil Kim

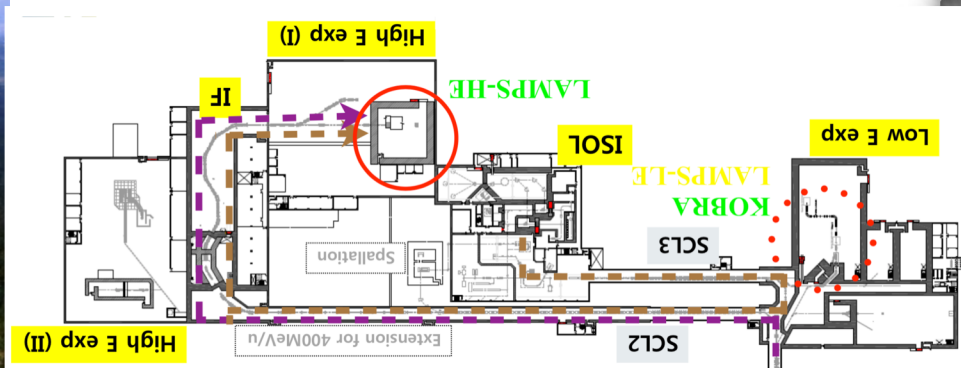
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Oct. 19<sup>th</sup>, 2018, HaPhy

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# RAON

Mar. 2018





중이온가속기 시설건설사업 건립공사 시공 현황 (건물/구역별)

(18.9.12, 현재)



SCL2구역 (터널 벽체 철근배근 및 Conc 타설)



SCL3구역 (갤러리 벽체 철근배근)



저에너지AB구역 (B1F 벽체 철근배근 및 Conc 타설)



ISOL구역 (B1F 벽체 철근배근 및 Conc 타설)



IF/고에너지A구역 (B1F 벽체 철근배근 및 Conc 타설)



고에너지B구역 (B1F 벽체 철근배근 및 Conc 타설)



SRF시험동 (내부 마감 공사)



초전도조립동 (내부 마감 공사)



수전설비동 (하지철물 설치)



중앙제어센터 (2F 벽체 철근배근)

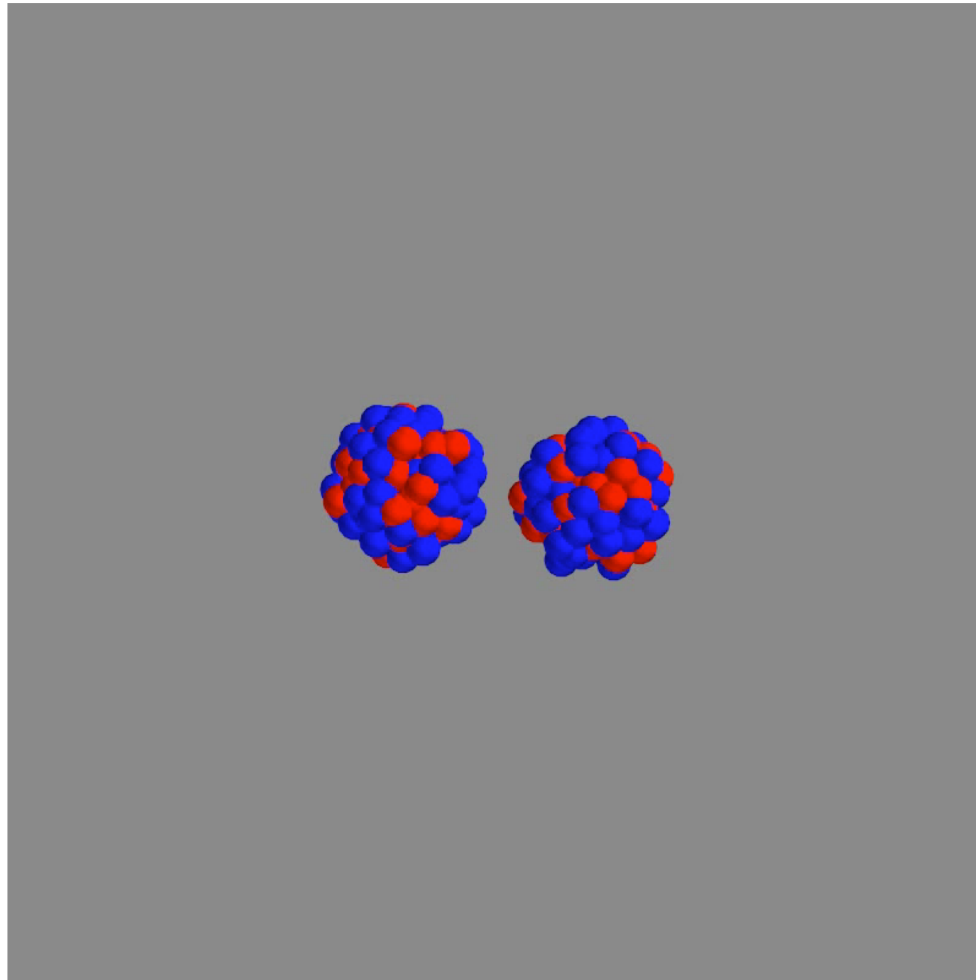


유틸리티동 (1F 벽체 철근배근 및 Conc 타설)



본부동 (터파기)

# Transport Model

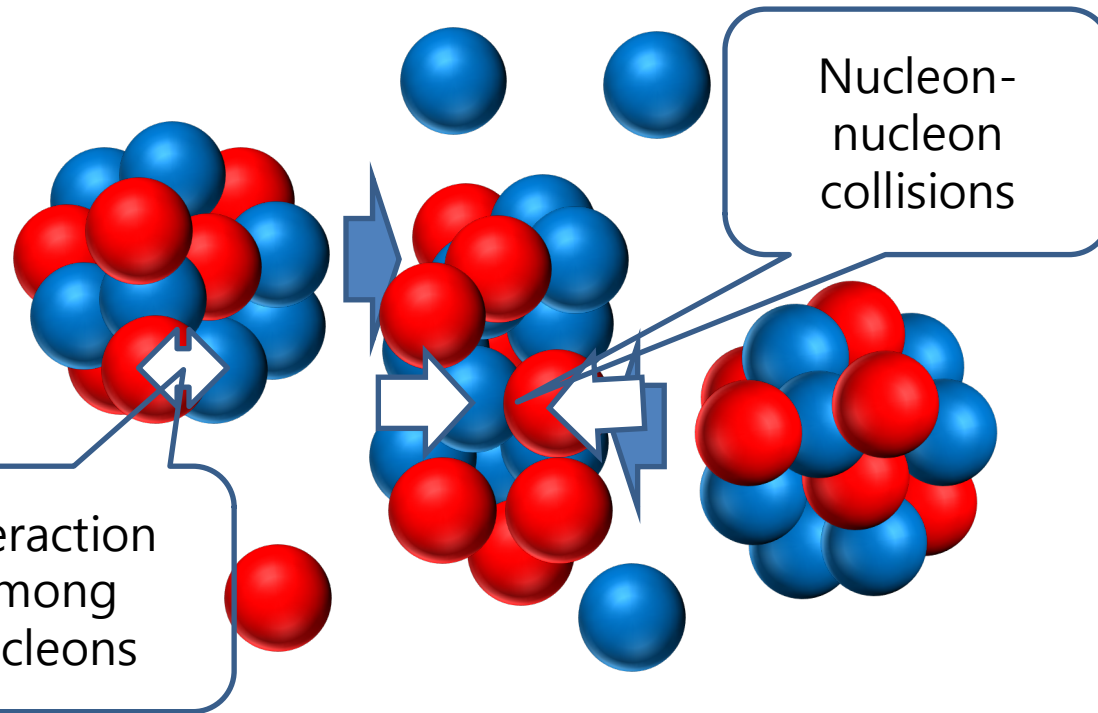


$^{129}\text{Xe} + ^{120}\text{Sn}$

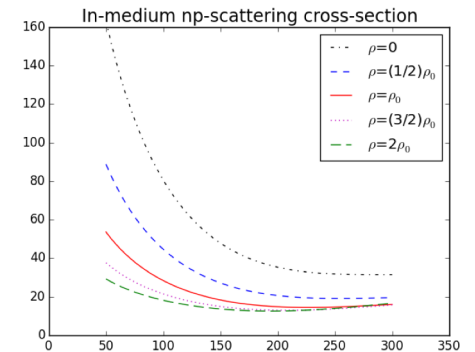
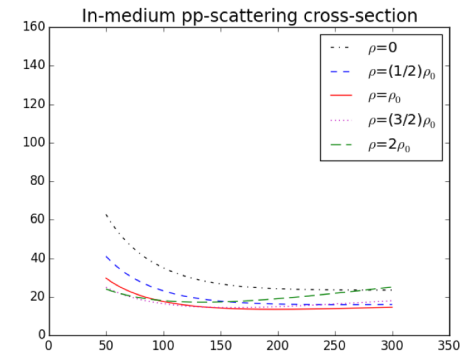
$E_{\text{beam}} = 65 \text{ MeV/n}$

$b = 1 \text{ fm}$

# Transport Model



In-medium cross-section :



$$\text{Wavefunction : } \psi_i(\vec{r}, t) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{4\sigma_r^2} + \frac{i}{\hbar}(\vec{p}_i \cdot \vec{r})\right)$$

$$\text{Potential : } H = \sum_i \frac{\vec{p}_i^2}{2m} + U^{2body} + U^{3body} + U^{surf} + U^{sym} + U^{Coul}$$

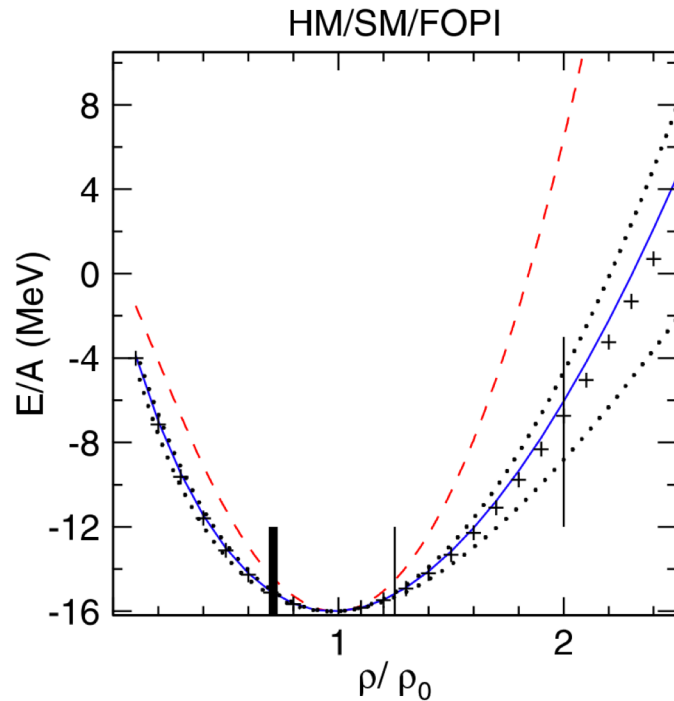
# What Can I Do with QMD?

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- Reproduction of experimental results
- Test of various potentials
- Test of EOS parameters
- Finding new observables
- ...



# Nuclear Equation of State



<Incompressibility>

$$K = -V \frac{\partial p}{\partial V} = 9\rho^2 \frac{\partial^2 E/A(\rho)}{(\partial\rho)^2} \Big|_{\rho=\rho_0}$$

*ref. A. Le Fevre, et al. NPA 945 (2016) 112*



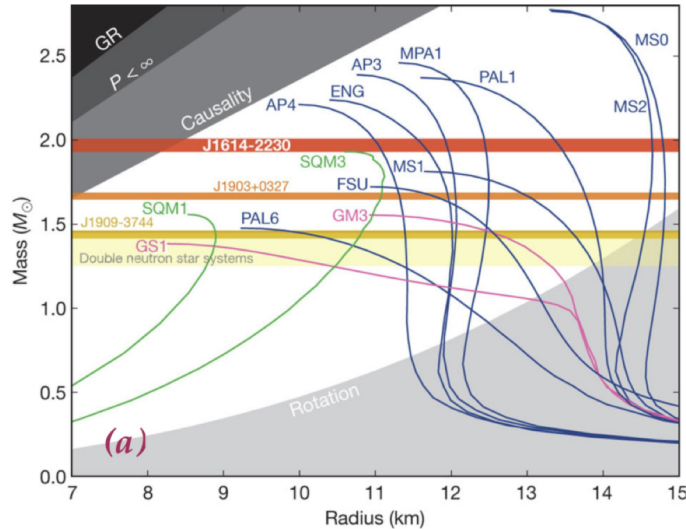
$K=380$  MeV

$K=200$  MeV



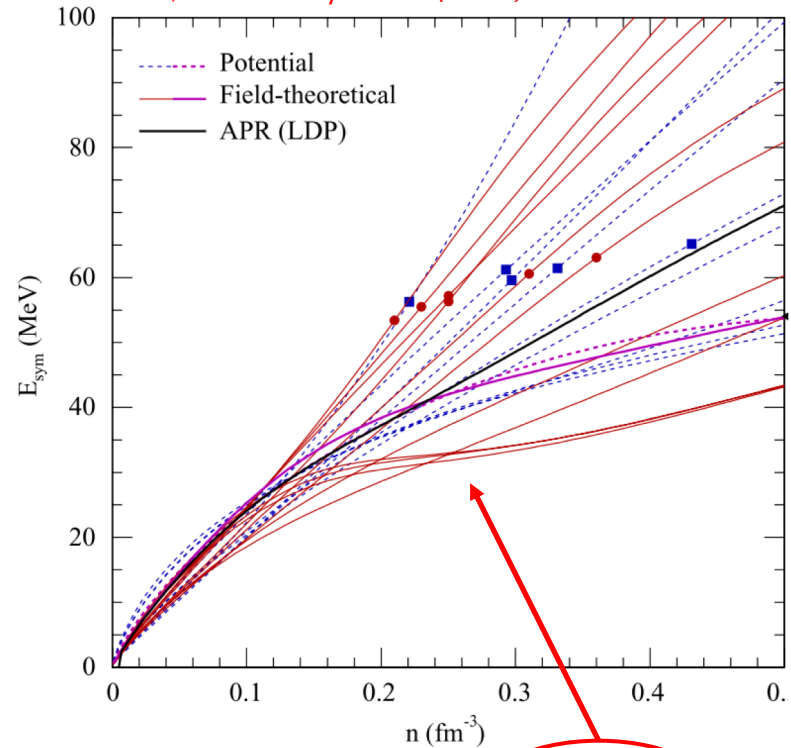
# Nuclear Symmetry Energy

<Neutron star>



various symmetry energy assumption

*A. W. Steiner, et al. PRept 411 (2005) 325*



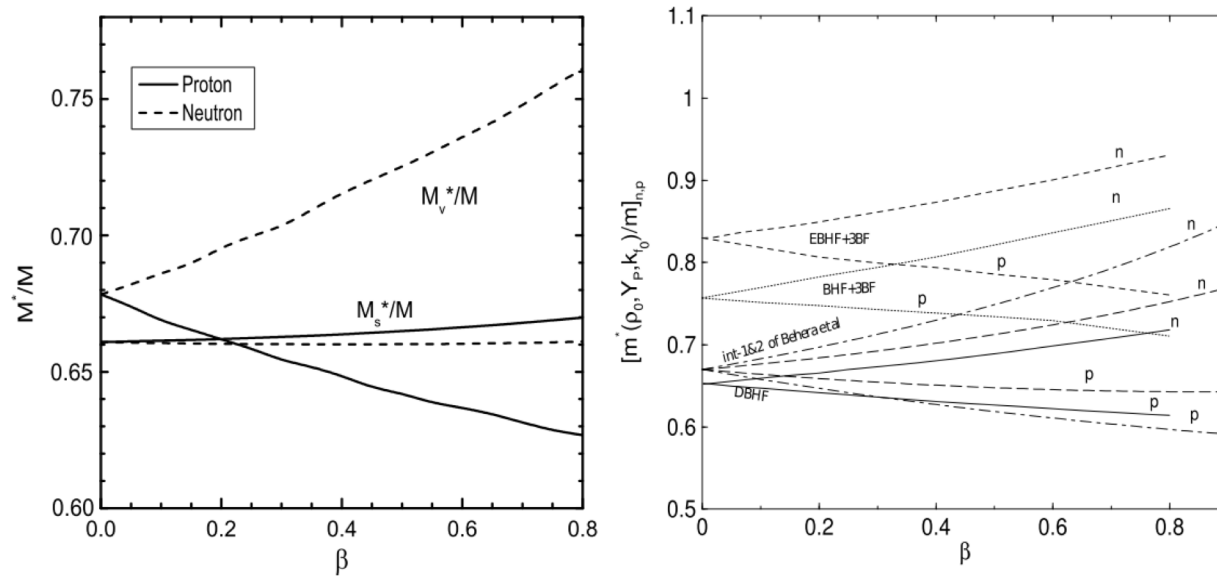
$$E/A(\rho, \delta) = E/A(\rho, \delta = 0) - E_{\text{sym}}(\rho) \cdot \delta^2 + \mathcal{O}(\delta^4).$$

