Chiral soliton lattice in QCD

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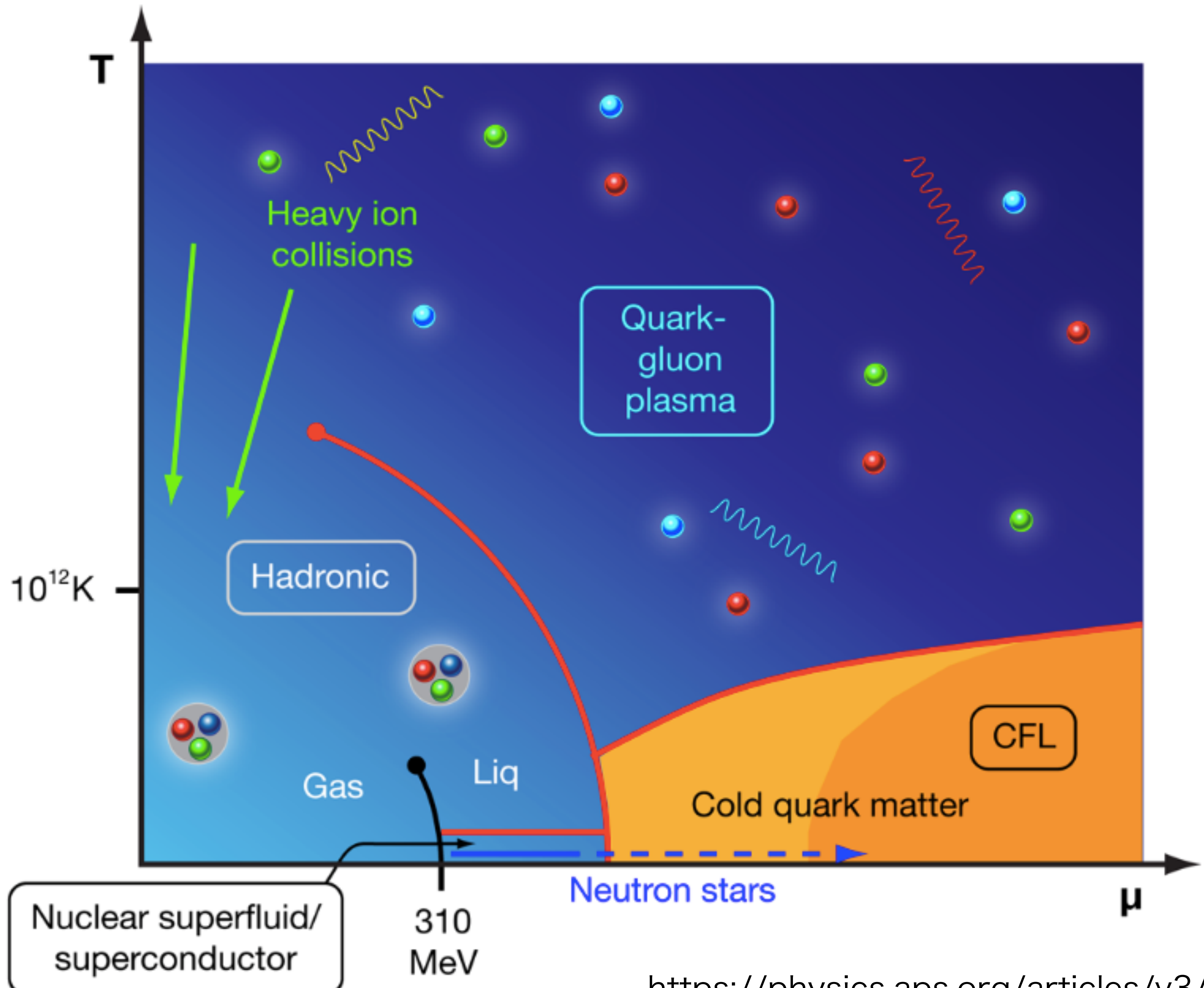
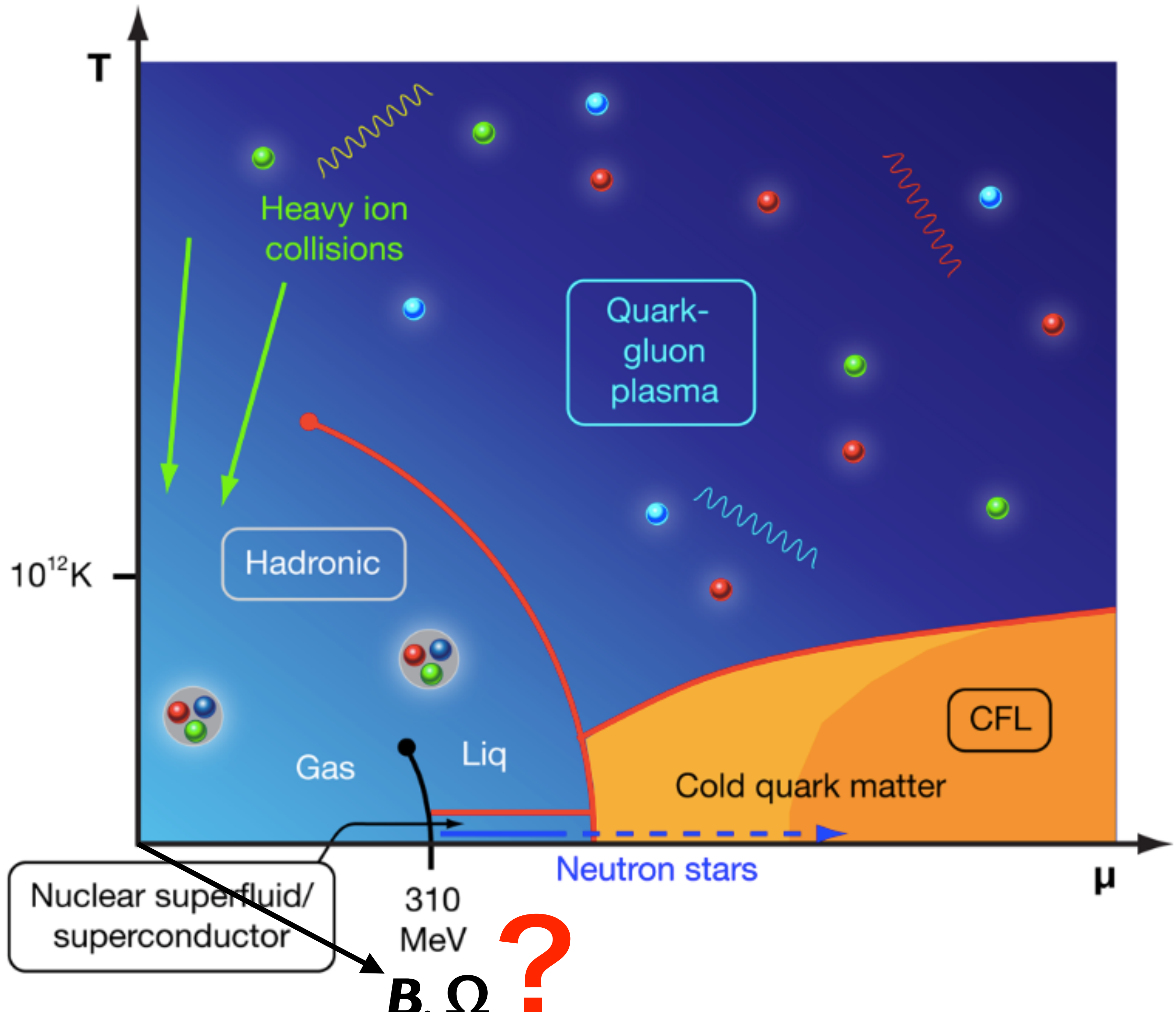
QCD phase diagram

