Inhomogeneous chiral phases in dense matter

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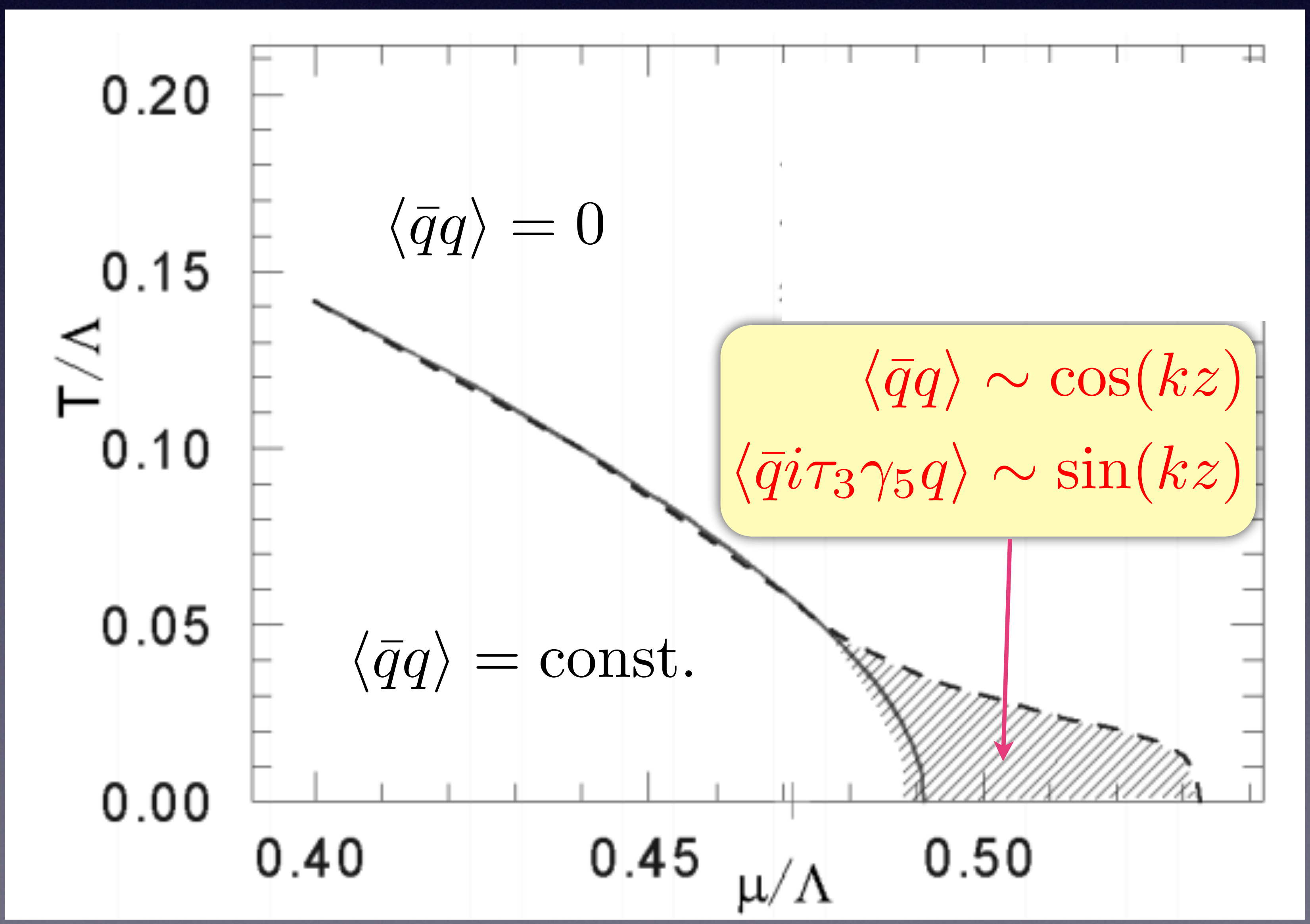
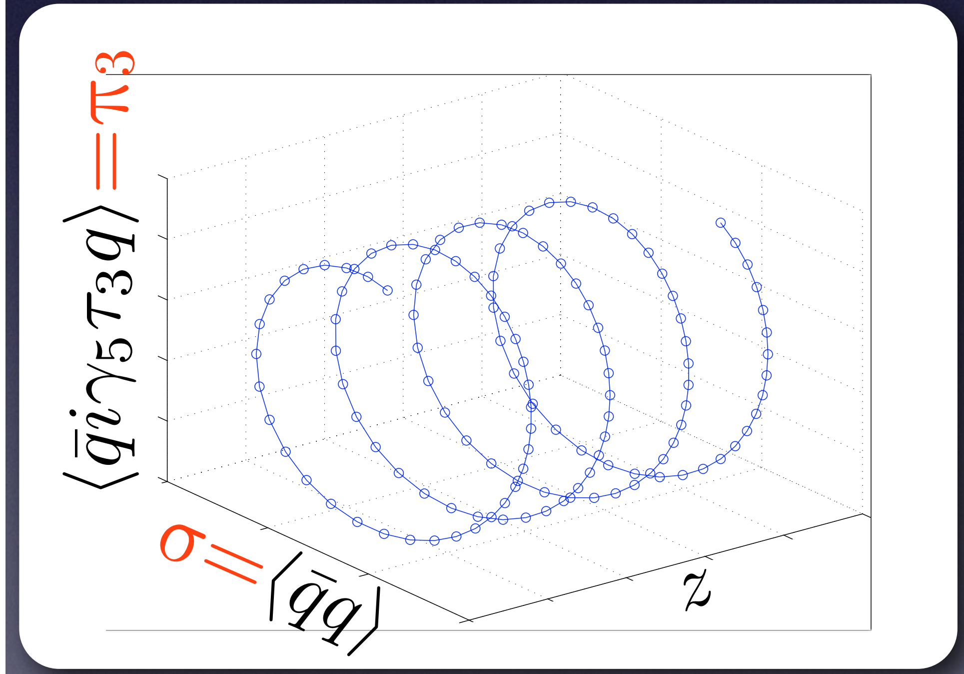
Plan

- inhomogeneous chiral phases
- generalized Ginzburg-Landau approach to Chiral crystals in the chiral limit
- Magnetically induced DCDW (spiral)
- Summary and Outlook

DCDW; aka Chiral Spiral

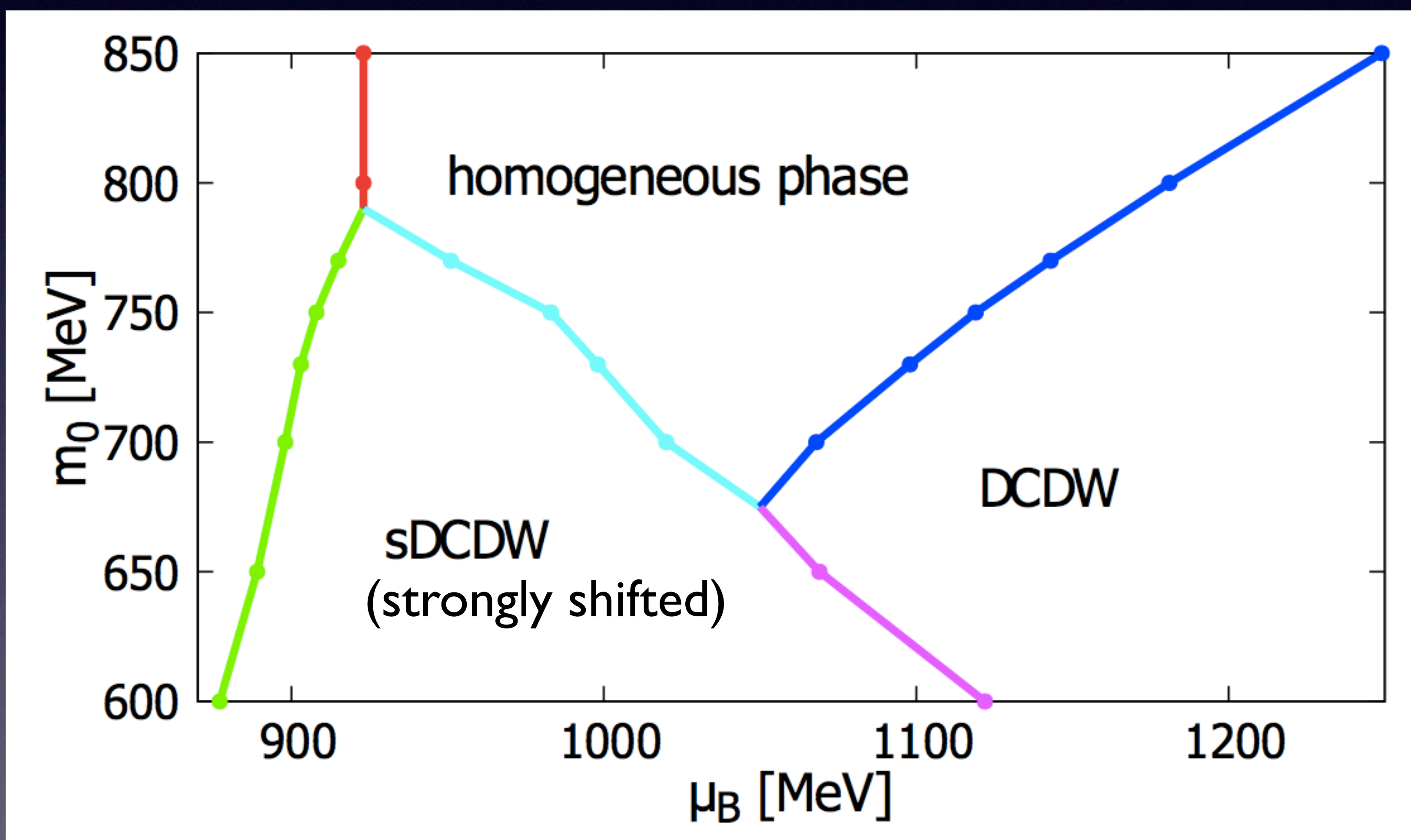
Nakano, Tatsumi, PRD71(2005)114006





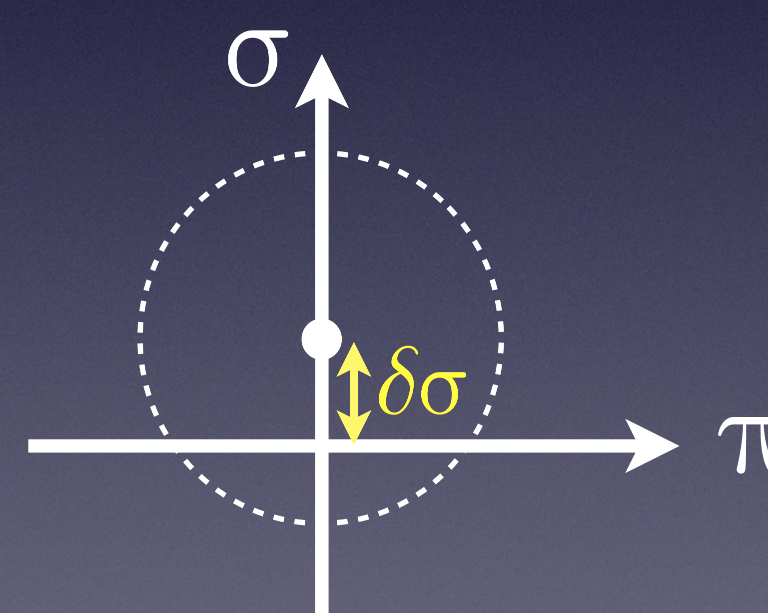
DCDW; variations

Y. Takeda, HA, M. Harada, PRD97 (2018)



$$\langle \sigma \rangle = \sigma_0 \cos(2fz) + \delta\sigma$$
$$\langle \pi \rangle = \sigma_0 \sin(2fz)$$