*P. Demorest et al. Nature 467, 1081–1083 (2010)*

# Nucleon Effective Mass

B.-A. Li et al. / Progress in Particle and Nuclear Physics 99 (2018) 29–119

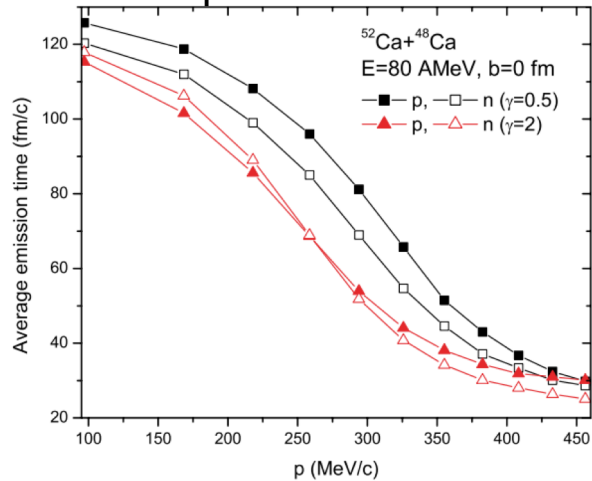
45



**Fig. 9.** Left: Nucleon Lorentz-scalar (Dirac) effective mass  $M_s^*$  and Lorentz-vector effective masses  $M_v^*$  at a nucleon energy of  $E = 50$  MeV in neutron-rich matter of isospin-asymmetry  $\beta$  with  $k_F = 1.36 \text{ fm}^{-1}$ . Taken from Ref. [59]. Right: Comparisons of nucleon effective masses as a function of isospin asymmetry  $\beta$  at saturation density predicted by (1) the DBHF theory [123], (2) the BHF+3-body force, (3) the extended BHF+3-body force [58] as well as the energy density functional using two forms (int-1&2) of isospin-dependent interactions [28]. Modified from a figure in Ref. [28].

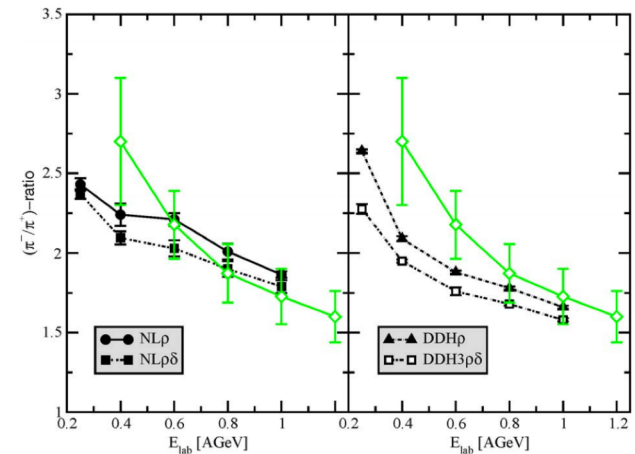
# Observables

## neutron-proton correlation

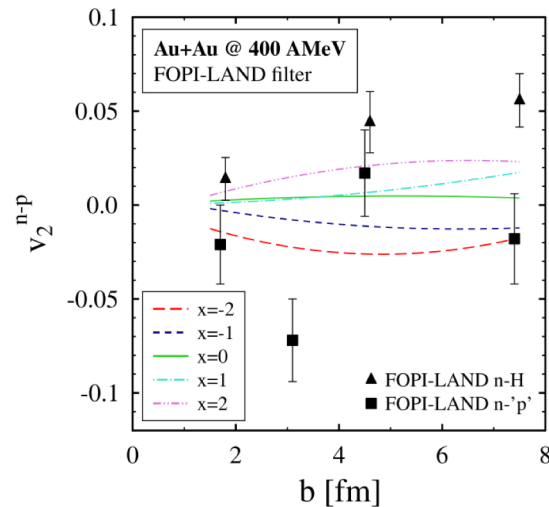


ref. Lie-Wen Chen, et al. PRL 90 (2003) 162701

## pion production ratio



ref. T. Gaitanos, et al. NPA 732 (2004) 24



## Flow observables

ref. P. Russotto, et al. EPJA 50 (2014) 38

# New Observables

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- neutron-to-proton ratio
- neutron-proton differential flow
- neutron-proton correlation function
- $t/{}^3\text{He}$ ,  $\pi^+/\pi^-$ ,  $K^0/K^-$ , ...
- ...

***What is the new observable at RAON?***

# Transport Model

## Development of a new QMD code

Transport model

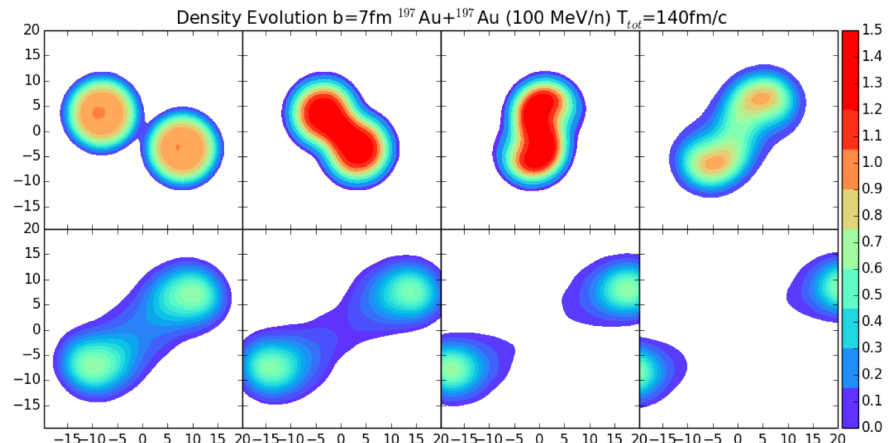
### Boltzmann-Uehling-Uhlenbeck(BUU)

- ✓ Less fluctuation, Good to study EOS
- ✓ Not good for event generator
  - Code(DjBUU) developed by external experts: **S. Jeon** (McGill Univ), **C.-H. Lee**(Pusan Nat'l Univ.)

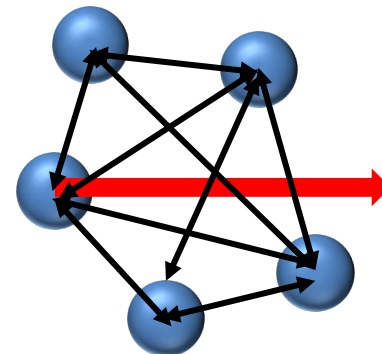
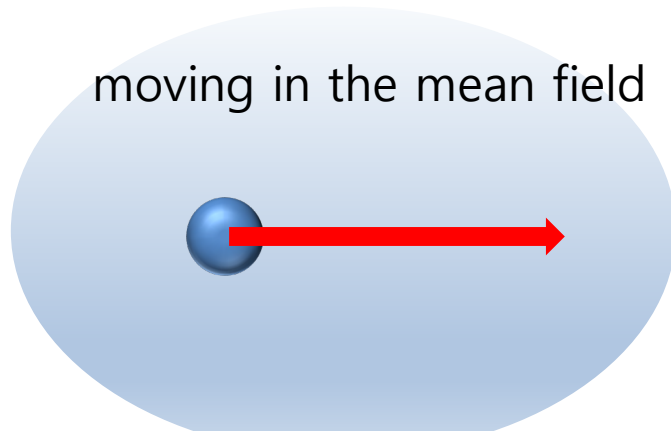
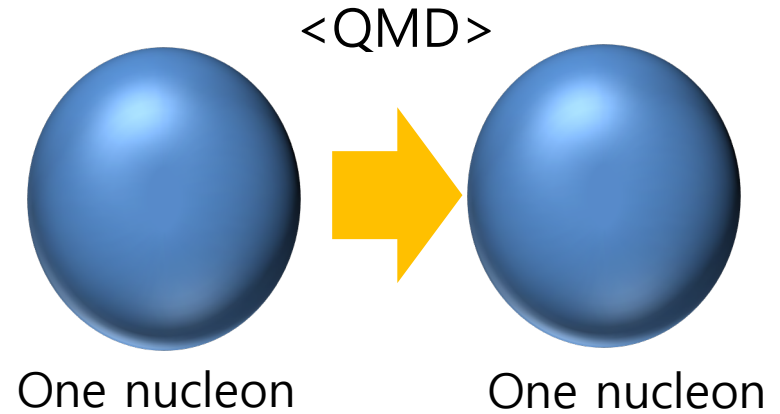
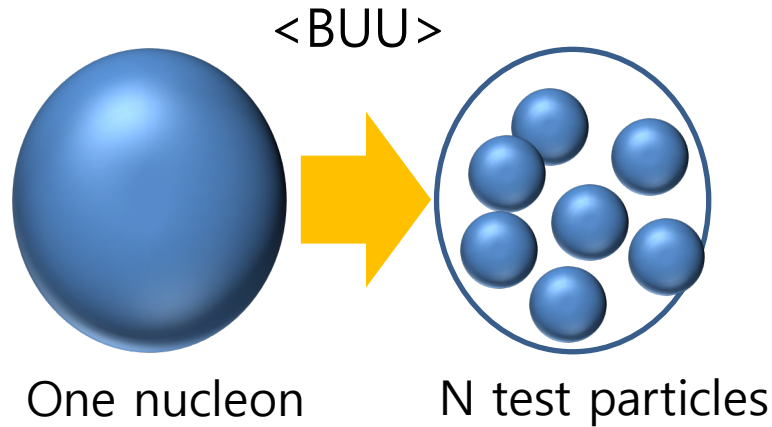
### Quantum Molecular Dynamics (QMD)

- ✓ Good for event generator (event by event analysis)
- ✓ More fluctuation, More CPU time
  - Code development mainly by KK with **K.S. Lee**(Chonnam Nat'l Univ)

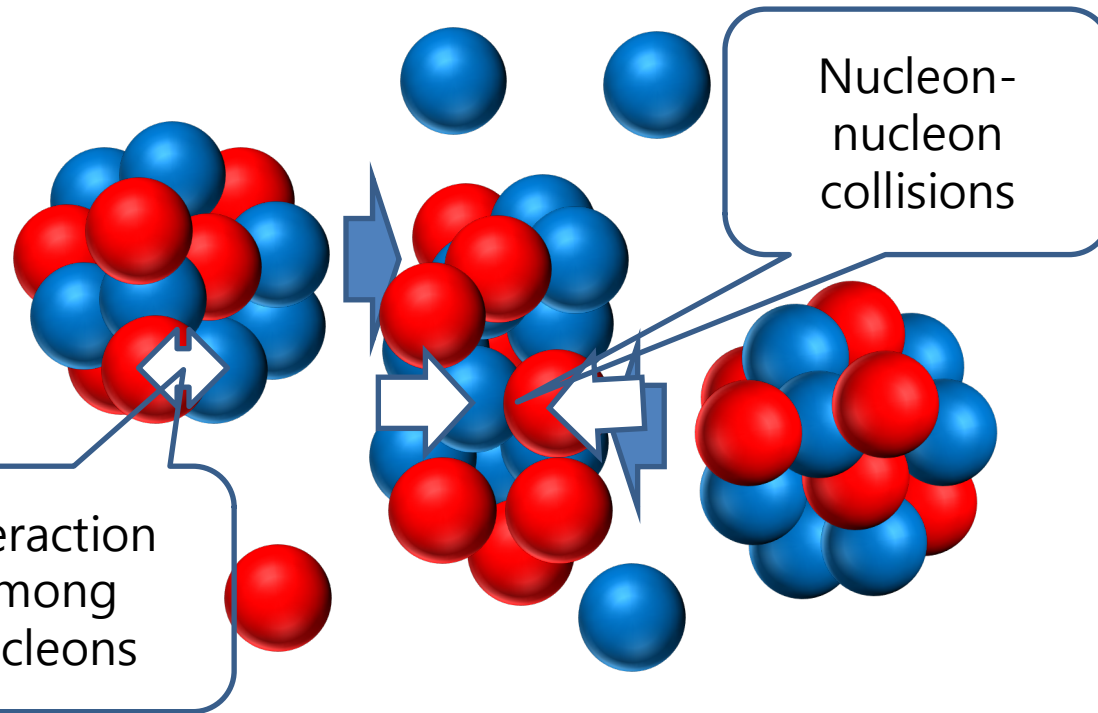
Simulation sample by QMD  
 $^{197}\text{Au} + ^{197}\text{Au}$  at  $b=7\text{fm}$   
with the beam energy  $100\text{MeV}/n$   
(averaging 1000 events)



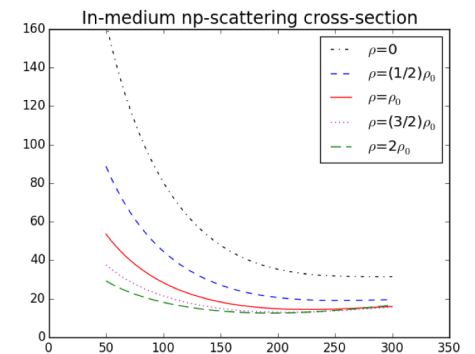
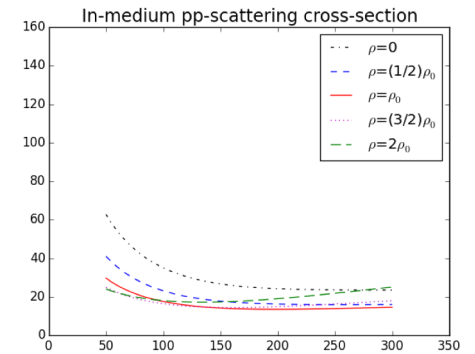
# BUU vs QMD



