

QCD vacuum and nuclear matter

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Contents

- **Motivations/backgrounds**
- **Chiral quarks in Savvidy vacuum**
- **Discussion**

Motivation 0

- Any remnants of QCD vacuum/confinement in dense matter or finite nuclei?
- QM and NM on the same footing?

Motivation I

A simple holographic QCD model study has claimed that a typical scale of QCD changes in nuclei.

A	$1/z_m$
20	72.8 MeV
30	77.5 MeV
50	79.0 MeV
70	78.5 MeV
100	77.0 MeV

hQCD is macroscopic description of something!

$$1/z_m \sim 320 \text{ MeV}$$

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_{1/2})/a}}$$

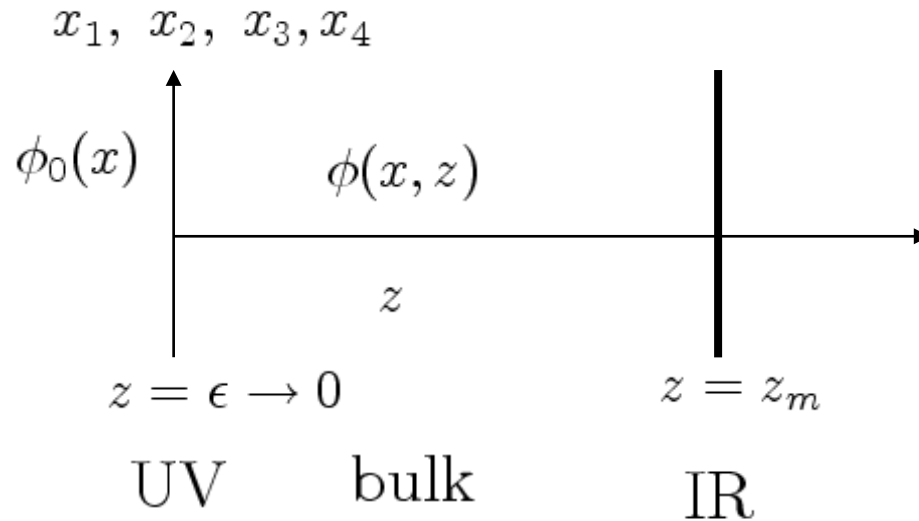
Reminder: Holographic QCD

4D generating functional : $Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i \int \phi_0(x)\mathcal{O}\}$,
5D (classical) effective action : $\Gamma_5[\phi(x, z) = \phi_0(x)]$; $\phi_0(x) = \phi(x, z = 0)$.

AdS/CFT correspondence : $Z_4 = \Gamma_5$.

$$S_I = \int d^4x dz \sqrt{g} \mathcal{L}_5,$$

$$\mathcal{L}_5 = \text{Tr} \left[-\frac{1}{4g_5^2} (L_{MN} L^{MN} + R_{MN} R^{MN}) + |D_M \Phi|^2 - M_\Phi^2 |\Phi|^2 \right],$$



J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

Motivation II

quarks



3 x 5 grams

proton



1 kilogram

Origin of nucleon mass?

Can nuclear matter and nuclei do anything for this?

Nucleon mass (in the chiral limit) in the linear sigma model

$$\delta\mathcal{L} = -g_\pi \left[(i\bar{\psi}\gamma_5\vec{\tau}\psi) \vec{\pi} + (\bar{\psi}\psi) \sigma \right]$$

$$\langle \sigma \rangle = \sigma_0 = f_\pi$$

$$\langle \pi \rangle = 0$$

$$M_N = g_\pi \sigma_0 = g_\pi f_\pi$$

Parity doublet model in dense matter

Introduce two nucleon fields that transform in a mirror way under chiral transformations:

$$SU_L(2) \times SU(2)_R$$

$$\psi_{1R} \rightarrow R\psi_{1R}, \quad \psi_{1L} \rightarrow L\psi_{1L},$$

$$\psi_{2R} \rightarrow L\psi_{2R}, \quad \psi_{2L} \rightarrow R\psi_{2L}.$$

$$\begin{aligned} & m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\ &= m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L}) \end{aligned}$$

