QCS2019@Busan

Bayesian Analysis with KIDS Energy Density Functional

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in collaboration with BUD :

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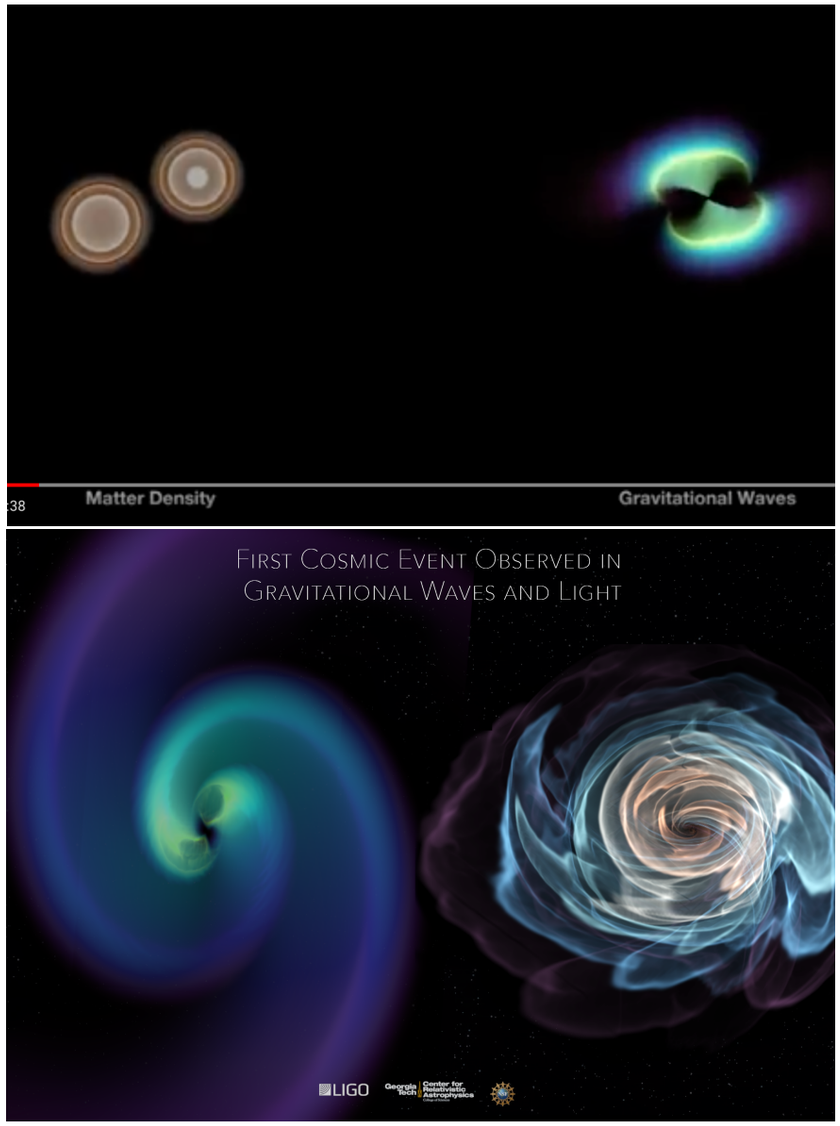
C.-H. Lee, M. Kim (Pusan Nat'l Univ.), , Y. Kim (IBS), S. Jeon (McGill Univ.)

1st detection of GW from a BNS merger

GW170817



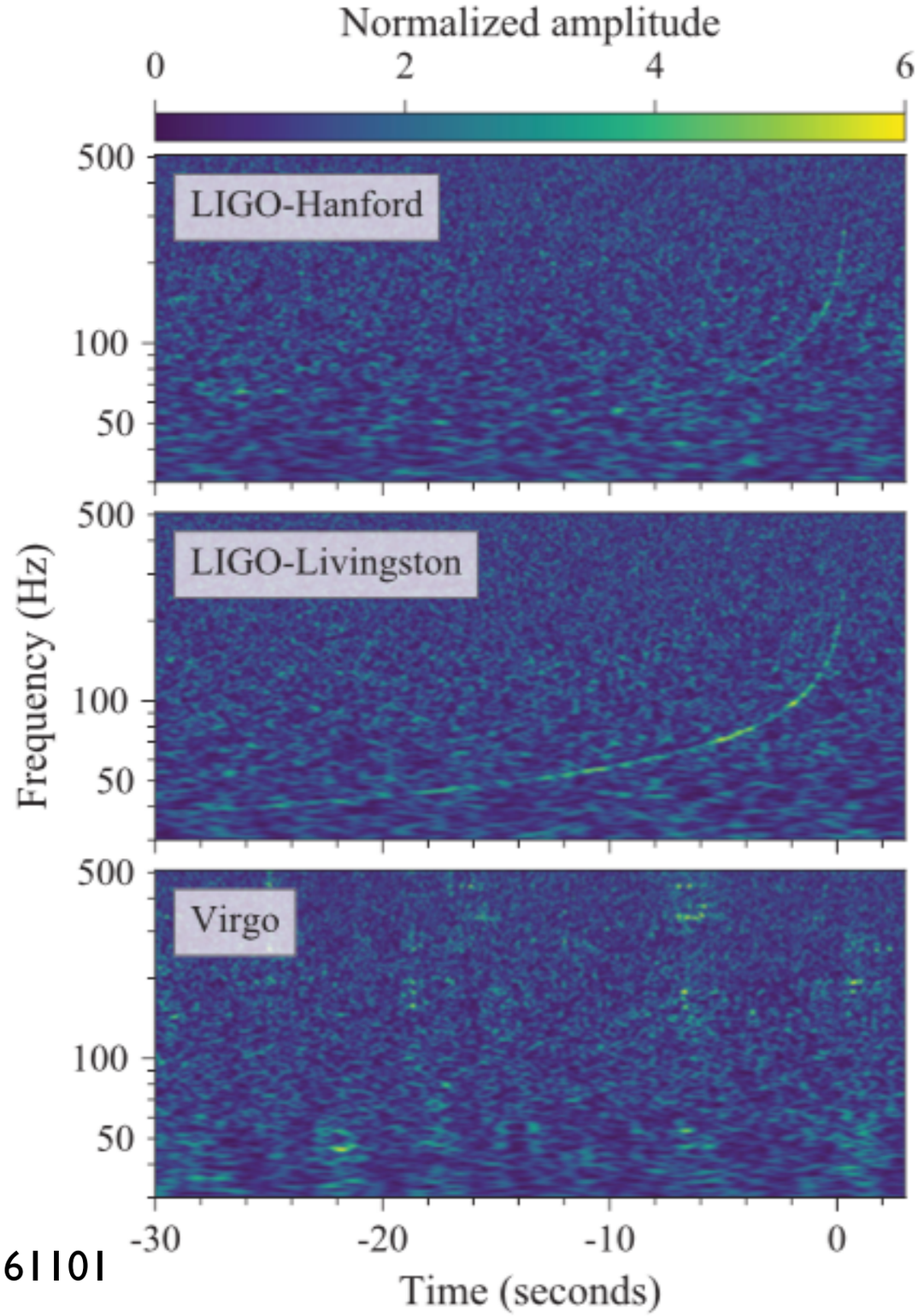


Image credit: Karan Jani/Georgia Tech.

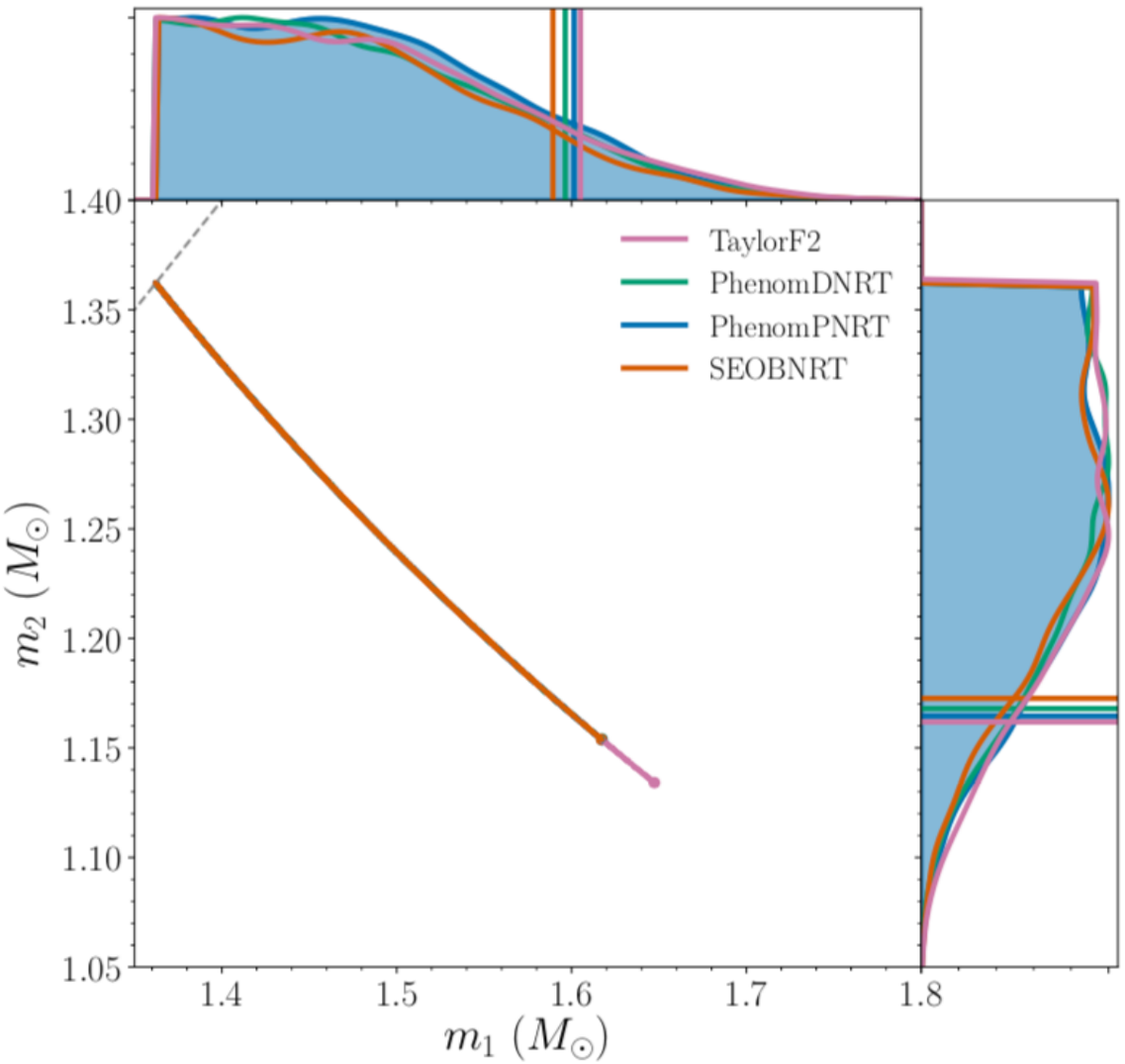
PhysRevLett.119.161101

Bayesian Analysis on GW observation (1)

low spin prior : χ <= 0.05

m1=1.36~1.60 $M_\odot$

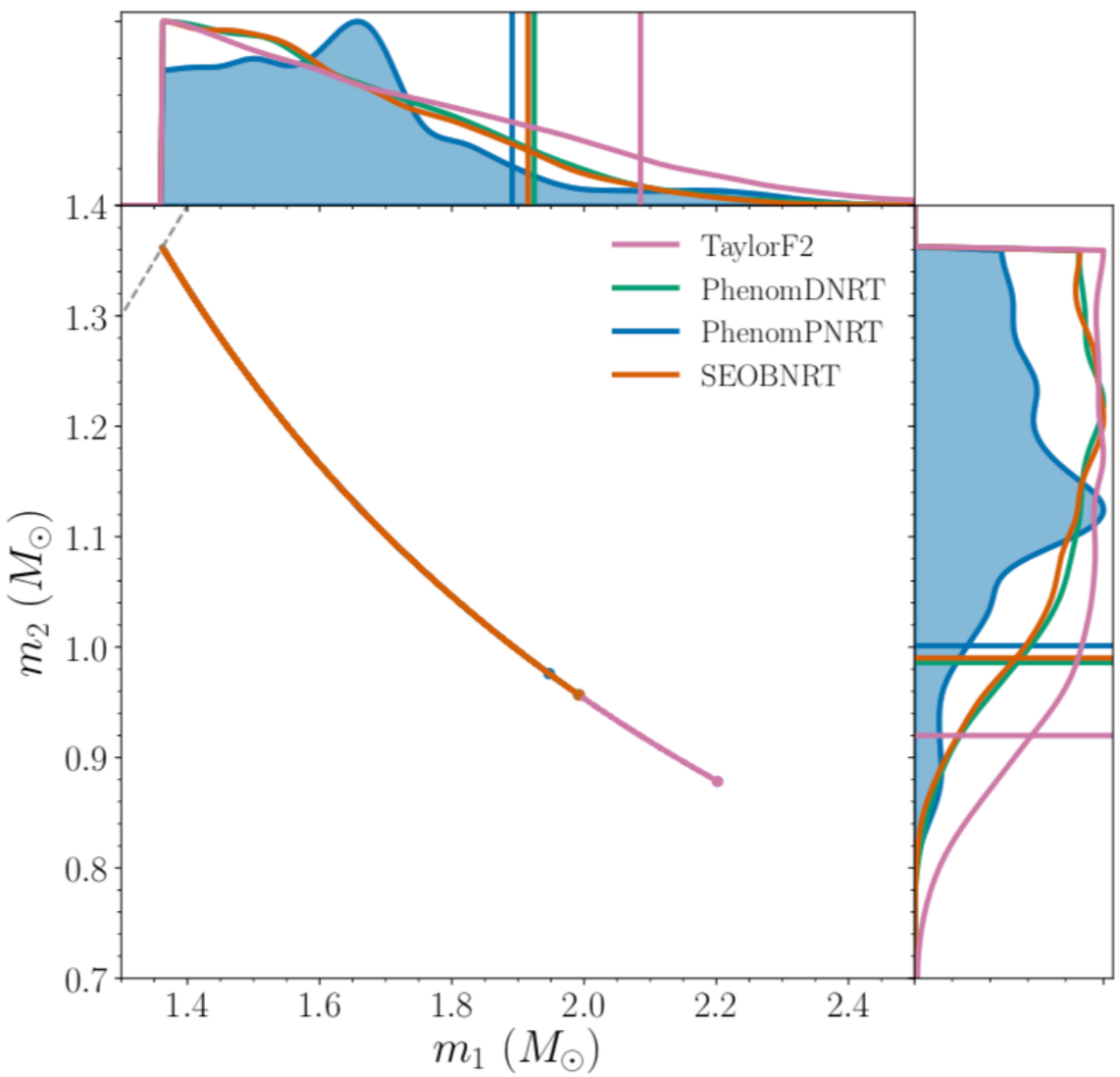
m2=1.16~1.36 $M_\odot$



high spin prior: :χ <= 0.89

m1=1.36~1.89 $M_\odot$

m2=1.00~1.36 $M_\odot$

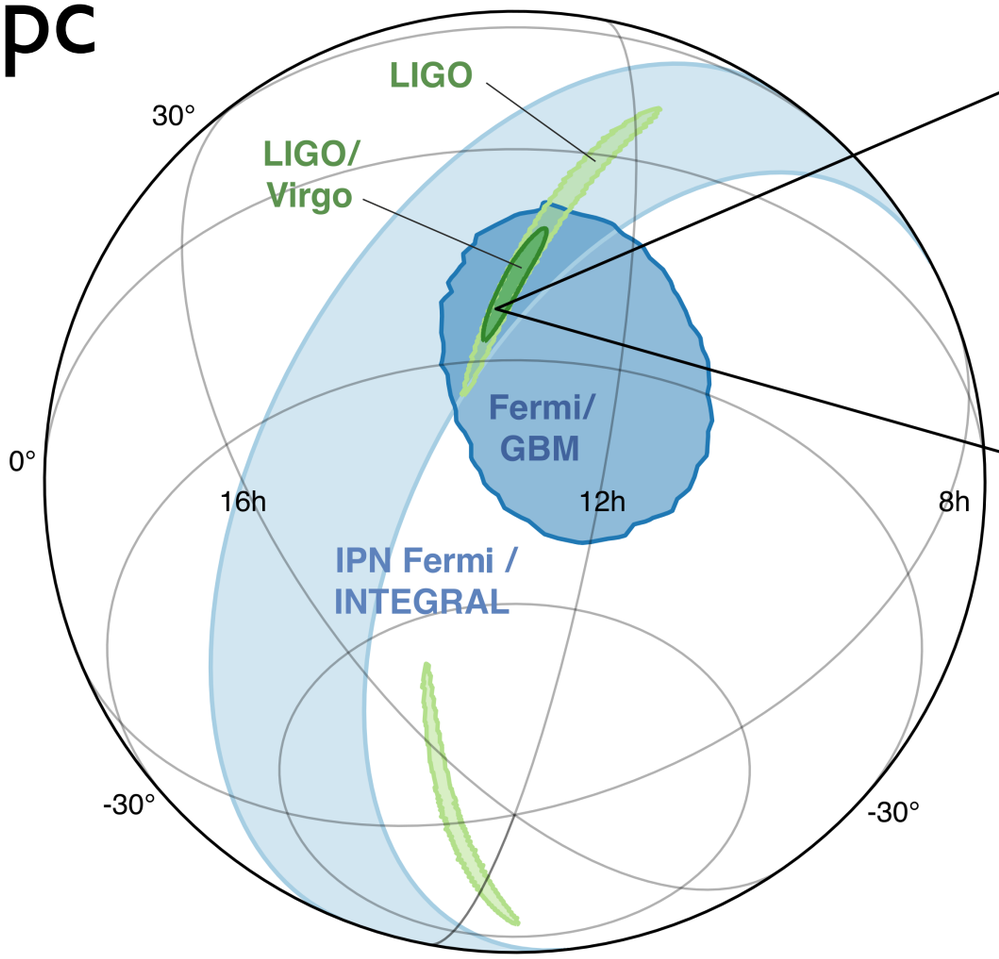


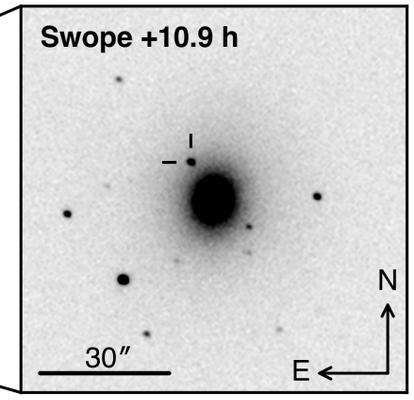
Abbott et al. (LSC and Virgo), arxiv:1805.11579 (PhysRevX.9.011001)

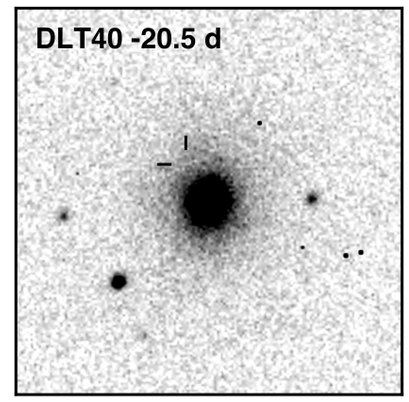
Bayesian Analysis on GW observation (2)

