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Hyperons in nuclear matter

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“Hadrons in various environments”
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Motivation

To construct a theoretical framework which describes at same footing

the single nucleon properties

- in separate state considering it as a structure-full system
- in the community of their partners (possible structure changes)

as well as the properties of the whole nucleonic systems

- infinite nuclear matter properties (EOS, volume and symmetry energy properties)
- matter under extreme conditions (e.g. neutron stars)
- few/many (ordinary/exotic) nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)
- nucleon knock-out reactions (lepton-nucleus scattering experiments)
- possible changes in in-medium NN interactions
- etc

Motivation

How to construct a theoretical framework?

- the best way is to start from QCD and to arrive some an effective framework (it is not completely understood yet)
- therefore, as much as possible main peculiarities of QCD must be taken into account in arriving an effective theory or in constructing a phenomenological approach which describe the hadrons
- at low energies main peculiarities are
  - large Nc behaviour
  - chiral symmetry and its breaking
  - quark confinement
- in addition one should take into account the structure changes of nucleons in constructing the nuclear systems

Content

- Topological models
- Medium modifications
- Nucleons in nuclear matter
- SU(3) generalisations
- In-medium hyperons
- Summary

Topological models

Why topological models?

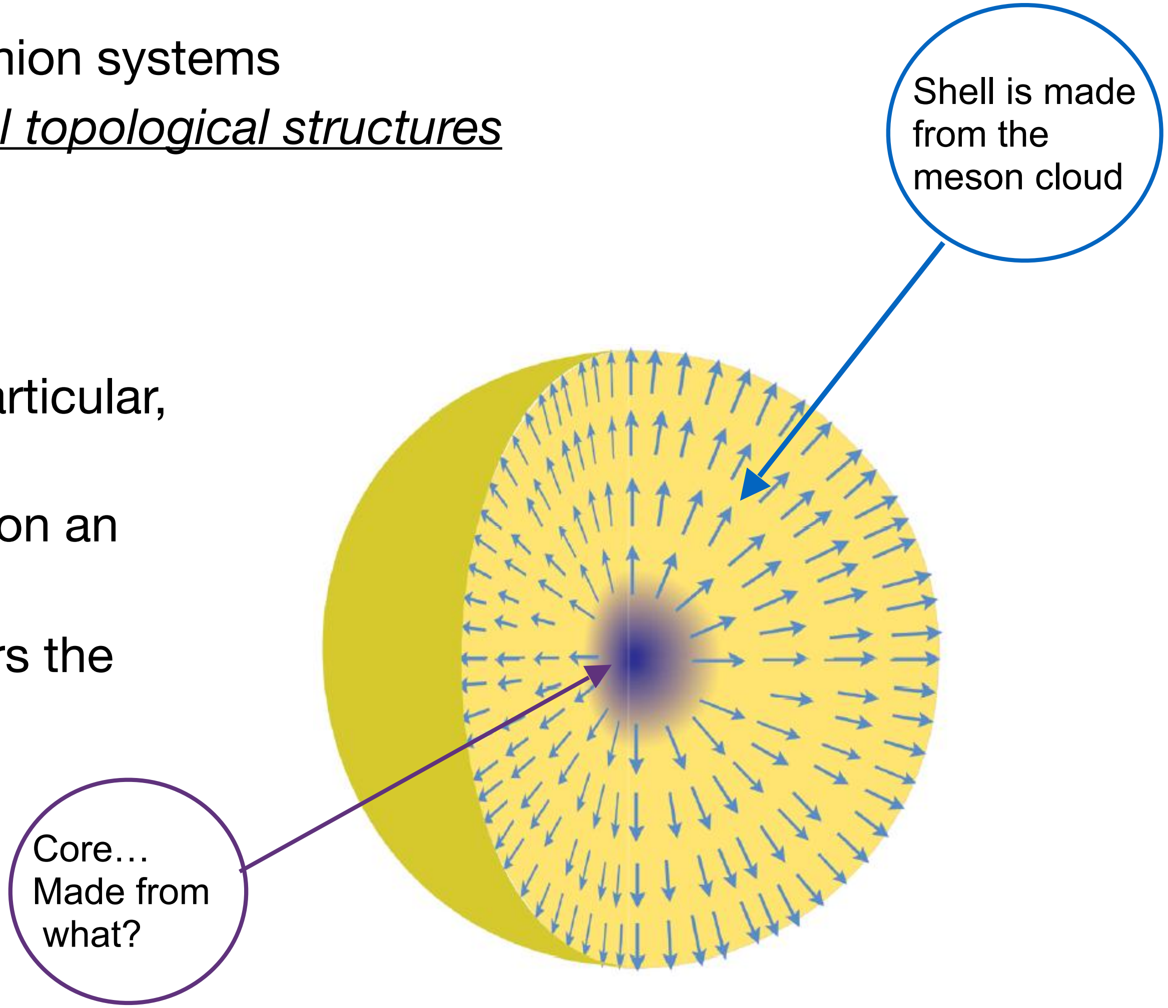
At fundamental level we may have

- fermions -> bosons are trivial fermion systems
- bosons -> fermions are *nontrivial topological structures*

