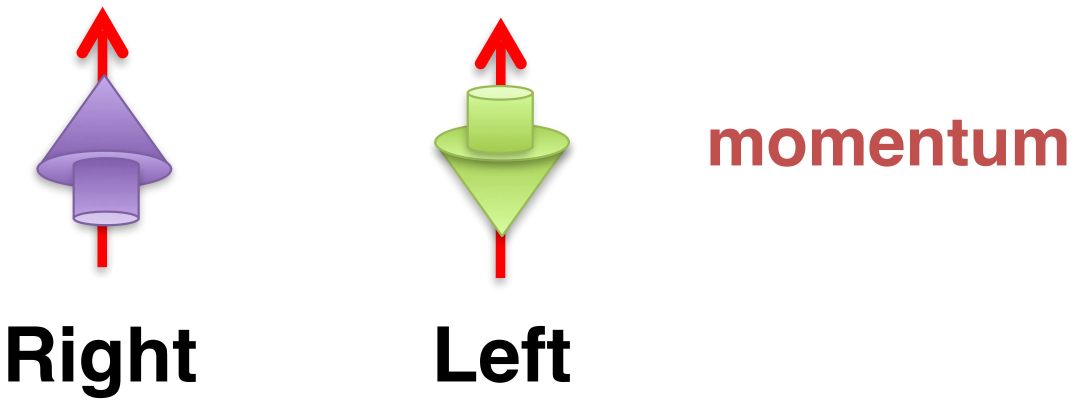
Non-dissipative transport effects induced by chiral anomaly in heavy-ion collisions

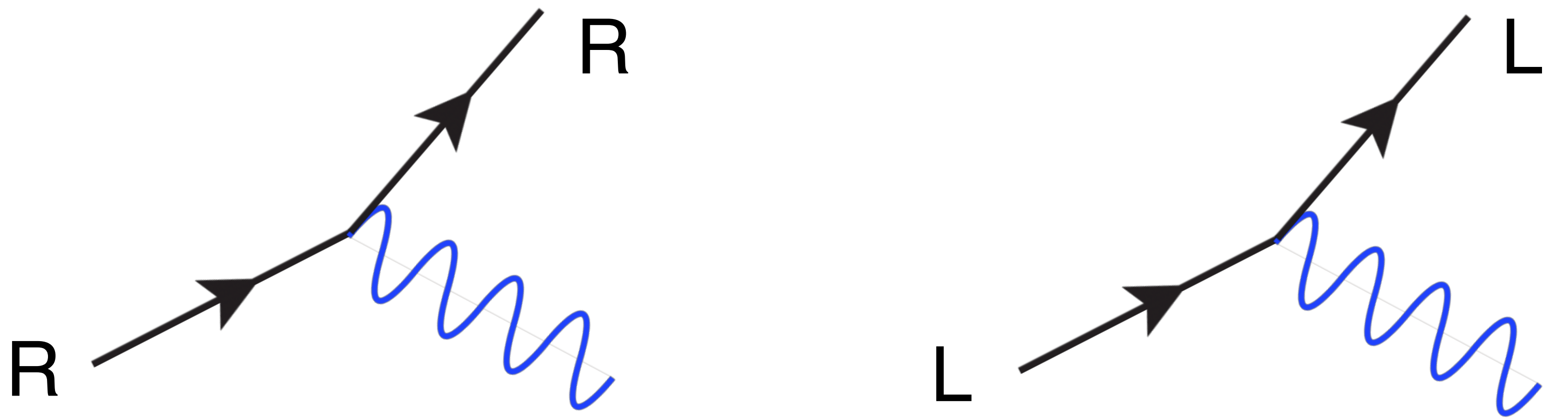
Yuji Hirono

apctp

Chiralities of massless fermions



- Chirality does not change through interaction classically



- **Quantum effects** breaks the chirality conservation

Chiral anomaly **[Adler, Bell-Jackiw (1969)]**

Chiral anomaly

Axial current $j_{\mathrm{A}}^{\mu} \equiv j_{\mathrm{R}}^{\mu} - j_{\mathrm{L}}^{\mu}$

$$\partial_\mu j_{\mathrm{A}}^{\mu} = C_{\mathrm{A}} \boldsymbol{E} \cdot \boldsymbol{B}$$

$$C_{\mathrm{A}} = \frac{e^2}{2\pi^2}$$

Chiral Magnetic Effect (CME)

[Kharzeev-Warringa-Mclerran (2007)]

$$\boldsymbol{j}_{\mathrm{CME}} = C_{\mathrm{A}} \mu_{\mathrm{A}} \boldsymbol{B}$$

- Macroscopic transport
- Dissipationless (no heat production)
- Transport coefficient is **universal**

- Where does it happen?
  - Heavy-ion collisions
  - 3D Dirac semimetals

LETTERS
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nature physics

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

TOPOLOGICAL MATTER

SCIENCE sciencemag.org

Evidence for the chiral anomaly in the Dirac semimetal Na_3Bi

Jun Xiong,[1] Satya K. Kushwaha,[2] Tian Liang,[1] Jason W. Krizan,[2] Max Hirschberger,[1] Wudi Wang,[1] R. J. Cava,[2] N. P. Ong[1*]

Giant negative magnetoresistance induced by the chiral anomaly in individual Cd_3As_2 nanowires

nature COMMUNICATIONS

Cai-Zhen Li[1,*], Li-Xian Wang[1,*], Haiwen Liu[2], Jian Wang[2,3], Zhi-Min Liao[1,3] & Da-Peng Yu[1,3]

Signatures of the Adler–Bell–Jackiw chiral anomaly in a Weyl fermion semimetal

Cheng-Long Zhang[1,*], Su-Yang Xu[2,*], Ilya Belopolski[2,*], Zhujun Yuan[1,*], Ziquan Lin[3], Bingbing Tong[1], Guang Bian[2], Nasser Alidoust[2], Chi-Cheng Lee[4,5], Shin-Ming Huang[4,5], Tay-Rong Chang[2,6], Guoqing Chang[4,5], Chuang-Han Hsu[4,5], Horng-Tay Jeng[6,7], Madhab Neupane[2,8,9], Daniel S. Sanchez[2], Hao Zheng[2], Junfeng Wang[3], Hsin Lin[4,5], Chi Zhang[1,10], Hai-Zhou Lu[11], Shun-Qing Shen[12], Titus Neupert[13], M. Zahid Hasan[2] & Shuang Jia[1,10]

nature COMMUNICATIONS

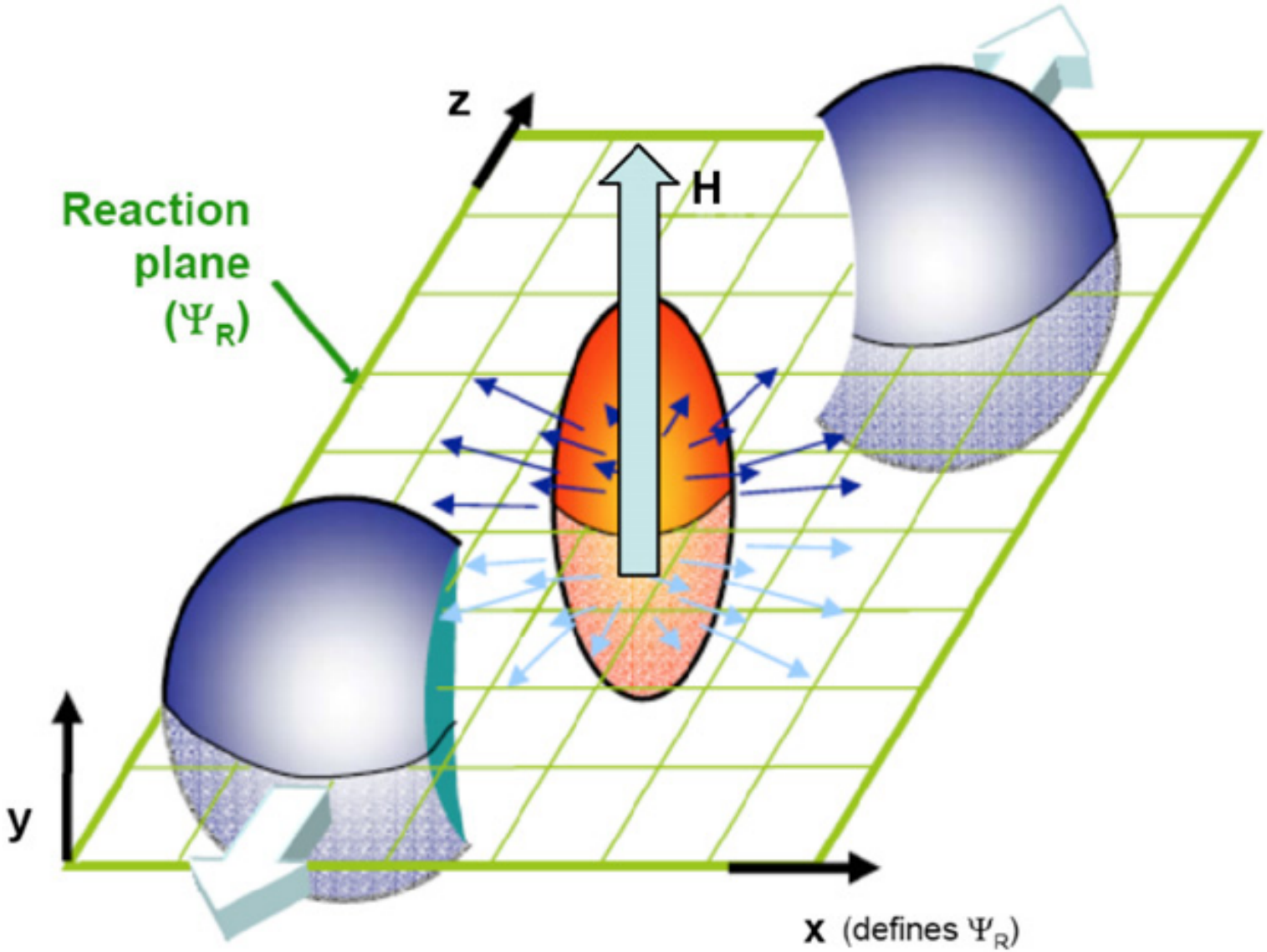
Chiral magnetic effect in heavy-ion collisions

CME in heavy-ion collisions?