Cold, dense nuclear matter in a SU(2) parity doublet model

$$\begin{aligned}\mathcal{L} = & \bar{\psi}_1 i \not{\partial} \psi_1 + \bar{\psi}_2 i \not{\partial} \psi_2 + m_0 (\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2) \\ & + a \bar{\psi}_1 (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_1 + b \bar{\psi}_2 (\sigma - i \gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_2 \\ & - g_\omega \bar{\psi}_1 \gamma_\mu \omega^\mu \psi_1 - g_\omega \bar{\psi}_2 \gamma_\mu \omega^\mu \psi_2 + \mathcal{L}_M,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_M = & \frac{1}{2} \partial_\mu \sigma^\mu \partial^\mu \sigma_\mu + \frac{1}{2} \partial_\mu \vec{\pi}^\mu \partial^\mu \vec{\pi}_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + g_4^4 (\omega_\mu \omega^\mu)^2 \\ & + \frac{1}{2} \bar{\mu}^2 (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + \epsilon \sigma,\end{aligned}$$

Asymmetric nuclear matter in a parity doublet model with hidden local symmetry

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We construct a model to describe dense hadronic matter at zero and finite temperatures, based on the parity doublet model of DeTar and Kunihiro [C. E. DeTar and T. Kunihiro, *Phys. Rev. D* **39**, 2805 (1989)], including the isosinglet scalar meson σ as well as ρ and ω mesons. We show that, by including a six-point interaction of the σ meson, the model reasonably reproduces the properties of normal nuclear matter with the chiral invariant nucleon mass m_0 in the range from 500 to 900 MeV. Furthermore, we study the phase diagram based on the model, which shows that the value of the chiral condensate drops at the liquid-gas phase transition point and at the chiral phase transition point. We also study asymmetric nuclear matter and find that the first-order phase transition for the liquid-gas phase transition disappears in asymmetric matter and that the critical density for the chiral phase transition at nonzero density becomes smaller for larger asymmetry.

Parity doublet model in relativistic mean field theory

- ✓ Spherical code was provided by Jie Meng (Peking Univ.).
- ✓ Main difference is the behavior of sigma mean field.
- ✓ Revised the code to incorporate the difference.
- ✓ With no Delta baryons

	BE (MeV)			R_C (fm)	
	PDM	PC-PK1	Exp.	PDM	Exp.
^{16}O	8.04	7.96	7.98	2.76	2.70
^{24}O	7.06	7.12	7.04	2.82	—
^{32}Mg	7.83	7.91	7.80	3.14	—
^{38}Si	7.59	7.90	7.89	3.28	—
^{38}Ar	8.51	8.62	8.61	3.39	3.40
^{40}Ca	8.57	8.58	8.55	3.46	3.48
^{48}Ca	8.42	8.65	8.67	3.52	3.48
^{42}Ti	8.16	8.32	8.26	3.58	—
^{58}Ni	8.12	8.69	8.73	3.84	3.78
^{72}Kr	8.22	8.31	8.43	4.11	4.16
^{208}Pb	7.86	7.87	7.87	5.53	5.50

We observed that our results, especially the binding energies, are closest to the experiments when we take $m_0 = 700$ MeV.

Constraint to chiral invariant masses of nucleons from GW170817 in an extended parity doublet model

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We construct nuclear matter based on an extended parity doublet model including four light nucleons, $N(939)$, $N(1440)$, $N(1535)$, and $N(1650)$. We exclude some values of the chiral invariant masses by requiring the saturation properties of normal nuclear matter: saturation density, binding energy, incompressibility, and symmetry energy. We find a further constraint on the chiral invariant masses from the tidal deformability determined by the observation of the gravitational waves from neutron star merger GW170817. Our result shows that the chiral invariant masses are larger than about 600 MeV. We also give some predictions on the symmetry energy and the slope parameters in the high density region, which will be measured in future experiments.