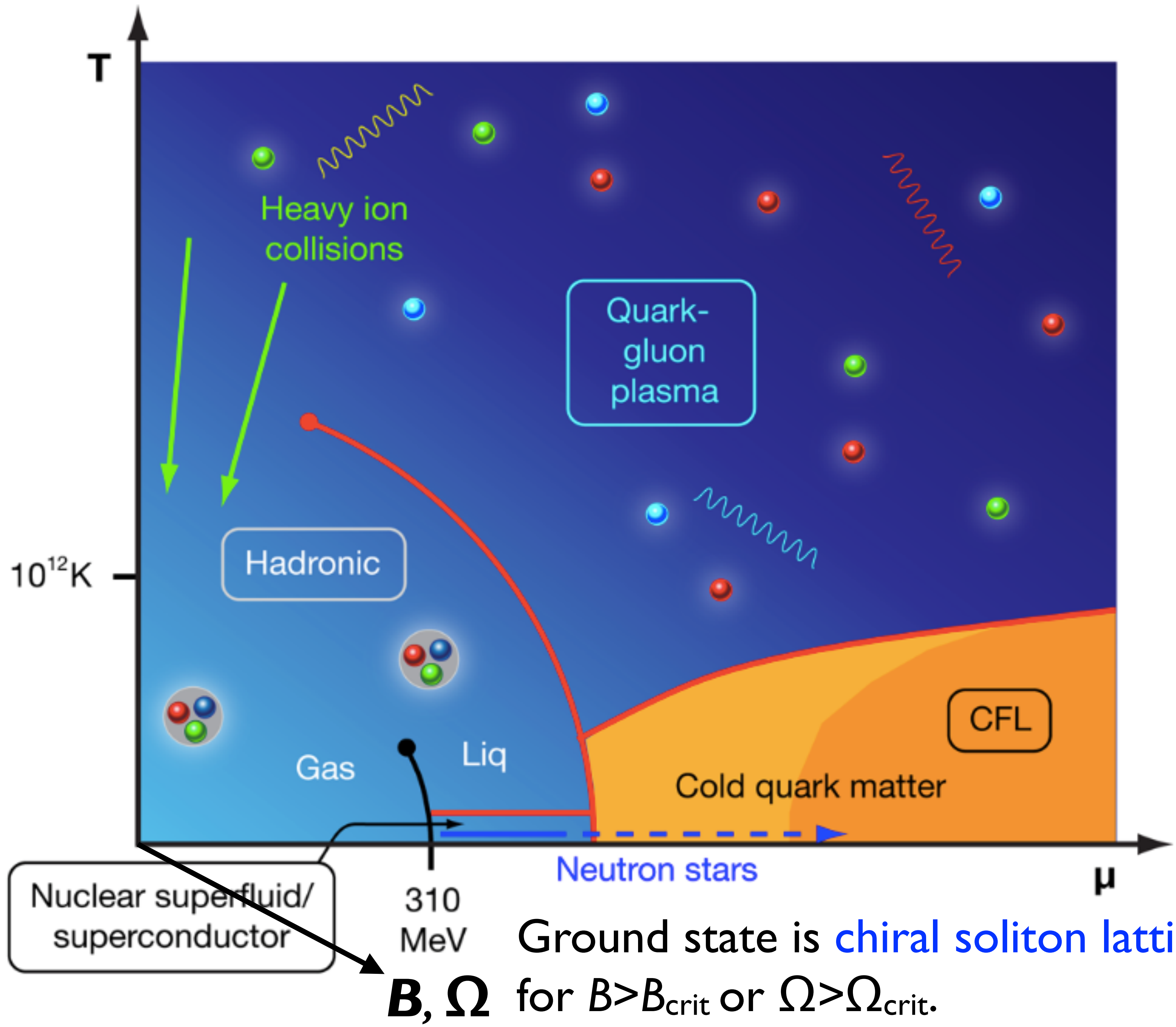


https://physics.aps.org/articles/v3/44

QCD phase diagram



QCD phase diagram



Ground state is chiral soliton lattice (CSL) for $B>B_{crit}$ or $\Omega>\Omega_{crit}$.

Chiral Soliton Lattice

- Violates parity

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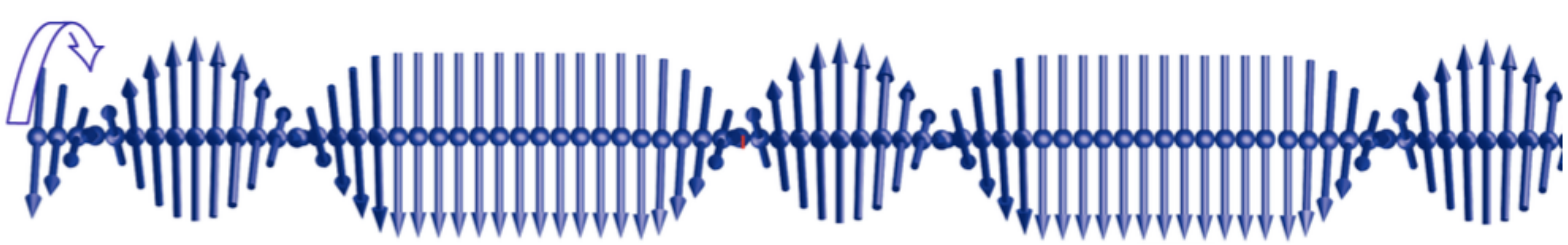
CSL is realized as a ground state of QCD at finite μ and $\boldsymbol{B}$ or **Ω**.

