Inhomogeneous Nuclear Matter in Strong Magnetic Field

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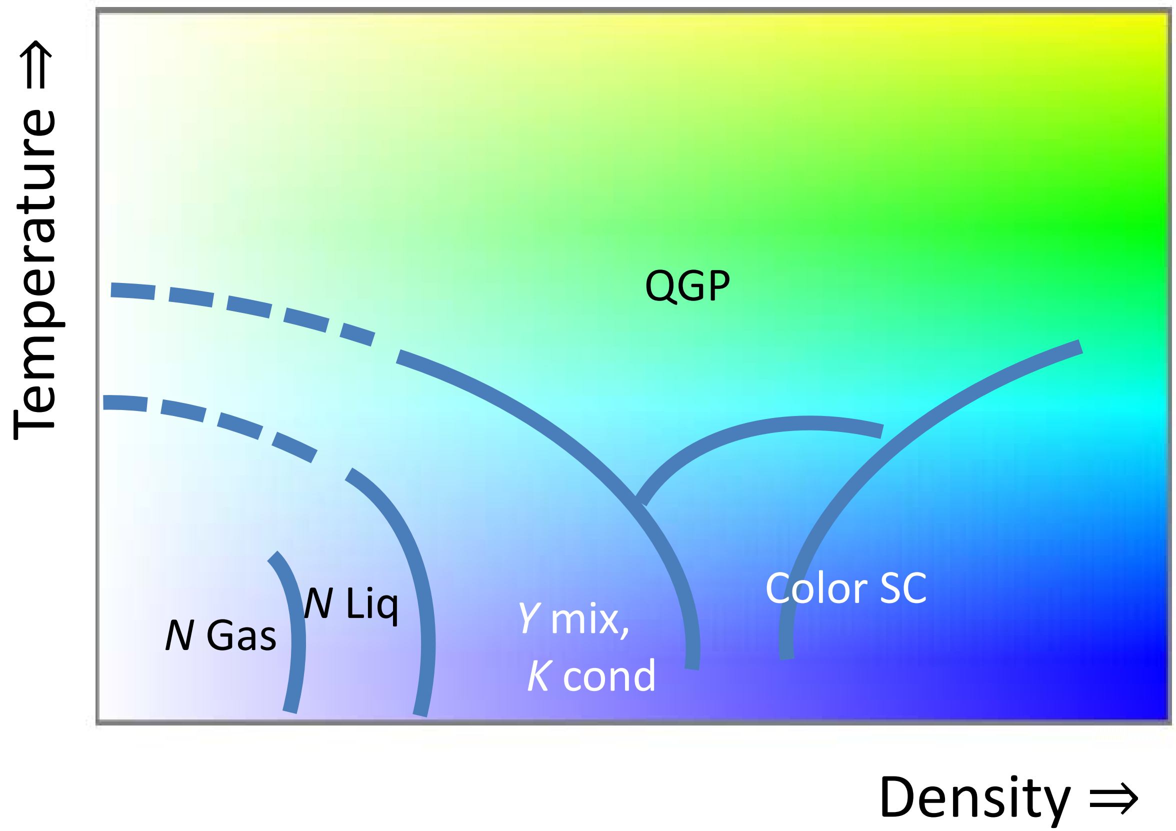
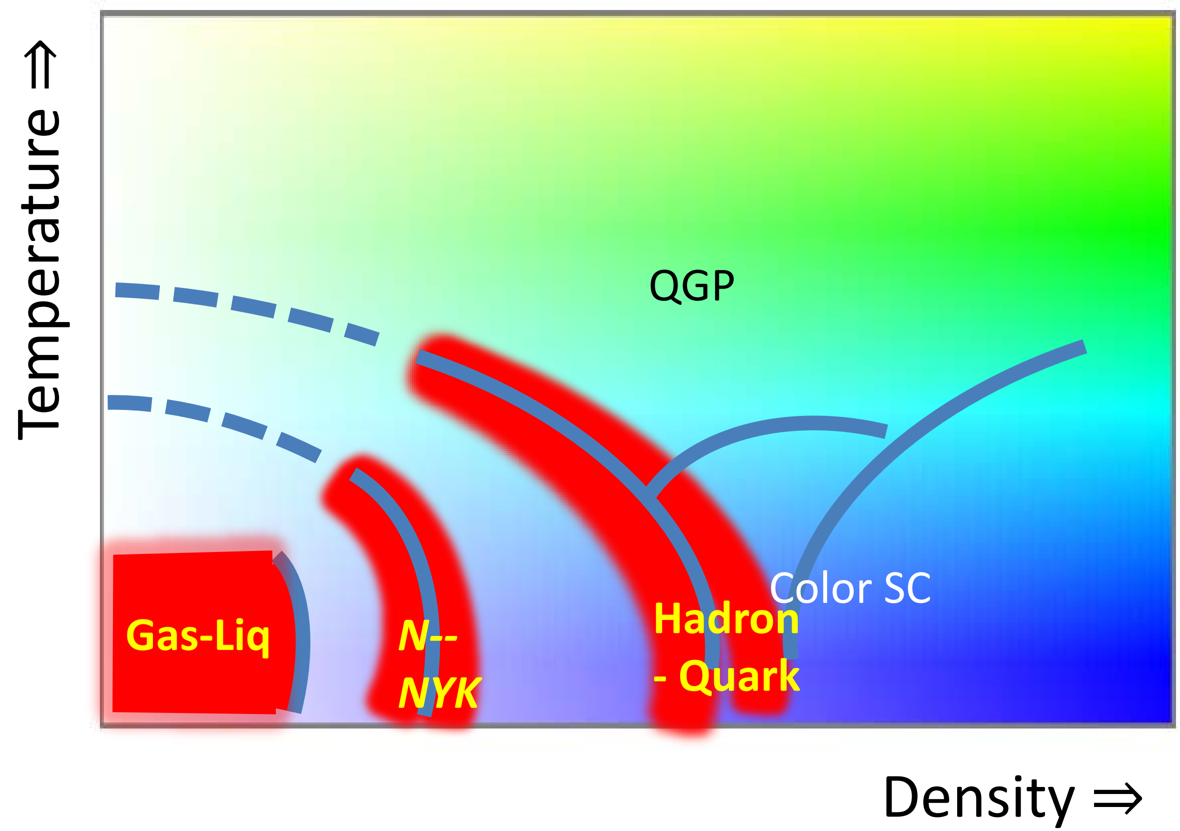
Possible QCD phase diagram



Possible QCD phase diagram



The area of mixed phase in rho-T plane is considerably large.

➔to know the structure of neutron stars, need to know the EOS of mixed phase as well as that of single phase.

The EOS of mixed phase

Maxwell construction applies only to the most simple cases with a single chemical component.

Nuclear matter in compact stars:

- chemically composite →Maxwell const. not satisfy Gibbs cond.
- charged phases → balance between Coulomb & surface tension
  →geometrical structure (pasta)
  → different from bulk Gibbs calculation
- Possibility of strong magnetic field

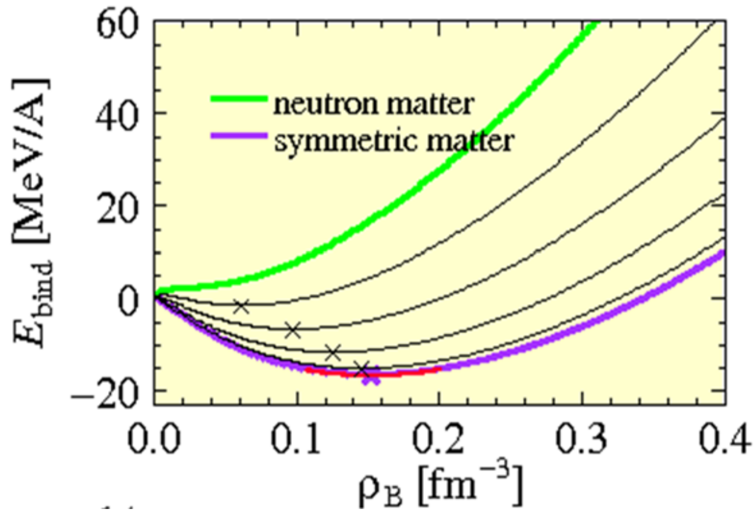
Our goal: inhomogeneous matter

- from low-density nuclear matter to high-density quark matter
- fully 3-dimensional structure
- with and without strong magnetic field

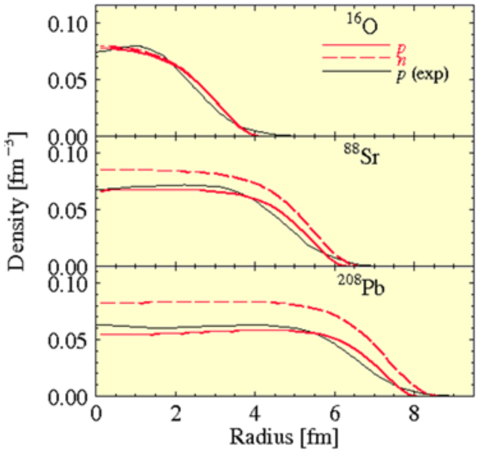
Low-density nuclear matter (Supernova & Neutron star crust)

RMF + Thomas-Fermi model

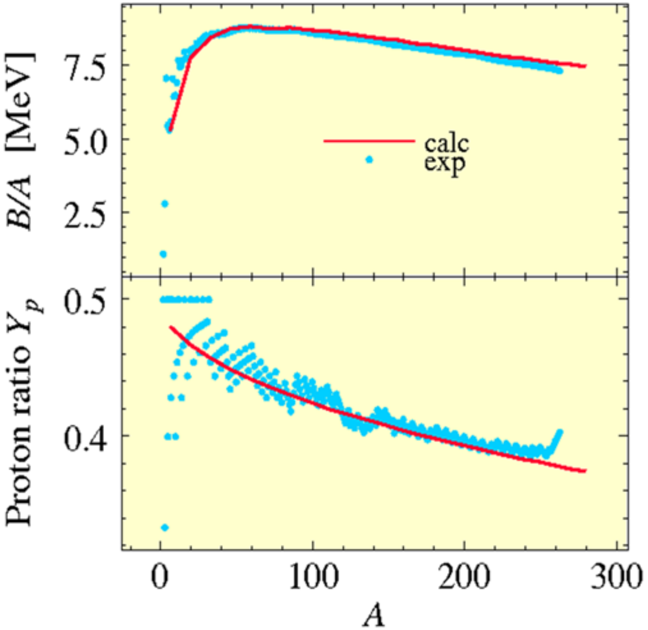
Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but realistic enough.



Saturation property of symmetric nuclear matter : minimum energy $E/A \approx -16$ MeV at $\rho_B \approx 0.16\ \mathrm{fm}^{-3}$.



Binding energies, proton fractions, and density profiles of nuclei are well reproduced.



Details of the model

RMF Lagrangian

$$L = L_N + L_M + L_e,$$

$$L_N = \bar{\Psi}\left[i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N}\gamma^\mu \omega_\mu - g_{\rho N}\gamma^\mu \vec{\tau}\vec{b}_\mu - e\frac{1+\tau_3}{2}\gamma^\mu V_\mu \right]\Psi$$

$$L_M = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}_\mu\vec{R}^\mu,$$

$$L_e = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \bar{\Psi}_e\left[i\gamma^\mu\partial_\mu - m_e + e\gamma^\mu V_\mu \right]\Psi_e, \qquad (F_{\mu\nu} \equiv \partial_\mu F_\nu - \partial_\nu F_\mu)$$

$$m_N^* = m_N - g_{\sigma N}\sigma, \qquad U(\sigma) = \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}c(g_{\sigma N}\sigma)^4$$

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but feasible!

