

Di-Higgs day @ Konkuk University 2019. 06. 27. Thursday

Probing the trilinear Higgs boson coupling in the di-Higgs production at the LHC and its performance improvement using machine learning

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- Ref : 1. An exploratory study of Higgs-boson pair production **(JHEP 1508(2015) 133)**
	- 2. Higgs-boson-pair production $H(\rightarrow b\overline{b})H(\rightarrow y\gamma)$ from gluon fusion at the HL-LHC and HL-100 TeV hadron collider **(Arxiv :1804.07130)**
	- 3. [Higgs-boson-pair production](http://inspirehep.net/record/1668935) $H(\rightarrow b\overline{b})H(\rightarrow \gamma\gamma)$ [from gluon fusion with multivariate techenique](http://inspirehep.net/record/1668935) **(Work in Progress)**
	- 4. Introduction to Machine Learning with Python (Andreas Muller)

Contents

- Higgs in the SM (Prof. Song's presentation)
- How to probe it ? : Higgs pair production (From the experimental view, Dr. Yu's presentation) (**From the intermediate(?), this talk**) (From the theoretical view, Prof. M. Park's and Prof. S. Park's talks
- Simulation
- Analysis (TMVA, BDT) BDTL1 scenario
- Expected yields and significance(Z)
- Conclusion
- Recent updates

Preliminary

Recent updates

- HH NLO generator POWHEG BOX V2 Use of NLO kinematic distributions (or variables)
- Improved yields and significance(Z)
- Improved Likelihood fit using the M_{HH} kinematic distribution

Higgs In the SM

- Higgs field (h) : responsible for ① the spontaneous EW symmetry breaking ② the generation of masses of all the SM particle
- The potential is characterized by only two parameters : $\circled{1}$ vacuum expectation value ν (2) the Higgs mass m_H

$$
v = \frac{1}{\sqrt{2} G_F} \approx 246 \text{ GeV} \qquad V_{SM}(h) = \frac{1}{2} m_H^2 h^2 + \lambda_3 v h^3 + \frac{1}{4} \lambda_4 h^4
$$

Trilinear and Quartic Higgs boson coupling in the SM

$$
\lambda_3^{SM} = \lambda_4^{SM} = \frac{m_H^2}{2 v^2}
$$

New Physics can affect the Higgs potential form

$$
\lambda_3 = \lambda_3^{SM} + \delta \lambda_3^{SM} , \qquad \lambda_4 = \lambda_4^{SM} + \delta \lambda_4^{SM}
$$

Trilinear Higgs boson coupling

 $\lambda_3 = \lambda_3^{SM} + \delta \lambda_3^{SM}$

How to probe it ?

We now focus on

Higgs pair production

Why Higgs pair production so interesting?

Allows accessing crucial components of the Higgs sector !!!

can help to reconstruct the electroweak symmetry breaking potential

can probe the Higgs self-coupling

may reveal the doublet nature of the Higgs by means of the hhVV coupling

Higgs pair productions at the LHC **Production modes**

Gluon Fusion Vector Boson Fusion $\frac{1}{\epsilon}$ $\frac{1}{H}$

Top associated productions

Higgs strahlung

Why Higgs pair production so difficult?

In the leading gluon fusion production mode, the cross section at 14 TeV is only 45 fb (in the SM), further suppressed by each decay branching fractions.

45 fb \leftrightarrow NNLO accuracy including NNLL gluon resummation in the infinite top quark mass approximation.

Why Higgs pair production so difficult?

Full top-quark mass effect at the NLO

45.05 fb 36.69 fb

Recently, the NLO corrections considering full top-quark mass dependence have been available. We observe that **20 % reduction** at 14 TeV compared to the cross sections used.

Strong QCD backgrounds

Which search mode would be better to use?