cf) CSL known to appear in chiral magnets and liquid crystals



Y. Togawa, *et al.*, PRL (2012)

Anomalous effects at finite μ

Son, Zhitnitsky; PRD (2004); Son, Stephanov, PRD (2008)

- Anomalous term in the vacuum: $\mathcal{L}_{\rm anom} = C\pi^0 \boldsymbol{E}\cdot\boldsymbol{B}, \quad C = \dfrac{1}{4\pi^2}$

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- Anomalous term in the vacuum: $\mathcal{L}_{\mathrm{anom}} = C\pi^0 \boldsymbol{E}\cdot\boldsymbol{B}, \quad C = \dfrac{1}{4\pi^2}$
- Anomalous term at finite μ and B: $\mathcal{L}_{\mathrm{anom}} = C\mu\boldsymbol{\nabla}\pi^0\cdot\boldsymbol{B}$

$$\because \; S_{\mathrm{anom}} = C\int \mathrm{d}^4x\;\pi^0(-\boldsymbol{\nabla}\phi)\cdot\boldsymbol{B} \sim C\int \mathrm{d}^4x\;\phi\boldsymbol{\nabla}\pi^0\cdot\boldsymbol{B}$$

scalar potential

Chiral perturbation theory

- Low-energy effective theory of QCD (model-independent)
- Constructed based on chiral symmetry breaking
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- Low-energy effective theory of QCD (model-independent)
- Constructed based on chiral symmetry breaking
- Systematic expansion in derivatives and quark masses
- π^0 sector to leading order:

$$\mathcal{H} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\pi^0)^2 - \underbrace{C\mu \boldsymbol{B}\cdot\boldsymbol{\nabla}\pi^0}_{\text{anomalous term}} - \underbrace{m_\pi^2 f_\pi^2 \cos\pi^0}_{\text{mass term}} + \text{const.}$$

QCD vs. cond-mat

- QCD at finite μ and ***B*** (***B***=B***z***):

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - \underbrace{C\mu B \partial_z \pi^0}_{\text{anomalous}} - \underbrace{m_\pi^2 f_\pi^2 \cos\pi^0}_{\text{mass}} + \text{const.}$$

- Chiral magnets: Kishine, Ovchinnikov, Solid State Phys. **66** (2015)

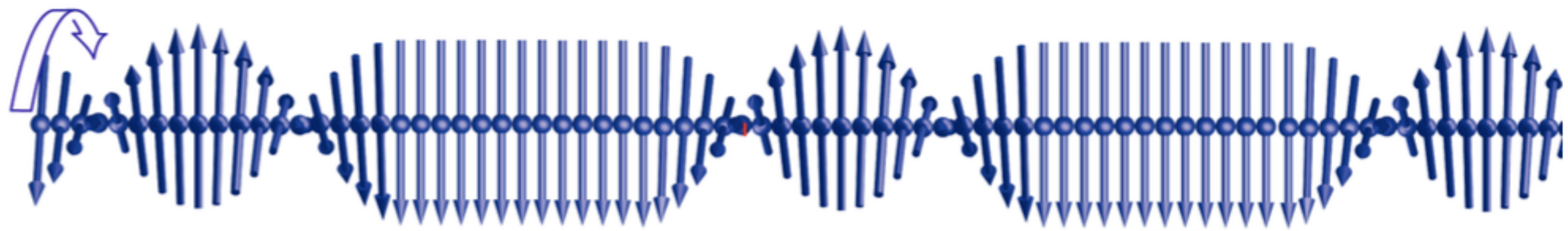
$$\mathcal{H} = JS^2 a \left[\frac{1}{2}(\partial_z \phi)^2 - \underbrace{q_0 \partial_z \phi}_{\text{Dzyaloshinskii-Moriya}} - \underbrace{m^2 \cos\phi}_{\text{Zeeman}}\right] + \text{const.}$$

Two Hamiltonians are mathematically equivalent.

