“Superfluid” neutron star dynamics: From oscillations to tidal deformation

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based on collaboration with
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Superfluidity and superconductivity in neutron stars

Internal temperature

Newborn neutron star ~ 10^{12} K

After a few decades ~ 10^{8} K

Expected transition temperature for neutrons and protons to become superfluid and superconducting

~ 10^{9} K

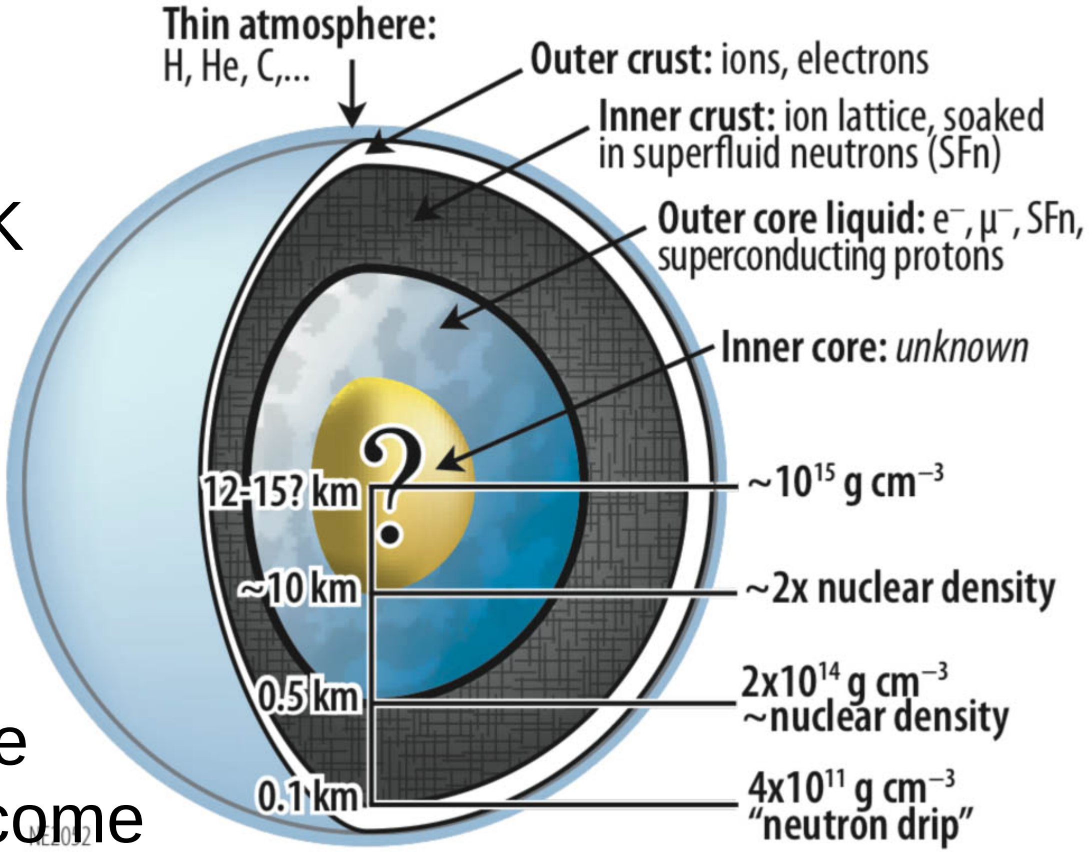
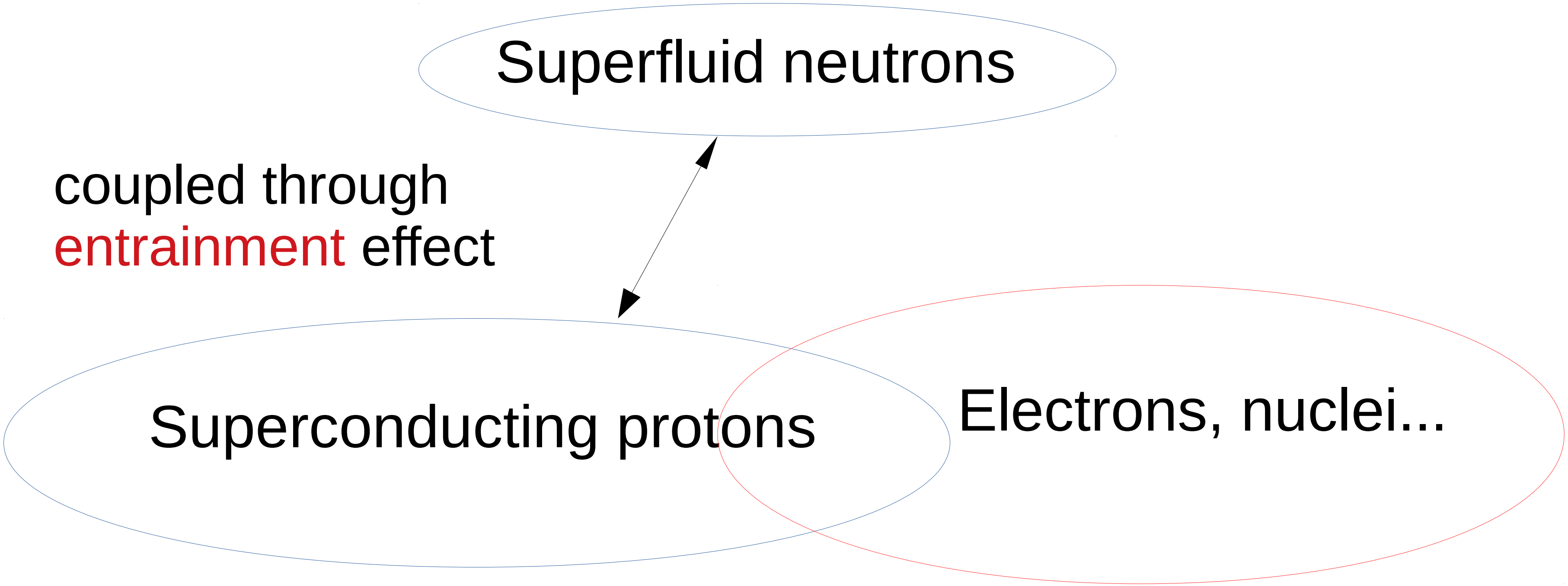