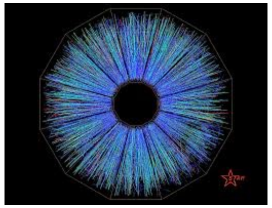
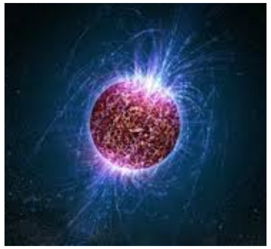
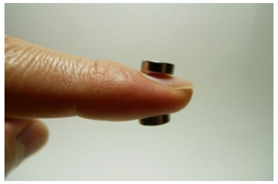
$$\boldsymbol{j}_{\rm CME} = C_{\rm A}\mu_{\rm A}\boldsymbol{B}$$

- Magnetic field
- Chirality imbalance

Magnetic field in heavy-ion collisions



Comparison of B field strength

	**Heavy-ion collisions**	**10^17 Gauss**
	Magnetors Neutron stars	10^15 Gauss 10^13 Gauss
	Neodymium magnet	5000 Gauss
	Earth’s magnetic field	0.6 Gauss

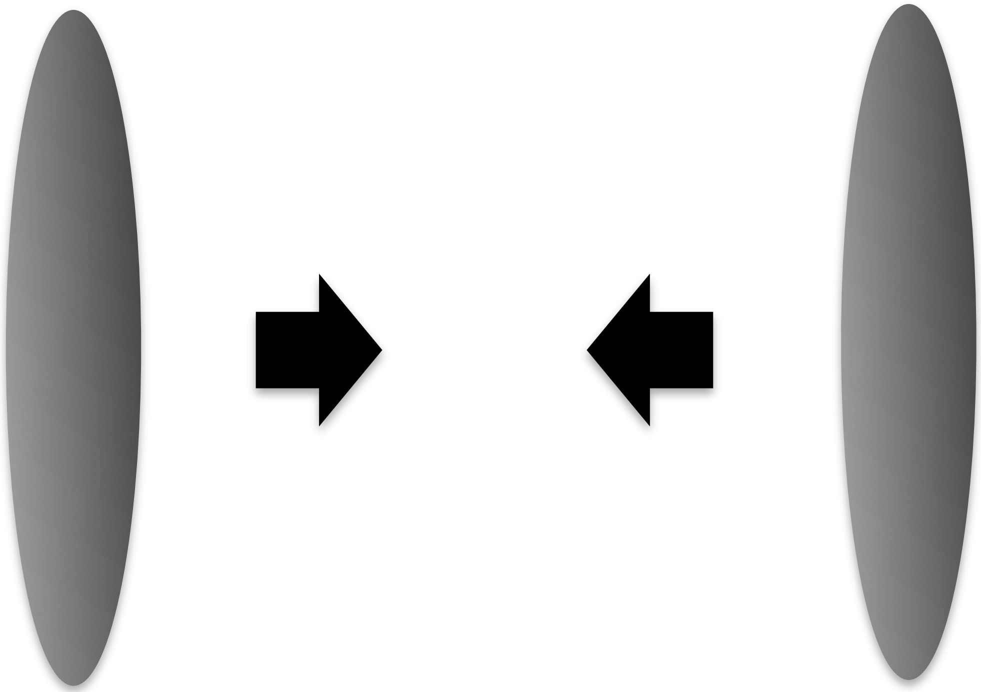
CME in heavy-ion collisions?

$$j_{\mathrm{CME}} = C_{\mathrm{A}} \mu_{\mathrm{A}} \boldsymbol{B}$$

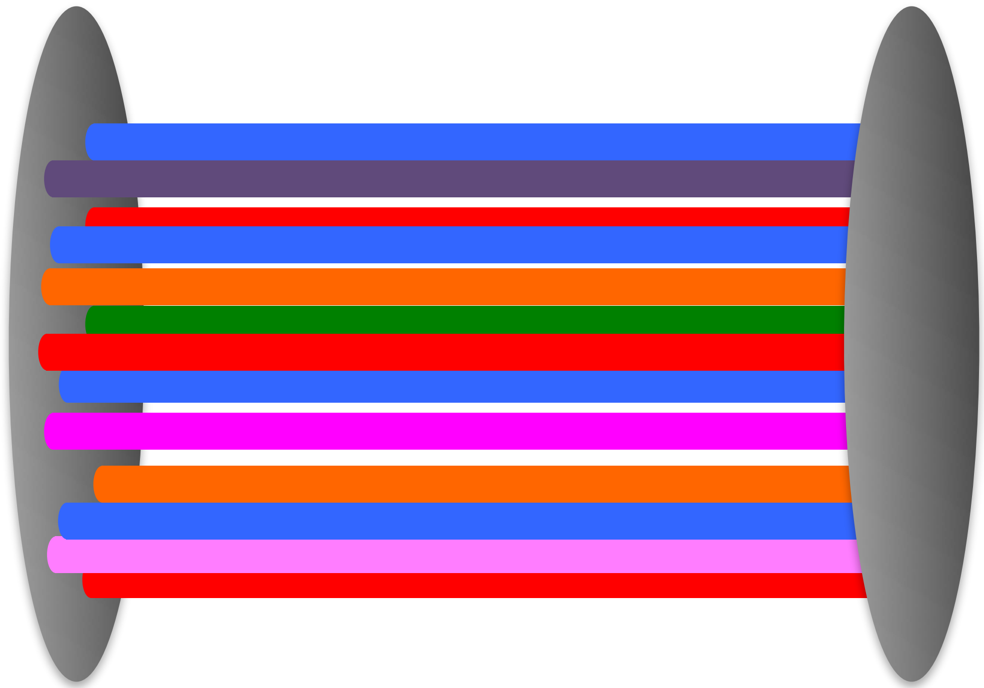
- Magnetic field

  Strong B field from electric charges

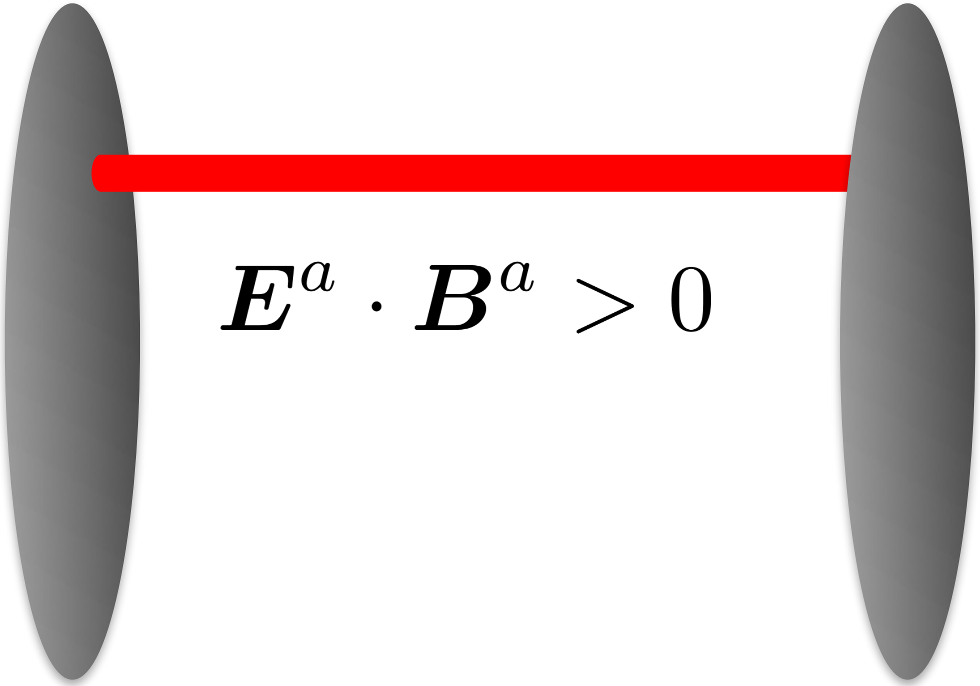
- Chirality imbalance



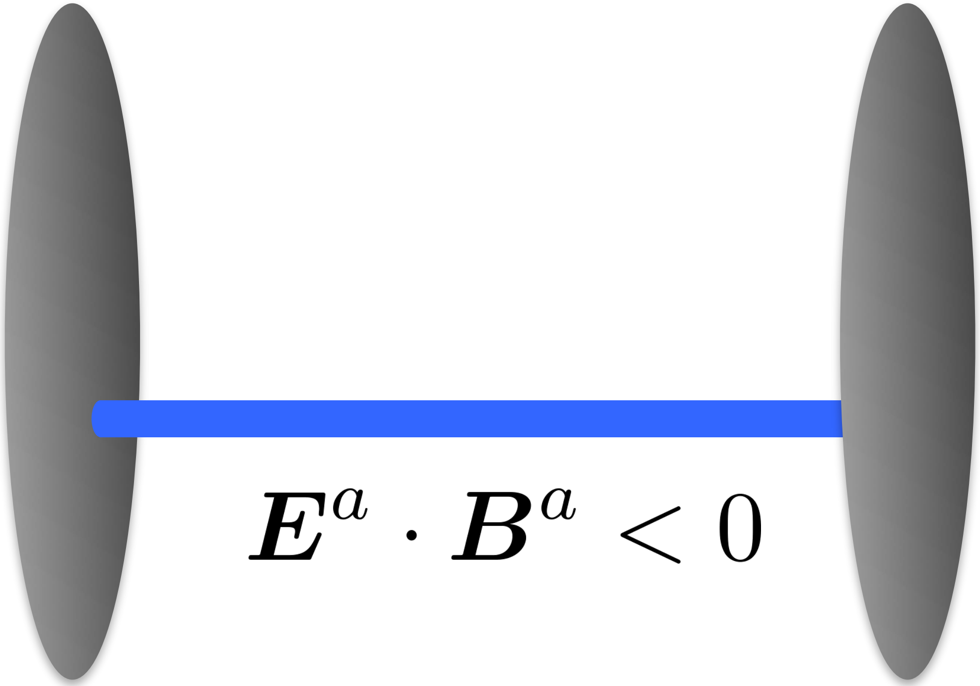
$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$



$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$



$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} \boldsymbol{E}^a \cdot \boldsymbol{B}^a$$

