Gamma-Vortex Generation through Synchrotorn Radiation in Strong Magnetic fields in Relativistic Quantum Approach

Tomoyuki Maruyama **BRS, Nihon University**

Collaborators

***Toshitaka* Kajino** **NaO, Japan**

Takehito Hayakawa

***Myong-Ki* Cheoun** **Soongsil Univ., Korea.**

T.M, T. Hayakawa,.M.K.Cheoun, T.Kajino arXiv : 1908.11545

Quark and Compact Stars 2019

Sep. 26 – 28, Busan (釜山), Korea（大韓民国）

§1 Introduction

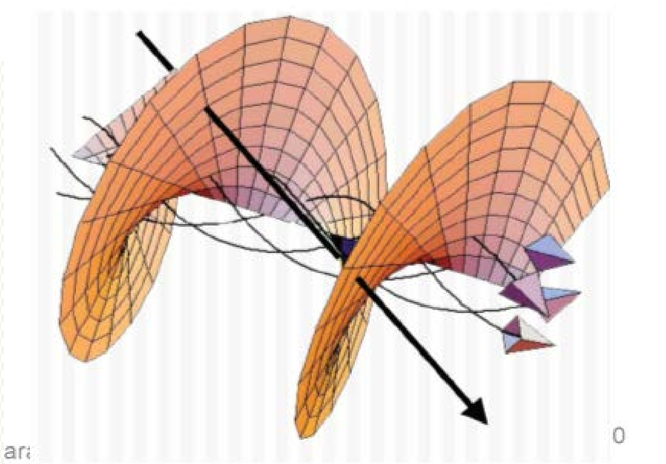
Light Vortex： Light with Orbital Angular Momentum along Beam Direction

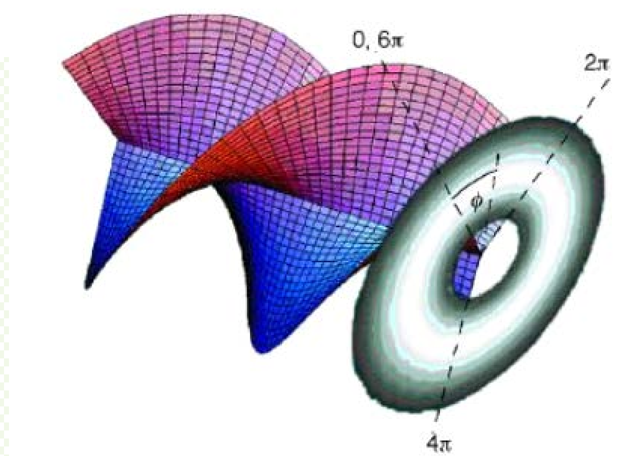
PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

Laser light with a Laguerre-Gaussian amplitude distribution is found to have a well-defined orbital angular momentum. An astigmatic optical system may be used to transform a high-order Laguerre-Gaussian mode into a high-order Hermite-Gaussian mode reversibly. An experiment is proposed to measure the mechanical torque induced by the transfer of orbital angular momentum associated with such a transformation.





Photon Vortex (Twisted Photon)

Higher Harmonic Wave
with Orbital Angular Momentum (OAM) along Beam-Direction
⇒ z-direction

$A(x)$: Solution of Maxwell Eq. ⇔ Solution of Klein Gordon Eq.

⇒ **Wave Function of Photon**

Eigen State of L_z ⇒ Vortex Photon

1) **Bessel Beam (Wave)** **Eigen State of p_z**

$$A(\mathbf{r}) = \epsilon J_L(k_T r_T)\exp(iL\phi + ik_z - iet)$$

$$-\nabla^2 A = \left(-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{L^2}{r^2} + k_z^2\right) A = e^2 A \qquad \left(e^2 = k_T^2 + k_z^2\right)$$

Klein-Gordon Equation

2) **Laguerre Gaussian Beam (Wave)** **Not Eigen State of p_z**

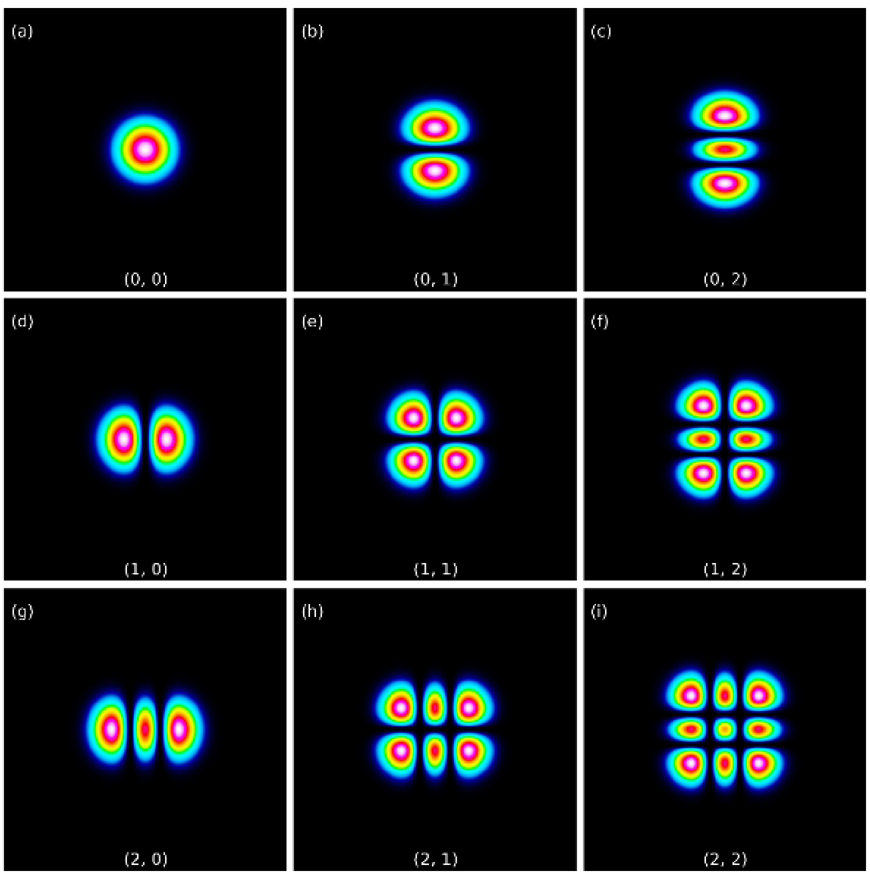
cf. Hermit Gaussian Beam Wave

Papa-Axial Approximation ⇒ Finite Size of Beam Cross-Section

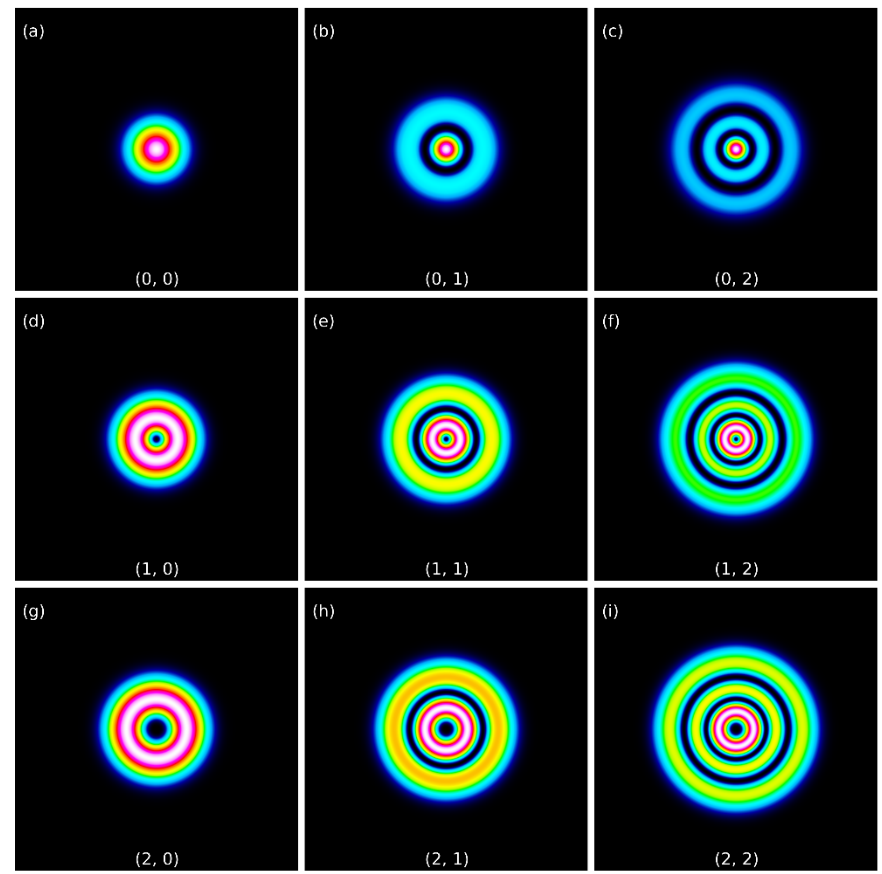
Vortex Light： Light with Orbital Angular Momentum