Fig source: heasarc.gsfc.nasa.gov/

Could we probe the existence of nucleon superfluid from neutron star dynamics?
(eg, f-mode and tidal deformability)

What we need in order to model superfluid neutron stars….at least from a hydrodynamics perspective? (dirty nuclear physics ignored….Sorry!)



Superfluid neutrons

coupled through
entrainment effect

Superconducting protons

Electrons, nuclei...

Charged particles locked together

Minimal model: Two inter-penetrating fluids coupled through gravity and possibly entrainment effect

In general relativity, the hydrodynamics equations follows from

$$\nabla_\alpha T^{\alpha\beta} = 0$$

But it is not enough when there are 2 or more fluids!

We need (at least) two independent number density currents:

n^μ for superfluid neutrons

p^μ for “protons”
(protons, electrons etc)

We can form 3 scalars out of them:

$$n^2 = -n_\alpha n^\alpha \quad , \quad p^2 = -p_\alpha p^\alpha \quad , \quad x^2 = -p_\alpha n^\alpha$$

General Relativistic Two-Fluid Formalism

- Brandon Carter and his collaborators have developed a variational formalism to study the hydrodynamics of relativistic superfluids:

The central quantity: Master function $\Lambda\left(n^2, p^2, x^2\right)$

(Take $\Lambda = -$ energy density)

$$n^2 \equiv -n_\alpha n^\alpha \quad , \quad p^2 \equiv -p_\alpha p^\alpha \quad , \quad x^2 \equiv -p_\alpha n^\alpha$$

n = neutron number density

p = proton number density

The master function contains all information about the local thermodynamic state of the fluid.
It is the two-fluid analog of the equation of state.

A general variation of the master function:
(that spacetime metric fixed)

$$\Lambda(n^2, p^2, x^2) \longrightarrow \delta\Lambda = \mu_\alpha \delta n^\alpha + \chi_\alpha \delta p^\alpha$$

Chemical potential vectors:

$$\mu_\alpha = B n_\alpha + A p_\alpha \quad , \quad \chi_\alpha = C p_\alpha + A n_\alpha$$

magnitude = neutron chemical potential

magnitude = “proton” chemical potential

$$\boxed{A \equiv -\frac{\partial \Lambda}{\partial x^2}} \quad , \quad B \equiv -2\frac{\partial \Lambda}{\partial n^2} \quad , \quad C \equiv -2\frac{\partial \Lambda}{\partial p^2}$$

The chemical potential vectors are the momentum canonically conjugate to the corresponding number density currents.
Note that they do not parallel to the corresponding currents when $A \neq 0$ (this is the entrainment effect)

One-fluid vs Two-fluid

$$G_{\alpha\beta}=8\pi T_{\alpha\beta}$$

$$P=P(\rho)$$

$$T^{\alpha\beta}=(\rho+P)u^{\alpha}u^{\beta}+P g^{\alpha\beta}$$

$$\nabla_{\alpha}T^{\alpha\beta}=0$$

$$-\rho=\Lambda(n^2,p^2,x^2)$$

$$T^{\alpha}_{\beta}=\Psi\,\delta^{\alpha}_{\beta}+n^{\alpha}\mu_{\beta}+p^{\alpha}\chi_{\beta}$$