Chiral Soliton Lattice (CSL)

1D lattice of topological solitons w/ parity and translational symm. breaking

cf) chiral magnet

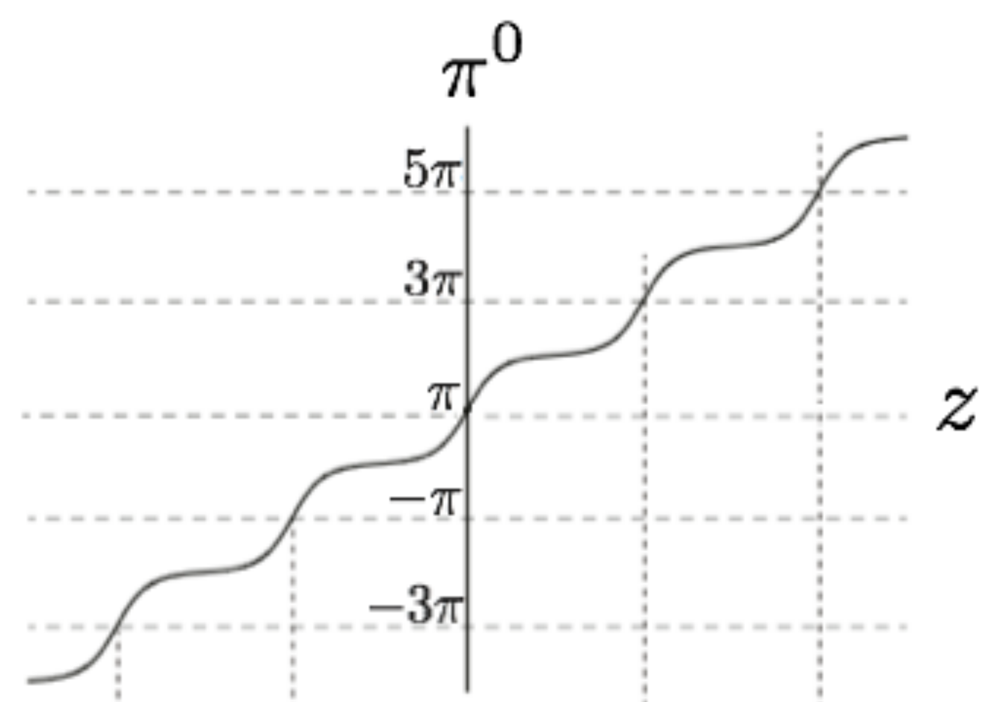


Y. Togawa, *et al.*, PRL (2012)

$$\cos\frac{\pi^0(\bar{z})}{2} = \mathrm{sn}(\bar{z}, k)\,, \quad \bar{z} = \frac{z m_\pi}{k}$$

Jacobi elliptic function

elliptic modulus



Brauner, Yamamoto, JHEP (2017)

Rough picture

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - C\mu B \partial_z \pi^0 - \cancel{m_\pi^2 f_\pi^2 \cos\pi^0} + \text{const.}$$

- **2nd term ≫ 3rd term:** $\langle \pi^0 \rangle = \dfrac{C\mu B}{f_\pi^2} z + \text{const.}$

Rough picture

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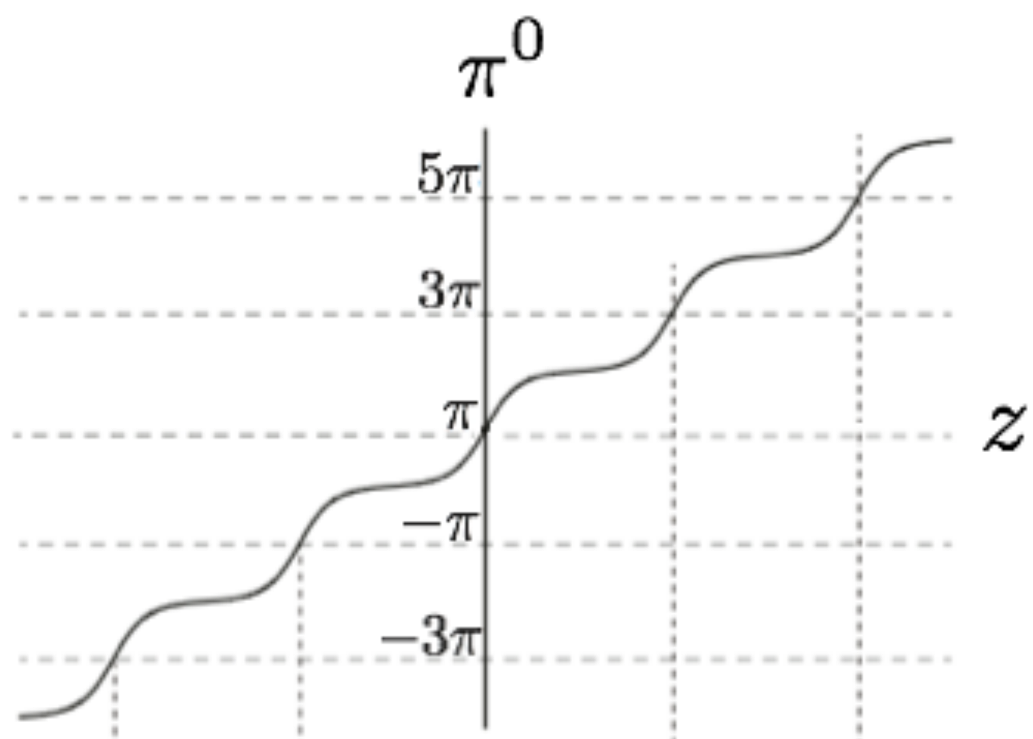