Stable spherically symmetric monopole field background in a pure QCD

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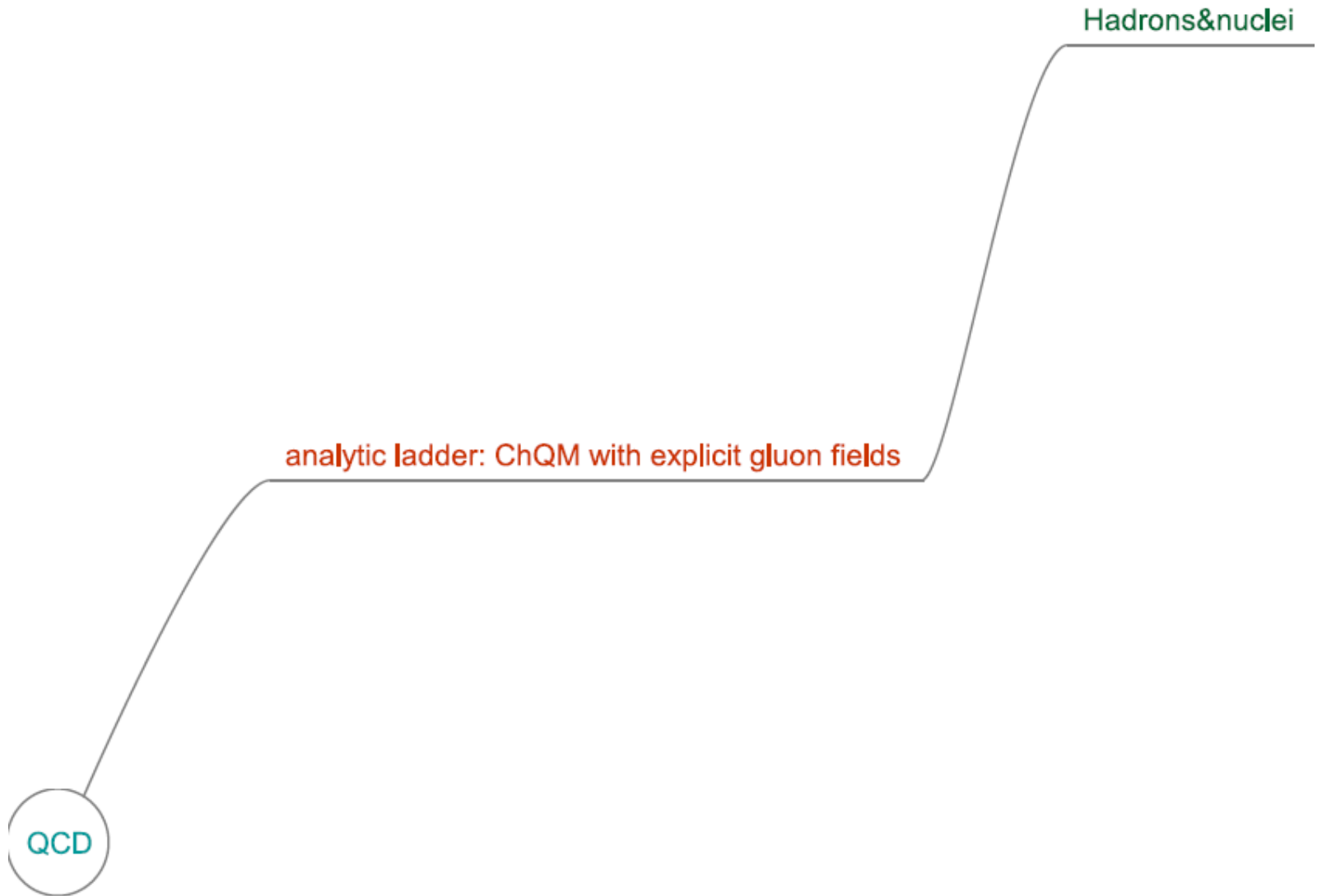
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We consider a stationary spherically symmetric monopole-like solution with a finite energy density in a pure quantum chromodynamics (QCD). The solution can be treated as a static Wu–Yang monopole dressed in a time-dependent field corresponding to off-diagonal gluons. We have proved that such a stationary monopole field represents a background vacuum field of the QCD effective action which is stable against quantum gluon fluctuations. This resolves a long-standing problem of the existence of a stable vacuum field in QCD and opens a new avenue towards a microscopic theory of the vacuum.
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Chiral quarks in Savvidy vacuum



CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

The success of the NR QM?

First of all the quarks should be massive, having constituent mass.

So, we assume that the bulk of the light quark mass is the effect of chiral symmetry breaking.

The leading contribution to the baryon mass is just sum of the constituent quark masses.

$$\begin{aligned} \mathcal{L} = & \bar{\psi} (i\not{D} + V) \psi + g_A \bar{\psi} A \gamma_5 \psi - m \bar{\psi} \psi \\ & + \frac{1}{4} f^2 \text{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma - \frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \dots, \end{aligned}$$

where

$$\begin{aligned} m & \simeq g^2 \frac{\langle \bar{\psi} \psi \rangle}{q^2}, & V_\mu & = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ \xi & = e^{i\pi/f}, & \Sigma & = \xi \xi, & A_\mu & = \frac{1}{2} i (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \end{aligned}$$

$$\pi = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{bmatrix},$$

$$D_\mu = \partial_\mu + igG_\mu$$

Pions are perturbative!

$$(2\pi)^4 \delta^4(\Sigma p_i) \left(\frac{\pi}{f}\right)^A \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{gG_\mu}{\Lambda}\right)^C \left(\frac{p}{\Lambda}\right)^D f^2 \Lambda^2 \left[f^{-2L} (4\pi)^{-2L} \Lambda^{2L} \right].$$

How about gluons?

$$\alpha_s \approx 0.28,$$

which was computed by looking at the ratio of color and electromagnetic hyperfine splittings of the baryon spectrum

- Then no room for the gluon in this model to develop any non-trivial v.e.v?
- No, if the mass was not fully from quark-antiquark condensate.
- After all, Copenhagen (Spagetti) vacuum has also a small gauge coupling constant!

Background gluon field?

Reminder: fermions in external field

$$(i\mathcal{D} - m)\psi = 0.$$

$$\mathcal{D} = \gamma^\mu D_\mu = \gamma^\mu (\partial_\mu + ieA_\mu) = \gamma^0 (\partial_0 + ieA_0) + \vec{\gamma} (\vec{\nabla} - ie\vec{A}).$$

$$\partial_\mu = (\partial_0, \vec{\nabla}), A_\mu = (A_0, -\vec{A}).$$

γ -matrices in Weyl (chiral) representation

$$\gamma^0 = \begin{pmatrix} 0 & -1_2 \\ -1_2 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}.$$

$$A_0 = A_1 = A_3 = 0, A_2 = -Bx_1.$$

$$\psi(x) = \exp(-iEt + ip_2x_2 + ip_3x_3) \begin{pmatrix} f_1(x_1) \\ f_2(x_1) \\ f_3(x_1) \\ f_4(x_1) \end{pmatrix}.$$

$$\begin{pmatrix} -m & 0 & -(E + p_3) & -\xi_- \\ 0 & -m & -\xi_+ & -(E - p_3) \\ -(E - p_3) & \xi_- & -m & 0 \\ \xi_+ & -(E + p_3) & 0 & -m \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = 0,$$

where $\xi_+ = -i\frac{\partial}{\partial x_1} + i(p_2 + eBx_1)$, $\xi_- = -i\frac{\partial}{\partial x_1} - i(p_2 + eBx_1)$.