Structure

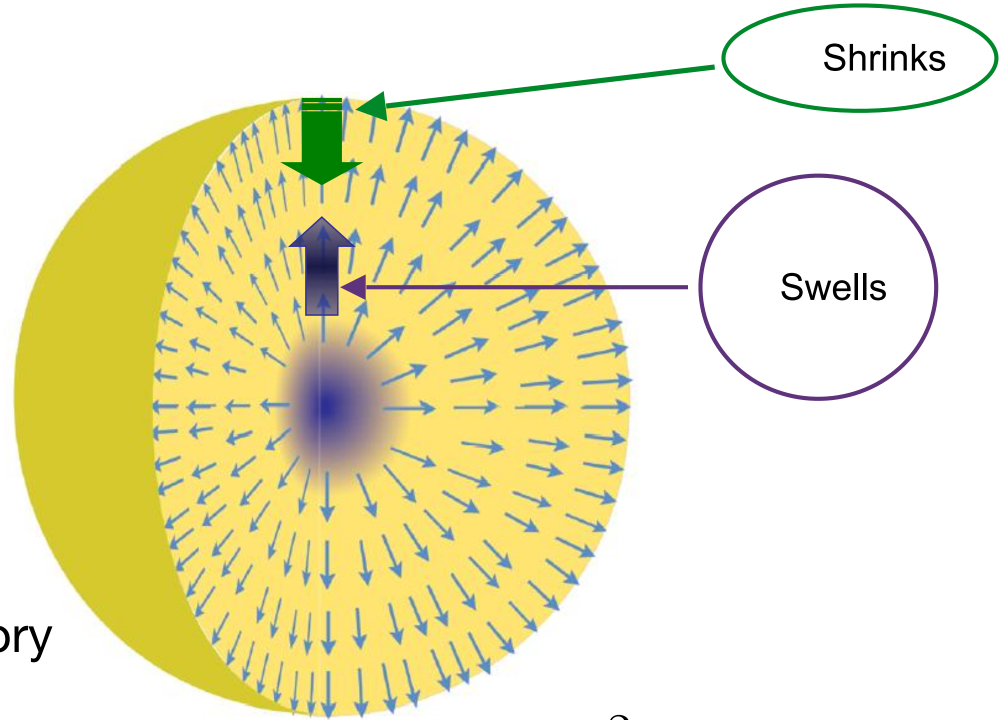
From what made a nucleon and, in particular, its core?

- The structure treatment depends on an energy scale
- At the limit of large number colours the core still has the mesonic content



Stabilisation mechanism

- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian



Prototype: **Skyrme model**

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2$$

Shrinking term — Swelling term

- *Hedgehog* solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\vec{\tau}\,\vec{\pi}}{2F_\pi}\right\} = \exp\{i\vec{\tau}\,\vec{n}F(r)\}$$

Topological models

The free space Lagrangian (was widely in use)
[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}\left(U + U^\dagger - 2\right)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) ***A***

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

- Nucleon is quantized state of the classical soliton-skyrmion

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|\,S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

What happens in the nuclear medium?