$$\Psi=\Lambda-n^{\alpha}\mu_{\alpha}-p^{\alpha}\chi_{\alpha}$$

$$\nabla_{\alpha}n^{\alpha}=0\quad,\quad\nabla_{\alpha}p^{\alpha}=0$$

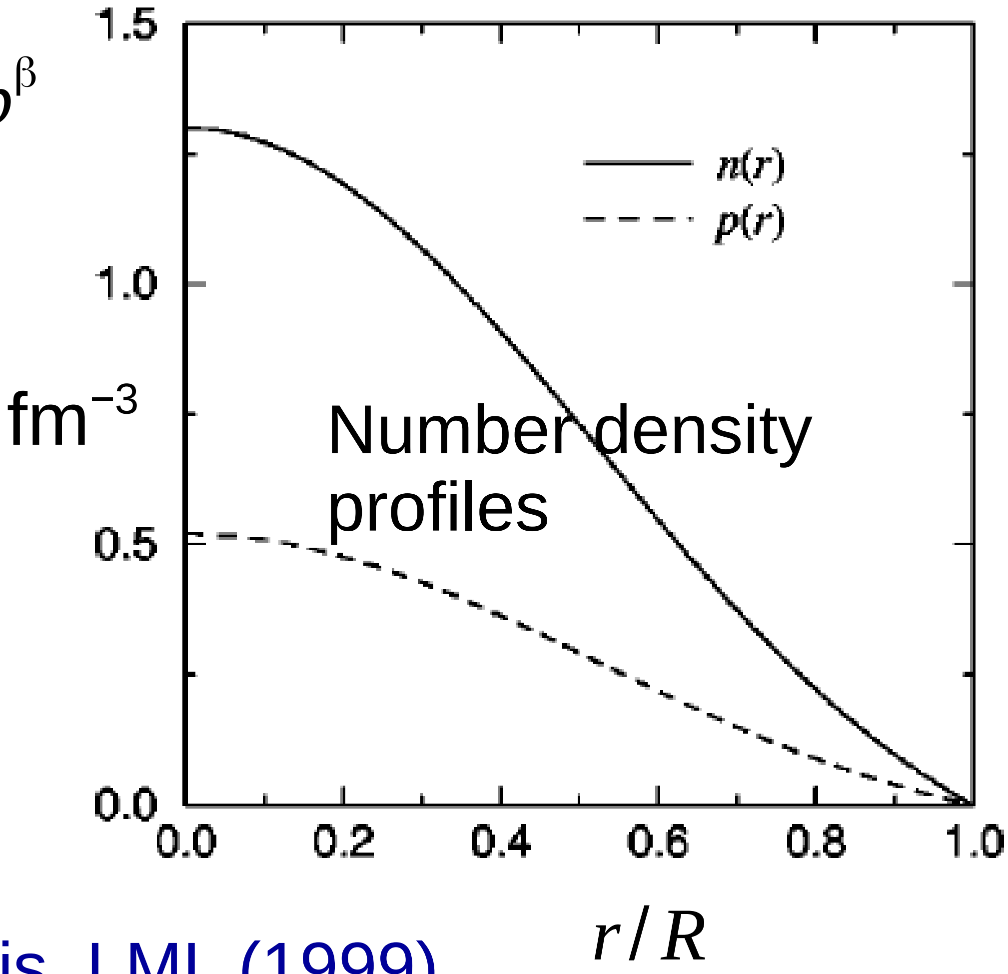
$$n^{\alpha}\nabla_{[\alpha}\mu_{\beta]}=0\quad,\quad p^{\alpha}\nabla_{[\alpha}\chi_{\beta]}=0$$

- First general relativistic two-fluid model for (Toy) superfluid neutron stars and oscillation-mode calculation was studied in 1999.

Two-fluid analog of polytropic model:

$$\Lambda \equiv -m_n n - \sigma_n n^{\beta} - m_p p - \sigma_p p^{\beta}$$

(serves as EOS input)

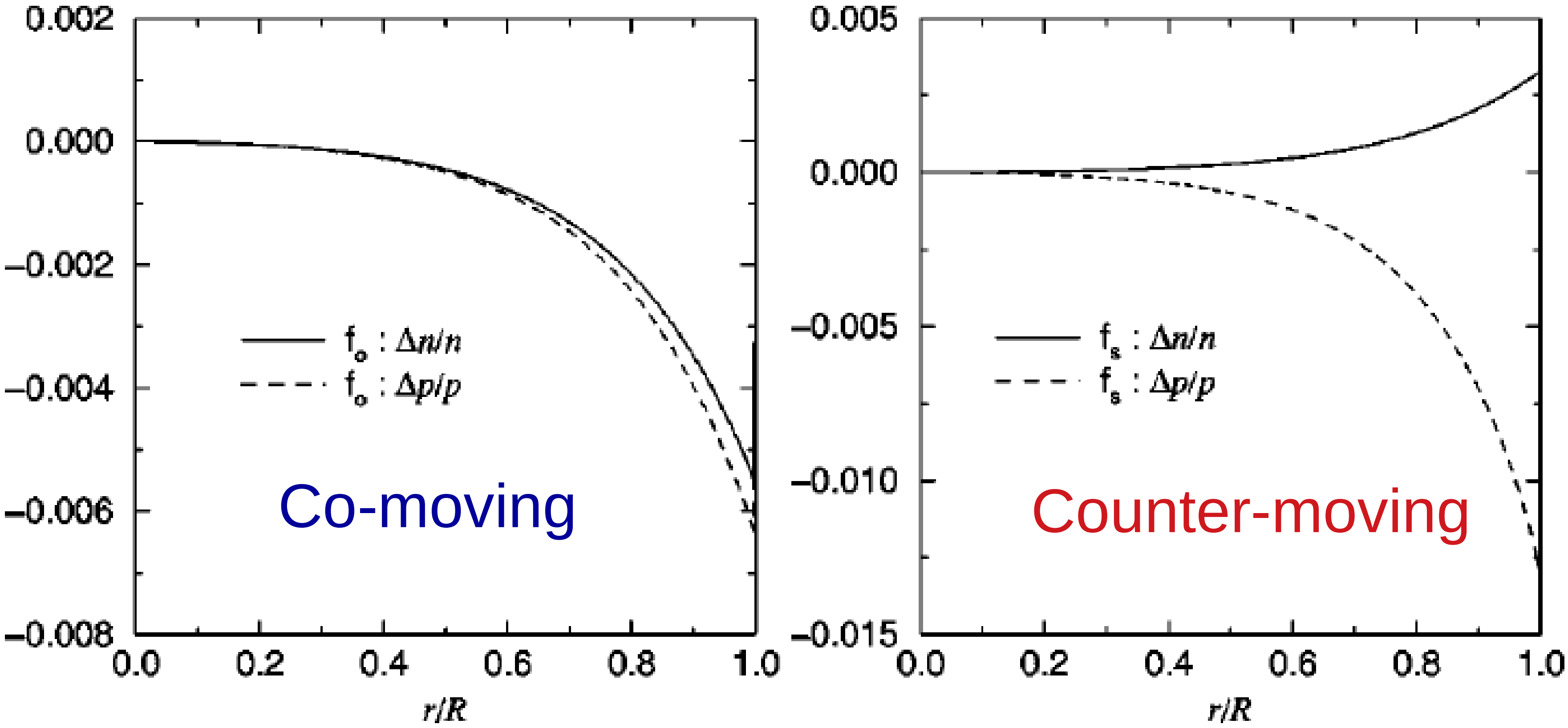


Number density profiles

Comer, Langlois, LML (1999)

• Two different f-modes in superfluid neutron stars

Ordinary f-mode (f_o) Superfluid f-mode (f_s)



Co-moving

Counter-moving

Lagrangian variations in number densities

- The tidal deformability of superfluid neutron stars based on the GR two-fluid formalism has recently been studied by Char and Datta (2018).