Electric Field Distribution in Cross-Section

Hermit-Gaussian (HG) Mode



Laguerre-Gaussian (LG) Mode



http://www.dataray.com/blog-m2-high-order-modes.html

Production of Gamma-Ray Vortex

Inverse Compton Scattering low energy photons to ultrarelativistic electrons

U. D. Jentschura, V. G. Serbo
PRL 106, 013001 (2011)

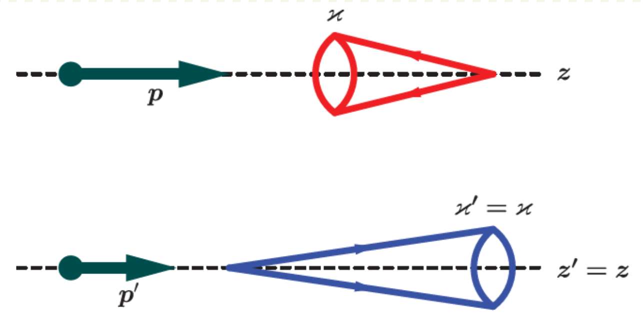
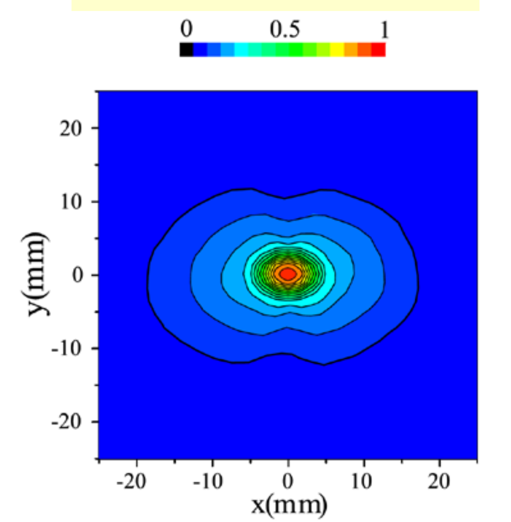


FIG. 2 (color). Initial (above) and final (below) states for the head-on Compton backscattering geometry of a twisted photon.

Initial Photon



U. D. Jentschura, V. G. Serbo
PRL 106, 013001 (2011)

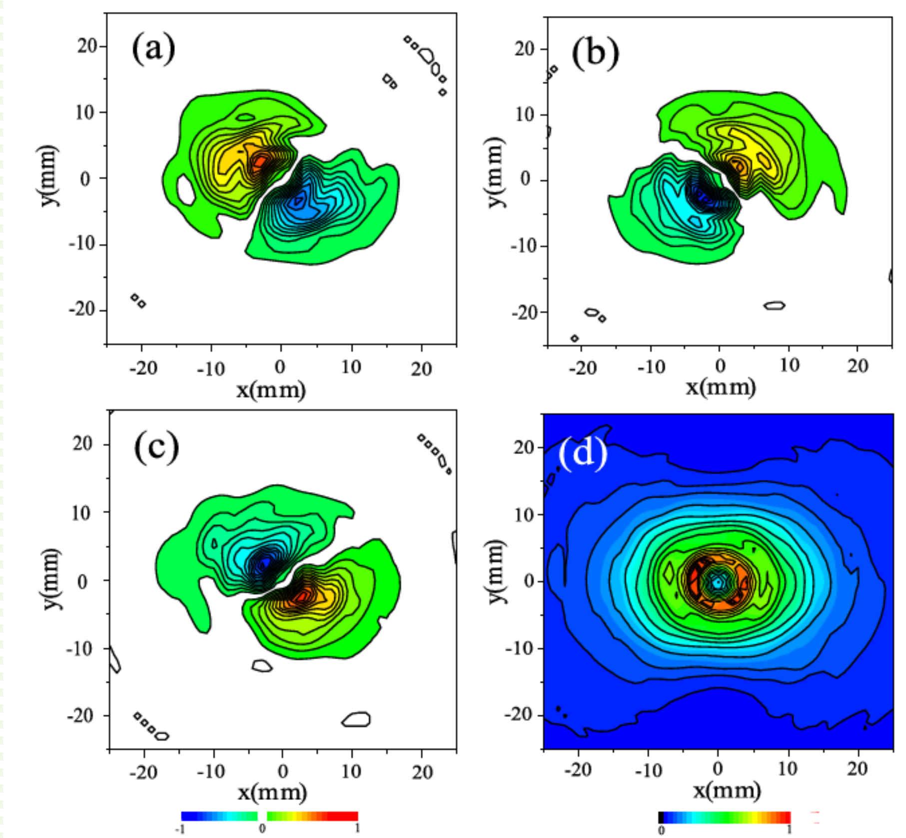
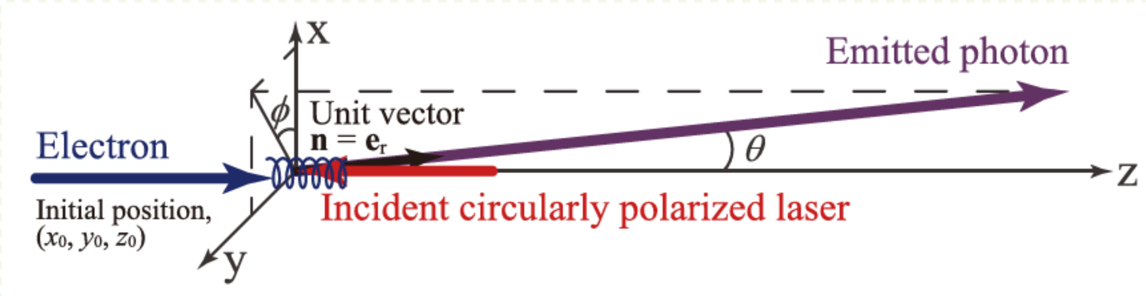