$$I_{np_2}(x_1) = \left(\frac{eB}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{1}{2}eB\left(x_1 + \frac{p_2}{eB}\right)^2\right) \frac{1}{\sqrt{n!}} H_n\left(\sqrt{2eB}\left(x_1 + \frac{p_2}{eB}\right)\right),$$

where $H_n(x)$ are “probabilistic” Hermite polynomials. Their Rodrigues formula

$$H_n = (-1)^n e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2/2},$$

orthogonality is

$$\int_{-\infty}^{\infty} H_n(x) H_{n'}(x) e^{-x^2/2} dx = \sqrt{2\pi n!} \delta_{nn'}.$$

ξ_{\pm} act to I_{np_2} by following rules:

$$\xi_+ I_{np_2} = i\sqrt{2eB(n+1)} I_{n+1,p_2}, \quad \xi_- I_{np_2} = -i\sqrt{2eBn} I_{n-1,p_2}$$

$$\psi_{1,np_2p_3}^{(+)} = \frac{\exp(-iEt + ip_2x_2 + ip_3x_3)}{\sqrt{2E_n(E_n - p_3)}} \begin{pmatrix} -i\sqrt{2eBn}I_{n-1,p_2}(x_1) \\ (E_n - p_3)I_{np_2}(x_1) \\ 0 \\ -mI_{np_2}(x_1) \end{pmatrix},$$

$$E_n = \sqrt{m^2 + p_3^2 + 2eBn}.$$

Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

$$m_{\text{dyn}} = \bar{\sigma} \sim |eB|^{1/2}.$$

RG analysis of magnetic catalysis in dynamical symmetry breaking

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(Received 25 March 1996)

We perform the renormalization group analysis on dynamical symmetry breaking under a strong external magnetic field studied recently by Gusynin, Miransky, and Shovkovy. We find that any attractive four-Fermi interaction becomes strong at low energy, thus leading to dynamical symmetry breaking. When the four-Fermi interaction is absent, the β function for the electromagnetic coupling vanishes in the leading order in $1/N$. By solving the Schwinger-Dyson equation for the fermion propagator, we show that in the $1/N$ expansion, for any electromagnetic coupling, dynamical symmetry breaking occurs due to the presence of a Landau energy gap by the external magnetic field. [S0556-2821(96)03724-1]

$$k_z \rightarrow s k_z, \quad \omega \rightarrow s \omega \quad \text{with } s < 1,$$

$$\psi_0(k_z, \omega) \rightarrow s^{-3/2} \psi_0(\omega, k_z),$$

$$\psi_n(k_z, \omega) \rightarrow s^{-1} \psi_n(\omega, k_z) \quad \text{for } n > 0.$$

$$m_{\text{dyn}} \sim \sqrt{|eB|} e^{-4\pi^2/Ng(|eB|)}.$$

Fermions at the LLL are 1+1 dimensional !

INFRARED INSTABILITY OF THE VACUUM STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

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A constant chromomagnetic field is a non-trivial classical solution of the SU(2) Yang-Mills equation of motion. The real part of the one-loop vacuum energy in a homogeneous chromomagnetic field is given by

$$\text{Re } \epsilon = \frac{1}{2}H^2 + \frac{11}{48\pi^2}g^2 H^2 \left(\ln \frac{gH}{\Lambda^2} - \frac{1}{2} \right)$$

AN UNSTABLE YANG-MILLS FIELD MODE

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Soon after this interesting finding, a subsequent study showed that the Savvidy vacuum is unstable due to the imaginary part in the one-loop vacuum energy

$$\text{Im } \epsilon = -\frac{(gH)^2}{8\pi^2} .$$

It is straightforward to identify the origin of the instability [4]. For SU(2) Yang-Mills theory, with a specific choice of the constant chromomagnetic field $A_y^3 = Hx$, the eigenvalue of the gluon field, $W_\mu = (A_\mu^1 + A_\mu^2)/\sqrt{2}$ becomes

$$E_n = \sqrt{2gH\left(n + \frac{1}{2}\right) + k_3^2} \pm 2gH .$$

Extension to $SU(3)$ and $SU(4)$ were done in Ref. [5].