Using two-fluid approach can change the Love number by up to 10% for massive stars

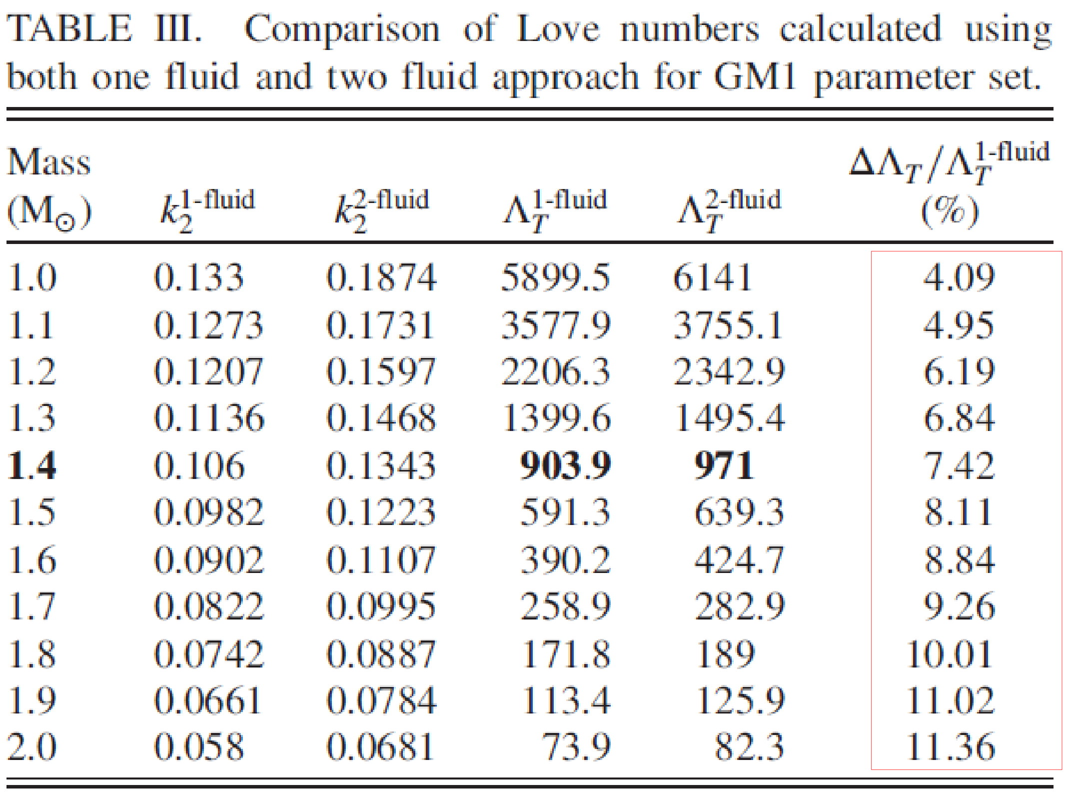


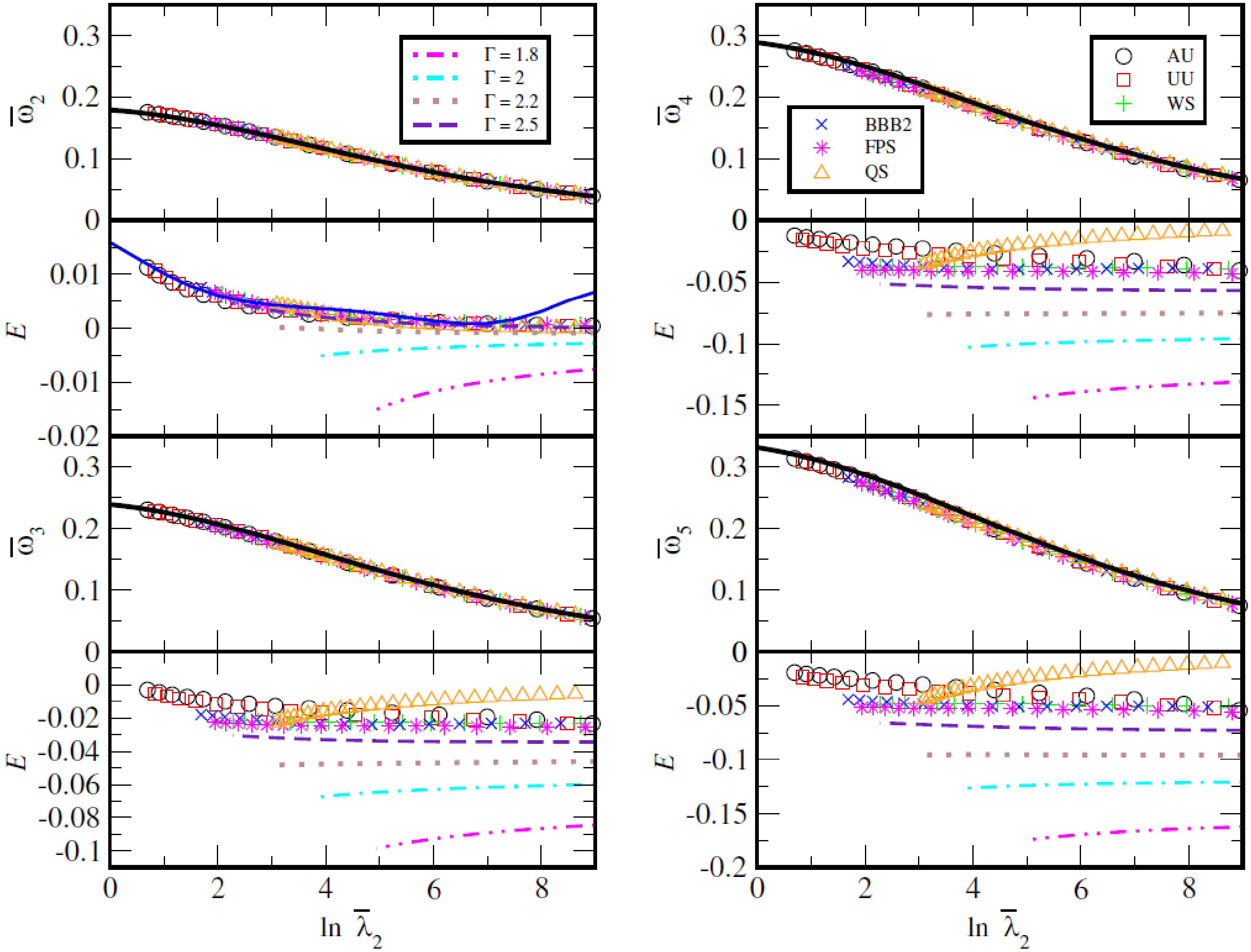
TABLE III. Comparison of Love numbers calculated using both one fluid and two fluid approach for GM1 parameter set.