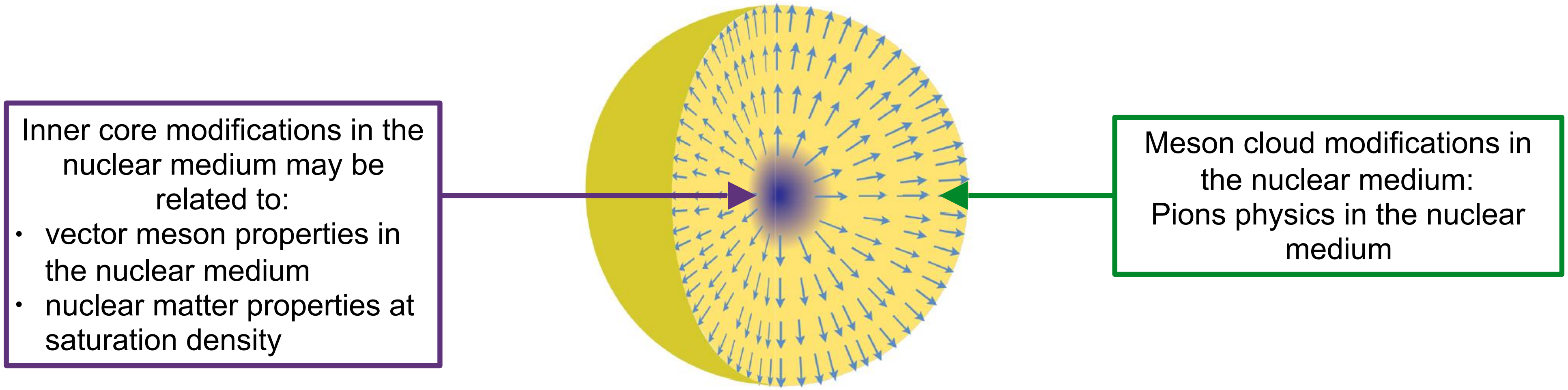
The possible medium effects

- Deformations (swelling or shrinking, multipole deformations) of nucleons
- Characteristic changes in: effective mass, charge distributions, all possible form factors
- NN interactions may change
- etc.

One should be able to describe all those phenomena

Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)



Medium modifications

“Outer shell” modifications

- In free space three types of pions can be treated separately: isospin breaking

$$(\partial^\mu \partial_\mu + m_\pi^2)\vec{\pi}^{(\pm,0)} = 0$$

- In nuclear matter: three types of polarization operators

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\rm opt} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Optic potential approach: parameters from the pion-nucleon scattering (including the isospin dependents)

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

"Outer shell" modifications in the Lagrangian [U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16} \alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16} \alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16} \alpha_m \mathrm{Tr}\left(2 - U - U^\dagger\right)$$

$$\hat{\Pi}^0 = 2\omega U_{\mathrm{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0) \vec{\nabla}$$

- Due to the nonlocality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters the following parts of the kinetic term are modified in different forms:
  - Temporal part
  - Space part

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
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g'	0.47	0.47

Medium modifications

"Inner core" modifications

[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau}\mathrm{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

may be related to

- Vector meson properties in nuclear matter
- Nuclear matter properties

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \text{parameters})$$

Medium modifications

Final Lagrangian
[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\text{sym}} + \mathcal{L}^*_{\text{asym}}$$

- Isoscalar part

$$\mathcal{L}^*_{\text{sym}} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_m$$

- Isovector part

$$\mathcal{L}^*_{\text{asym}} = \mathcal{L}^*_{\delta m} + \mathcal{L}^*_{\delta\rho}$$

- **Nuclear matter stabilization** → $\mathcal{L}^*_2$, $\mathcal{L}^*_4$
- **Asymmetric matter properties** → $\mathcal{L}^*_{\delta\rho}$

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\alpha_\tau \text{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \text{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}\text{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\text{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

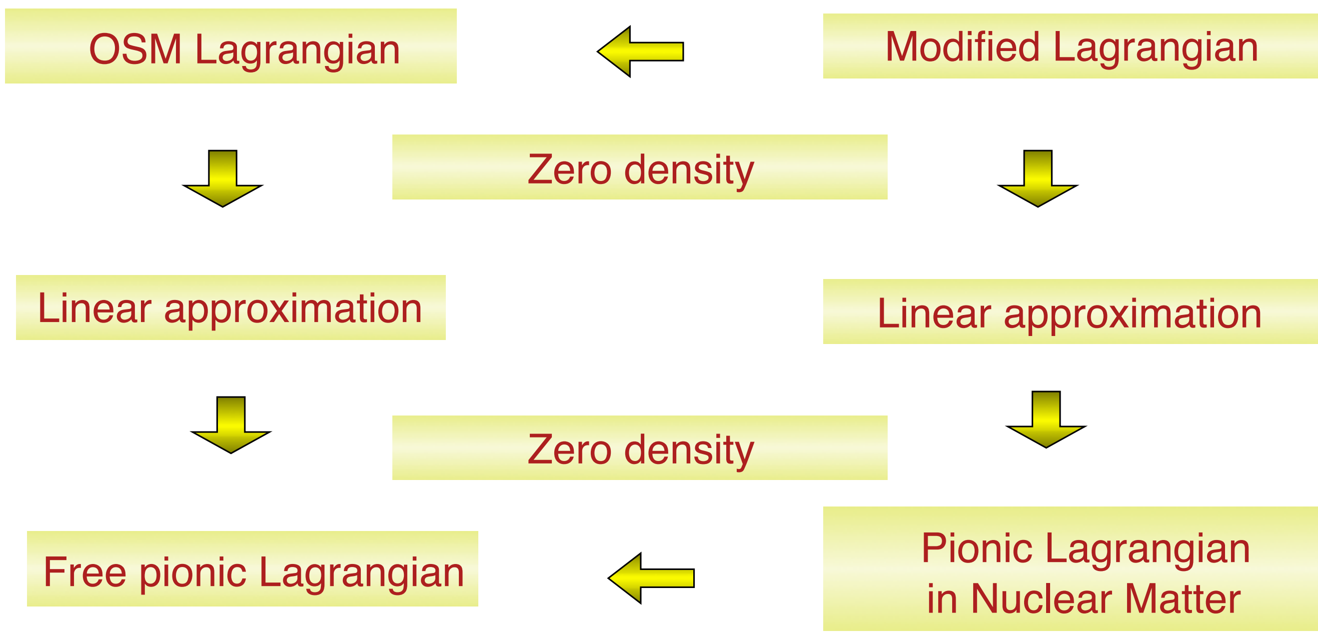
$$\mathcal{L}^*_m = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \text{Tr}\left(2 - U - U^+\right)$$