It is obvious that the constant magnetic field cannot be the true QCD vacuum because it is unstable and breaks rotational and Lorentz invariance. It is argued that the instability can be removed by the formation of domains.

It was shown in Ref. [6] that the chromomagnetic field has locally a domain-like structure: the field has different orientation in different domains. At long distance the orientations of domains is random so that both gauge and rotational symmetries of the vacuum can be restored in the infrared regime.

The domain structure introduces an infrared cutoff that prevents the momenta from taking the smaller values causing the instability (it is still controversial, though ...)

In Ref. [7], an interesting observation was made that the coupling constant that minimizes the vacuum energy is unexpectedly small $\alpha_s = g^2/4\pi = 0.37$.

- [4] N. K. Nielsen and P. Olesen, Nucl. Phys. B **144**, 376 (1978).
- [5] H. Flyvbjerg, Nucl. Phys. B **176**, 379 (1980).
- [6] H. B. Nielsen and P. Olesen, Nucl. Phys. B **160**, 380 (1979).
- [7] J. Ambjorn and P. Olesen, Nucl. Phys. B **170**, 60 (1980).

As a first step, we consider a simple choice of the constant chromomagnetic field $A_y^3 = Hx$ to calculate the effective action with finite chemical potential for the equation of state and the shift of the quark mass to discuss the chiral invariant nucleon mass, etc.

At least for the evaluation of the effective action, we can use the results from dense matter with external U(1) magnetic field. The difference is that the gluon field has a color matrix T^3 whose eigenvalue is $+1/2, -1/2$.

$$S_F(x'; x|m)_{ab} = \sum_{n=0}^{\infty} \int \frac{d\omega dk_y dk_z}{(2\pi)^3} \exp[-i\omega(t' - t) + ik_y(y' - y) + ik_z(z' - z)] \\ \times \frac{1}{\omega^2 - k_z^2 - m^2 - 2eBn + i\epsilon} S_{ab}(n; \omega, k_y, k_z).$$

The matrix $S(n; \omega, k_y, k_z)$ entering above is given by

$$S(n; \omega, k_y, k_z) \equiv \begin{pmatrix} mI_{n,n} & 0 & -(\omega + k_z)I_{n,n} & -i\sqrt{2eBn}I_{n,n-1} \\ 0 & mI_{n-1,n-1} & i\sqrt{2eBn}I_{n-1,n} & -(\omega - k_z)I_{n-1,n-1} \\ -(\omega - k_z)I_{n,n} & i\sqrt{2eBn}I_{n,n-1} & mI_{n,n} & 0 \\ -i\sqrt{2eBn}I_{n-1,n} & -(\omega + k_z)I_{n-1,n-1} & 0 & mI_{n-1,n-1} \end{pmatrix}$$

$$S_F^{\beta,\mu}(x'; x|m)_{ab} = \sum_{n=0}^{\infty} \int \frac{d\omega dk_y dk_z}{(2\pi)^3} \exp[-i\omega(t' - t) + ik_y(y' - y) + ik_z(z' - z)] \\ \times 2\pi i \delta(\omega^2 - k_z^2 - m^2 - 2eBn) f_F(\omega) S_{ab}(n; \omega, k_y, k_z),$$

