

ELECTROWEAK PROPERTIES AND FORM FACTORS OF THE PION AND KAON IN A NUCLEAR MEDIUM

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Workshop on Hadron in various environments, Pohang, October, 19-20, 2018



OUTLINE

- 1 PION & KAON PROPERTIES IN THE BSE-NJL MODEL
- 2 QUARK-MESON COUPLING (QMC) MODEL
- 3 PION & KAON PROPERTIES IN A NUCLEAR MEDIUM
 - In-medium pion masses
 - In-medium quark-quark coupling of the pion
 - In-medium pion decay constants
- 4 PION & KAON FORM FACTORS IN A NUCLEAR MEDIUM
 - Medium modifications pion form factor
 - Medium modifications kaon form factor
- 5 CONCLUSION AND OUTLOOK

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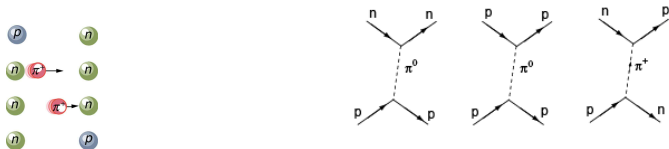
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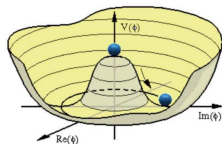
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INTRODUCTION

- Pion was introduced by Yukawa as a mediator (carrier) of the nuclear strong force (Proc. Phys. Math. Soc. Jpn. 17 (1935))

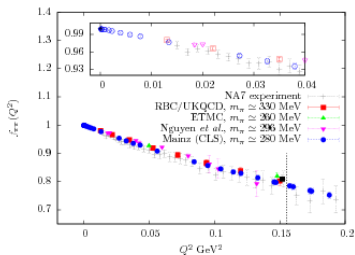
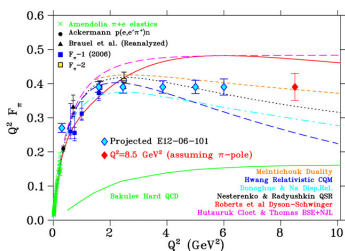


- Pion, the Goldstone bosons emerged as consequence of spontaneously breaking of global chiral symmetry in the favor SU(2), has a special place in QCD \iff Nambu-Jona-Lasinio introduced chiral symmetry and its breaking for generating mass and appearing pion (Phys. Rev. 124 & 122 (1961))



INTRODUCTION

- There have been many studies devoted to understand the internal structure of the pion in free space based on phenomenological approaches such as NJL, Instanton vacuum, Light-front, and Dyson-Schwinger models as well as lattice QCD simulations (Courtesy: Garth Huber Slide, EIC Meeting 2018 & Bastian. B. Brant, Int. J. Mod. Phys. E22 (2013))



- However, in nuclear medium, only few theoretical studies have been reported so far on the pion & kaon structures and no experimental data available (only nucleon in medium \iff EMC effect)

INTRODUCTION

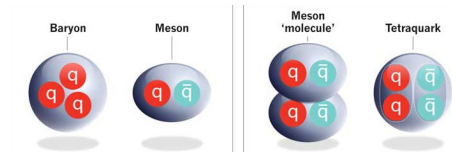
- Recent work of Ref.¹ used a light-front constituent quark model to describe the pion in vacuum as well as in medium. There are a few aspects which require further investigations:
 - ▶ In the LF constituent quark model, the dressed mass value in vacuum is an input and treated as a parameter
 - ▶ There is no quark condensate which cannot explain the connection with spontaneous breaking of chiral symmetry of the vacuum
- We address this point in our work by using the NJL model which describes the spontaneous breaking of chiral symmetry and offers the dynamically generated quark mass through quark condensates
- Some observations such as EMC effect indicates the internal structure of hadrons may change in nuclear medium. The phenomena of medium modifications is therefore one the most interesting subject in nuclear and hadron physics²

¹J. P. B. C. de Melo, *et al.*, Phys. Rev. C90 (2014)