$$\mathcal{L}^*_{\delta m} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m^2_{\pi^\pm} - m^2_{\pi^0})\text{Tr}\left(\tau_a U\right)\text{Tr}\left(\tau_a U^\dagger\right)$$

$$\mathcal{L}^*_{\delta\rho} = -\frac{F_\pi^2}{16} m_\pi \alpha_e\ \varepsilon_{ab3}\text{Tr}\left(\tau_a U\right)\text{Tr}\left(\tau_b \partial_0 U^\dagger\right)$$

Medium modifications

- Modifications of the mesonic sector modifies the baryonic sector
- Lagrangian satisfies some limiting conditions



Medium modifications

Reparametrization
[UY, PRC88 (2013)]

- Five density dependent parameters

- Rearrangment (technical simplification to describe nuclear matter)

Shell modifications

Core modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad e_\tau \to e^*_\tau, \quad m_\pi \to m^*_\pi,$$

$$F_{\pi,s} \to F^*_{\pi,s}, \quad e_s \to e^*_s$$

$$+C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$+C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$+C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Medium modifications

In preliminary summary SU(2) version of present model describes at same footing

the single nucleon properties

- in separate state considering it as a structure-full system
- in the community of their partners (EM and EMT form factors)

as well as the properties of the whole nucleonic systems

- infinite nuclear matter properties (volume and symmetry energy properties)
- matter under extreme conditions (e.g. neutron stars)
- few/many nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)
- nucleon knock-out reactions (lepton-nucleus scattering)
- possible changes in in-medium NN interactions
- etc

SU(3) generalizations

In-medium modified form [UTY PRC88 (2013)]

$$\mathcal{L} = \alpha_1(\rho)\mathcal{L}_{WZ} - \frac{F_\pi^2}{16}\alpha_2^s(\rho)\mathrm{tr}(\partial_i U^\dagger \partial^i U) + \frac{F_\pi^2}{16}\alpha_2^t(\rho)\mathrm{tr}(\partial_0 U^\dagger \partial^0 U)$$
$$+ \frac{\alpha_4^s(\rho)}{32e^2}\mathrm{tr}[(\partial_i U)U^\dagger, (\partial_j U)U^\dagger]^2 - \frac{\alpha_4^t(\rho)}{16e^2}\mathrm{tr}[(\partial_0 U)U^\dagger, (\partial_i U)U^\dagger]^2 + \frac{F_\pi^2}{16}\alpha_{\chi SB}(\rho)\mathrm{tr}M(U + U^\dagger - 2)$$

$\alpha_1(\rho), \alpha_2^{s,t}(\rho), \alpha_4^{s,t}(\rho), \alpha_{\chi SB}(\rho)$: They describe an influence of surrounding environment to the pion properties

U is defined as $U = \mathcal{A}U_0\mathcal{A}^\dagger$, where $\mathcal{A}(t)$ is a rotational matrix and U_0 is a unitary matrix describing the mesonic mean fields.

Hedgehog ansatz:

$$U_0 = \begin{bmatrix} e^{2i\frac{\vec{\pi}\cdot\vec{\tau}}{f_\pi}} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{iF(r)\vec{\tau}\cdot\hat{n}} & 0 \\ 0 & 1 \end{bmatrix}$$

$F(r)$ is a profile function satisfying boundary conditions $F(r) = \pi$ at $r = 0$ and $F(r) = 0$ at $r \to \infty$.