From $\partial_\mu\left[\partial L/\partial(\partial_\mu\phi)\right] - \partial L/\partial\phi = 0$,젏?

$$(\phi = \sigma, \omega_\mu, R_\mu, V_\mu, \Psi),$$

$$-\nabla^2\sigma(\mathbf{r}) + m_\sigma^2\sigma(\mathbf{r}) = g_{\sigma N}(\rho_n^{(s)}(\mathbf{r}) + \rho_p^{(s)}(\mathbf{r})) - \frac{dU}{d\sigma}(\mathbf{r}),$$

$$-\nabla^2\omega_0(\mathbf{r}) + m_\omega^2\omega_0(\mathbf{r}) = g_{\omega N}(\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})),$$

$$-\nabla^2 R_0(\mathbf{r}) + m_\rho^2 R_0(\mathbf{r}) = g_{\rho N}(\rho_p(\mathbf{r}) - \rho_n(\mathbf{r})),$$

$$\nabla^2 V_C(\mathbf{r}) = 4\pi e^2\rho_{\mathrm{ch}}(\mathbf{r}),$$

For Fermions, we employ Thomas-Fermi approx. with finite T

$$f_{i=n,p}(\mathbf{r};\mathbf{p},\mu_i) = \left(1 + \exp\left[\left(\sqrt{p^2 + m_i^*(\mathbf{r})^2} - \sqrt{p_{Fi}(\mathbf{r})^2 + m_i^*(\mathbf{r})^2}\right)\Big/T\right]\right)^{-1},$$

$$f_e(\mathbf{r};\mathbf{p},\mu_e) = \left(1 + \exp\left[\left(p - (\mu_e - V_C(\mathbf{r}))\right)/T\right]\right)^{-1},$$

$$\rho_{i=p,n,e,\nu}(\mathbf{r}) = 2\int_0^\infty \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r};\mathbf{p},\mu_i),$$

$$\mu_n = \sqrt{p_{Fn}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r}), \qquad \mu_n = \mu_p + \mu_e,$$

$$\mu_p = \sqrt{p_{Fp}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_C(\mathbf{r}),$$

$$\int_V d^3r\left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})\right] = \text{const}, \qquad \int_V d^3r\,\rho_p(\mathbf{r}) = \int_V d^3r\,\rho_e(\mathbf{r}),$$

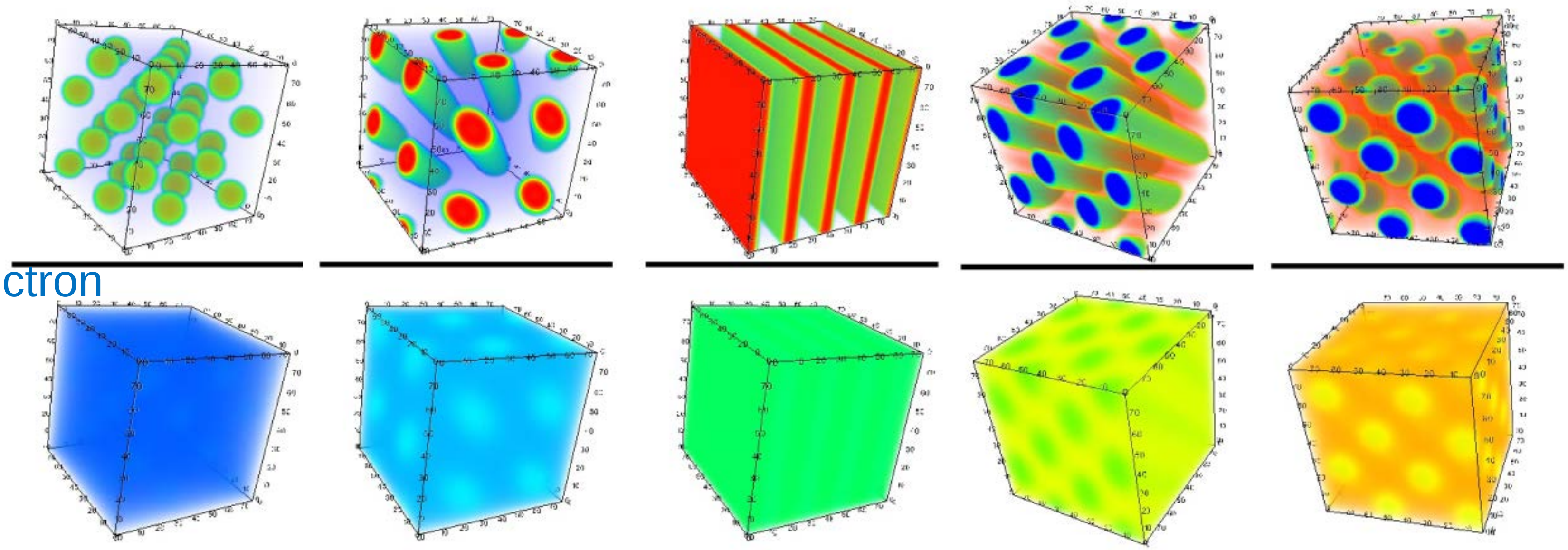
Result of fully 3D calculation

[**Phys.Lett. B713 (2012) 284**]

Symmetric nuclear matter
$Y_p = Z/A = 0.5$
(supernova matter)

proton

electron

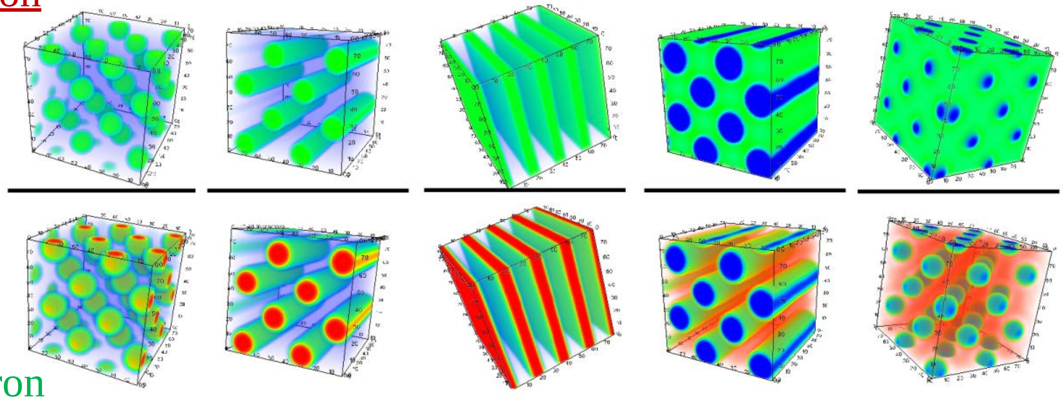


"droplet" [fcc]	"rod" [honeycomb]	"slab"	"tube" [honeycomb]	"bubble" [fcc]
$\rho_B = 0.012$ fm^{-3}	0.024 fm^{-3}	0.05 fm^{-3}	0.08 fm^{-3}	0.094 fm^{-3}

Confirmed the appearance of pasta structures.

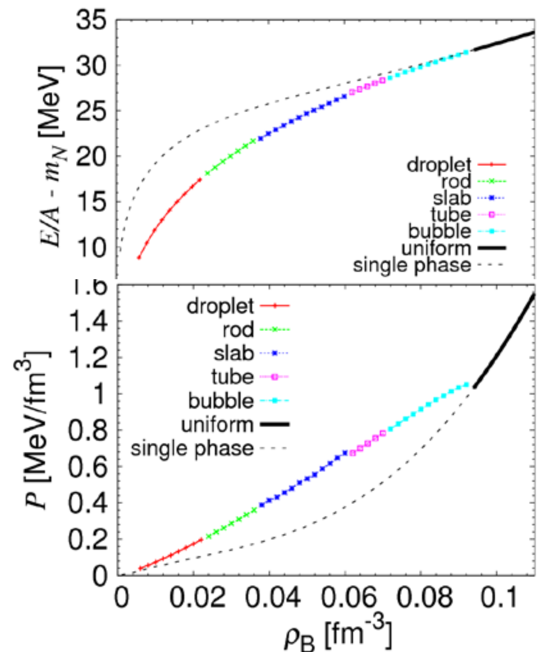
$Y_p = 0.3$

proton



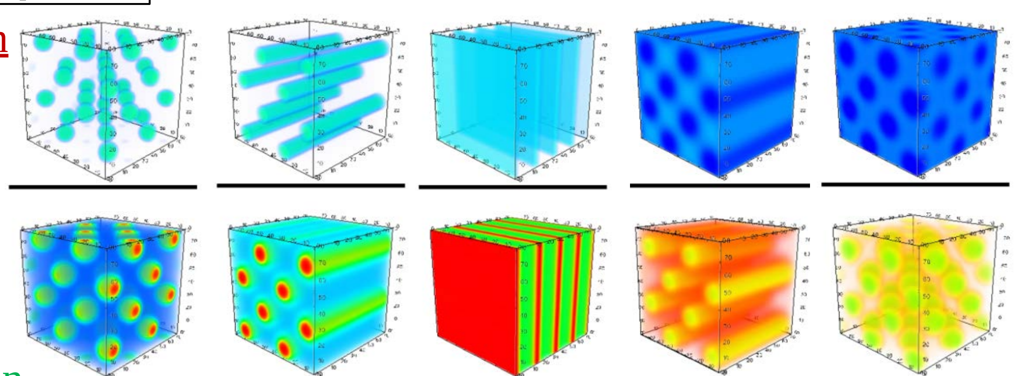
neutron

"droplet" [fcc]	"rod" [simple]	"slab"	"tube" [simple]	"bubble" [fcc]
ρ_B = 0.016 fm^{-3}	0.030 fm^{-3}	0.05 fm^{-3}	0.066 fm^{-3}	0.080 fm^{-3}



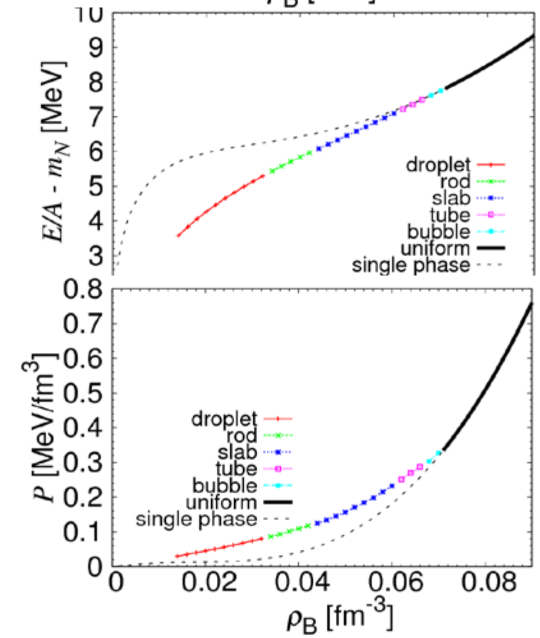
$Y_p = 0.1$

proton



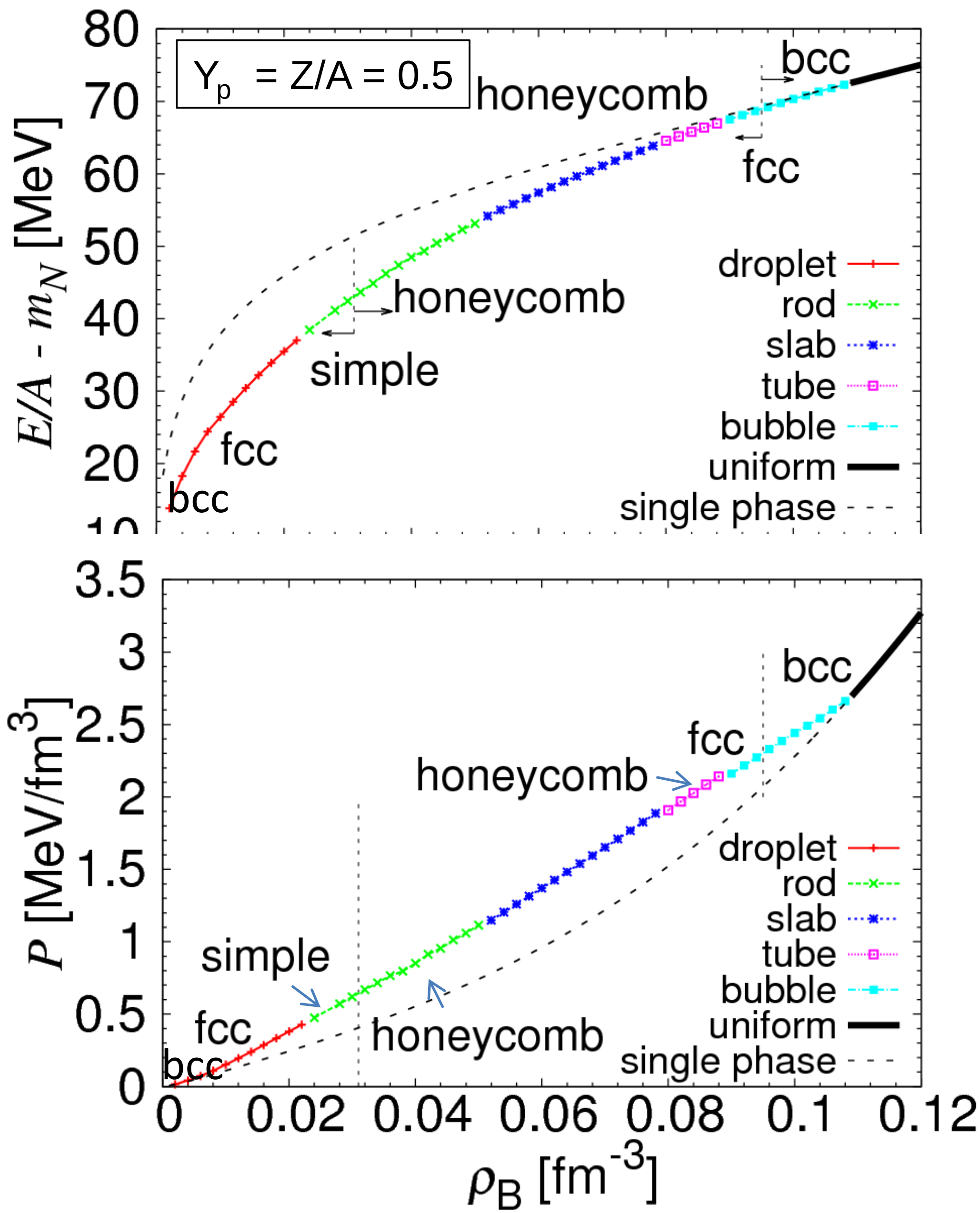
neutron

"droplet" [fcc]	"rod" [simple]	"slab"	"tube" [simple]	"bubble" [fcc]
ρ_B=0.020 fm^{-3}	0.040fm^{-3}	0.05 fm^{-3}	0.066 fm^{-3}	0.070 fm^{-3}