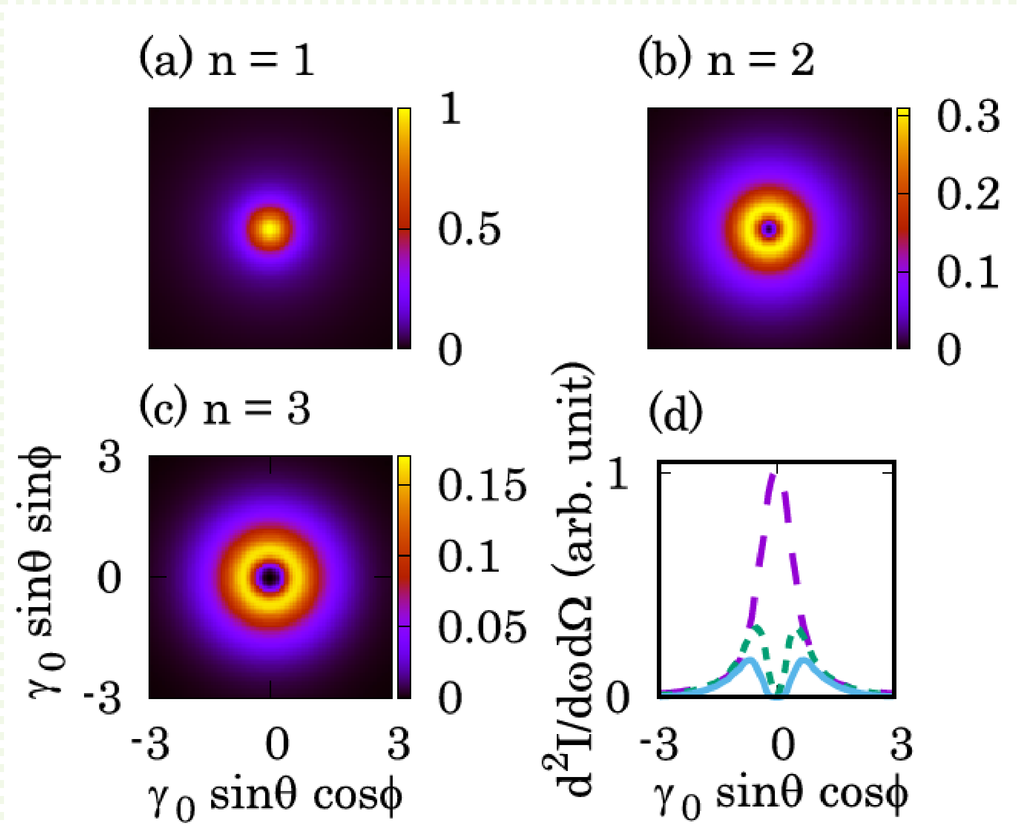
FIG. 4. y component of the electric field $\underline{E}$ of the radiation at different times. (a) $t = 0.2$, (b) $t = 0.4$, (c) $t = 0.6$ ps. (d) average intensity for $m = 1$, $\varepsilon = 1$.

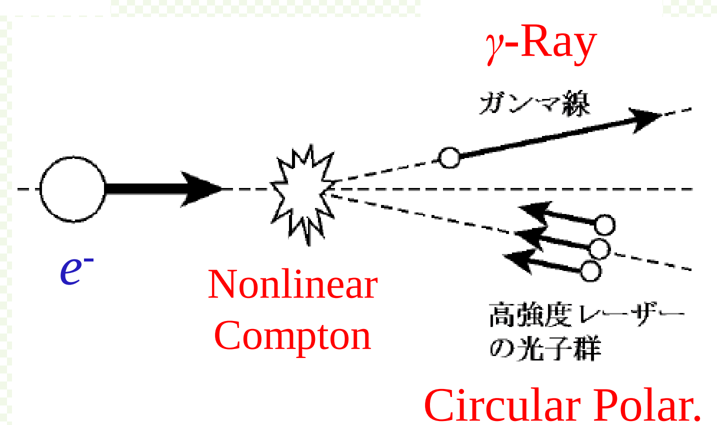
Nonlinear Compton Scatt.

Multiphoton → Single Photon

Y.Taira , T. Hayakawa, M. Katoh, Sci. Rep. 2017







High Intencity Lazer Beam

Short Pulse

Photon with Large OAM

Method to identify Photon Vortex

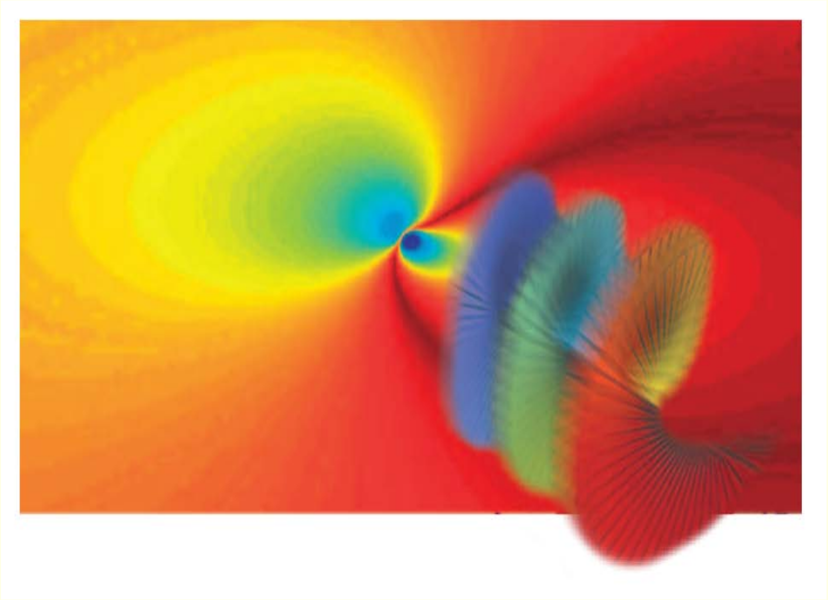
Compton Scattering

T.M., et al, T.Kajino, Sci. Rep. 9, 51 (2019)

Light Vortex Generation in Astronomical System

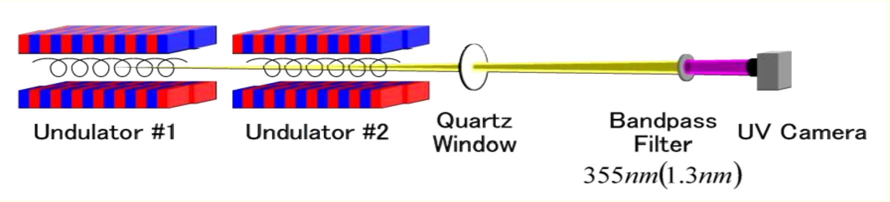
Production in Rotating Black Hole

Fabrizio Tamburini et al.
Nature-Phys., Vol.7, **195 (2011)**



Radiation from Electron rotating in Strong Magnetic Field

M. Katoh et al., PRL 118, 094801 (17)

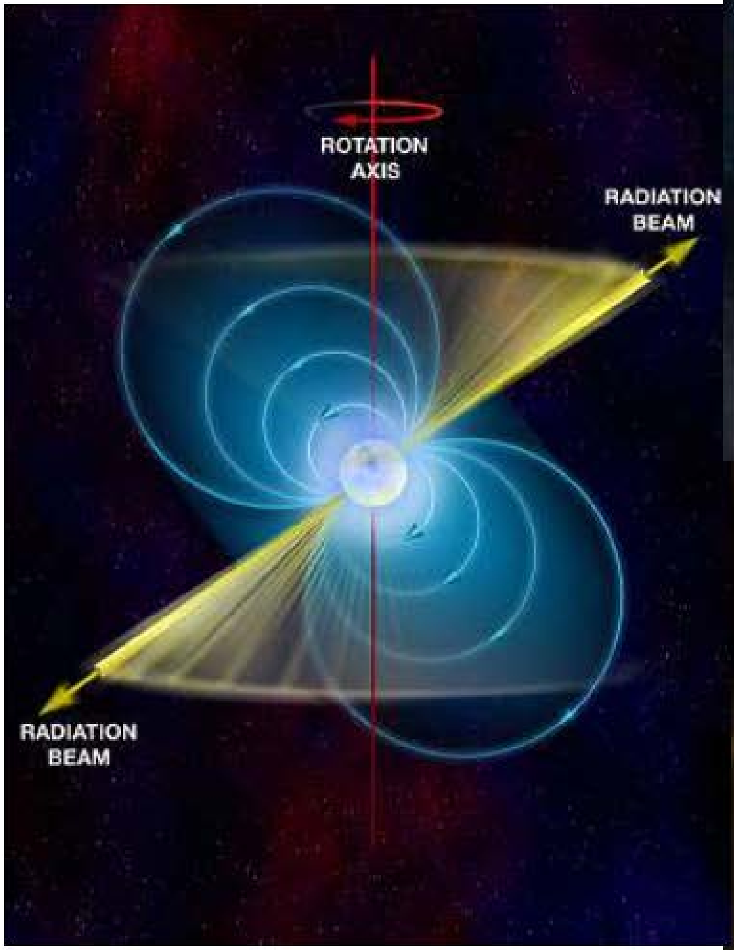


Vortex Photons may be radiated from Stars with Strong Mag. Fld.