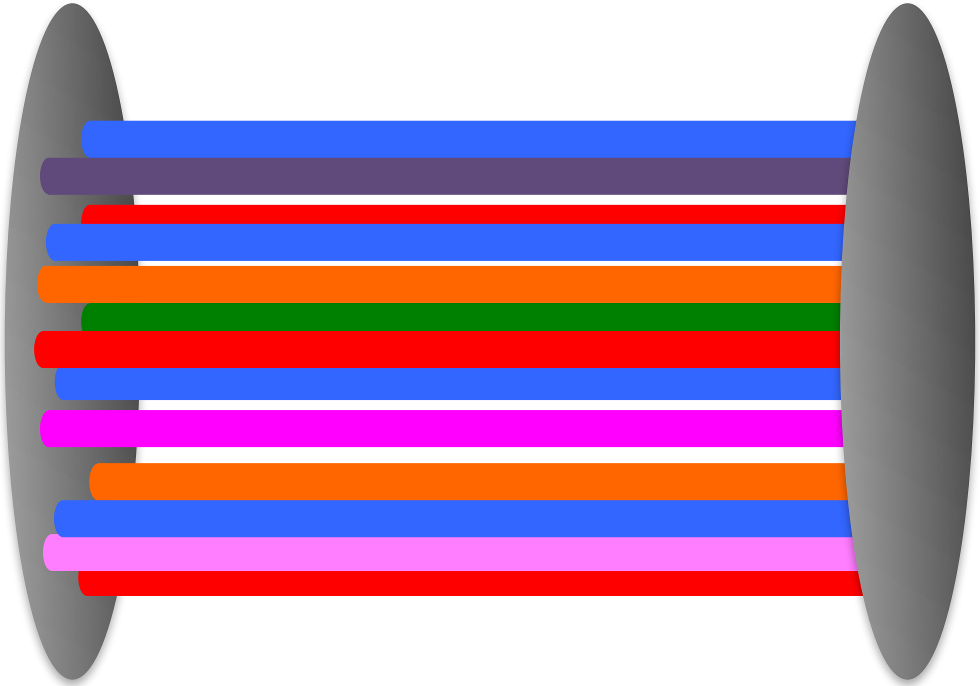


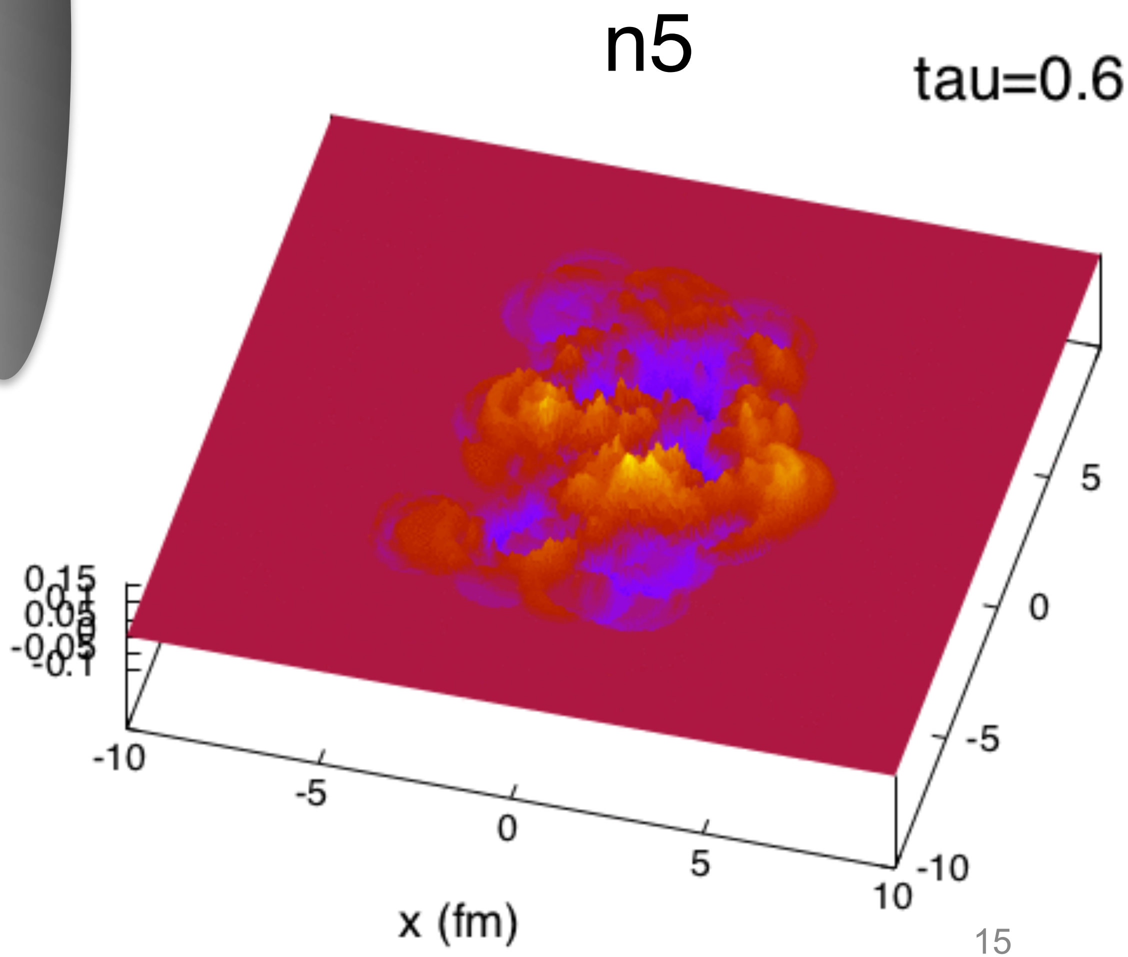
Chirality imbalance from color flux tubes

[Kharzeev-Krasnitz-Venugopalan ’02]

[Mace-Schlichting-Venugopalan ’16]

………

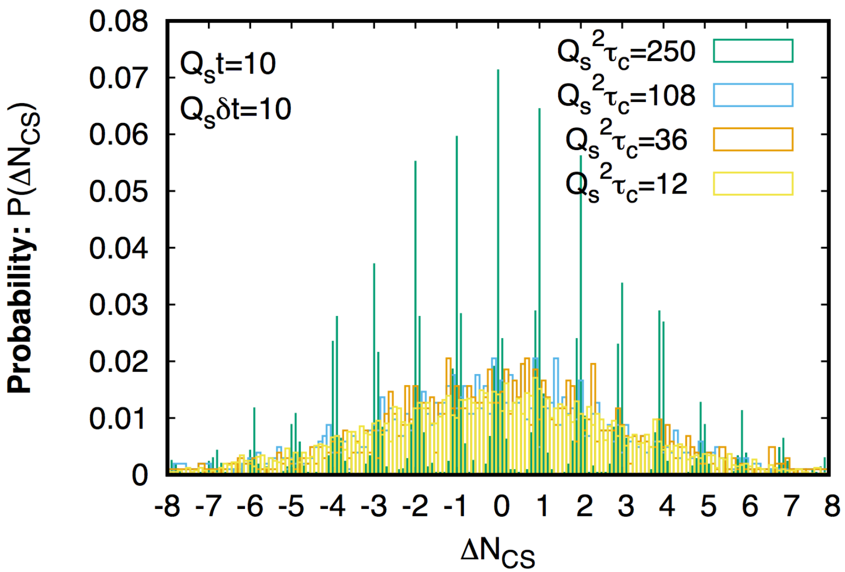




[Hirono-Hirano-Kharzeev ’14]

Enhanced n5 generation in glasma

[Mace-Schlichting-Venugopalan PRD'16]



Probability dist. of ΔN_{CS}

Enhanced sphaleron rate in non-eq

→ **Efficient n5 generation**

CME in heavy-ion collisions?

$$j_{\mathrm{CME}} = C_{\mathrm{A}} \mu_{\mathrm{A}} \boldsymbol{B}$$

- Magnetic field

  Strong B field from electric charges

- Chirality imbalance

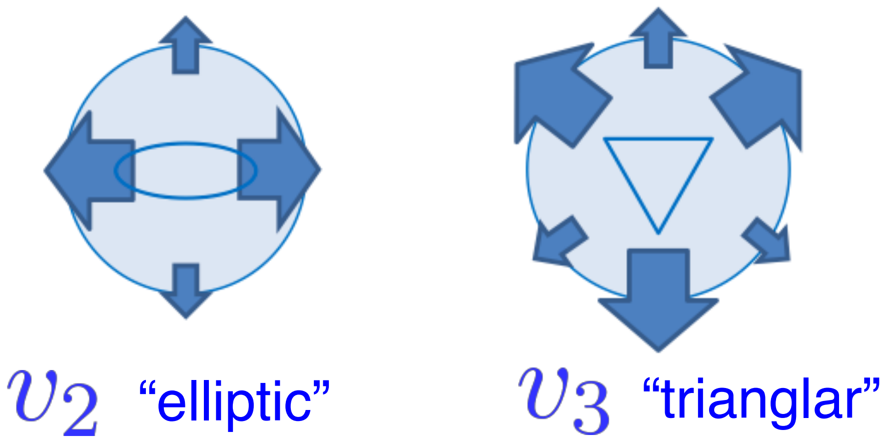
  Created from color fields (glasma)

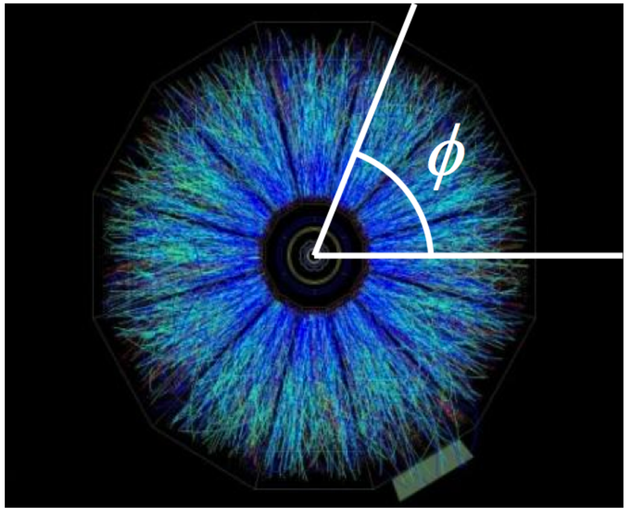
Harmonics v_n

- Azimuthal angle distribution of observed particles

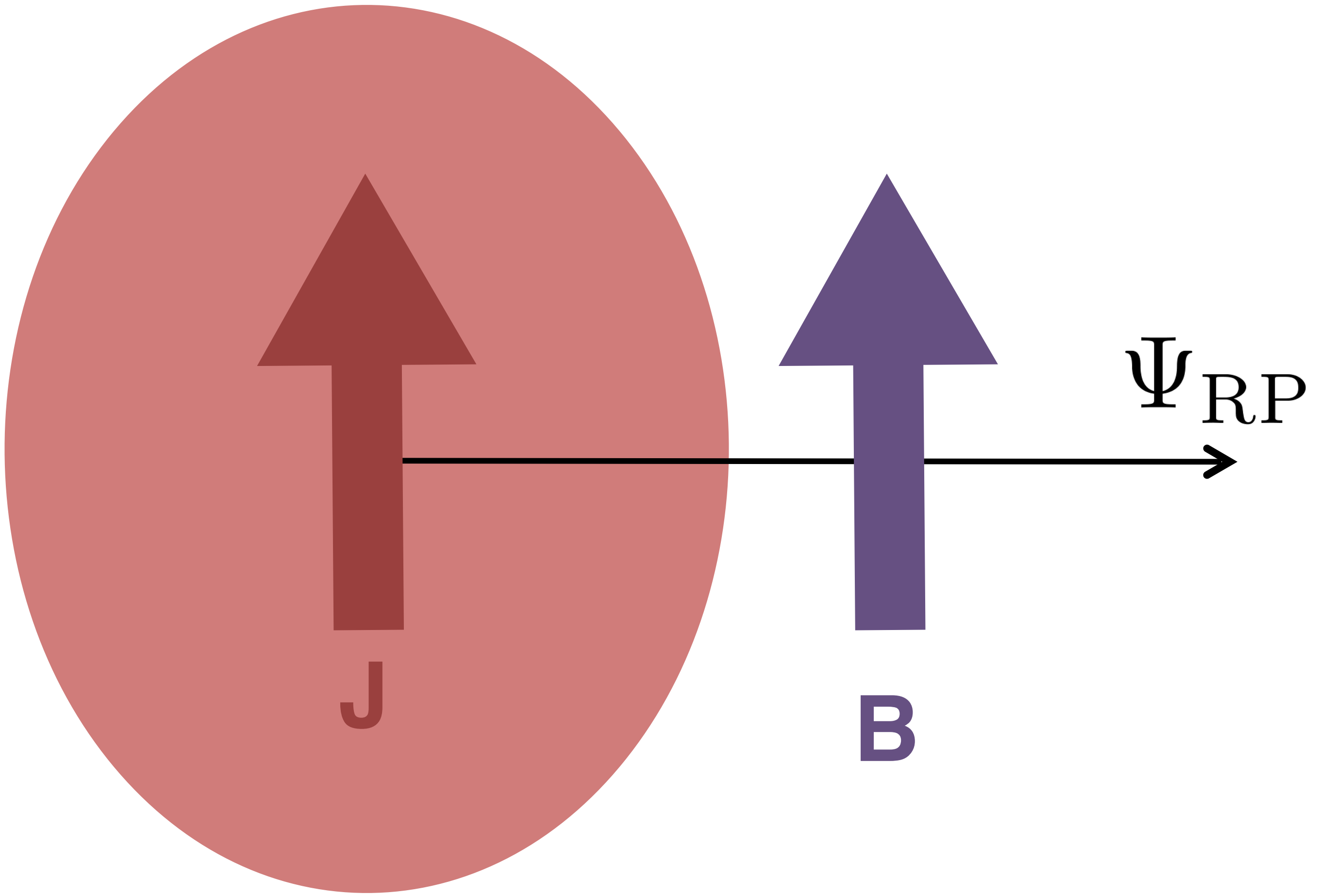
$$\frac{dN}{d\phi} = \bar{N}\left[1 + \sum_{n=1}^{\infty} 2 v_n \cos n(\phi - \Psi_n)\right]$$

