Baryons at finite density

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1. Few words on Baryons in sum rule approach
2. Nucleon in symmetric and asymmetric nuclear medium
3. Hyperon in medium
4. Delta in medium

Acknowledgements:

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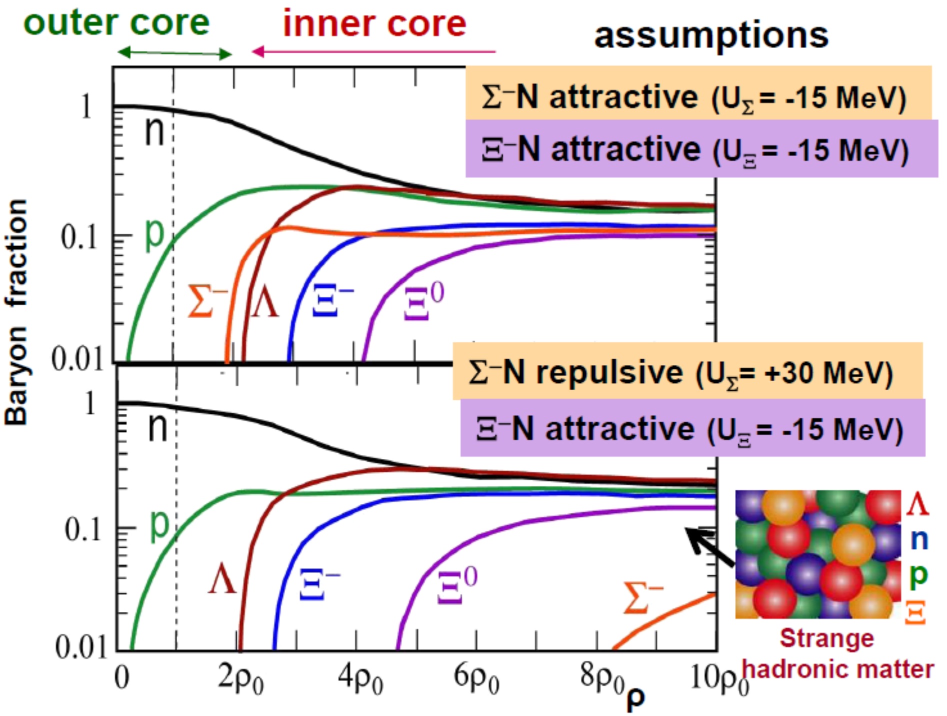
Hyperon puzzle: Slide by Tamura



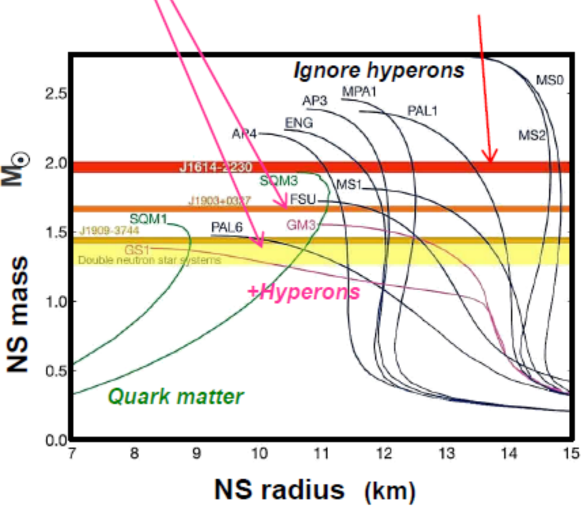
C. Ishizuka et al., J.Phys. G35 (2008) 085201

"Hyperon puzzle" in neutron stars

- Hyperons (Λ at least) should appear at $\rho \sim 2\text{-}3\ \rho_0$

  EOS's with hyperons or kaons too soft => cannot support $M > 1.5\ M_{sun}$
- Heavy NS's (~2.0 M_{sun}) were observed.

PSR J1614-2230 (2010) 1.97±0.04 M_{sun}
PSR J0348-0432 (2013) 2.01±0.04 M_{sun}



=> Unknown repulsion at high ρ

- Strong repulsion in three-body force including hyperons, NNN, YNN, YYN, YYY ?