Search channels for

at Collider

Our Channel reconstruct τ **/ W**

b-tagging, QCD BG

Huge $t\bar{t}$ **BG** Huge hadronic BG small BR relatively clean channel dominate BGs comes from fake photon or b-jet

Simulation

Outline of Simulations

2015 MadGraph school on Collider Phenomenology November 23-27 @ Shanghai

$\sigma \cdot Br(H \to \gamma \gamma)Br(H \to b\overline{b}) = 0.119$ fb \longrightarrow 0.096 fb ① Updated Signal Cross Section (about 20% reduction)

② Updated non-resonant BG with the recent PDF

 $CTEQ6L1 PDF$ $CT14LO PDF$ $(+ about 20% reduction)$

Cuts at the generator level on the non-resonant BGs

 $P_{T_i} > 20$ GeV, $P_{T_b} > 20$ GeV, $P_{T_{\gamma}} > 25$ GeV, $P_{T_i} > 10$ GeV, $|\eta_j| < 5, |\eta_\gamma| < 2.7, |\eta_l| < 2.5, \ \Delta R_{jj, ll, \gamma\gamma, \gamma j, il, \gamma l} > 0.4,$ $M_{ij} > 25 \text{ GeV}, M_{bb} > 45 \text{ GeV}, 60 < M_{\gamma\gamma} < 200 \text{ GeV}.$

CTEQ6L1 v.s. CT14LO CT14LO contains the LHC RUN I results

They show similar kinematic distributions.

Analysis

Analysis Methods

• Cut Based Analysis

• Machine Learning Multi-Variate Analysis : BDT (Boosted Decision Tree) (work in progress)

TMVA (Toolkit for Multivariate Data Analysis with ROOT)

A. Hoecker, P. Speckmayer, J. Stelzer, J. Therhaag, E. von Toerne, and H. Voss, *TMVA - Toolkit for Multivariate Data Analysis*, PoS ACAT 040 (2007), arXiv:physics/0703039

Machine LearningTMVA package

Pre-selection cuts for TMVA study

We use TMVA analysis to improve our previous result

TMVA Inputs : 8 (kinematic) variables

 $\bm{P_T}_{\bm{\gamma} \bm{\gamma} \bm{\gamma}}$, $\bm{P_T}_{\bm{b} \bm{b}}$, $\bm{M}_{\bm{\gamma} \bm{\gamma}}$, $\bm{M}_{\bm{b} \bm{b}}$, $\bm{M}_{\bm{\gamma} \bm{\gamma} \bm{b} \bm{b}}$, $\Delta \bm{R}_{\bm{\gamma} \bm{b}}$, $\Delta \bm{R}_{\bm{\gamma} \bm{b}}$

Correlation Matrix among variables

Linear correlation coefficients in % 100 34 21 21 24 M aabb $\overline{4}$ 8 100 80 32 DR ab -19 -1 23 -34 -16 100 8 60 40 20 DR aa -52 -16 6 -21 100 20 DR bb 66 -50 100 20 -34 $\overline{2}$ -24 $\overline{4}$ 0 PT aa 39 18 -24 -52 23 24 100 -20 18 5 100 21 M aa 7 $\overline{2}$ -1 -40 -60 PT bb 100 $5₅$ 39 -50 -21 32 21 -80 M bb -19 34 100 $\overline{7}$ 66 6 -100 DR_ab M_aabb $P\mathcal{T}_{\neg a_d}$ DR $_{bb}$ DR $_{aa}$ M_{bb} $P_{\text{L}}b_{\text{L}}$ $M_{\text{L}}a_{\text{R}}$

Correlation Matrix (background)

Correlation Matrix (signal)

Various ML methods in TMVA

Background rejection versus Signal efficiency

Background rejection

ROC = Receiver Operating Characteristic curve

: a good way to illustrate the performance of given classifier

> **AUC = Area under the ROC curve**

Background rejection versus Signal efficiency

BDT (Boosted Decision Tree)

DT (Decision Tree)

22. A decision tree to distinguish among several animals

Decision Trees

Introduction to Machine Learning with Python (Andreas Muller)

- Decision trees are widely used models for classification and regression tasks. Essentially, they learn a hierarchy of if/else questions, leading to a decision.
- This series of questions can be expressed as a decision tree, as shown in

Figure 2-22. A decision tree to distinguish among several animals

<One example>

How to classify this set ?

Figure 2-23. Two-moons dataset on which the decision tree will be built

False

counts = $[1, 10]$

Figure 2-25. Decision boundary of tree with depth 2 (left) and corresponding decision tree (right)

Finally…..

Figure 2-26. Decision boundary of tree with depth 9 (left) and part of the corresponding tree (right); the full tree is quite large and hard to visualize

However,

If we don't restrict the depth of a decision tree, the tree can become arbitrarily deep and complex.

Easily Overfitting !!