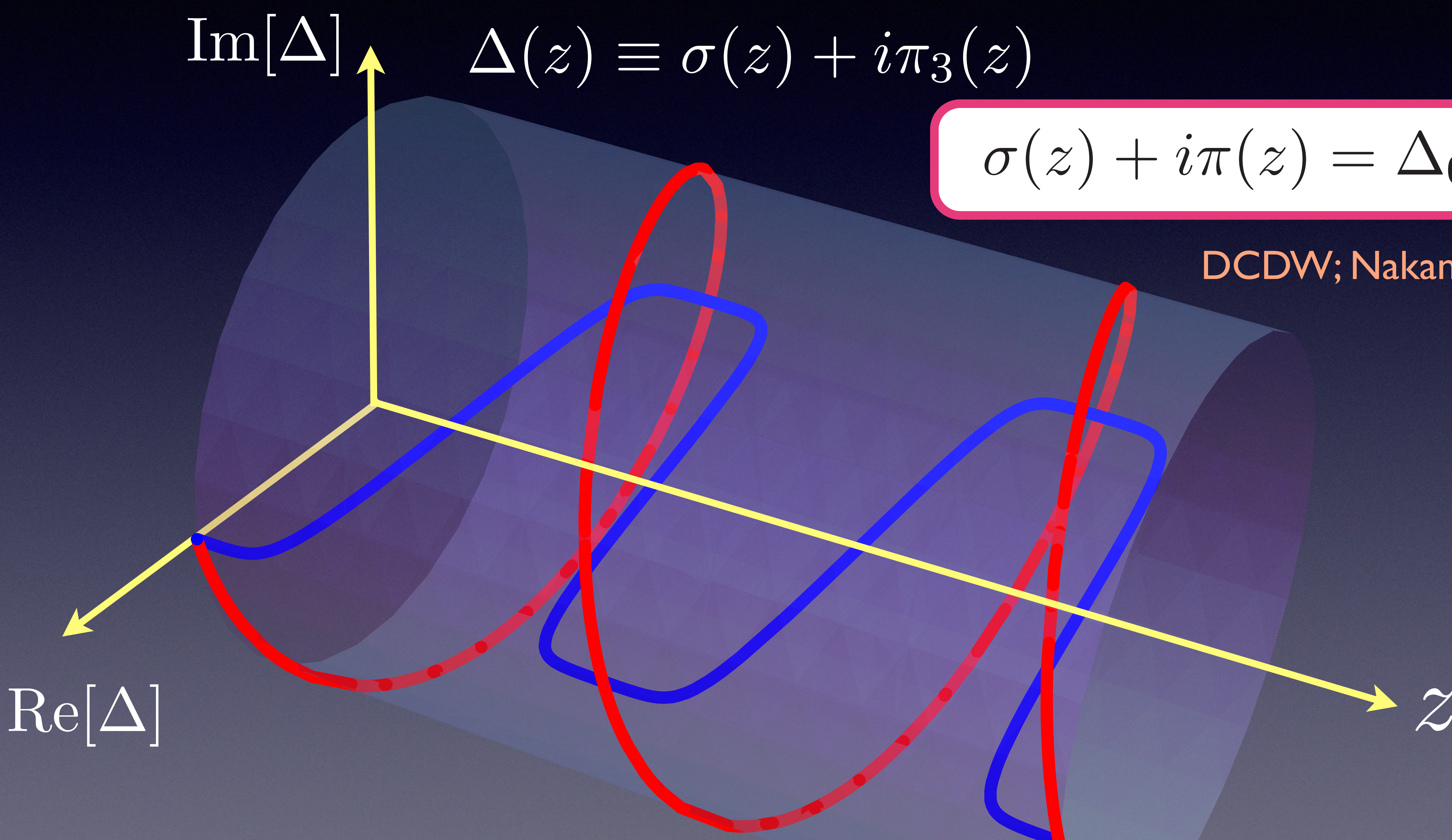
standard DCDW ansatz



Nucleon-meson model with parity doublet structure
tuned so as to reproduce saturation properties of sym. nuclear matter

c.f. sY. Motohiro, Y. Kim, M. Harada, , PRC92 (2015), RRC95 (2017)

DCDW or RKC?



$$\Delta(z) \equiv \sigma(z) + i\pi_3(z)$$

$$\sigma(z) + i\pi(z) = \Delta_0 e^{iqz}$$

DCDW; Nakano, Tatsumi

$$\sigma(z) = \sqrt{\nu} q \, \mathrm{sn}(qz, \nu)$$

Real Kink Crystal; Nickel, PRL09, PRD09

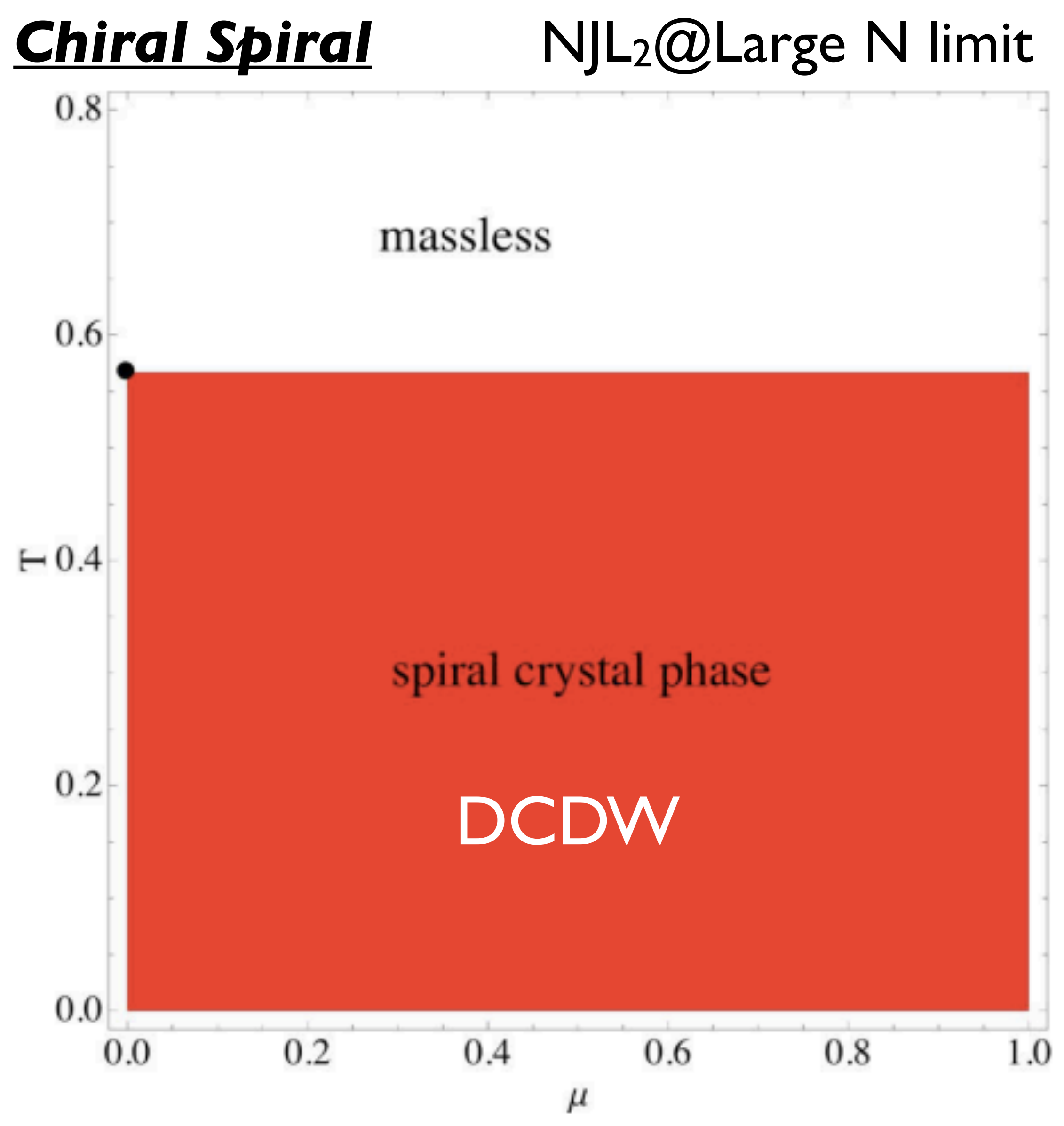
1+1 dim. NJL/GN model

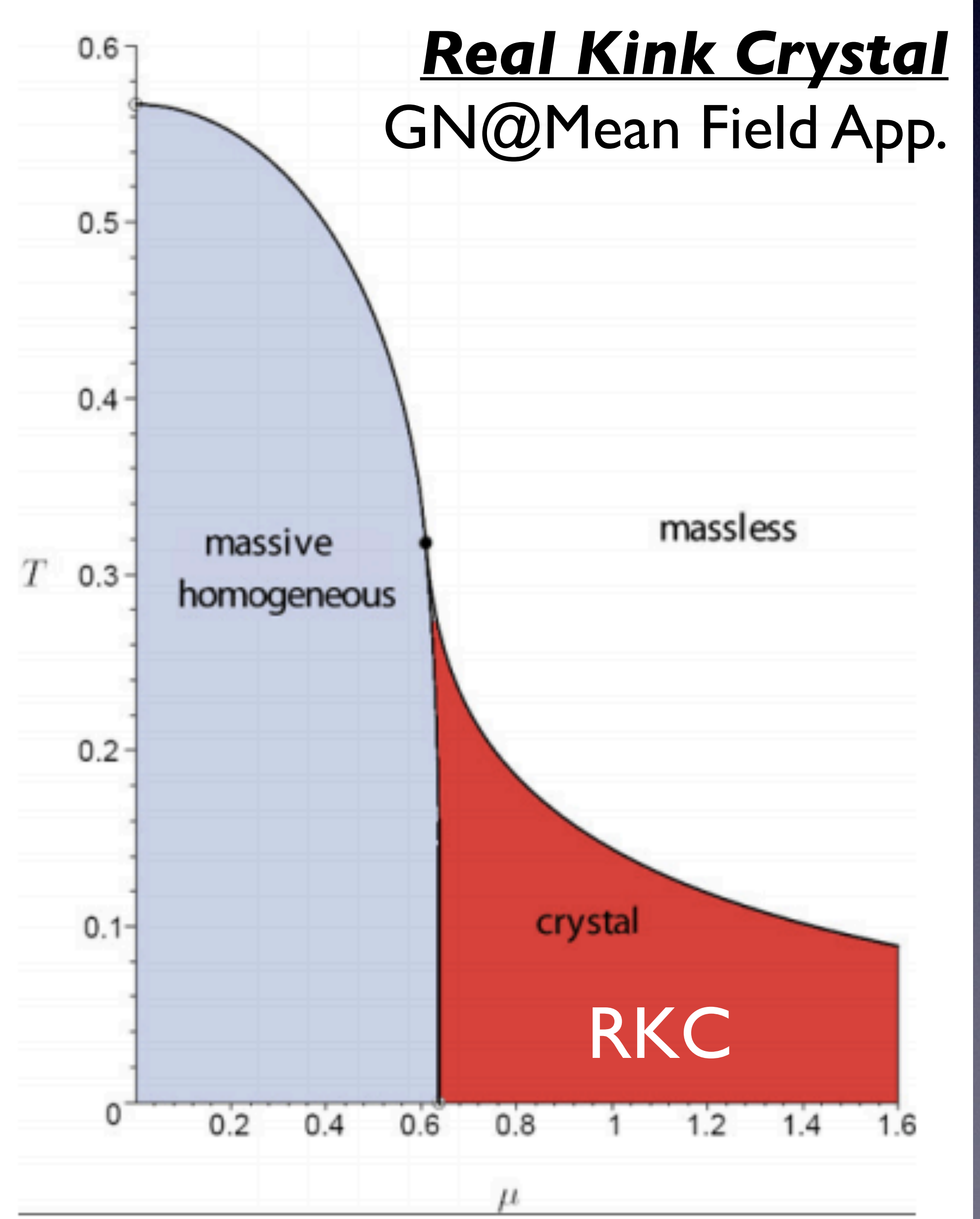
M. Thies, J. Phys. A2006

$$\sigma = \langle \bar{\varphi}\varphi \rangle = \cos(2\mu z)\Delta$$

$$\pi = \langle \bar{\varphi} i\gamma_5 \varphi \rangle = \sin(2\mu z)\Delta$$

$$\sigma = \langle \bar{\varphi}\varphi \rangle = \sqrt{\nu} q \mathrm{sn}(qz, \nu)$$





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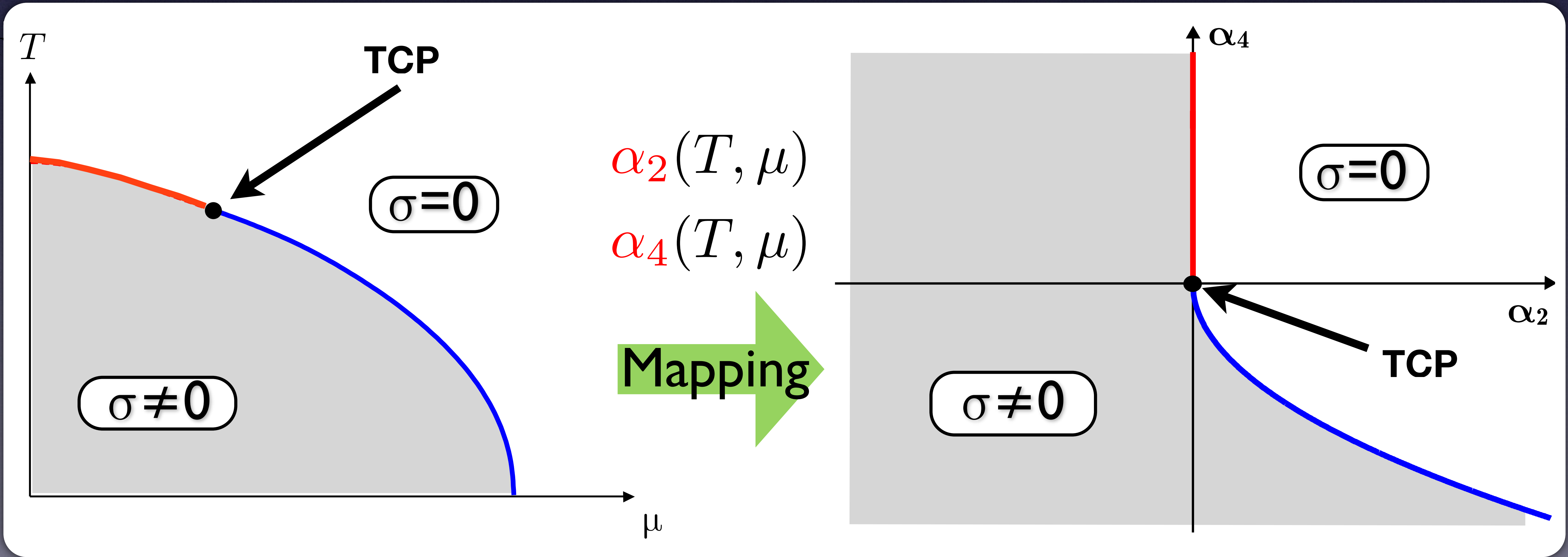
Ginzburg-Landau @TCP

Minimal GL to describe the tricritical point (TCP)

$$\Omega_{\rm GL} = \frac{\alpha_2}{2}\sigma(\mathbf{x})^2 + \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{1}{6}\sigma(\mathbf{x})^6$$



generalized Ginzburg-Landau (gGL) expansion

Nickel, PRL09
Abuki (2014)