- Reflects the shape of the flow



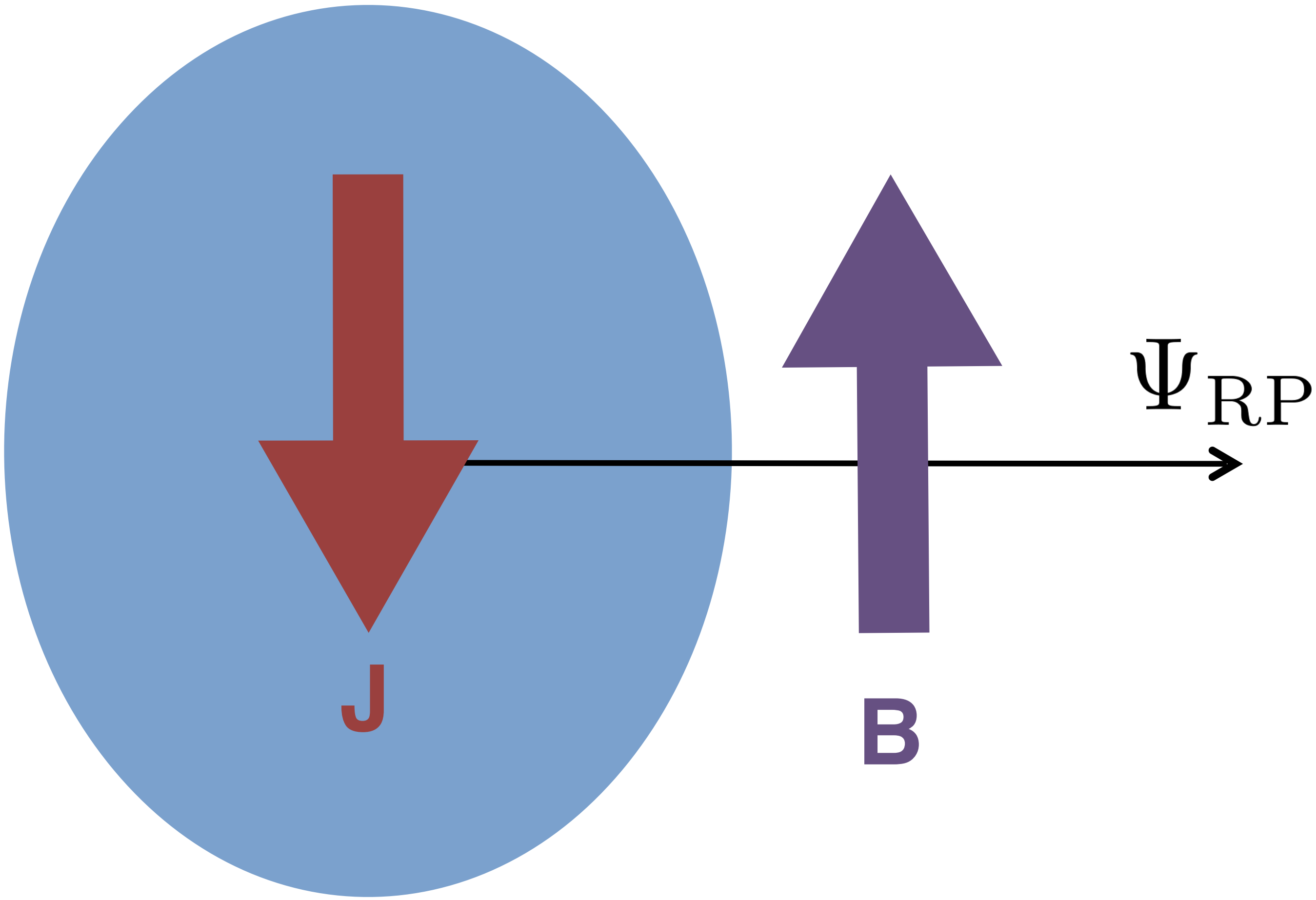


$n_5 > 0$



$a_1^+ > 0 \quad a_1^- < 0$

$$n_5 < 0$$



$$a_1^+ < 0 \qquad a_1^- > 0$$

Expectation from CME

$$\langle a_1^+ \rangle = \langle a_1^- \rangle = 0$$

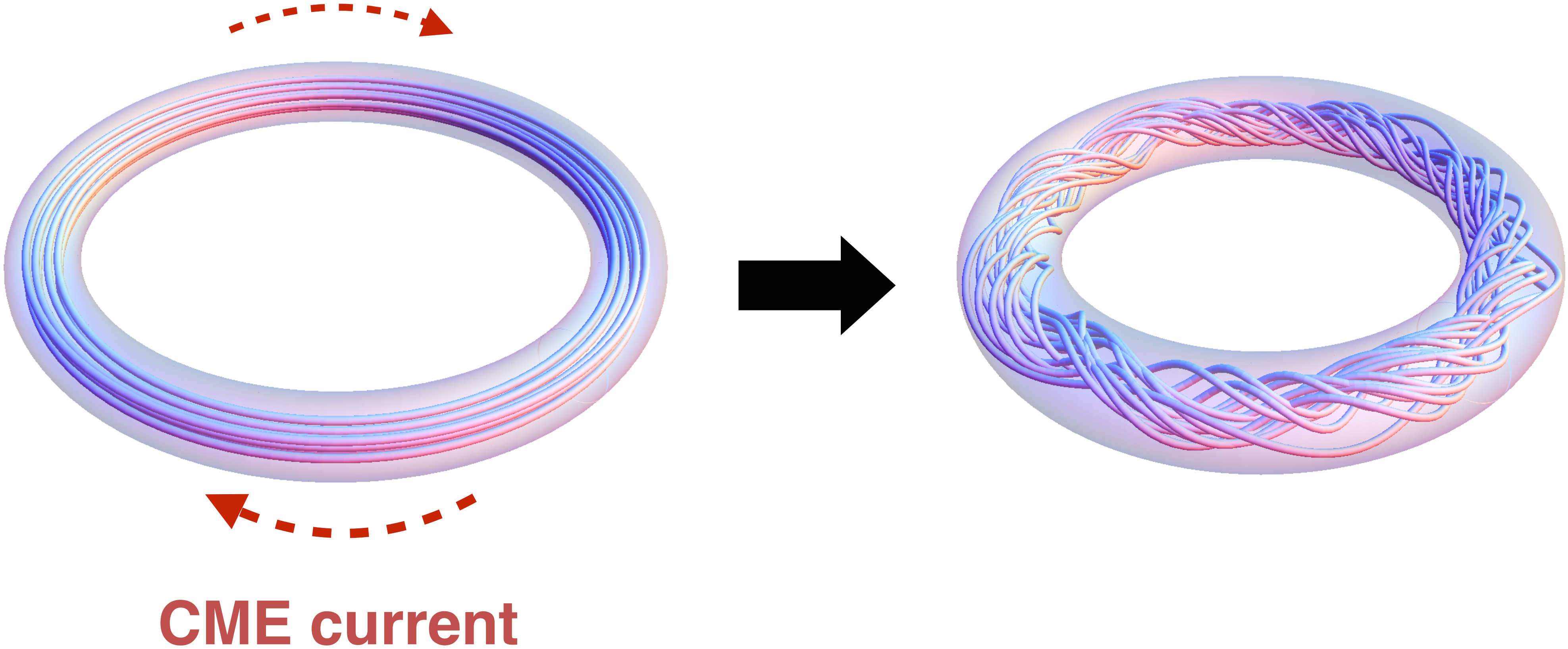
$$\langle (a_1^+)^2 \rangle > 0 \quad \langle (a_1^-)^2 \rangle > 0$$

$$\langle a_1^+ a_1^- \rangle < 0$$

Chiral fluids & dynamical EM fields

Chiral Fluid

EM fields


CME current

Effects of dynamical EM

- Chiral magnetohydrodynamics (MHD) as a hydrodynamic derivative expansion

[Hattori-Hirono-Yee-Yin ‘17]

- Vortex filament motions in chiral fluids

[Hirono-Kharzeev-Sadofyev PRL’18]

Hydrodynamics

- Degree of freedom: **conserved densities**
  - Particle number, energy, momentum, …

$$\{n, e, \boldsymbol{v} \cdots\}$$

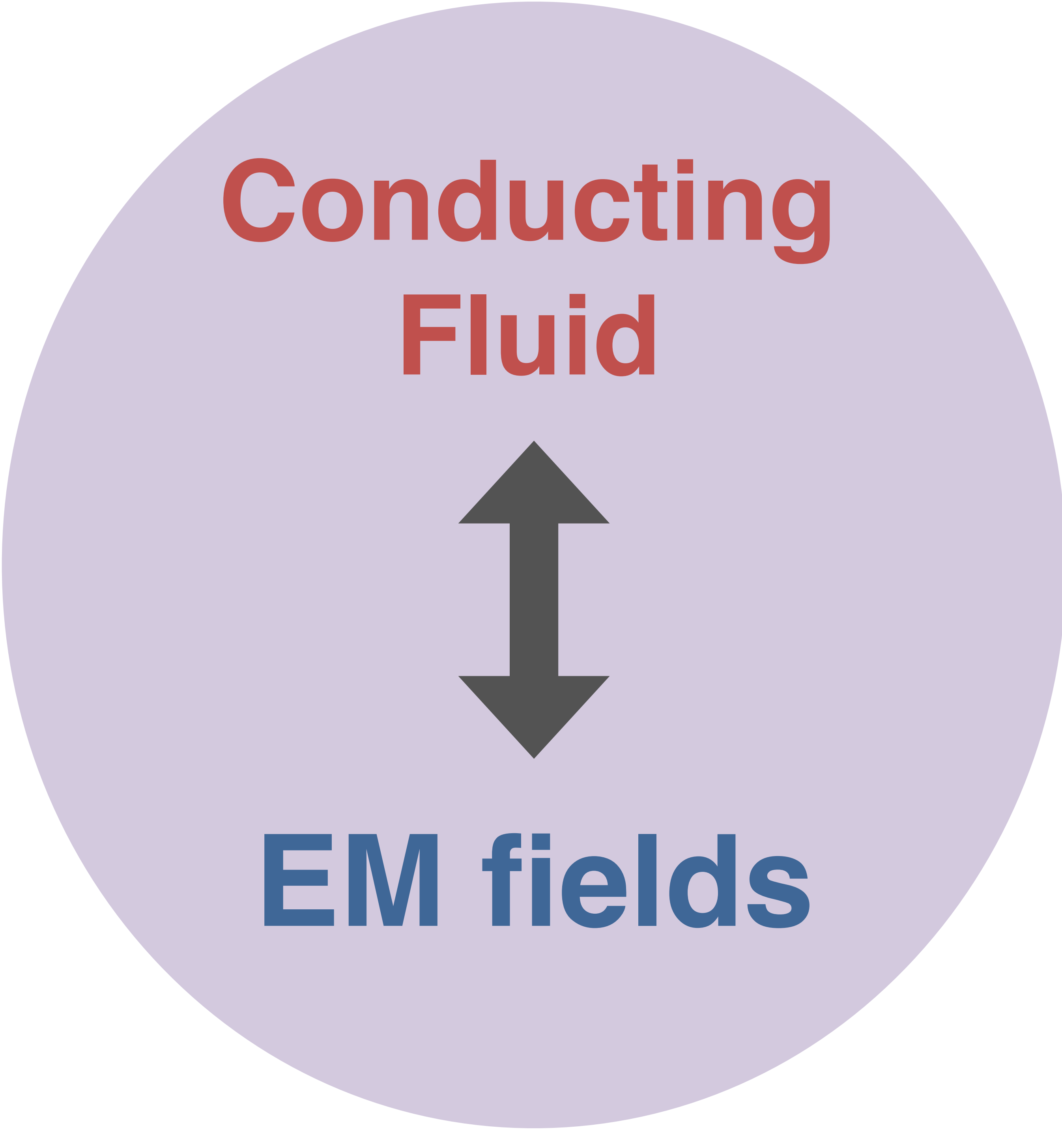
  - “hydrodynamic variables”