Normal Neutron Stars	**10^{12-13} G**
Magnetars	**10^{14-15}G**

In future we may observe Light Vortex from Universe

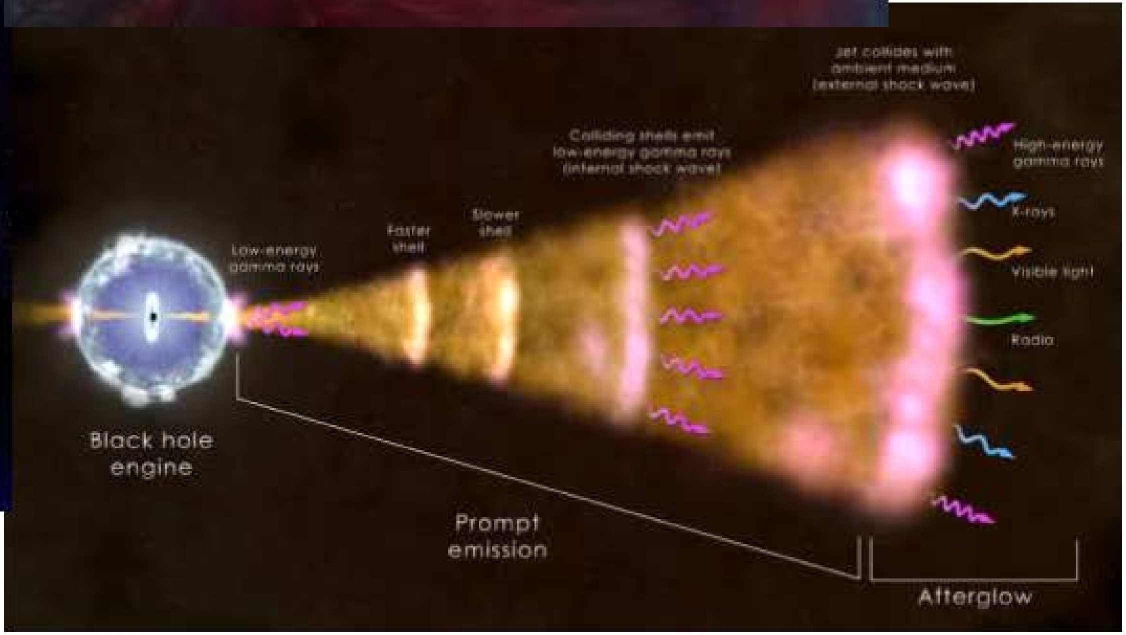
High Energy Vortex Photons in Universe?



https://public.nrao.edu/gallery/parts-of-a-pulsar/

Reaction
Photon-Vortex
+ Nucleus

https://en.wikipedia.org/wiki/Quasar



https://www.nasa.gov/feature/goddard/nasas-swift-spots-its-thousandth-gamma-ray-burst

§ 2 Gamma Vortex Generation in Strong Magnetic Field

Photon Vortex :

carring Orbital Angular Momentum along Beam Dir.

(1) **Radiation from Rapid Rotating Black Hole**

F.Tamburini et al. , Nat. Phys. 7, 195□197 (2011).

(2) **Radiation from Electron** Rotating in Strong Magnetic Field

M. Katoh, PRL 118, 094801 (2017)

Vortex Photons may be radiated

from Stars with Strong Mag. Fld. ???

Norma Neutron Stars	10^{12-13} G
Magnetars	10^{14-15} G

Synchrotron Radiation

Quantum Process

Electron in Strong Magnetic Field

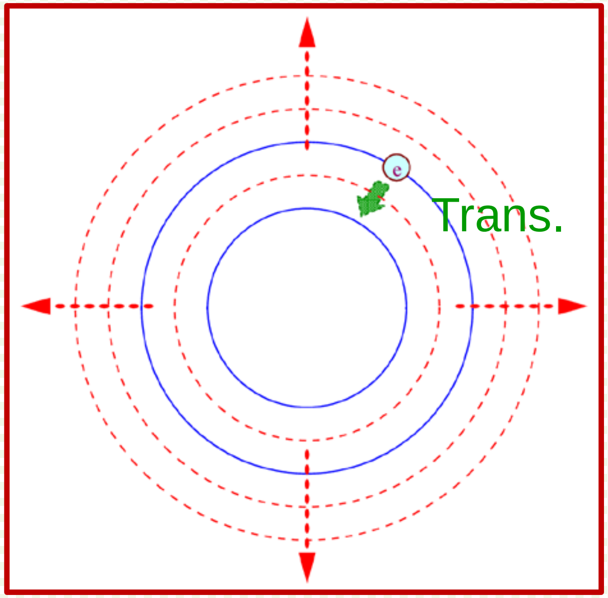
Circular Motion

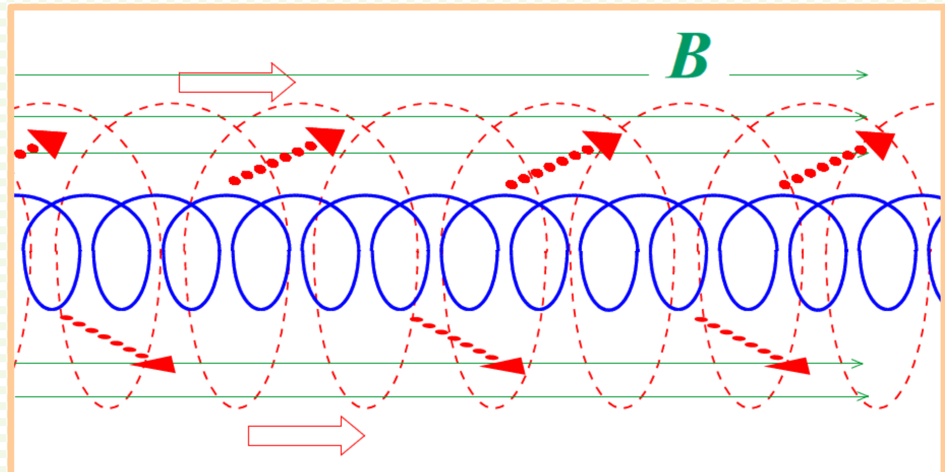
Landau Levels

High Speed Electron

Helical Motion along z-Direction

Eigen States of L_z & p_z





Transition of Electron

⇒ **One-Photon Emission**

Eigen States of OAM

Photon is **Cylindrical Wave**

Lorentz Trans.