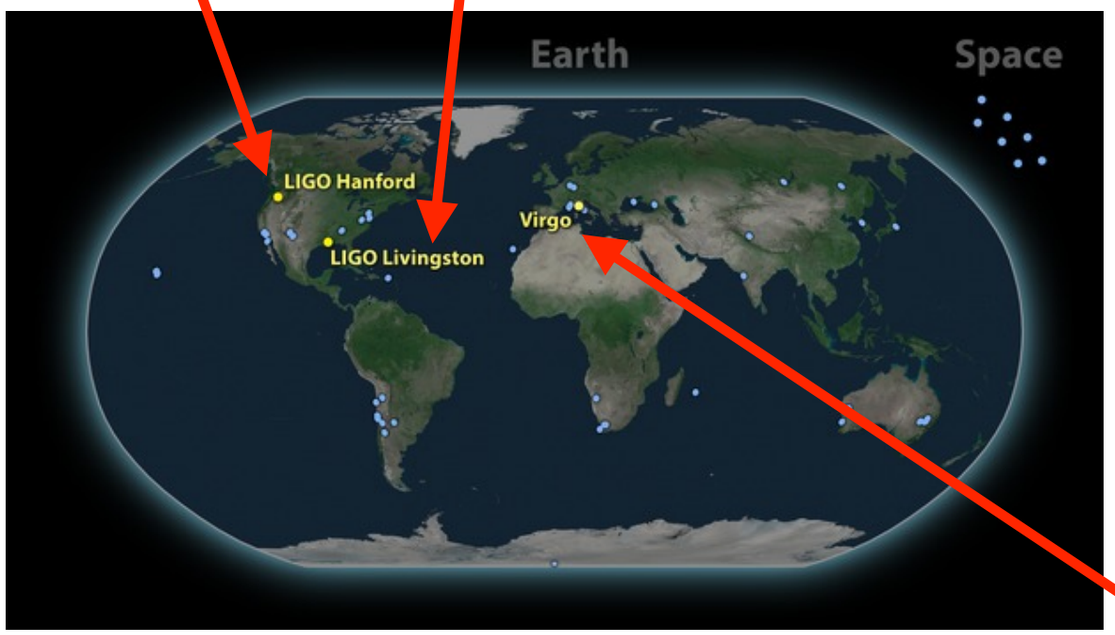
D_L ~ 40 Mpc

ApJL.848.L12(2017)





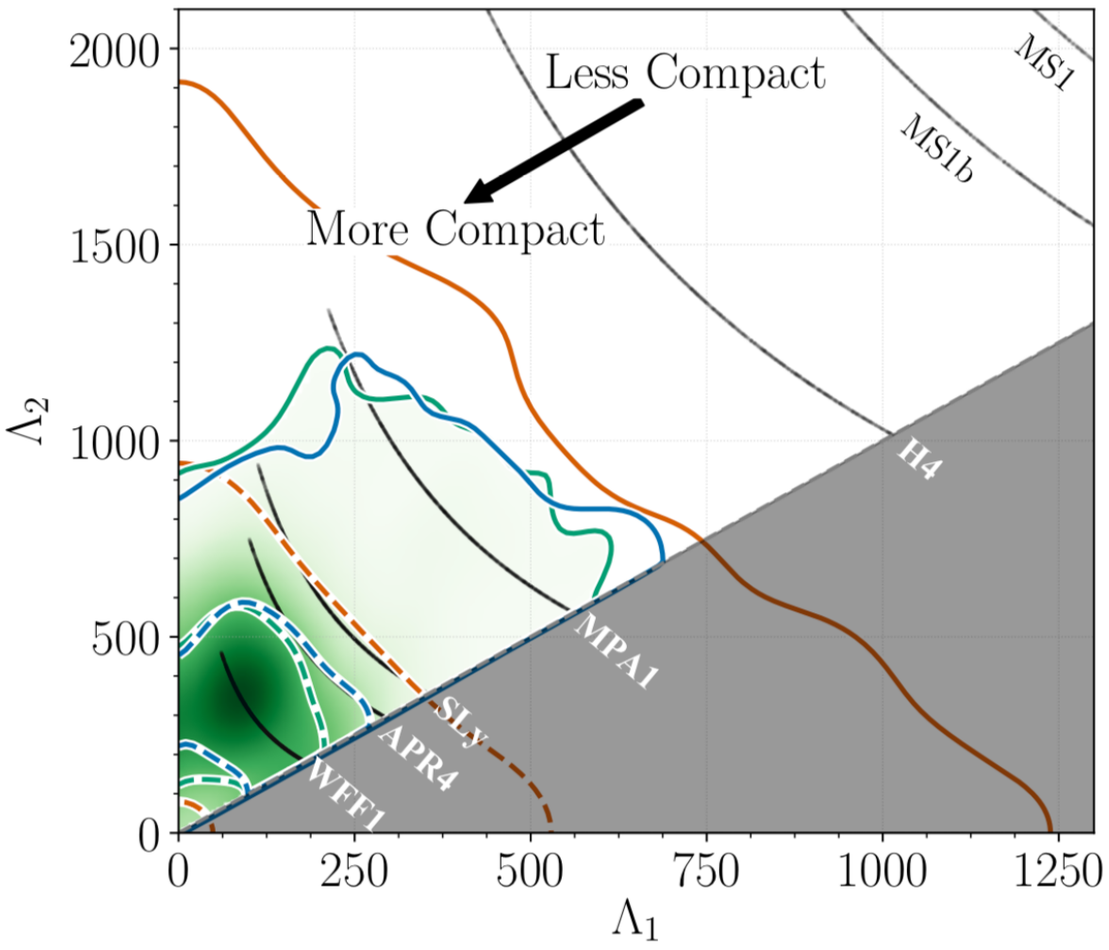




AT2017gfo in NGC4993

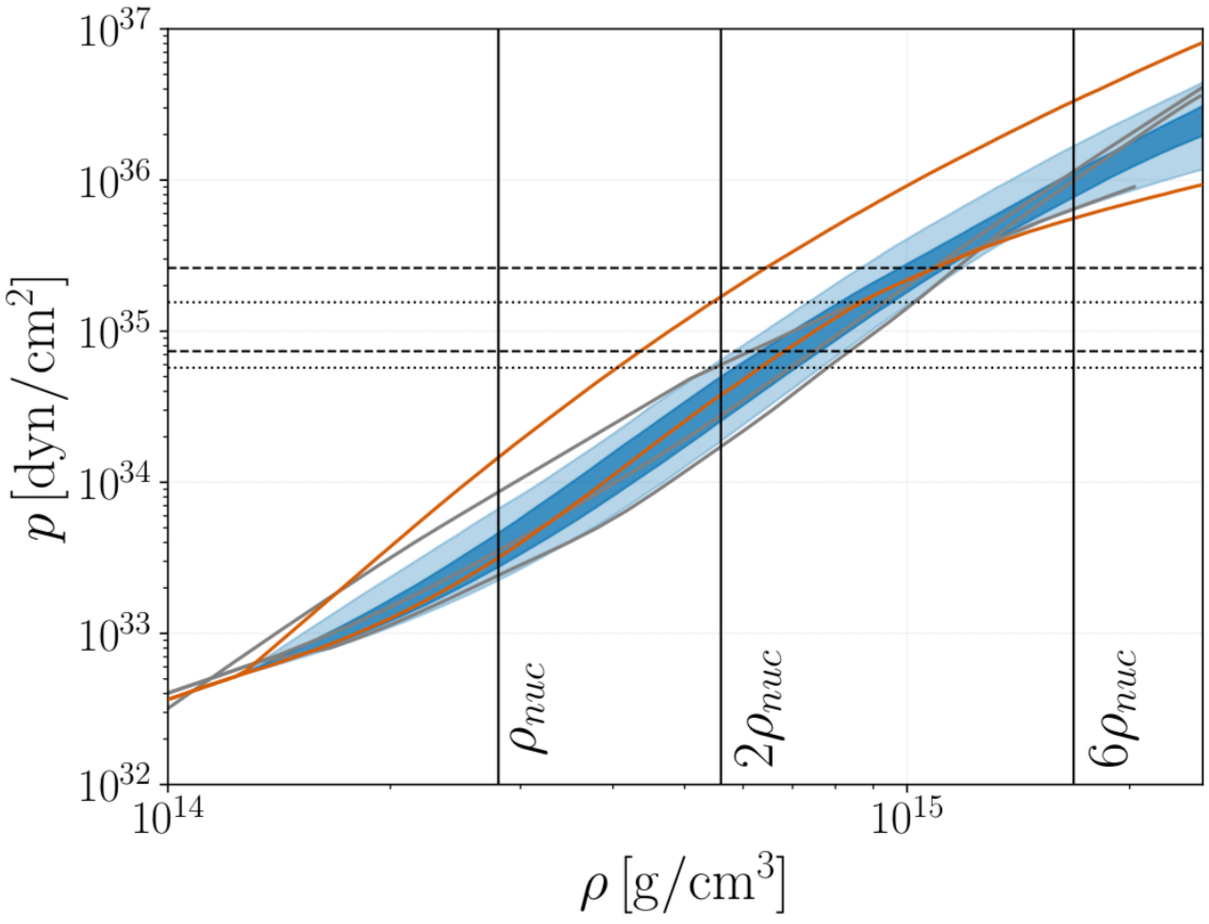
Bayesian Analysis on GW observation (3)

$\Lambda(1.4M_\odot)=190^{+390}_{-120}$



$P(2\ \rho_{nuc})= 3.5^{+2.7}_{-1.7} \times 10^{34}$ dyne/cm^2

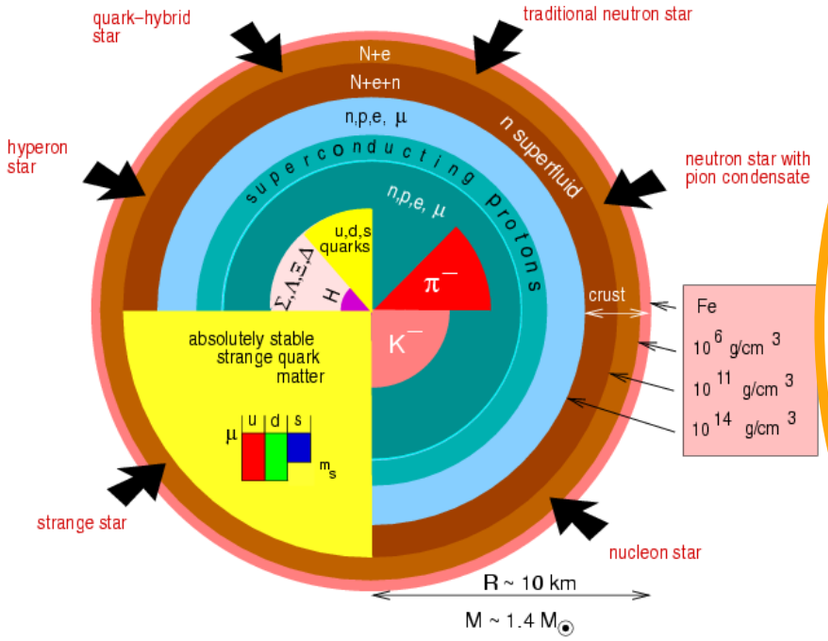
$P(6\ \rho_{nuc})= 9.0^{+7.9}_{-2.6} \times 10^{35}$ dyne/cm^2

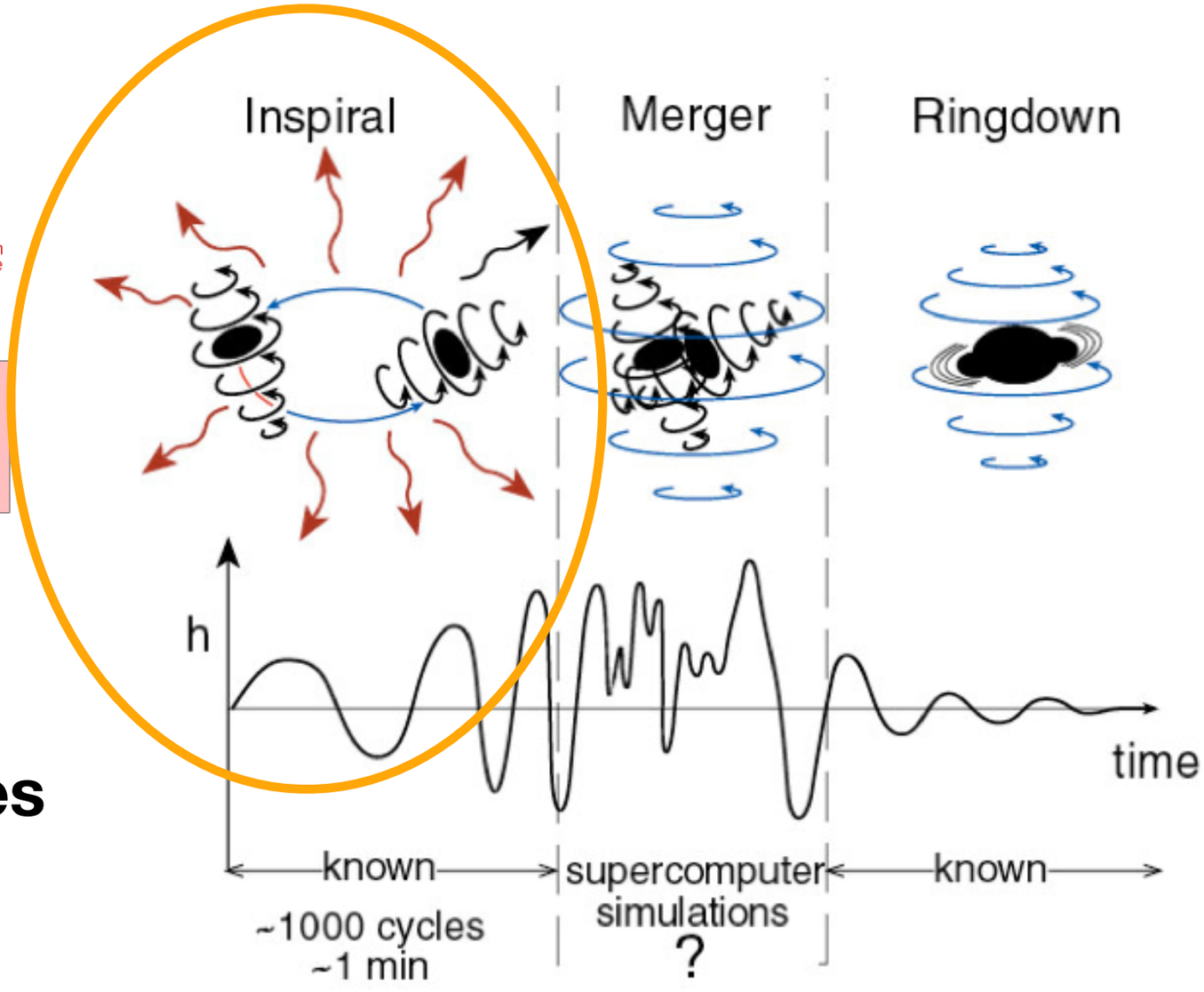


Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PhysRevLett.121.161101)

$\rho_{nuc}= 2.8 \times 10^{14}$ g/cm^3

NS Tidal deformability





perturbative approaches

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ :Tidal deformability
k2 :Tidal Love number

KIDS Energy Density Functional

KIDS Energy density functional form

Phys. Rev. C 97, 014312 (2018)

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3} \qquad \delta = \frac{\rho_n - \rho_p}{\rho}$$

$$\mathcal{T}(\rho,\delta) = \frac{3}{5}\left[\frac{\hbar^2}{2m_p}\left(\frac{1-\delta}{2}\right)^{5/3} + \frac{\hbar^2}{2m_n}\left(\frac{1+\delta}{2}\right)^{5/3}\right](3\pi^2\rho)^{2/3}$$

$c_i(\delta) = \alpha_i + \beta_i\delta^2$ to be determined by fitting to the observables

at zero temperature $k_F = (3\pi^2\rho/2)^{1/3}$ $k_{F_\tau} = k_F(1+\tau\delta)^{1/3}$

Constraints on Nuclear EoS

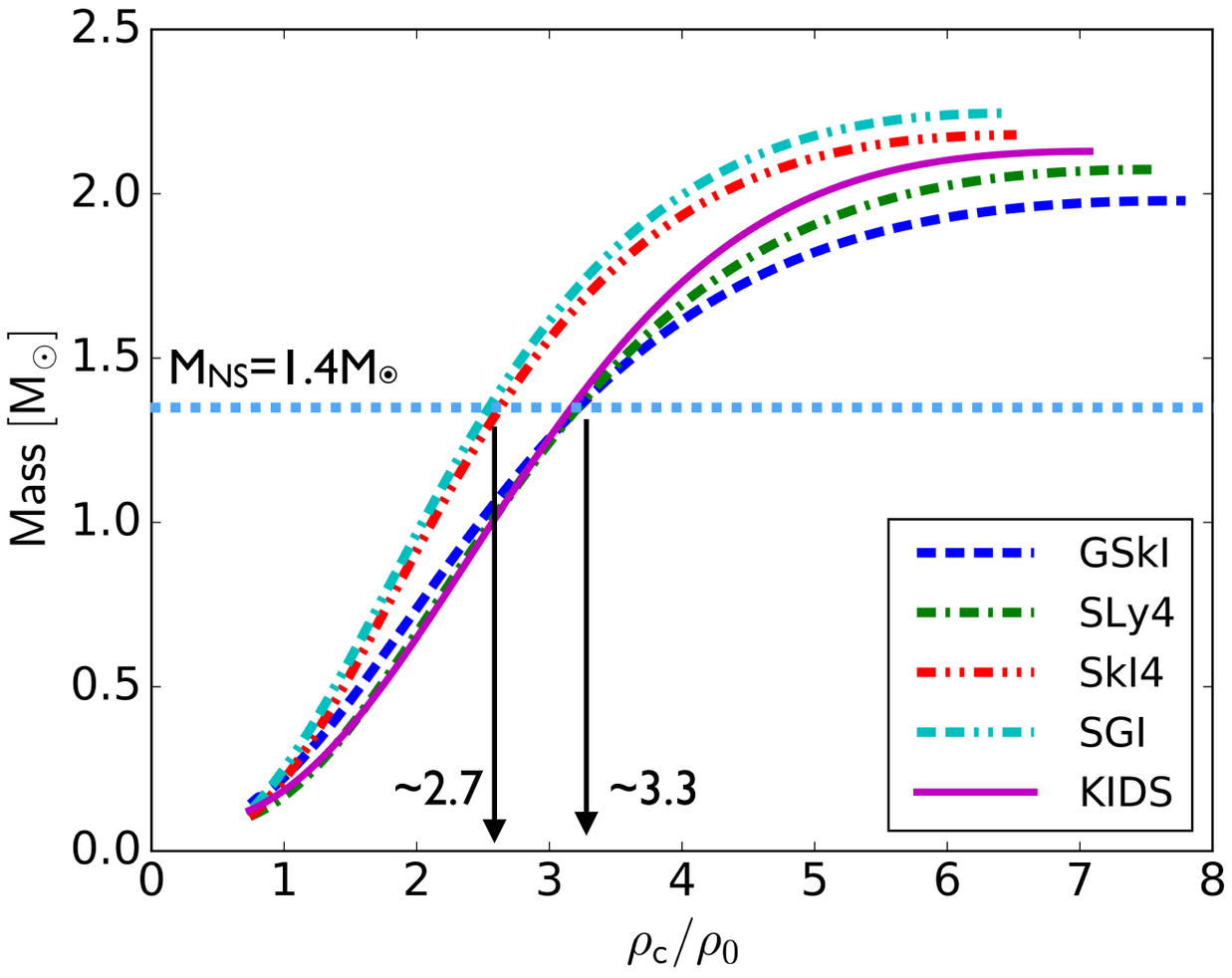
- Nuclear data: hundreds of models (Skyrme force, RMF, …)
- Neutron star maximum mass