- Time evolution is given by conservation law

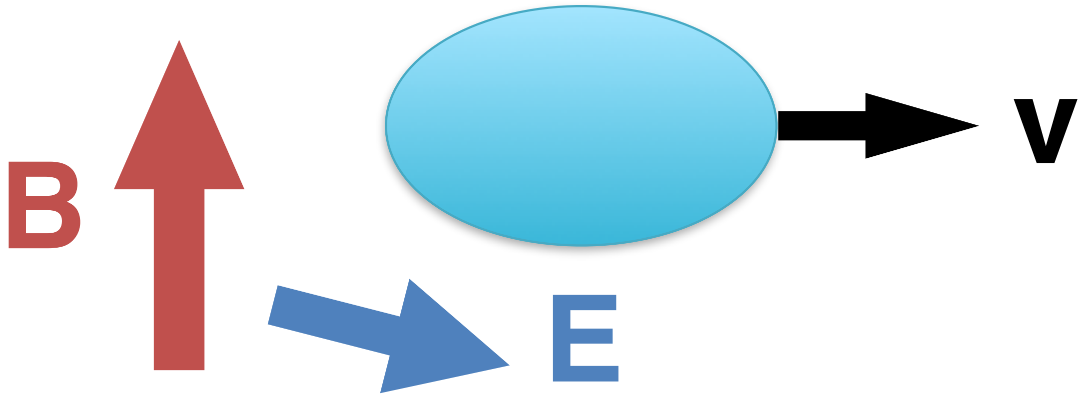
$$\partial_t n + \nabla \cdot \boldsymbol{j} = 0$$

- “Constitutive relations”

$$\boldsymbol{j} = n\boldsymbol{v} - D\nabla n + \cdots$$


Conducting
Fluid
EM fields

Fluid under external **B** & **E**



- Ohm's law $\boldsymbol{j} = \sigma \left(\boldsymbol{E} + \boldsymbol{v}_\perp \times \boldsymbol{B} \right)$

- From $\partial_\mu T^{\mu\nu}_{\text{fluid}} = F^{\nu\rho} j_\rho$

$$\partial_t \boldsymbol{v}_\perp = \frac{1}{e+p} \boldsymbol{j} \times \boldsymbol{B} = -\frac{\sigma \boldsymbol{B}^2}{e+p} \left(\boldsymbol{v}_\perp - \bar{\boldsymbol{v}}_\perp \right) \equiv -\frac{1}{\tau_v} \left(\boldsymbol{v}_\perp - \bar{\boldsymbol{v}}_\perp \right)$$

$$\bar{\boldsymbol{v}}_\perp \equiv \frac{\boldsymbol{E} \times \boldsymbol{B}}{\boldsymbol{B}^2}$$

The transverse velocity becomes "massive"

E in the presence of fluid

- Ohm's law $$\boldsymbol{j} = \sigma \left(\boldsymbol{E} + \boldsymbol{v}_{\perp} \times \boldsymbol{B} \right)$$

- Maxwell equations

$$\partial_t \boldsymbol{E} = -\boldsymbol{j} + \nabla \times \boldsymbol{B} = -\sigma(\boldsymbol{E} - \bar{\boldsymbol{E}}) + \nabla \times \boldsymbol{B}$$

$$\bar{\boldsymbol{E}} = -\boldsymbol{v} \times \boldsymbol{B}$$

- Electric field slaves v & B after $$\tau_E = \frac{1}{\sigma}$$

Two time scales

$$\tau_v = \frac{e+p}{\sigma B^2} \qquad \tau_E = \frac{1}{\sigma}$$

- $\tau_v \ll \tau_E \quad \longleftrightarrow \quad B^2 \gg e+p$

  EM fields are **non-dynamical**

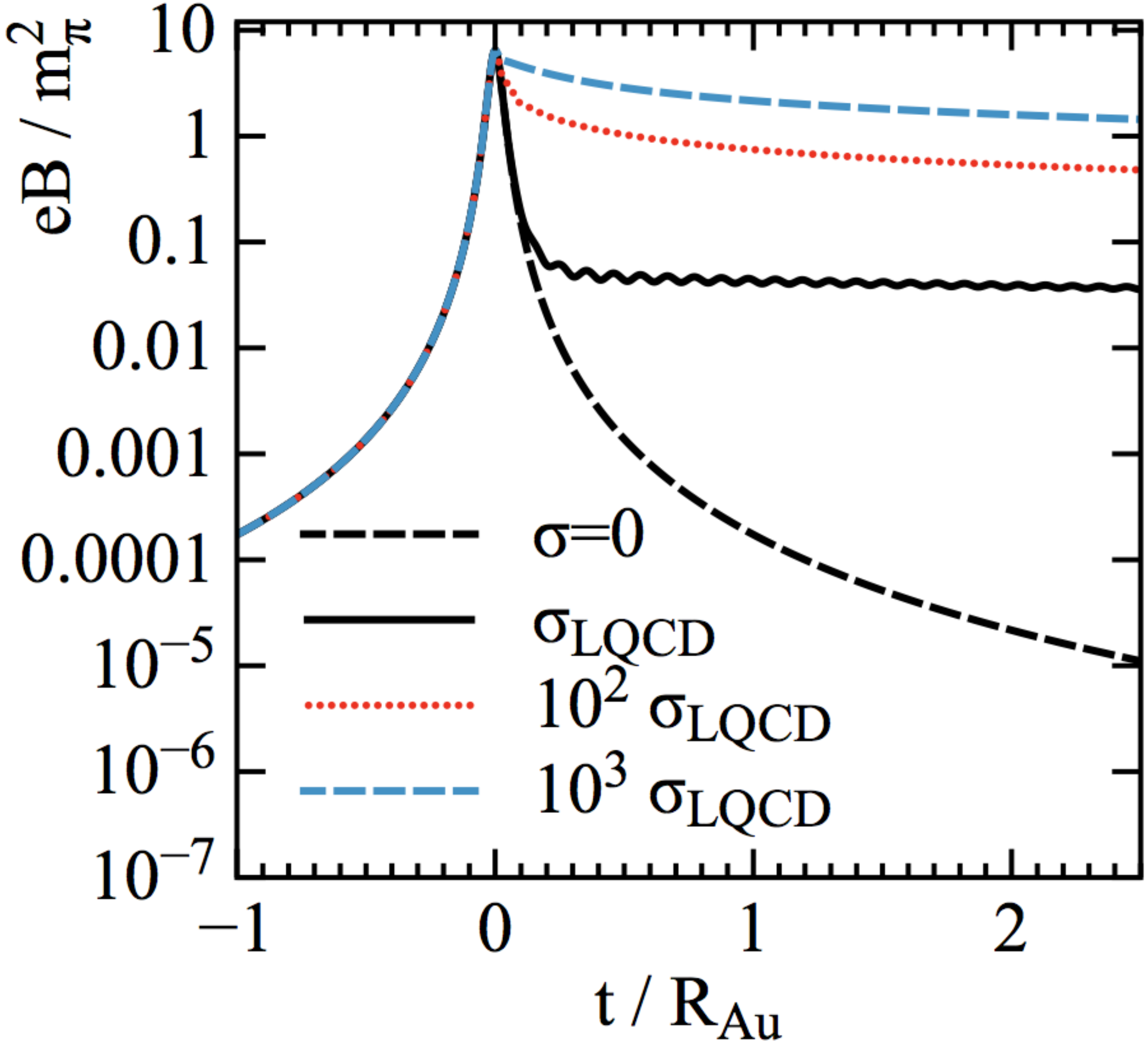
  $\boldsymbol{v}_\perp$ is not a hydrodynamic variable

- $\tau_v \gg \tau_E \quad \longleftrightarrow \quad B^2 \ll e+p$

  EM fields are **dynamical - MHD-like**

  E & **n** is not a hydrodynamic variable

B field in heavy-ion collisions



[L. McLerran, V. Skokov]

MHD

- Hydrodynamic variables

$$\{e(x), u^{\mu}(x), B^{\mu}(x)\}$$

$$E^{\mu} \equiv F^{\mu\nu} u_{\nu}, \quad B^{\mu} \equiv \tilde{F}^{\mu\nu} u_{\nu} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- EOM

$$\partial_{\mu} T^{\mu\nu}_{\text{tot}} = 0 \qquad \partial_{\mu} \tilde{F}^{\mu\nu} = 0$$

Chiral MHD

- Hydrodynamic variables

$$\{e(x), u^\mu(x), B^\mu(x), n_{\rm A}(x)\}$$

- EOM

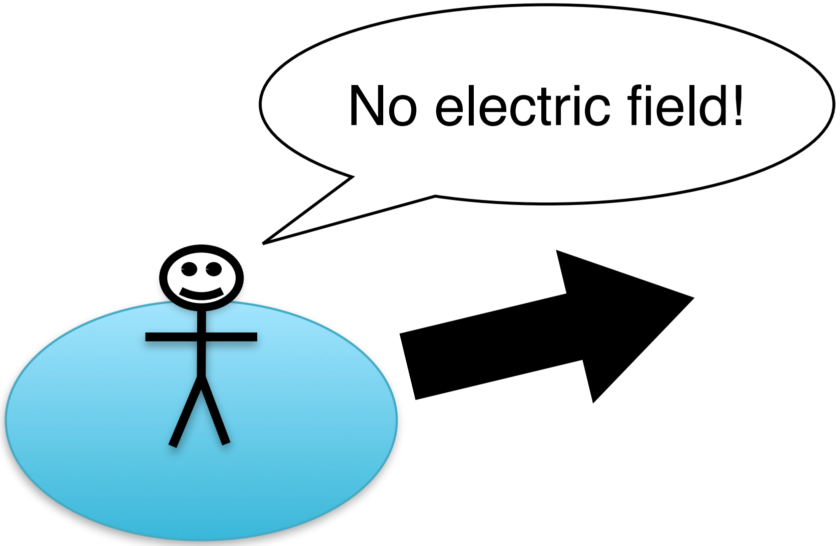
$$\partial_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\partial_\mu J^\mu_{\rm A} = -C_{\rm A} E_\mu B^\mu$$