²G. E. Brown, PRL 66 (1991), K.Saito, Prog. Part. Nucl. Phys. 58 (2007), Hayano, Rev.Mod. Phys. 82 (2010), Leupold, Int. J. Mod. Phys. E19 (2010), and Metag, EPJ Web Conf. 34 (2017)

INTRODUCTION

- Since chiral symmetry has a big impact on the low-lying hadron mass spectrum, the partial restoration of chiral symmetry in a strongly interaction medium is important to understand the change of hadrons properties in nuclear medium
- As pions are the lightest bound states composed of dressed and quark-antiquark pair



- We focus on electroweak properties in nuclear medium in this study by calculating the weak decay constant of the in-medium pion, the pion-quark coupling constant in symmetric nuclear matter, and quark condensate in medium as well as the medium modifications of the pion & kaon form factors

PION AND KAON IN THE BSE-NJL MODEL

The three flavor NJL Lagrangian – containing only four fermion interactions

$$\begin{aligned}\mathcal{L}_{NJL} = & \bar{\psi}[i\partial - \hat{m}_q]\psi + G_\pi \sum_{a=0}^8 \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2 \right] \\ & - G_\rho \sum_{a=0}^8 \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2 \right]\end{aligned}\quad (1)$$

- $\psi = (u, d, s)^T$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \dots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \text{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

PION AND KAON IN THE BSE-NJL MODEL

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction by integrating out the gluon field and absorbing into the coupling constant \iff *quark effective theory*



- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole). Therefore we regularize using the proper time regularization to simulate confinement (IC.Cloet, PRC90 (2014), PTPH, PRC94 (2016))

$$\begin{aligned} \frac{1}{\mathcal{X}^n} &= \frac{1}{(n-1)!} \int_0^\infty d\tau \tau^{(n-1)} e^{-\tau \mathcal{X}} \\ &\rightarrow \frac{1}{(n-1)!} \int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{(n-1)} e^{-\tau \mathcal{X}} \end{aligned} \quad (2)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24 \text{ GeV}$ and Λ_{UV} is determined.

PION AND KAON IN THE BSE-NJL MODEL

- NJL Gap Equation is determined using quark propagator in momentum space $S_q^{-1}(p) = \not{p} - M_q + i\epsilon$

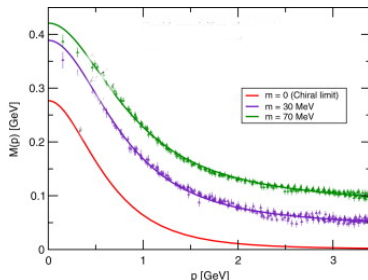
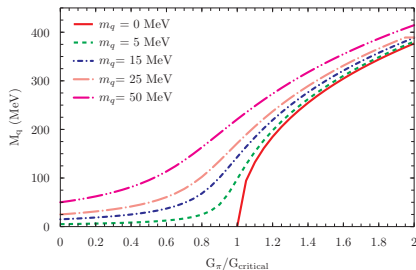
$$\begin{aligned} M_q &= m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\ &= m_q - 2G_\pi \langle \bar{\psi}\psi \rangle \end{aligned} \quad (3)$$

- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -\frac{3M_q}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL GAP EQUATION

NJL and DSE gap equations [PTPH et al., PRC94 \(2016\)](#), [C.D.Roberts, PPNP 61 \(2008\)](#)

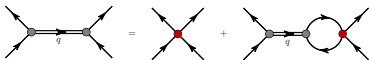
- The NJL constituent quark mass is a constant up to certain $p \sim 0.6$ GeV and it drops in higher p region



- The NJL model can be used for low momentum p and low energy E

BETHE SALPETER EQUATION FOR THE PION AND KAON

Mesons in the NJL model are quark-antiquark bound states whose properties are determined by solving the BSE



- In the NJL model, \mathcal{T} -matrix is given by

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4 k}{(2\pi)^4} \mathcal{K} S(q+k) \mathcal{T}(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$\mathcal{T}_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab} t_\alpha(q) [\gamma_t \lambda_\alpha^\dagger] \quad (4)$$