SU(3) Generalizations

Minimal embedding

Time dependent Rotations: $$\mathcal{A}(t) = \begin{pmatrix} A(t) & 0 \\ 0^\dagger & 1 \end{pmatrix} S(t)$$

$$A(t) = k_0(t)\mathbf{1} + i\sum_{a=1}^{3} \tau_a k_a(t),$$

$$S(t) = \exp\left\{ i\sum_{p=4}^{7} k_p\lambda_p \right\}$$

$$\equiv \exp(i\mathcal{D}) = \exp\left\{ \begin{pmatrix} 0 & i\sqrt{2}D \\ i\sqrt{2}D^\dagger & 0 \end{pmatrix} \right\},$$

$$D(t) = \frac{1}{\sqrt{2}} \begin{pmatrix} k_4(t) - ik_5(t) \\ k_6(t) - ik_7(t) \end{pmatrix}.$$

D. B. Kaplan and I. R. Klebanov, Nucl. Phys. B 335, 45 (1990)

Quantization & Hamiltonian

Conjugate Momenta	Commutation Relations
$(J_{ud})_i = \frac{\partial L}{\partial \omega^i}$ $\Pi^\gamma = \frac{\partial L}{\partial \dot{D}^\dagger_\gamma}$ $\Pi^\dagger_\gamma = \frac{\partial L}{\partial \dot{D}^\gamma}$	$[(J_{ud})_i, \omega^j] = \frac{1}{i}\delta^j_i$ $[\Pi^\gamma, D^\dagger_\beta] = [\Pi^\dagger_\beta, D^\gamma] = \frac{1}{i}\delta^\gamma_\beta.$

$$M_0^2 \equiv \frac{N^2}{16\Phi\Gamma}$$

$$D = \frac{1}{\sqrt{N}}\left(1 + \frac{m_K^2}{M_0^2}\right)^{-1/4}(a + b^\dagger)$$

$$\Pi^\dagger = \frac{i}{2}\sqrt{N}\left(1 + \frac{m_K^2}{M_0^2}\right)^{1/4}(a^\dagger - b)$$

$$H = E_0^* + \left(\Gamma^* m_K^2 + \frac{N^2\alpha_1^2(\rho)}{16\Phi^*}\right) D^\dagger D - \frac{iN\alpha_1(\rho)}{8\Phi^*}(D^\dagger\Pi - \Pi^\dagger D)$$
$$\frac{1}{2\Omega^*}\left\{\vec{J}_{ud,\alpha} + \frac{i}{2}(D^\dagger\vec{\sigma}\Pi - \Pi^\dagger\vec{\sigma}D)\left(1 - \frac{\Omega^*}{2\Phi^*}\right) + \frac{N\alpha_1(\rho)\Omega^*}{4\Phi^*}D^\dagger\vec{\sigma}D\right\}^2.$$

SU(3) Generalizations

➢ Eigenstate : $|n_s, n_{\bar{s}}\rangle |I, J\rangle$

➢ Eigenvalues of Operators

$$S = b^\dagger b - a^\dagger a \qquad \vec{J}_s = \frac{1}{2}(a^\dagger \vec{\sigma} a - b \vec{\sigma} b^\dagger)$$

$$\vec{J}_{ud}^2 = I(I+1), \qquad \vec{J}^2 = J(J+1) \qquad , \vec{J}_s^2 = \frac{1}{4}(a^\dagger a)^2 + \frac{1}{2}a^\dagger a = \frac{1}{4}S^2 - \frac{1}{2}S = \frac{1}{4}S(S-2)$$

$$\vec{J} = \vec{J}_{ud} + \vec{J}_s, \qquad 2\vec{J}_{ud} \cdot \vec{J}_s = \vec{J}^2 - \vec{J}_{ud}^2 - \vec{J}_s^2$$

$a^\dagger$ and $b^\dagger$ are creation operators of strange quark and anti strange quark, respectively

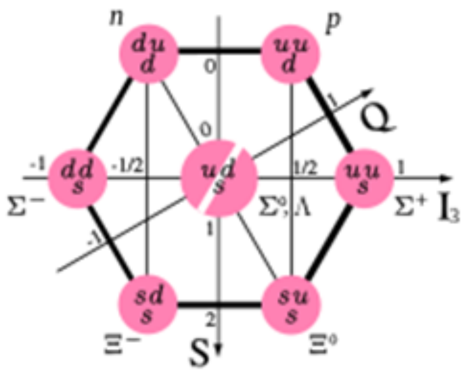
Masses of multiplets

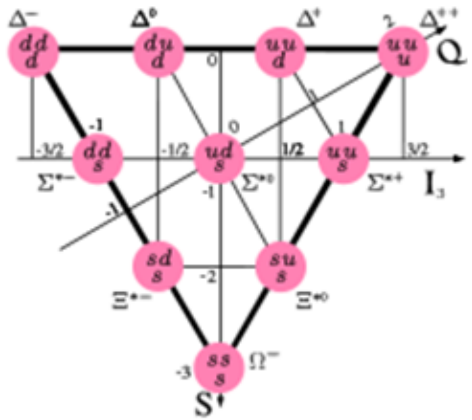
$$F_\pi m_K \to F_K^* m_K^* = F_\pi m_K (1 - C\lambda)$$