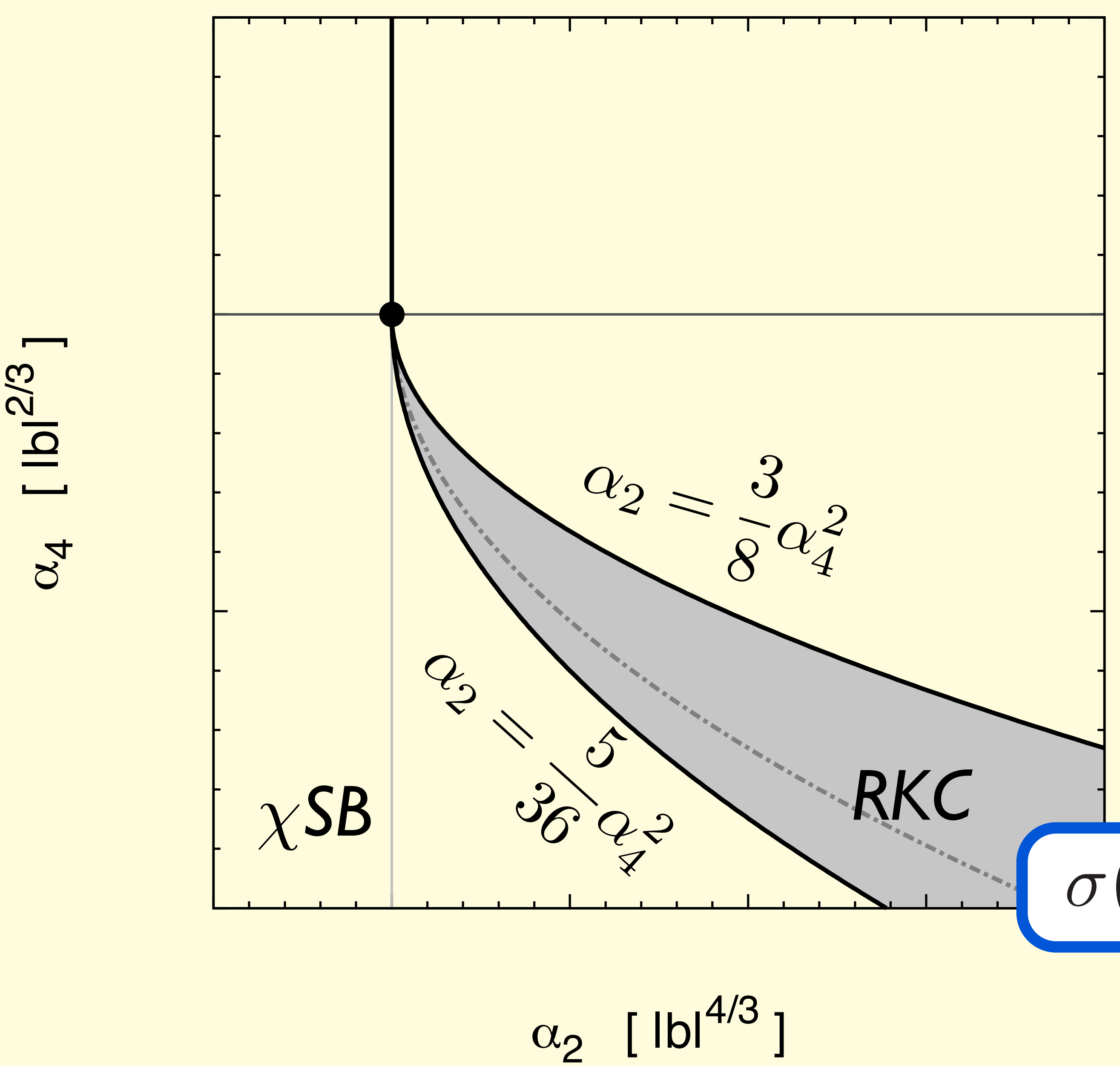
gradient terms

$$\Omega_{\mathrm{GL}} = +\frac{\alpha_2}{2}\sigma(\mathbf{x})^2 + \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{\alpha_{4b}}{4}(\nabla\sigma(\mathbf{x}))^2 + \frac{\alpha_6}{6}\sigma(\mathbf{x})^6 + \frac{\alpha_{6b}}{6}\sigma^2(\nabla\sigma)^2 + \frac{\alpha_{6c}}{6}(\Delta\sigma)^2$$

$\alpha_{4b} = 0$: Lifshitz point (LP)

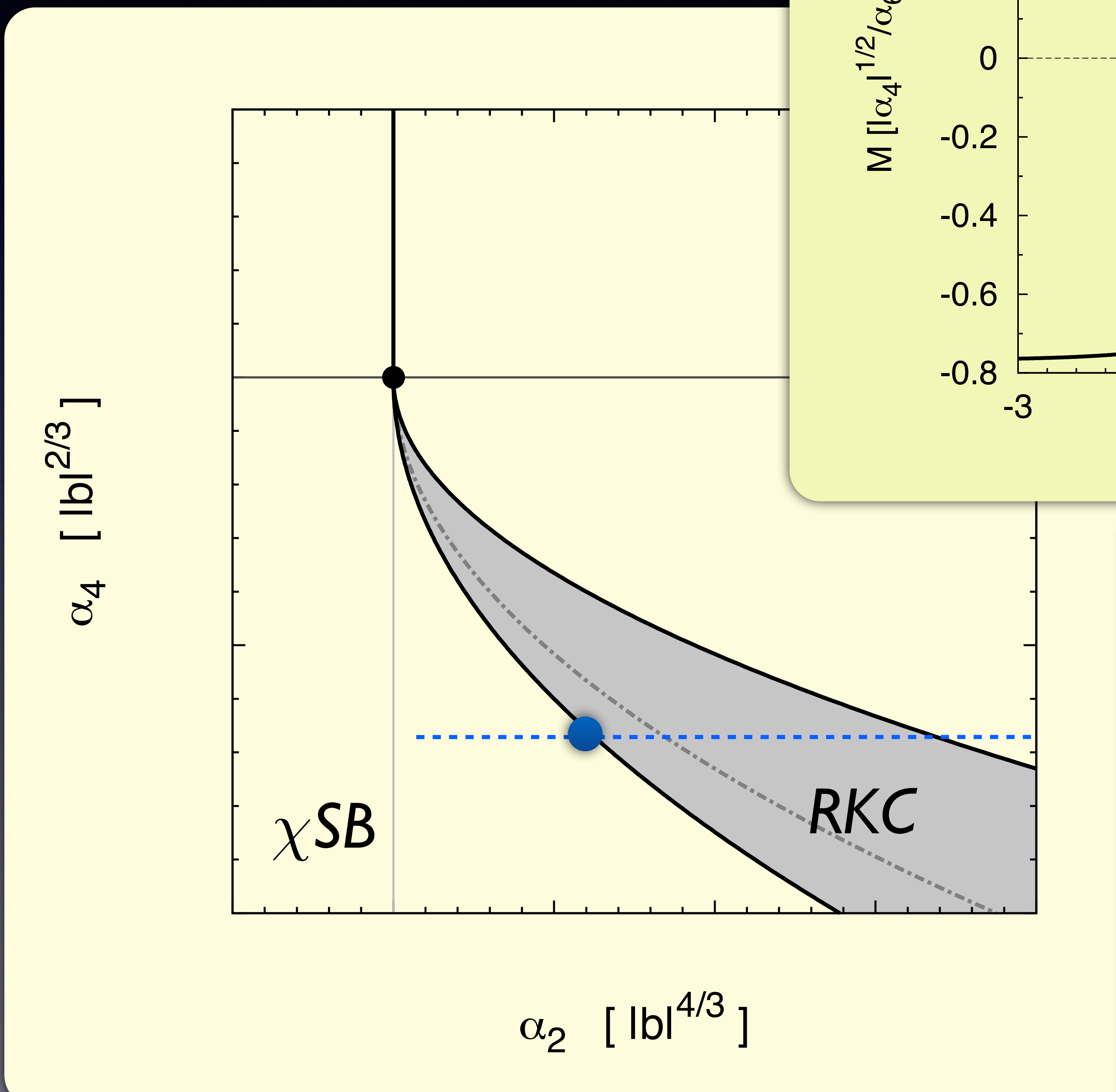
$\alpha_2 = \alpha_4 = 0$: Tricritical Point (TCP)

RKC: Solitonic CDW!

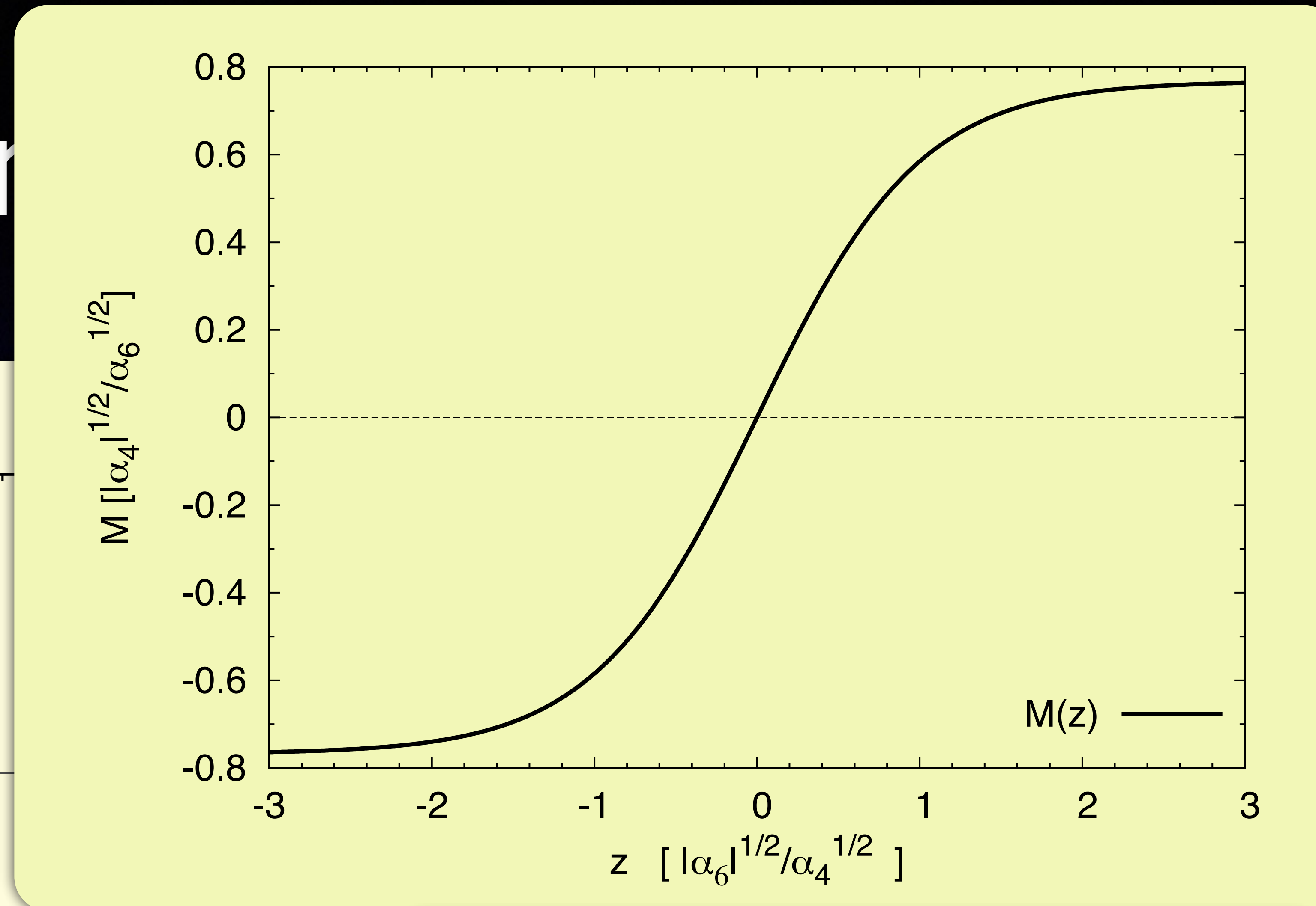


$$\sigma(z) = \sqrt{\nu} q \, \mathrm{sn}(qz, \nu)$$

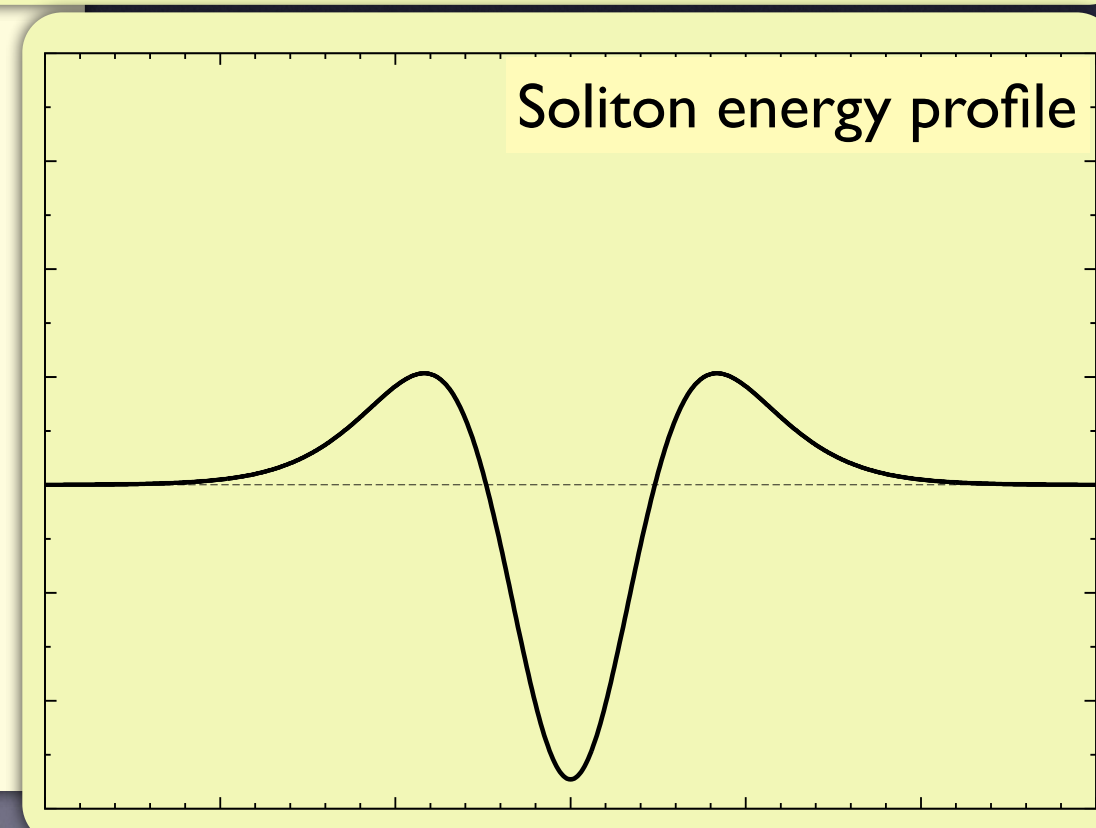
Solitor



χSB
RKC
α4 [|b|^2/3]
α2 [|b|^4/3]