Emitted Photon:

Eigen States of : L_z、p_z

⇒ ***Bessel Wave***

§ 2-1 Electron Wave Function in Magnetic Field

Mag. Fld. $\boldsymbol{B} = (0, 0, B)$, $\boldsymbol{A} = \frac{B}{2}(-y, x\ \ 0)$ **Symmetry Gauge**

Dirac Eq.

$$\{\boldsymbol{\alpha}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) + \beta m_e c^2 - E\}\,\psi(\boldsymbol{r}) = \begin{pmatrix} m_e c^2 - E & \boldsymbol{\sigma}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) \\ \boldsymbol{\sigma}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) & -m_e c^2 - E \end{pmatrix} \begin{pmatrix} \psi_U \\ \psi_L \end{pmatrix} = 0.$$

S.P. Energy : $E = \sqrt{p_z^2 + 2eB\hbar^2\left(n + \frac{L+|L|}{2}\right) + m_e^2 c^2} = \sqrt{p_z^2 + 2eBn_L + m_e^2 c^2}$

L : z-Comp. of Orbital Ang. Mom. (zOAM) , n : Node Number in xy-plane

Landau Level Number

Wave Function $(L \geq 0)$

2D HO W.F.

$$\psi(\boldsymbol{r}) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} = \sqrt{\frac{E + m_e}{2E}}\, e^{ip_z z} \begin{pmatrix} \begin{bmatrix} \lambda_1 G_n^{L-1}(\boldsymbol{r}_T) \\ \lambda_2 G_n^{L}(\boldsymbol{r}_T) \end{bmatrix} \\ \dfrac{-\sqrt{2(n+|L|)}\sigma_y + p_z\sigma_z}{E + m_e} \begin{bmatrix} \lambda_1 G_n^{L-1}(\boldsymbol{r}_T) \\ \lambda_2 G_n^{L}(\boldsymbol{r}_T) \end{bmatrix} \end{pmatrix}$$

Eigen State of J ⇒ z-Comp of Total AM , $L = J \pm \frac{1}{2}$ is Mixed

Single Particle Energy :

$$E = \sqrt{p_z^2 + 2eB\hbar^2\left(n + \frac{L+|L|}{2}\right) + m_e^2c^2} = \sqrt{p_z^2 + 2eBn_L + m_e^2c^2}$$

Landau Number : $n_\mathrm{L} = n + L$ (when $L \geq 0$), $n_\mathrm{L} = n$ (when $L < 0$)

L: z-comp. of OAM, n : Node Number in xy-Plain

$L \leq -1$ is impossible in Classical Theory

Choice of the Rotation Axis is arbitrary

States with $n \geq 1$ and/or $L \leq -1$ ⇒ Shift of the Central Position

K.Kubo S.J Miyake, N.Hashitsume, Solid State Physics 17, 269 (1965)

We consider only $n = 0$ (Circular Motion around Origin)

Photon Emission

Transition between Two Landau Levels

⇒ **Emitted Photon : Eugen State of L_z → Photon Vortex**

Phototn Field $(A_0, \boldsymbol{A})$ Gauge: $A_0 = 0$, $\nabla \cdot \boldsymbol{A} = 0$

Eugen State of L_z

$$\boldsymbol{A}(\boldsymbol{r}) \propto \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \int d^3\boldsymbol{r}' e^{-i\mathbf{q}\cdot\mathbf{r}'} \psi_f^\dagger(\boldsymbol{r}') \boldsymbol{\alpha} \psi_i(\boldsymbol{r}')$$

$$e^{i\mathbf{q}\cdot\mathbf{r}} = e^{iq_z z} \sum_{n=-\infty}^{\infty} i^n J_n(q_T r) e^{in(\phi_r - \phi_q)}.$$

$$\boldsymbol{A}(\boldsymbol{r}) \sim \boldsymbol{\epsilon} J_L(q_T r) e^{iL\phi} e^{iq_z z}$$

⇒ **Bessel Wave**

Eugen State of L_z

§ 2-2 Bessel Wave

Gauge : $A_0=0$, $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$

z-Comp. of Momentum q_z

z -Comp. of Ang. Mom. κ

Eigen State

$$\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{\epsilon} J_K(q_T r) e^{iK\phi} e^{ip_z z} \qquad \boldsymbol{\epsilon}_h = (1, \pm h)/\sqrt{2}$$

$$\boldsymbol{A}_{K,h}(\boldsymbol{r}) = \frac{1}{\sqrt{q_0^2+q_z^2}} e^{iq_z z}\left[-i(1,ih)q_z\tilde{J}_{K-h}, hq_T\tilde{J}_K\right]$$

Adding A_z for Gauge

$$\tilde{J}_L = J_L(q_T r)e^{iL\phi}, \quad K = J_i - J_f, \quad q_0^2 = q_z^2 + q_T^2 = q_z^2 + q_x^2 + q_y^2, \quad q_0 = E_i - E_f$$

κ : z-Comp of Ang, Momentum

$|q_z| \gg q_T$ ⇒ Bessel Wave （OAM: $\kappa - h$, Spi:h ）

$|q_z| \ll q_T$ ⇒ **Cylindrical Wave** (OAM: κ , Polarized in z-Dir.)

Two Waves are connected in Lorentz Transformation

$\boldsymbol{A}(h{=}+1)$ & $\boldsymbol{A}(h{=}-1)$ are not orthogonal ⇒

$$\boldsymbol{A}^{(\pm)} \propto \boldsymbol{A}(h=+1) \pm \boldsymbol{A}(h=-1)$$

Orthogonal Wave-Functions

§2-3 Emission Probability and Decay Width

Decay Width of Electron

$$\Gamma_e = \frac{e^2}{8\pi^2}\sum_{f,\alpha}\int \frac{dq_z dq_T q_T}{|\boldsymbol{q}|}\delta(E_i - E_f - |\boldsymbol{q}|)\left|\int d\boldsymbol{r}\overline{\psi}_f(\boldsymbol{r})\not{A}_\alpha^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

$$= \frac{\alpha_e}{2\pi}\sum_{f,\alpha}\int dq_z \left|\int d\boldsymbol{r}\overline{\psi}_f(\boldsymbol{r})\not{A}^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