  Chiral EFT is successful in NNN force.
  Extension to include hyperons requires **high quality YN scattering data.**
- Phase transition to quark matter ? (quark star or hybrid star)

We need to know YN, YY, $K^{bar}N$ interactions both <u>in free space</u> and <u>in nuclear medium</u>

Baryon in nuclear matter : QCD sum rule approach

- *Baryon correlation function in medium* $u=(1,0,0,0)$

$$\Pi(p)=\int d^4x\, e^{ipx}\left[\ \left\langle (q^T\Gamma q)q(x),(\bar{q}\Gamma\bar{q}i)\bar{q}(0)\right\rangle\right]=\Pi_p\cdot \not{p}+\Pi_s\cdot 1+\Pi_V\cdot \not{u}$$

→ Phenomenological side: Positive parity states

$$\Pi \Rightarrow \frac{\not{p}+M_+^*-\Sigma_V\not{u}}{(p_0-E_p)(p_0-\bar{E}_p)} \qquad E_p \xrightarrow{\vec{p}=0} =\Sigma_V+M_+^*$$

→ Operator Product Expansion

$$\Pi_p=-\frac{C_p}{64\pi^4}p^4\ln(-p^2)+\ldots$$

$$\Pi_s=\frac{C_s}{4\pi^2}p^2\ln(-p^2)\langle\bar{q}q\rangle+\ldots$$

$$\Pi_V=\frac{C_V}{3\pi^2}p^2\ln(-p^2)\langle\bar{q}\gamma_0 q\rangle+\ldots$$

....

Scalar attraction and vector repulsion

Cohen, Griegel, Furnsthal 91

$$M_+^*=-\frac{8\pi^2C_S}{M^2C_P}\langle\bar{q}q\rangle+\ldots$$

$$\Sigma_V=\frac{64\pi^2C_V}{3M^2C_P}\langle\bar{q}\gamma_0 q\rangle+\ldots$$

$$C_{P,S,V}=1 \ \text{ for } \ \eta^{\text{Ioffe}}=\varepsilon^{abc}\left(q_{1a}^T C\gamma_\mu q_{1b}\right)\gamma_5\gamma^\mu q_{2c}$$

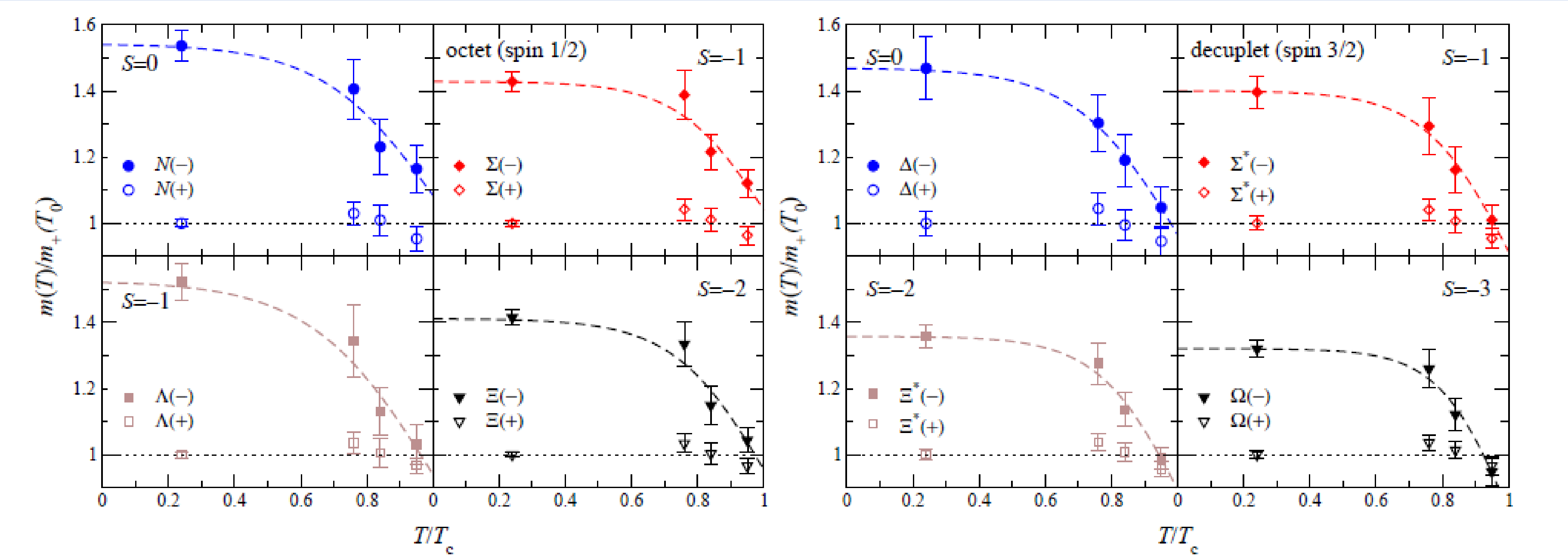
- *Consequence of chiral symmetry restoration*