Mass ($M_\odot$)	$k_2^{\text{1-fluid}}$	$k_2^{\text{2-fluid}}$	$\Lambda_T^{\text{1-fluid}}$	$\Lambda_T^{\text{2-fluid}}$	$\Delta\Lambda_T/\Lambda_T^{\text{1-fluid}}$ (%)
1.0	0.133	0.1874	5899.5	6141	4.09
1.1	0.1273	0.1731	3577.9	3755.1	4.95
1.2	0.1207	0.1597	2206.3	2342.9	6.19
1.3	0.1136	0.1468	1399.6	1495.4	6.84
1.4	0.106	0.1343	**903.9**	**971**	7.42
1.5	0.0982	0.1223	591.3	639.3	8.11
1.6	0.0902	0.1107	390.2	424.7	8.84
1.7	0.0822	0.0995	258.9	282.9	9.26
1.8	0.0742	0.0887	171.8	189	10.01
1.9	0.0661	0.0784	113.4	125.9	11.02
2.0	0.058	0.0681	73.9	82.3	11.36

Table taken from Char and Datta (2018)

Why f-mode and tidal deformability?

Ordinary fluid neutron stars: f-mode-Love universal relations



$$\bar{\omega}_l \equiv M\,\omega_l$$

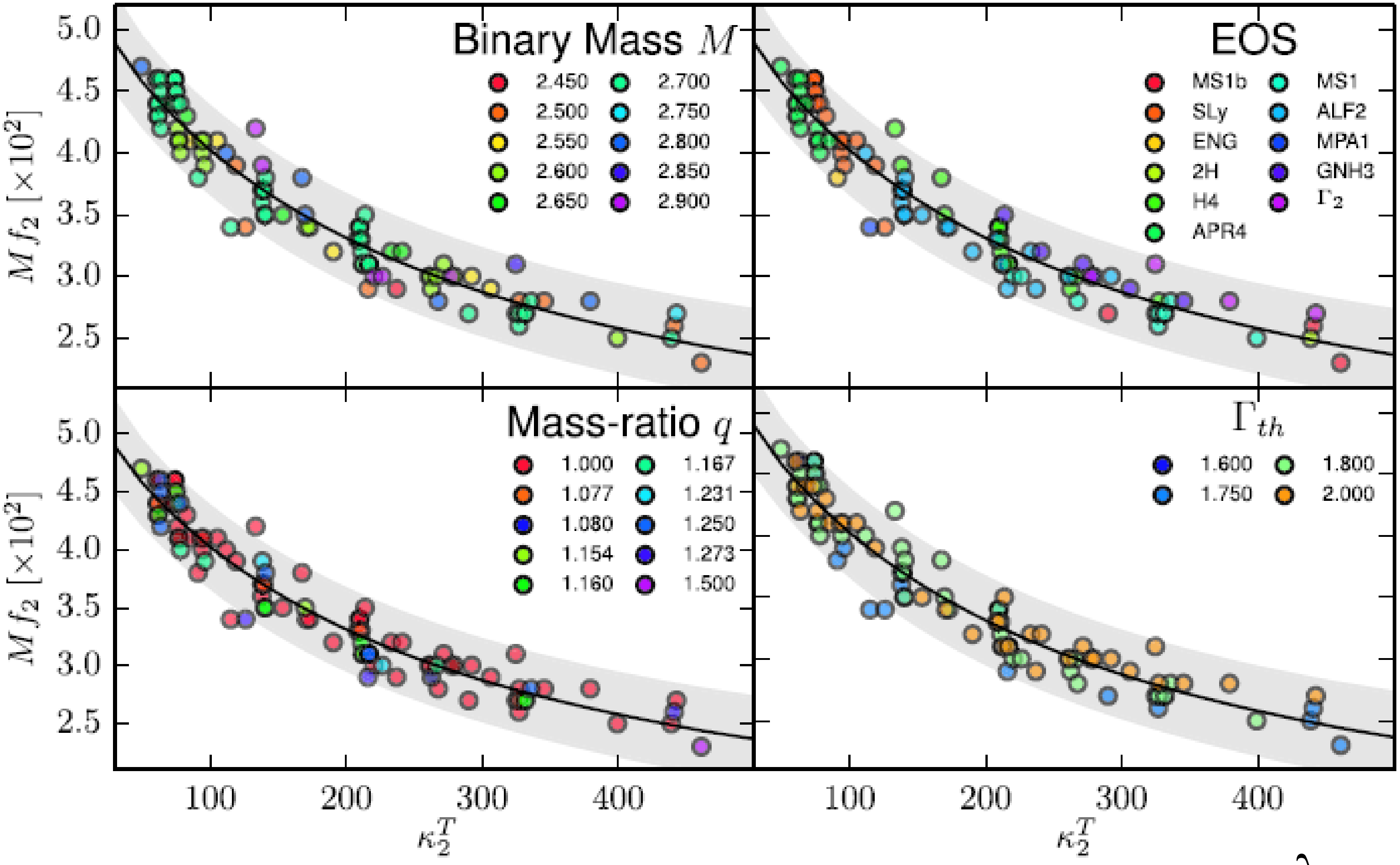
$$\bar{\lambda}_l \equiv \frac{\lambda_l}{M^{2l+1}}$$