# Transport Model



In-medium cross-section :



$$\text{Wavefunction : } \psi_i(\vec{r}, t) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{4\sigma_r^2} + \frac{i}{\hbar}(\vec{p}_i \cdot \vec{r})\right)$$

$$\text{Potential : } H = \sum_i \frac{\vec{p}_i^2}{2m} + U^{2body} + U^{3body} + U^{surf} + U^{sym} + U^{Coul}$$



# QMD Model - Potential

Nucleon -> Gaussian wave packet

$$\psi_i(\vec{r}, t) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp\left(-\frac{(\vec{r} - \vec{r}_i)^2}{4\sigma_r^2} + \frac{i}{\hbar}(\vec{p}_i \cdot \vec{r})\right)$$



$$H = \sum_i \frac{\vec{p}_i^2}{2m} + U^{2body} + U^{3body} + U^{surf} + U^{sym} + U^{Coul}$$

$$U^{2body} = \frac{\alpha}{2\rho_0} \sum_{i,j \neq i} \rho_{ij},$$

$$U^{3body} = \frac{\beta}{\gamma + 1} \sum_i \left( \sum_{j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^\gamma,$$

$$U^{surf} = \frac{g_{surf}}{2\rho_0} \sum_{i,j \neq i} \nabla_{r_i}^2(\rho_{ij}),$$

$$U^{sym} = \frac{g_{sym}}{2\rho_0} \sum_{i,j \neq i} [2\delta_{\tau_i \tau_j} - 1] \rho_{ij},$$

$$U^{Coul} = \frac{e^2}{2} \sum_{i,j \neq i} \frac{1}{|\vec{r}_i - \vec{r}_j|} \operatorname{erf}\left(\frac{|\vec{r}_i - \vec{r}_j|}{2\sigma_r}\right) \quad (i, j \text{ for protons})$$

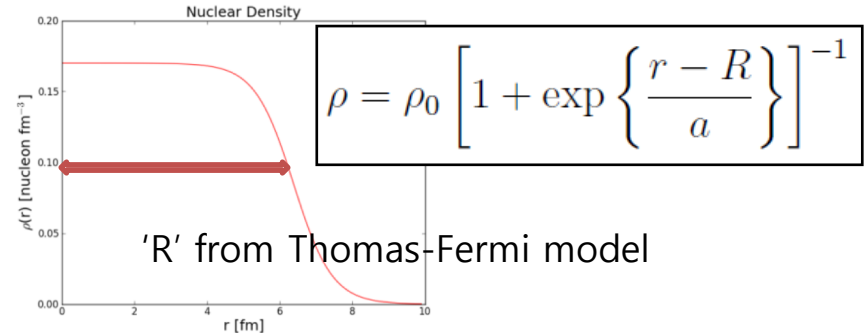
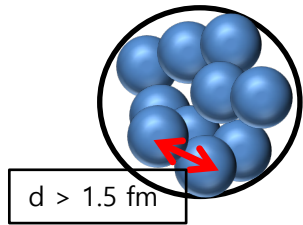
**Skyrme parametrization for NN potential**

| K(MeV) | $\alpha$ (MeV) | $\beta$ (MeV) | $\gamma$ | EOS  |
|--------|----------------|---------------|----------|------|
| 200    | -356           | 303           | 7/6      | Soft |
| 380    | -124           | 70.5          | 2        | Hard |

Ref.) M. Papa PRC 64(2010)024612

# QMD Model - Initialization

<Initialization>



$$f_i = \frac{1}{(2\pi\sigma_r\sigma_p)^3} \exp \left[ -\frac{(\vec{r} - \vec{r}_i)^2}{2\sigma_r^2} - \frac{(\vec{p} - \vec{p}_i)^2}{2\sigma_p^2} \right]$$

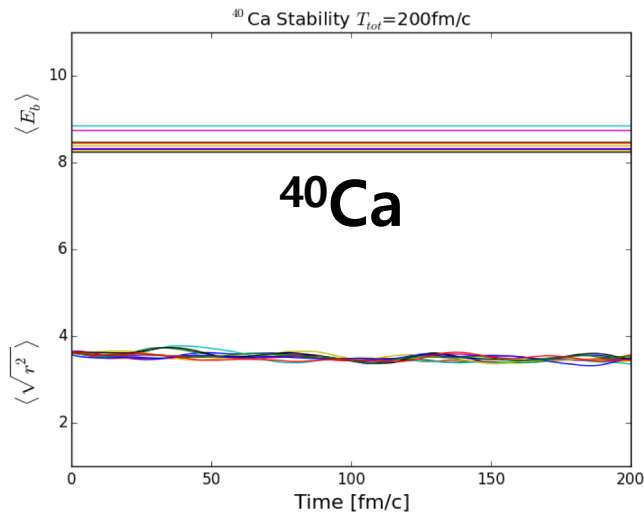
momentum of a nucleon is also chosen in the range of  $0 \sim k_f = \sqrt{2m_i U_{tot}^i}$ ,



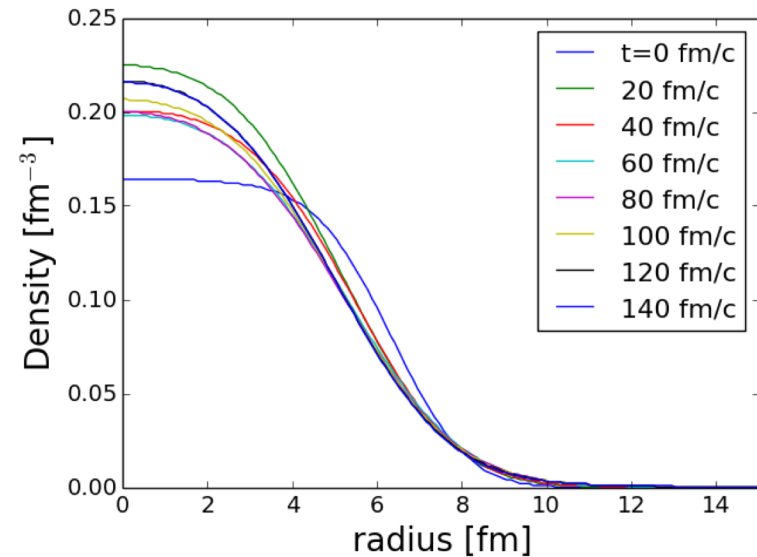
Only the set which gives the binding energy within 5% difference with experimental (or theoretical) data will be selected.

# QMD Model - Initialization

<Stability check>

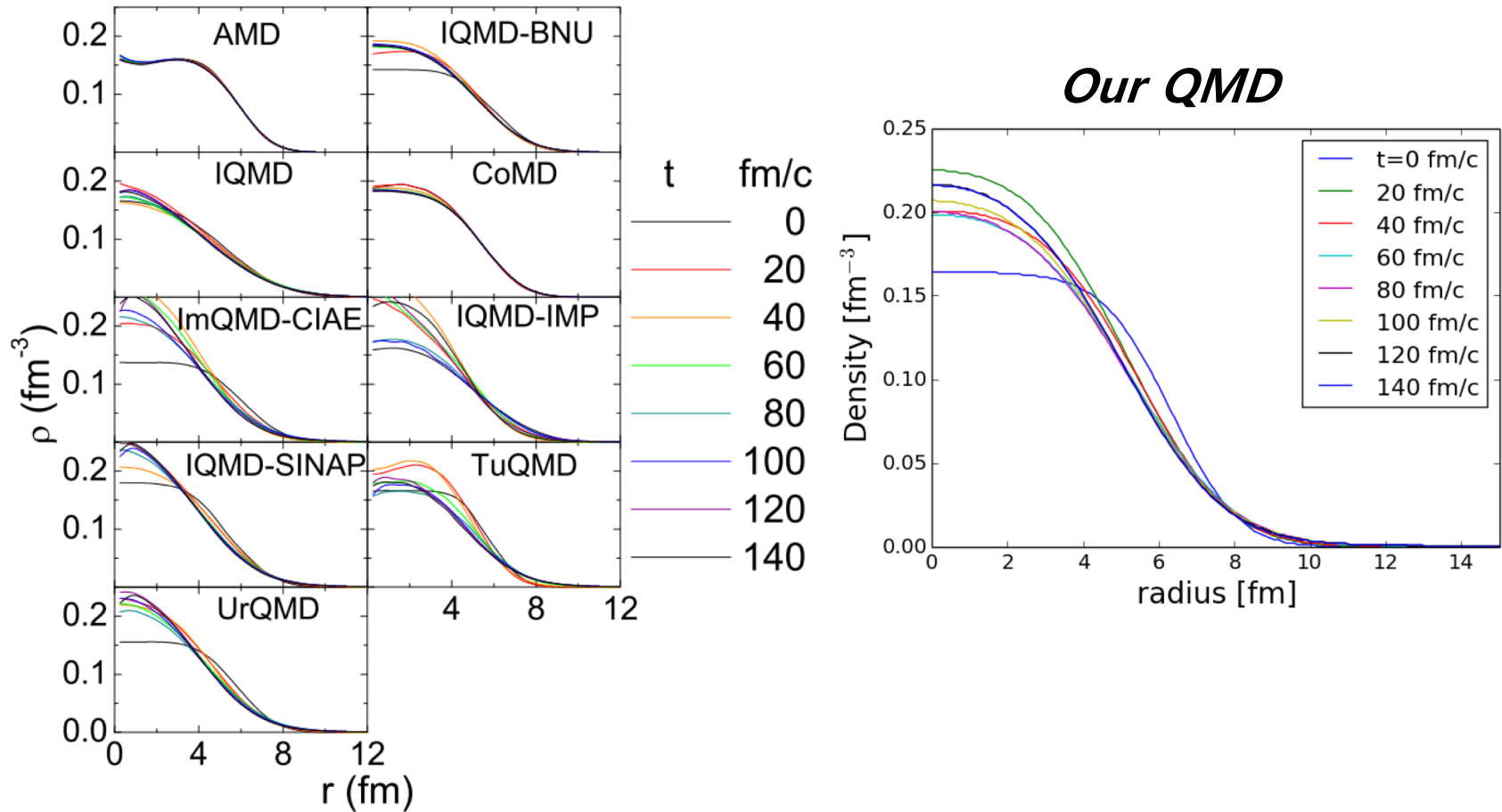


<sup>197</sup>Au



# QMD Model - Comparison

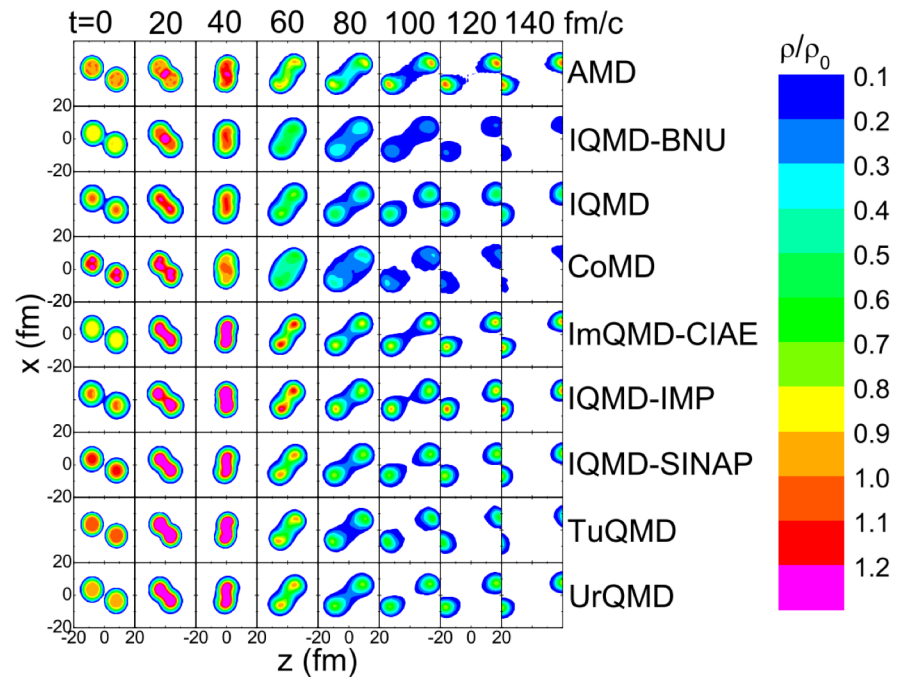
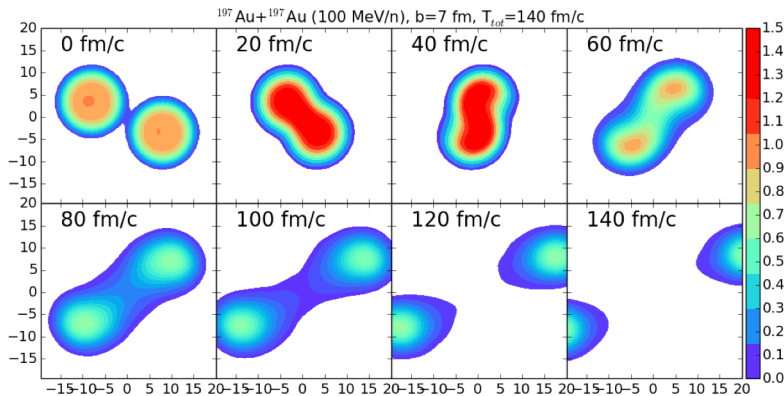
ref. J. Xu, et al. PRC 93 (2016) 044609



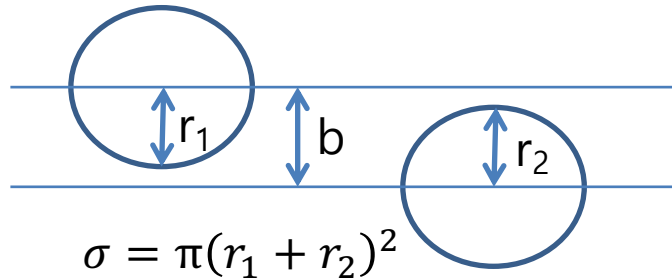
# QMD Model - Propagation

<Equation of Motion>

$$\frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{\mu} \quad \frac{d\vec{p}_i}{dt} = -\nabla_{r_i} U$$



# QMD Model - Collisions



*In classical scattering,*

$$b < r_1 + r_2$$

➔ *Two particles are always scattered.*

In our model,

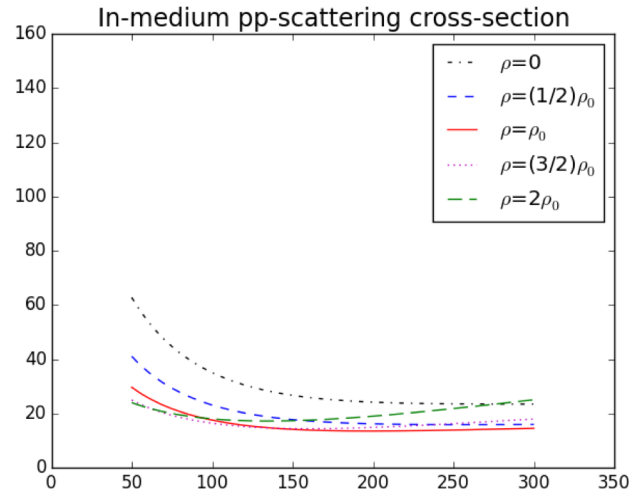
$$b = \sqrt{\sigma_{tot}/(10 \times \pi)}, \quad (b: [\text{fm}], \sigma: [\text{mb}])$$