$$\Pi(p) = \Pi_p \cdot \not{p} + \Pi_s \cdot 1 + \Pi_u \cdot \not{u}$$

$$\Pi \Rightarrow \frac{\not{p} + M_+^* - \Sigma_v \not{u}}{(p_0 - E_p)(p_0 - \bar{E}_p)} + \frac{\not{p} - M_-^* - \Sigma_v \not{u}}{(p_0 - E_p)(p_0 - \bar{E}_p)}$$

$$\Pi_s \propto \mathrm{Tr}[S(x,0)] = \langle \bar{q} q \rangle + .. \to 0 \qquad M_\pm^* \to 0 \quad \text{or} \quad (M_-^* - M_+^*) \to 0$$

- Finite T Lattice result (Aarts et al. arXiv:1710.00566)



Nucleons in symmetric and asymmetric matter (K. Jeong, SHL 2013)

- *Quark operators at finite density with isospin asymmetry*

$$\langle \hat{O}_{u,d} \rangle_{\rho,I} = \langle \hat{O}_{u,d} \rangle_{vac} + \langle \hat{O}_0 \rangle_p \rho \pm \langle \hat{O}_1 \rangle_p I\rho$$

$$\hat{O}_0 = \frac{1}{2}(\hat{O}_u + \hat{O}_d), \quad \hat{O}_1 = \frac{1}{2}(\hat{O}_u - \hat{O}_d)$$

$$\rho = \rho_n + \rho_p, \quad I\rho = \rho_n - \rho_p$$

→ Important operators

symmetric

$$\langle [\bar{q}q]_0 \rangle_p = \frac{\sigma_N}{2m_q} : \; 4.5$$

$$\langle \bar{u}\gamma_0 u \rangle_p = 2$$

asymmetric

$$\langle [\bar{q}q]_1 \rangle_p = \frac{1}{2}\left[\langle \bar{u}u \rangle_p - \langle \bar{d}d \rangle_p \right] = \frac{1}{2}\left[\frac{(m_n - m_p)_{\text{strong}}}{m_d - m_u} \right] : \; 0.1 \langle [\bar{q}q]_0 \rangle_p < 0.5$$

$$\langle [\bar{q}\gamma_0 q]_1 \rangle_p = \frac{1}{2}\left[\langle \bar{u}\gamma_0 u \rangle_p - \langle \bar{d}\gamma_0 d \rangle_p \right] = \frac{1}{3} \langle [\bar{q}\gamma_0 q]_0 \rangle_p = 0.5$$

$$\langle [\bar{q}\gamma_0 D_0 q]_1 \rangle_p = 0.35 \langle [\bar{q}\gamma_0 D_0 q]_0 \rangle_p$$