$$M = f_1(\lambda)E_0^* - S\omega_-^* + \frac{f_3(\lambda)}{2\Omega}\left\{c^* J(J+1) + (1-c^*)I(I+1) + \frac{c^*(c^*-1)}{4}S(S-2)\right\}$$

$$\omega_-^* = \frac{N\alpha_1(\rho)f_3(\lambda)}{8\Phi}\left(\sqrt{1 + \frac{16 f_1(\lambda) f_2^2(\lambda)\Phi\Gamma}{N^2\alpha_1^2(\rho)f_3(\lambda)}m_K^2} - 1\right)$$

$$c^* = 1 - \frac{4\Omega\omega_-^*}{8\Phi\omega_-^* + N\alpha_1(\rho)f_3(\lambda)}$$





- Mass splitting within the representation : [blue box]
- Octet & Decuplet Splitting : [red box]

In-medium hyperons

Skyrmion functionals	Free space values	Values at $\rho = \rho_0$ $C = 0$	$C = 0.2$
E_0^* [MeV]	865.60	665.04	665.04
Ω^* [MeV^{-1}]	5.116×10^{-3}	1.453×10^{-3}	1.453×10^{-3}
Φ^* [MeV^{-1}]	1.852×10^{-3}	5.000×10^{-4}	5.000×10^{-4}
Γ^* [MeV^{-1}]	3.995×10^{-3}	5.442×10^{-3}	5.442×10^{-3}
ω_-^* [MeV]	213.63	241.98	162.29
c^*	0.291	0.646	0.742

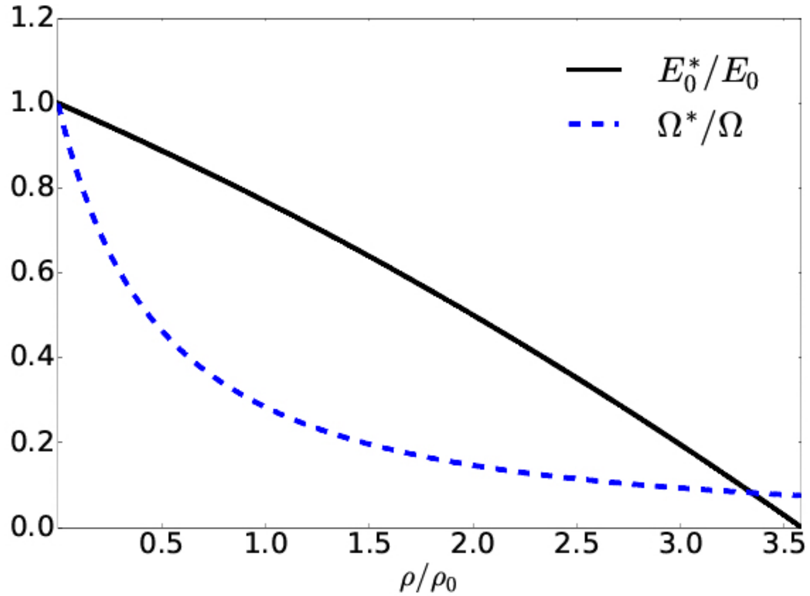
Table.1 : We list the values of the density-dependent skyrmion functionals at normal nuclear matter density ρ0, comparing them with those in free space.