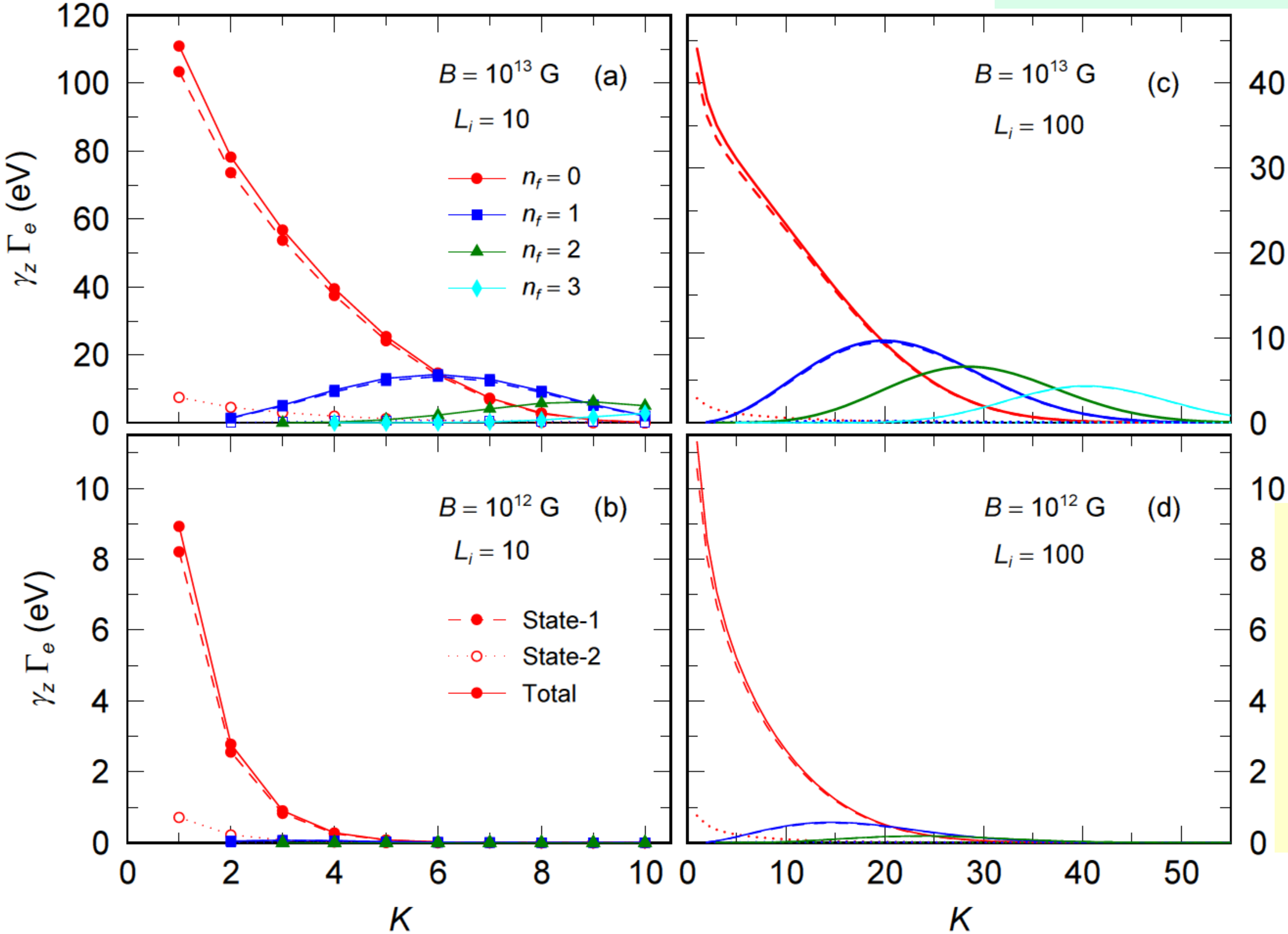
Emission Probability

$$P_{if}^{(\alpha)} = \left|e\int d\boldsymbol{r}\overline{\psi}_n(\boldsymbol{r})\not{A}^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

§ 3 Results

$p_{iz} = 10\text{MeV/c}$

$B = 10^{13}$ G, $\sqrt{eB} = 0.243\text{MeV}$



State-1

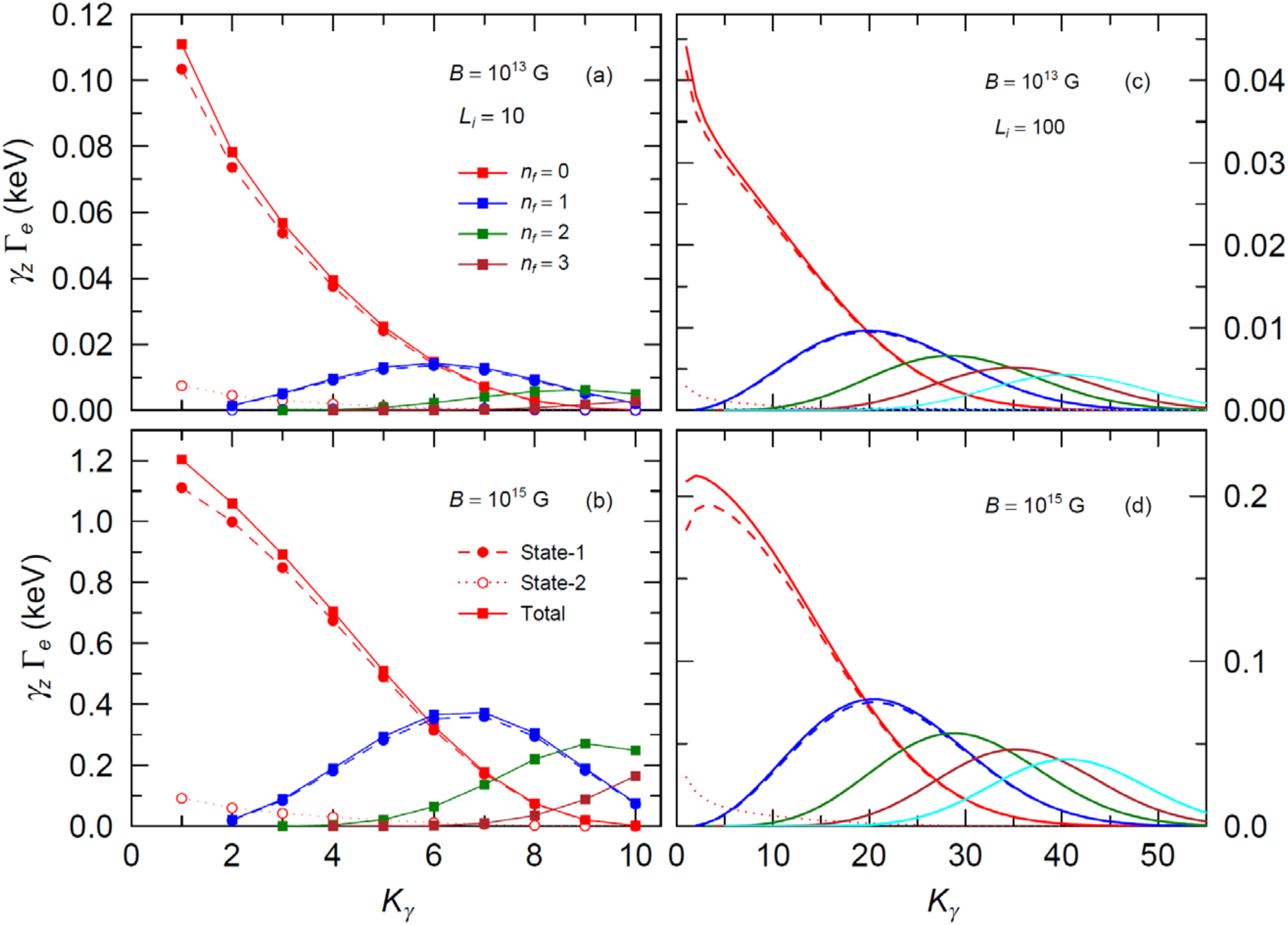
⇓

dominant contribution

Small Photon Ang. Mom.

Small Contribution from $\Delta n = n_i - n_f \geq 1$

$K = J_i - J_f$: zTAM of Photon $K = 1$: Fundamental, ≥ 2 : **Higher Harmonic**



$B = 10^{13}$ G (a)
$L_i = 10$
$n_f = 0$
$n_f = 1$
$n_f = 2$
$n_f = 3$
$B = 10^{13}$ G (c)
$L_i = 100$
$B = 10^{15}$ G (b)
State-1
State-2
Total
$B = 10^{15}$ G (d)
$\gamma_z \Gamma_e$ (keV)
K_γ

Transition Probability

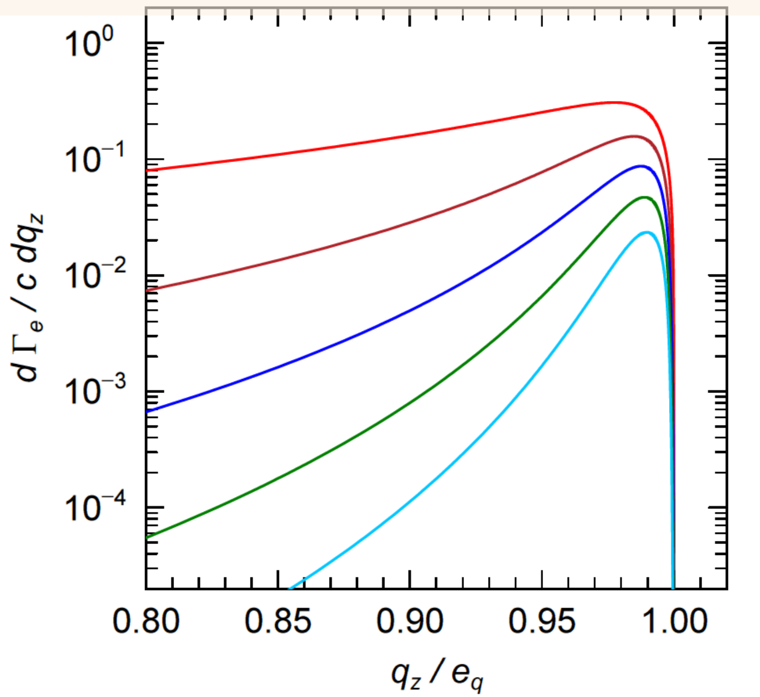
$$\frac{d\Gamma_e}{dq_z} = \frac{\alpha_e}{2\pi} \left| \int d\boldsymbol{r} \overline{\psi}_f(\boldsymbol{r}) \not{A}^{(\alpha)*}(\boldsymbol{r}) \psi_i(\boldsymbol{r}) \right|^2$$