[Chan, Sham, Leung, and LML (2014)]

Subscript = spherical harmonic index *l*

“Universal” relation in binary neutron star simulations

Bernuzzi, Dietrich, and Nagar (2015)



f_2 = post-merger GW peak frequency

$$\lambda_l \equiv \frac{2}{(2l-1)!!} k_l R^{2l+1}$$

$$\kappa_2^T = 2\left(\frac{q^4}{(1+q)^5}\frac{k_2^A}{C_A^5} + \frac{q}{(1+q)^5}\frac{k_2^B}{C_B^5}\right),$$

k_2 = Love number

C = compactness

q = mass ratio

Does the f-mode-Love universal relation still hold for superfluid (two-fluid) neutron stars?

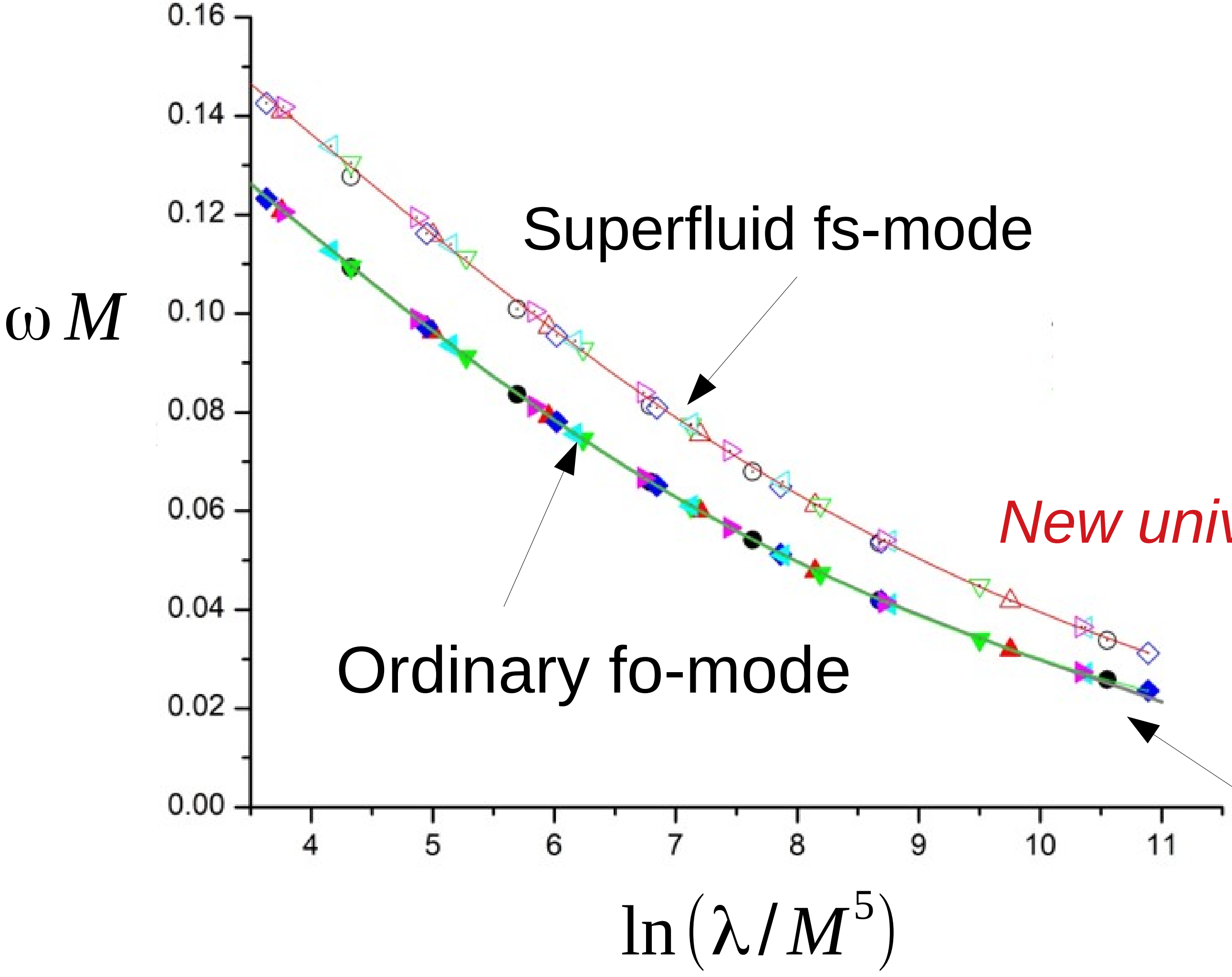
Wait….we have two different f-modes

fo = Ordinary f-mode (co-moving motion)
fs = Superfluid f-mode (counter-moving motion)