Supervised Learning

- Classification and Regression
- Generalization, Overfitting, and Underfitting

To avoid the overfitting,

Ensembles of Decision Trees

• There are two ensemble models that have proven to be effective on a wide range of datasets for classification and regression, both of which use decision trees as their building blocks: **random forests** and **gradient boosted decision trees**.

Use the **ensemble average** ! Use the **strong pre-pruning** !

Gradient boosted trees often use very shallow trees, of depth one to five,

> Shallow tree + Shallow tree + Shallow tree + ………

Other methods (algorithms)

- k-Nearest Neighbors
- Linear Models (Linear Fitting)
- Decision Trees

• ……

- Support Vector Machines
- Neural Networks (Deep Learning)

Decision boundaries created by the nearest neighbors model for different values of n_neighbors
Neural Network (Briefly……)

Decision functions for different numbers of hidden units and different settings of the alpha parameter

BDT setup in TMVA

- Pre-Selection Cuts (1) (5) → TMVA inputs
- NTrees=800
- MinNodeSize=2.5%
- MaxDepth=4
- BoostType=AdaBoost
- AdaBoostBeta=0.5
- UseBaggedBoost
- BaggedSampleFraction=0.5
- SeparationType=GiniIndex
- nCuts=20

BDT machine optimized with $\lambda_{3H} = 1$

 $AUC = 0.996$: Area under ROC (receiver operating characteristic curve) curve

Expected Yields and

Significance(Z)

C&C

- **# of signal = 9.15 # of bg = 81.70**
- ∴ **significance Z = 0.99**

BDT λ_1 **# of signal = 7.64 # of bg = 31.27**

∴ **BDT-improved Z = 1.32**

33% enhancement on Z

Significance of the signal over the background versus λ_{3H} at the HL-LHC

Significance of the signal over the background versus λ_{3H} at the HL-LHC

Z v.s. $\lambda_{3H}=1$ in the cases of C&C and BDTL1

: It may be possible to measure at the LH-LHC with $6ab^{-1}$.

If we can measure(?) the signal and bg. numbers for the case of $\lambda_{3H} = 1$ at the LH-LHC with **6** ab^{-1}

M_{hh} kinematic distributions

Log-Likelihood Ratios with the $\lambda_{3H} = 1$ nominal data set and the different λ_{3H} template sets

Impact of NLO

Preliminary

Recent updates

- HH NLO generator POWHEG BOX V2 Use of NLO kinematic distributions (or variables)
- Improved (NLO) yields and significance(Z)
- Improved Likelihood fit using the (NLO) M_{HH} kinematic distribution

Preliminary

HH NLO generator - POWHEG BOX V2

Validation of the NLO sample

At the parton level

After the decay of the Higgs boson

Validation of the NLO sample

At the detector level

TMVA (NLO) input variables

BDTL1 scenario

Improved (NLO) yields and significance(Z)

Improved Likelihood fit using the (NLO) M_{HH} kinematic distribution

Conclusion

- 1. We examine the impact of the full NLO corrections considering full top-quark mass dependence, and the recent CT14LO PDF.
- 2. (CT14LO) At 14 TeV with 3000 fb^-1, the trilinear coupling is constrained to be $-1.1 <$ λ_{3H} < 7.0 at 95% CL taking account of the uncertainties associated with the top-Yukawa coupling and the estimation of backgrounds.
- 3. (CT14LO) Taking the central line, the 95% CL sensitivity region for λ_{3H} is -0.3 < λ_{3H} < 6.7
- 4. (BDTL1) The trilinear coupling is constrained to be -0.7< λ_{3H} < 7.5 at 95% CL taking account of the uncertainties associated with the top-Yukawa coupling and the estimation of backgrounds.
- 5. (BDTL1) Taking the central line, the 95% CL sensitivity region for λ_{3H} is 0.2 < λ_{3H} < 7.1
- 6. (BDTL1 + NLO) Taking the central line,
the 95% CL sensitivity region for λ_{3H} is 0.6 < λ_{3H} < 6.7

\cdot HL-LHC : constraint the λ_{3H}

1. Cut-Based Analysis : $-1.1 < \lambda_{3H}$ < 7.0 at 95% CL,

Z v.s. λ_{3H} =1 in the cases of C&C and BDTL1

Last, but not least

….