- The reduced t -matrix in this channel take a form

$$t_\alpha(q) = \frac{-2i G_\pi}{1 + 2 G_\pi \Pi_\pi(q^2)}$$
$$t_\beta^{\mu\nu}(q) = \frac{-2i G_\rho}{1 + 2 G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2 G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2} \right) \quad (5)$$

BETHE SALPETER EQUATION OF THE PION AND KAON

- The bubble diagrams appearing read

$$\begin{aligned}\Pi_{\pi}(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D [\gamma_5 S_l(k) \gamma_5 S_l(k+q)], \\ \Pi_K(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D [\gamma_5 S_l(k) \gamma_5 S_s(k+q)], \\ \Pi_{\nu}^{aa}(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D [\gamma^{\mu} S_a(k) \gamma^{\nu} S_a(k+q)]\end{aligned}\quad (6)$$

- The kaon and pion masses is given by the pole of the t-matrix

$$\begin{aligned}1 + 2G_{\pi}\Pi_{\pi}(k^2 = m_{\pi}^2) &= 0 \\ 1 + 2G_{\pi}\Pi_K(k^2 = m_K^2) &= 0\end{aligned}\quad (7)$$

PION AND KAON MASSES

The meson masses are defined by the pole in the corresponding t -matrix and therefore the kaon and pion masses are given by

$$\begin{aligned} m_\pi^2 &= \left[\frac{m}{M_l} \right] \frac{2}{G_\pi \mathcal{I}_{II}(m_\pi^2)} \\ m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l} \right] \frac{1}{G_\pi \mathcal{I}_{Is}(m_K^2)} + (M_s - M_l)^2 \end{aligned} \quad (8)$$

where \mathcal{I}_{II} and \mathcal{I}_{Is} in the proper time regularization scheme are defined by

$$\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \quad (9)$$

THE MESON-QUARK-QUARK COUPLING CONSTANTS AND PION AND KAON DECAY CONSTANTS

The residue at a pole in the $\bar{q}q t$ -matrix defines the effective meson-quark-quark coupling constants:

$$\begin{aligned}Z_{\pi}(q^2) &= -\frac{\partial \Pi_{\pi}(q^2)}{\partial q^2} \Big|_{q^2=m_{\pi}^2} \\Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2} \Big|_{q^2=m_K^2} \\Z_{\rho}(q^2) &= -\frac{\partial \Pi_{\rho}(q^2)}{\partial q^2} \Big|_{q^2=m_{\rho}^2}\end{aligned}\tag{10}$$

Pion and kaon decay constant in the proper time regularization is given by

$$\begin{aligned}f_{\pi} &= \frac{N_C \sqrt{Z_{\pi}} M}{4\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\f_K &= \frac{N_C \sqrt{Z_K}}{4\pi^2} [(1-x)M_2 + xM_1] \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)}\end{aligned}\tag{11}$$

QUARK-MESON COUPLING (QMC) MODEL

LAGRANGIAN QMC MODEL

The effective Lagrangian for a symmetric nuclear matter in the QMC model:

$$\mathcal{L}_{\text{QMC}} = \bar{\psi} [i\gamma_{\mu}\partial^{\mu} - M_N^*(\sigma) - g_{\omega}\gamma_{\mu}\omega^{\mu}] \psi + \mathcal{L}_m, \quad (12)$$

The free meson Lagrangian density:

$$\mathcal{L}_m = \frac{1}{2} \left(\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^2\sigma^2 \right) - \frac{1}{2} \partial_{\mu}\omega_{\nu} \left(\partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu} \right) + \frac{1}{2} m_{\omega}^2\omega^{\mu}\omega_{\mu}$$

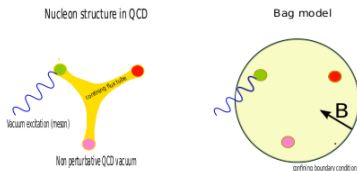


FIGURE: The QCD picture of the nucleon and the bag model ³

³J.Stone *et al.*, Prog.Part.Nucl.Phys. 100 (2018)