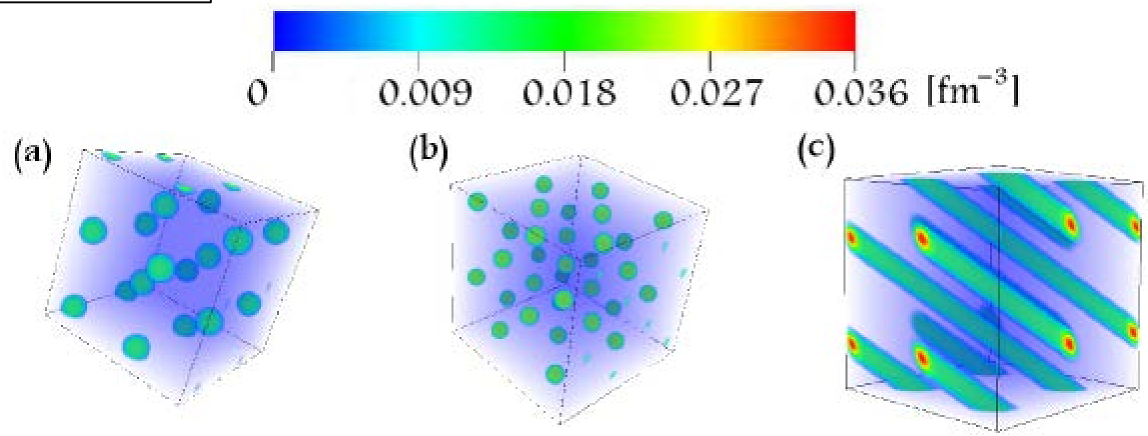
EOS (full 3D) is different from that of uniform matter.
The result is similar to that of the conventional studies with Wigner-Seitz approx.

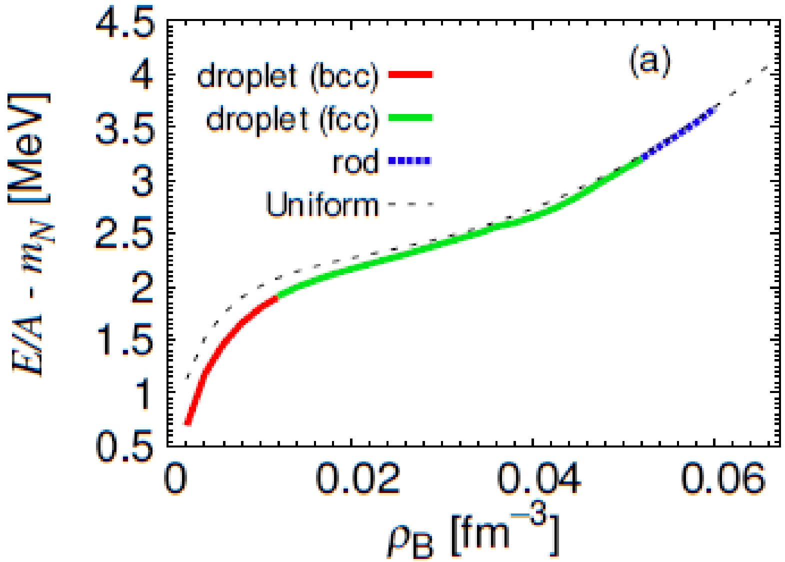
Novelty:
fcc lattice of droplets can be the ground state at some density.
← Not the Coulomb interaction among “point particles” but the change of the droplet size is relevant.



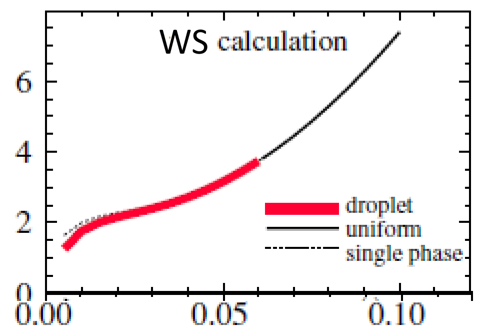
Beta equilibrium case
(neutron star crust)

[Phys. Rev. C **88**, 025801]





Slightly different result from the WS approx.
Crystalline structures bcc & fcc.
Rod phase appears.



4. low-density nuclear matter with magnetic field
(Neutron star crust)

RMF + Thomas-Fermi Model with Magnetic Field

De Lima, et al, PRC88, 035804 , C.J. Xia et al, in preparation
Thomas-Fermi in parallel direction and Landau level in perpendicular direction for charged particle

We have added anomalous magnetic moments of *p* and *n*.

$$L = \sum_i \Psi_i\left[i\gamma^\mu\partial_\mu - m_i - g_{\sigma i}\sigma - g_{\omega i}\gamma^\mu\omega_\mu - g_{\rho i}\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu - q_i\gamma^\mu A_\mu\right]\Psi_i$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - \frac{1}{4}\boldsymbol{A}_{\mu\nu}\boldsymbol{A}^{\mu\nu} + L_{AMM}$$

$$L_{AMM} \equiv -\frac{1}{2}\sum_i \overline{\Psi_i}\mu_N\kappa_i\frac{i}{2}[\gamma^\mu,\gamma^\nu]A_{\mu\nu}\ \Psi_i$$
$$\kappa_n = 1.91, \qquad \kappa_p = 1.79$$

For charged particles, momenta are quantized as

$$p_\perp^2 = 2n|q|B, \qquad n = l + \frac{1}{2} - \frac{s}{2}\frac{q}{|q|}$$

Single particle energy

$$\varepsilon_i = g_{\omega_i}\omega_0 + g_{\rho_i}R_0 + q_i A_0 + \sqrt{p_\parallel^2 + m_i^{*2}}$$

Change of the integral by inclusion of B is

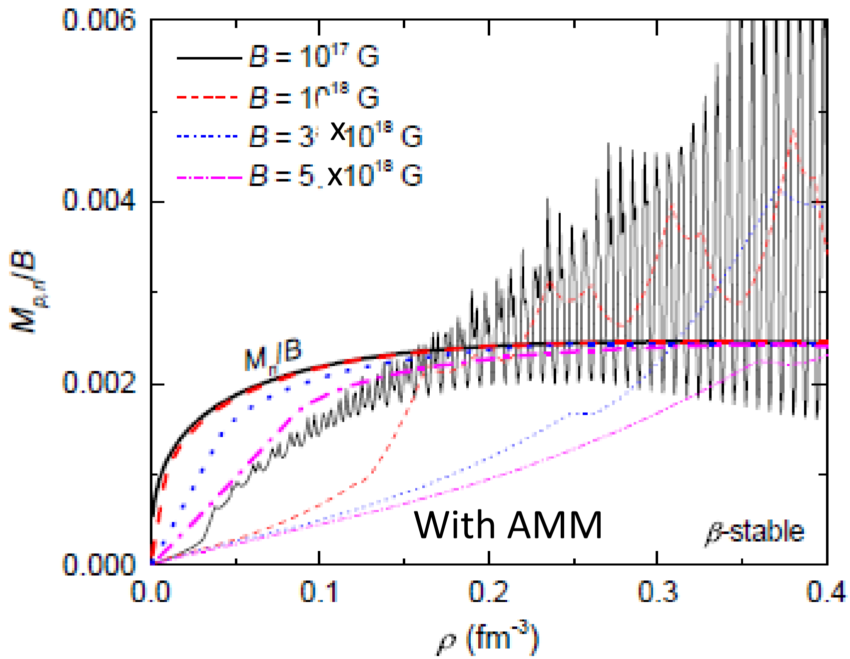
$$2\int \frac{d^3p}{(2\pi)^3} \to \frac{|q_i|B}{2\pi^2} \sum_{s=\pm1} \sum_{l}^{n \le n_i^{\mathrm{max}}} \int_0^{\nu_i} dp_\parallel$$

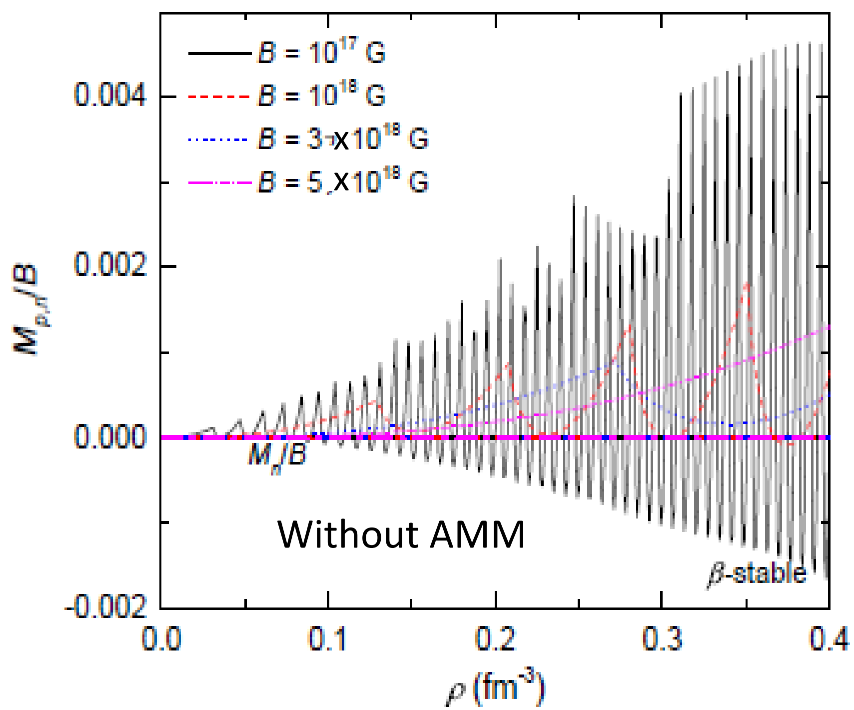
$$n_i^{\mathrm{max}} \equiv int\left[\frac{\left(E_i^f + s\mu_N\kappa_i B\right)^2 - m_i^{*2}}{2|q_i|B}\right]$$

$$\nu_i(n,s) \equiv \sqrt{\left(E_i^f\right)^2 - \bar{m}_i(n,s)^2}\,,\ \ \bar{m}_i = \sqrt{m_i^{*2} + 2n|q_i|B} - s\mu_N\kappa_i B$$