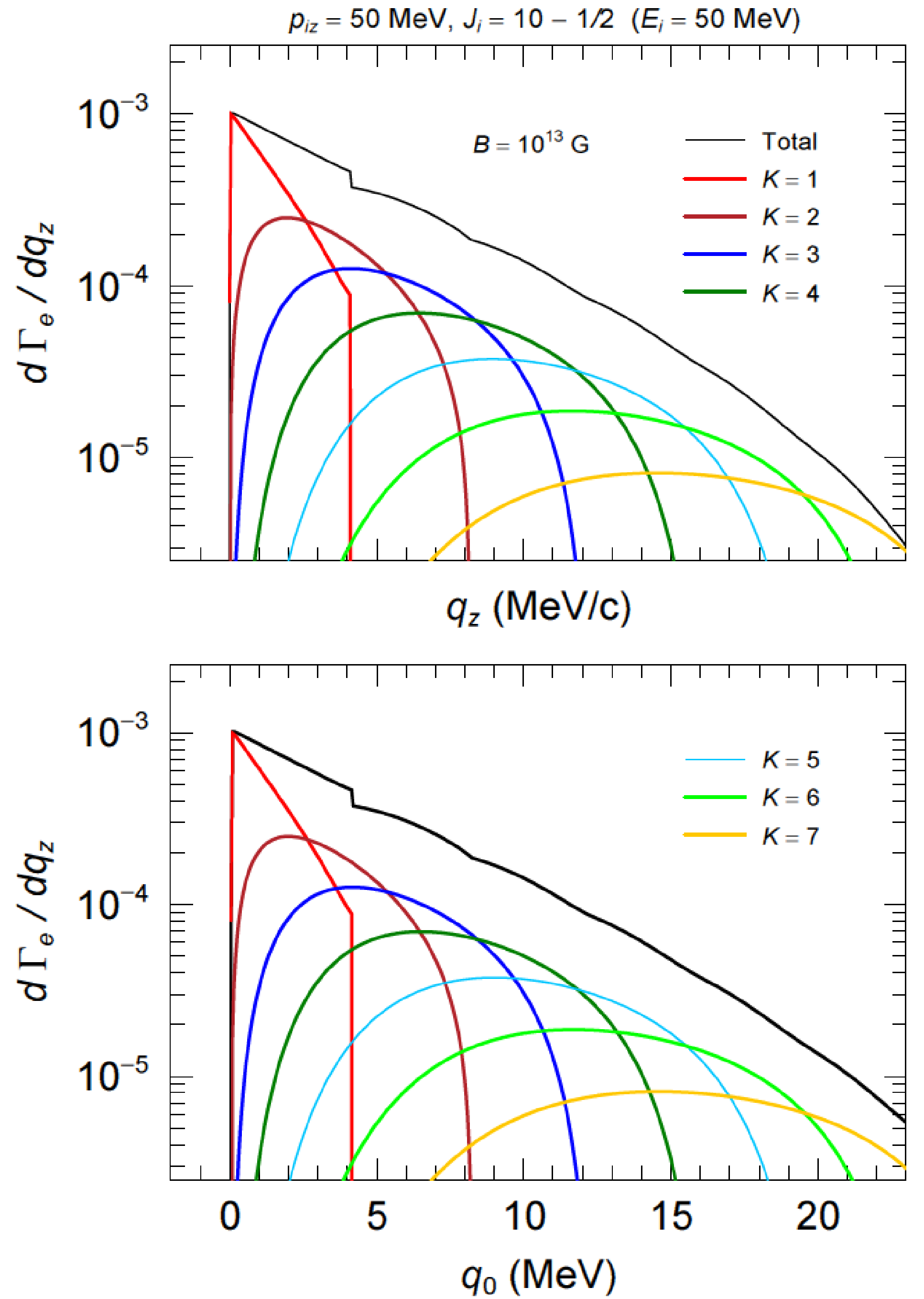
$K = 1$: Fundamental   Largest

($L_z = 0$ and 2 are Mixed)

$K \geq 2$: **Higher Harmonic**   **Not Small**

(Photon Vortex)





Question

Synchrotron Radiation

Emitted Photon is

cylindrically polarized

(Eigen State of Helicity)

Twisted Photons are at State-1

(h = 1, −1, **equal Probability**)