LAGRANGIAN QMC MODEL

- In the QMC model, the nuclear matter is treated as a collection of the nucleons that are assumed to be non-overlapping MIT bags
- The Dirac equation for the light quarks inside the bags are given by

$$\begin{aligned} \left[i\gamma \cdot \partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q + \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} &= 0 \\ \left[i\gamma \cdot \partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q - \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} &= 0 \\ [i\gamma \cdot \partial_x - m_s] \begin{pmatrix} \psi_s(x) \\ \psi_{\bar{s}}(x) \end{pmatrix} &= 0, \end{aligned} \quad (13)$$

where the effective current quark mass m_q^* is defined as

$$m_q^* \equiv m_q - V_\sigma^q, \quad (14)$$

where m_q is the current quark mass, where $q = (u, d, s)$ and V_σ^q is the scalar potential.

LAGRANGIAN QMC MODEL

The effective nucleon mass:

$$M_N^*(\sigma) \equiv M_N - g_\sigma(\sigma)\sigma, \quad (15)$$

Total energy per nucleon:

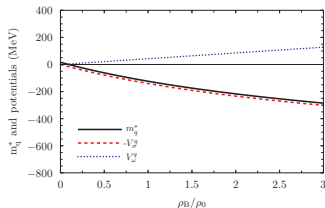
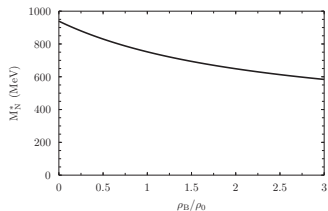
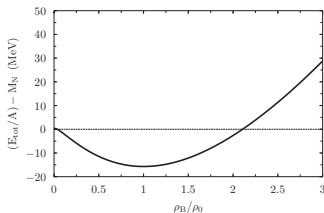
$$E^{\text{tot}}/A = \frac{4}{(2\pi)^3 \rho_B} \int d\mathbf{k} \theta(k_F - |\mathbf{k}|) \sqrt{M_N^{*2}(\sigma) + \mathbf{k}^2} + \frac{m_\sigma^2 \sigma^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2m_\omega^2}. \quad (16)$$

TABLE: Parameters of the QMC model and the obtained nucleon properties at saturation density $\rho_0 = 0.15 \text{ fm}^{-3}$ for two quark mass values in free space, $m_q = 5.0$, and 16.4 MeV . The m_q , M_N^* , and K are given in units of MeV. The parameters are fitted to the free space nucleon mass $M_N = 939 \text{ MeV}$ with $R_N = 0.8 \text{ fm}$ (input), and the nuclear matter saturation properties.

m_q	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	$B^{1/4}$	z_N	M_N^*	K
5	5.393	5.304	170.0	3.295	754.6	279.3
16.4	5.438	5.412	169.2	3.334	752.0	281.5

LAGRANGIAN QMC MODEL

Energy per nucleon ($E^{\text{tot}}/A - M_N$), effective nucleon mass M_N^* and effective quark mass (m_q^*) and the quark potentials (V_σ^q and V_ω^q) for symmetric nuclear matter in the QMC model for the current quark mass $m_q = 16.4$ MeV (PTPH, Yongseok Oh and K. Tsushima, in preparation (2018))



PION AND KAON PROPERTIES IN A NUCLEAR MEDIUM

IN-MEDIUM PION PROPERTIES (NJL+QMC)

- Using the in-medium properties corresponding to $m_q = 16.4$ MeV calculated in the QMC model,
- we calculate the effective quark mass M_u^* , in-medium pion decay constant, in-medium quark condensate, and in-medium πqq coupling constant using the NJL model.
- The in-medium dressed quark propagator:

$$S_q^*(k^*) = \frac{\not{k} + V^0 + M_q^*}{(k + V^0)^2 - M_q^{*2} + i\epsilon}, \quad (17)$$

where the medium modification enter as the shift of the quark momentum through $(k^*)_\mu \rightarrow k^\mu + V^\mu$ where vector potential, $V^\mu = (\delta_0^\mu V^0, \vec{0})^4$. The asterisk denotes the in-medium quantity

- The in-medium NJL gap mass in the proper-time regularization scheme:

$$M_q^* = m_q^* + \frac{3G_\pi M_q^*}{\pi^2} \int_{\frac{1}{\Lambda_{UV}^2}}^{\infty} \frac{d\tau}{\tau^2} e^{(-\tau(M_q^*)^2)} \quad (18)$$