0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3
-2
-1
0
1
2
3
M [|α4|^1/2/α6^1/2]
z [|α6|^1/2/α4^1/2]
M(z)



Soliton energy profile

Remark (3D versus 1D)

Why is DCDW disfavored in 1+3dim

Boehmer, Thies, Urlichs, PRD75 (2007)

$$\Delta(z) \equiv \sigma(z) + i\pi(z)$$

$$\Omega_{\mathrm{NJL}_2} = \frac{\alpha_2}{2}|\Delta(z)|^2 + \frac{\alpha_3}{3}\mathrm{Im}\left[\Delta^*\Delta'\right] + \frac{\alpha_4}{4}\left(|\Delta(z)|^4 + |\Delta'(z)|^2\right) + \frac{\alpha_5}{5}\mathrm{Im}\left((\Delta'' - 3|\Delta|^2\Delta)\Delta'^*\right) + \frac{1}{6}\Delta(\mathbf{x})^6$$

For chiral spiral :

$$\Delta(z) \equiv \Delta_0 e^{iqz}$$

$$-\frac{\alpha_3}{3}\Delta_0^2 q$$

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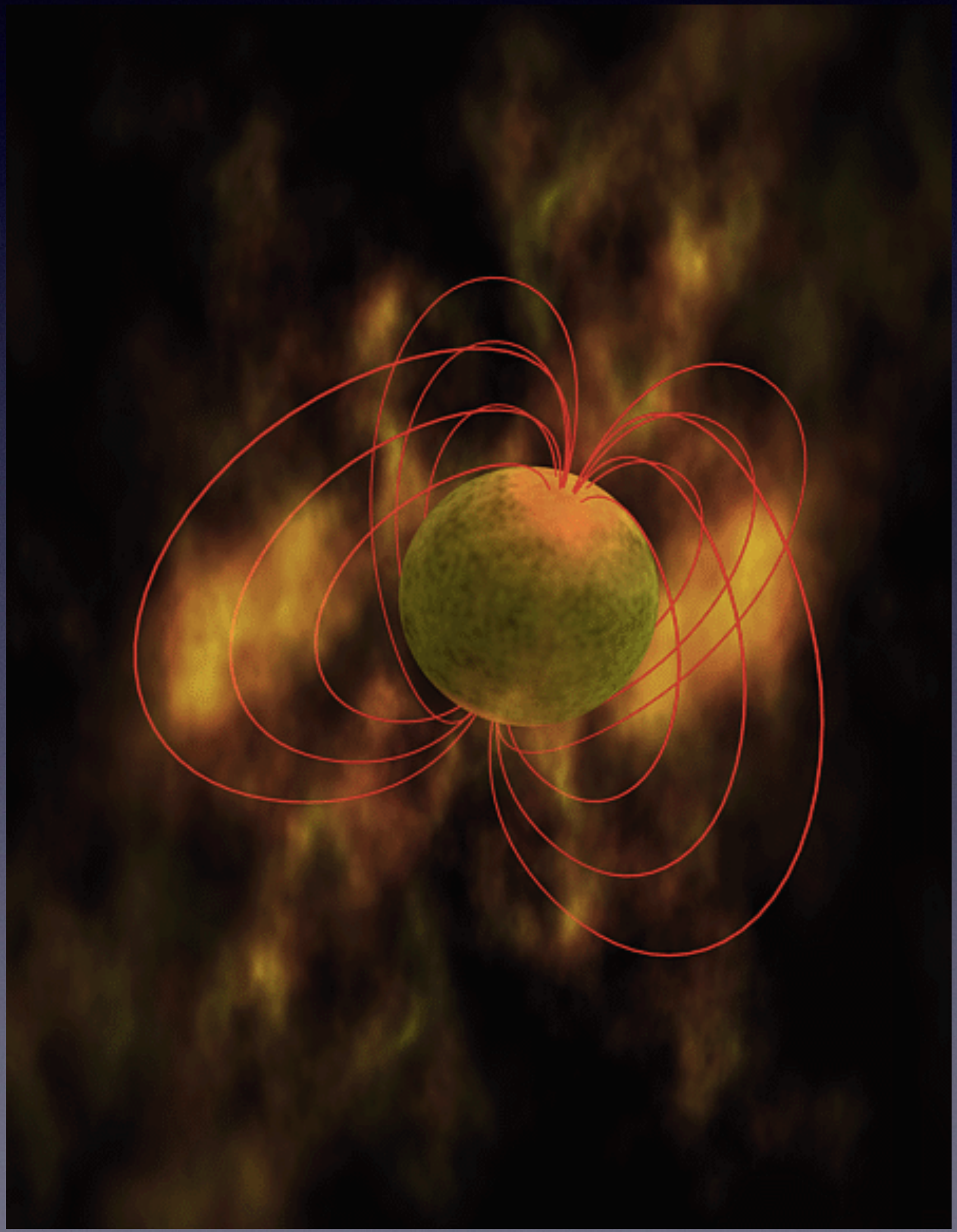
Magnetic field



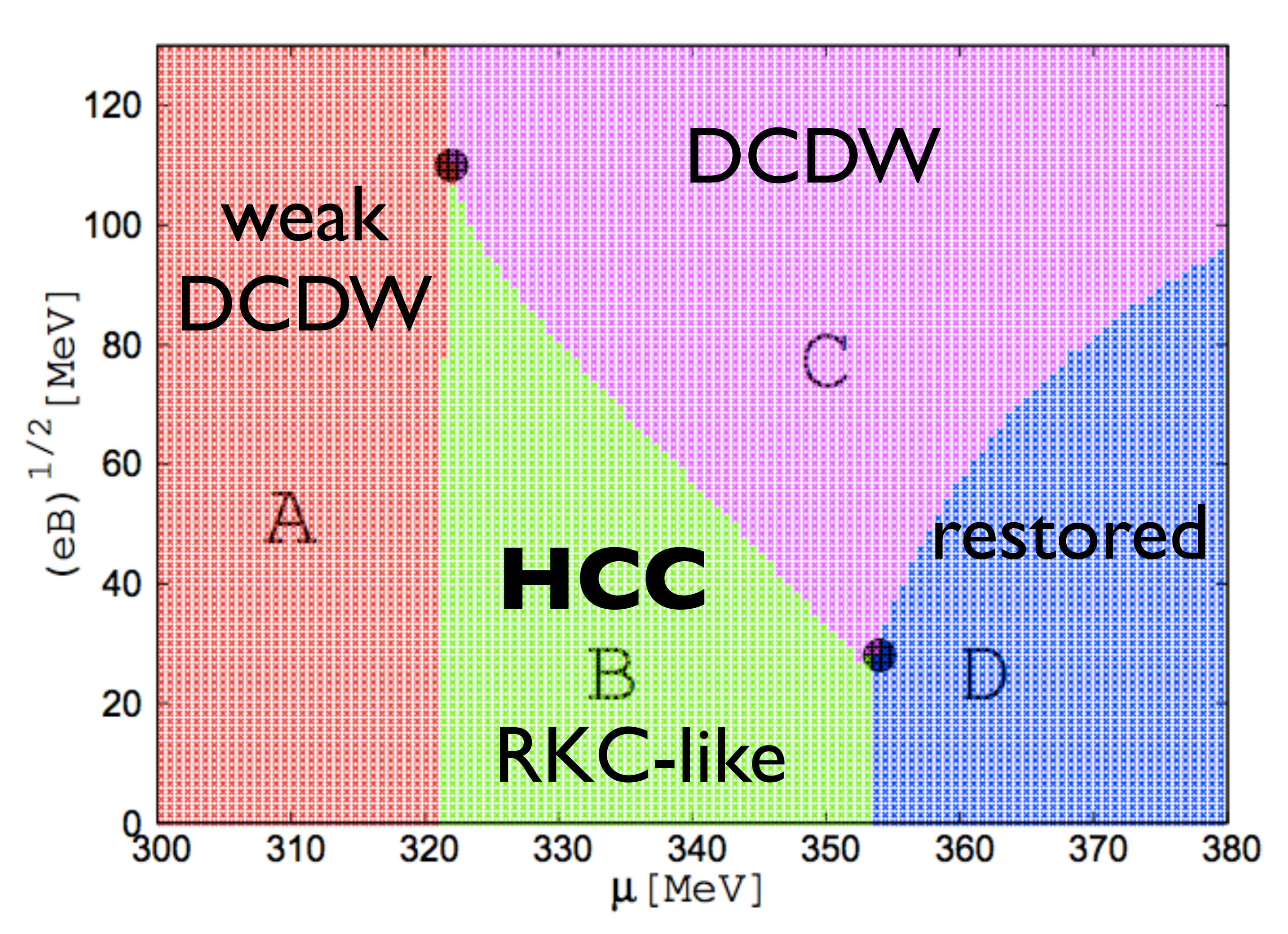
Compact stellar objects
with super strong magnetic field

Magnetar (B～10^{15} Gauss)

$\sqrt{eB} \sim 2.5\,\mathrm{MeV}$

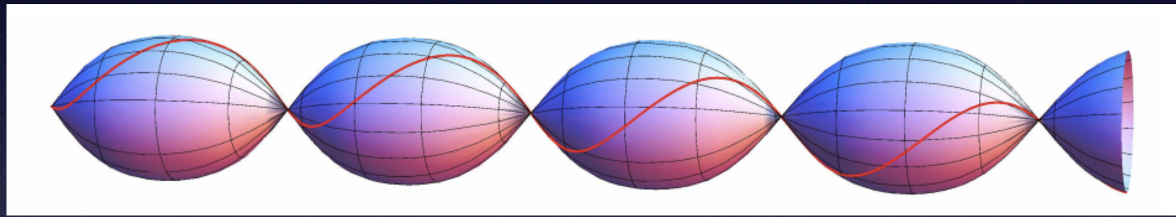
Magnetic Field & Chiral symmetry breaking

K. Nishiyama, S. Karasawa, T. Tatsumi, PRD92 (2015)

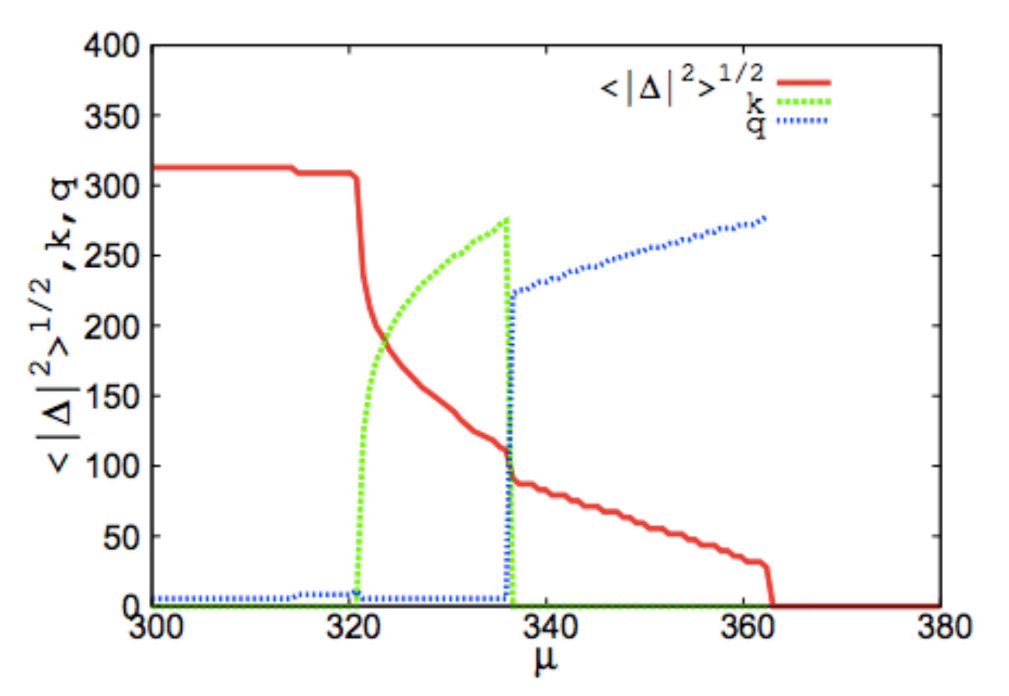


NJL@T=0 & Chiral Limit

Hybrid Chiral Condensate



$$M(z) = \frac{2m\nu}{1+\sqrt{\nu}}\mathrm{sn}\left(\frac{2mz}{1+\sqrt{\nu}};\nu\right)e^{iqz}$$