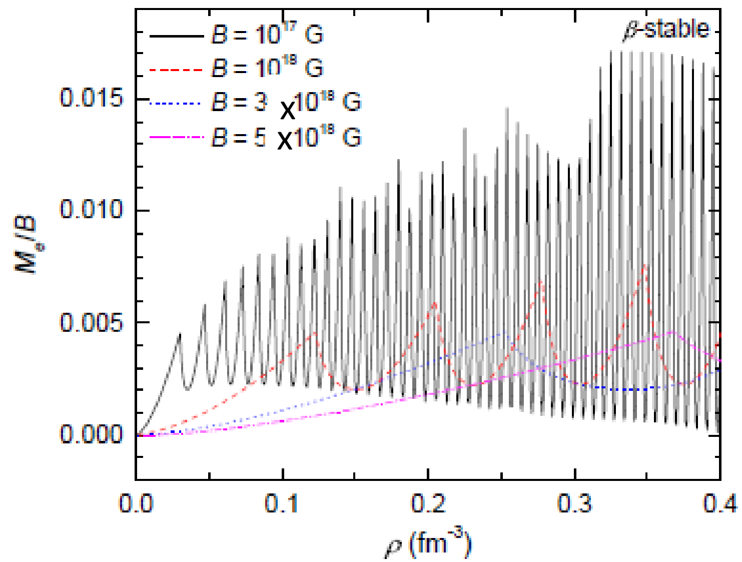
Uniform matter

Magnetization against *B*

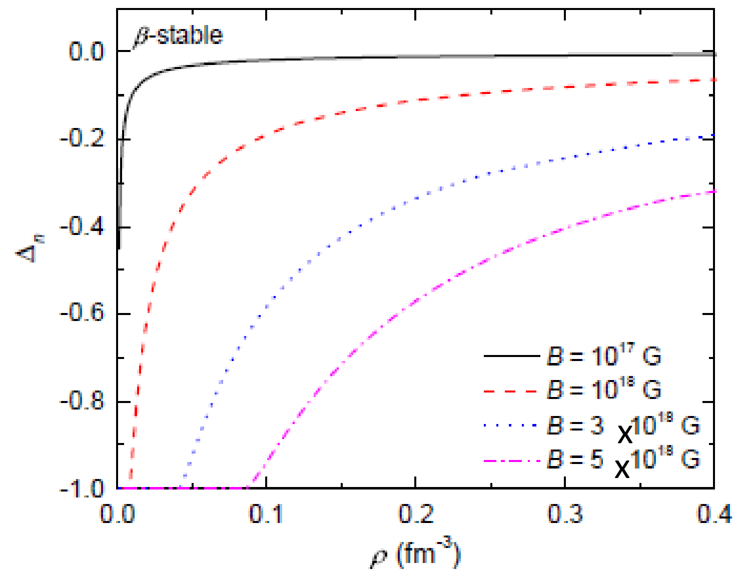




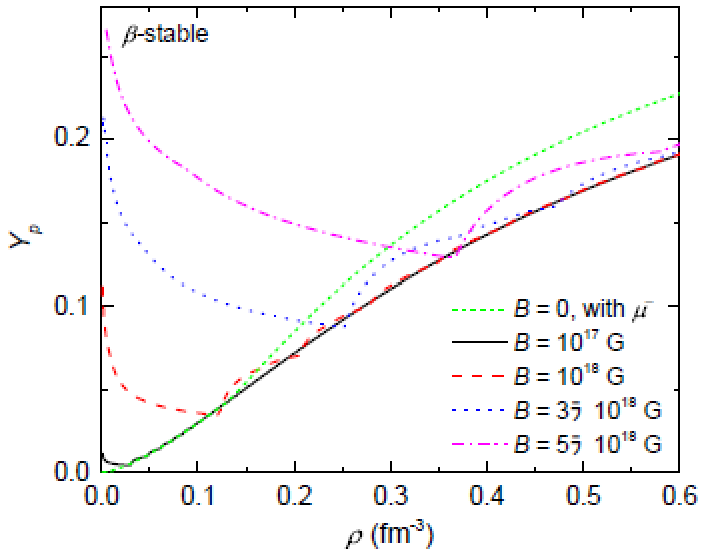
Electron magnetization



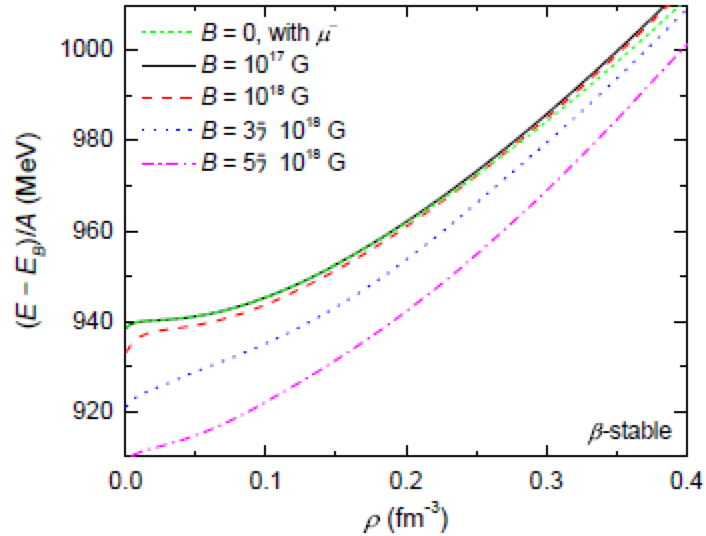
Neutron polarization



Proton fraction

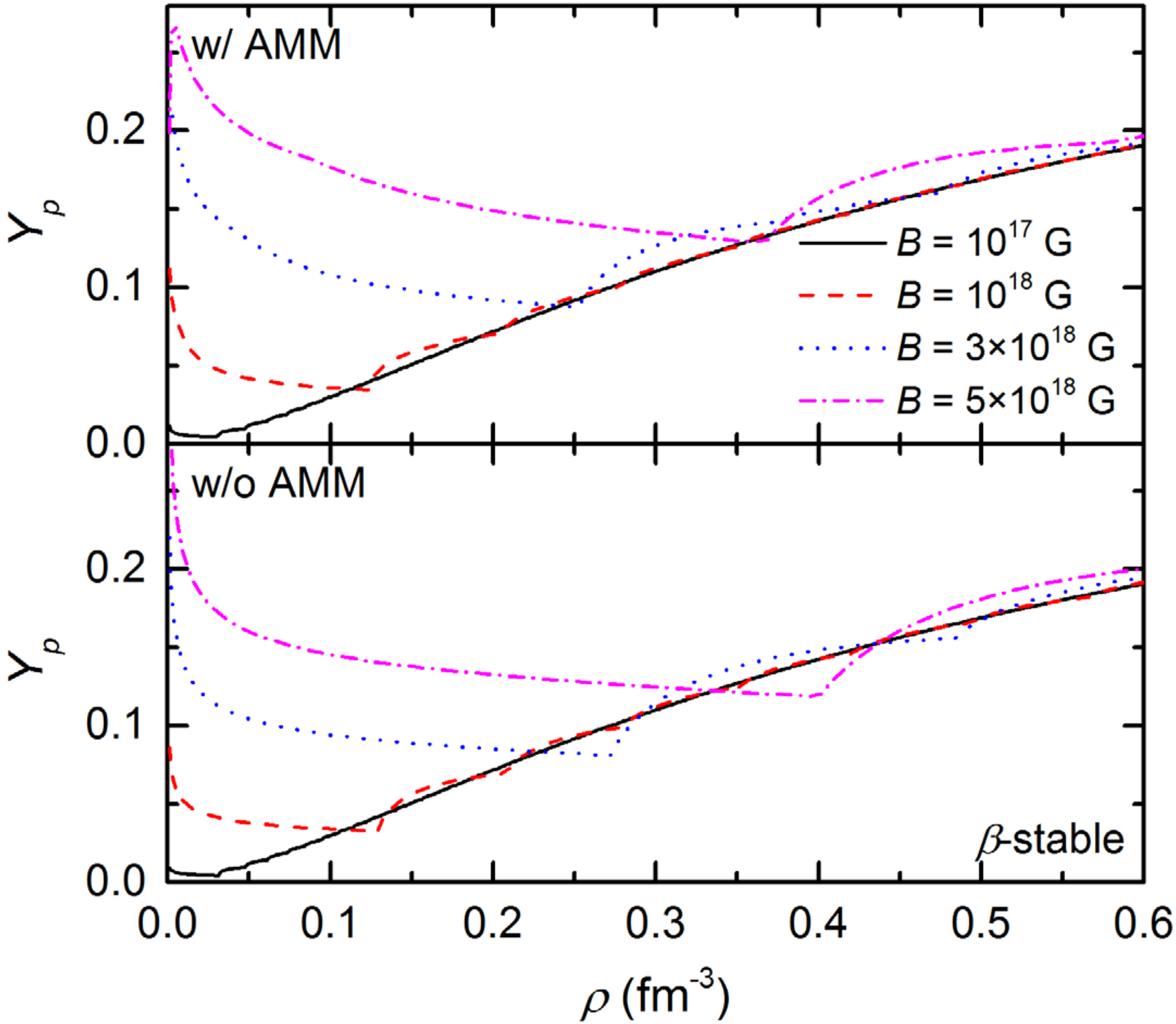


Binding energy

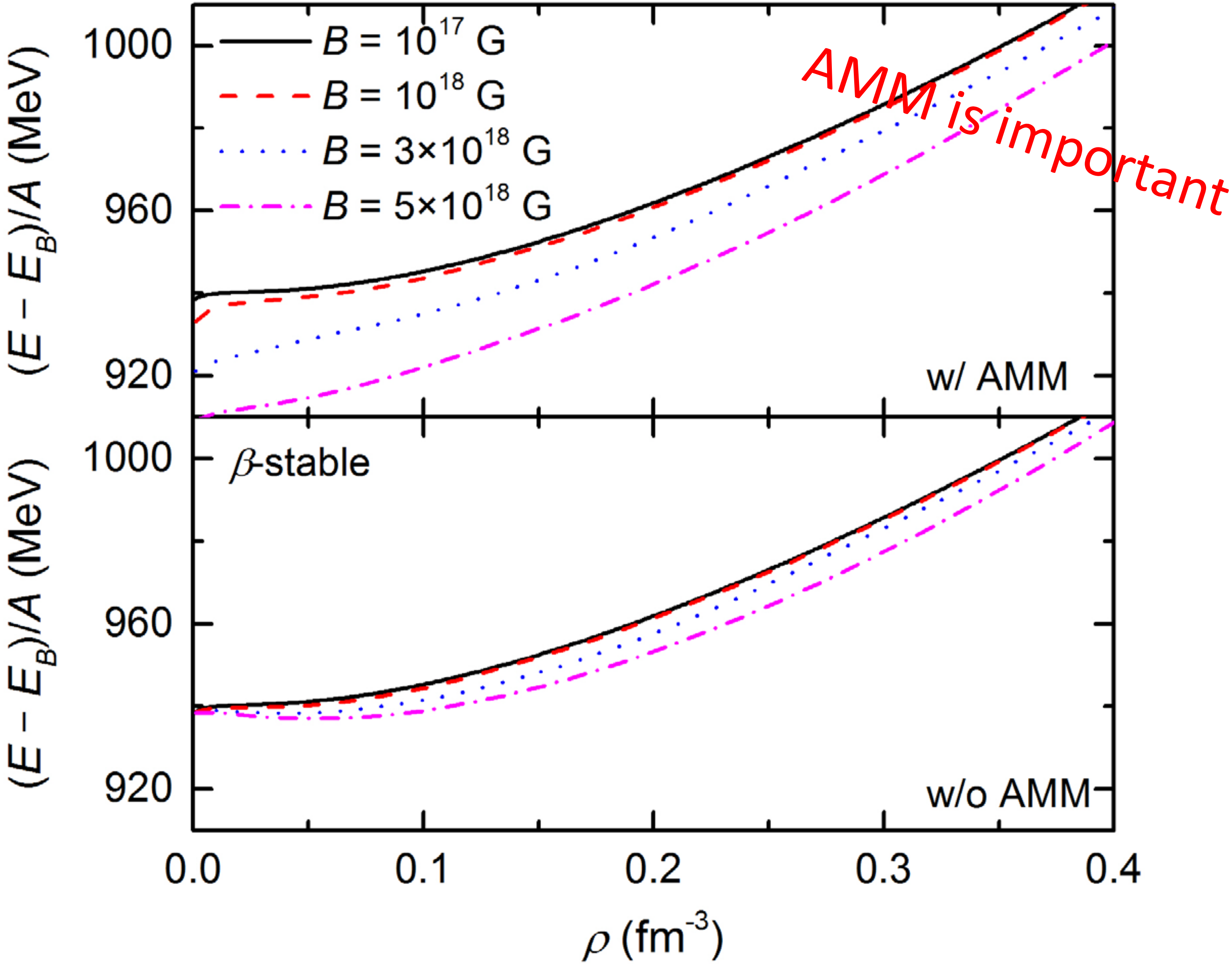


Behavior similar to inhomogeneous nuclear matter

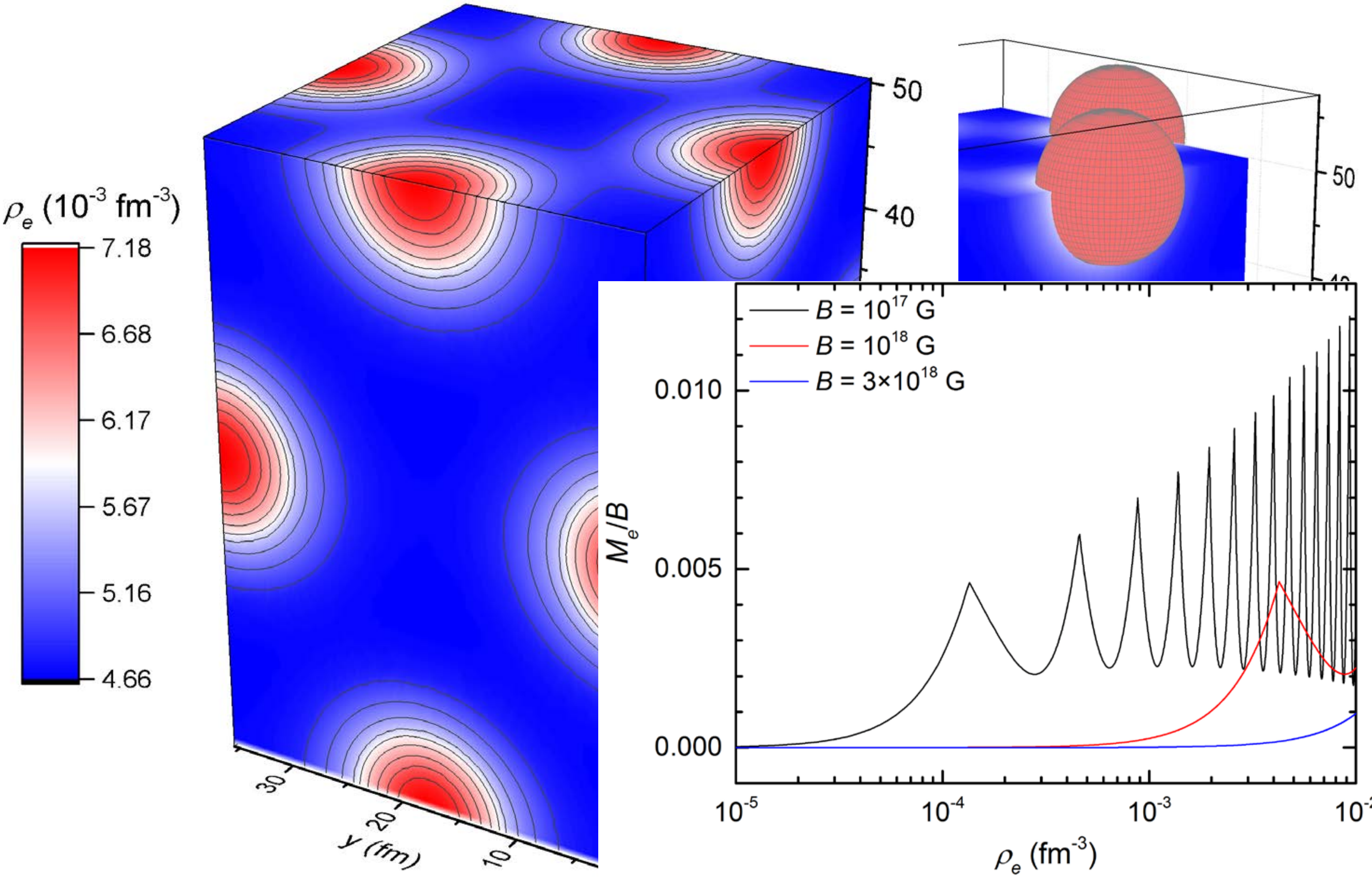
Proton fraction



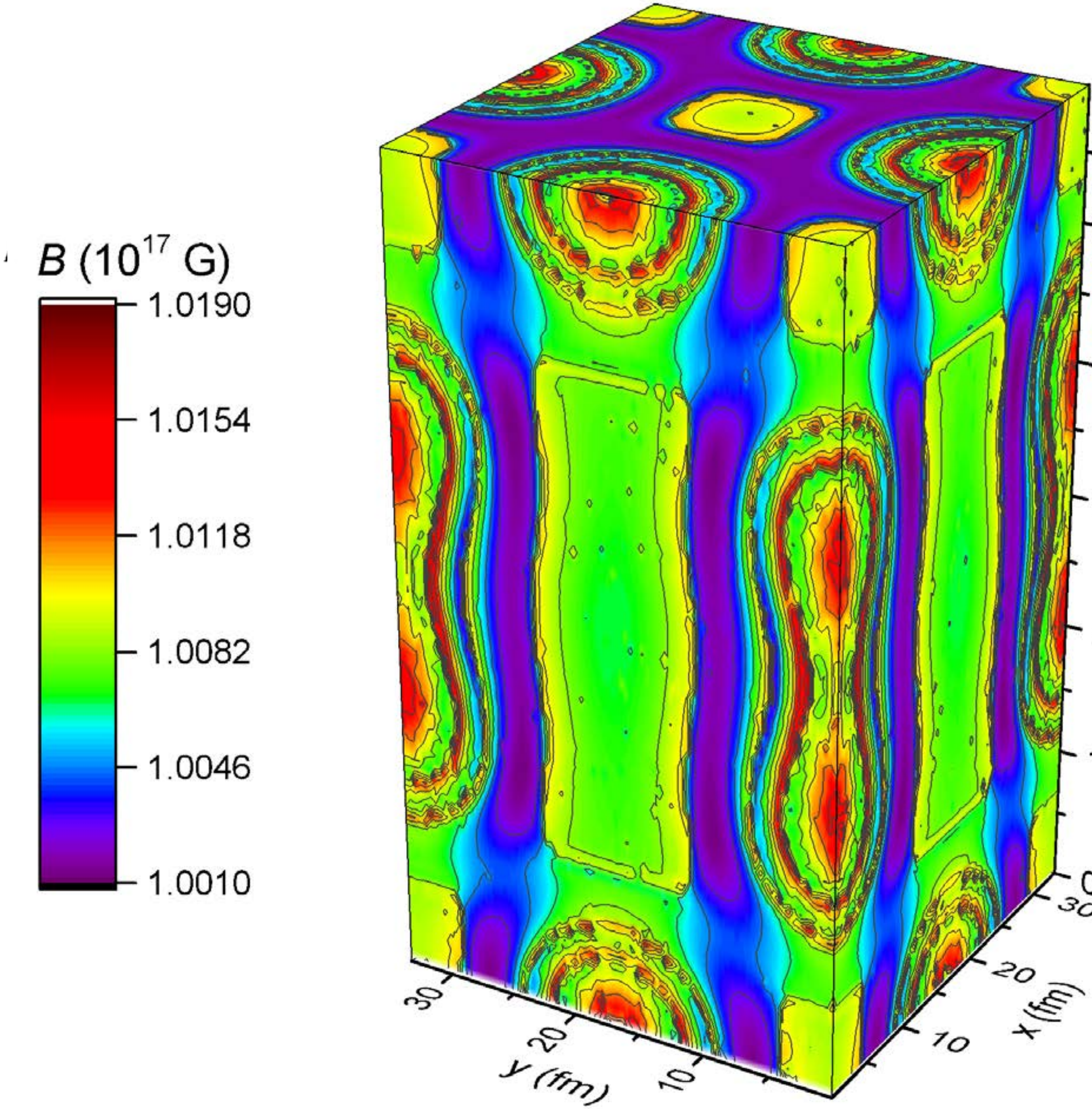
Energy per baryon

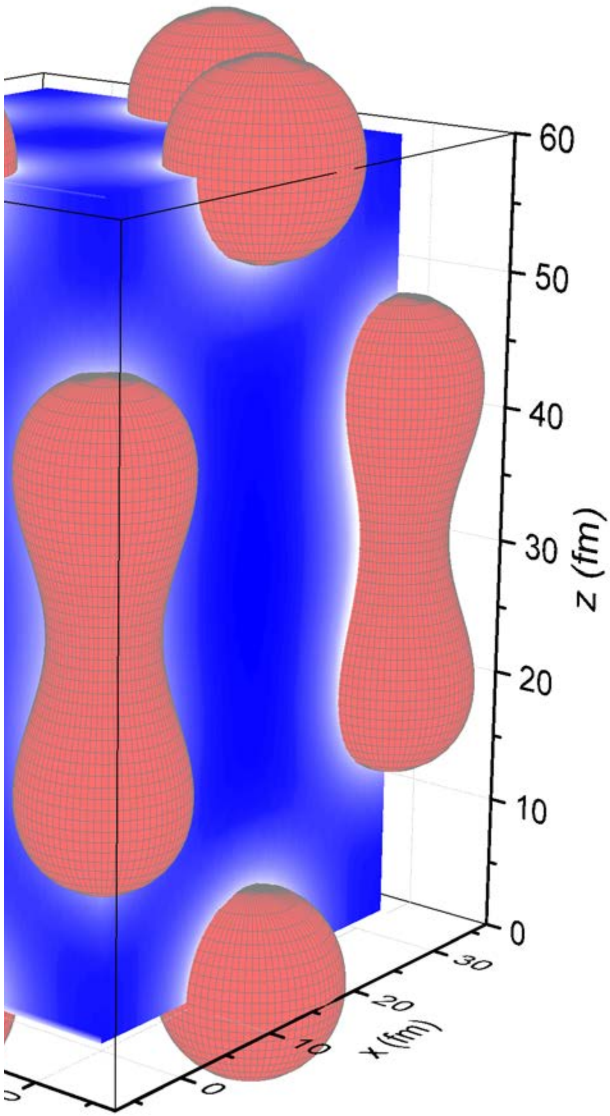


3D calculation at ρ = 0.01 fm^{-3}, Y_p= 0.5, and B_0 = 10^{17} G



3D calculation at ρ = 0.016 fm^{-3}, Y_p= 0.5, and B_0 = 10^{17} G





Summary

--- The EOS of mixed phase

Maxwell construction applies only to the most simple cases with a single chemical component.

Nuclear matter in compact stars:

- chemically composite →Maxwell const. not satisfy Gibbs cond.
- charged phases → balance between Coulomb & surface tension
  →geometrical structure (pasta)
  → different from bulk Gibbs calculation
- Possibility of strong magnetic field

Our goal: inhomogeneous matter

- from low-density nuclear matter to high-density quark matter
- fully 3-dimensional structure
- with and without strong magnetic field

Numerical procedure

Wigner-Seitz approx is often used. But we have performed 3D calc.

To **equilibrate** $\mu_i(r)$ **in** ***r*** and **among species** ***i***

$$\left.\begin{array}{ll}\text{remove } r\text{–dependence} & \mu_i(r) = \mu_i \\ \text{satisfy chemical balances} & \mu_n = \mu_p + \mu_e\end{array}\right\} \#$$

- Divide whole space into equivalent and neutral cubic cells with **periodic boundary conditions**
- Distribute fermions (p, n, e) **randomly** but $\int d^3 r\, \rho_i(r) = \text{given}$