⁴Miller, Phys. Rev. Lett. **103** (2009)

IN-MEDIUM PION PROPERTIES (NJL+QMC)

Using the in-medium properties corresponding to $m_q = 16.4$ MeV calculated in the QMC model,

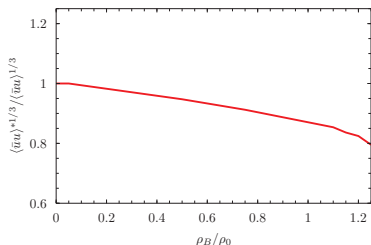
- in-medium pion decay constant $f_\pi \rightarrow f_\pi^*$
- in-medium quark condensate $\langle \bar{u}u \rangle \rightarrow \langle \bar{u}u \rangle^*$
- in-medium πqq coupling constant $g_{\pi qq} \rightarrow g_{\pi qq}^*$

TABLE: Properties of in-medium pion calculated in the NJL model with the in-medium inputs from the QMC model for $m_q = 16.4$ MeV. The units for M_u^* , m_π^* , f_π^* , and $-\langle \bar{u}u \rangle^{*1/3}$ are GeV.

ρ_B/ρ_0	M_u^*	m_π^*	f_π^*	$g_{\pi qq}^*$	$-\langle \bar{u}u \rangle^{*1/3}$
0.00	0.400	0.140	0.093	4.225	0.171
0.25	0.370	0.136	0.092	3.964	0.167
0.50	0.339	0.134	0.089	3.720	0.162
0.75	0.307	0.132	0.086	3.494	0.156
1.00	0.270	0.131	0.081	3.265	0.149
1.25	0.207	0.136	0.069	2.948	0.136

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum quark condensates as a function of ρ_B/ρ_0 with $\rho_0 = 0.15 \text{ fm}^{-3}$

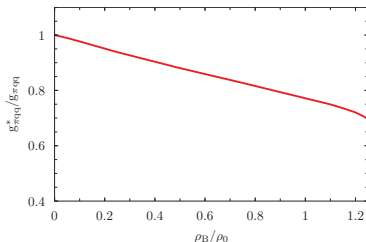


- The ratio of the in-medium to vacuum quark condensates decreases with increasing nuclear matter density
- The ratio at saturation nuclear matter density is estimated about 0.87
- This is somehow higher than obtained in Ref⁵ which gives 0.63-0.57 via the relation $\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle \sim 1 - (0.37 \sim 0.43)\rho_B/\rho_0$

⁵Jido *et al.*, PLB 670 (2008)

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum pion-quark coupling constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15 \text{ fm}^{-3}$



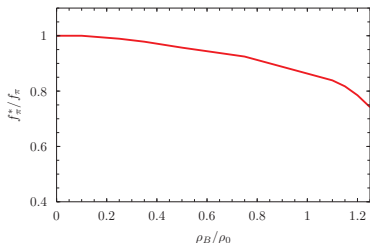
- We observe $g_{\pi qq}^*/g_{\pi qq}$ decreases with increasing density, which is consistent with the results of Refs.⁶
- At normal density, we obtain $g_{\pi qq}^*/g_{\pi qq} = 0.77$ which is smaller than the value obtained in Ref.⁷

⁶V. Bernard *et al.*, PRD **38** (1988)

⁷Ramalho *et al.*, J. Phys. G**40** (2013)

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum pion decay constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15 \text{ fm}^{-3}$



- The ratio is found to decrease as density increases
- At normal density, we obtained $f_\pi^*/f_\pi = 0.87$, which is in a good agreement with results of Ref⁸ $f_\pi^*/f_\pi = 0.80$
- Ratio is larger than the values obtained in Refs.^{9 10} by about 10-20 %

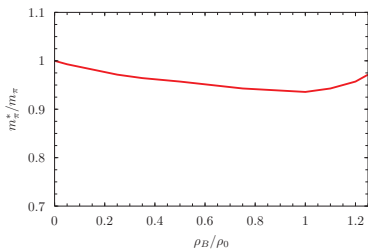
⁸P. Kienle *et al.*, Prog. Part. Nucl. Phys. **52** (2004)