Nucleons in symmetric and asymmetric matter

- *Symmetry energy from QCD sum rule:* K. Jeong, SHL, PRC87, 015204 (2013)

$$E_{V,\rho}^{\rm sym} = \frac{1}{2}\left[-\frac{N^p_{(\rho,I)}}{D^p_{(\rho^0,I^0)}} + \frac{N^p_{(\rho^0,I^0)}}{\left(D^p_{(\rho^0,I^0)}\right)^2} D^p_{(\rho,I)} \right]$$

where

Ioffe Current

$$D^p_{(\rho^0,I^0)} = \frac{1}{32\pi^4}\left(M^2\right)^3 ..$$

$$D^p_{(\rho,I)} = \frac{1}{3\pi^2} M^2 \left\langle [\bar{q}\gamma_0 i D_0 q]_1 \right\rangle_p ..$$

$$N^p_{(\rho,I)} = -\frac{1}{2\pi^2}\left(M^2\right)^2 \left\langle [\bar{q}\gamma_0 q]_1 \right\rangle_p ..$$

Larger vector repulsion + Smaller scalar attraction < 50 MeV

→ Feature similar to Relativistic mean field models

$$L = \bar{q}_N \left[\gamma_\mu \left(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\vec{\rho}^\mu \right) - \left(m_N - g_{\sigma N}\sigma - g_{\delta N}\vec{\delta} \right) \right] \rho_B$$

$$E_{V,\rho}^{\rm sym} = \frac{1}{2}\left[\frac{g^2_{\rho N}}{m^2_\rho} - \frac{g^2_{\delta N}}{m^2_\delta}\left(\frac{m^*}{E^*_F} \right) \right] \rho_B$$

Hyperons in symmetric and asymmetric matter

- *QCD sum rule: Interpolating field determines how well OPE converges*

- *Needed additional input*

$$\langle \bar{s}s \rangle_p = \frac{\sigma_{sN}}{m_s} \rightarrow \text{Will take } 0.1 \times \langle \bar{u}u \rangle_p \qquad \text{Lattice } \sigma_{sN} \rightarrow 17 \text{ MeV (JLQCD)}$$

$$: \; 105 \text{ MeV (BMW)}$$

$$\langle s\gamma_0 s \rangle_p = 0$$

$$\langle q^a_\alpha \bar{q}^b_\beta q^c_\gamma \bar{q}^d_\delta \rangle_{\rho,I} = \langle q^a_\alpha \bar{q}^b_\beta \rangle_{\rho,I} \langle q^c_\gamma \bar{q}^d_\delta \rangle_{\rho,I} - \langle q^a_\alpha \bar{q}^d_\delta \rangle_{\rho,I} \langle q^c_\gamma \bar{q}^b_\beta \rangle_{\rho,I}$$

- Λ *in nuclear matter* (Jeong, Gye, SHL PRC94 (2016)065201)

Operator Product Expansion when the current has a strong u-d scalar diquark

Nucleon with Ioffe current

$$M_+^* = -\frac{8\pi^2}{M^2}\langle \bar{q}q \rangle + \dots$$

$$\Sigma_V = \frac{64\pi^2 2}{3M^2}\langle \bar{q}_1\gamma_0 q_1 \rangle + \dots$$

Λ with strong u-d diquark current

$$M_+^* = -\frac{8\pi^2}{M^2}\left(2\langle \bar{s}s \rangle + \text{small}\langle \bar{q}q \rangle\right) + \dots$$

$$\Sigma_V = \frac{64\pi^2 2}{3M^2}\left(\frac{1}{4}\langle \bar{q}_1\gamma_0 q_1 \rangle \dots\right) + \dots$$

→ Both Smaller than 2/3 from Quark counting but similar total binding

Vector repulsion < Scalar attraction

$$\frac{\Sigma_V(\Lambda)}{\Sigma_V(N)} \approx 0.26 \qquad \frac{\Sigma_S(\Lambda)}{\Sigma_S(N)} \approx 0.3$$

Σ in symmetric nuclear matter

- *Σ in nuclear matter* (Jeong, Gye, SHL PRC94 (2016)065201)

Operator Product Expansion is similar with Ioffe current with d→s

Nucleon with Ioffe current

$$M_+^* = -\frac{8\pi^2}{M^2}\langle \bar{q}q \rangle + \ldots$$

$$\Sigma_V = \frac{64\pi^2 2}{3M^2}\langle \bar{q}_1 \gamma_0 q_1 \rangle + \ldots$$