No electric field in the fluid frame in ideal MHD

$$E^{\mu}_{(0)} = 0$$

Correspond to large conductivity



Constitutive relation for ideal MHD

$$T^{\mu\nu}_{\mathrm{tot}(0)} = (e+p)u^\mu u^\nu - p\eta^{\mu\nu} + \boldsymbol{B}^2\left[u^\mu u^\nu - b^\mu b^\nu - \frac{1}{2}\eta^{\mu\nu}\right]$$

$$B^\mu = |\boldsymbol{B}|b^\mu \quad b_\mu b^\mu = -1$$

$$F^{\mu\nu}_{(0)} = \epsilon^{\mu\nu\rho\sigma}u_\rho B_\sigma$$

Chiral anomaly doesn't play any role at this order

First order in derivative expansion

Using the second law,

$$T^{\mu\nu}_{\mathrm{tot}(1)} = \zeta \Delta^{\mu\nu} \partial \cdot u + 2\eta \nabla^{<\mu} u^{\nu>}$$

E^{μ} : C-odd, P-odd

$$E^{\mu}_{(1)} = \frac{1}{\sigma\beta} \epsilon^{\mu\nu\alpha\beta} u_{\nu} \partial_{\alpha} (\beta B_{\beta}) - \epsilon_B B^{\mu}$$

CME

β : inverse temperature

σ : electric conductivity

$$\epsilon_B \equiv \frac{C_{\mathrm{A}} \mu_{\mathrm{A}}}{\sigma}$$

First order in derivative expansion

$$T^{\mu\nu}_{\mathrm{tot}(1)} = \zeta \Delta^{\mu\nu} \partial\cdot u + 2\eta \nabla^{<\mu} u^{\nu>}$$

$$J^{\mu}_{\mathrm{A}(1)} = D_{\mathrm{A}} \nabla^{\mu} \bar{\mu}_{\mathrm{A}} \qquad \tilde{F}^{\mu\nu}_{(1)} = \epsilon^{\mu\nu\rho\sigma} E_{(1)\rho} u_{\sigma}$$

$$\partial_\mu \left[T^{\mu\nu}_{\mathrm{tot}(0)} + T^{\mu\nu}_{\mathrm{tot}(1)} \right] = 0$$

$$\Rightarrow \quad \partial_\mu \left[\tilde{F}^{\mu\nu}_{(0)} + \tilde{F}^{\mu\nu}_{(1)} \right] = 0$$

$$\partial_\mu \left[J^{\mu}_{\mathrm{A}(0)} + J^{\mu}_{\mathrm{A}(1)} \right] = -C_{\mathrm{A}} E^{\mu}_{(1)} B_{\mu}$$

Waves in chiral MHD

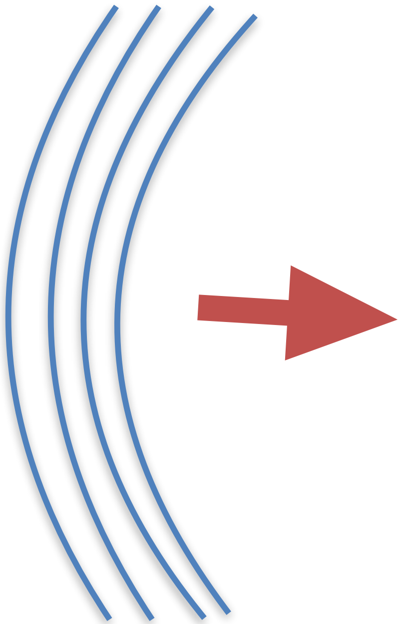
- Linear fluctuations

$$e \to e + \delta e$$

$$B^\mu \to B^\mu + \delta B^\mu$$

$$u^\mu \to u^\mu + \delta u^\mu$$

Alfven wave



restoring force

magnetic field line

momentum along the background **B**

Dispersion relation $\omega = \pm v_A k_{||}$

$$v_A^2 = \frac{\boldsymbol{B}^2}{e + p + \boldsymbol{B}^2}$$ Alfven velocity

Alfven wave in dissipative MHD

Dispersion relation

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2$$

damping

$$\lambda = \frac{1}{\sigma}$$

σ : electric conductivity

$$\bar{\eta} \equiv \frac{\eta}{e + p + \boldsymbol{B}^2}$$

η : shear viscosity

Alfven wave in dissipative MHD

& Chiral

Dispersion relation

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2 + \underbrace{\frac{i}{2} s \epsilon_B k_{||}}_{\text{CME}}$$

CME

$$\epsilon_B \equiv \frac{C_\mathrm{A}\mu_\mathrm{A}}{\sigma}$$

$s = \pm 1$: handedness of the mode

Instability in one of the helicity modes!

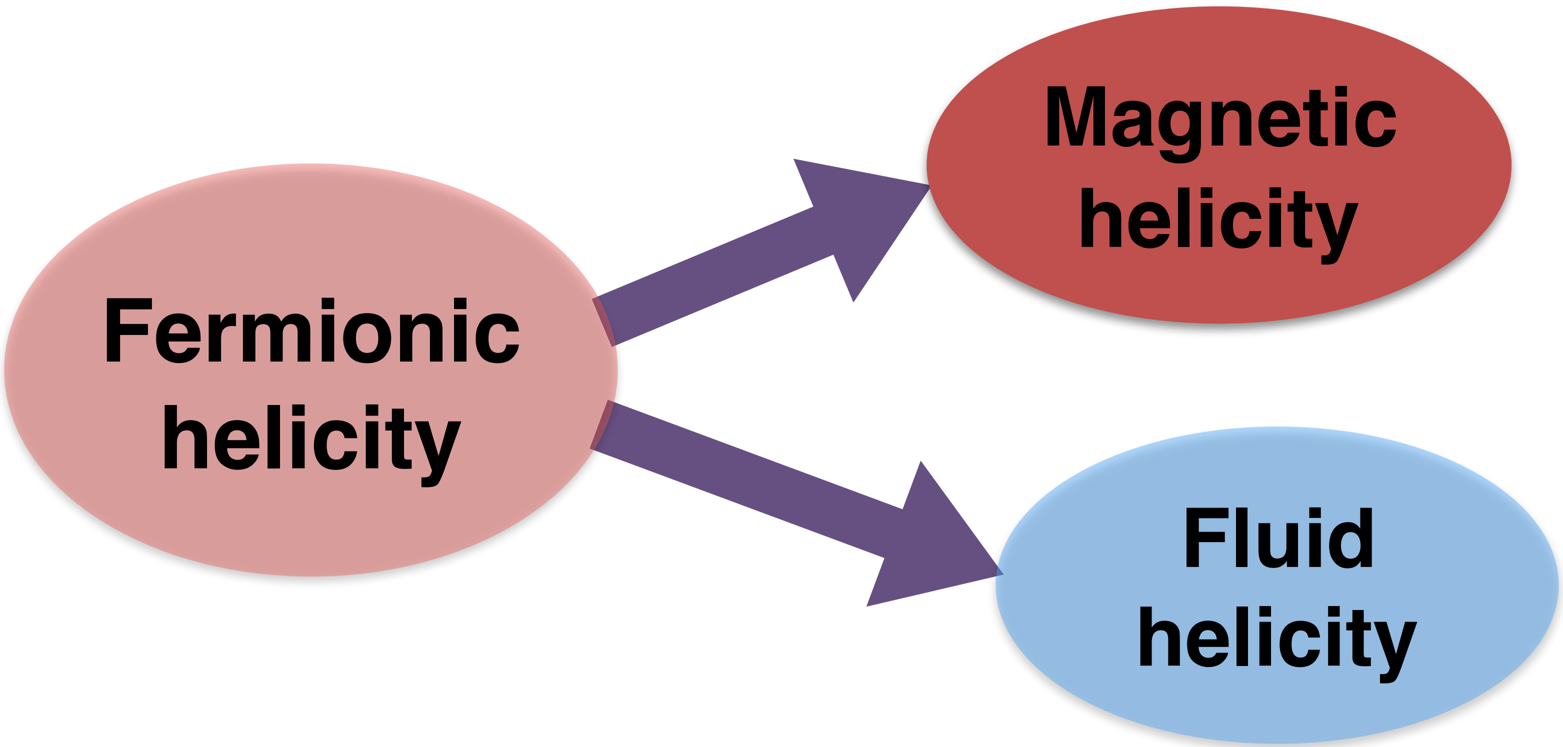
Alfven wave in dissipative MHD

& Chiral

Dispersion relation

$$\omega = \pm v_A k_{||} - \frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2 + \underline{\frac{i}{2} s \epsilon_B k_{||}}$$

CME



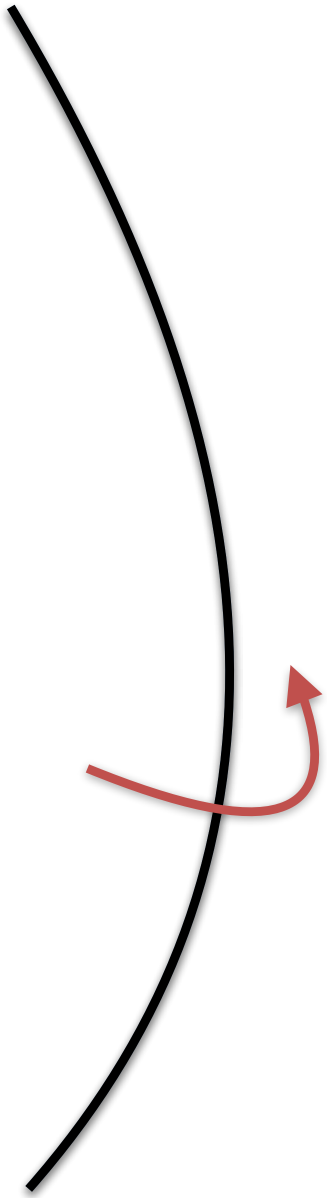
Chiral Fluids

Vortex filaments

[Hirono-Kharzeev-Sadofyev PRL'18]

EM fields

Dynamics of a thin vortex



$\boldsymbol{X}(t,s)$: vortex coordinate

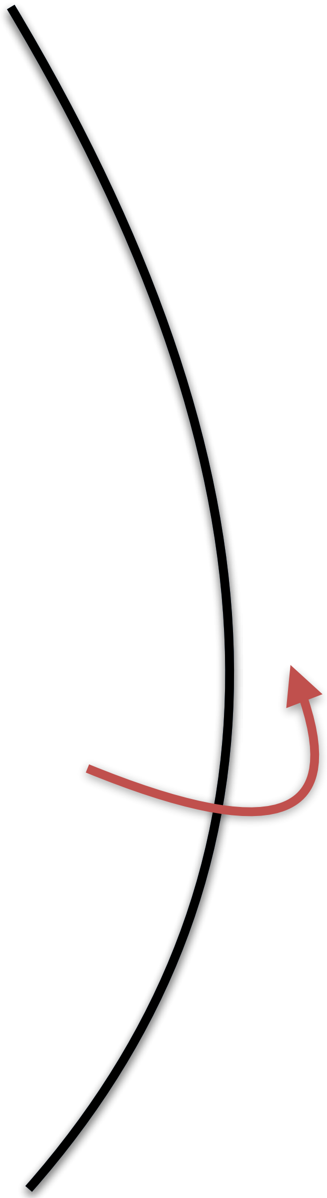
s : arc-length parameter

$$\boldsymbol{\omega}(t,\boldsymbol{x}) = \gamma \int \boldsymbol{X}'(t,s)\delta(\boldsymbol{x}-\boldsymbol{X}(t,s))ds$$