➔ **If a distance,  $d$ , between two nucleons is smaller than  $b$ ,  $d < b$ , a collision is always tried.**

*Here,  $\sigma_{tot}$  is in-medium cross-section.*

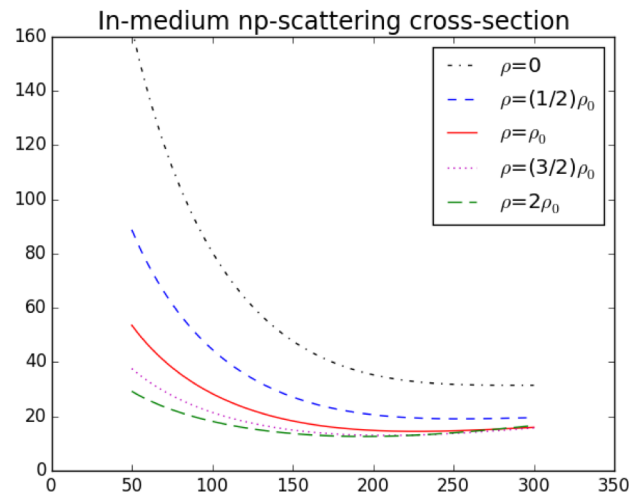
$$\left| \frac{\Delta \vec{r} \cdot \vec{p}}{p} \right| < \left( \frac{p}{2m_1} + \frac{p}{2m_2} \right) \frac{\delta t}{2}$$

# QMD Model - Collisions



*Ref.) G.Li and R.Machleidt PRC 48, 1702, PRC 49, 566*

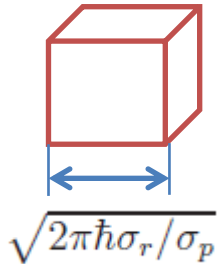
$$\sigma_{pp-tot}(E_{lab}, \rho) = [23.5 + 0.00256(18.2 - E_{lab}^{0.5})^{4.0}] \times \frac{1.0 + 0.1667 E_{lab}^{1.05} \rho^3}{1.0 + 9.704 \rho^{1.2}}$$



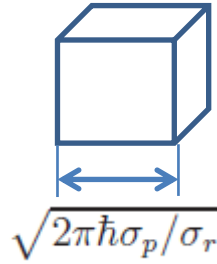
$$\sigma_{np-tot}(E_{lab}, \rho) = [31.5 + 0.092|20.2 - E_{lab}^{0.53}|^{2.9}] \times \frac{1.0 + 0.0034 E_{lab}^{1.51} \rho^2}{1.0 + 21.55 \rho^{1.34}}$$

# QMD Model – Pauli Blocking

<x-space>



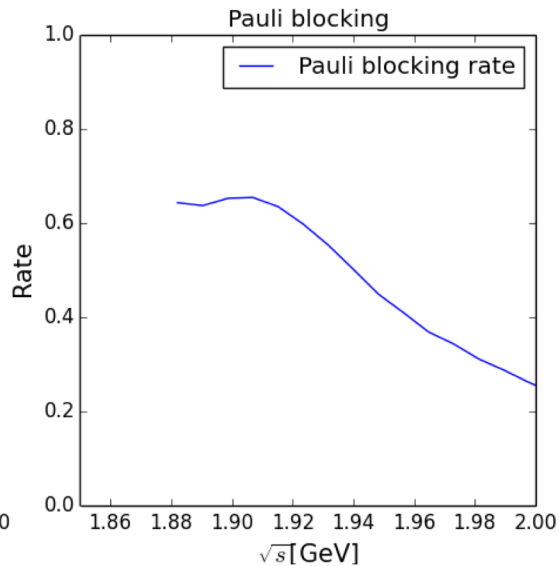
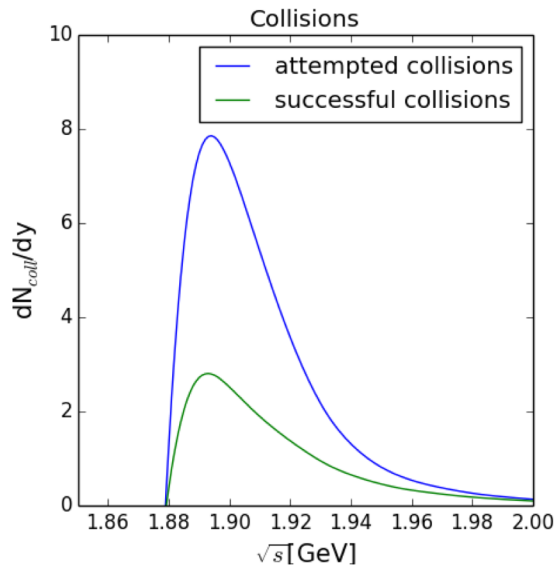
<p-space>



<Phase space density for  $i$ th particle>

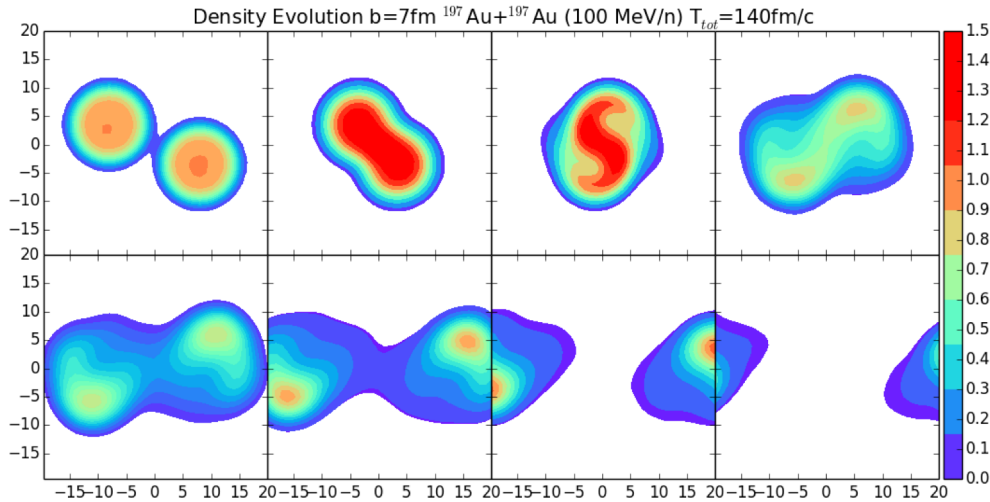
$$\bar{f}_i \equiv \sum_j \delta_{\tau_i, \tau_j} \delta_{s_i, s_j} \int_{h^3} f_i(\vec{r}, \vec{p}) d^3r d^3p$$

↓  
**Occupation number**

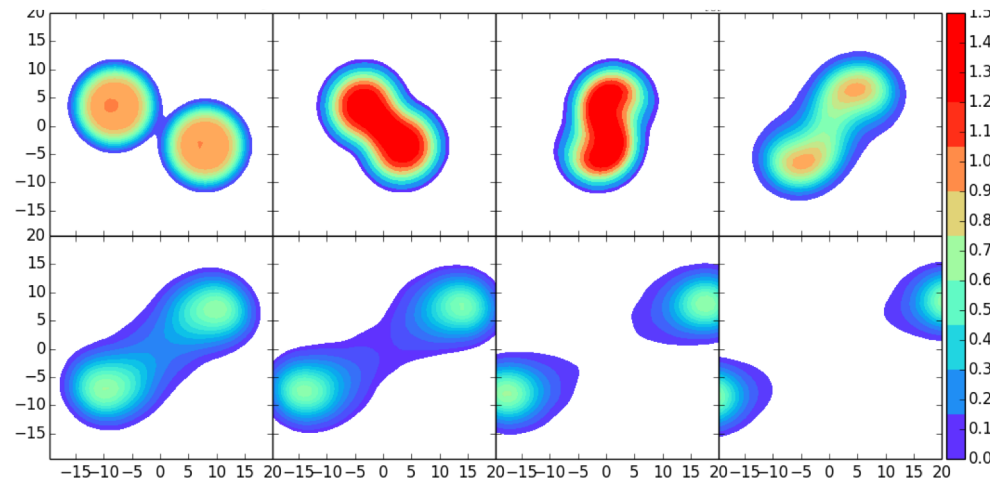




# Role of NN Collision



w/o collisions



w/ collisions

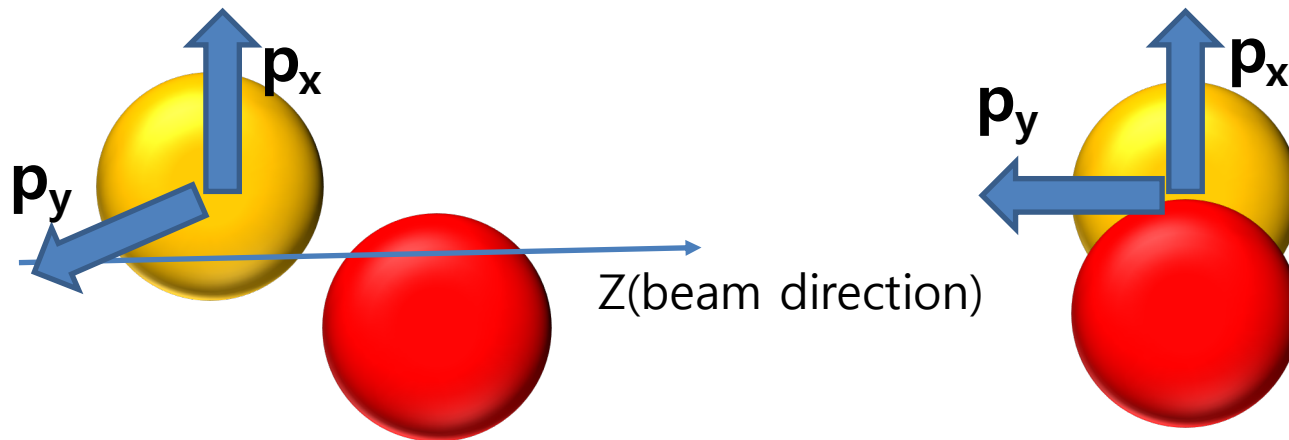
# Differential Flow

$$u = (\gamma, \vec{\beta}\gamma); \quad u_t = \beta_t \gamma$$

$$\frac{dN}{u_t du_t dy d\phi} = v_0 [1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi)]$$

$$v_0 = v_0(y, u_t); \quad v_1 = v_1(y, u_t); \quad v_2 = v_2(y, u_t)$$

$$v_1 = \left\langle \frac{p_x}{p_t} \right\rangle = \langle \cos(\phi) \rangle; \quad v_2 = \left\langle \left( \frac{p_x}{p_t} \right)^2 - \left( \frac{p_y}{p_t} \right)^2 \right\rangle = \langle \cos(2\phi) \rangle$$



# Sn+Sn Simulation

- ❖  $^{112}\text{Sn} + ^{112}\text{Sn}$  and  $^{132}\text{Sn} + ^{124}\text{Sn}$  at  $E_{\text{lab}} = 200 \text{ MeV/n}$
- ❖ impact parameter,  $b = 5 \text{ fm}$
- ❖ number of total event = 20,000
- ❖ asy-soft ( $\sigma=0.5$ ) and asy-stiff ( $\sigma=2$ ) EOS

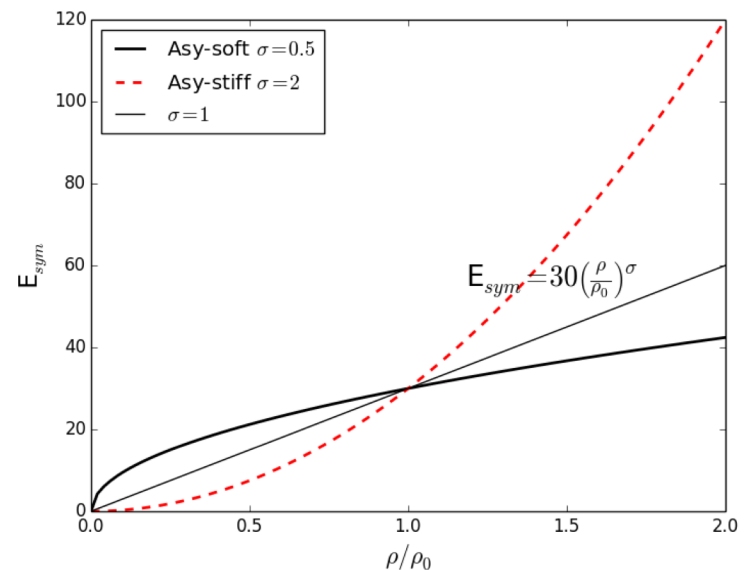
$$E_{\text{sym}} = 30 \left( \frac{\rho}{\rho_0} \right)^\sigma$$

- ❖ under the pion threshold energy

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma,$$

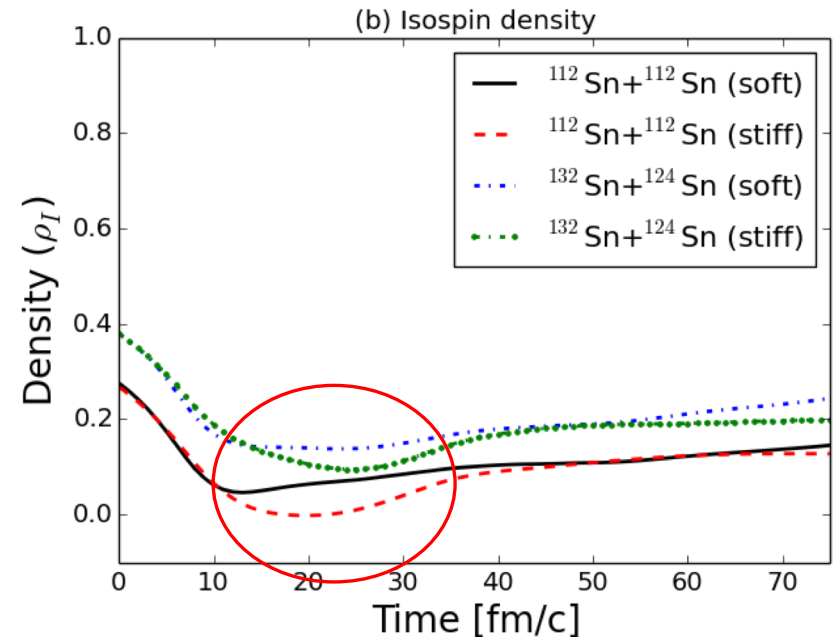
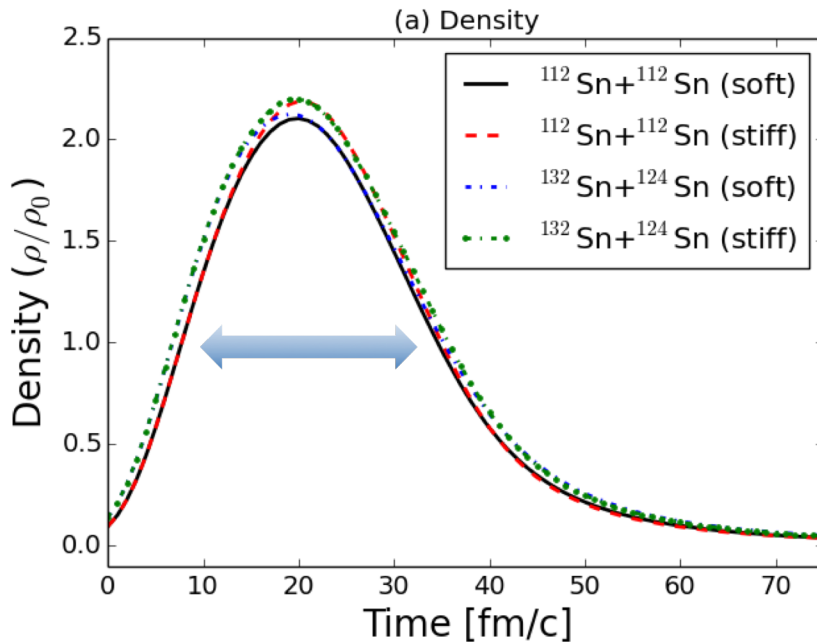
when  $\alpha = -218 \text{ MeV}$ ,  $\beta = 164 \text{ MeV}$  and  $\gamma = 4/3$ .