⁹M. Kirchbach *et al.*, Nucl. Phys. **A616** (1997)

¹⁰U. G. Meissner *et al.*, Ann. Phys. (N.Y)**297** (2002)

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum pion mass as a function of ρ_B/ρ_0 with $\rho_0 = 0.15 \text{ fm}^{-3}$



- we confirm that the pion is almost unchanged up to $1.25\rho_0$ which is consistent with the results obtained by Bernard in the low density region
- The difference between the in-medium and free pion masses is within 6% up to nuclear density of $1.25\rho_0$
- This justifies the assumption (deeply bound pionic atom) that $m_\pi^* \sim m_\pi$ up to normal nuclear density or $1.25\rho_0$

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum isovector nucleon axial-vector coupling constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15 \text{ fm}^{-3}$.

- The Goldberger-Treiman relation at nucleon level is given by $g_A \equiv g_{\pi NN} f_\pi / M_N$. at the quark level, the ratio of the in-medium to vacuum isovector nucleon axial-vector coupling constant:

$$\left(\frac{g_A^*}{g_A} \right) = \left(\frac{g_{\pi qq}^*}{g_{\pi qq}} \right) \left(\frac{f_\pi^*}{f_\pi} \right) \left(\frac{M_q}{M_q^*} \right). \quad (19)$$

- we estimate $g_A^*/g_A = 0.99$, which is consistent with the quenching of g_A^* but *less amount of quenching* compared with the result of Ref.¹¹ that gives $g_A^*/g_A = 0.9$

¹¹D. H. Lu *et al.*, arXiv:nucl-th/0112001

IN-MEDIUM KAON PROPERTIES (NJL+QMC)

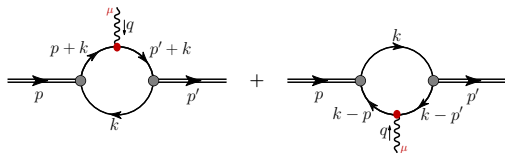
TABLE: Result for the in-medium constituent quark mass and kaon properties in the NJL model. The effective constituent quark mass M_q^* is calculated in the NJL model combined with the inputs of the effective quark mass $m_u^* = 0.0164$ GeV and $m_s^* = 0.356$ GeV calculated in the QMC model. The effective constituent quark masses, M_q^* are in units of GeV.

ρ_B/ρ_0	M_u^*	M_s^*	m_K^*	f_K^*	g_{Kqq}^*
0.0	0.400	0.611	0.495	0.097	4.575
0.25	0.370	0.611	0.465	0.099	4.536
0.50	0.339	0.611	0.437	0.101	4.495
0.75	0.307	0.611	0.411	0.104	4.455
1.00	0.270	0.611	0.386	0.106	4.408
1.25	0.207	0.611	0.359	0.110	4.332

IN-MEDIUM MODIFICATION FORM FACTOR OF THE PION

IN-MEDIUM MODIFICATIONS FORM FACTOR IN NJL MODEL

Diagrammatic representation of the electromagnetic current for the pion and kaon

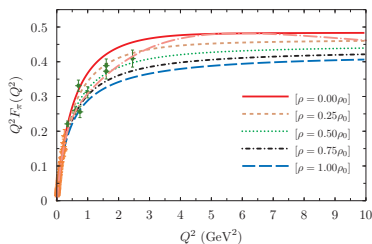
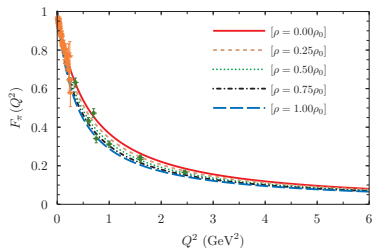