- **Combined analysis** : $b\bar{b}b\bar{b} + b\bar{b}\gamma\gamma + b\bar{b}\tau\tau \rightarrow Z$ ($\geq 3\sigma$) ATL-PHYS-PUB-2018-053
- **Advanced technology in the future** … Increased luminosity, improved tagging efficiency, improved resolution and so on….
- **More precise simulation, higher order QCD correction ..** Improved MC Event generators (at the NLO, NNLO QCD order), QCD NLO, NNLO, NNNLO corrections …

\cdot HL-LHC : constraint the λ_{3H}

END

Backup Slides I

Further improvements to be made

- New low and high-level kinematic variables : ex. 8 var. -> 22 var.
- Finding the optimized pre-selection cuts (Hyperopt)

Impurity

$$
Purity: p = S/(S+B)
$$

Gini Index:

$$
p\cdot (1-p)
$$

Cross entropy:

$$
-p \cdot ln(p) - (1-p) \cdot ln(1-p)
$$

Misclassification error:

$$
1 - \max(p, 1 - p)
$$

Statistical significance:

$$
S/\sqrt{S+B}
$$

Boost and Bagging

- A way of enhancing the classification performance
- Increasing the stability with respect to statistical fluctuations in the training sample
- TMVA Provides :
- **Boosting** original data weighted data • Adaptive Boost (boosting tree) weighted sum • Gradient Boost (GBDT) random subsets • Bagging (random forest) **Majority Vote TMVA Users Guide**

Adaptive Boost

- Starting with the original event weights when training the first decision tree
- Subsequent tree is trained using a modified event sample (previous misclassified events multiplied by a boost weight a)

$$
\alpha = \frac{1 - \text{err}}{\text{err}} \quad \text{err = misclassified / tot.}
$$

• The boosted event classification : (small – background-like; large – signal-like event)

$$
y_{\text{Boost}}(\mathbf{x}) = \frac{1}{N_{\text{collection}}} \cdot \sum_{i}^{N_{\text{collection}}} \ln(\alpha_i) \cdot h_i(\mathbf{x}) \quad h(x) = \pm 1 \quad \text{Signal}
$$

• The learning rate $\alpha \rightarrow \alpha^{\beta}$

M_hh distributions II

Log-Likelihood Ratios v.s Lambda_3H

(NLO-) Correlation Matrix

Correlation Matrix (signal)

Correlation Matrix (background)

Event selection on Cut-and-Count Analysis

These red conditions of cuts were very important to distinguish signal and background on the Cut-and-Count Analysis !

Result at the HL-LHC

Timeline of LHC

LHC RUN II

Machine Learning

From Wikipedia

- **Machine learning** (ML) is a field of [artificial intelligence](https://en.wikipedia.org/wiki/Artificial_intelligence) that uses statistical techniques to give <u>computer systems</u> the abili[ty to](https://en.wikipedia.org/wiki/Data) "learn" (e.g., progressively
improve perf[orm](https://en.wikipedia.org/wiki/Machine_learning#cite_note-2)a[nce on a specific ta](https://en.wikipedia.org/wiki/Computer_systems)sk) from <u>[data](https://en.wikipedia.org/wiki/Data)</u>, without being explicitly
programmed.^{[\[2\]](https://en.wikipedia.org/wiki/Machine_learning#cite_note-2)}
- The name *machine learning* was coined in 1959 by <u>[Arthur Samuel.](https://en.wikipedia.org/wiki/Arthur_Samuel)[1</u>] Machine learning explores the study and construction of **[algorithms](https://en.wikipedia.org/wiki/Algorithm)** that can learn from and make predictions on data^[3] – such algorithms overcome following stri[ctl](https://en.wikipedia.org/wiki/Machine_learning#cite_note-bishop2006-4)y
static <u>[program instructions](https://en.wikipedia.org/wiki/Computer_program)</u> by making data-driven predictions or decisions,^{[4]:2} through building a <u>model</u> from sample inputs. Machine learning is employed
in a range of comp[uting t](https://en.wikipedia.org/wiki/Mathematical_model)asks where designing and programming explicit algorithms with good performance is difficult or infeasible; example [applications inclu](https://en.wikipedia.org/wiki/Computer_vision)de <u>email filtering</u>, detection of network intruder's, and
<u>computer vision</u>.