$$E_{\text{sym}} = 30 \left( \frac{\rho}{\rho_0} \right)^\sigma$$



# Density and Isospin Density

Density (left) and isospin density (right) at the center of the Sn+Sn system

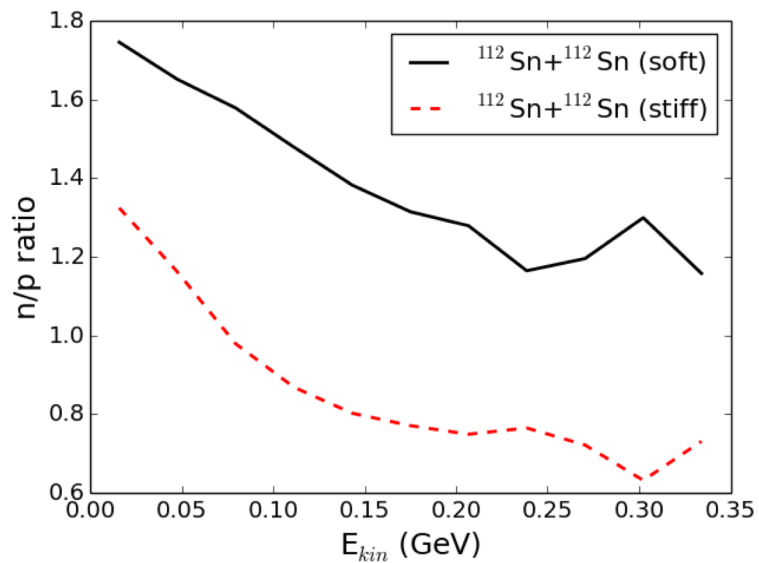


- The duration time for the supra-saturation density is  $\sim 20$  fm/c.
- The isospin density when the asy-soft EOS is used is larger.

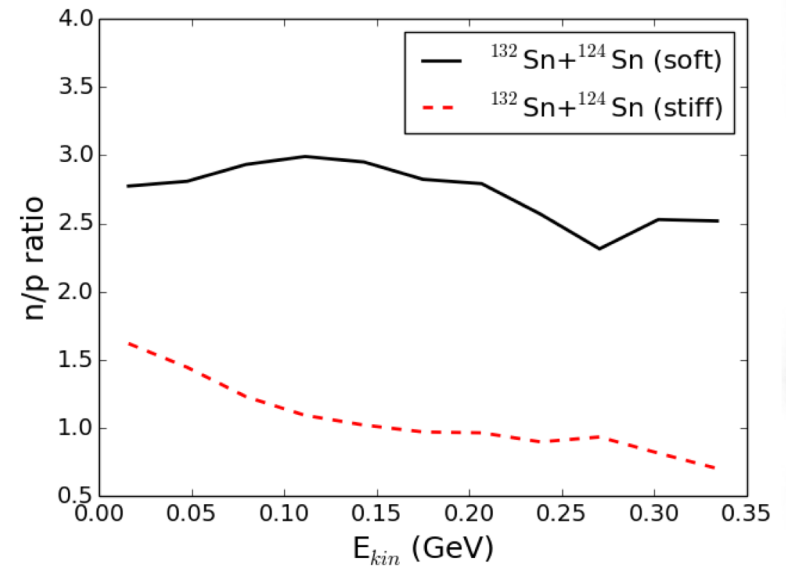
cf.) 
$$\rho_I = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$

# Observables – n/p Ratio

$\langle n/p \text{ ratio} - {}^{112}\text{Sn} + {}^{112}\text{Sn} \rangle$



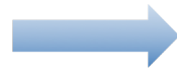
$\langle n/p \text{ ratio} - {}^{132}\text{Sn} + {}^{124}\text{Sn} \rangle$



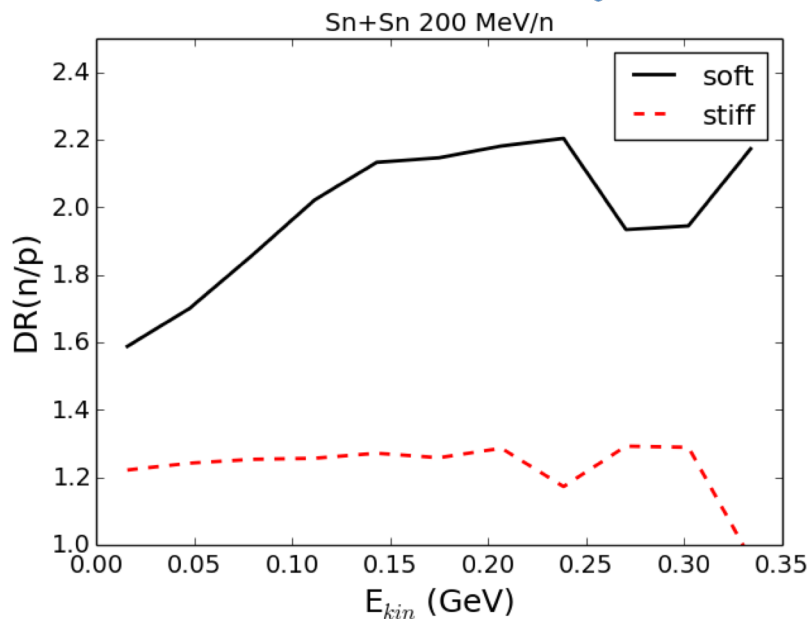
# Observables – Double n/p Ratio

$$DR(n/p) = \frac{n/p(^{124}\text{Sn} + ^{124}\text{Sn})}{n/p(^{112}\text{Sn} + ^{112}\text{Sn})}$$

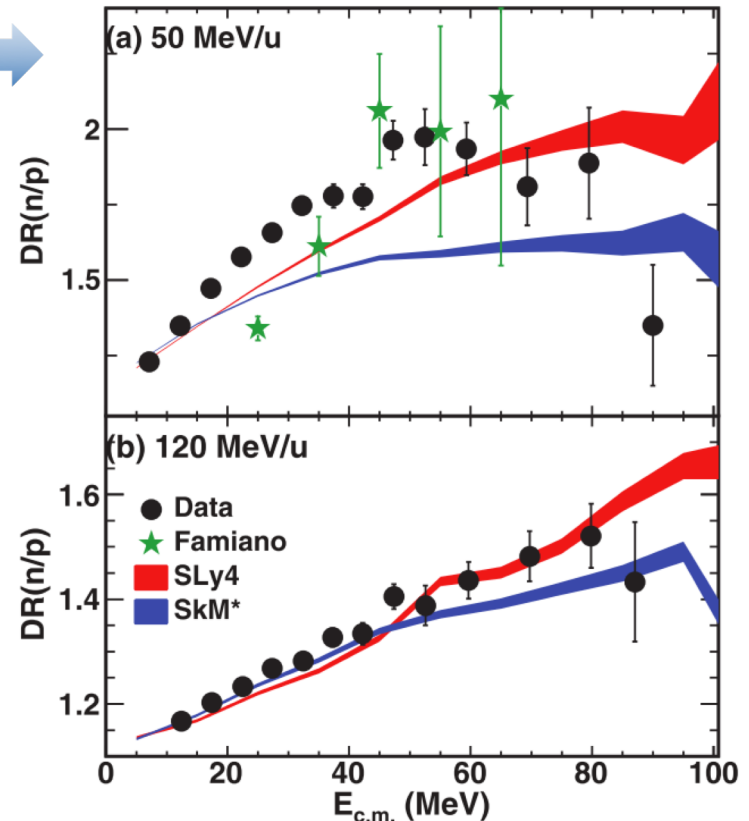
$$DR(n/p) = \frac{n/p(^{132}\text{Sn} + ^{124}\text{Sn})}{n/p(^{112}\text{Sn} + ^{112}\text{Sn})}$$



Double neutron-to-proton ratio from NSCL data



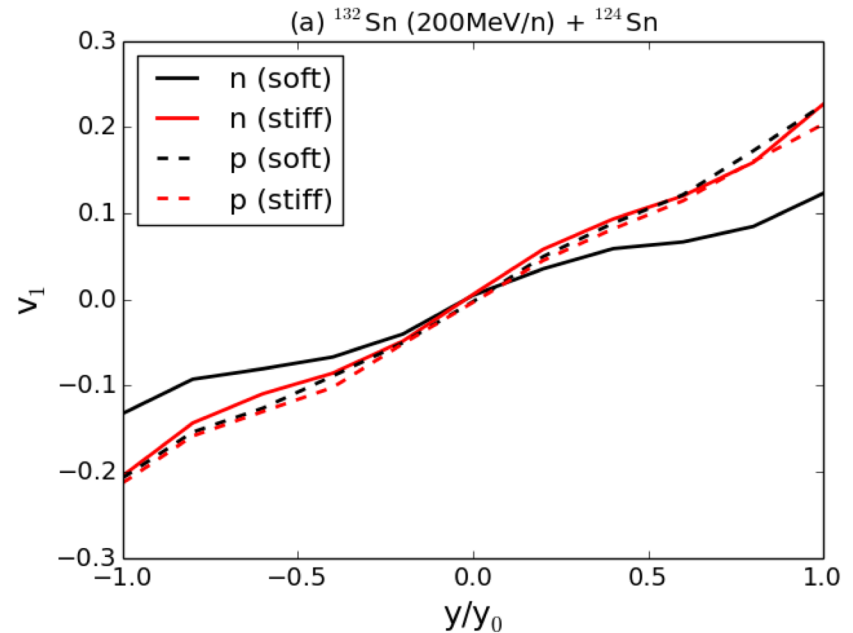
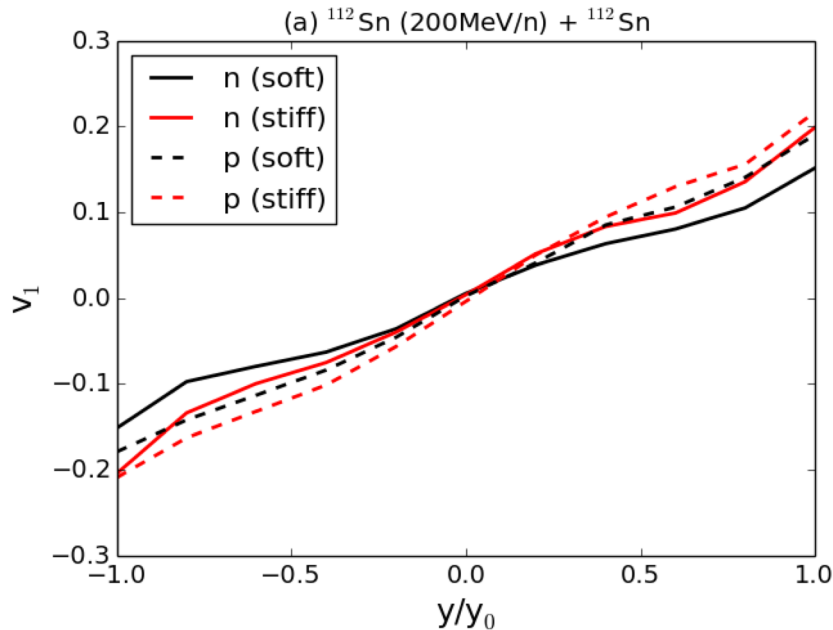
QMD simulation



ref. D. D. S. Coupland, et al. PRC 94 (2016) 011601

# Observables – $v_1$

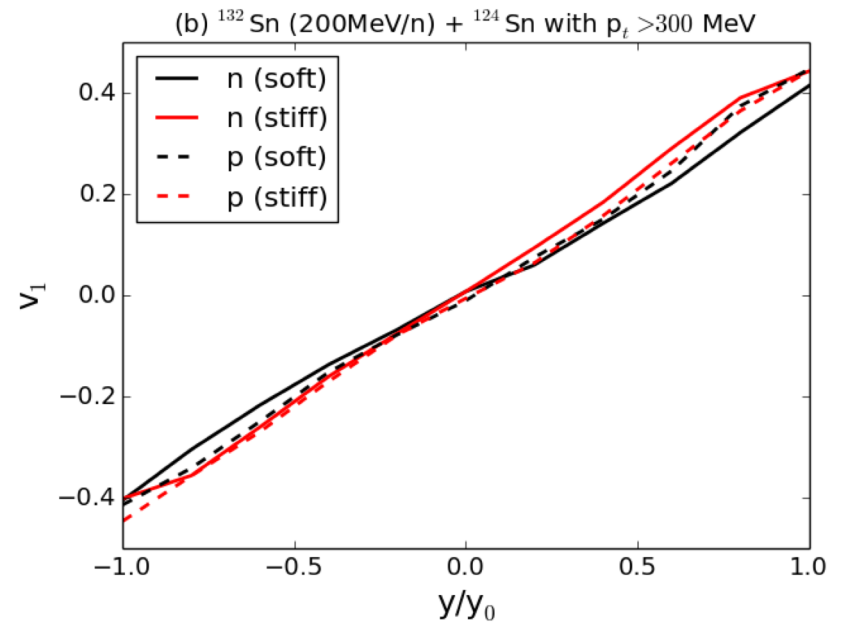
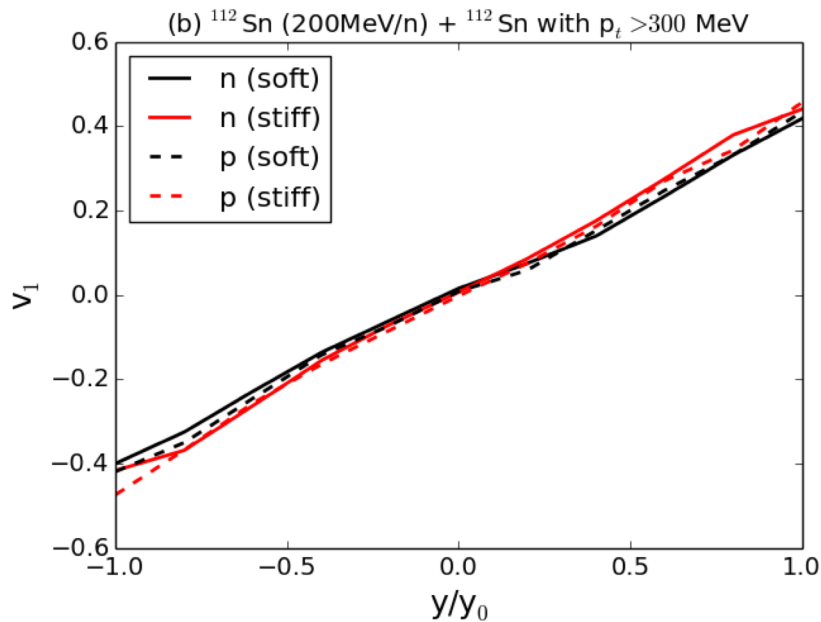
$v_1$  of the emitted nucleons from Sn+Sn collisions



Here,  $y_0$  is the rapidity of a nucleon of projectile in the center of mass frame.