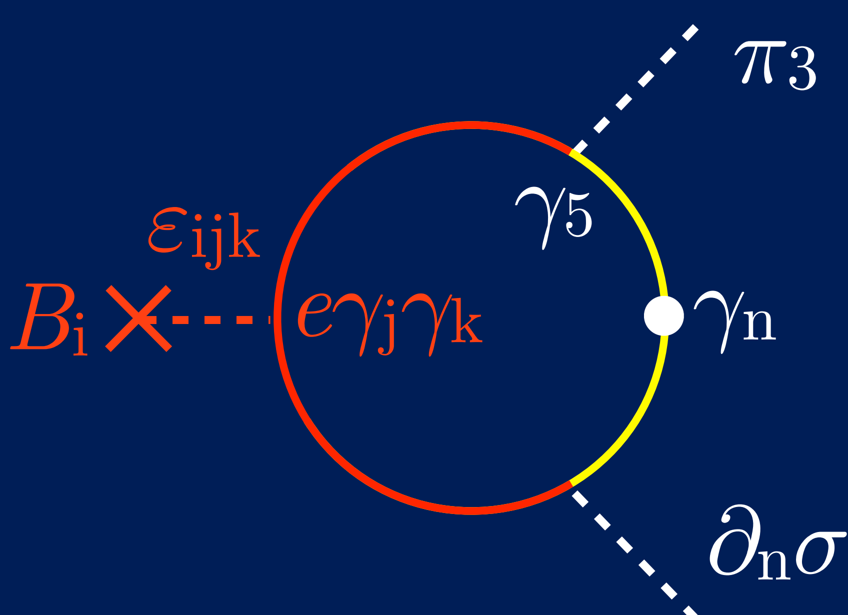
Including magnetic field

New couplings: 3D rotational symmetry breaking

$$\delta\Omega = -\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu}(e\mathbf{B})\cdot(\pi_{\mathbf{3}}\nabla\sigma - \sigma\nabla\pi_{\mathbf{3}})$$

$$= \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3)$$

Feynman graph contributing to new deriv. coupling



$$-iS(p) = \frac{\not{p}+\not{\mu}}{(p+\mu)^2}$$

$$+(e_q B_i)\frac{i\epsilon_{ijk}\gamma^j\gamma^k}{2}\frac{\not{p}_{\parallel}+\not{\mu}}{(p+\mu)^4}$$

Including magnetic field

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$$= \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3)$$

"*Universal*" coupling

$$-\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu} = \frac{N_c}{8\pi^2 T}f(e^{-\mu/T}),$$

$$f(e^{-\mu/T}) = \frac{1}{2\pi}\mathrm{Im}\psi^{(1)}\left(\frac{1}{2} - i\frac{\mu}{2\pi T}\right)$$

Tatsumi, Nishiyama, Karasawa, PLB (2015)

Mass vs Magnetic field

$$\delta\Omega \;=\; \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3) - h\sigma$$

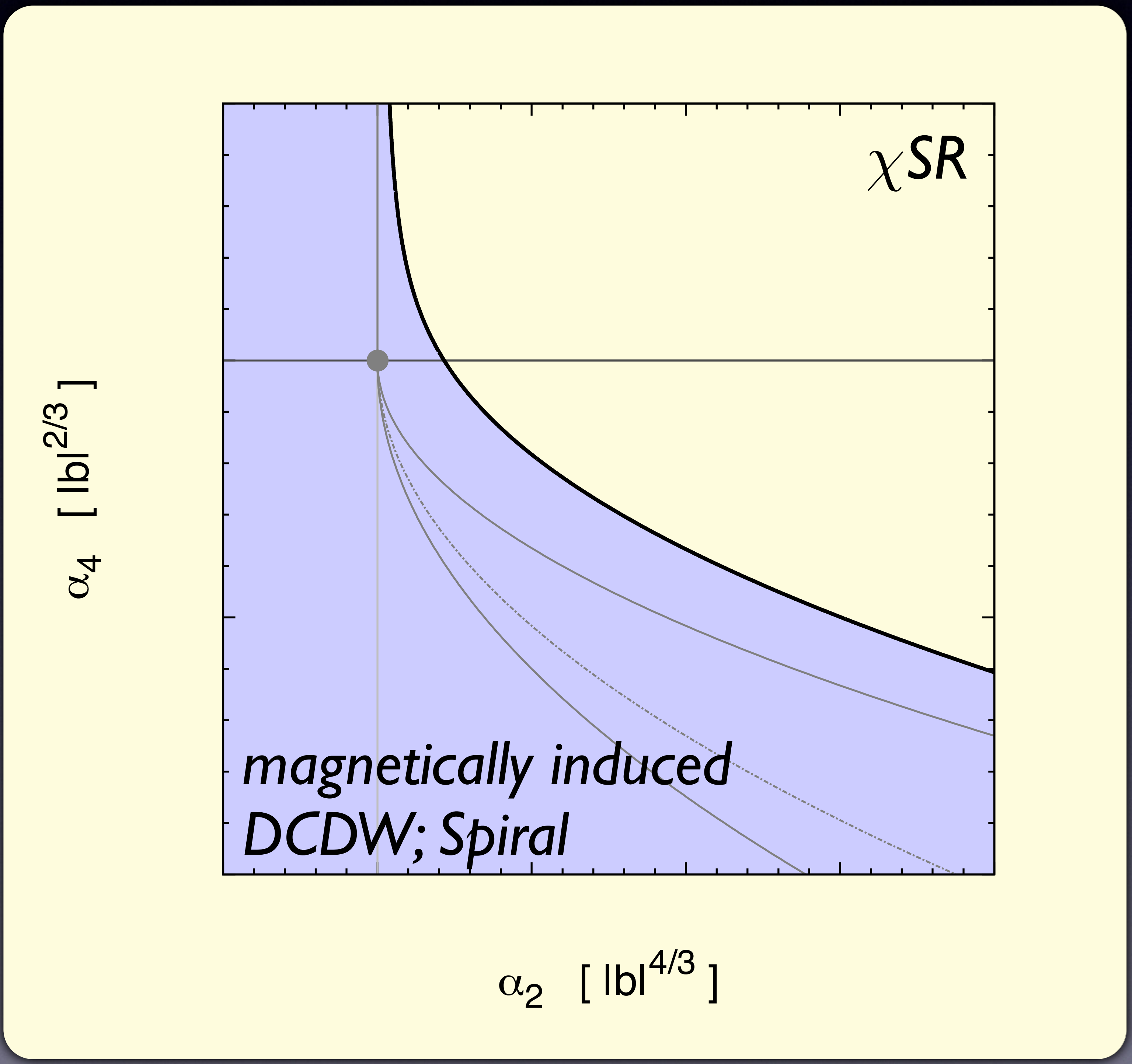
h : mass term

chiral SU(2)xSU(2) ⇒ isospin SU(2)

b : magnetic field

Rotational O(3) symmetry
Time reversal symmetry
Isospin I_3 symmetry

The fate of LTCP



i. *No magnetic field:*

Lifshitz TCP
& RKC phase

$$\sigma = k\nu \mathrm{sn}(kz, \nu)$$

ii. *Magnetic field on:*

Chiral spiral covers a whole region

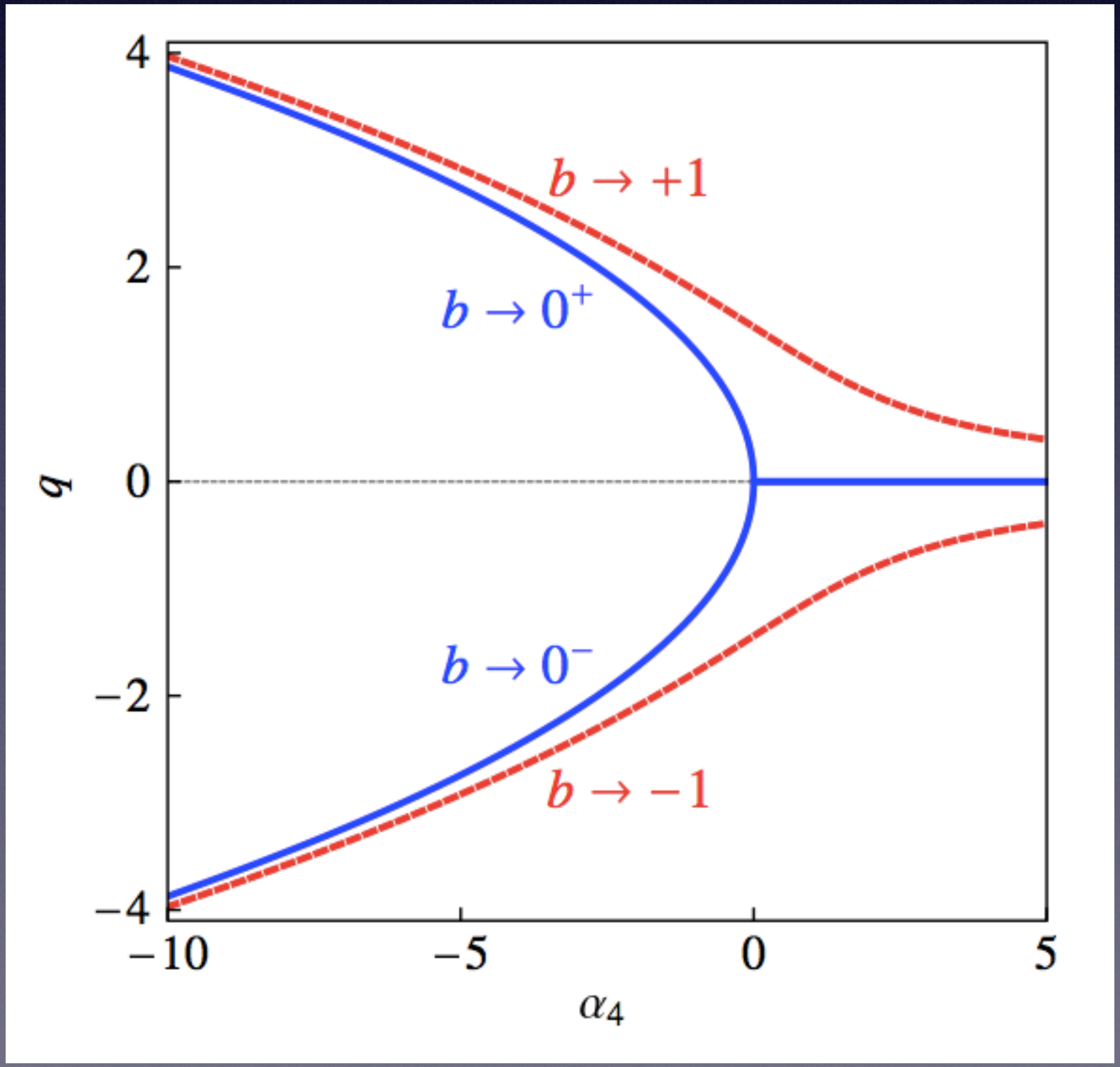
$$\sigma + i\pi_3 = \Delta_0 e^{iqz}$$

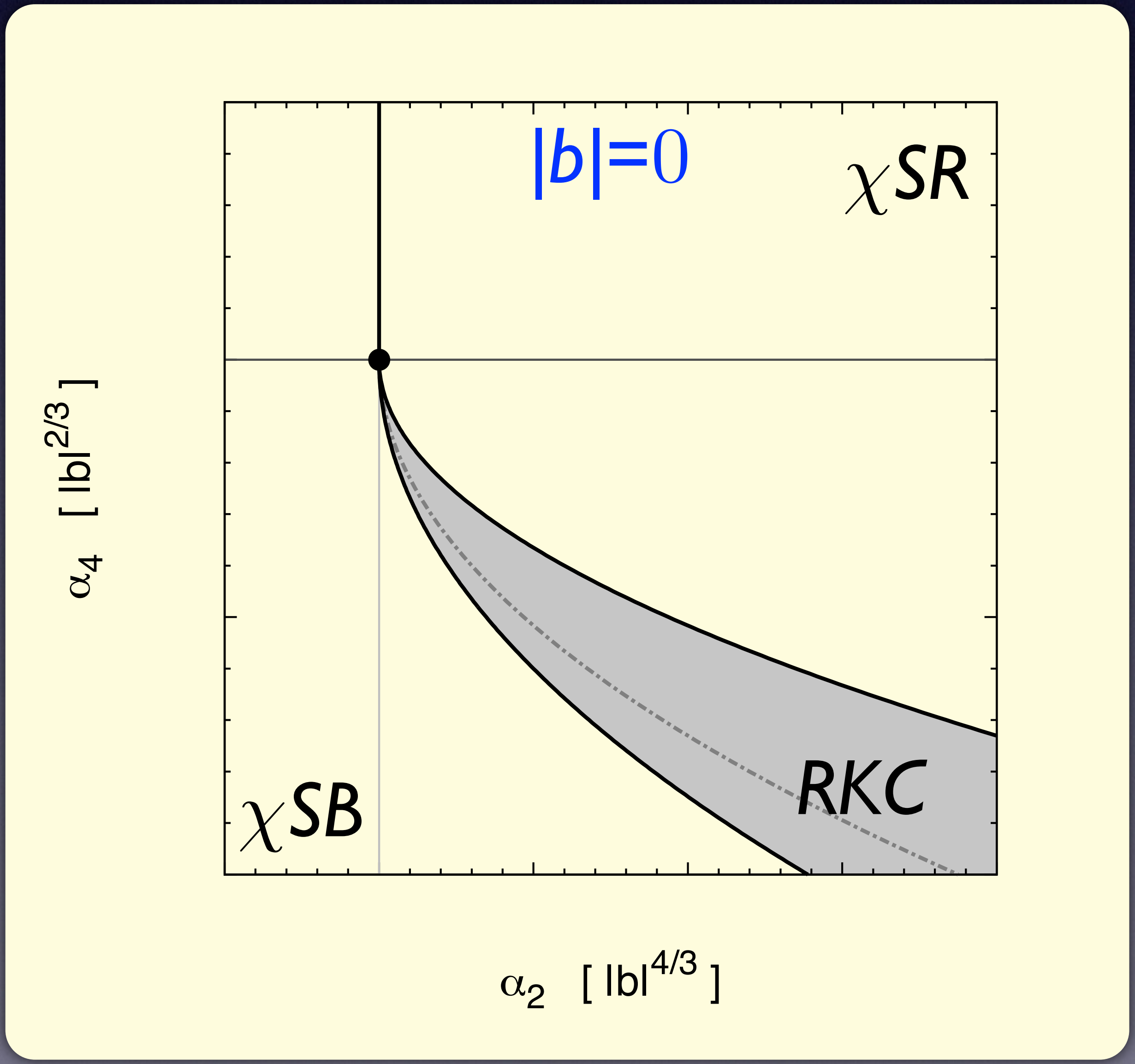
Only second order phase transition!