Topological charges

$$\mathcal{H} = \frac{f_\pi^2}{2}(\partial_z \pi^0)^2 - C\mu B \partial_z \pi^0 - m_\pi^2 f_\pi^2 \cos \pi^0 + \mathrm{const.}$$

- CSL carries two topological charge densities:
  - Baryon number: $n_{\mathrm{B}}(z) = -\dfrac{\partial \mathcal{H}}{\partial \mu} = CB\partial_z \pi^0(z)$
  - Magnetization: $m(z) = -\dfrac{\partial \mathcal{H}}{\partial B} = C\mu \partial_z \pi^0(z)$

Son, Stephanov, PRD (2008)

Topological charges

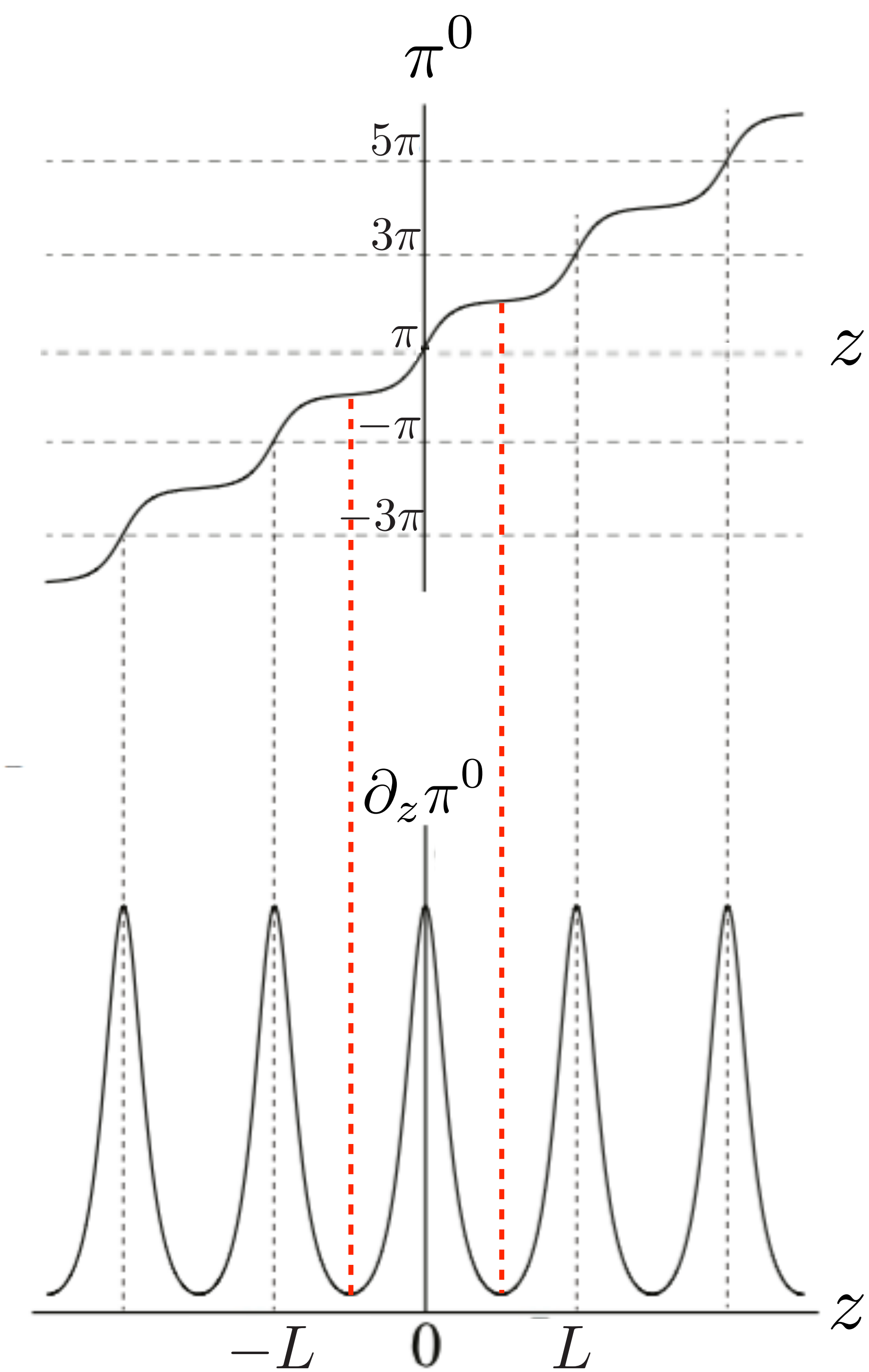
- Each domain wall has baryon charge:

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} n_{\mathrm{B}}(z) = CB\left[\underbrace{\pi^0\left(\frac{L}{2}\right)}_{2\pi} - \underbrace{\pi^0\left(-\frac{L}{2}\right)}_{0}\right] = \frac{B}{2\pi}$$

- Baryon number and magnetization:

$$\frac{N_{\mathrm{B}}}{S} = \frac{B}{2\pi}\,, \quad \frac{M}{S} = \frac{\mu}{2\pi}$$

independent of the detailed form of π^0



Ground state and excitations

- CSL is favored than vacuum and nuclear matter for $B>B_{\mathrm{CSL}}$:

$$\frac{E(k)}{k} = \frac{\mu B}{16\pi m_\pi f_\pi^2} \qquad \therefore\ B_{\mathrm{CSL}} = \frac{16\pi m_\pi f_\pi^2}{\mu}$$

$K(k)$, $E(k)$: complete elliptic integral of the 1st/2nd kind

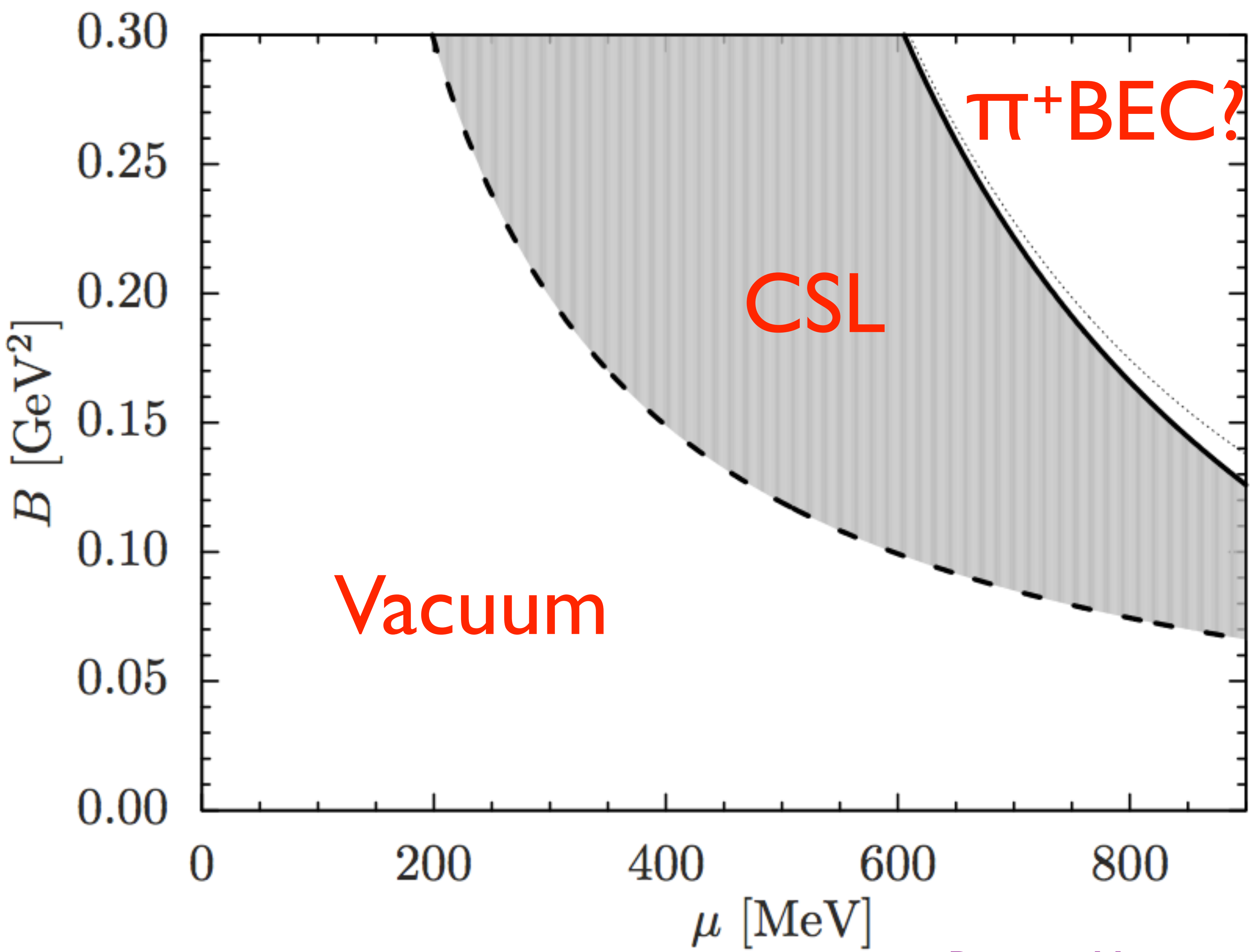
When m_q=0, QCD vacuum is unstable in an infinitesimally small B