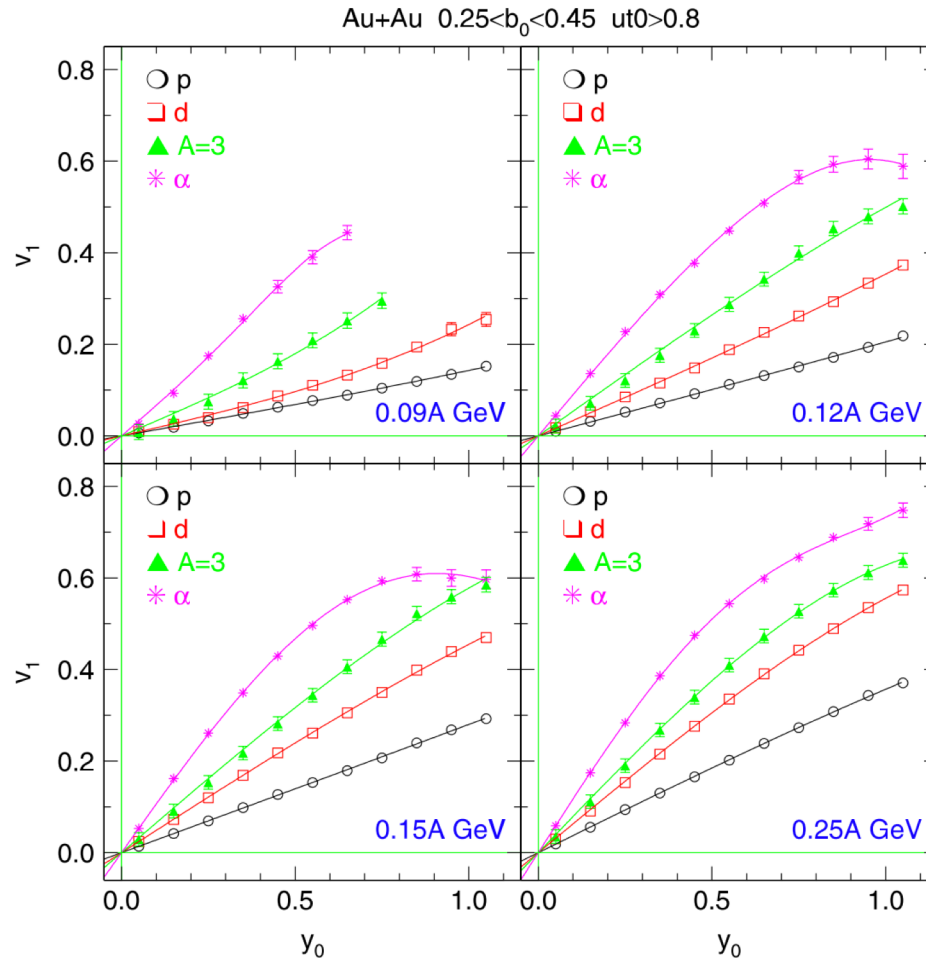
# Observables – $v_1$ with $p_t$ cut

$v_1$  of the emitted nucleons from Sn+Sn collisions with  $p_t > 300$  MeV





# Observables – $v_1$ from FOPI Data

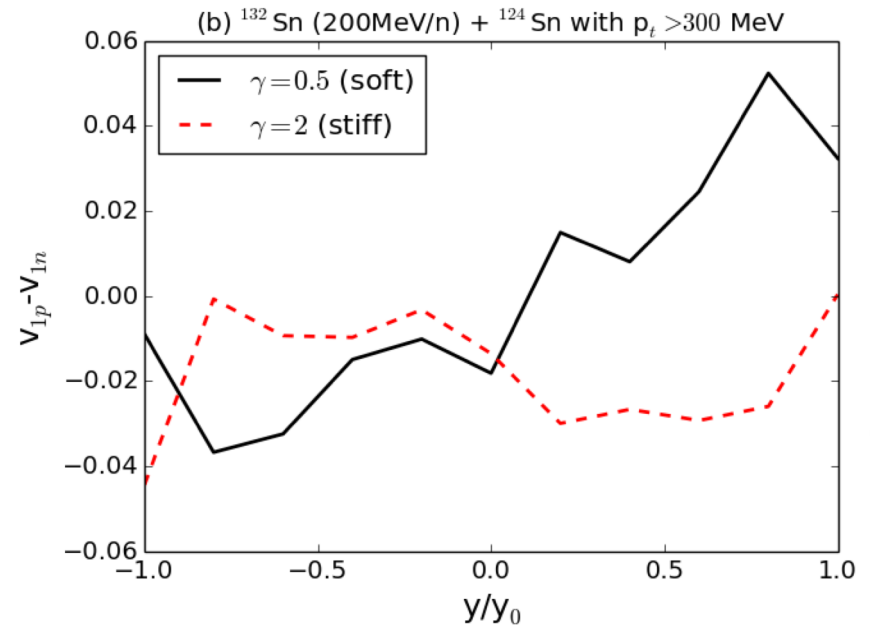
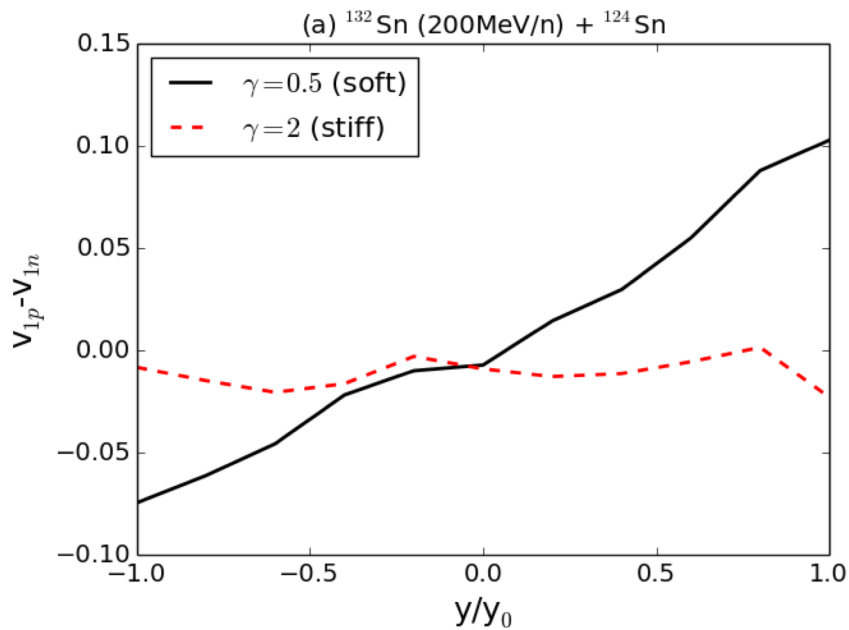


- $v_1$  of light particles of  $^{197}\text{Au}+^{197}\text{Au}$  collision from FOPI data
- $3.3 \text{ fm} < b < 6 \text{ fm}$  with  $p_t$  cut

*ref. Reisdorf, et al. NPA 876 (2012) 1*

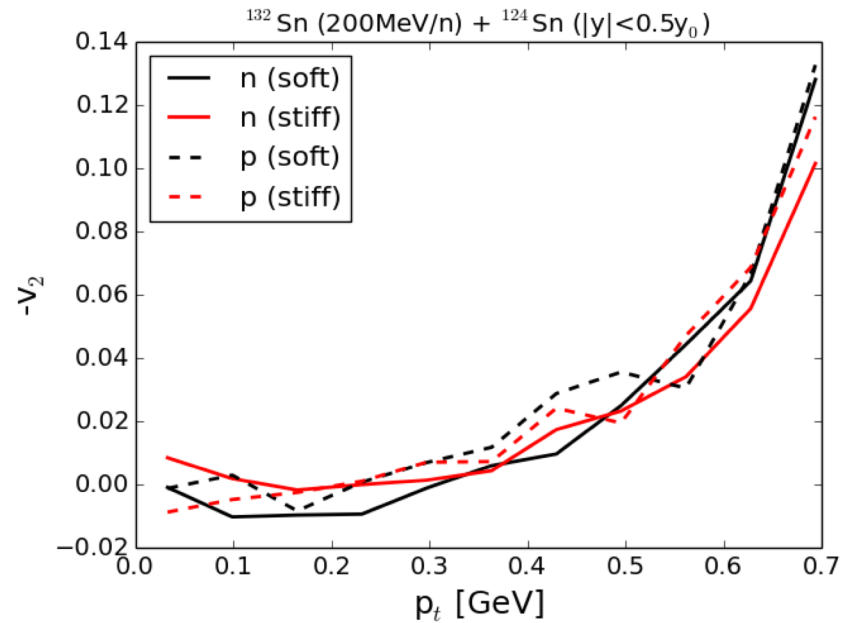
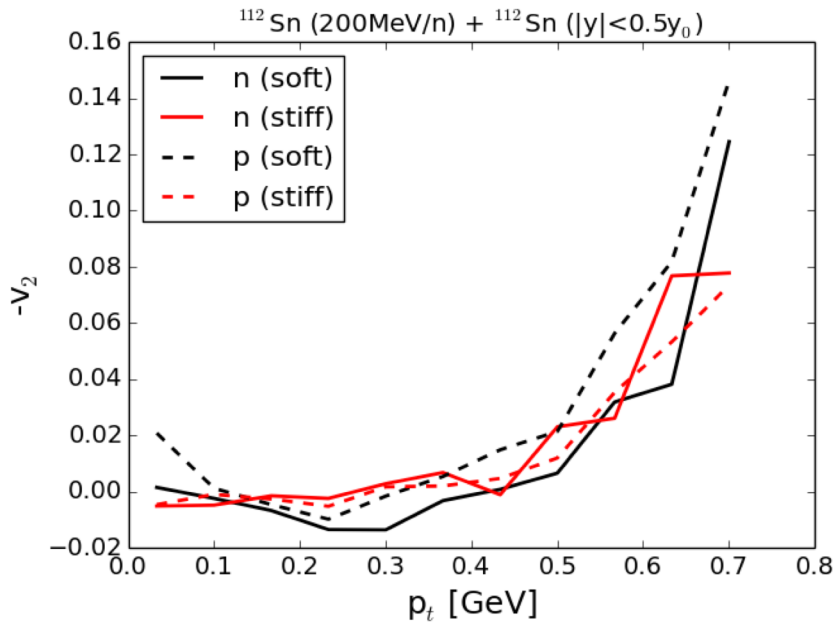
# Observables – $v_{1p} - v_{1n}$

The difference of  $v_1$  between protons and neutrons

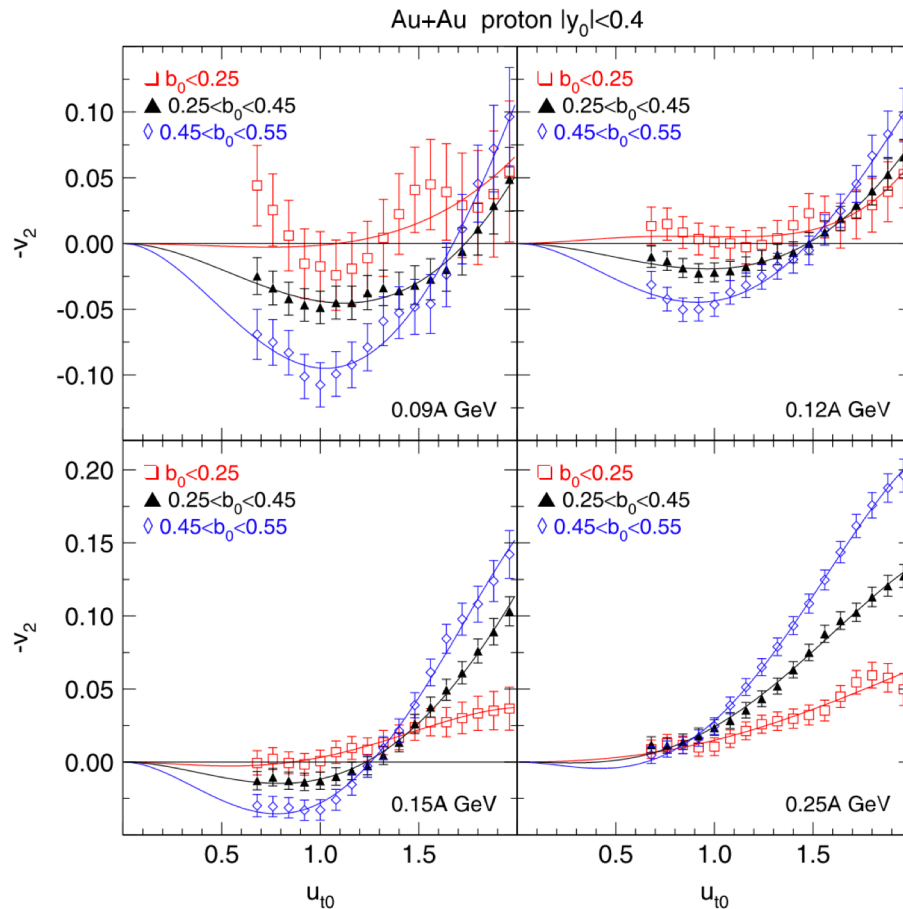


When the asy-soft EOS is used, the difference is larger.