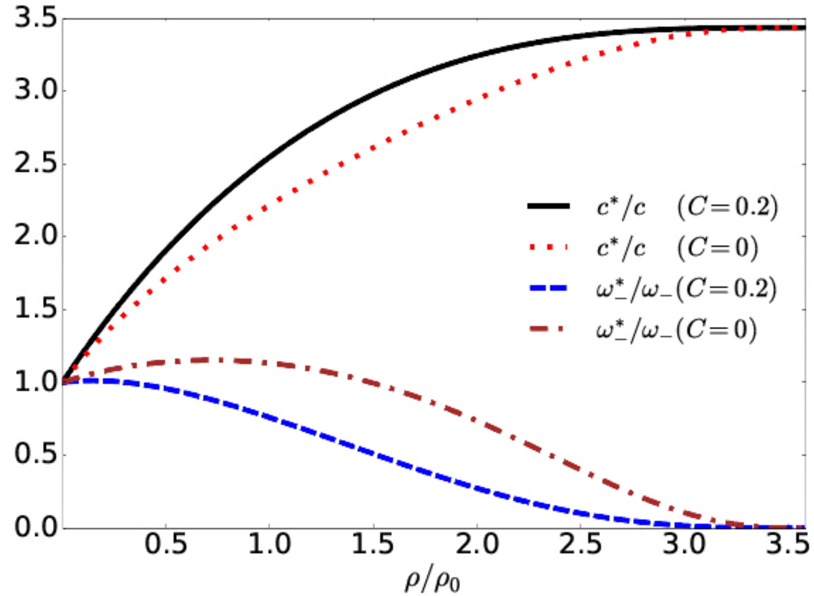
Baryon	Experimental mass	Free space mass	Mass at $\rho = \rho_0$ $C = 0$	$C = 0.2$
N	939	939^*	923^*	923^*
Λ	1115	1085	1015	969
Σ	1189	1224	1259	1147
Ξ	1315	1326	1250	1116
Δ	1232	1232^*	1956	1956
Σ^*	1385	1309	1925	1913
Ξ^*	1530	1411	1916	1881
Ω	1672	1537	1929	1862

Table.2 : Results of the masses of the baryon octet and decuplet both in free space and in nuclear matter at the saturation density ρ0 compared with the corresponding experimental data.

[K.H. Hong, UY, H.Ch.Kim, arXiv:1806.06504]

Tendency of Skyrmion Functionals





The classical mass and moments of inertia are decreasing and not affected by modified kaon mass. But other quantities are sensitively affected by modified kaon mass.

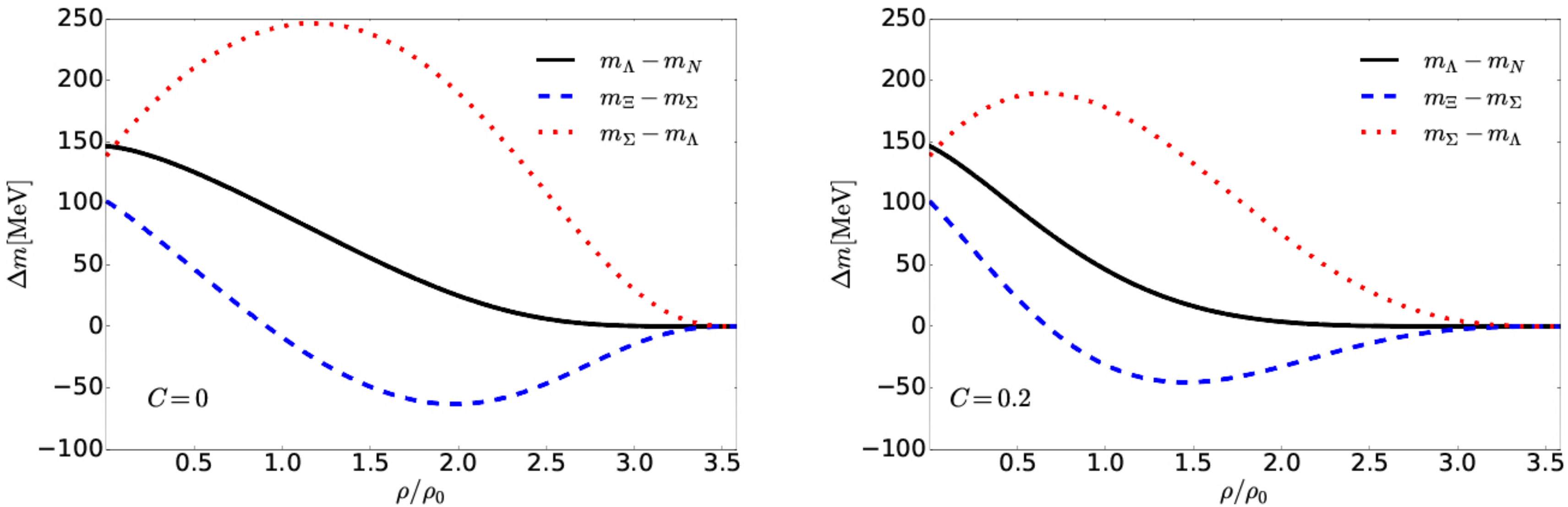
[K.H. Hong, UY, H.Ch.Kim, arXiv:1806.06504]