Feynman diagram for quark [left] and for the anti quark [right] The complete results for the pseudoscalar meson form factor in a nuclear medium – with a dressed quark-photon vertex – read

$$\begin{aligned}
 F_{\pi^+}^*(Q^2) &= \left[F_{1U}^*(Q^2) - F_{1D}^*(Q^2) \right] f_{\pi}^{*ll}(Q^2) \\
 F_{K^+}^*(Q^2) &= F_{1U}^*(Q^2) f_K^{*ls}(Q^2) - F_{1S}^*(Q^2) f_K^{*sl}(Q^2) \\
 F_{K^0}^*(Q^2) &= F_{1D}^*(Q^2) f_K^{*ls}(Q^2) - F_{1S}^*(Q^2) f_K^{*sl}(Q^2)
 \end{aligned} \tag{20}$$

IN-MEDIUM MODIFICATIONS PION FORM FACTOR

Results for the in-medium space-like electromagnetic form factors of the pion
(PTPH, Yongseok Oh, K. Tsushima, in preparation (2018))



- Our pion form factor results show that the in-medium pion electromagnetic form factor decreases with increasing density
- The medium effects on the suppression of the pion form factor are clearly seen and it is reduced by 20% at normal density, which would be large to be extracted empirically

IN-MEDIUM MODIFICATIONS PION FORM FACTOR

Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii

TABLE: Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii. This is calculated using the inputs from the QMC model. All charge radii in the medium along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_π	r_u	r_d	r^{expt}
0.00	0.629	0.629	0.629	0.672 ± 0.008
0.25	0.667	0.667	0.667	
0.50	0.705	0.704	0.705	
0.75	0.740	0.739	0.740	
1.00	0.771	0.771	0.771	

- The obtained charge radius of the pion in vacuum is in good agreement with the empirical data¹²

¹²C. Patrignani *et al.*, Chin. Phys. C40 (2016)

IN-MEDIUM MODIFICATIONS KAON FORM FACTOR

Results for the in-medium space-like electromagnetic form factors of the kaon
(PTPH, Yongseok Oh, K. Tsushima, in preparation (2018))

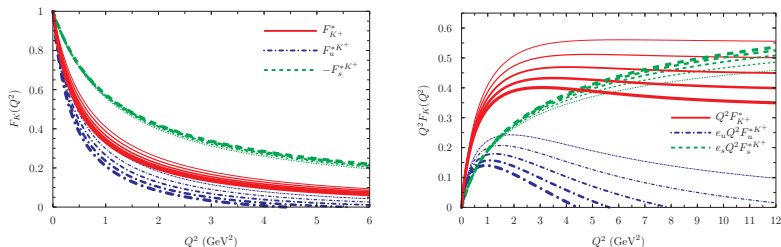


FIGURE: Results for the in-medium $Q^2 F_{K^+}^*(Q^2)$ (solid line) together with the charge-weighted quark-sector contributions. The in-medium form factors are calculated using the inputs from the calculated NJL model combined with the QMC model for $\rho_B/\rho_0 = [0.00, 0.25, 0.50, 0.75, 1.00]$. The difference densities are represented by thinner to thicker lines.

- Our kaon form factor results decrease with increasing density as the pion case

IN-MEDIUM MODIFICATIONS CHARGE RADIUS

Results for the charge radii of the charged kaon in the nuclear medium and its quark sector charge radii

TABLE: Results for the in-medium charge radii of the charged kaon and its in-medium quark sector charge radii. This is calculated in the NJL model using the inputs from the standard QMC model. All in-medium charge radii along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_K	r_u	r_s	r^{expt}
0.00	0.59	0.65	0.44	0.56 ± 0.03
0.25	0.62	0.69	0.44	
0.50	0.65	0.73	0.44	
0.75	0.68	0.77	0.44	
1.00	0.71	0.81	0.44	

CONCLUSION AND OUTLOOK

- We have studied a kaon and pion properties in vacuum as well as in medium.
- Our prediction on pion and kaon properties are in good agreement with other prediction
- We have extend our study on the in-medium modifications form factors of pion and kaon in order to understand the feature of form factors of the kaon and pion in the medium. The result looks very interesting and promising. we need experimental data to test our prediction
- It would be interesting to extend calculation to parton distributions (PDFs) or generalized parton distributions (GPDs) of the pion, kaon, and ρ meson in medium

THANK YOU VERY MUCH FOR ATTENTION !!