  1.97 ± 0.04 M⊙ [Nature 467, 1081 (2010)]

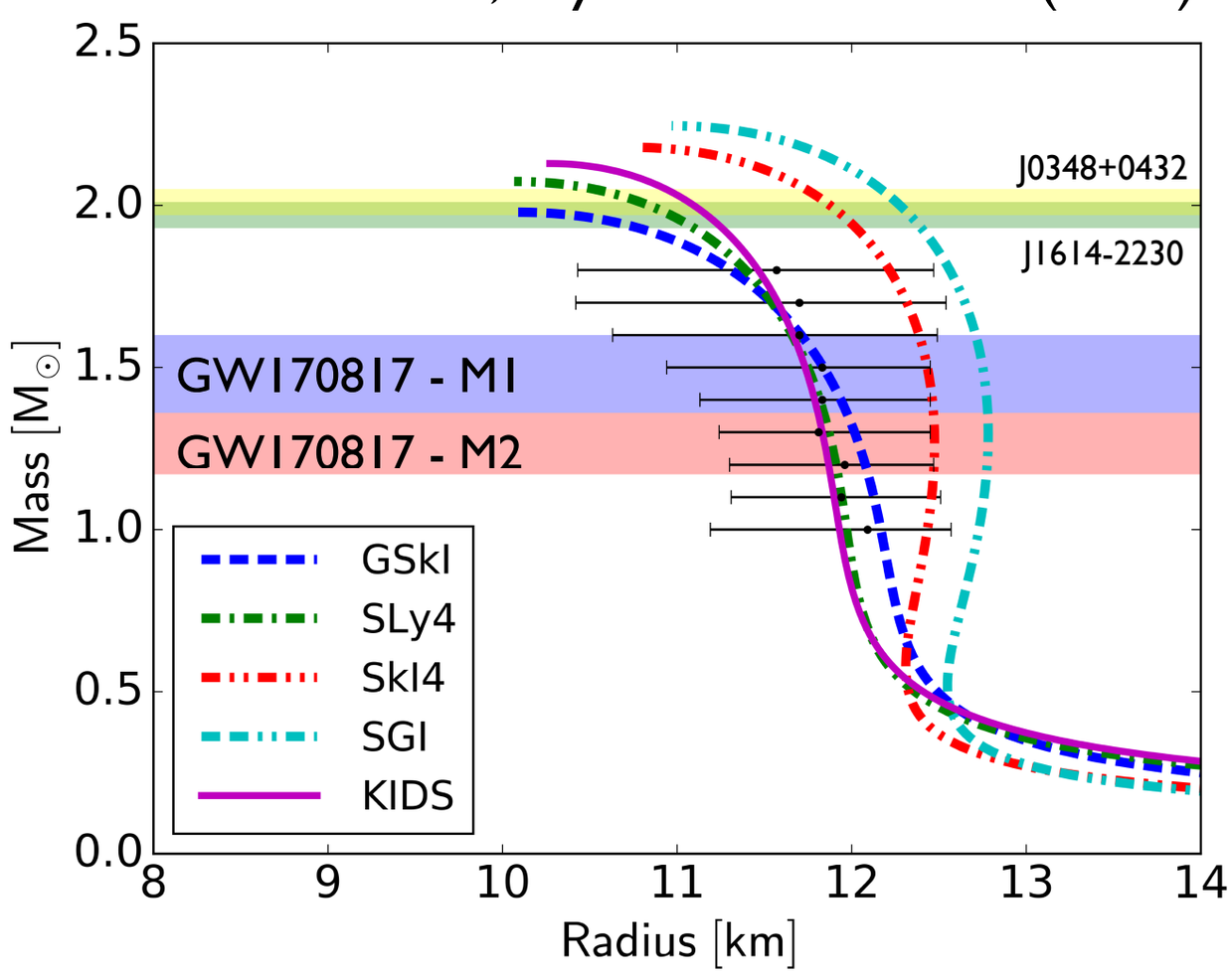
  2.01 ± 0.04 M⊙ [Science 340, 448 (2013)]
- 11 experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

Mass-Radius relations



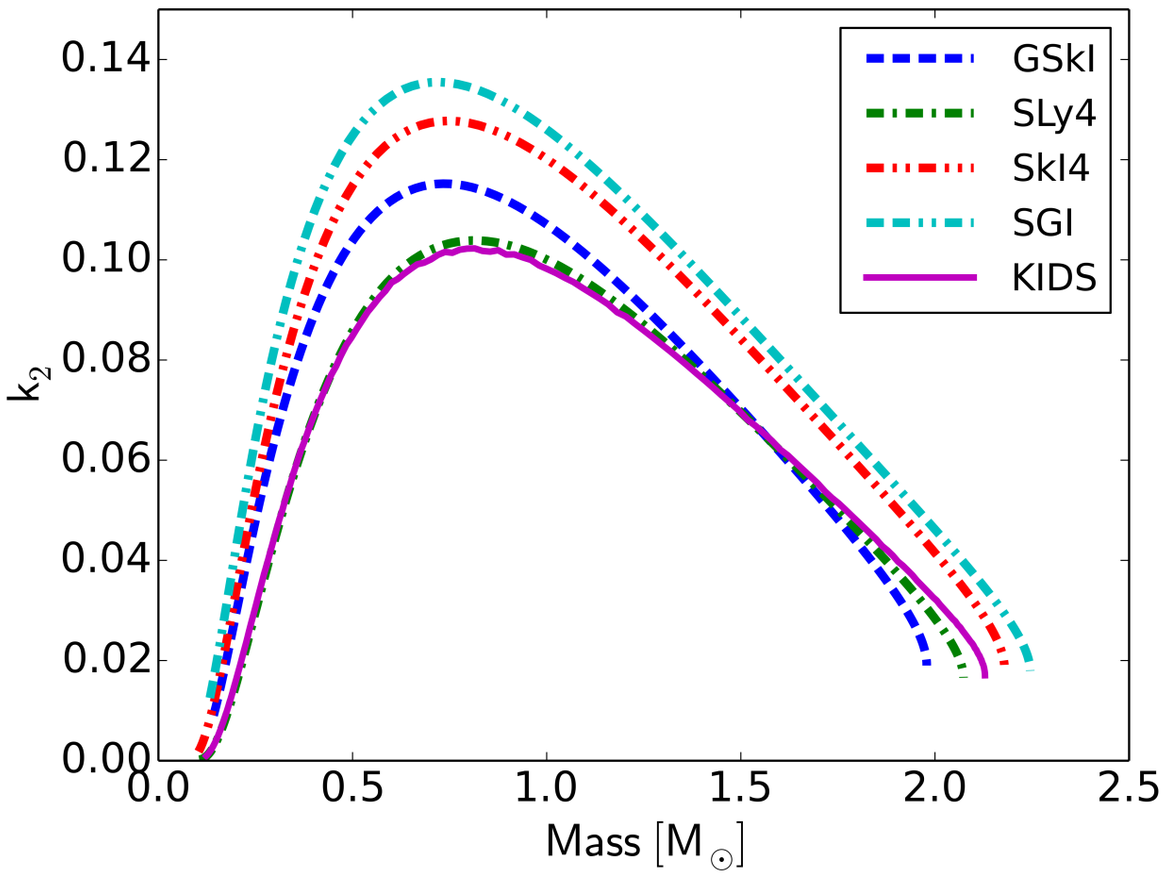
Kim et al., PhysRevC.98.065805 (2018)

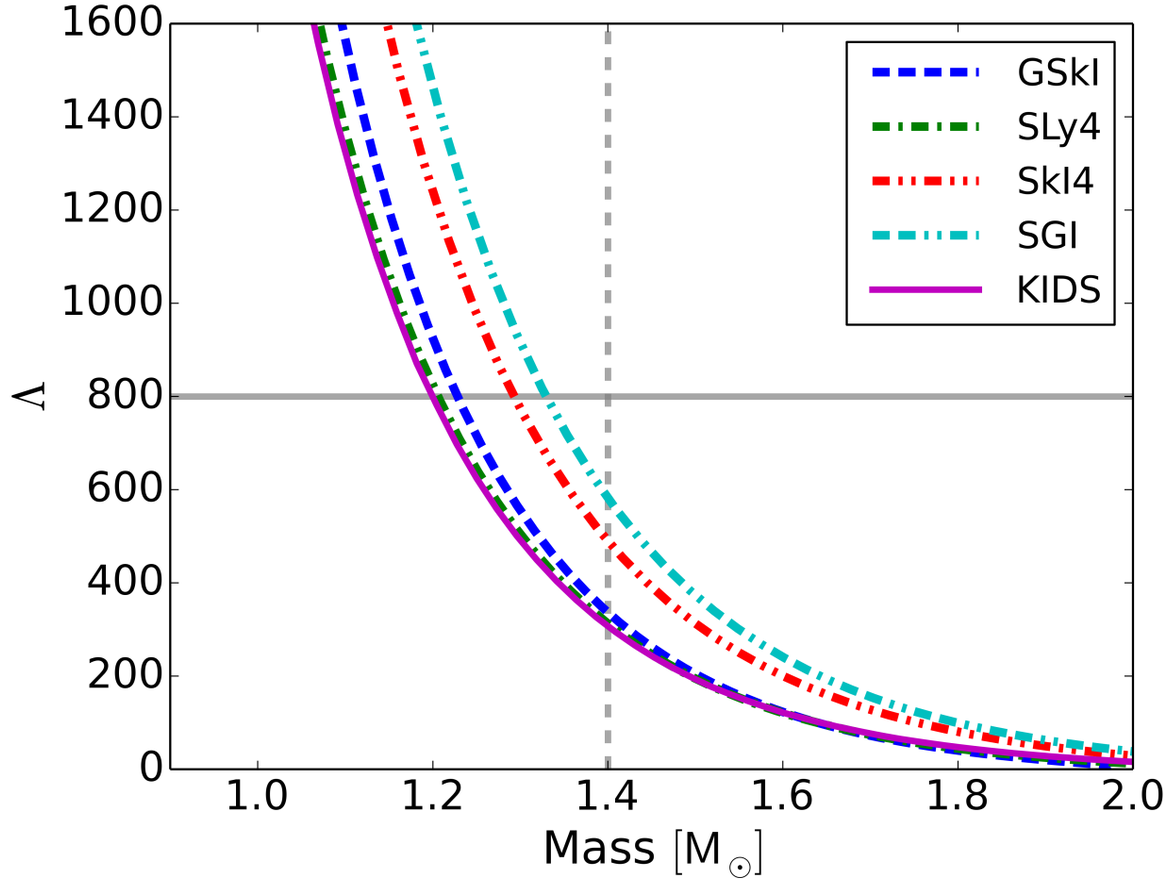


GW170817 - Abbott et al. (LSC and Virgo), arxiv:1805.11579
- Mchirp = 1.188 M⊙
- low spin prior : M_1=1.36~1.60 M⊙, M_2=1.16~1.36 M⊙
- high spin prior : M_1=1.36~1.89 M⊙, M_2=1.00~1.36 M⊙

Tidal deformability of a NS

Kim et al., PhysRevC.98.065805 (2018)

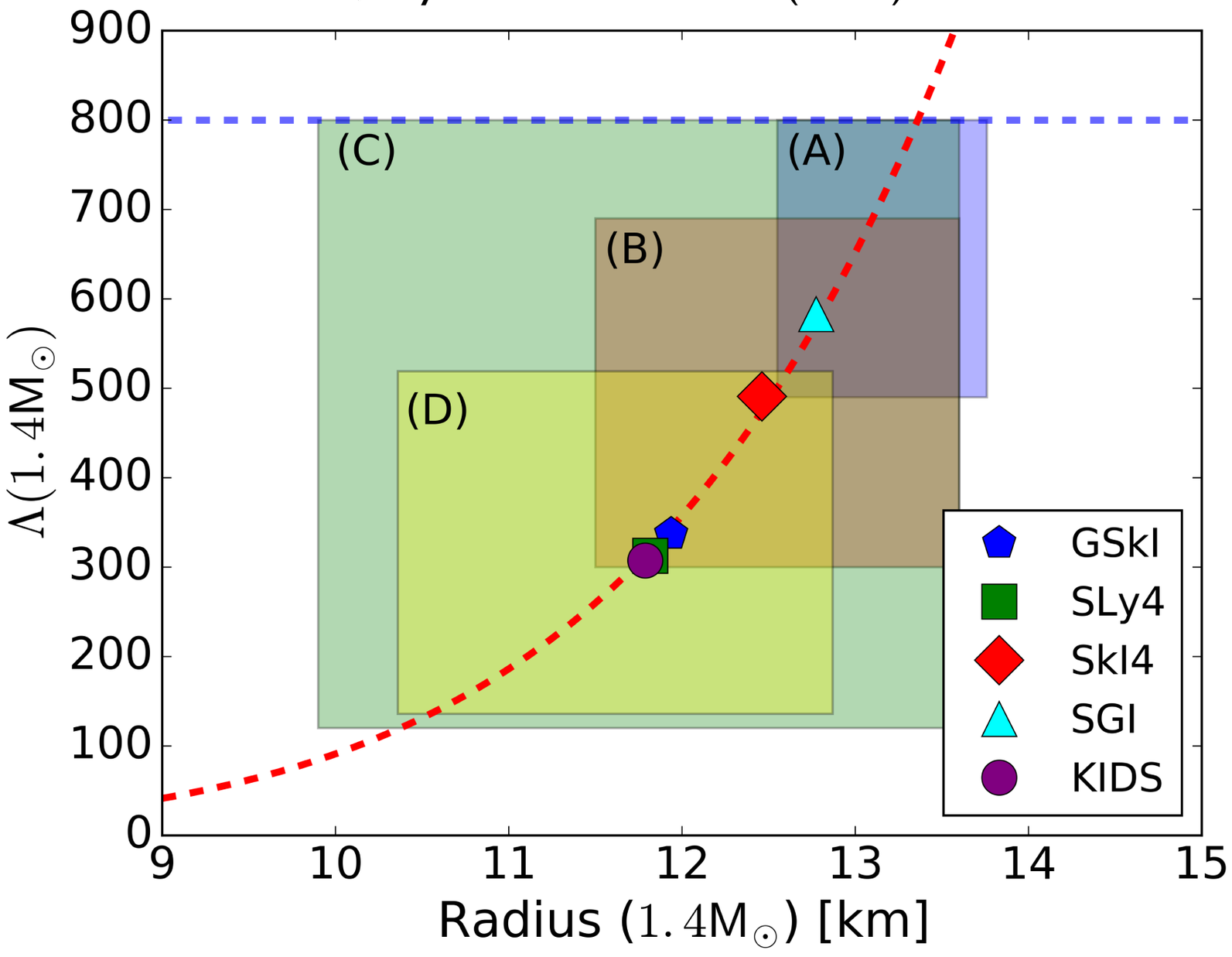




GW170817 - Abbott et al. (LSC and Virgo), arxiv:1805.11579
- Mchirp = 1.188 M⊙
- low spin prior : $\Lambda = 300^{+500}_{-190}$ (symmetric) / 300^{+420}_{-230} (HPD)
- high spin prior : $\Lambda = 0 \sim 630$

Tidal Deformability of 1.4 Msun

Kim et al., PhysRevC.98.065805 (2018)

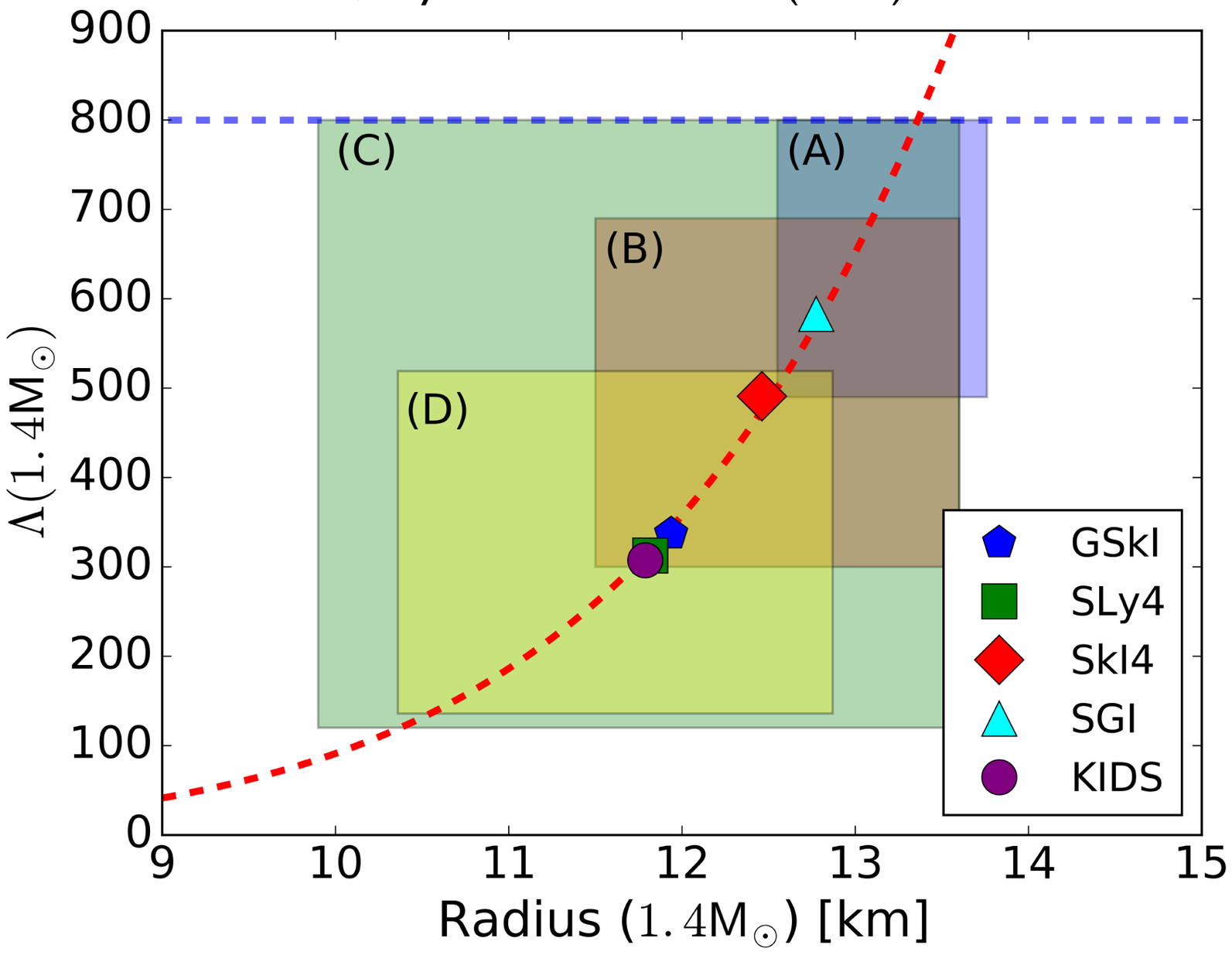


[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, PRL.120.172702 (arXiv: 1711.06615v2)
[B] P.G. Krastev, and B.-A. Li, arXiv: 1801.04620v1
[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, PRL.120.172703 (arXiv: 1711.02644v2)
[D] Y.Lim and J. Holt, PRL.121.062701 (arXiv:1803.02803v2)