Λ with Ioffe current d→s

$$M_+^* = -\frac{8\pi^2}{M^2}\left(\langle \bar{s}s \rangle + \text{small}\langle \bar{q}q \rangle\right) + \ldots$$

$$\Sigma_V = \frac{64\pi^2 2}{3M^2}\left(\frac{7}{8}\langle \bar{q}_1 \gamma_0 q_1 \rangle \ldots\right) + \ldots$$

Vector repulsion

Scalar attraction

$$\frac{\Sigma_V(\Lambda)}{\Sigma_V(N)} \approx 1 \quad \gg \quad \frac{\Sigma_S(\Lambda)}{\Sigma_S(N)} \approx 0.3$$

→ Total repulsion is larger than 100 MeV at nuclear matter

Delta in nuclear matter

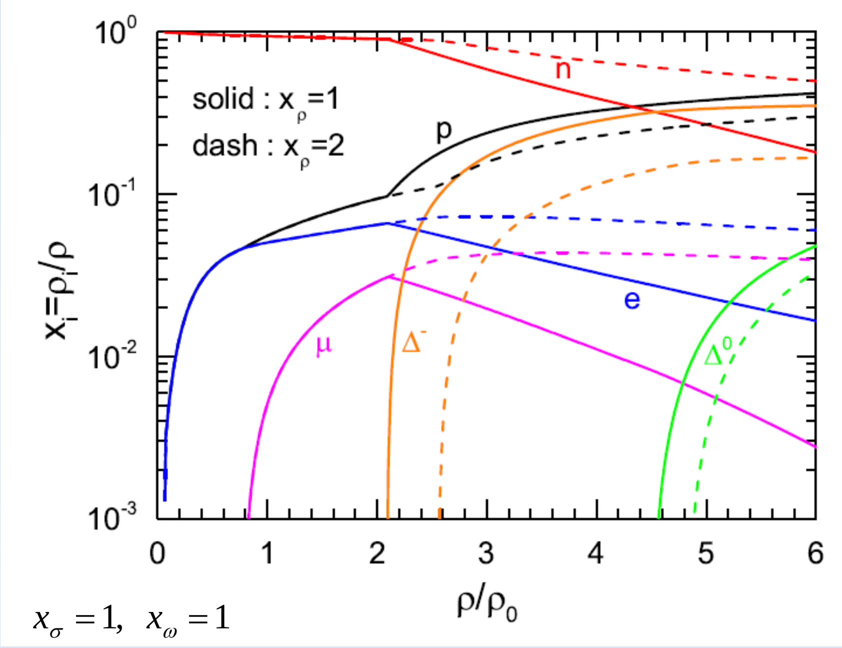
- *Baryon in Relativistic Mean Field models*

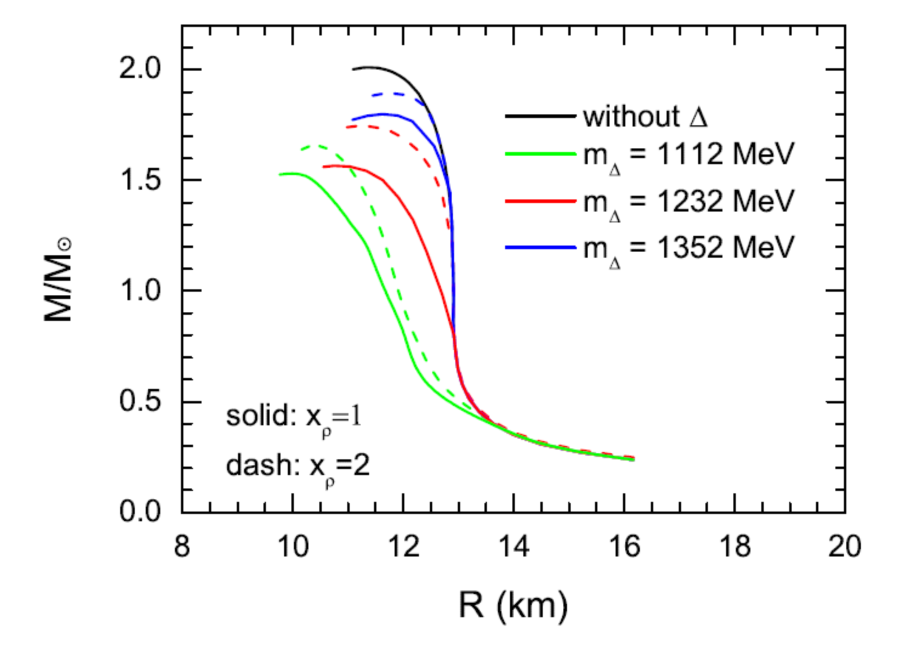
$$L = \bar{N}\left[\gamma_\mu\left(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\tau\rho^\mu\right) - \left(m_N - g_{\sigma N}\sigma\right)\right]N$$