[Ian Goodfellow, Yoshua Bengio a nd Aaron Courville] 'Deep Learning'

(In machine learning language) "machine" = 'model (from data)'

Learning $=$ Improving performance at a task (ex) with experience

(In machine learning language)

"Learning" = Optimization of (machine's/model's) parameters of a proper error function which represents performance at a task.

Therefore, "I am training a machine

 $=$ I am building a new model (from data)

- No labels
- No feedback
- "Find hidden structure"
- Decision process
- Reward system
- Learn series of actions

<http://solarisailab.com/archives/1785>, 솔라리스의 인공지능 연구실

Supervised Learning

Supervised Learning

- Classification and Regression
- Generalization, Overfitting, and Underfitting

This figure is really important !!!

Each λ_{3H} optimized BDT machine

$\sigma \times \mathcal{L} \times a \times \epsilon$

Each λ_{3H} optimized BDT machine

 $AUC = 0.992 - 0.996$

•
$$
\lambda_{3h} = 1
$$
 Optimized

 λ_{3h} Optimized \bullet

Recent Update I (on the signal)

• 1. We examine the impact of the NLO corrections considering full top-quark mass dependence.

(gg -> HH) = 45.05 fb (14 TeV), (gg -> HH) = 1749 fb (100 TeV) Previously,

NNLO accuracy including NNLL gluon resummation in the infinite top quark mass approximation.

Full top-quark mass effect at the NLO

1. Recently, the NLO corrections considering full top-quark mass dependence have been available. 2. We observe that 20 (30) % reduction at 14 (100) TeV compared to the cross sections used.

```
(gg -> HH) = 36.69 fb (14 TeV), 
(gg -> HH) = 1224 fb (100 TeV)
```
Double counting problem and their cross sections

- We have also checked the double counting problems between ME and PS on the non-resonant backgrounds.
- The final merged cross sections are reduced by about 20~30% (bbaa, zaa), about 30~40% (ccaa), about 50~60% (bbja, ccja), about $> 60\%$ (jjaa).
- However, the merged cross section of the "bbjj" is not reliable.

Faking Processes and rates

Preliminary

Preliminary

Understanding the process in the effective Lagrangian

$$
-\mathcal{L} = \frac{1}{3!} \left(\frac{3M_H^2}{v} \right) \left(\lambda_{3H} H^3 + \frac{m_t}{v} \bar{t} \left(g_t^S \right) + i \gamma_5 g_t^P \right) t H + \frac{1}{2} \frac{m_t}{v^2} \bar{t} \left(g_{tt}^S \right) + i \gamma_5 g_{tt}^P \right) t H^2
$$

 \star In the SM, $\lambda_{3H} = g_t^s = 1$ and $g_{tt}^s = g_{tt}^p = 0$

SM Higgs self couplings

$$
\mathcal{L} = -\frac{1}{2}m_H^2 H^2 - \frac{g_{HHH}}{3!}H^3 - \frac{g_{HHHH}}{4!}H^4
$$

In the gluon fusion process at the hadron collide $q(p_1)q(p_2) \rightarrow H(p_3)H(p_4)$

 g ннн $=$ $3m_H^2$ $\frac{1}{v}$, g _{HHHH} = $3m_H^2$ v^2

The differential cross section is given by

$$
\frac{d\hat{\sigma}(gg \to HH)}{d\hat{t}} = \frac{G_F^2 \alpha_s^2}{512(2\pi)^3} \left[\left| \lambda_{3H} g_t^S D(\hat{s}) F^S_{\triangle} + (g_t^S)^2 F^{SS}_{\square} \right|^2 + \left| (g_t^S)^2 G^{SS}_{\square} \right|^2 \right]
$$

Feynman diagrams

$$
\frac{d\hat{\sigma}(gg \to HH)}{d\hat{t}} = \frac{G_F^2 \alpha_s^2}{512(2\pi)^3} \left[\left| \lambda_{3H} g_t^S D(\hat{s}) F^S_{\triangle} + (g_t^S)^2 F^{SS}_{\square} \right|^2 + \left| (g_t^S)^2 G^{SS}_{\square} \right|^2 \right]
$$

In the heavy quark limit

$$
F_{\triangle}^S = + \frac{2}{3} + \mathcal{O}(\hat{s}/m_Q^2) \,, \qquad \qquad F_{\square}^{SS} = - \frac{2}{3} + \mathcal{O}(\hat{s}/m_Q^2) \,, \quad F_{\square}^{PP} = + \frac{2}{3} + \mathcal{O}(\hat{s}/m_Q^2) \label{eq:FS}
$$