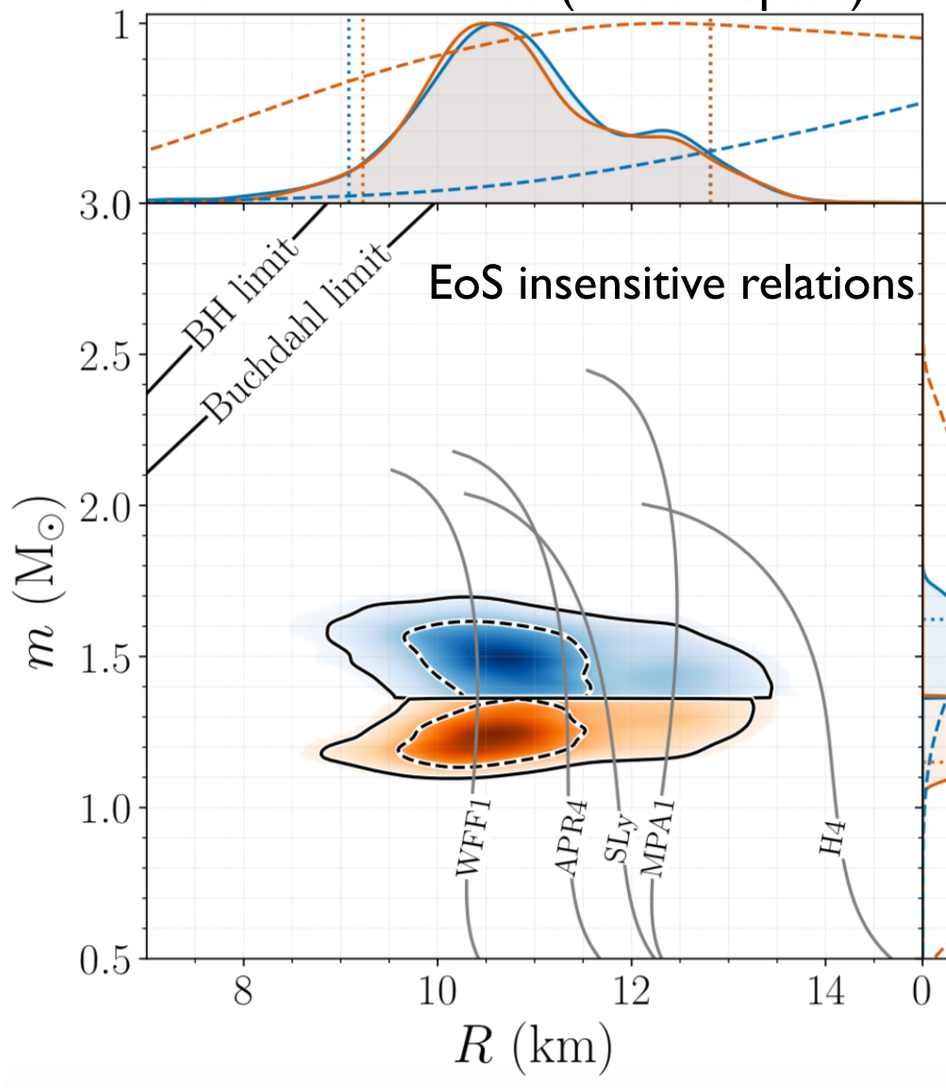
Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6} (R/km)^{7.5}$ (fitting function in [C])

Tidal Deformability of 1.4 Msun

Kim et al., PhysRevC.98.065805 (2018)



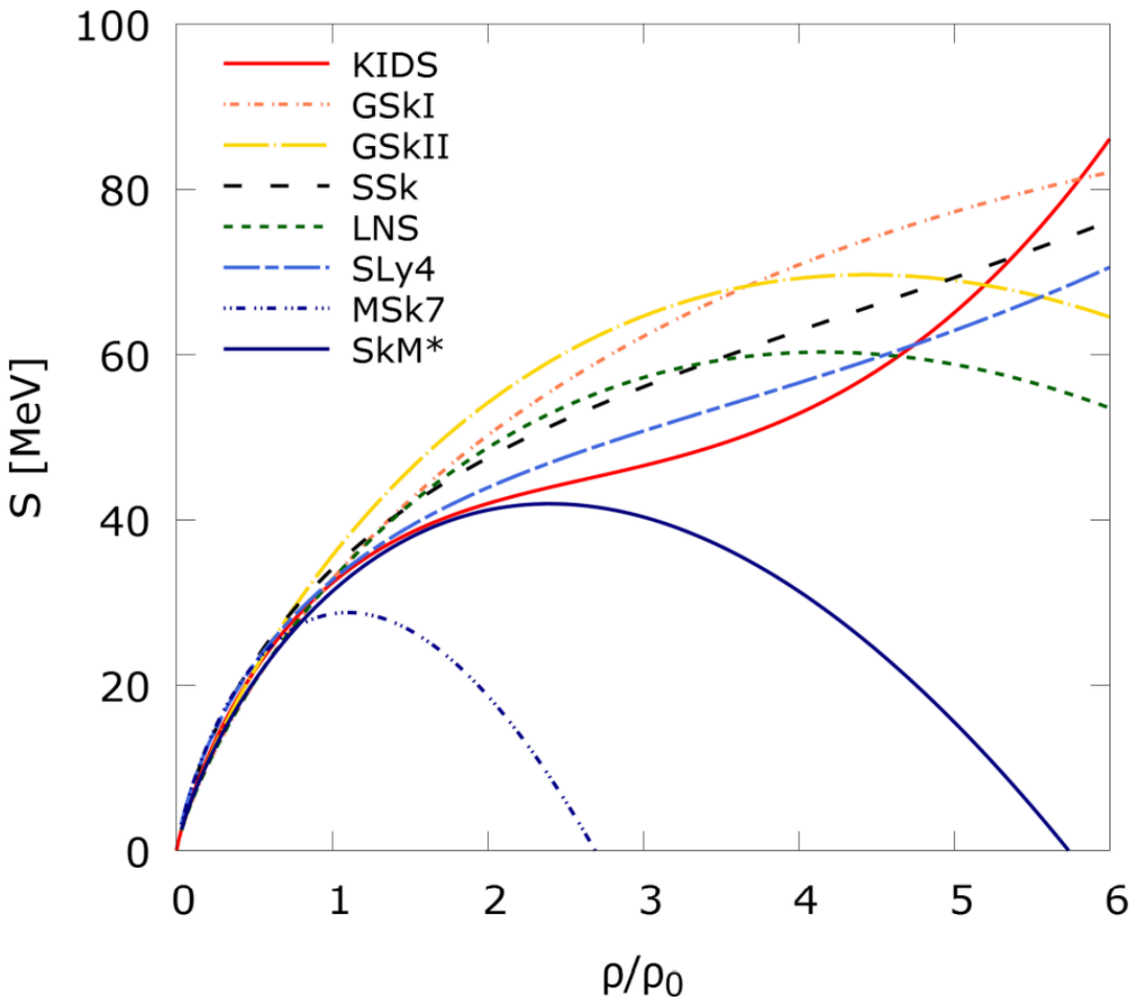
Abbott et al. (LSC and Virgo),
arxiv:1805.11581 (PRL accepted)



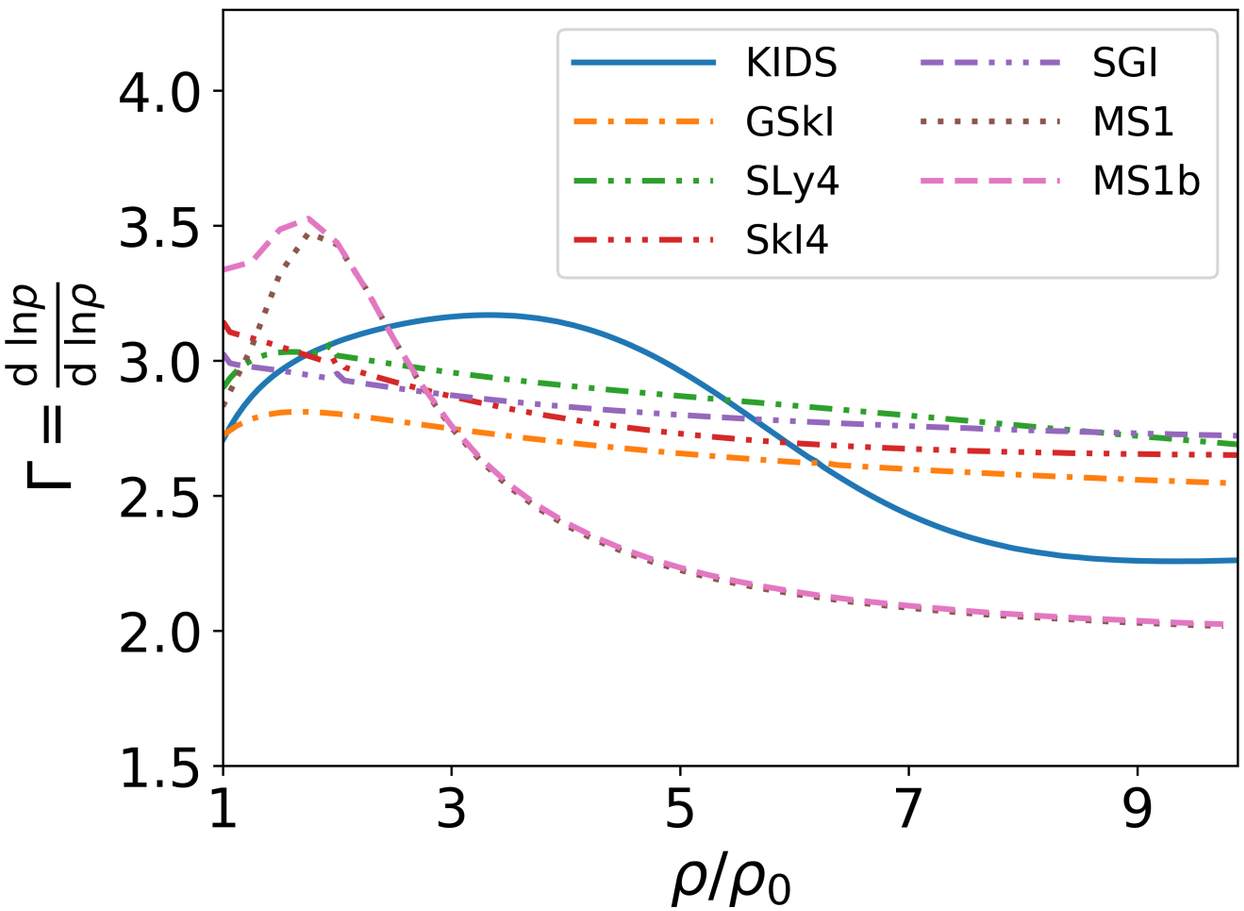
Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6} (R/km)^{7.5}$ (fitting function in [C])

Nuclear Properties

H. Gil, et al. , PRC, 99,064319 (2019)



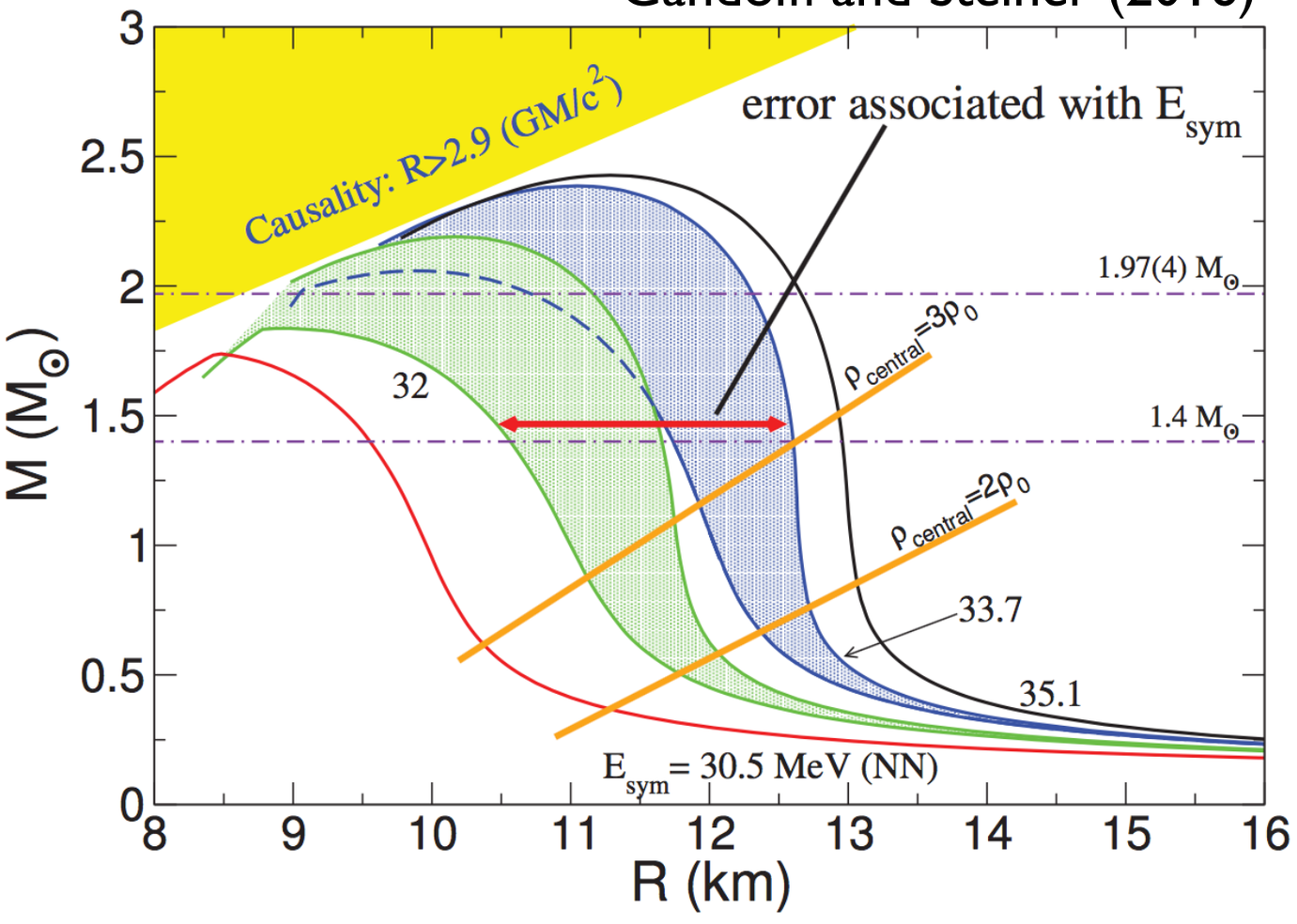
Y.-M. Kim et al., in preparation



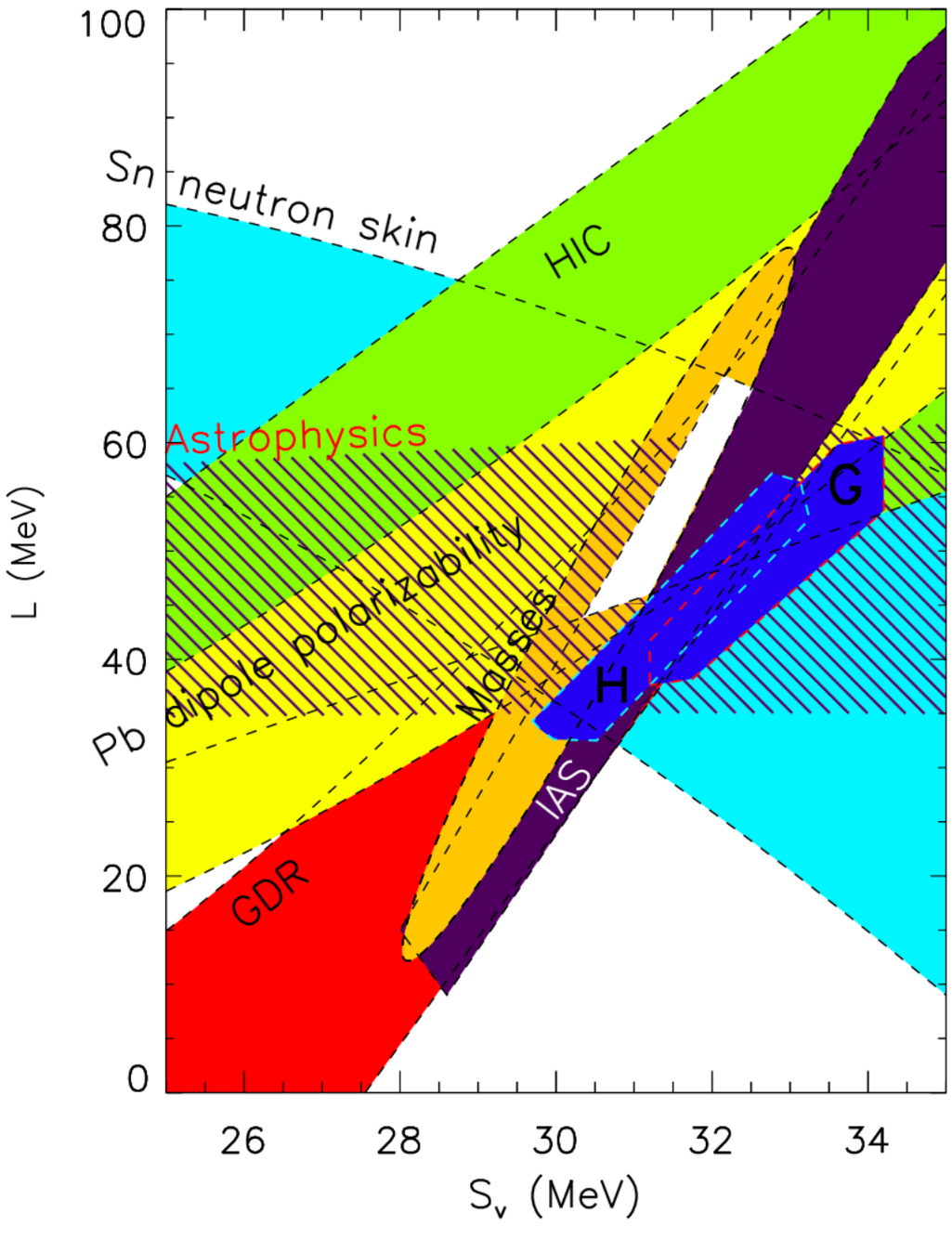
$$\mathcal{S}(\rho) = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}\mathscr{E}(\rho,\delta)\Big|_{\delta=0} = J + Lx + \tfrac{1}{2}K_{\rm sym}x^2 + \tfrac{1}{6}Q_{\rm sym}x^3 + \tfrac{1}{24}R_{\rm sym}x^4 + O(x^5), \quad \text{where } x = (\rho-\rho_0)/(3\rho_0).$$