Why so drastic?

$$\Omega_{\text{bin.}} = \Gamma^{-1}(\alpha_2, \alpha_4, q)\Delta_0^2 = \left(\frac{\alpha_2}{2} + \frac{\alpha_4 q^2}{4} + \frac{q^4}{12} - \mathbf{b}\cdot\mathbf{q}\right)\Delta_0^2$$

$$\min_q \Gamma^{-1}(\alpha_2, \alpha_4, q) = 0 \qquad (\Delta(\mathbf{x}) = \sigma + i\pi_3 = \Delta_0\, e^{i\mathbf{q}\cdot\mathbf{x}})$$





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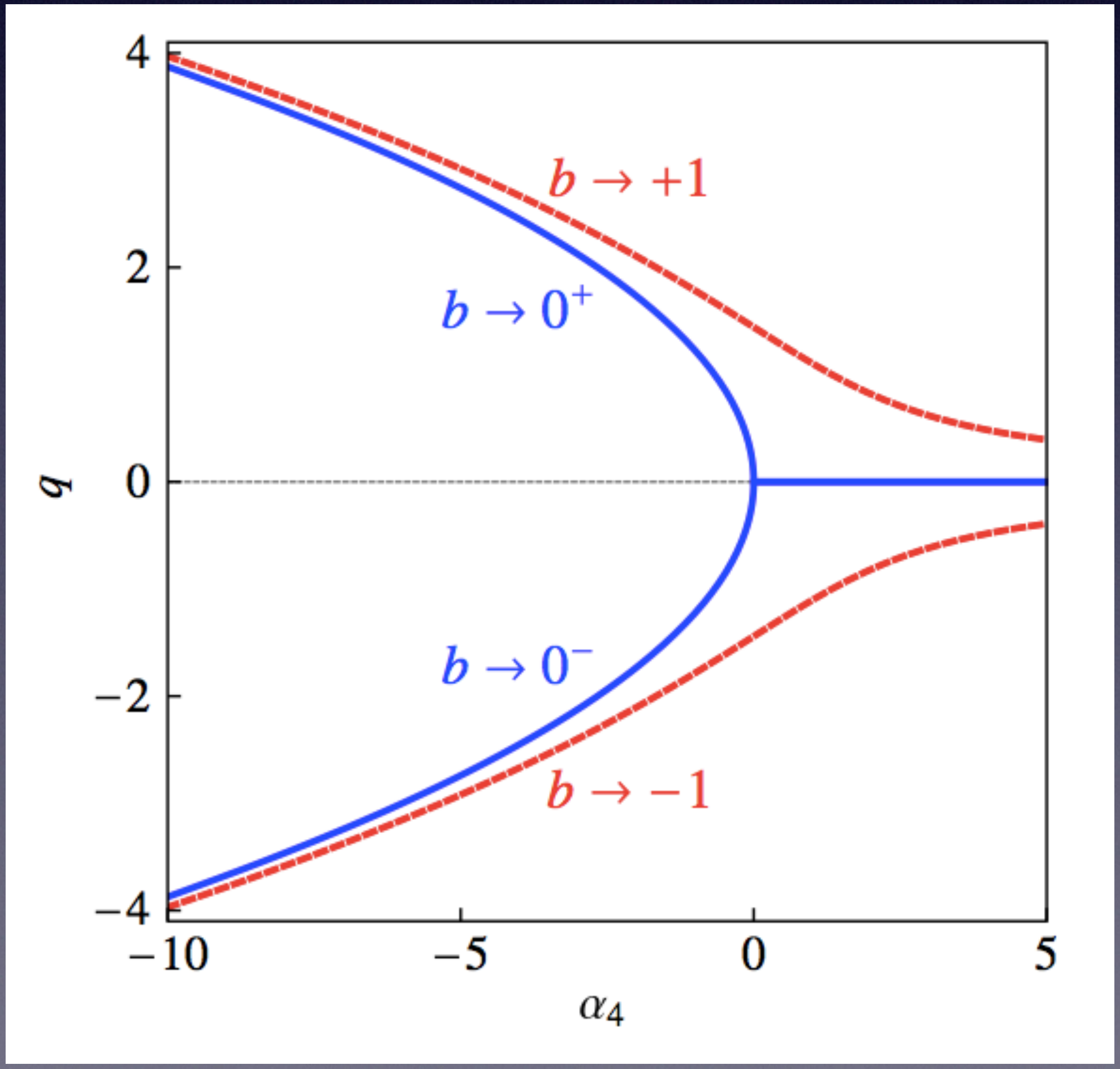

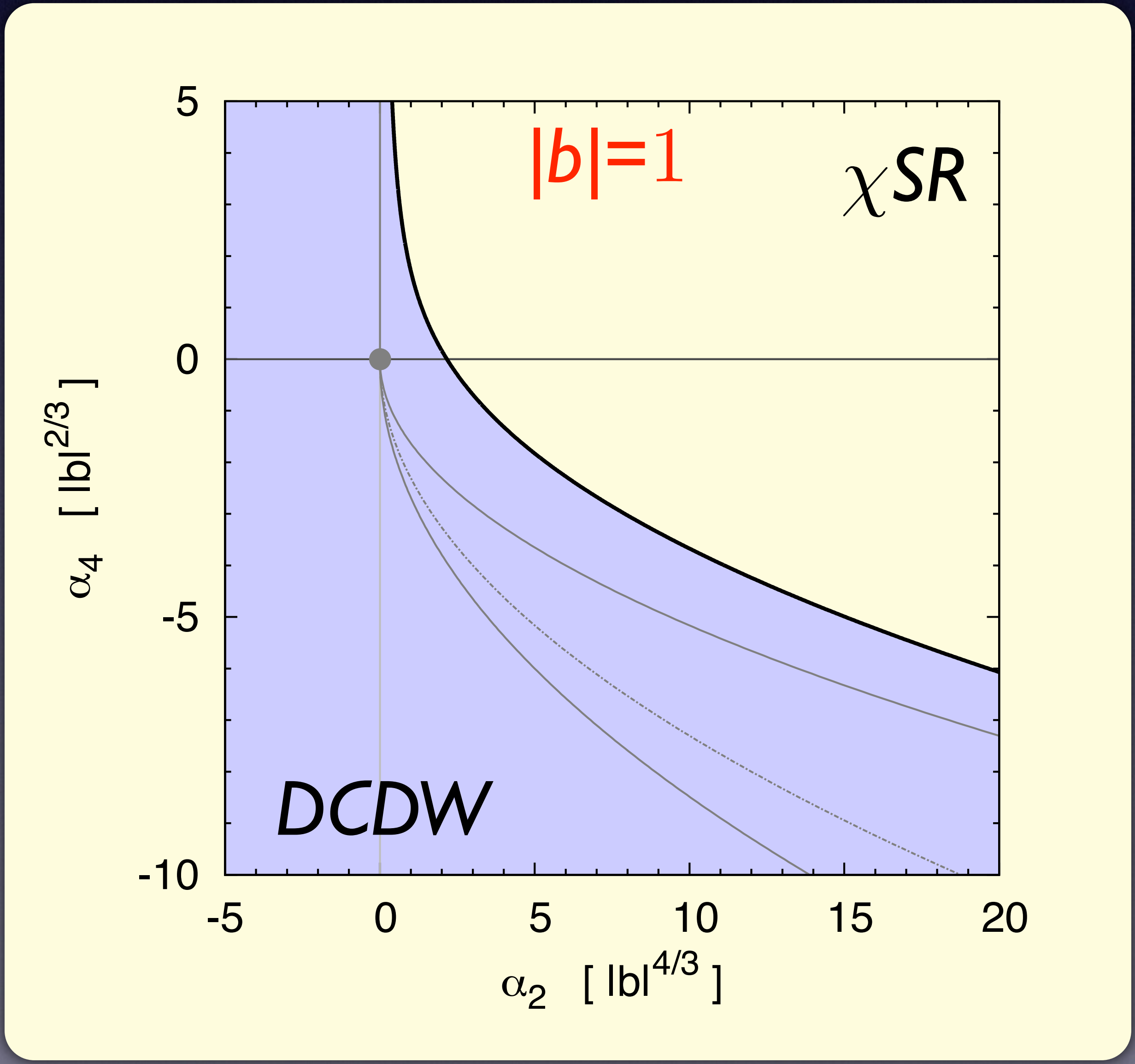

Mass vs Magnetic field

$$\delta\Omega \;=\; \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3) - h\sigma$$

h : mass term

chiral SU(2)xSU(2) ⇒ isospin SU(2)

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Rotational O(3) symmetry
Time reversal symmetry
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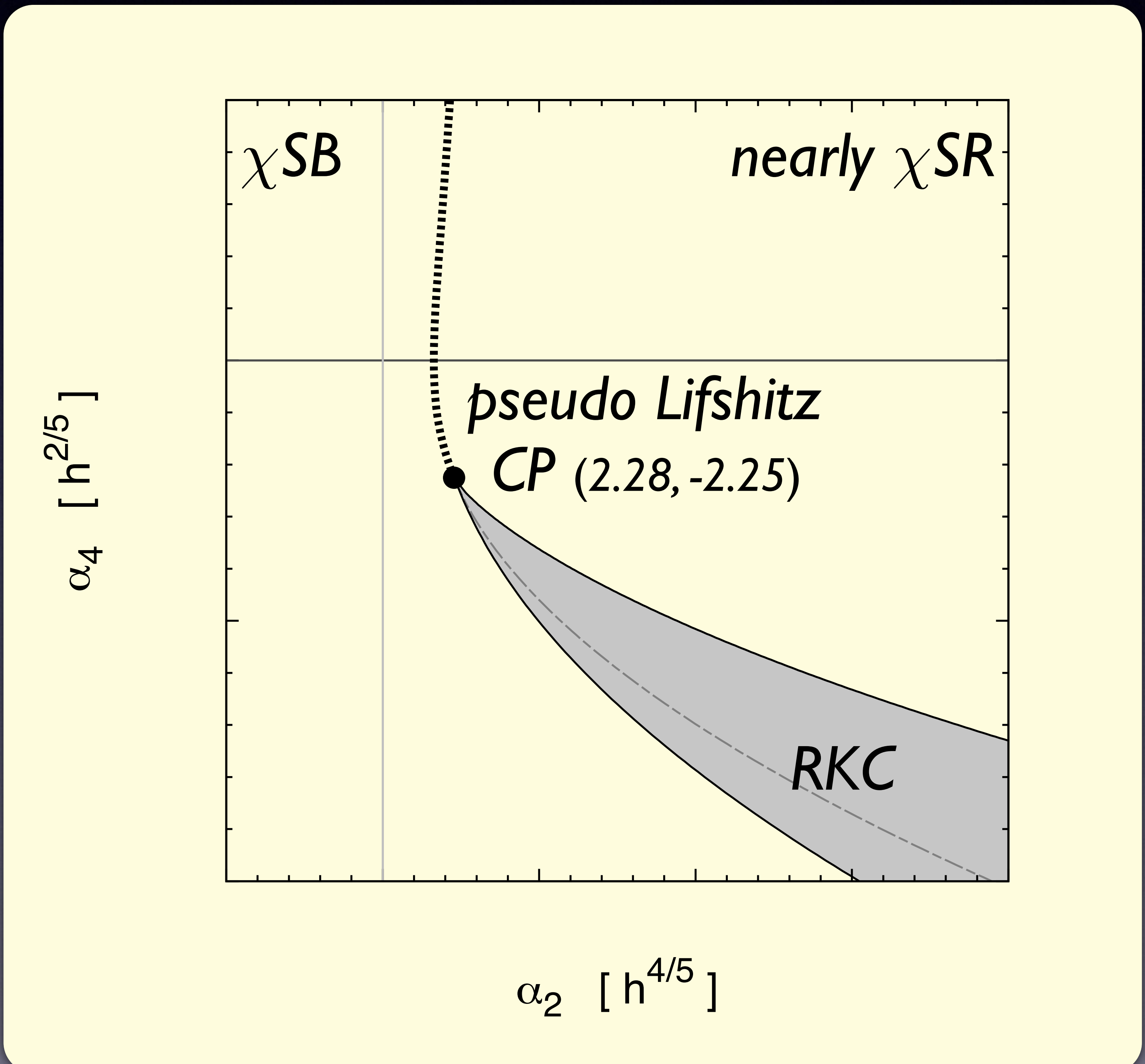
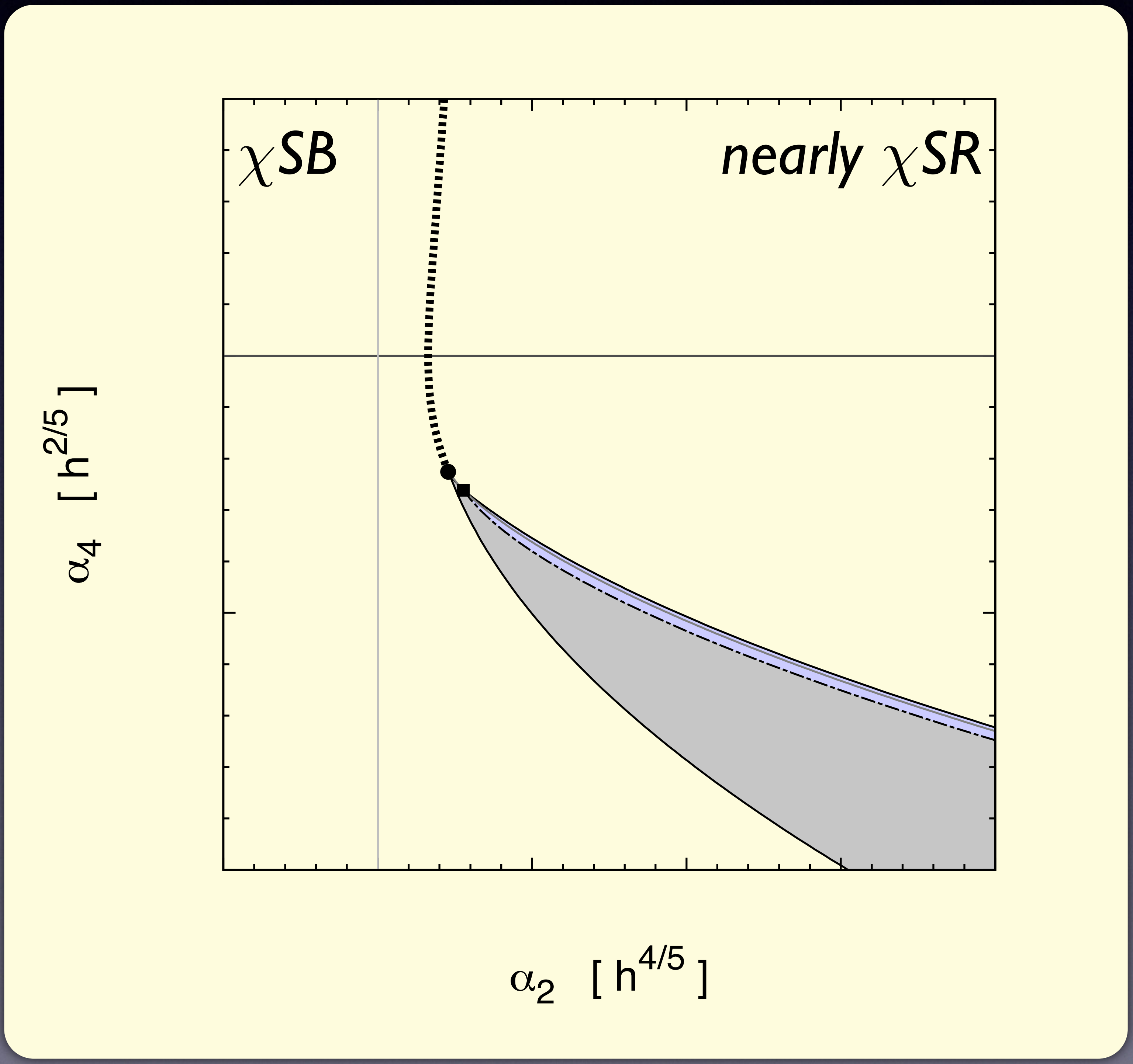
Mass vs Magnetic field