# Observables – $v_2$



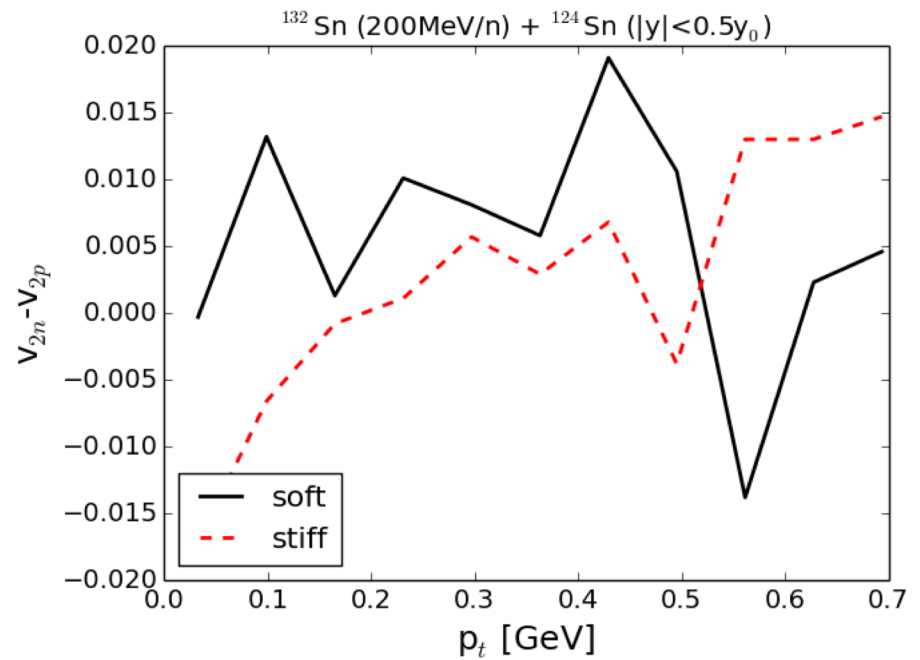
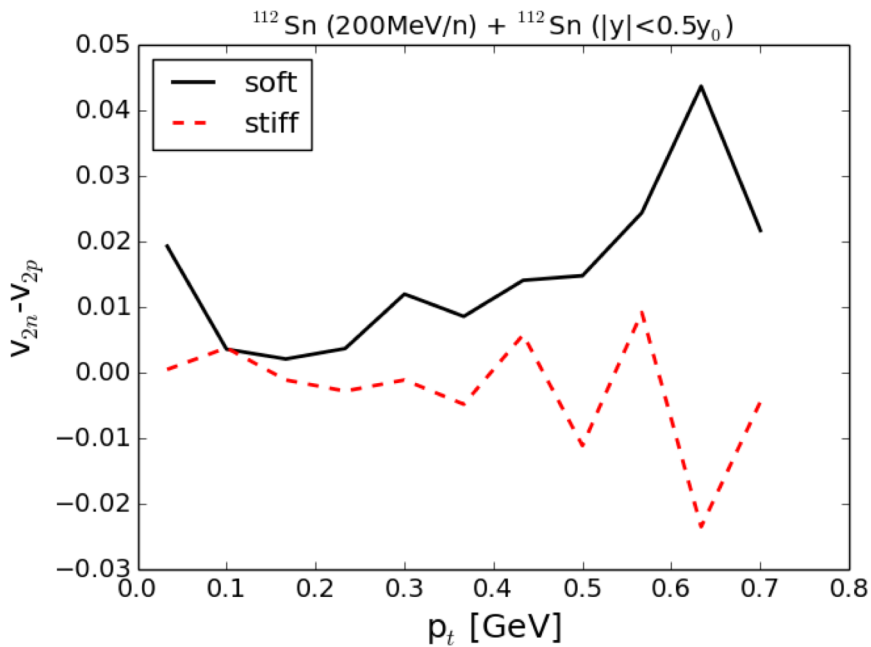
# Observables – $v_2$ from FOPI Data



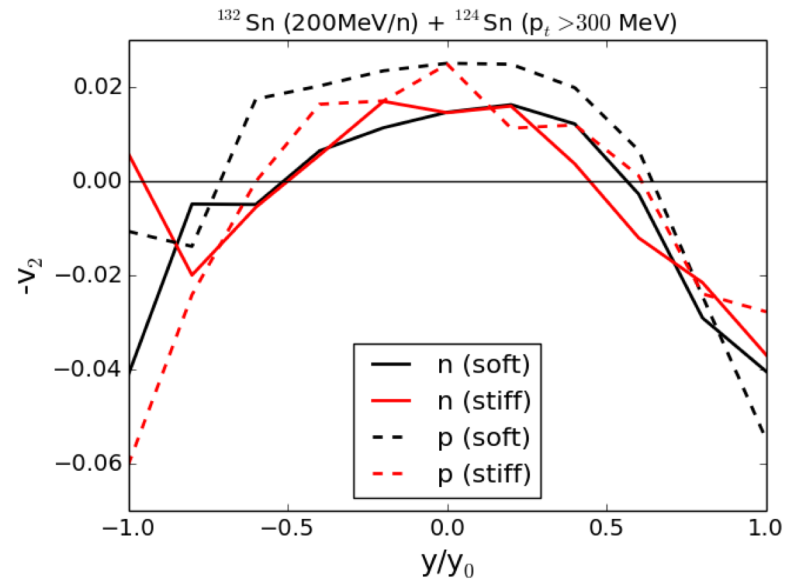
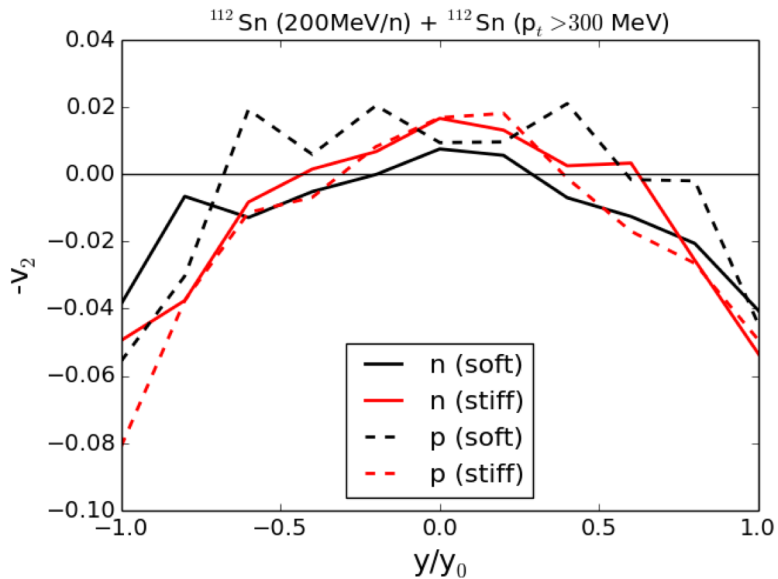
- $v_2$  of the emitted protons of  $^{197}\text{Au} + ^{197}\text{Au}$  collision from FOPI data with  $y_0$  range

*ref. Reisdorf, et al. NPA 876 (2012) 1*

# Observables – $v_{2n} - v_{2p}$

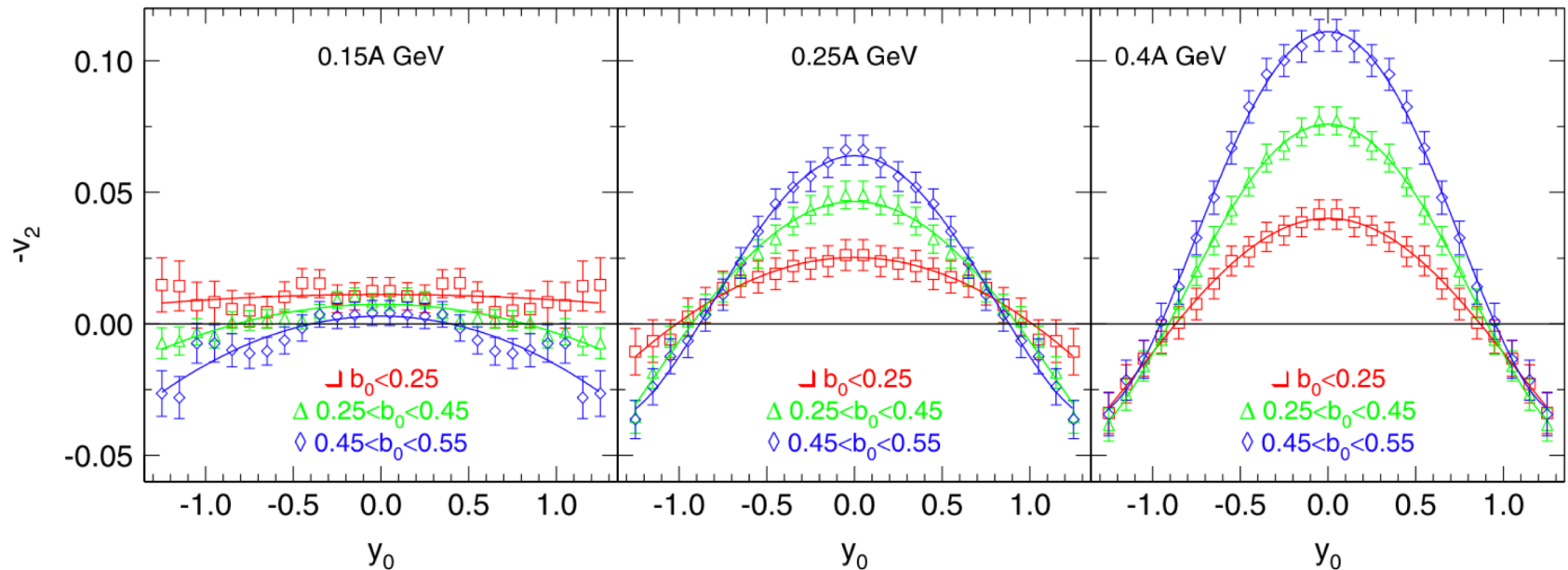


# Observables – $v_2$



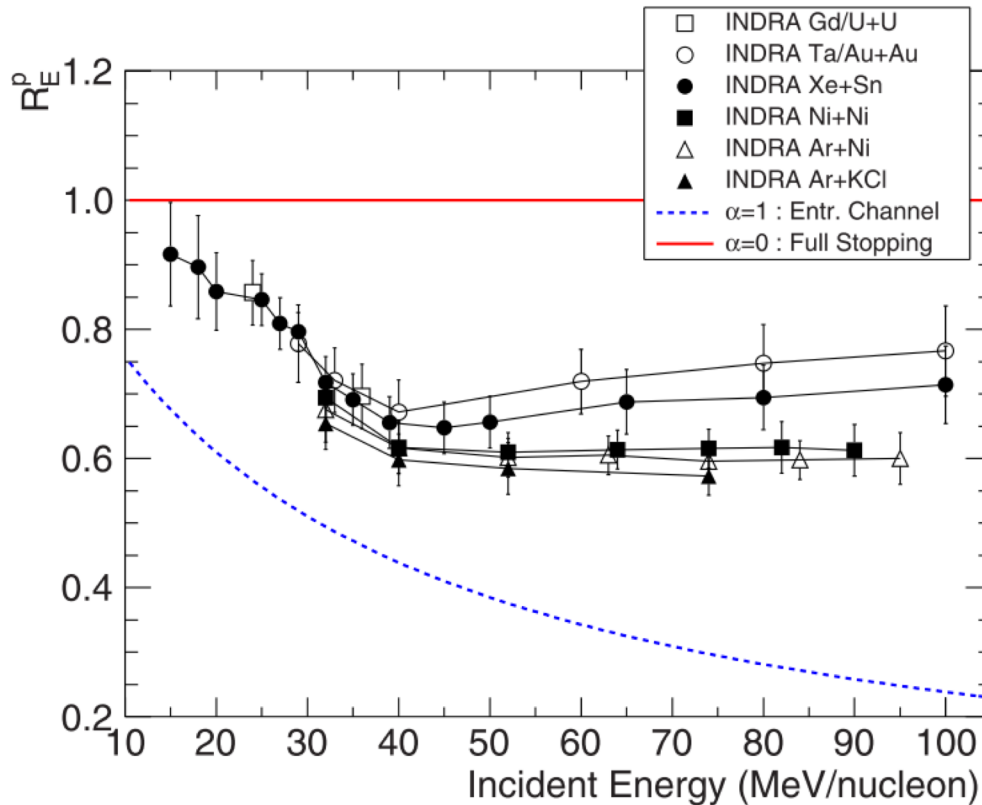
# Observables – $v_2$ from FOPI Data

Au+Au proton  $u_{t0} > 0.8$



*ref. Reisdorf, et al. NPA 876 (2012) 1*

# Stopping Ratio



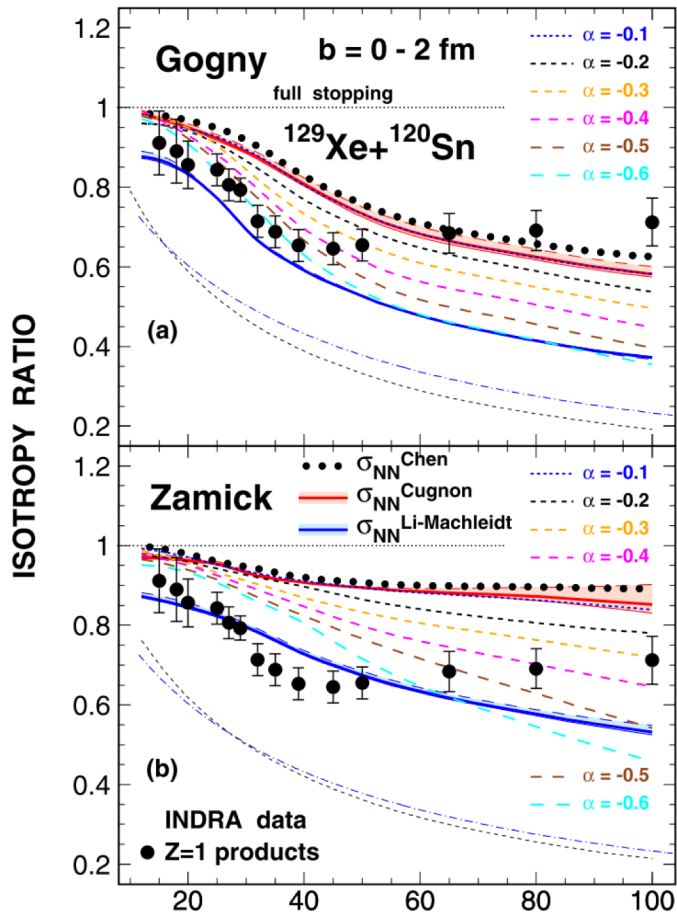
<Energy-based isotropy ratio>

$$R_E = \frac{\sum E_{trans}^i}{2 \sum E_{long}^i}$$

*O. Lopez et al. PRC 90, 064602 (2014)*

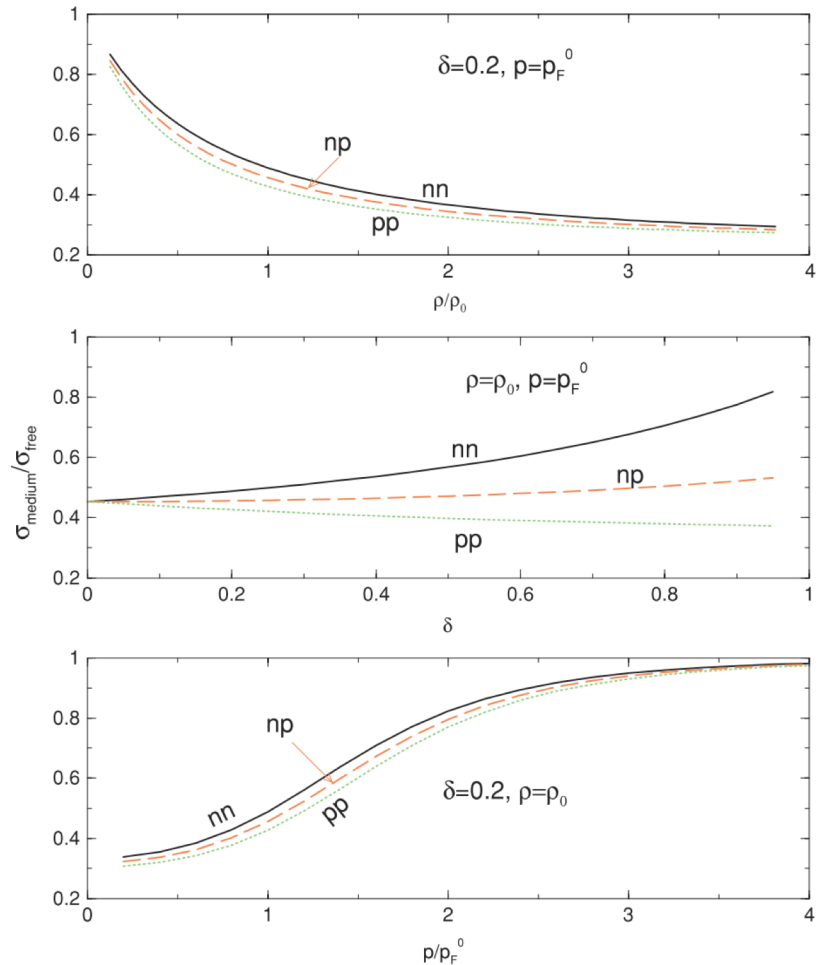


# NN Cross Section in Medium



LABORATORY ENERGY (MeV/nucleon)