Nuclear Symmetry Energy

Gandolfi and Steiner (2016)



J.M. Lattimer / Nuclear Physics A 928 (2014) 276–295



$$\frac{E}{A} = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) + \ldots,$$

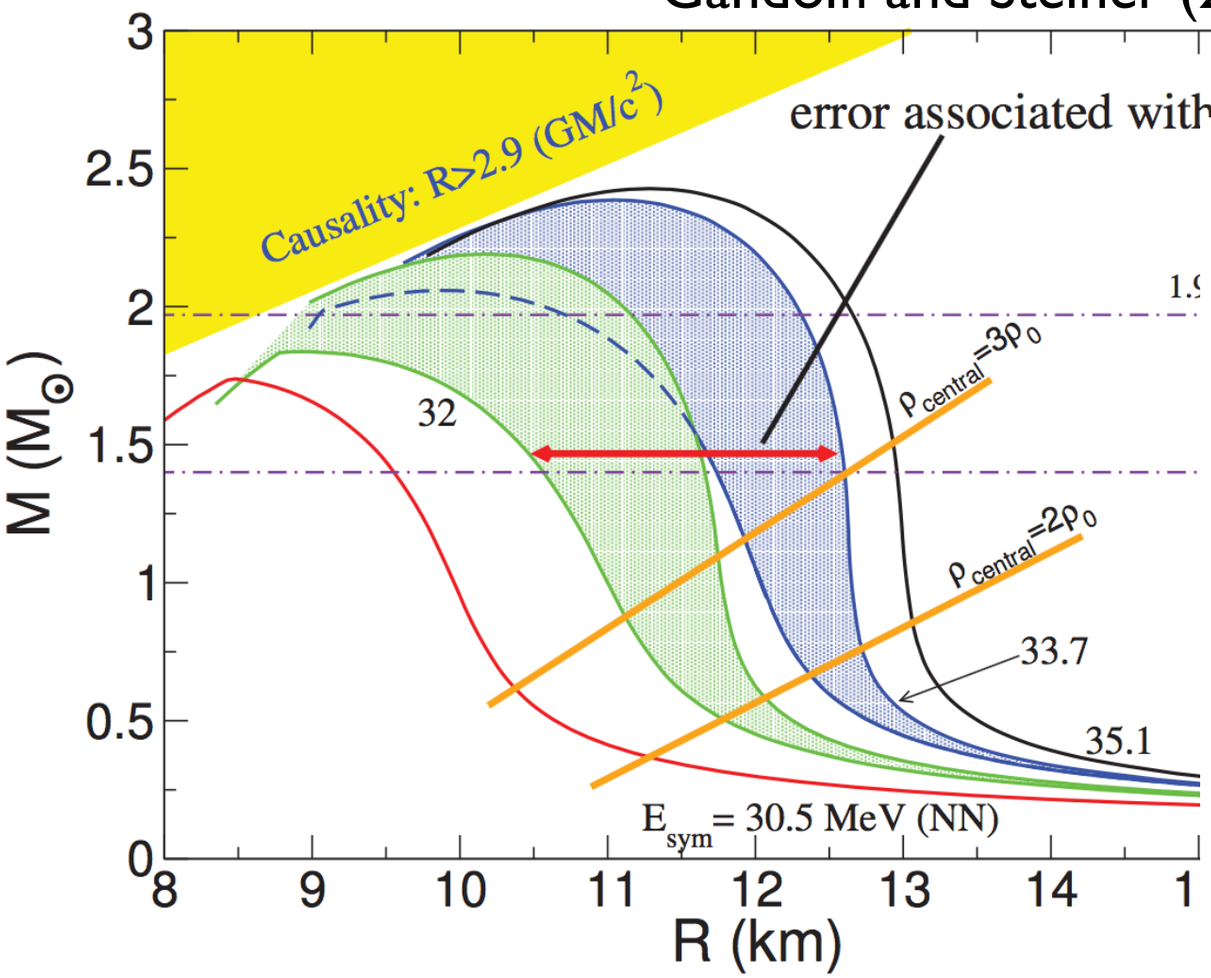
$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2$$

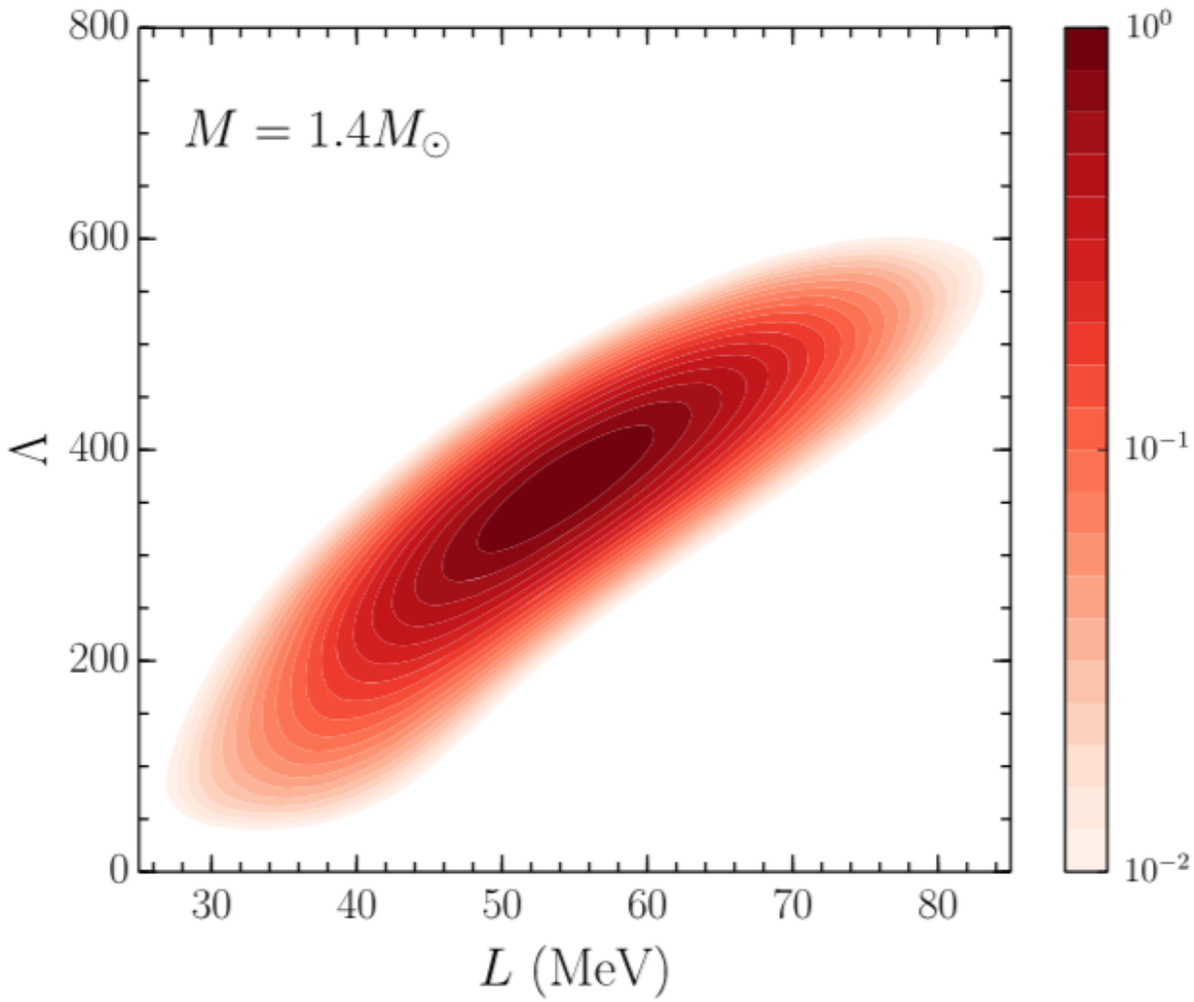
$$L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho}|_{\rho=\rho_0} \qquad K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}|_{\rho=\rho_0}.$$

Nuclear Symmetry Energy

J.M. Lattimer / Nuclear Physics A 928 (2014) 276–295

Gandolfi and Steiner (2016)





$$\frac{E}{A} = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) + \ldots,$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)$$

$$L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho}|_{\rho=\rho_0} \qquad K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}|_{\rho=\rho_0}.$$

isospin-asymmetry energy slope parameter L

[D] Y.Lim and J. Holt, PRL.121.062701 (arXiv:1803.02803v2)

Bayesian Analysis

Bayes' theorem

The likelihood could be the function of errors

Posterior $p(\theta|d) = \frac{p(d|\theta)}{p(d)} \cdot p(\theta)$

Prior choices can influence results

$$= \frac{p(d|\theta)}{\int p(d|\theta)p(\theta)d\theta} \cdot p(\theta)$$

The evidence is unimportant for parameter estimation (but not model selection !)

$$p(\theta|d) \sim p(d|\theta)p(\theta)$$

Prior, p(θ) : the distribution of the parameter(s) before any data is observed

Likelihood, p(d|θ) : the distribution of the observed data conditional on its parameters

Posterior, p(θ|d): the distribution of the parameter(s) after taking into account the observed data

Model evidence, p(d):the distribution of the observed data marginalized over the parameter(s)

Bayesian Analysis for KIDS - 1st approach

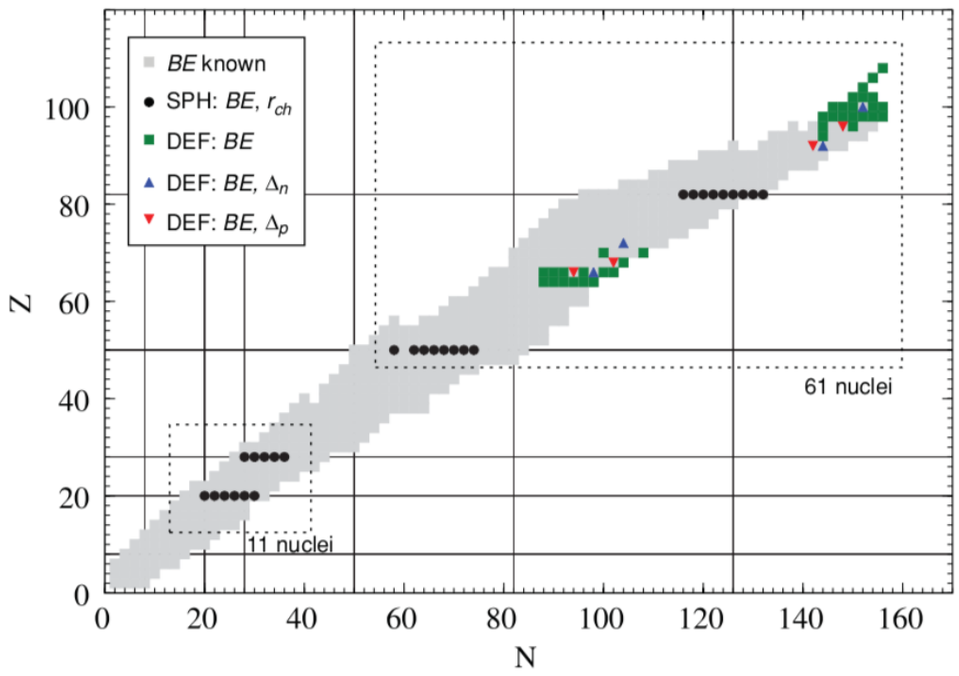
$$L \sim exp(-\Sigma(d_i - m_i)^2/2\sigma^2)$$

$$p(\theta|d) = \frac{p(\theta|d)}{p(d)} \cdot p(\theta)$$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$

+ $\langle R_{ch}^2 \rangle$

PHYSICAL REVIEW C **82**, 024313 (2010)



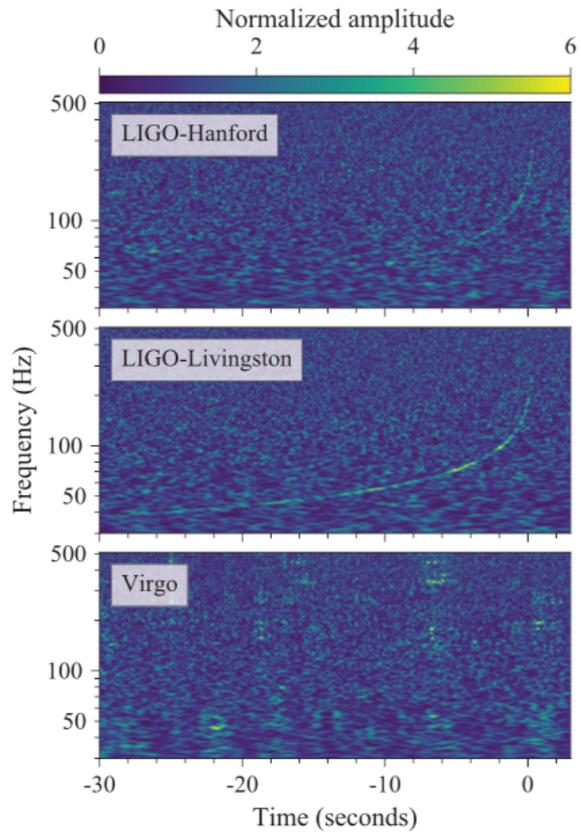
Do MCMC (Metropolis-Hasting algorithm)

Bayesian Analysis for KIDS - 2nd approach

$$L \sim exp(-\Sigma(d_i - m_i)^2/2\sigma^2)$$

$$p(\theta|d) = \frac{p(\theta|d)}{p(d)} \cdot p(\theta)$$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$



+ EM Obs. (LMXB, etc.)

Do MCMC (Metropolis-Hasting algorithm)

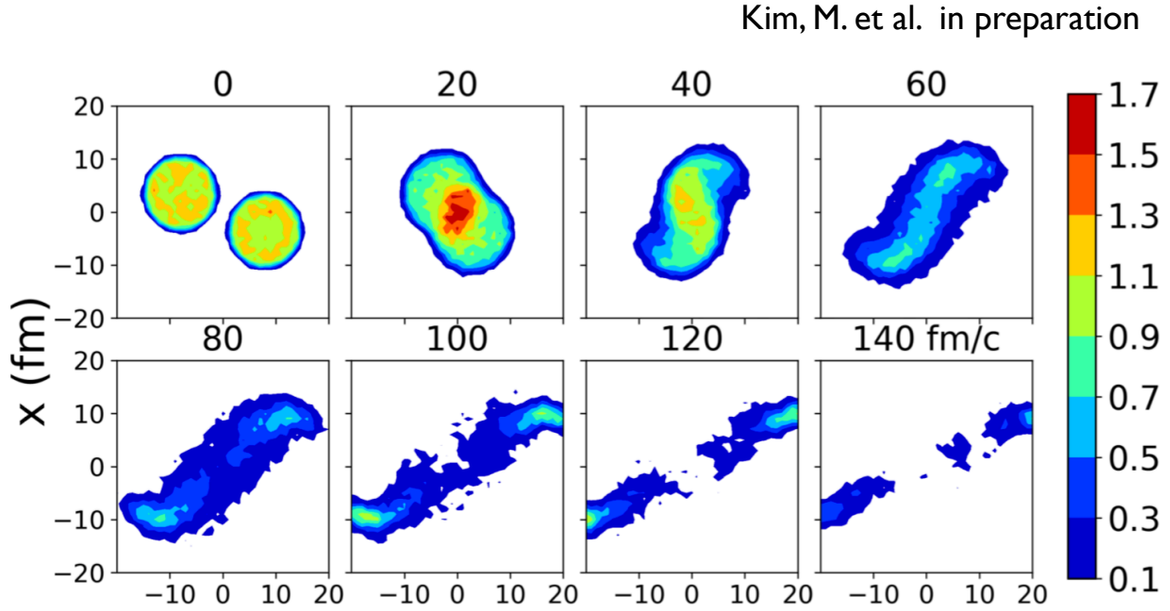
Bayesian Analysis for KIDS - 3rd approach

$$L \sim exp(-\Sigma(d_i - m_i)^2/2\sigma^2)$$

$$p(\theta|d) = \frac{p(\theta|d)}{p(d)} \cdot p(\theta)$$

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$

Kim, M. et al. in preparation



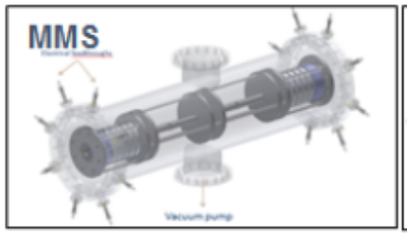
^{197}Au+^{197}Au@ 100 AMeV, b=7fm

Do MCMC (Metropolis-Hasting algorithm)