$$+\bar{\Delta}_\nu\left[\gamma_\mu\left(i\partial^\mu - g_{\omega\Delta}\omega^\mu - g_{\rho\Delta}\tau\rho^\mu\right) - \left(m_\Delta - g_{\sigma\Delta}\sigma\right)\right]\Delta^\nu$$

→ Effects in neutron star [Cai, Fattoyev, Bao-An Li, Newton (PRC92 (2015) 015802)]

$x_\sigma = g_{\sigma\Delta}/g_{\sigma N}$, $x_\omega = g_{\omega\Delta}/g_{\omega N}$, $x_\rho = g_{\rho\Delta}/g_{\rho N}$





Experimental results on Delta mass shift

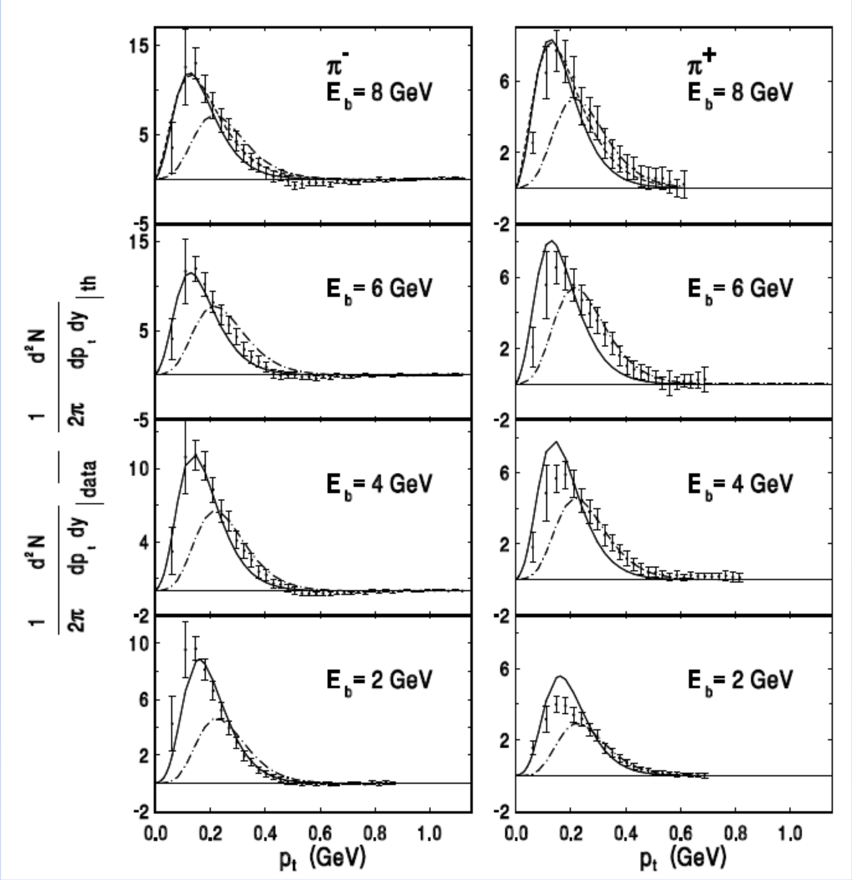
- *At symmetric Nuclear matter in γ -A*
  - → Broadening
  - → No clear observable Mass change?