- Phonon dispersion:

$$\omega^2 = p_x^2 + p_y^2 + (1-k^2)\left[\frac{K(k)}{E(k)}\right]^2 p_z^2 + \mathcal{O}(p_z^4)$$

Brauner, Yamamoto, JHEP (2017)

Phase diagram



Brauner, Yamamoto, JHEP (2017)

CSL under rotation

Huang, Nishimura, Yamamoto, JHEP (2018)

- (Non-renormalized) chiral vortical effect: $\boldsymbol{j}_5^{a=3} = \dfrac{\mu_{\mathrm{B}}\mu_{\mathrm{I}}}{\pi^2}\boldsymbol{\Omega}$
- "Anomaly matching" of CVE → new anomalous term for pions:

$$\mathcal{L}_{\mathrm{anom}} = \frac{\mu_{\mathrm{B}}\mu_{\mathrm{I}}}{2\pi^2 f_\pi}\boldsymbol{\nabla}\pi_0 \cdot \boldsymbol{\Omega}$$

- CSL is favored than vacuum and nuclear matter for $\Omega > \Omega_{\mathrm{CSL}}$:

$$\Omega_{\mathrm{CSL}} = \frac{8\pi m_\pi f_\pi^2}{\mu_{\mathrm{B}}|\mu_I|}$$

Axion electrodynamics in CSL

Electro-magnetism

- Effective theory for electromagnetic fields:

$$\mathcal{L} = \frac{\varepsilon}{2}\boldsymbol{E}^2 - \frac{1}{2\mu}\boldsymbol{B}^2 + C\langle\pi^0\rangle\boldsymbol{E}\cdot\boldsymbol{B} - j^\mu A_\mu$$

CSL solution

Axion electrodynamics

- Modified Maxwell's equations:

$$\epsilon \boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - C \langle \boldsymbol{\nabla} \pi^0 \rangle \cdot \boldsymbol{B},$$

$$\frac{1}{\mu} \boldsymbol{\nabla} \times \boldsymbol{B} = \epsilon \partial_t \boldsymbol{E} + \boldsymbol{j} + \underbrace{C \langle \boldsymbol{\nabla} \pi^0 \rangle \times \boldsymbol{E}}_{\text{Anomalous Hall effect}}$$

Properties of electromagnetic waves (photons) are modified

Non-relativistic photons

- For helicity +1, $\omega \sim \dfrac{f_\pi^2}{\mu B_{\rm ex}} k^2$: non-relativistic gapless photon
- For helicity -1, $\omega \sim \dfrac{\mu B_{\rm ex}}{f_\pi^2}$: gapped photon

Yamamoto, PRD (2016); Ozaki, Yamamoto, JHEP (2017);
Qiu, Cao, Huang, PRD (2017); Brauner, Kadam, JHEP (2017)

Non-relativistic photon = type-B NG mode of generalized global symmetry

See also Sogabe, Yamamoto, PRD (2019)

Summary

- New ground state of QCD: chiral soliton lattice (CSL)
- Axion electrodynamics and non-relativistic photons in CSL
- QCD phase diagram in (T, μ, B or Ω)? Physical observables?