i. *Only mass term on:*

Crossover, Lifshitz point & RKC phase

Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

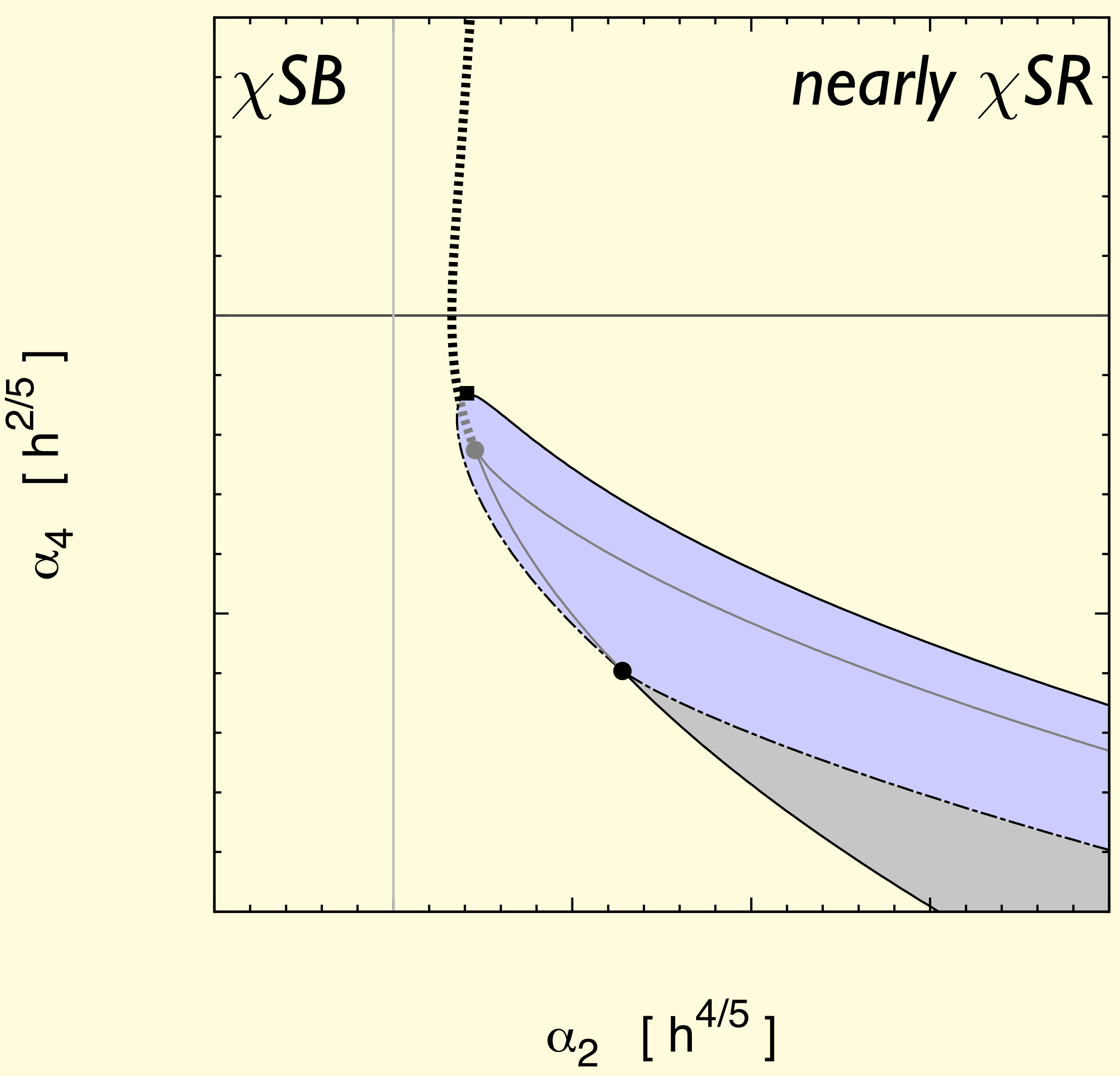
$$8b = 0.5 \times h^{3/5}$$
$$\left(\sqrt{eB} \sim 1.2\,\text{MeV}\right)$$

$10^{9}\text{T} \leftrightarrow 0.24\text{MeV}$

$10^{15}\text{G} \rightarrow 10^{11}\text{T} \leftrightarrow 2.4\text{MeV}$

$10^{13}\text{T} \leftrightarrow 24\text{MeV}$

Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

$$8b = 5.0 \times h^{3/5}$$

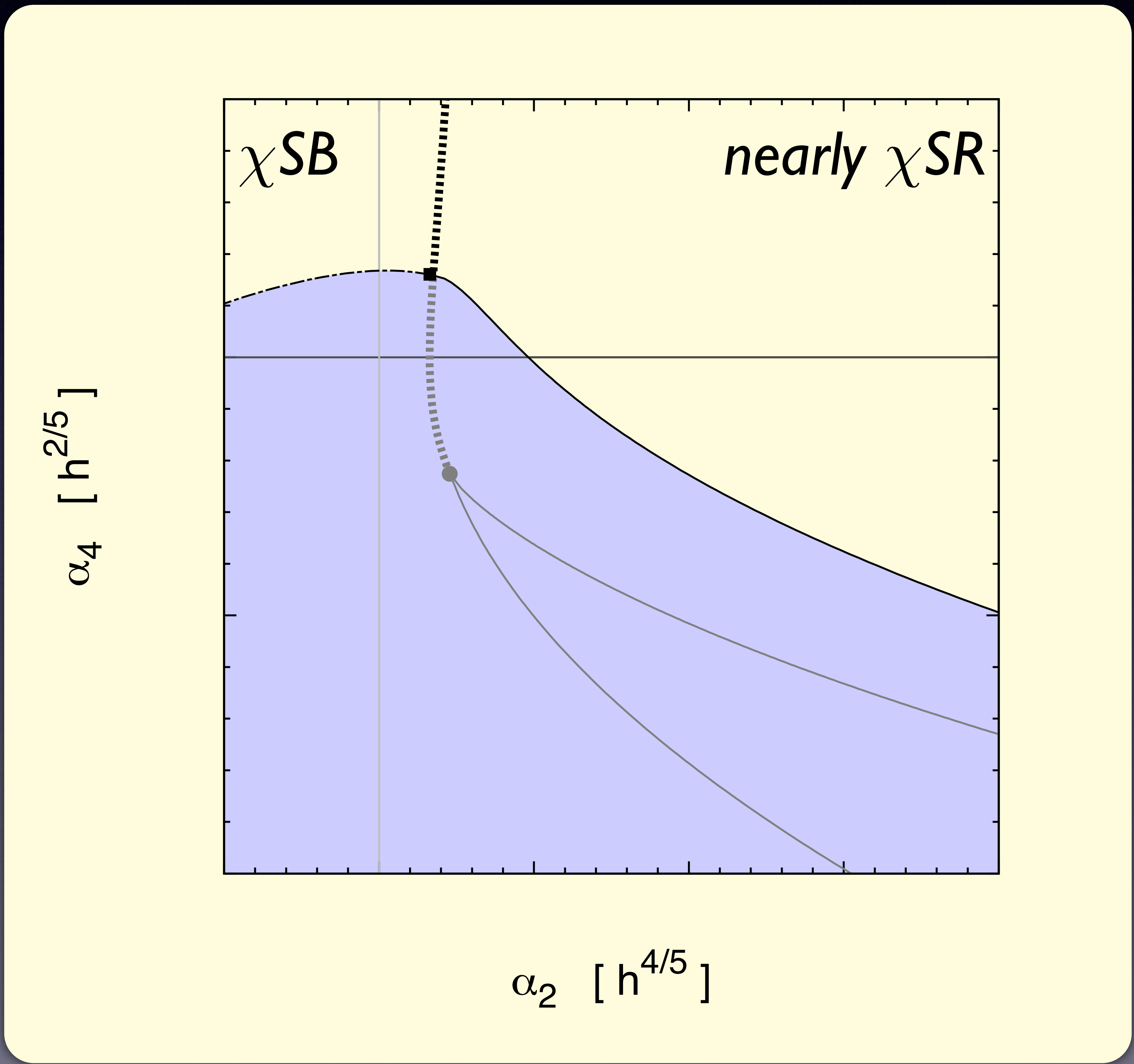
$$\left(\sqrt{eB} \sim 12\,\mathrm{MeV}\right)$$

$$10^9\mathrm{T} \leftrightarrow 0.24\mathrm{MeV}$$

$$10^{15}\mathrm{G} \rightarrow 10^{11}\mathrm{T} \leftrightarrow 2.4\mathrm{MeV}$$

$$10^{13}\mathrm{T} \leftrightarrow 24\mathrm{MeV}$$

Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

$$8b = 15 \times h^{3/5}$$

$$\left(\sqrt{eB} \sim 35\ \mathrm{MeV}\right)$$

$10^9\mathrm{T} \leftrightarrow 0.24\mathrm{MeV}$

$10^{15}\mathrm{G} \rightarrow 10^{11}\mathrm{T} \leftrightarrow 2.4\mathrm{MeV}$

$10^{13}\mathrm{T} \leftrightarrow 24\mathrm{MeV}$

Summary

- GL is powerful tool
- In 3D, RKC is the strongest structure
- In chiral limit, infinitesimal magnetic field makes RKC totally replaced by DCDW phase (spiral)

  (confirmed by stability analysis)
- Quark mass favors the RKC phase (soliton)
- Stability analysis revealed there is always an instability to $\Delta_0 \neq 0$ at finite momentum for b≠0

Outlook

- Use of lattice models (including fluctuations)

  Role of fluctuations:

  Lee, Nakano, Tsue, Tatsumi, Friman, PRD92 (2015)

  Hidaka, Kamikado, Kanazawa, Noumi, PRD92 (2015)

- Static and dynamic elastic properties of inhomogeneous chiral phase

  → Neutron star merger physics (Deformation?)

- Transport (thermal, and electric) properties of inhomogeneous phase: *Tatsumi-san*'s talk on Fri.