- *Heavy Ion Collisions*

  possible mass shift at higher density

  Analysis of pion Pt spectra (Rafelski 2007)

  BNL: Au+Au



- *Raon*

  Can produce through subthreshold production in heavy ion collision

Delta in nuclear matter

- Δ *interpolating currents* (Marques, SHL, A. Park, Matheus, Jeong PRC98 (2018)025206)

Nucleon Ioffe current

$$\eta^{\text{Ioffe}} = \varepsilon^{abc}\left(q_{1a}^{T} C \gamma_{\mu} q_{1b}\right)\gamma_5 \gamma^{\mu} q_{2c}$$

$$\Pi_p = -\frac{1}{64\pi^4} p^4 \ln\left(-p^2\right) + \ldots$$

$$\Pi_s = \frac{1}{4\pi^2} p^2 \ln\left(-p^2\right)\langle \bar{q} q\rangle + \ldots$$

$$\Pi_V = \frac{2}{3\pi^2} p^2 \ln\left(-p^2\right)\langle \bar{u}\gamma_0 u\rangle + \ldots$$

$$\rightarrow \quad \Pi_V / \Pi_s = \frac{8}{3} \times (\text{condensates})$$

Delta Ioffe current

$$\eta_{\mu}^{\text{Ioffe}} = \varepsilon^{abc}\left(q_{1a}^{T} C \gamma_{\mu} q_{1b}\right) q_{1c}$$

$$\Pi_p = \frac{1}{160\pi^4} p^4 \ln\left(-p^2\right) + \ldots$$

$$\Pi_s = -\frac{1}{3\pi^2} p^2 \ln\left(-p^2\right)\langle \bar{u} u\rangle + \ldots$$

$$\Pi_V = -\frac{1}{4\pi^2} p^2 \ln\left(-p^2\right)\langle \bar{u}\gamma_0 u\rangle + \ldots$$

$$\rightarrow \quad \Pi_V / \Pi_s = \frac{3}{4} \times (\text{condensates})$$

→ Δ *Has smaller vector repulsion compared to nucleon*

Δ *scalar attraction : 150 MeV,* Δ *Vector repulsion: 75 MeV*

Delta in asymmetric nuclear matter

- Δ *vector self energy* (Marques, SHL, A. Park, Matheus, Jeong PRC98 (2018)025206)

$$\Sigma_V(\rho, I) = \left(0.075 - 0.01\tau_\Delta^3 I\right) \text{ GeV} \qquad \tau^3_{\Delta^{++}} = 3, \quad \tau^3_{\Delta^{+}} = 1, \quad \tau^3_{\Delta^{0}} = -1, \quad \tau^3_{\Delta^{-}} = -3,$$

- *Nucleon vector self energy* (Marques, SHL, A. Park, Matheus, Jeong PRC98 (2018)025206)

$$\Sigma_V(\rho, I) = \left(0.32 - 0.08 I\right) \text{ GeV}$$

- *Nucleon Nucleon interaction at short distance is dominated by color spin interaction* (arXiv:1907.06351 A Park, SHL, Inoue, Hatsuda)

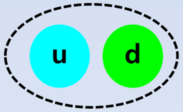
- Color-spin interaction

$$H = \sum_{i=1}^{N}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{N}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{N} \frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{SS}\left(r_{ij}\right)$$

☞ Color-spin interaction for 2 body:

$$K = -\sum_{i<j}^{N}\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i^s \sigma_j^s\right) \rightarrow$$

	q q				q q			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(1)	S(3)	S(3)	A(1)	1	1	3	3
K	-8	-4/3	8/3	4	-16	2	16/3	-2/3