- Solve field equations for $\sigma(r), \omega_0(r), \rho_0(r), V_{\mathrm{Coul}}(r)$
- Calculate local chemical potentials of fermions $\mu_i(r)$

$$\mu_i(r) = V_i(r) + \sqrt{{m_i}^2 + {p_F}_i(r)^2}$$

- Adjust densities $\rho_i(r)$ as

$$\mu_i(r_1) > \mu_i(r_2) \quad \rightarrow \quad \rho_i(r_1)\downarrow,\ \rho_i(r_2)\uparrow$$

$$\mu_n(r) > \mu_p(r) + \mu_e \quad \rightarrow \quad \rho_n(r)\downarrow,\ \rho_p(r)\uparrow \quad \text{beta equil.}$$

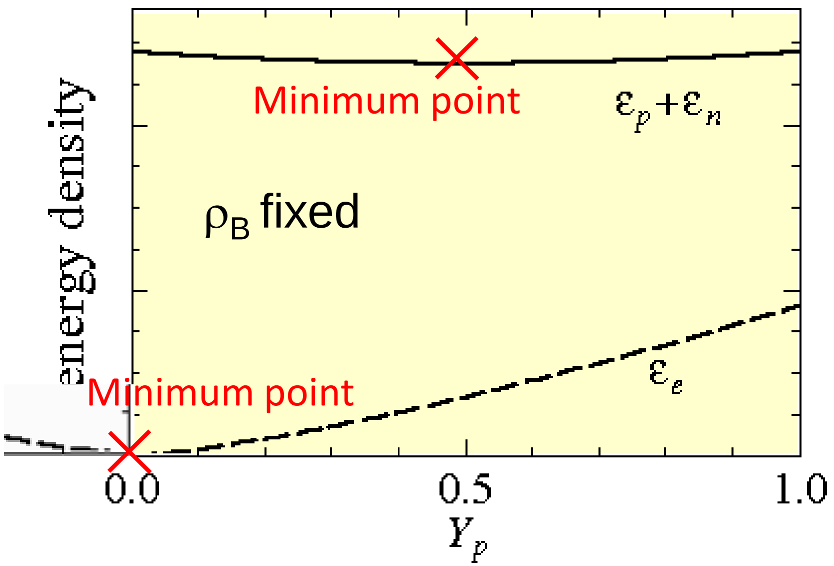
- repeat until #

Uniform electron, $T=0$

$$\rho_e = 2\frac{4\pi\left(p_{Fe}/2\pi\hbar\right)^3}{3} = \frac{p_{Fe}{}^3}{3\pi^2\hbar^3} = \frac{\mu_e{}^3}{3\pi^2\hbar^3}$$

$$\mu_e = p_{Fe} = \left(3\pi^2\rho_e\right)^{1/3}\hbar \quad \text{chemical potential}$$

$$\mathcal{E}_e = \int_0^{p_F}\frac{d^3p}{(2\pi)^3}p = \frac{p_{Fe}^4}{4\pi^2} = \frac{\left(3\pi^2\rho_e\right)^{4/3}}{4\pi^2} \quad \text{energy density}$$



Uniform nucleon, $T=0$

$$\rho_N = 2\frac{4\pi\left(p_{FN}/2\pi\hbar\right)^3}{3} = \frac{p_{FN}{}^3}{3\pi^2\hbar^3} \qquad (N=p,n)$$

$$\mu_N = \sqrt{p_{FN}{}^2+m_N^2}+U_N = \sqrt{\left(3\pi^2\hbar^3\rho_N\right)^{2/3}+m_N^2}+U_N \quad \text{chemical potential}$$

$$\mathcal{E}_N = 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[\sqrt{p^2+m_N^2}+U_N\right] \quad \text{energy density}$$

$$\approx 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[m_N+\frac{p^2}{2m_N}+U_N\right] \quad \text{non-relativistic approx}$$

$$= (m_N+U_N)\boldsymbol{\rho_N} + \frac{\left(3\pi^2\rho_N\right)^{5/3}\hbar^2}{10\pi^2\boldsymbol{m_N}} \approx (m_N+U_N(Y_p))\boldsymbol{\rho_N}$$

$\varepsilon_p+\varepsilon_n$ has minimum at $Y_p=0.5$.
But $\varepsilon=\varepsilon_p+\varepsilon_n+\varepsilon_e$ has minimum at $0<Y_p<0.5$. ➔ neutron-rich.
If symmetry energy is larger, Y_p is closer to 0.5.