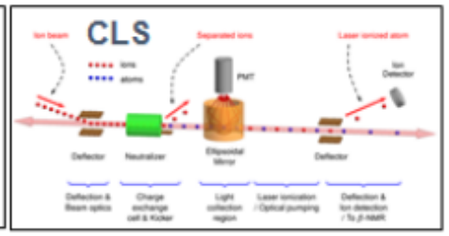
RAON - Rare Isotope Accelerator

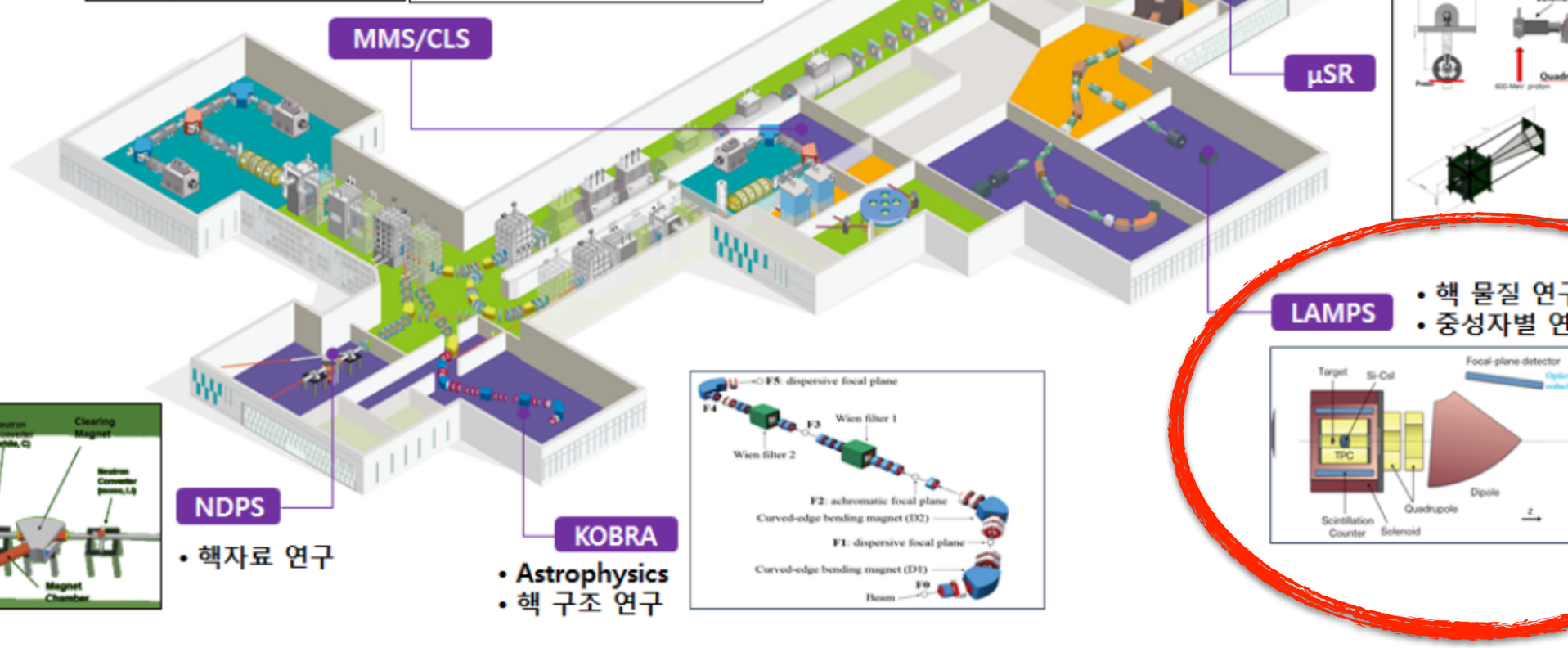
- **Experimental Systems**

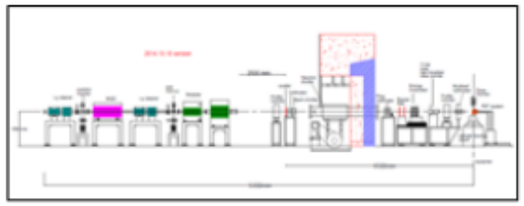
• 동위원소 질량 측정



• Charge Radius 측정

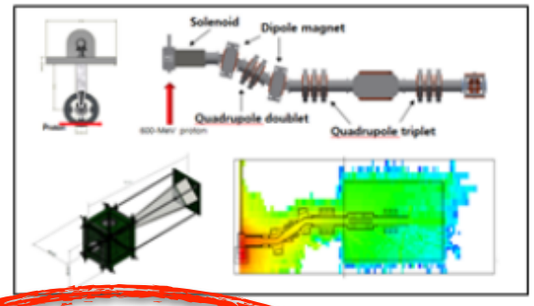


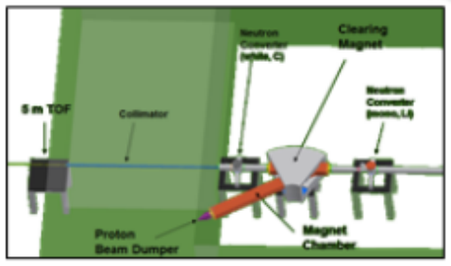




• 암 치료 연구
• 새로운 육종 연구

• 물성 연구





• 핵자료 연구

• Astrophysics
• 핵 구조 연구

• 핵 물질 연구
• 중성자별 연구

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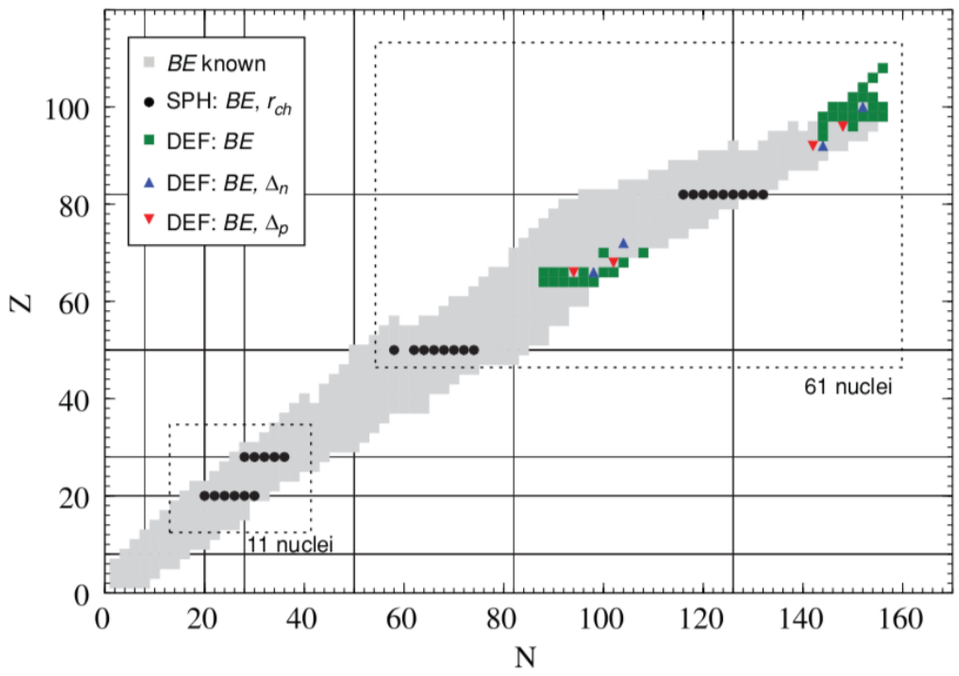
$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$

+ $\langle R_{ch}^2 \rangle$

PHYSICAL REVIEW C **82**, 024313 (2010)



Do MCMC (Metropolis-Hasting algorithm)

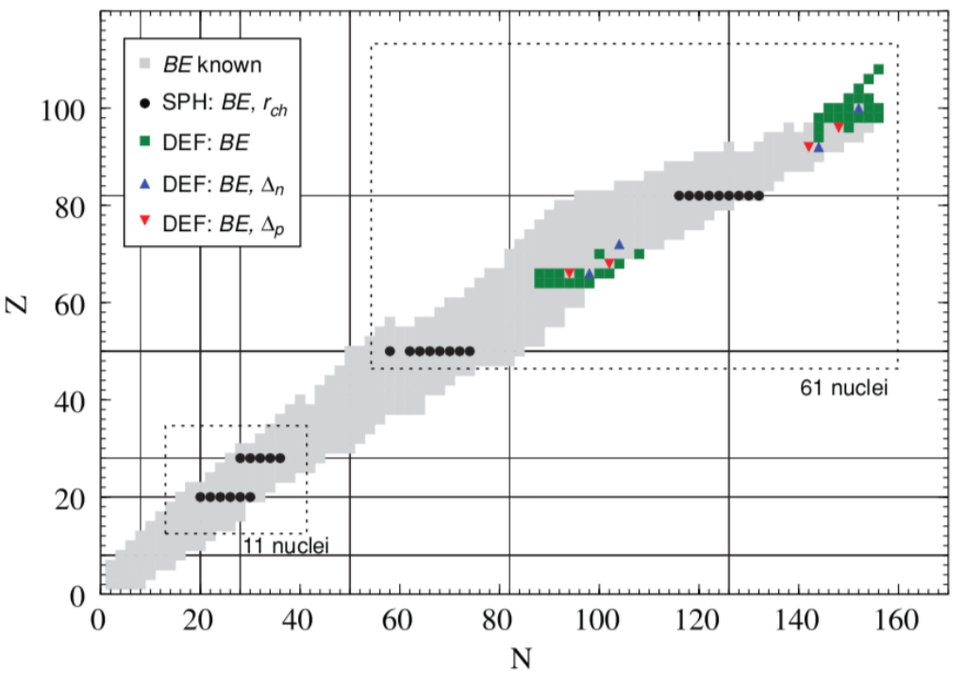
Bayesian Analysis for KIDS - details (1)

$$L \sim exp(-\Sigma(d_i - m_i)^2/2\sigma^2)$$

$$p(\theta|d) = \frac{p(\theta|d)}{p(d)} \cdot p(\theta)$$

Previous studies

PHYSICAL REVIEW C **82**, 024313 (2010)



+ $<R_{ch}^2>$

H. Gil, et al. PRC, 100, 014312 (2019)

Parameter	S3b	S4a	S4b	S4c
t_0 (MeV fm^3)	−1772.04	−1807.17	−1723.84	−1640.50
y_0 (MeV fm^3)	−127.52	−162.65	−79.32	4.02
t_1 (MeV fm^5)	275.72	262.17	288.94	303.28
t_2 (MeV fm^5)	−161.50	−167.94	−154.90	−146.98
t_{31} (10^4 MeV fm^4)	1.222	1.338	1.062	0.7855
y_{31} (10^4 MeV fm^4)	−1.197	−1.081	−1.357	−1.633
t_{32} (MeV fm^5)	571.0	−1310.7	3252.4	8043.0
y_{32} (10^4 MeV fm^5)	2.949	2.704	3.274	3.818
t_{33} (MeV fm^6)	–	1317.2	−1807.8	−4932.8
y_{33} (10^4 MeV fm^6)	−2.296	−2.164	−2.476	−2.789
ζ	0.1106	0.1281	0.0931	0.0729
W_0 (MeV fm^5)	108.35	106.79	109.88	111.55

Bayesian Analysis for KIDS - details (2)

H. Gil, et al. PRC, 100, 014312 (2019)

Nucleus	Binding energy per nucleon (E/A) (MeV)					Charge radius (R_c) (fm)				
	Expt.	S3b	S4a	S4b	S4c	Expt.	S3b	S4a	S4b	S4c
^{40}Ca	8.5513*	8.5565	8.5579	8.5544	8.5512	3.4776*	3.4781	3.4799	3.4758	3.4720
		(0.060%)	(0.078%)	(0.037%)	(0.001%)		(0.014%)	(0.066%)	(0.052%)	(0.161%)
^{48}Ca	8.6667*	8.6564	8.6569	8.6558	8.6549	3.4771*	3.4867	3.4882	3.4847	3.4813
		(0.120%)	(0.113%)	(0.126%)	(0.136%)		(0.277%)	(0.319%)	(0.220%)	(0.122%)
^{208}Pb	7.8675*	7.8809	7.8816	7.8800	7.8783	5.5012*	5.4887	5.4901	5.4870	5.4840
		(0.172%)	(0.179%)	(0.160%)	(0.138%)		(0.228%)	(0.201%)	(0.259%)	(0.313%)
^{16}O	7.9762	7.8684	7.8675	7.8686	7.8678	2.6991	2.7618	2.7643	2.7587	2.7541
		(1.35%)	(1.36%)	(1.35%)	(1.36%)		(2.322%)	(2.41%)	(2.209%)	(2.036%)
^{28}O	–	6.0646	6.0640	6.0650	6.0649	–	2.8371	2.8384	2.8351	2.8315
^{60}Ca	–	7.6561	7.6567	7.6552	7.6535	–	3.6465	3.6478	3.6445	3.6411
^{90}Zr	8.7100	8.7328	8.7345	8.7309	8.7282	4.2694	4.2476	4.2488	4.2459	4.2428
		(0.263%)	(0.281%)	(0.241%)	(0.209%)		(0.510%)	(0.482%)	(0.550%)	(0.622%)
^{132}Sn	8.3549	8.3563	8.3559	8.3565	8.3565	4.7093	4.7089	4.7100	4.7072	4.7044
		(0.017%)	(0.013%)	(0.020%)	(0.020%)		(0.009%)	(0.015%)	(0.044%)	(0.103%)

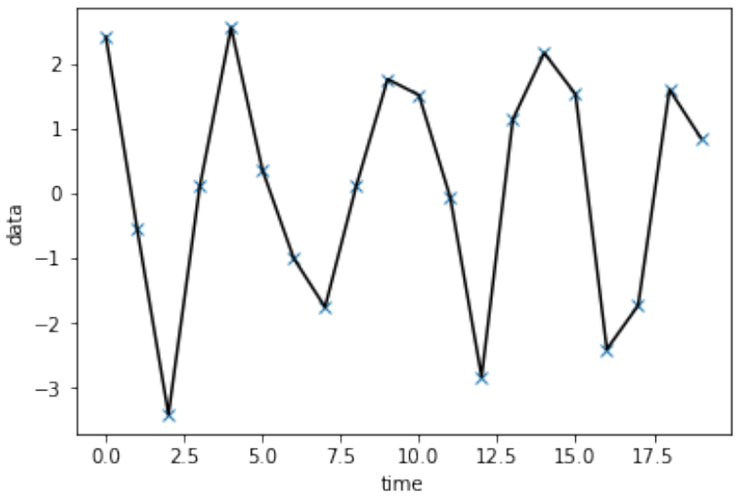
Skyrem-Hatree-Fock Calculation

My Hope is ….

1. A MCMC example : https://github.com/gw-odw/odw-2018/blob/master/parameter_estimation/IntroToMCMC.ipynb

$$d = A \cdot cos(f \cdot t + \phi)$$

<model>



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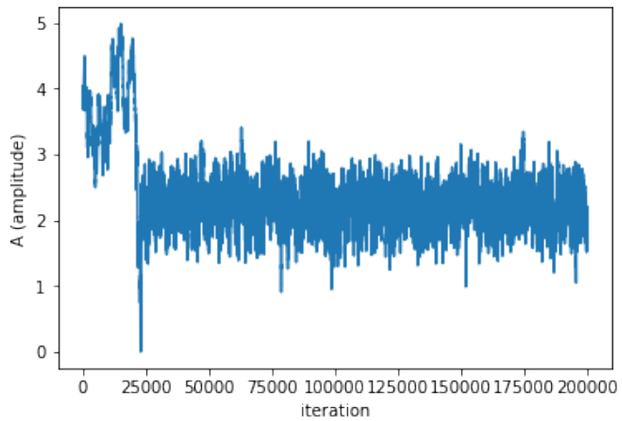
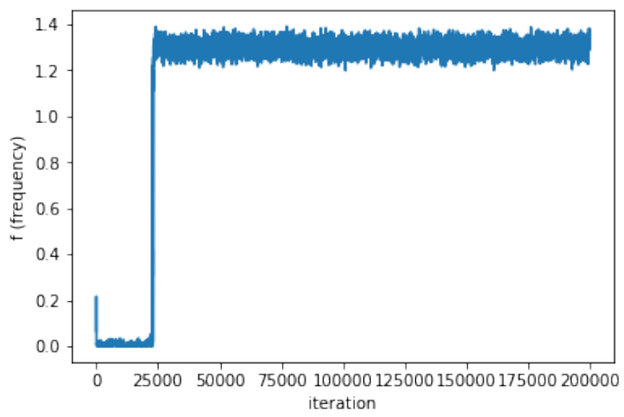
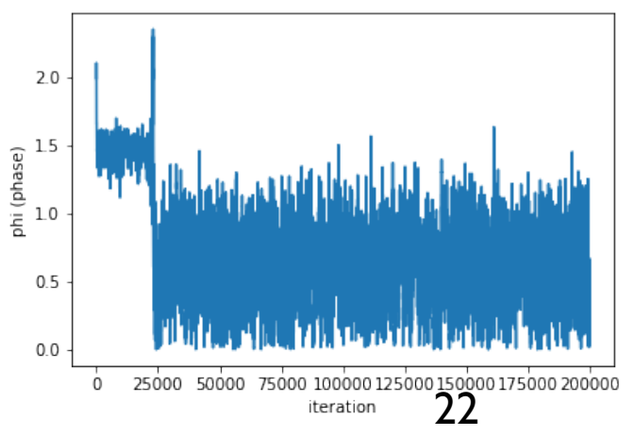
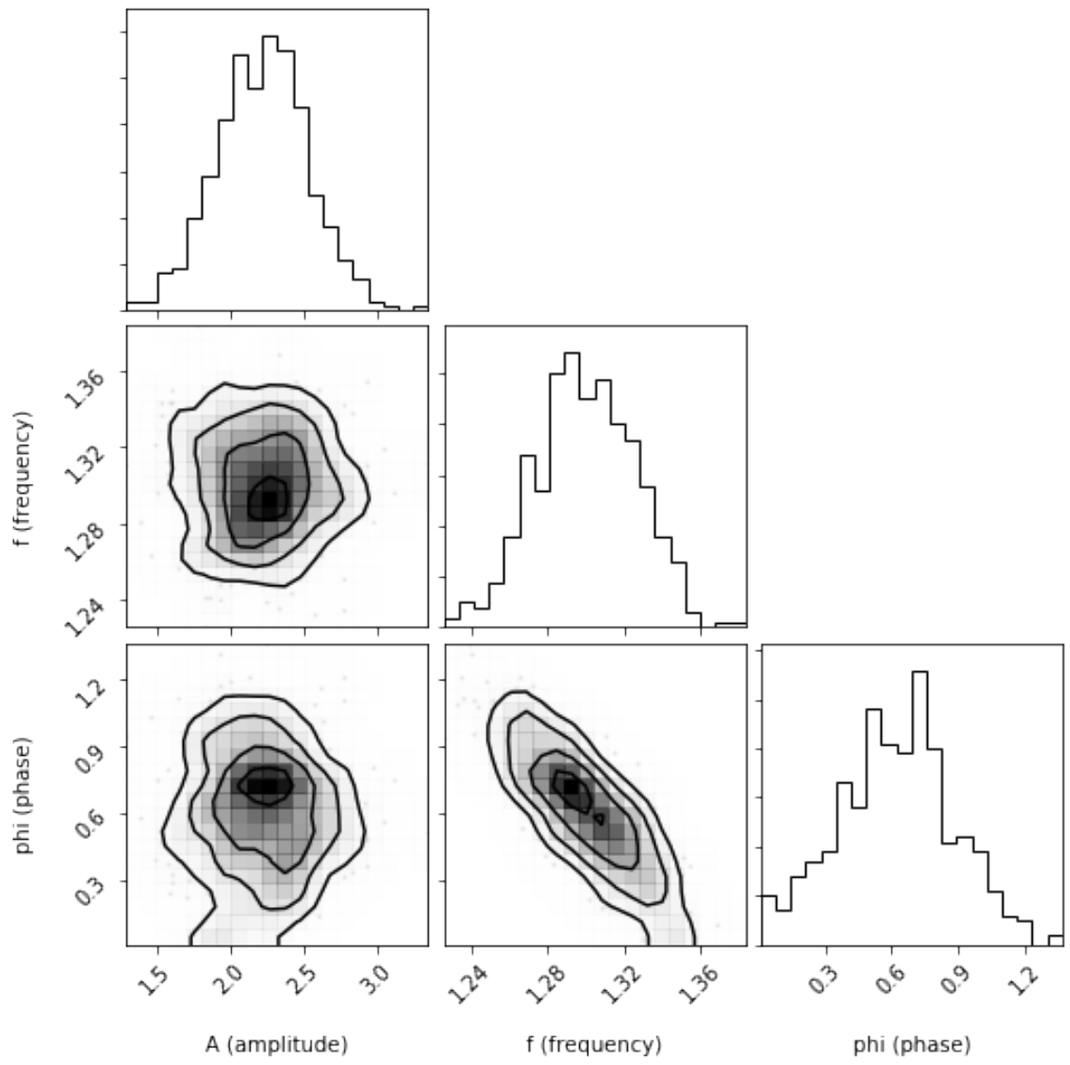
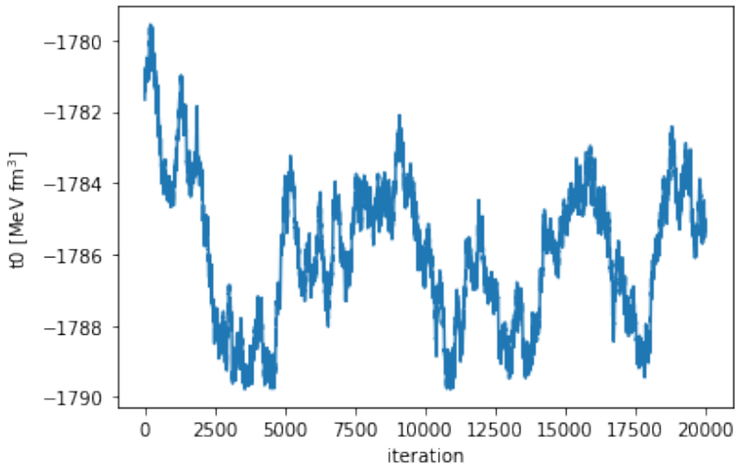