*Z. Basrak, P. Eudes, and V. de la Mota  
 PRC 93, 054609 (2016)*

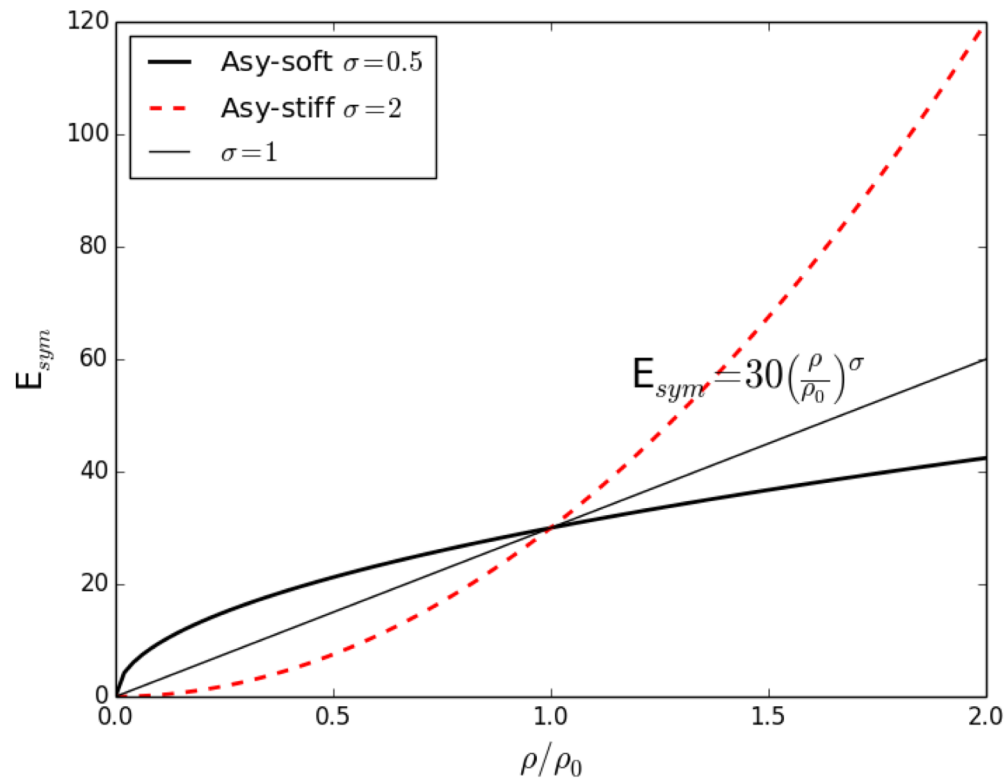


*L.Chen and B.Li PRC 72, 064611 (2005)*

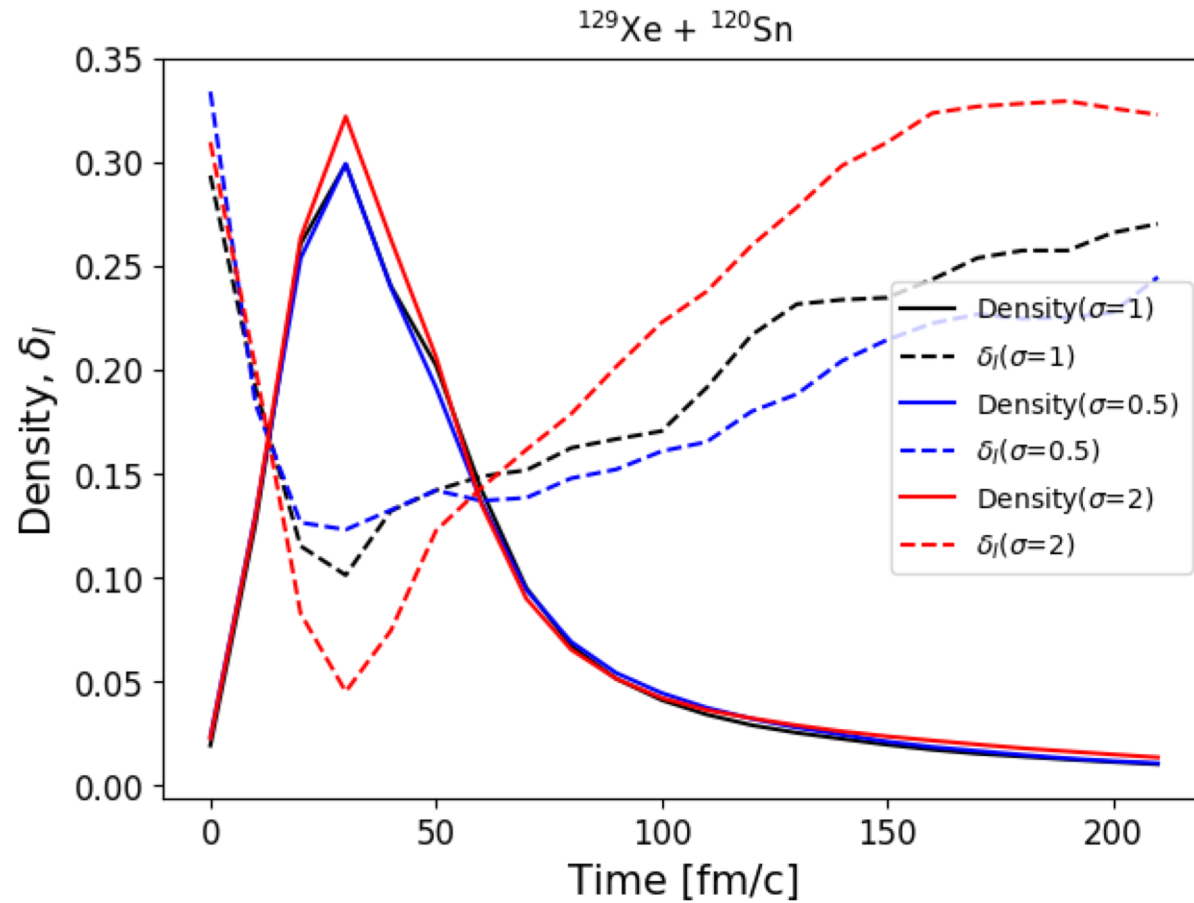
# $^{129}\text{Xe} + ^{120}\text{Sn}$ Simulation

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma, \text{ when } \alpha = -218 \text{ MeV}, \beta = 164 \text{ MeV} \text{ and } \gamma = 4/3.$$

$$E_{sym} = 30 \left( \frac{\rho}{\rho_0} \right)^\sigma$$

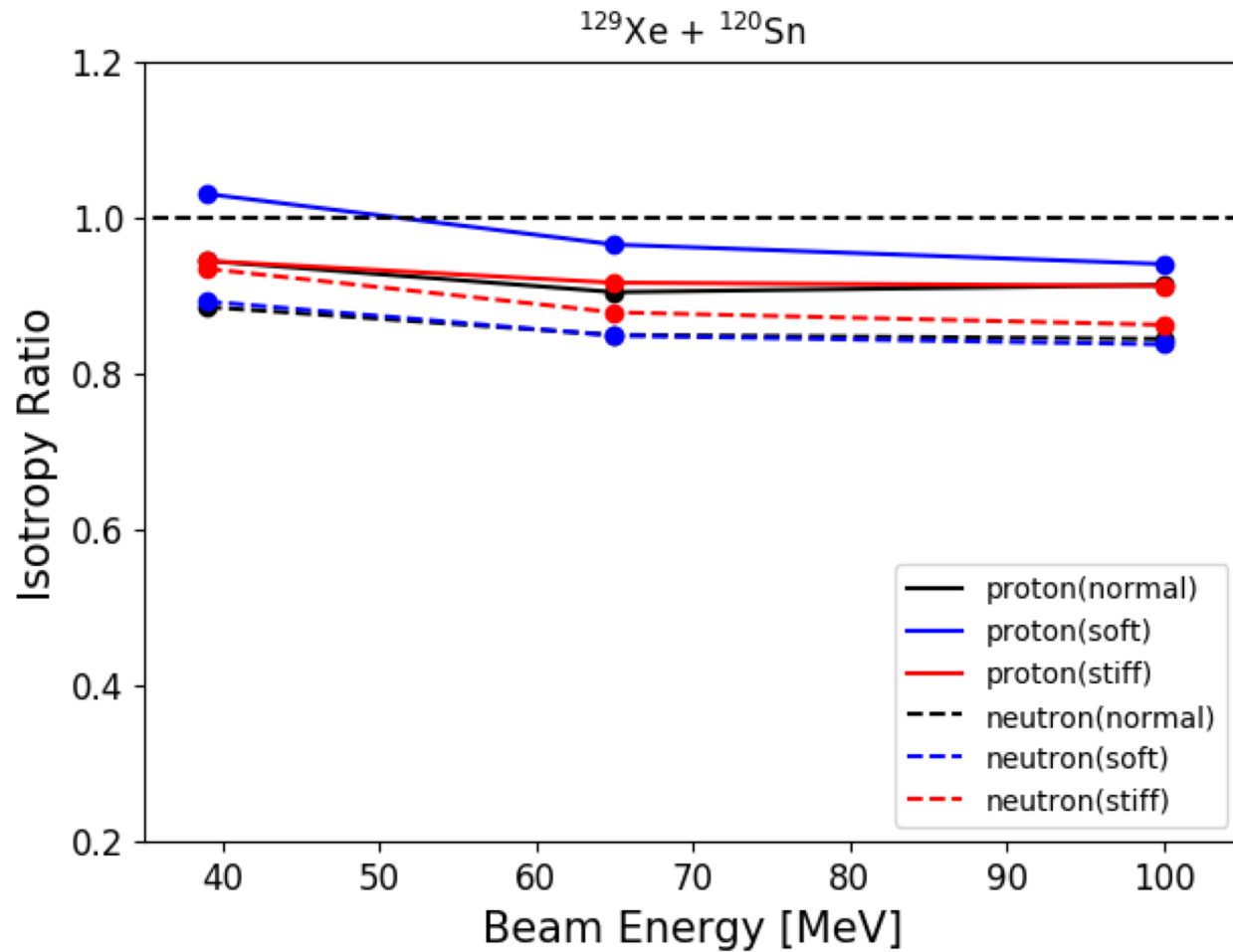


# Density

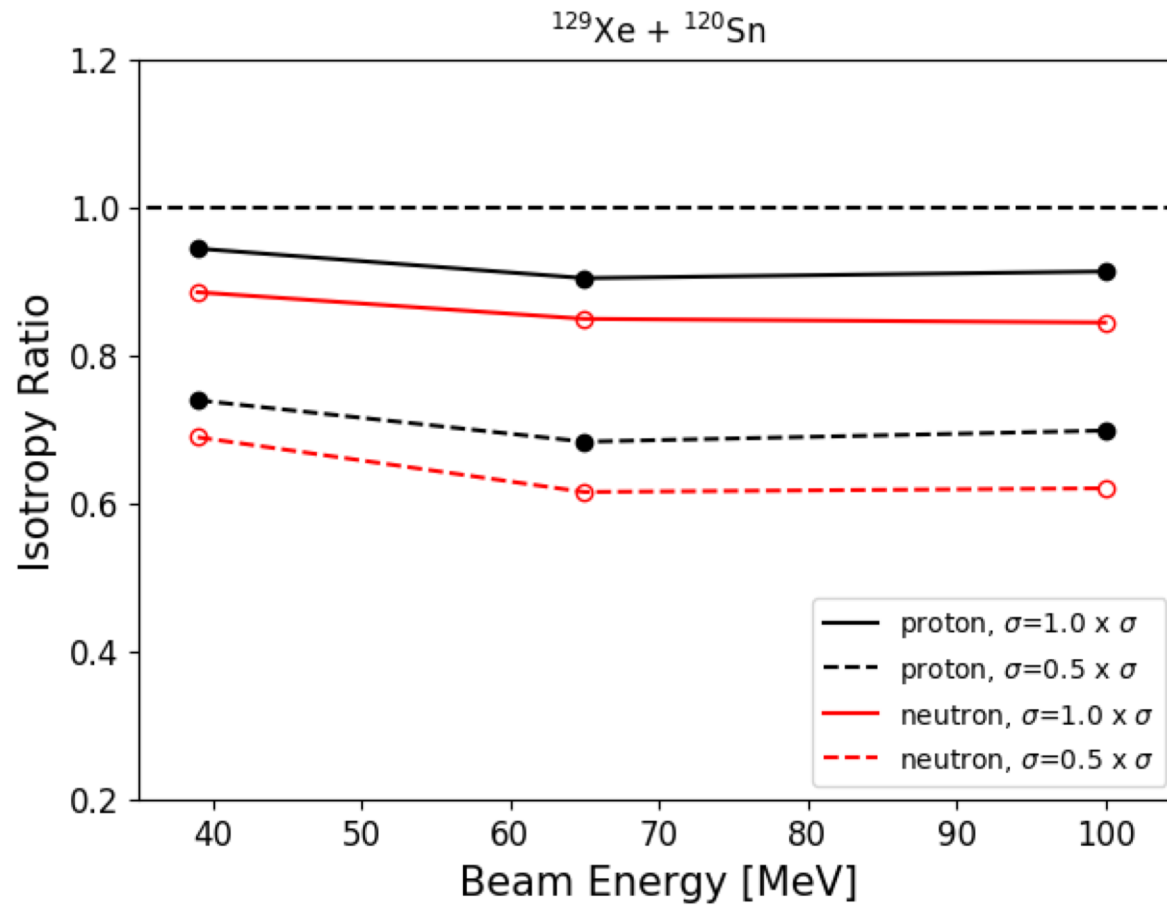


$$\delta_I = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

# Results



# Results



# Summary

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- We investigate the energy-based isotropy ratios of protons and neutrons in  $^{129}\text{Xe}+^{120}\text{Sn}$  with our QMD code.
- These ratios have a dependence of symmetry energy.
- As well as the potential, the in-medium NN cross-section gives large effect on the ratio.
- We have a plan to investigate the effect of nn- and pp-cross section splitting in asymmetric matter to the ratio.

***Thank you for your attention!!***