Uniform electron, $T = 0$

$$\rho_e = 2\frac{4\pi\left(p_{Fe}/2\pi\hbar\right)^3}{3} = \frac{p_{Fe}{}^3}{3\pi^2\hbar^3} = \frac{\mu_e{}^3}{3\pi^2\hbar^3}$$

$$\mu_e = p_{Fe} = \left(3\pi^2\rho_e\right)^{1/3}\hbar \quad \text{chemical potential}$$

$$\varepsilon_e = \int_0^{p_F}\frac{d^3p}{(2\pi)^3}\,p = \frac{p_{Fe}^4}{4\pi^2} = \frac{\left(3\pi^2\rho_e\right)^{4/3}}{4\pi^2} \quad \text{energy density}$$

Uniform nucleon, $T = 0$

$$\rho_N = 2\frac{4\pi\left(p_{FN}/2\pi\hbar\right)^3}{3} = \frac{p_{FN}{}^3}{3\pi^2\hbar^3} \qquad (N = p, n)$$

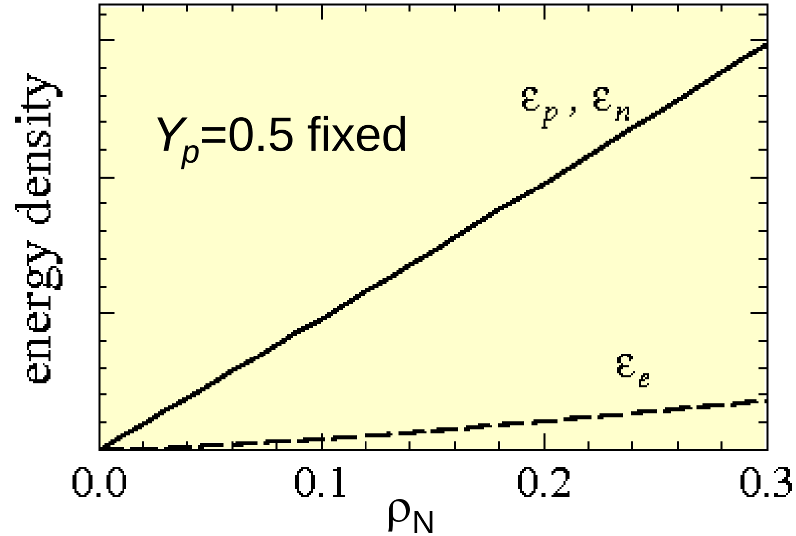
$$\mu_N = \sqrt{p_{FN}{}^2 + m_N^2} + U_N = \sqrt{\left(3\pi^2\hbar^3\rho_N\right)^{2/3} + m_N^2} + U_N \quad \text{chemical potential}$$

$$\varepsilon_N = 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[\sqrt{p^2 + m_N^2} + U_N\right] \quad \text{energy density}$$

$$\approx 2\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}\left[m_N + \frac{p^2}{2m_N} + U_N\right] \quad \text{non-relativistic approx}$$

$$= (m_N + U_N)\boldsymbol{\rho_N} + \frac{\left(3\pi^2\rho_N\right)^{5/3}\hbar^2}{10\pi^2\boldsymbol{m_N}} \approx (m_N + U_N(Y_p))\boldsymbol{\rho_N}$$

$$\approx m_N\rho_N.$$



Due to the linear dependence on the density,

ε_N increases more rapidly than ε_e.

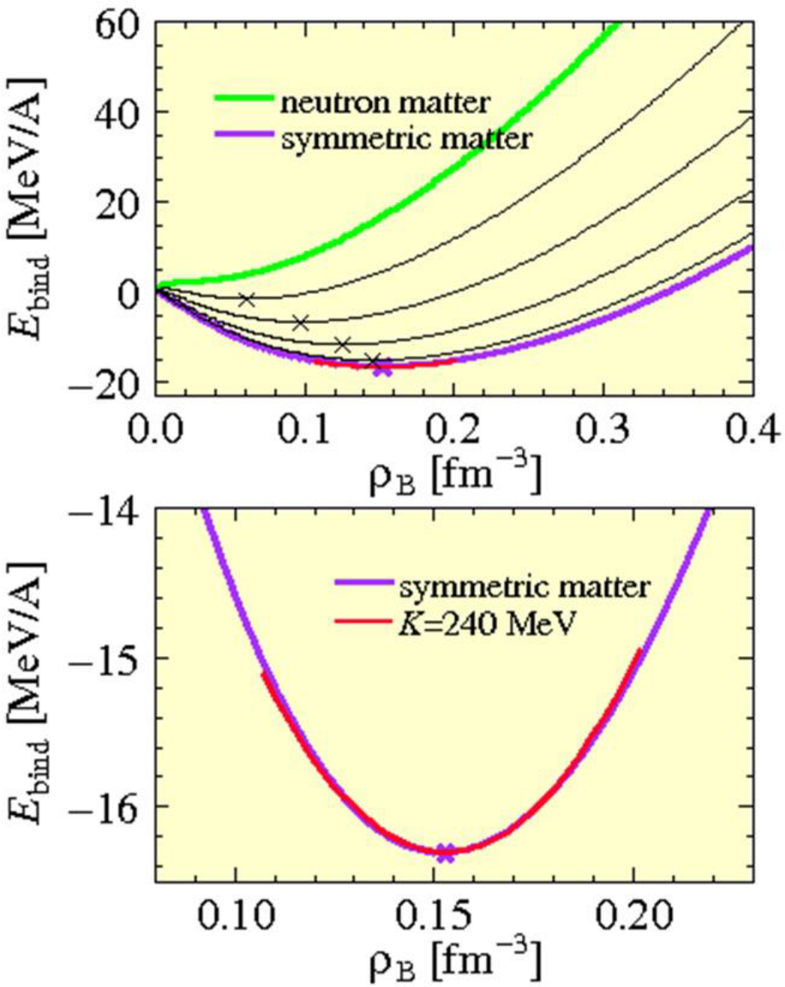
$$\varepsilon = \varepsilon_p + \varepsilon_n + \varepsilon_e$$
$$\approx m_N\rho_B + C\rho_e^{4/3}$$

At low densities, nucleons are stiffer than electron.

→ With increase of density, proton fraction increases.

0. basic properties of nuclear matter

Symmetric nuclear matter



Example by a RMF model:
Minimum energy at density $\rho = \rho_0$
with proton fraction $Y_p = 0.5$

Stiffness (incompressibility)

$$K = p_F^2 \frac{d^2\varepsilon}{dp_F^2} = 9\rho^2 \frac{d^2\varepsilon}{d\rho^2} = 9\frac{dP}{d\rho}$$

is important but not fixed yet.

Beta-equilibrium nuclear matter

Realistic macroscopic matter is neutral & beta-eq.

$$n \leftrightarrow p + e^- + \bar{\nu}$$

In the case of simple *npe* matter,

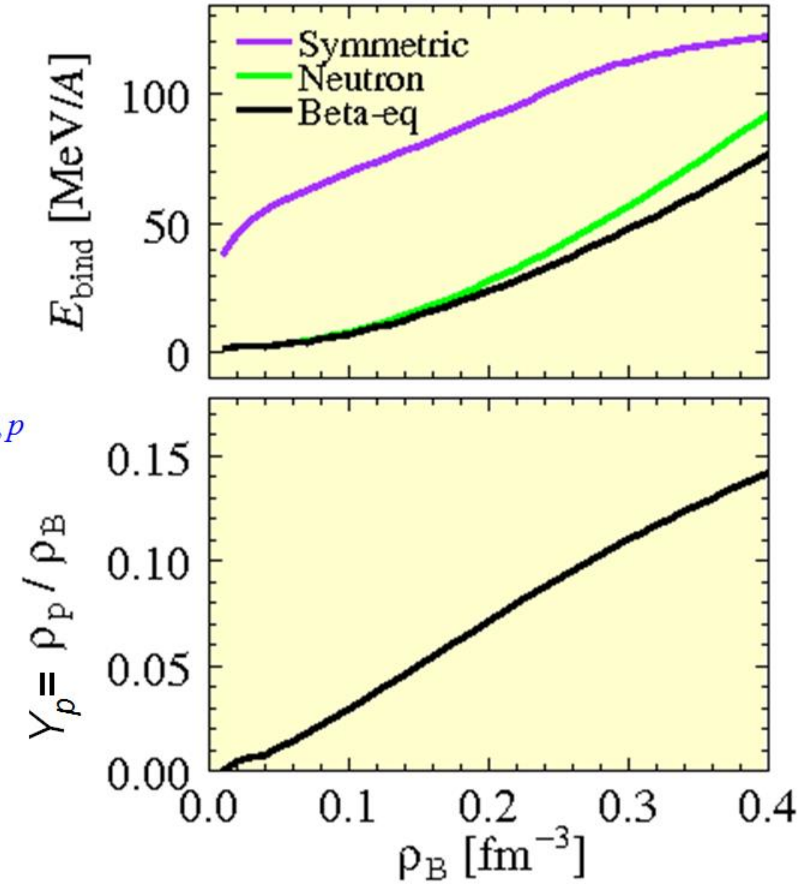
$$\rho_p = \rho_e \quad (荷電中性)$$

$$\mu_n = \mu_p + \mu_e \quad (ベータ平衡:\ n \leftrightarrow p + e + \nu)$$

$$\mu_\nu \approx 0$$

$$\mu_{n,p} = \sqrt{p_{F(n,p)}^2 + m^2} + U_{n,p} = \sqrt{(3\pi^3 \rho_{n,p})^{2/3} + m^2} + U_{n,p}$$

$$\mu_e = \left(3\pi^3 \rho_e\right)^{1/3}$$



No "saturation" due to electrons which are necessary for the charge neutrality.

e^- energy density $\varepsilon_e = (9\pi\rho_e^2 / 8)^{2/3}$

proton fraction of beta-eq. matter is small and monotonically increasing function of density if matter is uniform.

2. high-density nuclear matter
(Neutron star core)

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Kaon condensation

From a Lagrangian with chiral symmetry

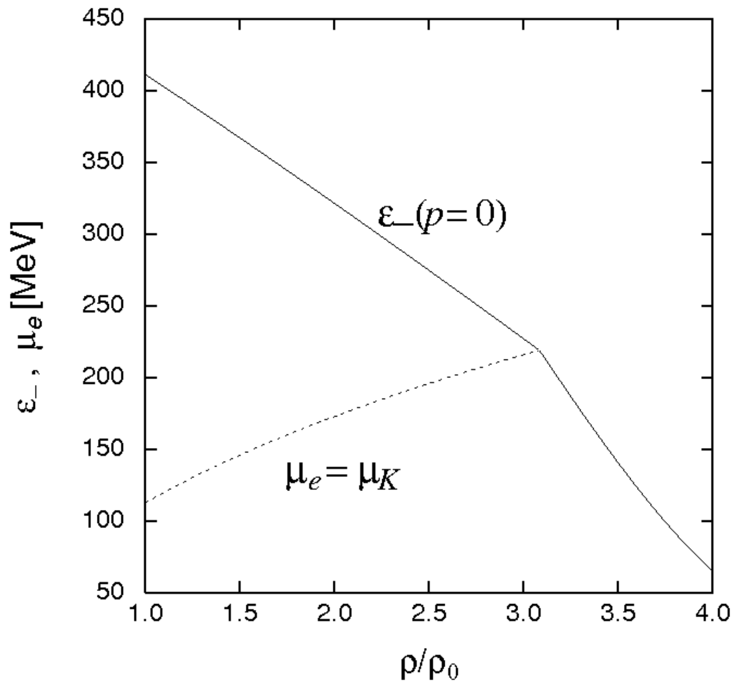
K single particle energy (model-independent form)

$$\varepsilon_{\pm}(\mathbf{p}) = \sqrt{p^2 + m_K^{*\,2} + \left(\left(\rho_n + 2\rho_p\right)/4f^2\right)^2} \pm \left(\rho_n + 2\rho_p\right)/4f^2,$$

$$m_K^{*\,2} = m_K{}^2 - \Sigma_{KN}\left(\rho_n + 2\rho_p\right)/4f^2,$$

$$\mu_K = \varepsilon_{-}(p=0) = \mu_n - \mu_p = \mu_e$$

(Threshold condition of condensation)



3. high-density hadron-quark mixed phase (NS core)