- **Our recent work:**
  We study the f-mode oscillations and tidal deformability based on a toy model master function (EOS)

$$\Lambda \equiv -m_n n - \sigma_n n^{\beta} - m_p p - \sigma_p p^{\beta}$$

(data based on 6 different sets of “EOS” parameters)



Superfluid fs-mode

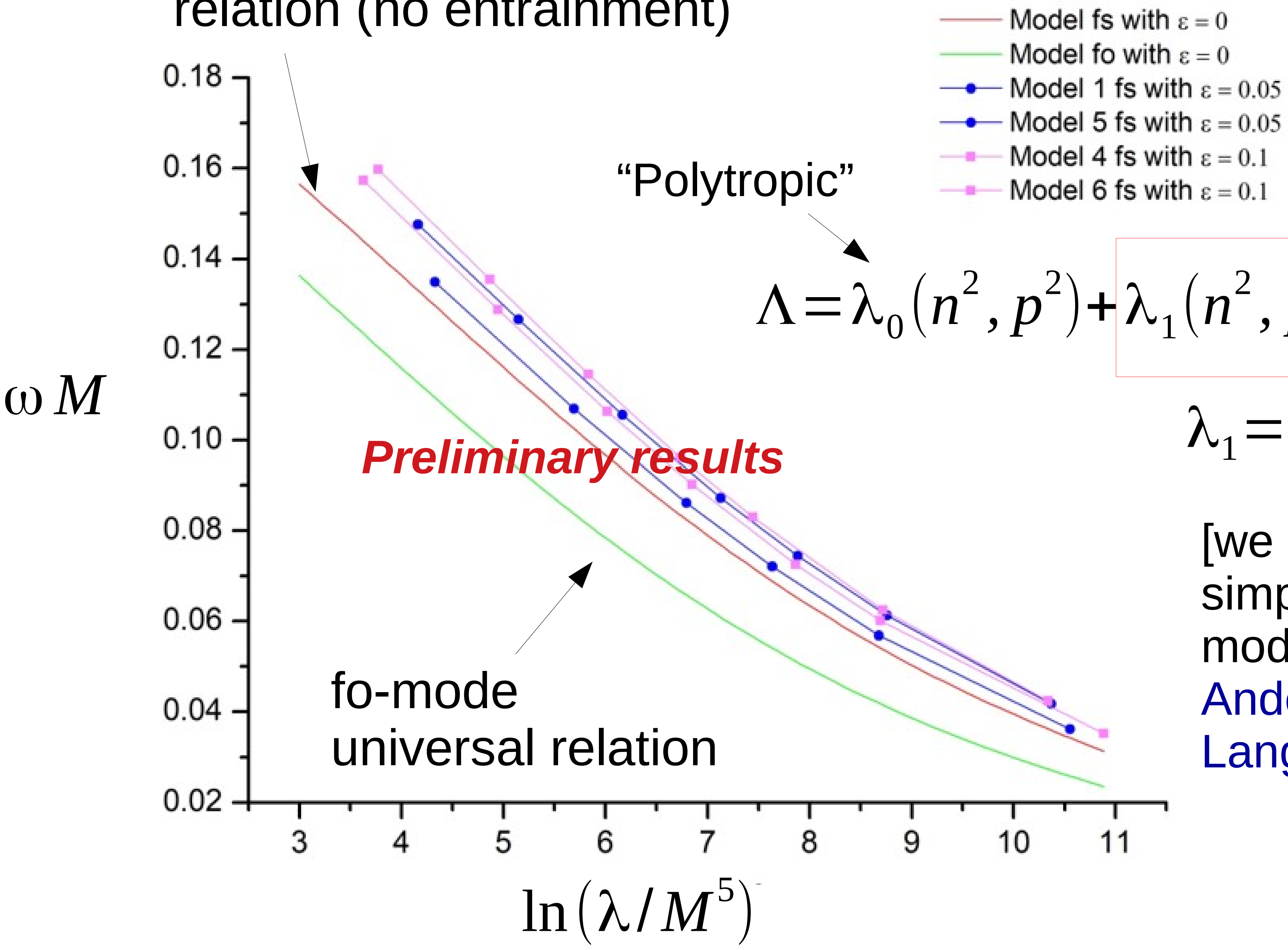
Ordinary fo-mode

Preliminary results
by
Cheung-Hei Yeung

New universal relation for fs-mode

Same universal relation for ordinary fluid stars

- Entrainment effect can break the universality



fs-mode universal relation (no entrainment)

Preliminary results

fo-mode universal relation

"Polytropic"