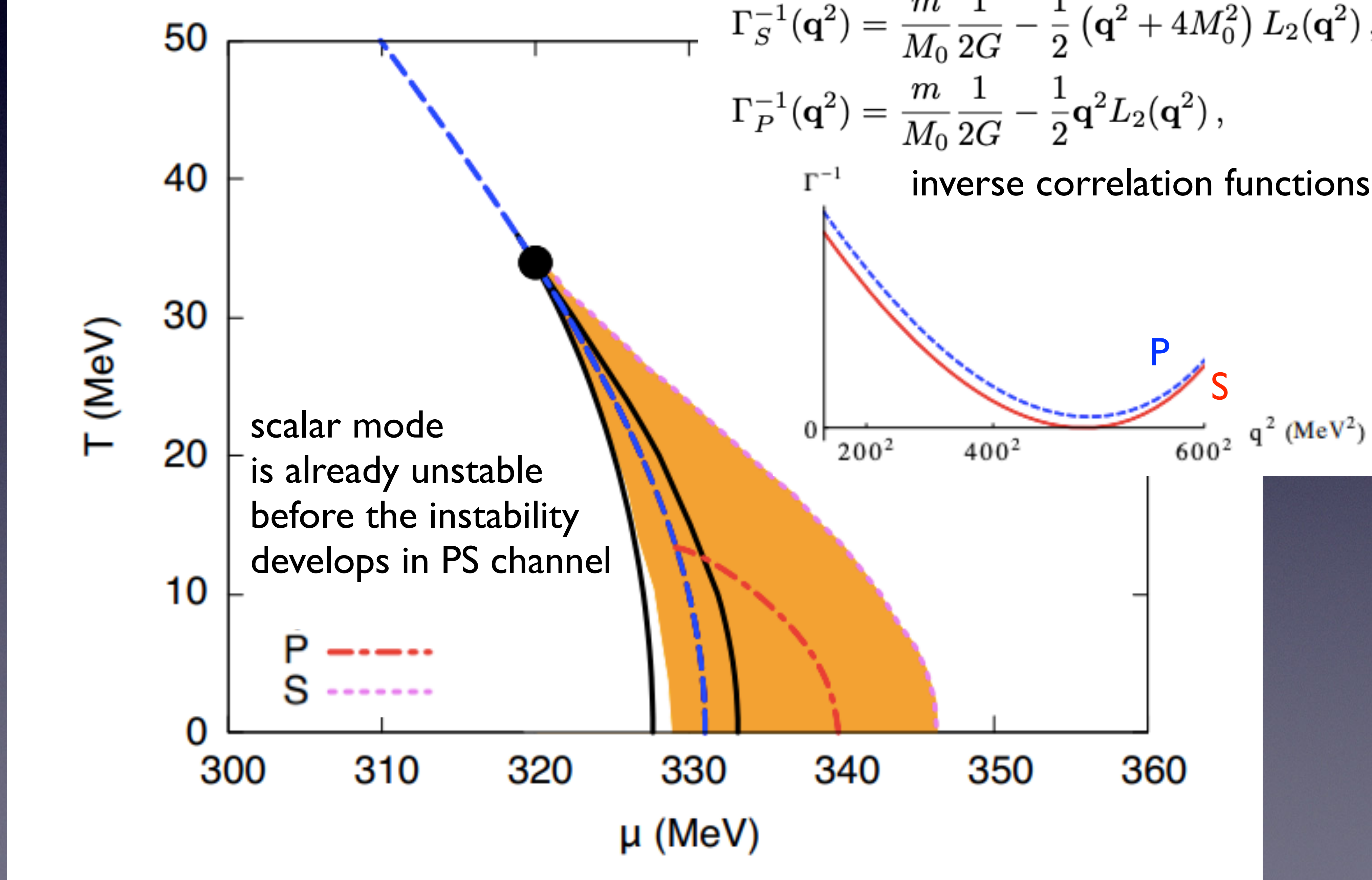
  → Cooling of proto-Neutron star?

Remark 2 (off chiral limit)

M. Buballa, S. Carignano, PLB791 (2019)

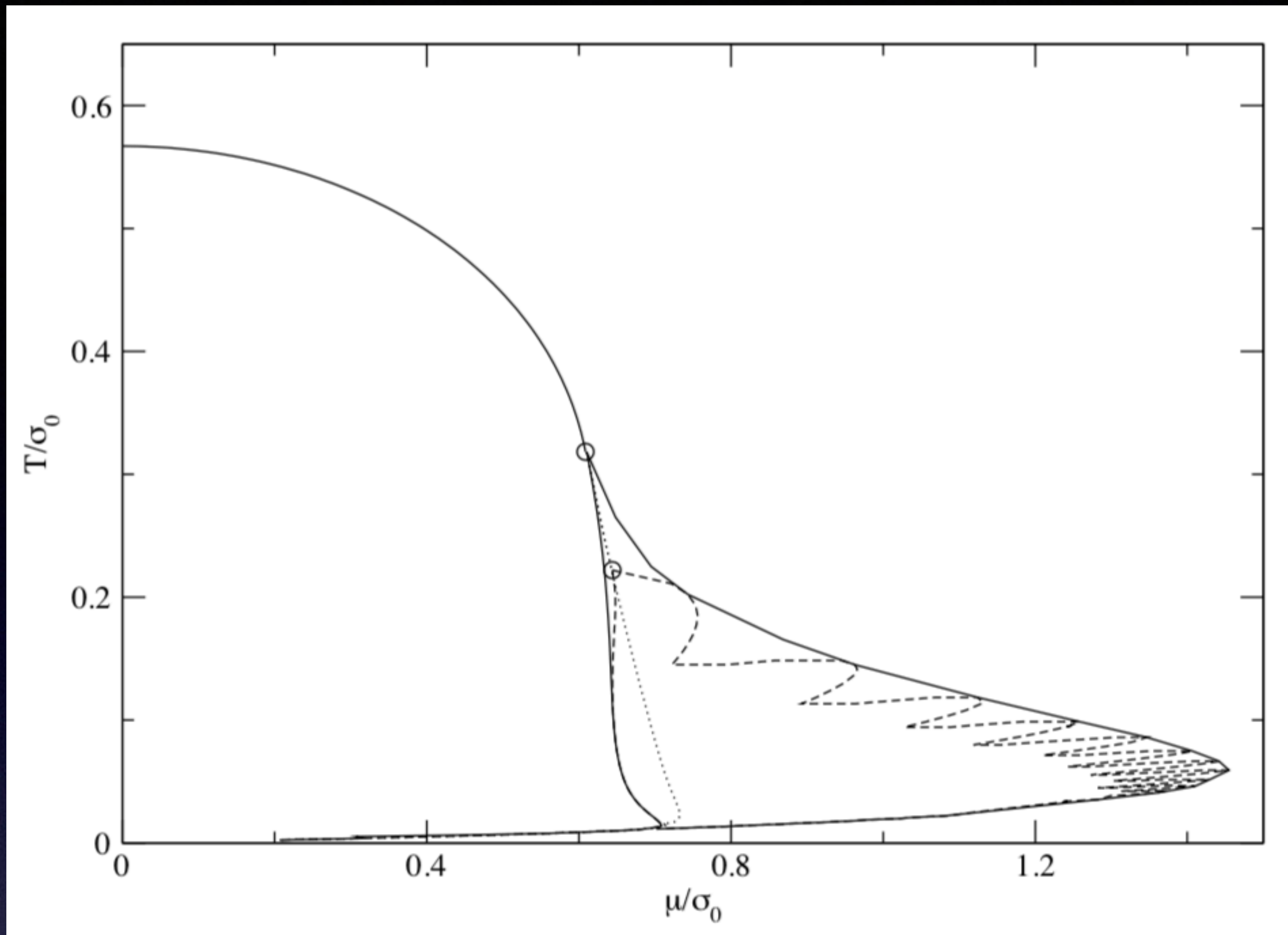
$$\Gamma_S^{-1}(\mathbf{q}^2) = \frac{m}{M_0}\frac{1}{2G} - \frac{1}{2}\left(\mathbf{q}^2 + 4M_0^2\right) L_2(\mathbf{q}^2)\,,$$

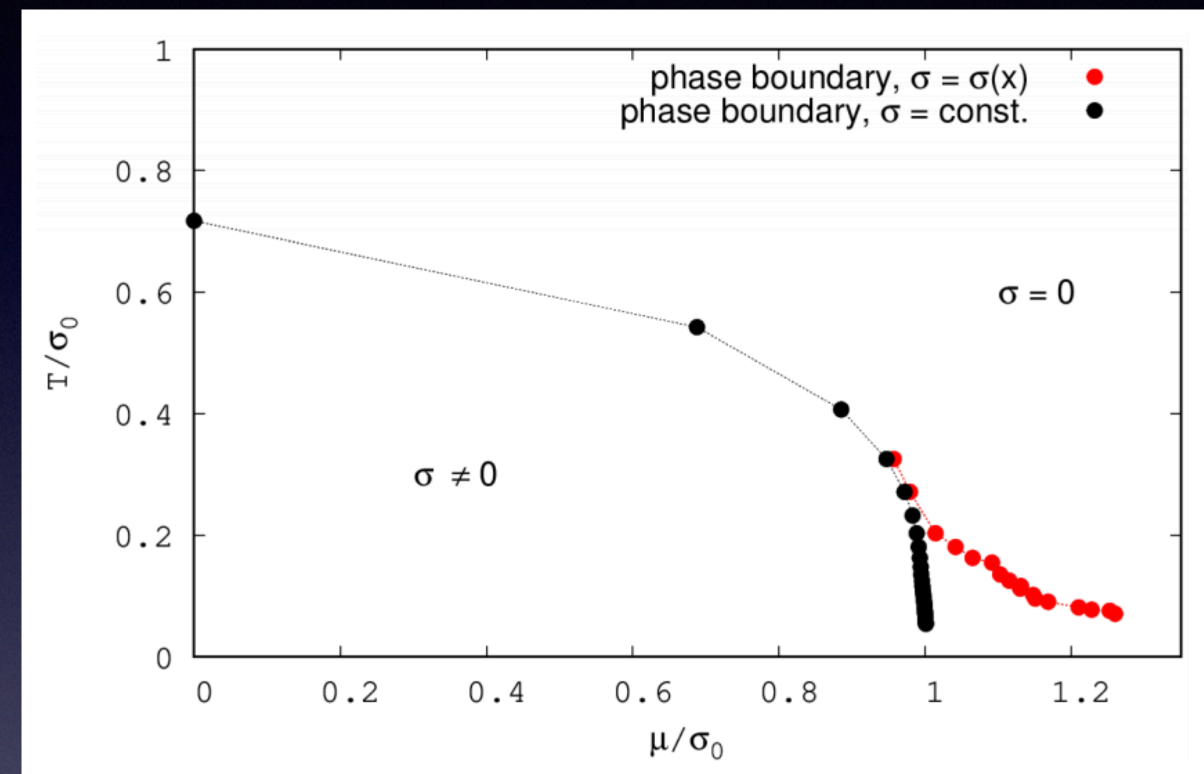
$$\Gamma_P^{-1}(\mathbf{q}^2) = \frac{m}{M_0}\frac{1}{2G} - \frac{1}{2}\mathbf{q}^2 L_2(\mathbf{q}^2)\,,$$



P. de Forcrand, U. Wenger, hep-lat/0610117

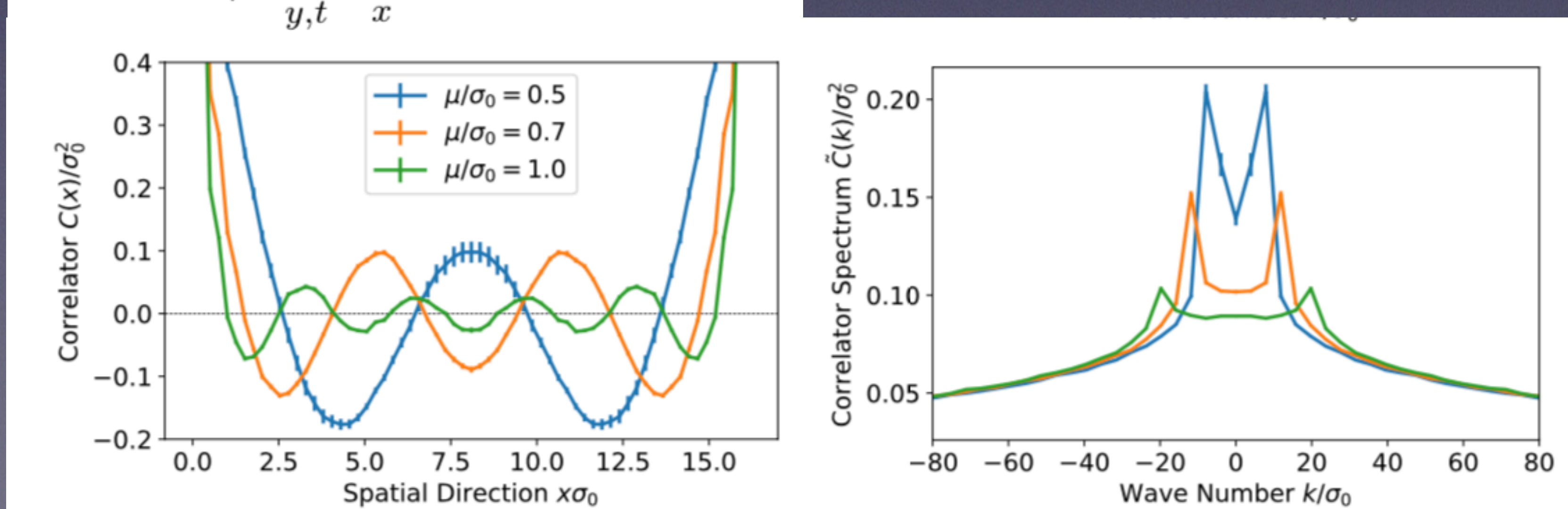
1+1 dim. Lattice GN model at large N limit
with staggered fermion, Nt = 80 (fixed)





2+1 dim. Lattice GN model at large N limit
M. Winstel, Jonas Stoll, M. Wargner,
arXiv:1909.00064

$$C(x) = \frac{1}{V}\sum_{y,t}\sum_{x}\sigma(y,t)\sigma(y+x,t).$$



1+1 dim. Lattice GN model at N≠∞
L. Pannullo, J. Lenz, M. Wagner, et al.,
arXiv:1902.11066