Quark-hadron mixed phase

$$\mu_u + \mu_e = \mu_d = \mu_s, \quad \mu_n = \mu_u + 2\mu_d, \quad \mu_p + \mu_e = \mu_n = \mu_\Lambda = \mu_\Sigma - \mu_e$$

$$\mu_i = \frac{\partial \varepsilon(\mathbf{r})}{\partial \rho_i(\mathbf{r})} \quad (i = u, d, s, p, n, \Lambda, \Sigma^-, e)$$

$$\varepsilon(\mathbf{r}) \equiv \varepsilon_B(\mathbf{r}) + \varepsilon_e(\mathbf{r}) + \left(\nabla V_C(\mathbf{r})\right)^2 / 8\pi e^2$$

$$\varepsilon_B(\mathbf{r}) = \begin{cases} \varepsilon_H(\mathbf{r}) & \text{(hadron phase : BHF)} \\ \varepsilon_Q(\mathbf{r}) & \text{(quark phase : MIT bag)} \end{cases}$$

$$\varepsilon_e(\mathbf{r}) = \left(3\pi^2 \rho_e(\mathbf{r})\right)^{4/3} / 4\pi^2$$

$$E/A = \frac{1}{\rho_B V}\left[\int_V d^3r\, \varepsilon(\mathbf{r}) + \tau S\right] \quad \begin{pmatrix} \rho_B = \text{average baryon density} \\ S = \text{Q - H boundary area} \\ V = \text{cell volume} \end{pmatrix}$$

$$\int_V d^3r \left[\rho_p(\mathbf{r}) - \rho_\Sigma(\mathbf{r}) + \frac{2}{3}\rho_u(\mathbf{r}) - \frac{1}{3}\rho_d(\mathbf{r}) - \frac{1}{3}\rho_s(\mathbf{r}) - \rho_e(\mathbf{r})\right] = 0 \quad \text{(total charge)}$$

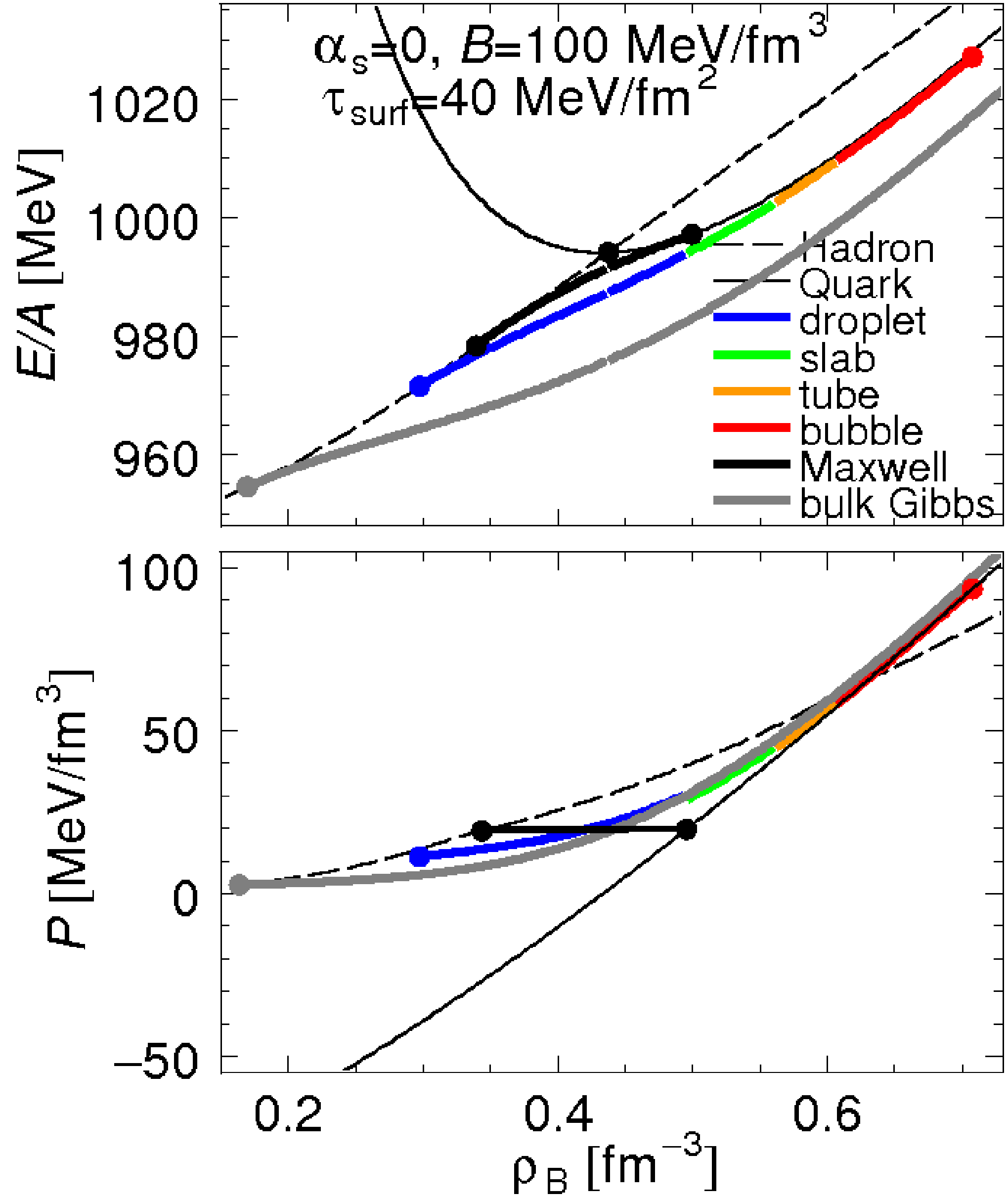
$$\frac{1}{V}\int_V d^3r \left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r}) + \rho_\Lambda(\mathbf{r}) + \rho_\Sigma(\mathbf{r}) + \frac{1}{3}\rho_u(\mathbf{r}) + \frac{1}{3}\rho_d(\mathbf{r}) + \frac{1}{3}\rho_s(\mathbf{r})\right] = \rho_B \quad \text{(given)}$$

EOS of matter

Full calculation is between the Maxwell construction (local charge neutral) and the bulk Gibbs calculation (neglects the surface and Coulomb).

Surface tension stronger
→ closer to the Maxwell.

→ N.Yasutake's talk



Structure of compact stars

TOV equation

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

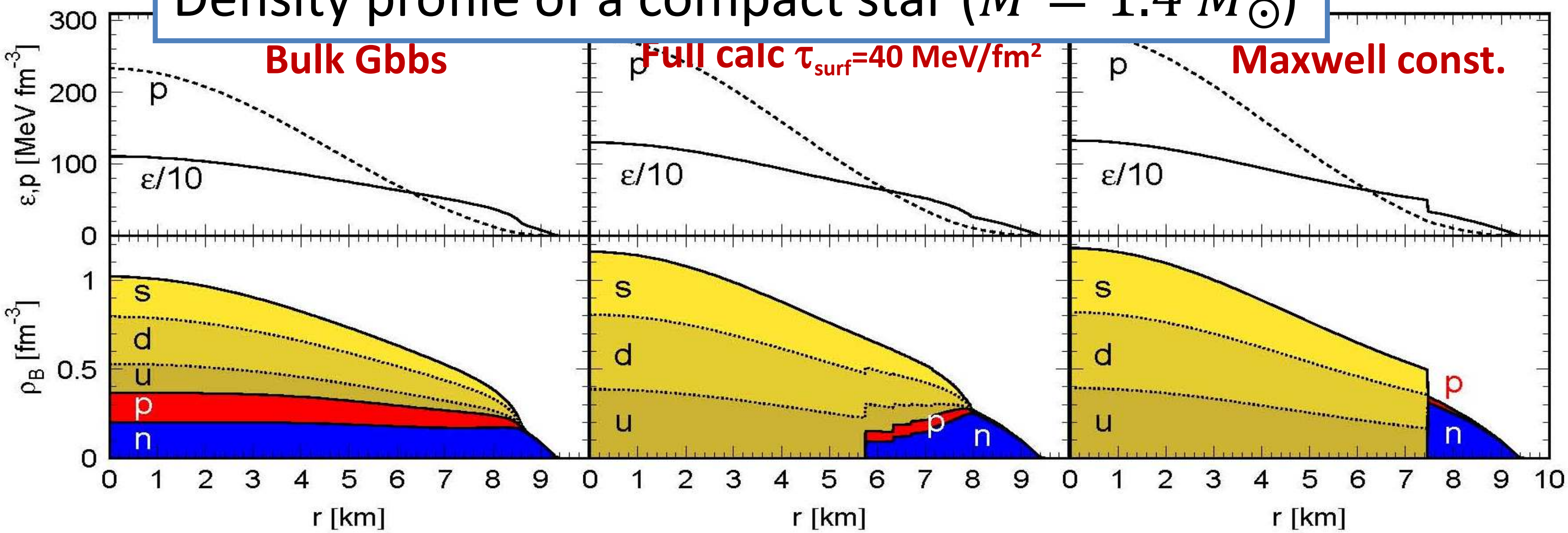
$P = P(\rho)$ Pressure (input of TOV eq.)

$\rho = \rho(r)$ Density at position r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ mass inside the position r

$M = m(R), \quad R = R(\rho \approx 0)$ total mass and radius.

Density profile of a compact star ($M = 1.4\ M_\odot$)



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EOM for fields (RMF model)

Kaon field $K(\mathbf{r})$ added.

$$\nabla^2\sigma = m_\sigma^2\sigma + \frac{dU}{d\sigma} - g_{\sigma N}\left(\rho_n^s + \rho_p^s\right) - 4g_{\sigma K} m_K f_K^2 K^2,$$

$$\nabla^2\omega_0 = m_\omega^2\omega_0 - g_{\omega N}\left(\rho_n + \rho_p\right) - 2g_{\omega K} m_K f_K^2 K^2\left(\mu_K - V_{\text{Coul}} + g_{\omega K}\omega_0 + g_{\rho K}R_0\right),$$

$$\nabla^2 R_0 = m_\rho^2 R_0 - g_{\rho N}\left(\rho_n - \rho_p\right) - 2g_{\rho K} m_K f_K^2 K^2\left(\mu_K - V_{\text{Coul}} + g_{\omega K}\omega_0 + g_{\rho K}R_0\right),$$

$$\nabla^2 K = \left[m_K^{*\,2} - \left(\mu_K - V_{\text{Coul}} + g_{\omega K}\omega_0 + g_{\rho K}R_0\right)^2\right]K,$$

$$\nabla^2 V_{\text{Coul}} = 4\pi e^2 \rho_{\text{ch}}, \qquad \rho_{\text{ch}} = \rho_p - \rho_e - \rho_K,$$

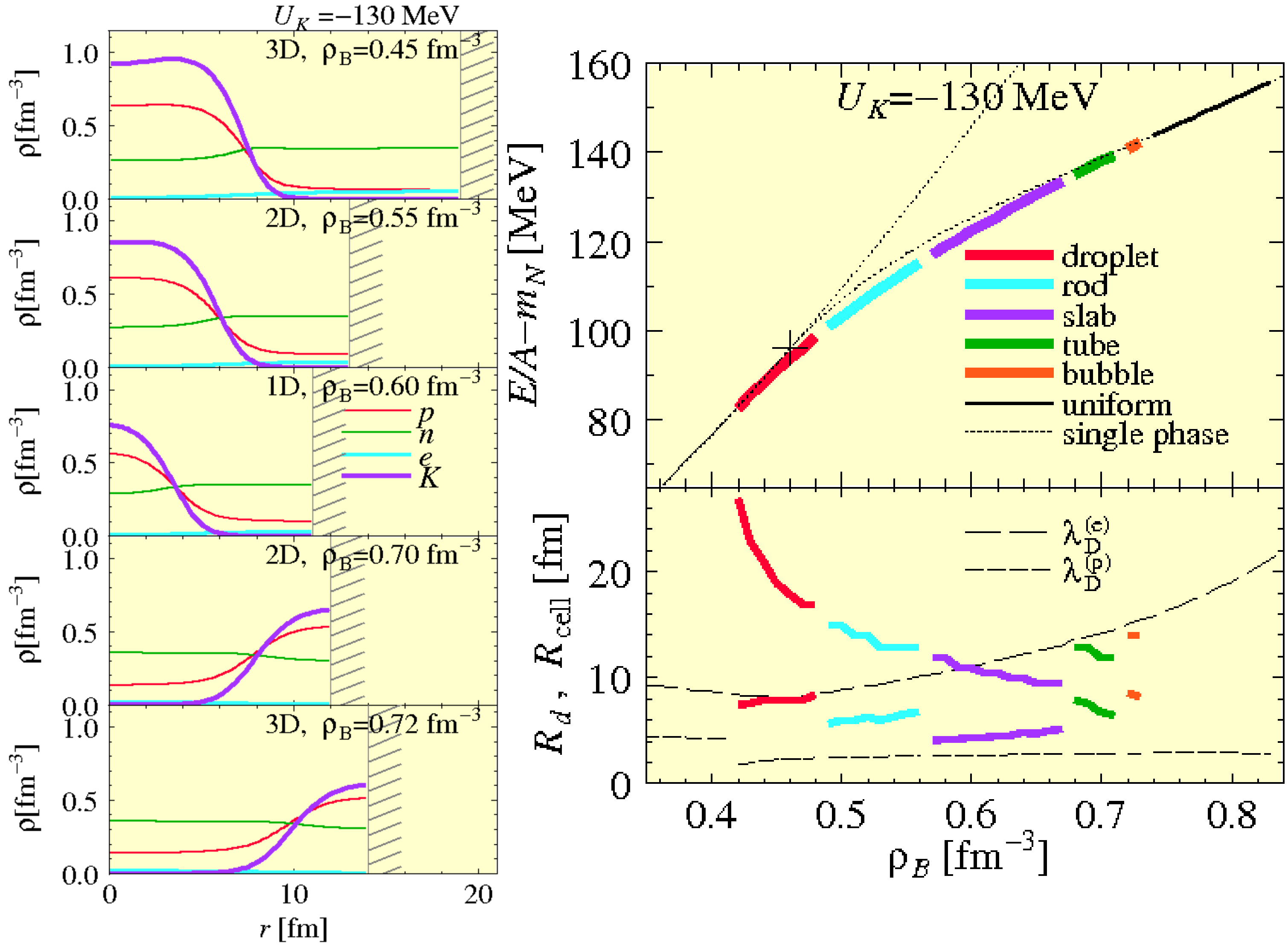
$$\rho_K = 2\left(\mu_K - V_{\text{Coul}} + g_{\omega K}\omega_0 + g_{\rho K}R_0\right)K^2,$$

$$\mu_e = \left(3\pi\rho_e\right)^{1/3} + V_{\text{Coul}},$$

$$\mu_n = \sqrt{k_{F,n}^{\ 2} + m_N^{*\,2}} + g_{\omega N}\omega_0 - g_{\rho N}R_0,$$