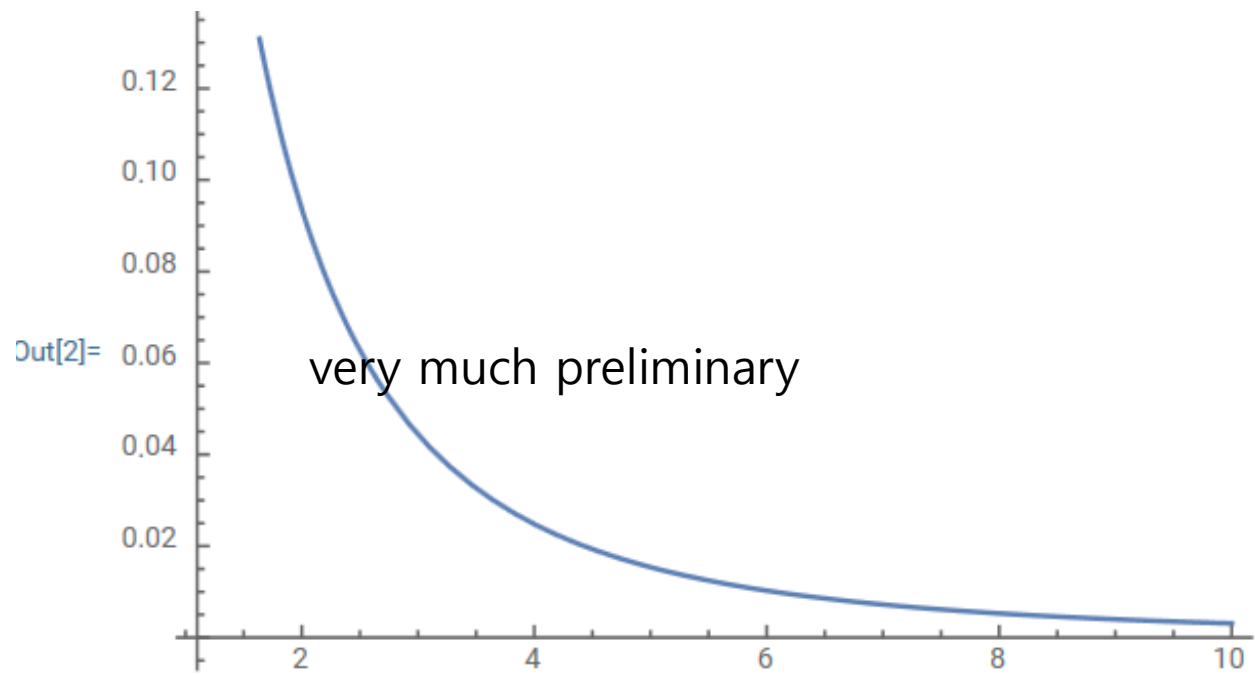
where $f_F(\omega)$ is the thermal distribution

$$f_F(\omega) = \theta(\omega) f_F^+(\omega) + \theta(-\omega) f_F^-(-\omega).$$

P. Elmfors, D. Persson, B.-S. Skagerstam, Astroparticle Physics (1994),

$$\mathcal{L}_{eff} = 2 \times \frac{4gH}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{\lambda=0}^2 \int_{-\infty}^{\infty} d\omega f_F(\omega) \cdot \int_0^{\infty} dk k^2 \delta(\omega^2 - k^2 - 2gH(n + \lambda - 1) - m^2)$$

P. Elmfors, D. Persson, B.-S. Skagerstam, *Astroparticle Physics* (1994),
D. Persson, V. Zeitlin, *Phys. Rev. D* 51 (1995), ...



Qualitative behavior of the symmetry energy from LLL

A proof of relevance of the randomness of the ensemble of chromomagnetic flux tubes to the area law for the spatial Wilson loop [P. Olesen, Nucl. Phys. B 200, 381 (1982)].

The confinement of quarks appears as a natural result of the randomization of the vortex domains: a single quark scatters on the vortices in each domain so that the quark's wave function acquires a random phase which varies from domain to domain. As a result of averaging over the domains, a single quark gets an infinitely large free energy which effectively forbids the existence of isolated quarks [M.N. Chernodub, T. Kalaydzhyan, J. V. Doorselaere, Phys.Rev. D89 (2014) 065021].

So, my guess is that ChQM itself is describing quark matter, while ChQM+ (extended/improved) Savvidy vacuum may represent nuclear matter.

Discussion

- **ChQM + (improved) Savvidy vacuum may give a chance to connect QCD vacuum with real world.**
- **Since the gluon background field ensures confinement, the combined one may also provide a handy platform for QM and NM.**