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$$

c.f. $\boldsymbol{J} = \nabla \times \boldsymbol{B}$

Dynamics of a thin vortex



$\boldsymbol{X}(t, s)$: vortex coordinate

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}''$$

[Arms-Hama '65]

Localized induction equation (LIE)

Can be mapped to
non-linear Schrodinger equation

[Hasimoto '72]

Non-linear Schrodinger Eq. (NLSE)

- Complex scalar field in 1+1D $\psi(t, s)$

$$i\dot{\psi} = -\psi'' - \frac{1}{2}|\psi|^2\psi$$

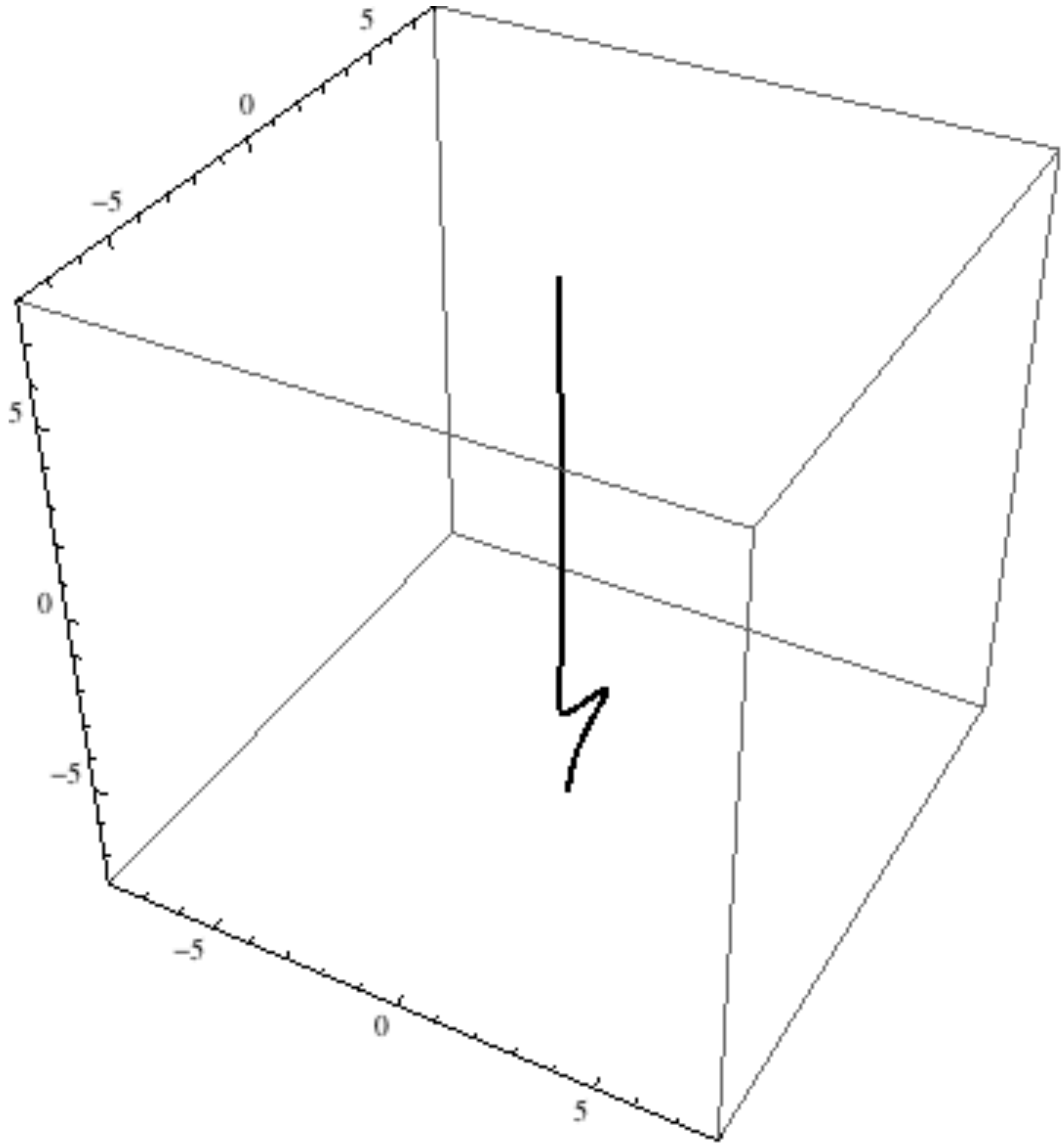
- Integrable system
  - Infinite number of conserved charges

$$Q_0 = \int ds\, \frac{1}{2}\psi^*\psi, \quad Q_1 = \int ds\, \frac{-i}{2}\psi^*\psi'$$

$$Q_2 = \int ds\, \left[\frac{-1}{2}\psi^*\psi'' + \frac{-1}{8}(\psi^*)^2\psi^2\right] \quad \cdots$$

- Solitons

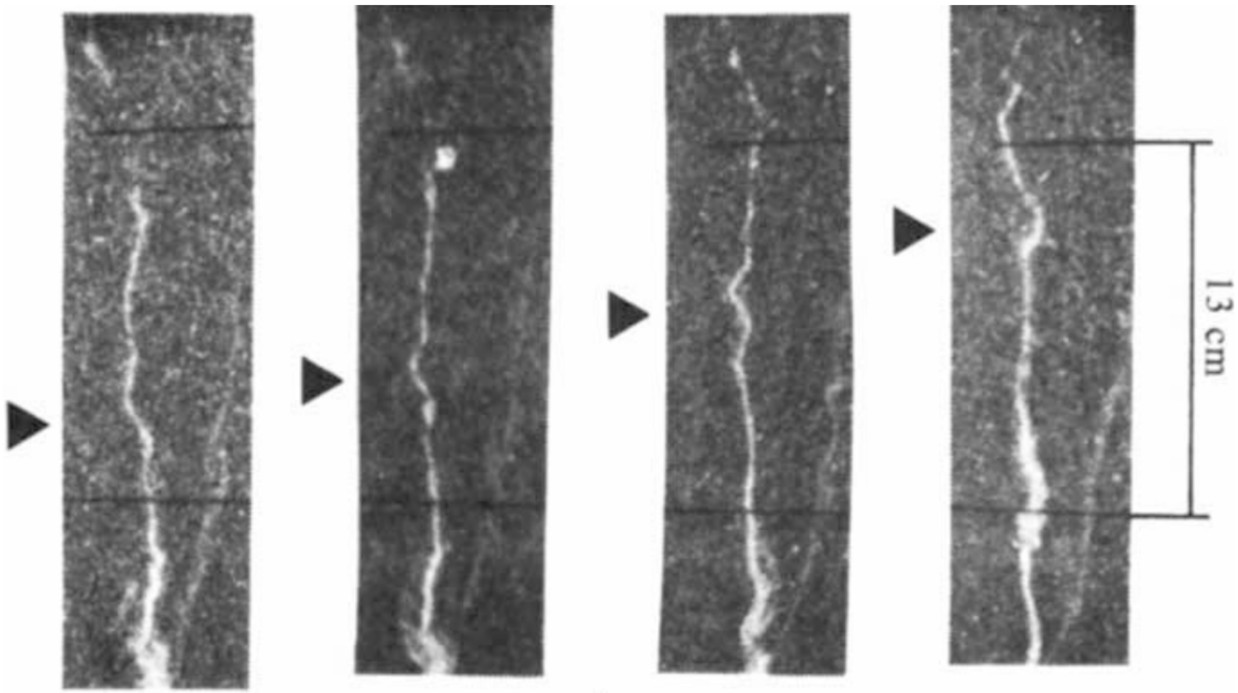
“Hasimoto solitons”



“Hasimoto solitons”

Nature Vol. 295 4 February 1982

Vortex solitary waves in a rotating, turbulent flow



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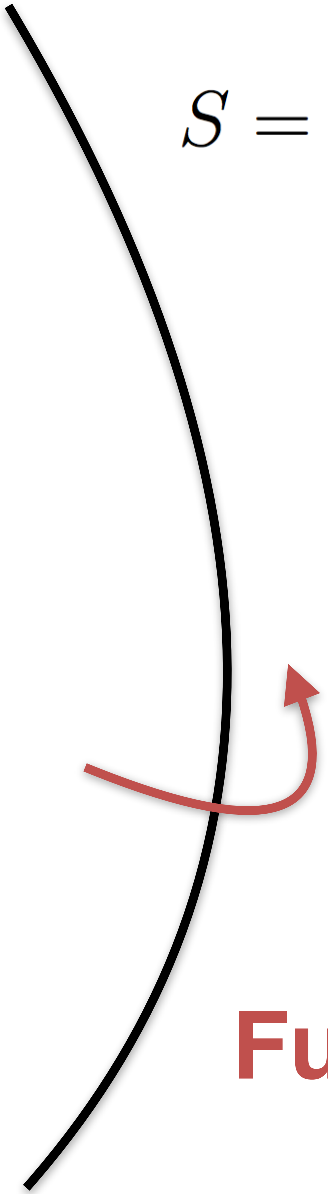
[Hopfinger-Browand ’82]

“Hasimoto solitons”

Dynamics of a thin magnetic flux in a chiral environment



$$S = S_{\text{Abelian-Higgs}} - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}d^4x + \int \mu \boldsymbol{A}\cdot\boldsymbol{B}d^4x$$

μ :chiral chemical potential

[Kozhevnikov '15]

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}''$$

$$+\mu\left[\boldsymbol{X}''' + \frac{3}{2}\left(\boldsymbol{X}''\right)^2\boldsymbol{X}'\right]$$

Fukumoto-Miyazaki Equation (FME)

Fukumoto-Miyazaki Equation

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}'' + \mu \left[\boldsymbol{X}''' + \frac{3}{2} (\boldsymbol{X}'')^2 \boldsymbol{X}' \right]$$

[Fukumoto-Miyazaki ‘91]

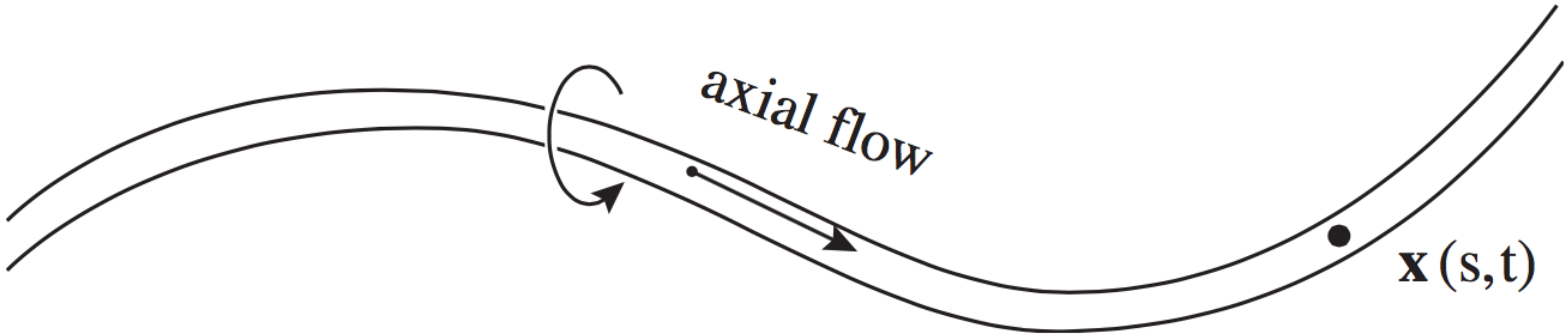


Figure from “Geometrical Theory of Dynamical Systems and Fluid Flows” by Kambe

Equivalence to an integrable system

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}''$$

LIE

$$\psi(t,s) = \kappa(t,s)\exp\left[i\int^{s}\tau(t,s')ds'\right]$$

Hasimoto transformation

$$i\dot{\psi} = -\psi'' - \frac{1}{2}|\psi|^2\psi$$

NLSE

Equivalence to an integrable system

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}'' + \mu \left[\boldsymbol{X}''' + \frac{3}{2} \left(\boldsymbol{X}'' \right)^2 \boldsymbol{X}' \right]$$

Fukumoto-Miyazaki eq.

$$\psi(t,s) = \kappa(t,s) \exp \left[i \int^s \tau(t,s') ds' \right]$$

Hasimoto transformation

$$i\dot{\psi} = -\psi'' - \frac{1}{2}|\psi|^2\psi + i\mu \left(\psi''' + \frac{3}{2}|\psi|^2\psi' \right)$$

Equivalence to an integrable system

$$\dot{\boldsymbol{X}} = \boldsymbol{X}' \times \boldsymbol{X}'' + \mu \left[\boldsymbol{X}''' + \frac{3}{2} \left(\boldsymbol{X}'' \right)^2 \boldsymbol{X}' \right]$$

Fukumoto-Miyazaki eq.

$$\psi(t,s) = \kappa(t,s) \exp \left[i \int^{s} \tau(t,s') ds' \right]$$

Hasimoto transformation

Hirota eq.