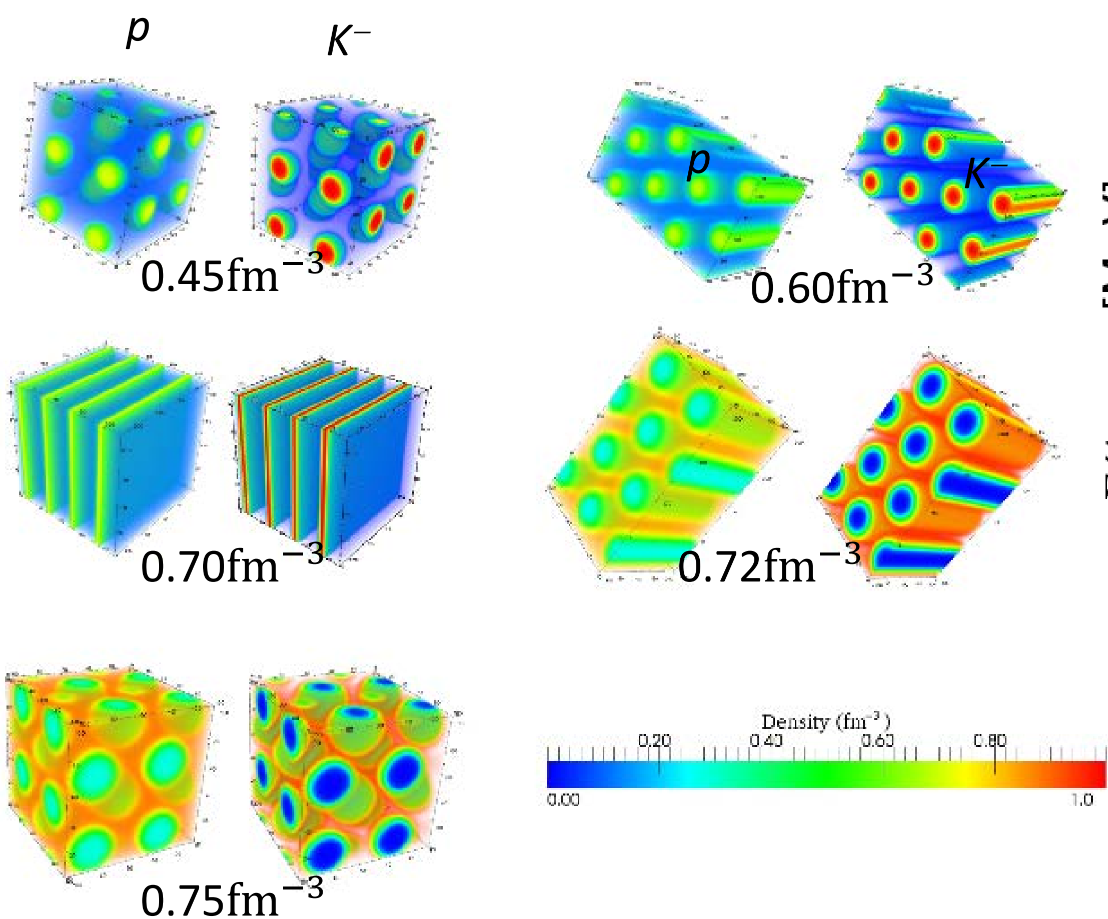
$$\mu_p = \sqrt{k_{F,p}^{\ 2} + m_N^{*\,2}} + g_{\omega N}\omega_0 + g_{\rho N}R_0 - V_{\text{Coul}},$$

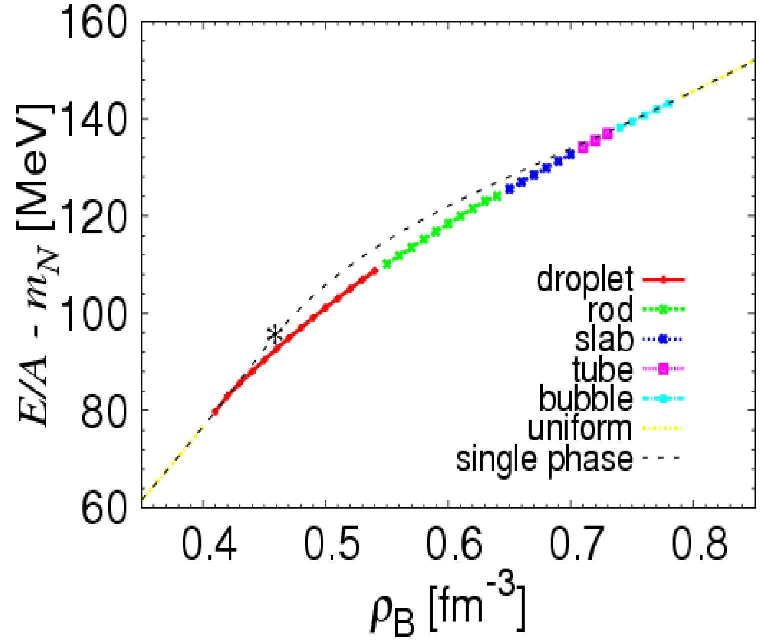
Kaonic pasta structure



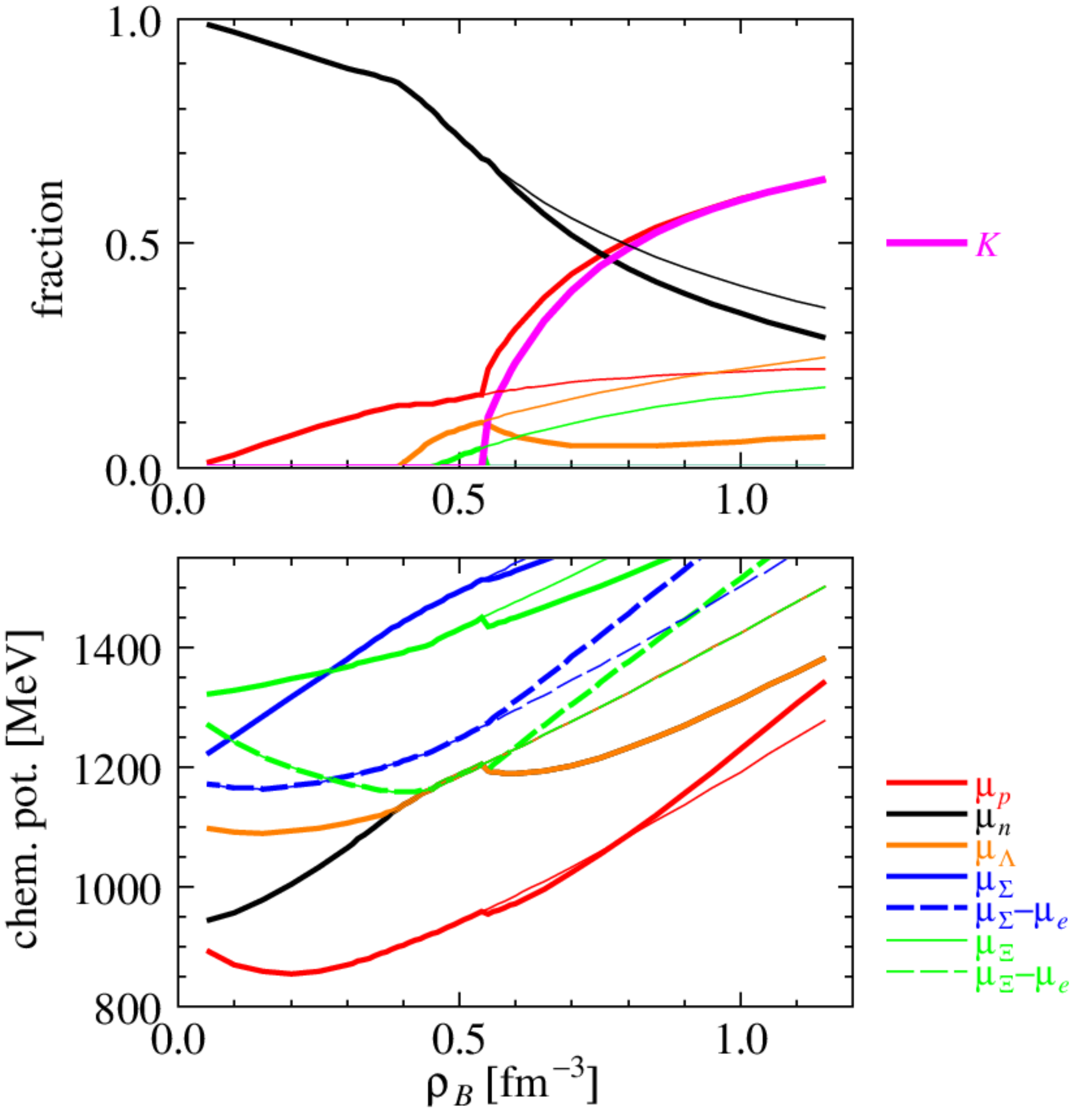
Fully 3D calculation

[unpublished yet]



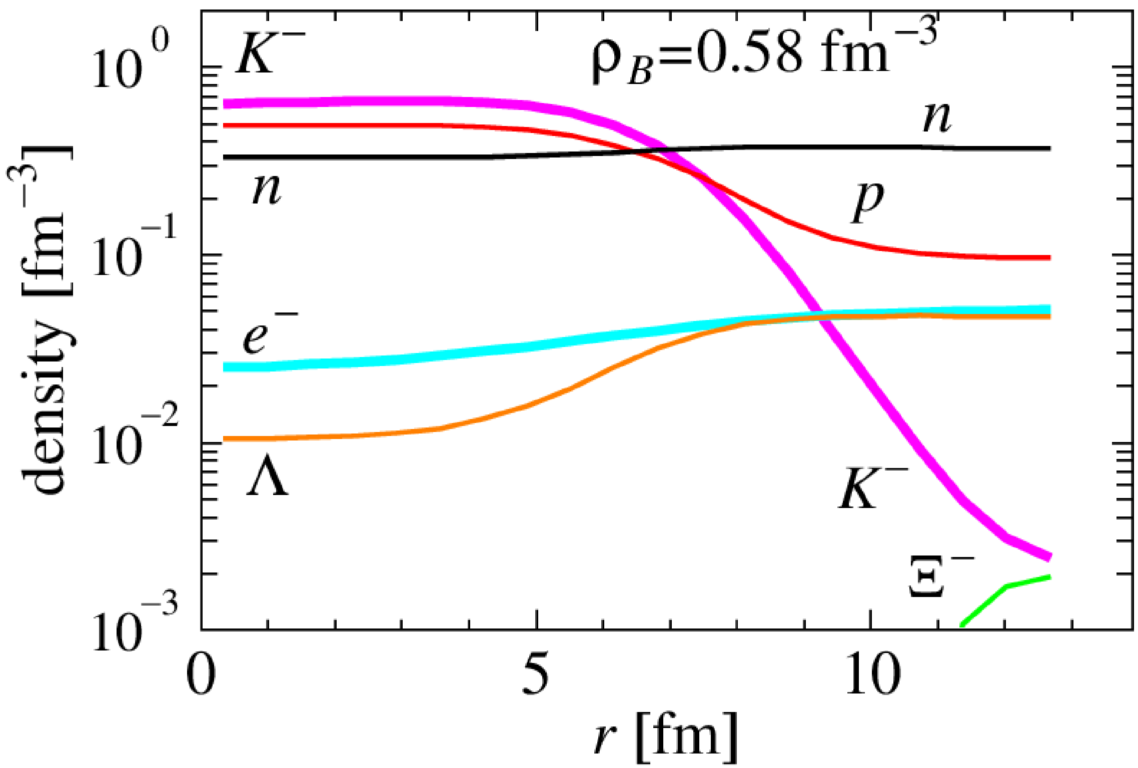


Kaon vs hyperon



Without kaon, with the present parameter set, first appears Λ and then Ξ^- in the case of uniform.

By the appearance of Kaon, Ξ^- disappears and Λ decreases.



Density profile in a WS cell with hyperons and kaons.
Segregation of kaons and hyperons, and attractive behavior between protons are seen.