↳ 

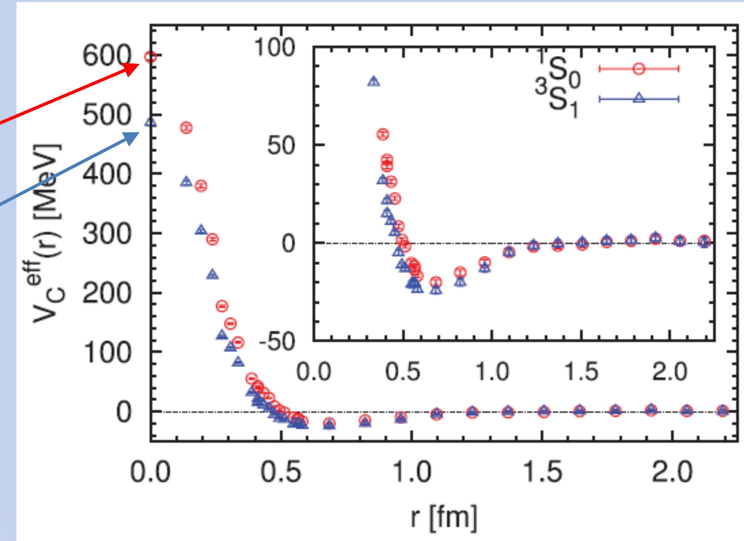
↳ meson

- Comparison with lattice (HAL QCD) – NN interaction

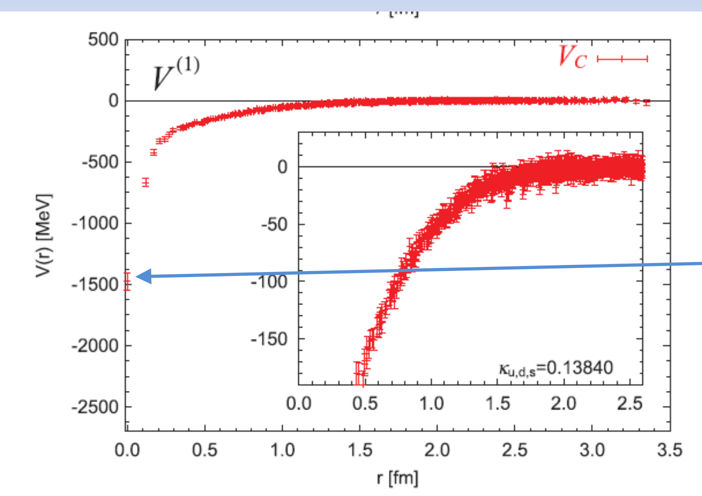
☞ NN force in SU(2) spin 1 vs spin 0 channel (W.Park, A. Park, Lee 2015)

$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N}\right)$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29$$ → comparison

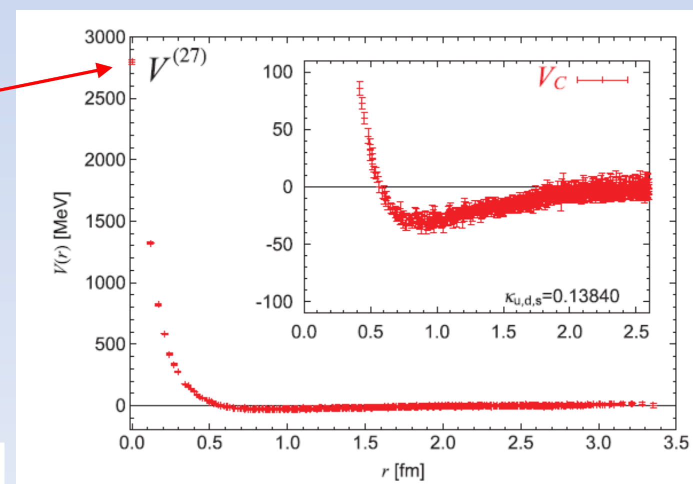


☞ H dibaryon channel: Flavor 1 vs Flavor 27



$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

(HAL QCD Collaboration)



- K factors in NN vs NΔ

☞ Spin and Isospin averaged NN interaction: repulsive with $K_{2-N} \approx 2.37$

☞ Spin and Isospin averaged NΔ interaction: repulsive with $K_{2-N} \approx 1.9$

Summary

1. QCD sum rule approach offers a way to calculate the potential part of the nucleon, delta and hyperons in medium→ similar to RMF models.
2. In sum rule, Symmetry energy is positive and due to larger vector repulsion and scalar attraction.
3. Lambda has smaller self energies than 2/3 of nucleon, but repulsion is even smaller
4. Sigma repulsion in matter is larger than naïve expectation. 100 MeV
5. Delta has smaller vector repulsion compared to nucleon
6. Further studies with different currents will be useful