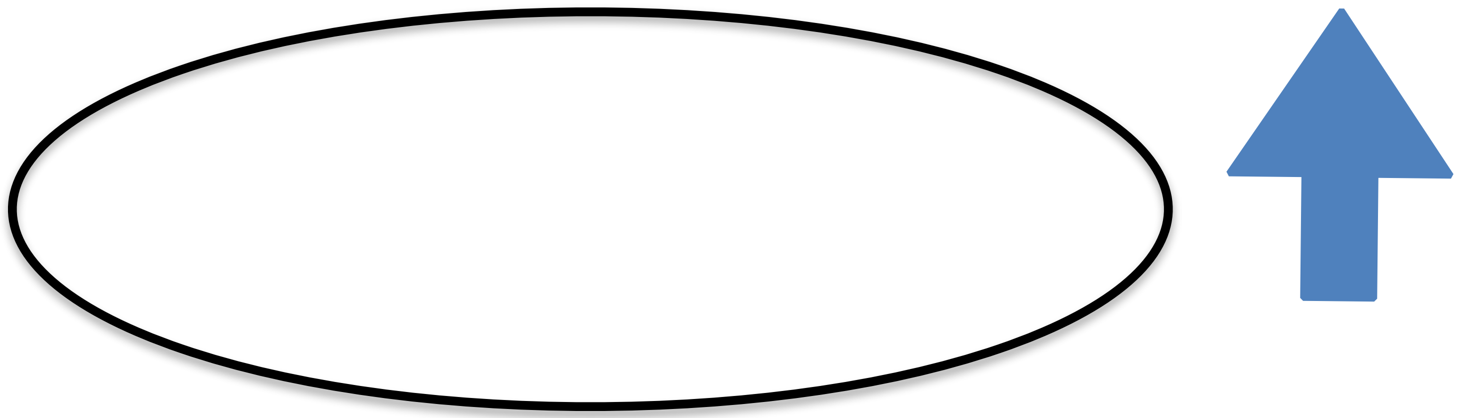
$$i\dot{\psi} = -\psi'' - \frac{1}{2}|\psi|^2\psi + i\mu \left(\psi''' + \frac{3}{2}|\psi|^2\psi' \right)$$

We can ask:

- How the solutions are modified by background chirality?
- Effects on fluctuations?
- Transport properties?

Physical properties of vortices in chiral media

Ring-like solution

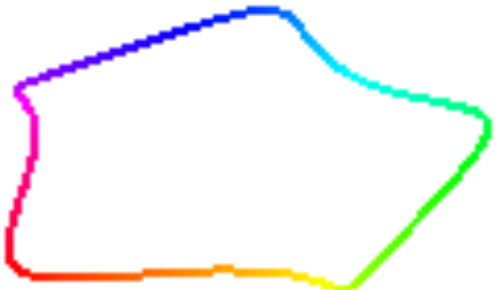


Excitation on a ring

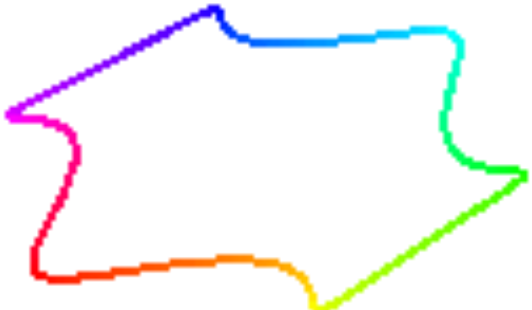
n=2

n=3

n=4



n=5



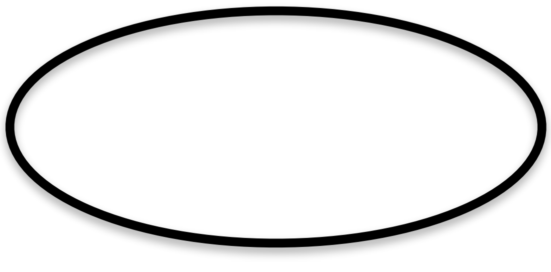
Excitation spectrum on a ring

$$\omega = \pm\kappa^2\sqrt{n^2(n^2-1)}$$ **LIE**

Zero modes

n=0 : radial expansion

n=1, -1 : translation



Excitation spectrum on a ring

$$\omega = \pm\kappa^2\sqrt{n^2(n^2-1)}$$ **LIE**

Zero modes

n=0 : radial expansion

n=1, -1 : translation

Symmetric between n & -n

Excitation spectrum on a ring

$$\omega = \pm\kappa^2\sqrt{n^2(n^2-1)} + \mu\kappa^3\, n\left(n^2 - \frac{3}{2}\right)$$

chiral correction

Zero modes

n=0 : radial expansion

~~n=1, -1 : translation~~

~~Symmetric between n & -n~~

"Chiral Hasimoto solitons"

$$\boldsymbol{X}_{\mathrm{sol}}(t,s) = \begin{pmatrix} -\frac{2\epsilon}{\epsilon^2+\tau_0^2}\ \mathrm{sech}[\epsilon\xi]\cos[\eta] \\ -\frac{2\epsilon}{\epsilon^2+\tau_0^2}\ \mathrm{sech}[\epsilon\xi]\sin[\eta] \\ s - \frac{2\epsilon}{\epsilon^2+\tau_0^2}\tanh[\epsilon\xi] \end{pmatrix}$$

$$\eta \equiv \tau_0 s + (\epsilon^2 - \tau_0^2)t + \underline{\mu\tau_0(3\epsilon^2 - \tau_0^2)}$$

$$\xi \equiv s - (2\tau_0 + \underline{\mu\left(3\tau_0^2 - \epsilon^2\right)})t$$

Parameters: ϵ, τ_0

"Chiral Hasimoto solitons"

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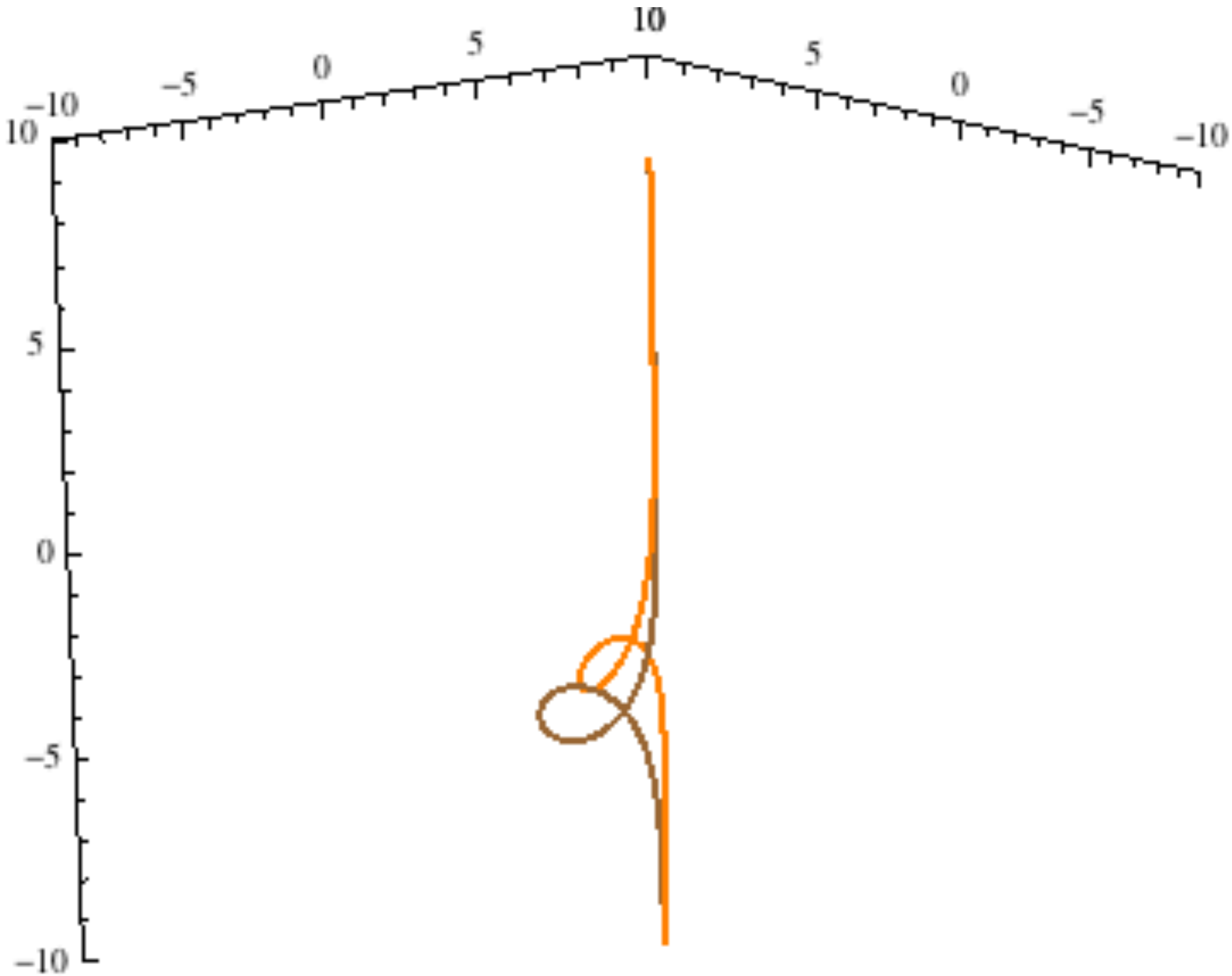
Momentum along the vortex

$$P_{\chi\text{Hasimoto}} = P_{\text{Hasimoto}} + \mu\bar{P}$$

"Chiral propulsion effect"

Chiral & ordinary Hasimoto solitons



Summary

- Chiral Magnetic Effect
- Chiral fluids dynamical EM fields
  - Chiral MHD
    - Instability leading to generation of helical fields
  - Vortices in chiral fluids
    - Mapping to an integrable system