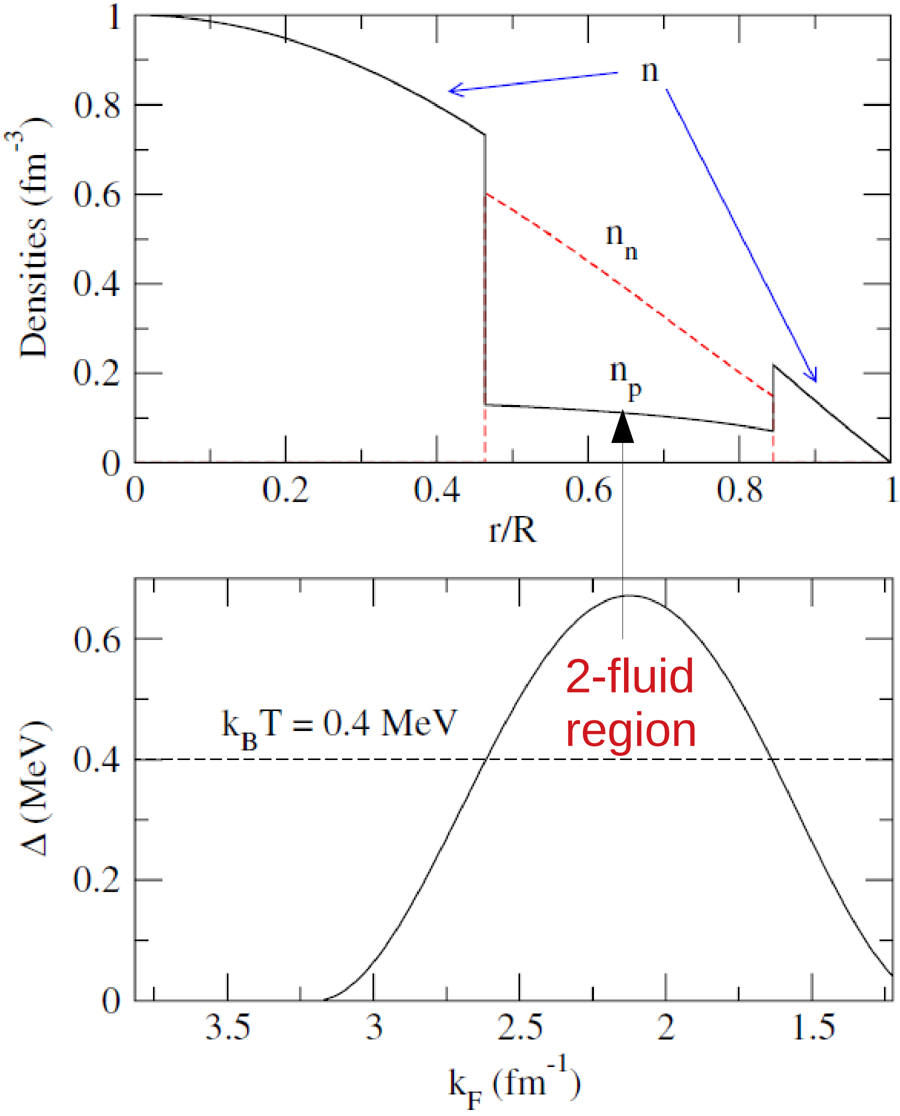
$$\Lambda = \lambda_0(n^2, p^2) + \boxed{\lambda_1(n^2, p^2)(x^2 - np)}$$

$$\lambda_1 = \frac{-\epsilon m}{p + \epsilon(n + p)}$$

[we adopted this simple entrainment model proposed by Andersson, Comer, Langlois (2002)]

What’s next?

- Try more realistic EOS and entrainment models…
- Try more realistic multi-layer structure….



2-fluid region

“Realistic” structure of a superfluid neutron star

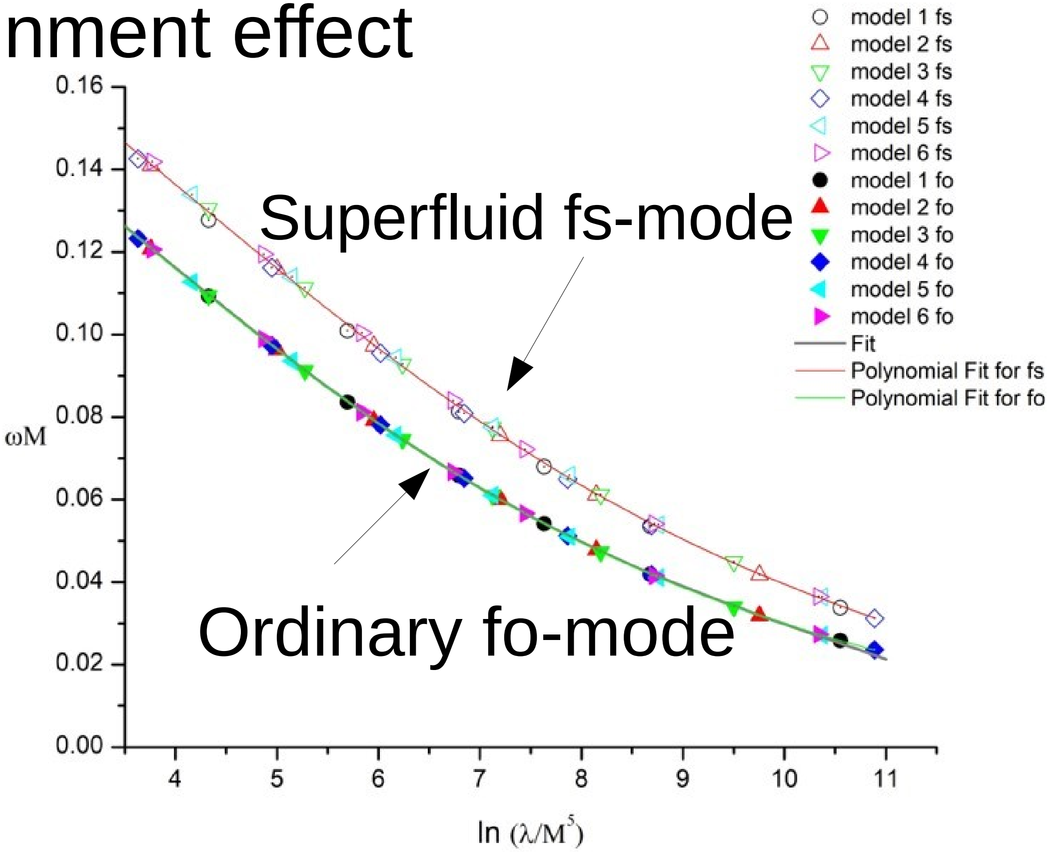
“Toy” model gap function

Lin, Andersson, Comer (2008)

Summary

- We have extended the study of f-mode-Love universal relation to superfluid neutron stars
(….so far only tested with simple two-fluid “polytropic” EOS)

- We found that the entrainment effect can break the fs-mode universal relation



Thank you!