Mass splitting of the Baryon Octet Particles

$$m^*_\Sigma - m^*_\Lambda = \frac{1-c^*}{\Omega^*},$$

$$m^*_\Xi - m^*_\Sigma = \omega^*_- + \frac{5(c^*-1)(c^*+1)}{8\Omega^*},$$

$$m^*_\Lambda - m^*_N = \omega^*_- + \frac{3(c^*-1)(c^*+1)}{8\Omega^*},$$

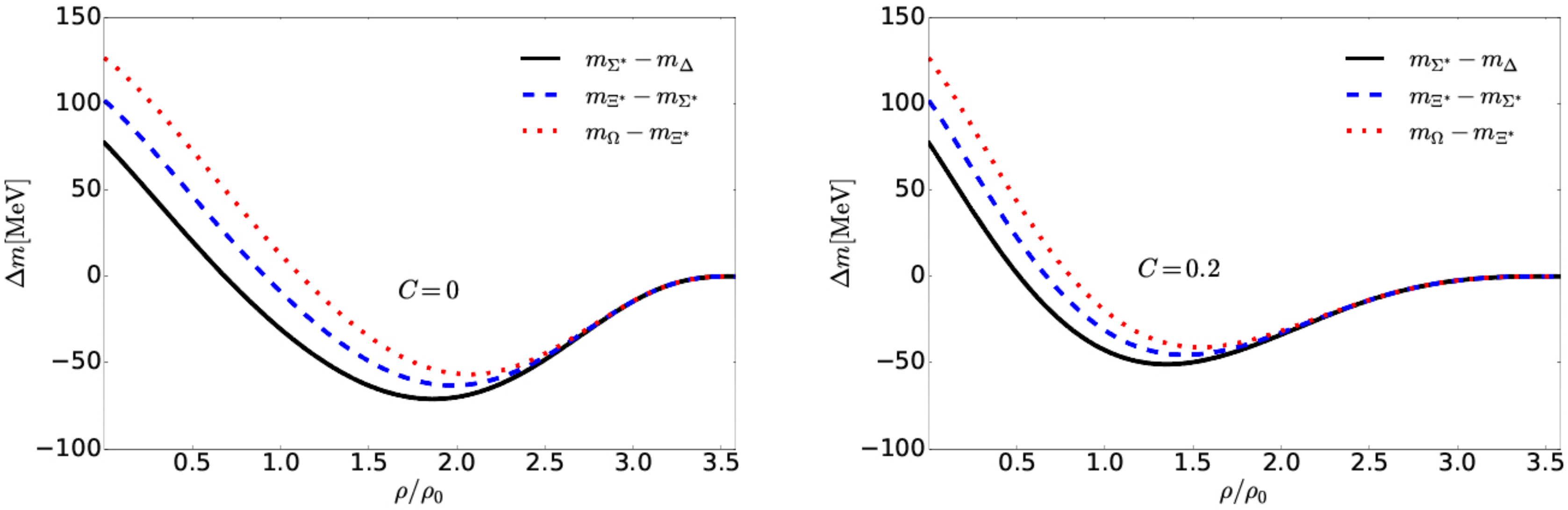


The changes of mass splitting of the baryon octet particles according to the density of nuclear matter

[K.H. Hong, UY, H.Ch.Kim, arXiv:1806.06504]

Mass splitting of the Baryon Decuplet Particles

$$m^*_{\Sigma^*} - m^*_{\Delta} = \omega^*_- + \frac{(c^*-1)(3c^*+7)}{8\Omega^*},$$
$$m^*_{\Xi^*} - m^*_{\Sigma^*} = \omega^*_- + \frac{5(c^*-1)(c^*+1)}{8\Omega^*},$$
$$m^*_{\Omega} - m^*_{\Xi^*} = \omega^*_- + \frac{(1+c^*)(7c^*-3)}{8\Omega^*}$$



The changes of mass splitting of the baryon decuplet particles according to the density of nuclear matter

[K.H. Hong, UY, H.Ch.Kim, arXiv:1806.06504]

Summary

Applicability and extensions of the approach so far

- Nucleon tomography in free space/nuclear medium
  - [H.Ch. Kim, P. Schweitzer, UY, PLB718 (2012)]
  - [H.Ch. Kim, UY, PLB726 (2013)]
  - [J.H.Jung, UY, H.Ch.Kim, Jour. Phys. G41 (2014)]
  - [J.H.Jung, UY, H.Ch.Kim, P. Schweitzer. PRD89 (2014)]
- Nucleon properties in asymmetric nuclear matter
  - [UY, Prog. Theor. Exp. Phys. 2014 (2014)]
- Isospin symmetric/asymmetric nuclear matter
  - [UY, PRC88 (2013)]
- Neutron stars
  - [UY, PLB749 (2015)]
- Vector mesons in nuclear matter
  - [J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013)]
- Hyperons in nuclear matter
  - [K.H. Hong, UY, H.Ch.Kim, arXiv:1806.06504]

Thank you very much for your attention!