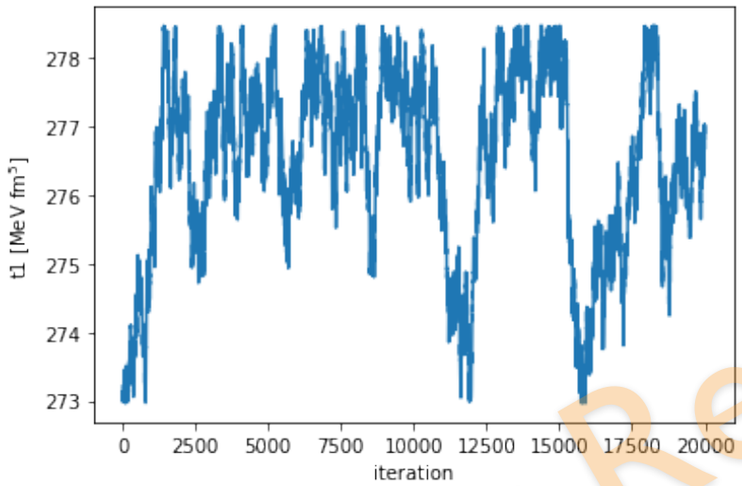


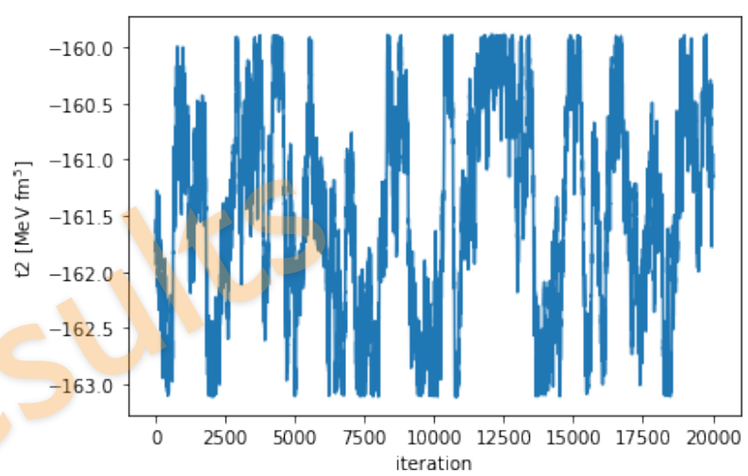


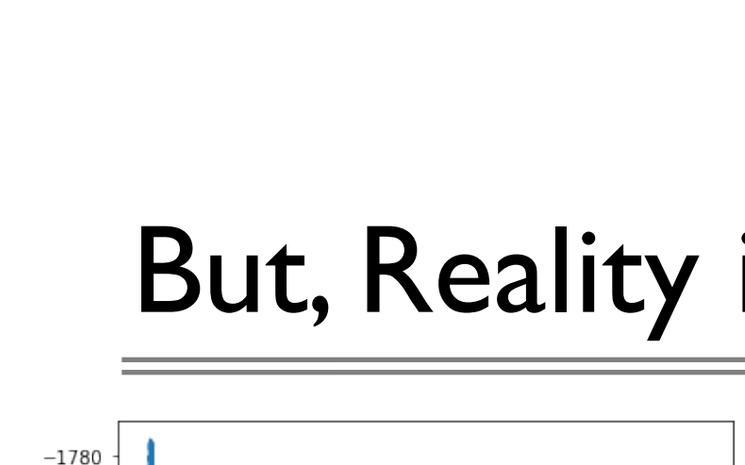


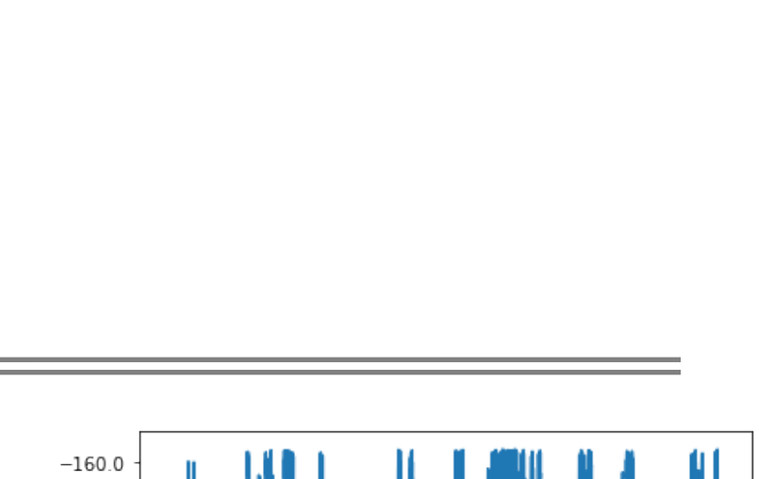
But, Reality is ….