There is large cancellation between the triangle and box diagrams

The production cross section normalized to the corresponding SM cross section :

$$
\frac{\sigma^{LO}(gg \to HH)}{\sigma_{\rm SM}^{LO}(gg \to HH)} = \underbrace{c_1(s)}_{0.263} \lambda_{3H}^2 (g_t^S)^2 + \underbrace{c_2(s)}_{-1.310} \lambda_{3H} (g_t^S)^3 + \underbrace{c_3(s)}_{2.047} (g_t^S)^4
$$
\n
$$
= 1.108
$$
\n1.900 100 TeV

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For the reference, there are various production modes

The gluon fusion production mode is dominant one !

(gg -> HH) = 45.05 fb,

 $(qq_0 \rightarrow HHqq_0) = 1.94$ fb, $(qq(0) \rightarrow V HH = 0.567(V = W) = 0.415(V = Z)$ fb, $(qq/qq \rightarrow$ ttHH $) = 0.949$ fb are calculated at NNLO+NNLL, NLO, NNLO, and NLO, respectively

 $0.4 < \Delta R_{\gamma\gamma} < 2.0$ $0.4 < \Delta R_{b\bar{b}} < 2.0$

 $P^{\gamma\gamma}_T > 80$ GeV

450

 $p_T^{bb}(GeV)$

500

Machine Learning approaches to the Higgs boson self coupling

① BDT(Boosted Decision Tree) : bbΥΥ

1. Phys.Rev. D96 (2017) no.3, 035022 (Alves, Alexandre et al.) arXiv:1704.07395 [hep-ph]

BDT + kinematic cuts \Box 5 σ (4.6 σ) significance with 10 %(20%) systematics and 3 ab^-1

② (Supervising) Deep Neural Networks (DNN) : bbWW + bbττ

1. "Supervising Deep Neural Networks with topological augmentation in search for di-Higgs production at the LHC (Dr. Won Sang Cho)

5 classes by the number of leptonic taus

Optimass & its compatibility distance with dim. Of vars \sim 40

AUC of ROC = 0.991

Eff(sig) $@$ (Background purity=0.01) = 0.84

Machine Learning approaches to the Higgs boson self coupling

③ DNN (ANN : a multi-layer feed-forward artificial neural network) : bbbb1. Eur. Phys. J. C (2016) 76:386 (Katharina Behr, Bortoletto et al.) arXiv:1512.08928 [hep-ph]

DNN + kinematic cuts $\frac{S}{\sqrt{B}} \sim 3$ o significance with 3 ab^-1

Summary Table

Summary Table

Summary Table

The ratio increases by about 10 (35) % at λ_{3H} =-1 (5)!

It is clear that the QCD corrections are less significant than the uncertainties associated with the top-Yukawa coupling.

About top Yukawa uncertainty !

Ratio of cross sections (gg -> HH)=(gg -> HH)sm versus $3H$ taking account of 10% uncertainty of the top-Yukawa coupling: $g_{st} = 1:1$ (black), 1 (blue), and 0:9 (red) for sqrt(s) = 14 TeV (left) and sqrt (s) = 100 TeV (right).

Y_t precision measurement

At the HL-LHC, the expected precision of measurement of the top-quark Yukawa coupling (Yt) is 10%.

- Rectangular cut optimisation (binary splits, Sec. 8.1).
- Projective likelihood estimation (Sec. 8.2).
- Multi-dimensional likelihood estimation (PDE range-search Sec. 8.3, PDE-Foam $-$ Sec. 8.4, and k-NN $-$ Sec. 8.5).
- Linear and nonlinear discriminant analysis (H-Matrix Sec. 8.6, Fisher –
Sec. 8.7, LD Sec. 8.8, FDA Sec. 8.9).
- Artificial neural networks (three different multilayer perceptron implementations – Sec. 8.10).
- Support vector machine (Sec. 8.12). Boosted/bagged decision trees (Sec. 8.13).
- Predictive learning via rule ensembles (RuleFit, Sec. 8.14).
- A generic boost classifier allowing one to boost any of the above classifiers (Sec. 10).
- A generic category classifier allowing one to split the training data into disjoint categories with independent MVAs.