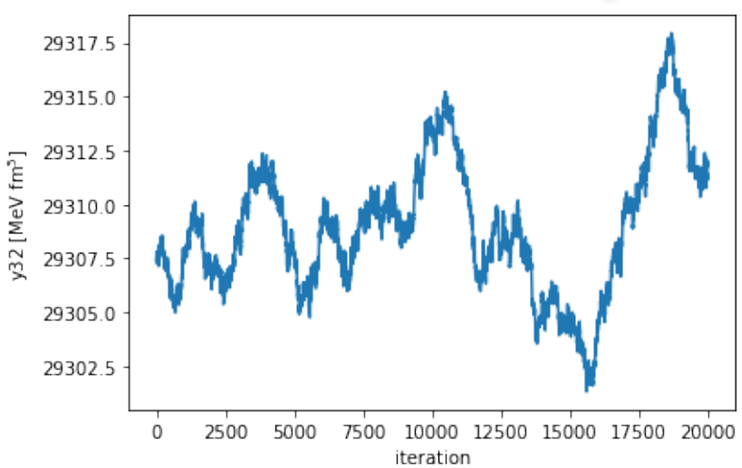


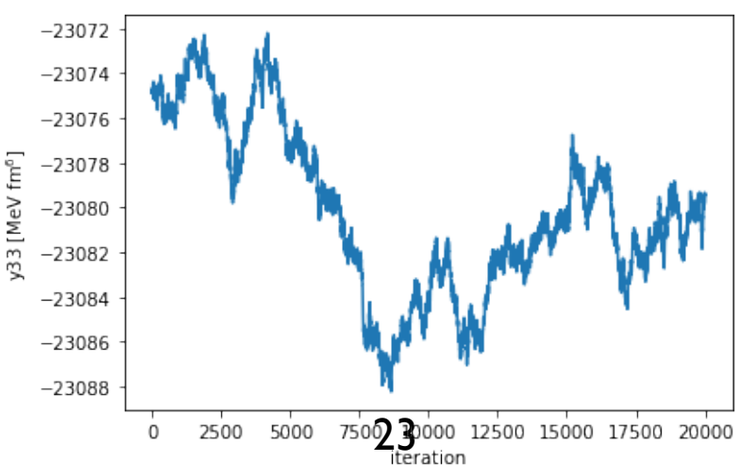


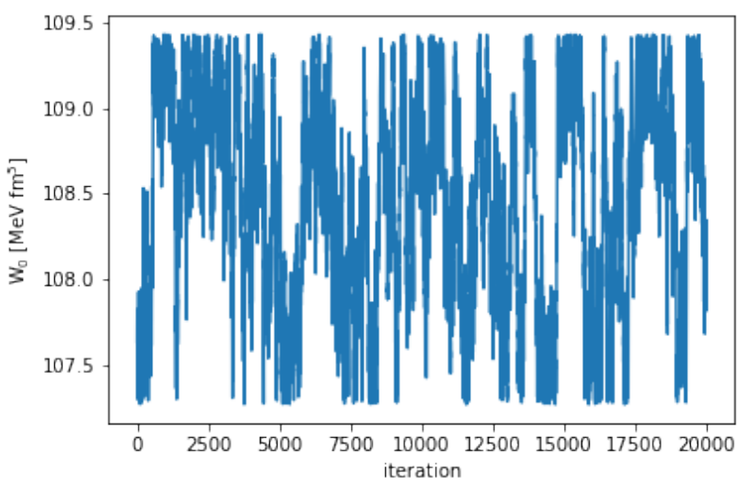




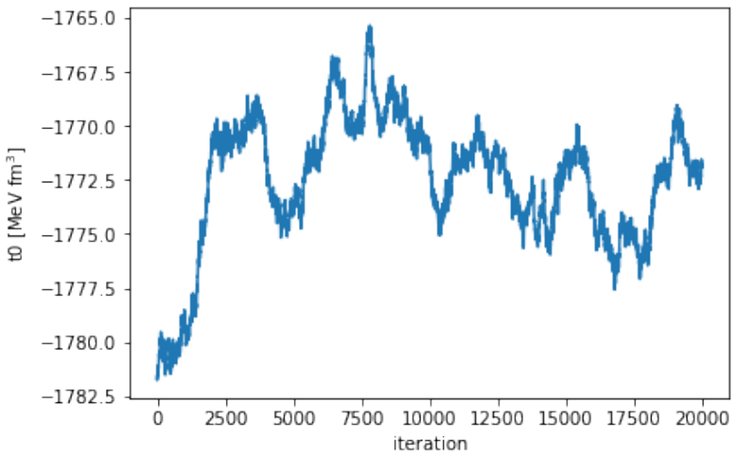


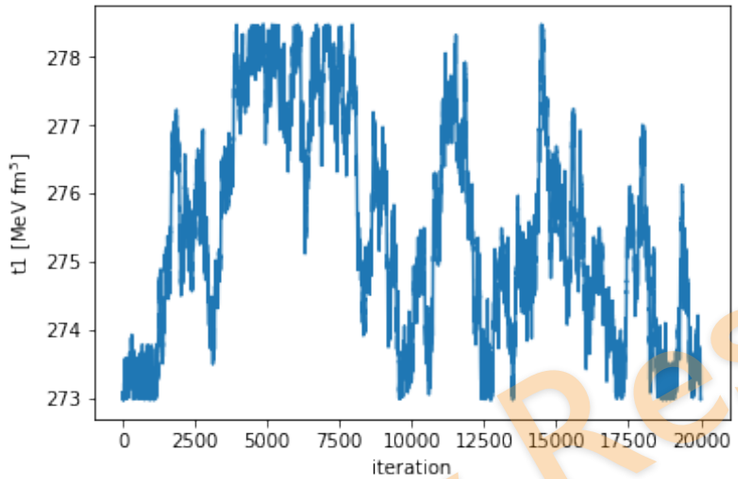


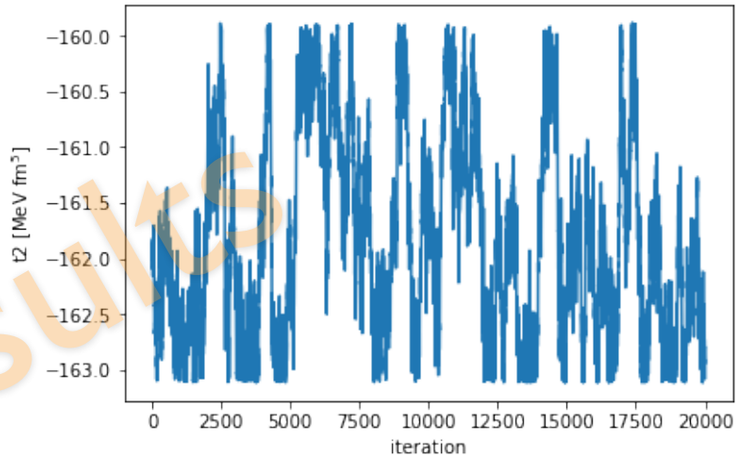


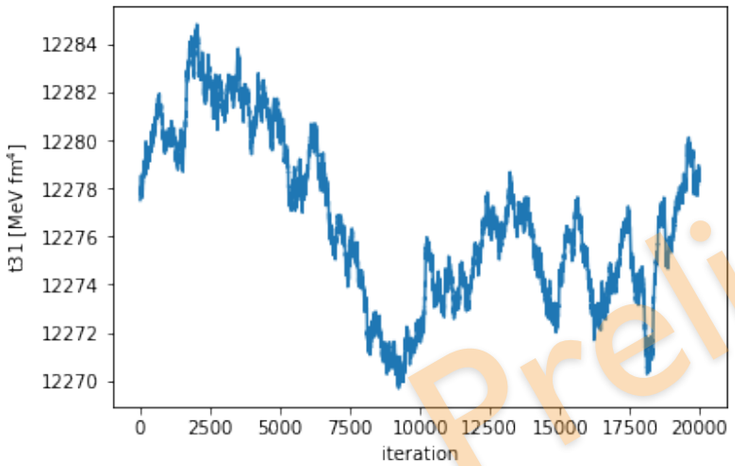


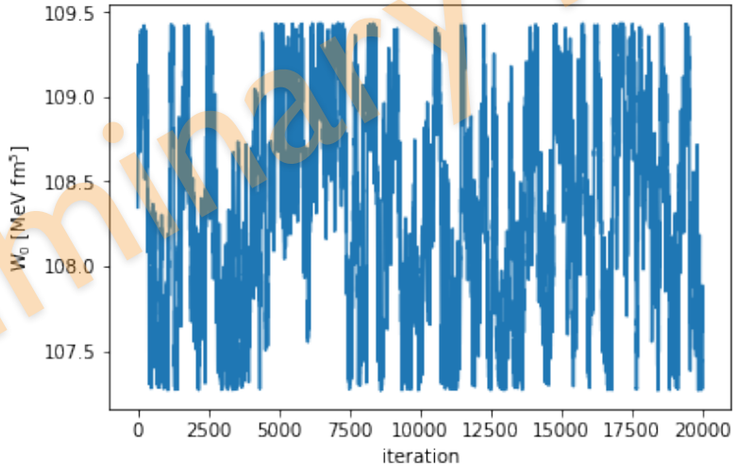
Variables : t0, t1, t2, t31, W0

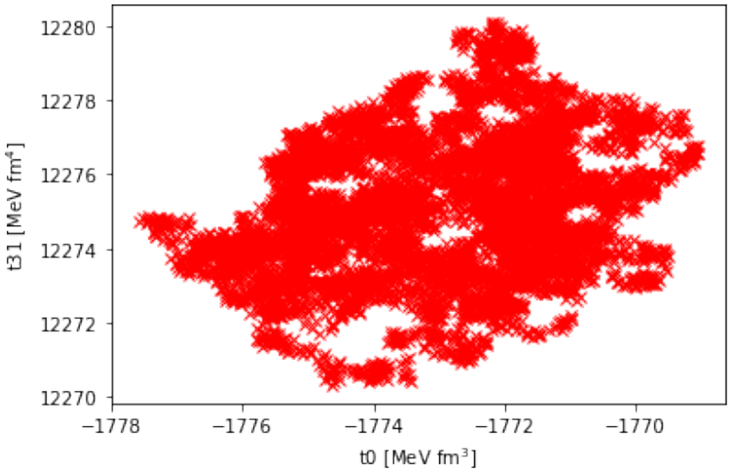


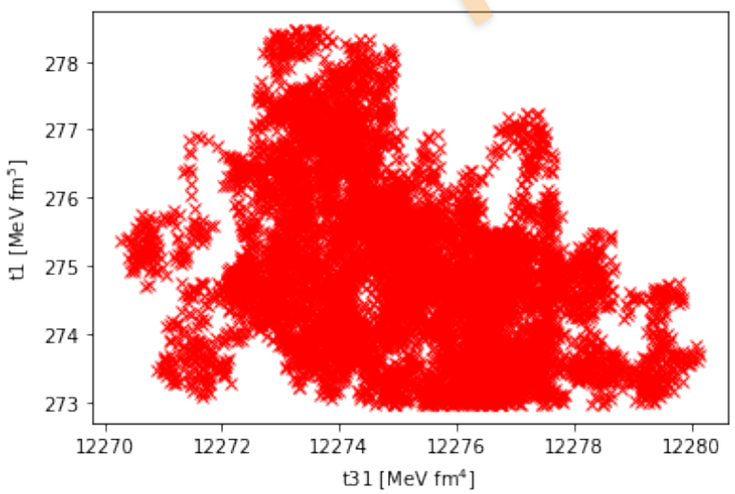


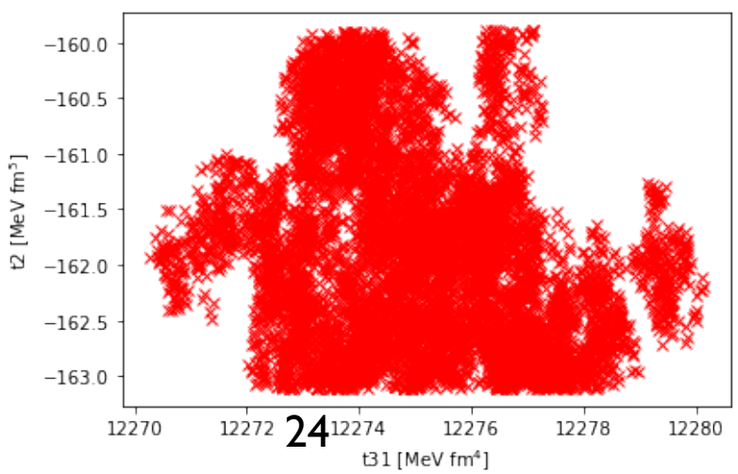


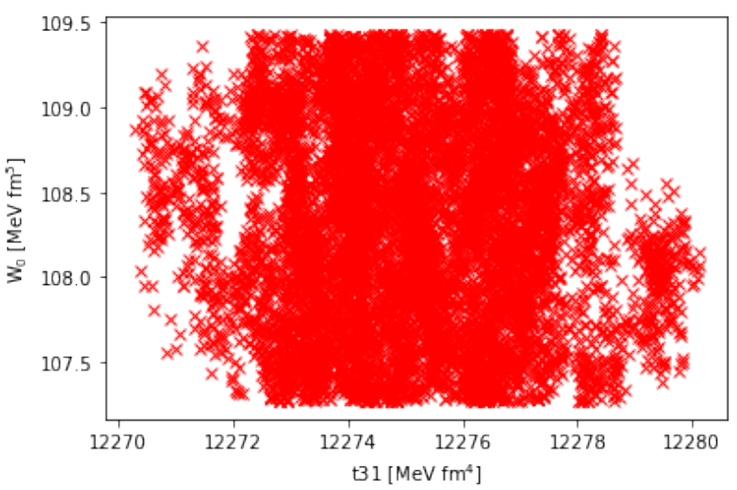




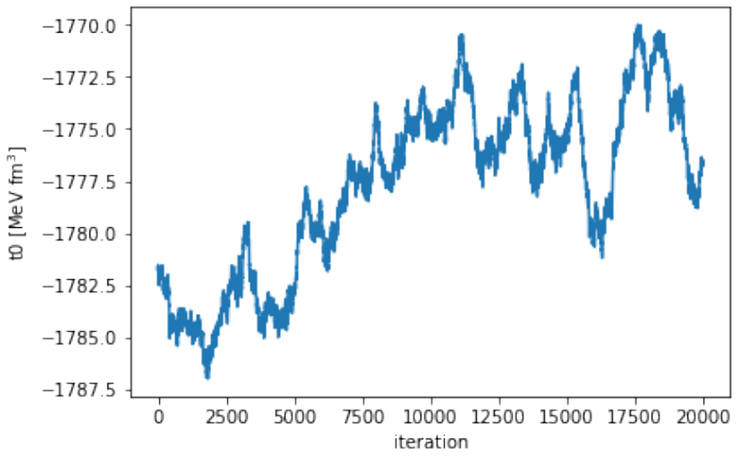


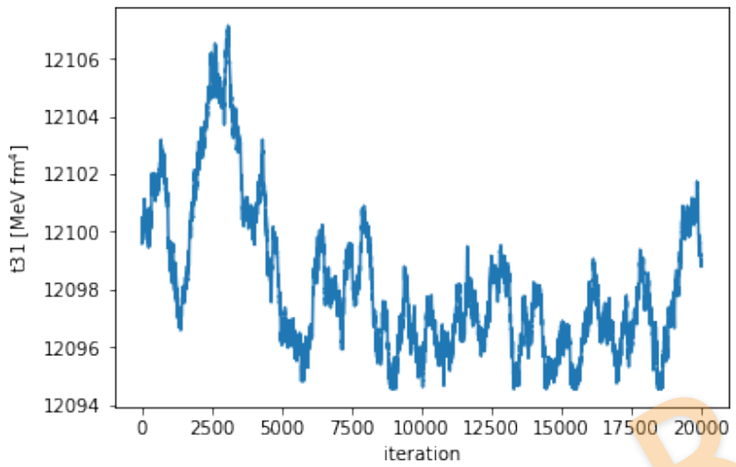


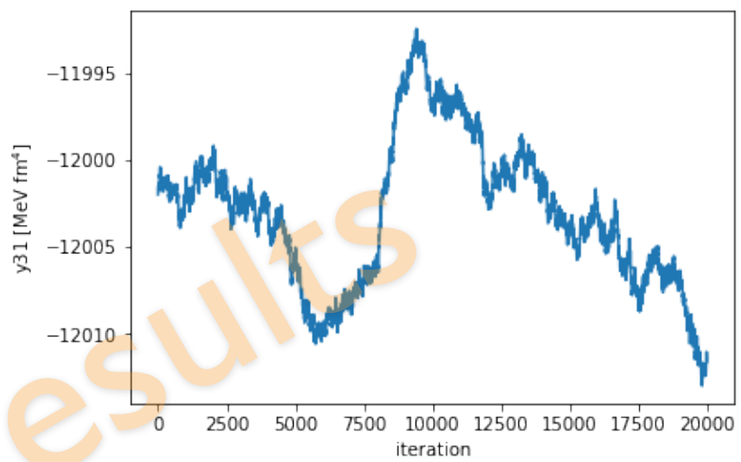


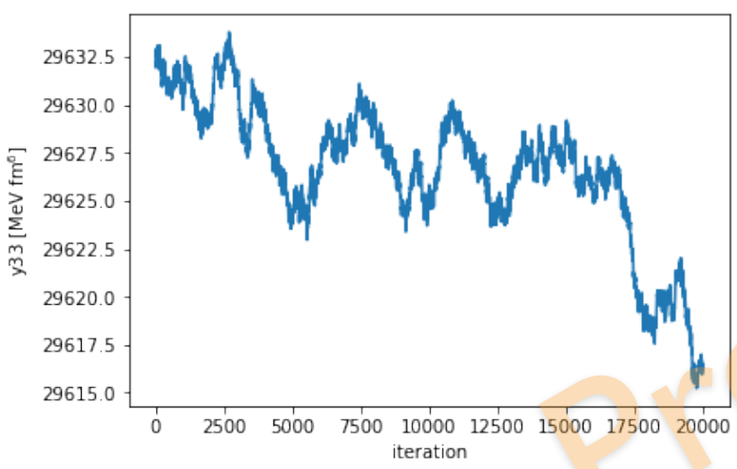


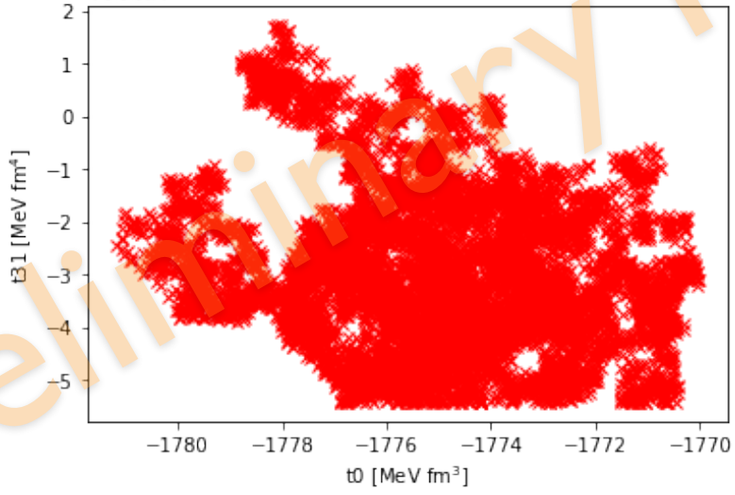
Variables : t0, t31, y31, y33

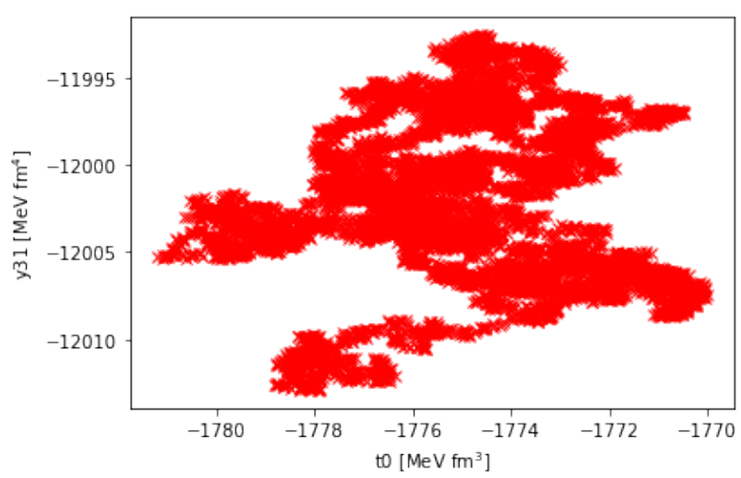


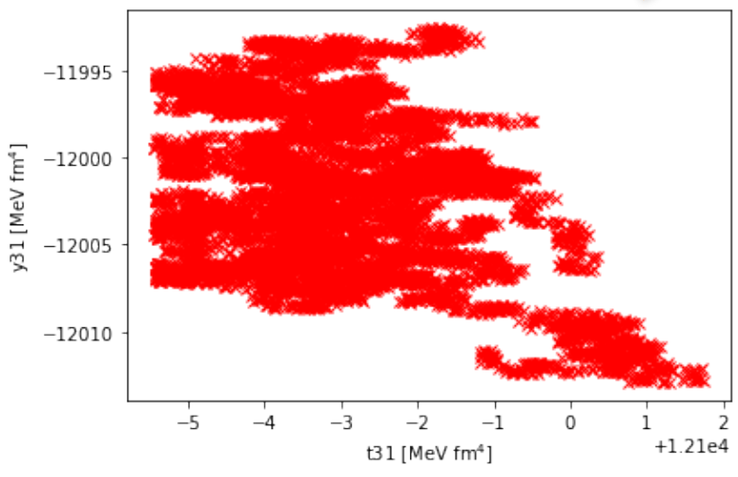


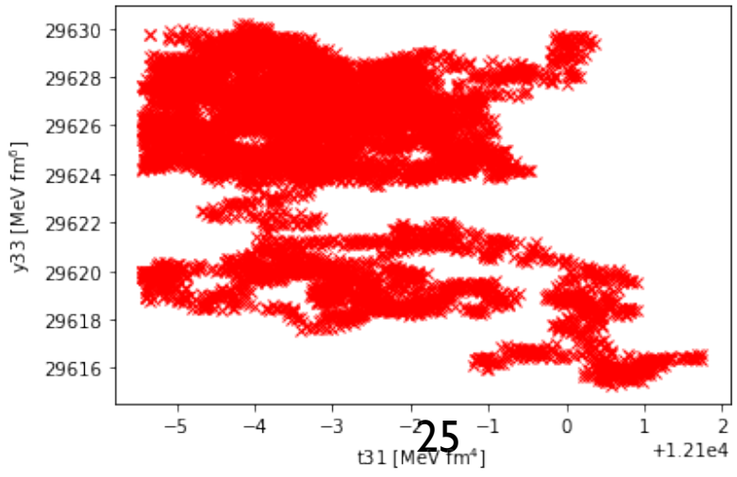


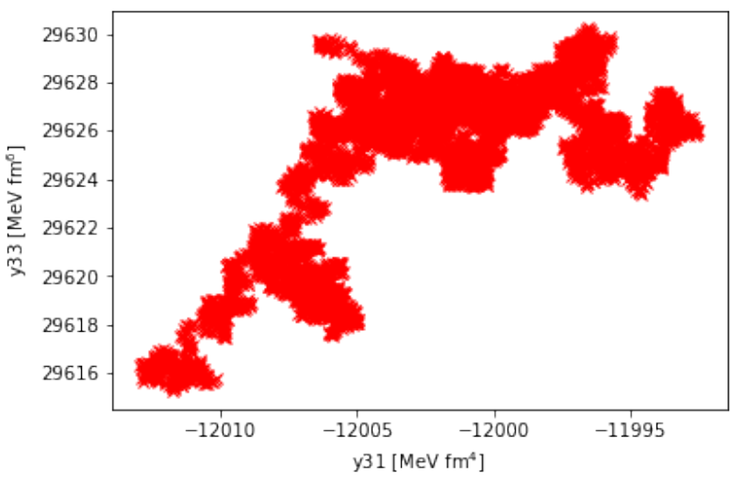








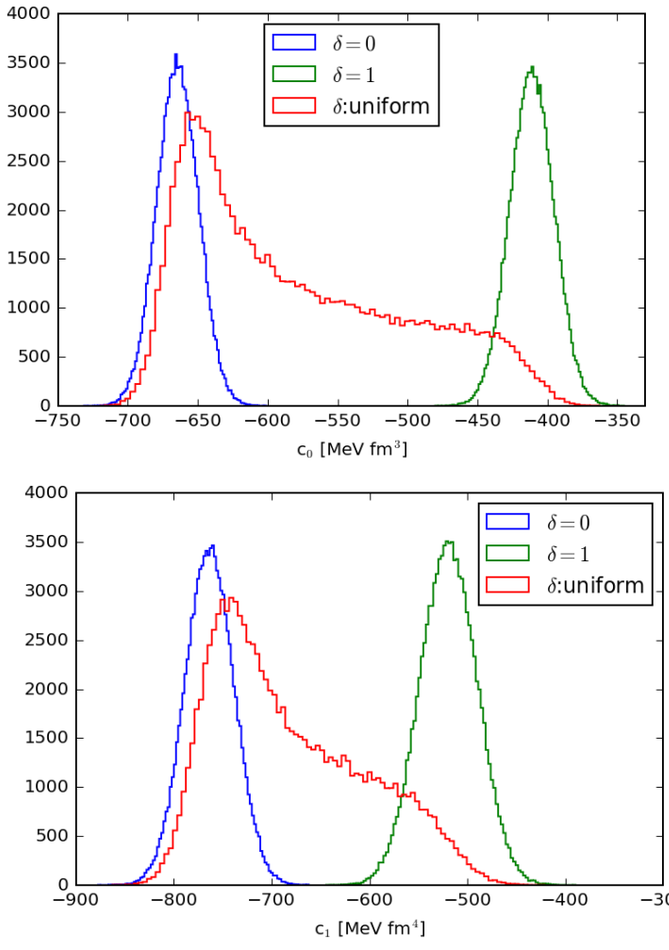


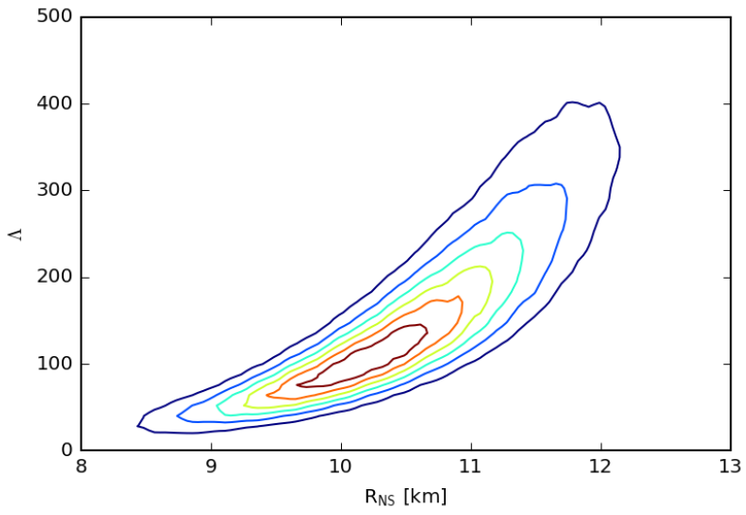


Future Plan

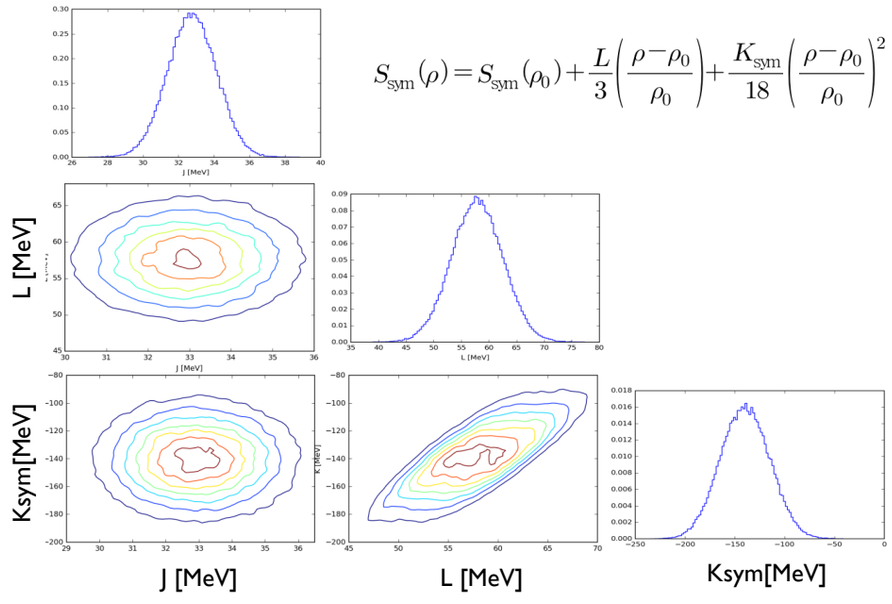
1. Bayesian Analysis on GWDA as well as RAON/LAMPS exp. + DJBUU

   - To Do better !!
   - MCMC code (C++ based) development for various applications

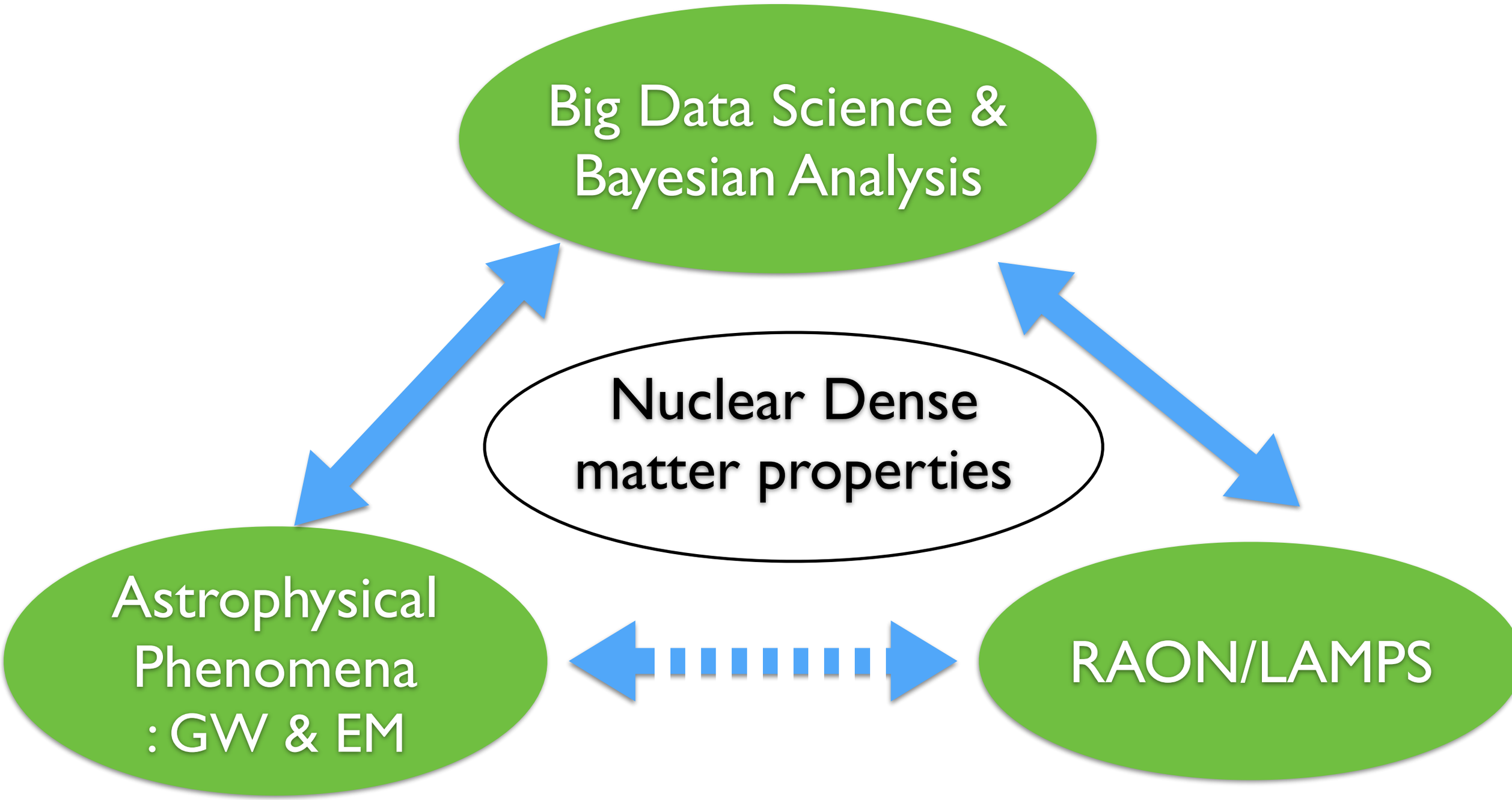




$$S_{sym}(\rho) = S_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2$$



What I want to do with Bayesian …



Thank you for your attention.