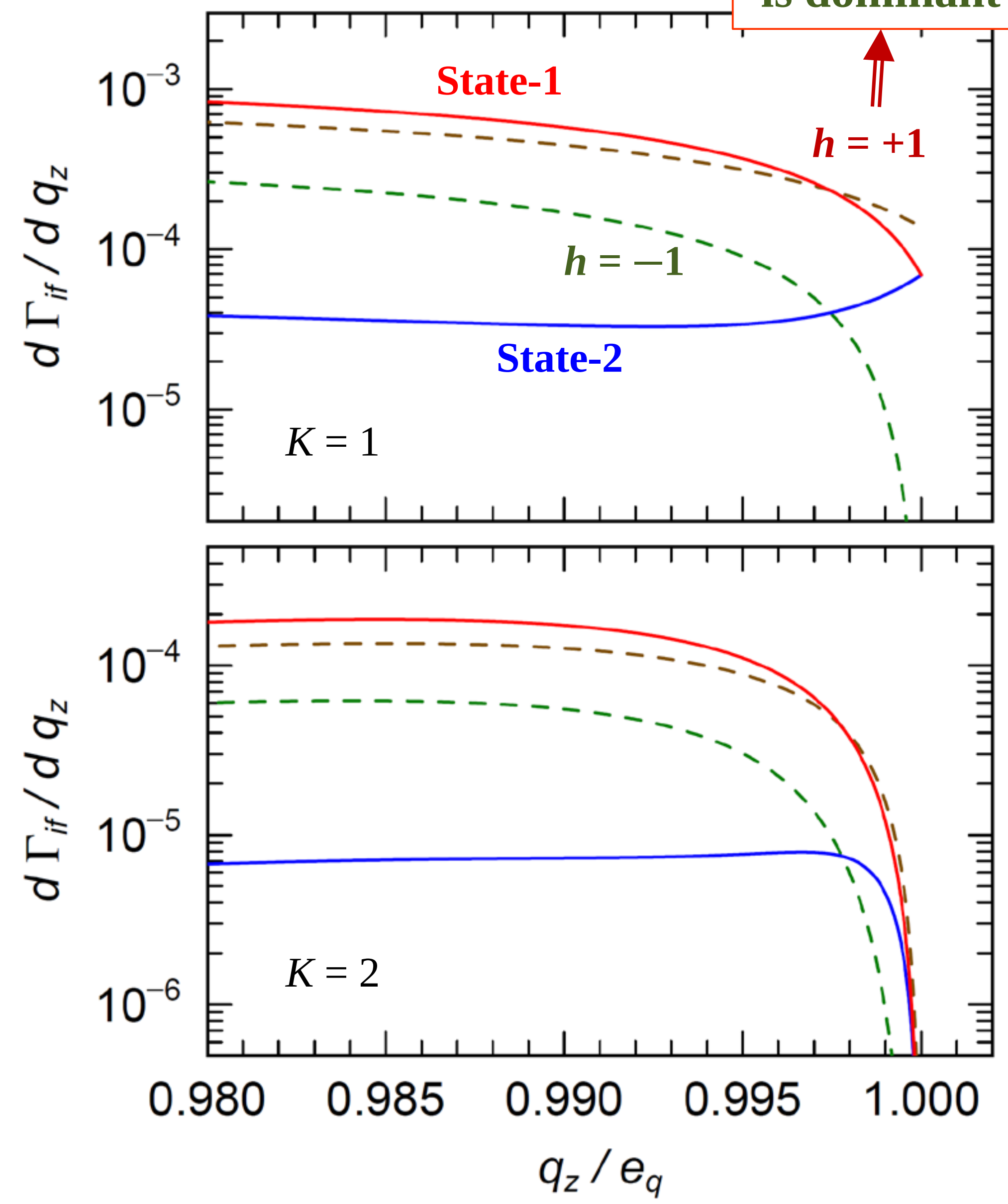
$h = +1$ at **$q_T \to 0$ Limit**

State-1 in $q_z/e_q \gtrsim 0.999$

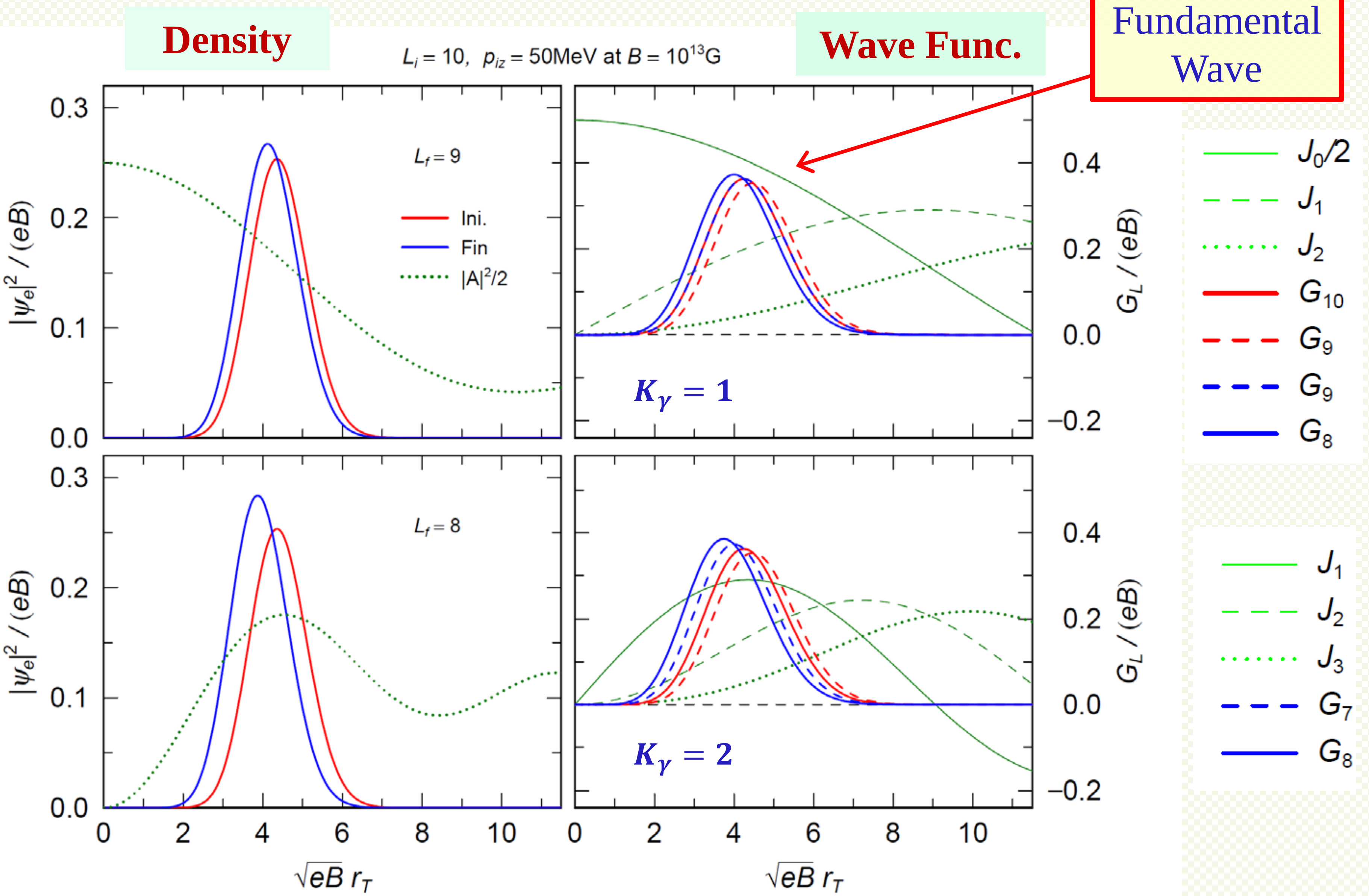
($q_T/e_q \gtrsim 0.045$)

$q_z/e_q \to 1$ ($q_T \to 0$) **Limit**

($h = +1$) is dominant



Density Distributions in Cross-section



§4 Summary

Electron in Strong Mag. Fld.   Synchrotron Motion   Photon Emission

Classical Process   Light Vortex Rad.   M. Katoh et al., PRL118, 094801 (17)

Quantum Process   Trans. Bettwo Landau Levels $\rightarrow$ 1-Photon Emission

$\Rightarrow$ *Present Work*

Electron Speed along Magnetic Field

Low $\rightarrow$ Cylindrical Wave

High $\rightarrow$ Bessel Wave (Gamma Vortex when $L \geq 1$)

Calculation   Symmetry Gauge $\Rightarrow$ Electron W,.F .   Eigen State of L_z

Emission Photon   Bessel Wave

Results   **Higher Hamonic Photons with OAM are also Emitted**

to Direction of Magnetic Field (Arctic or Antarctic)

Future Work

Photon Vortex in Super Novae

Photo Absorption Reaction : Selection Rule is changed

H-atom : A. Afanasev et al., PRA 88, 033841 (3)

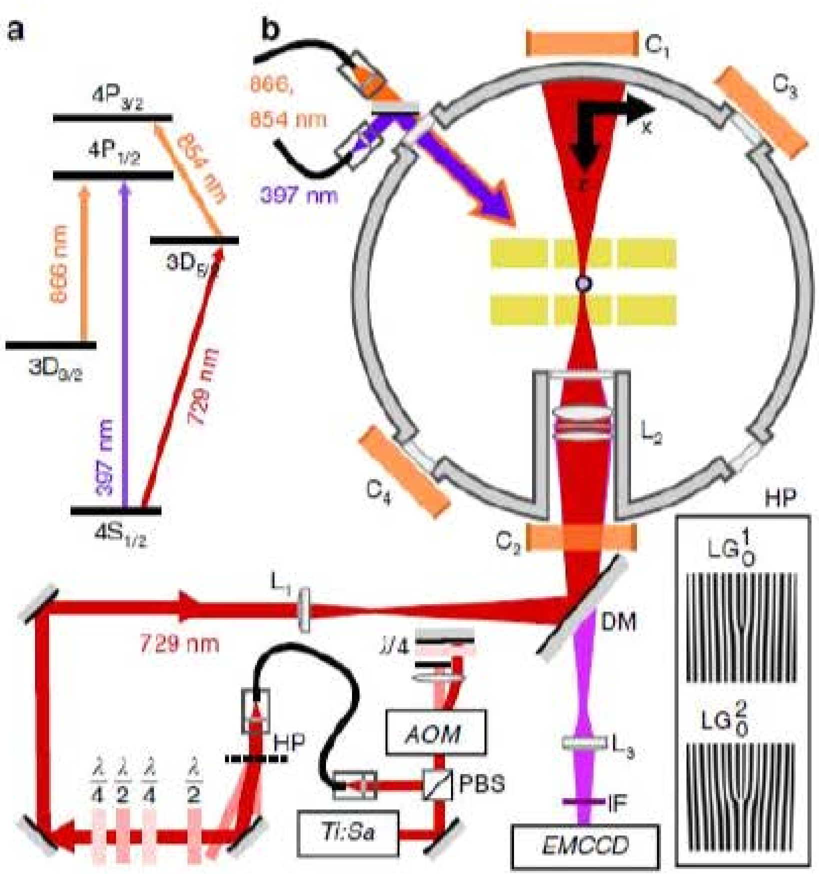
OAM ($L \geq 1$) + Spin (S=1) = Total AM ($J \geq 2$)

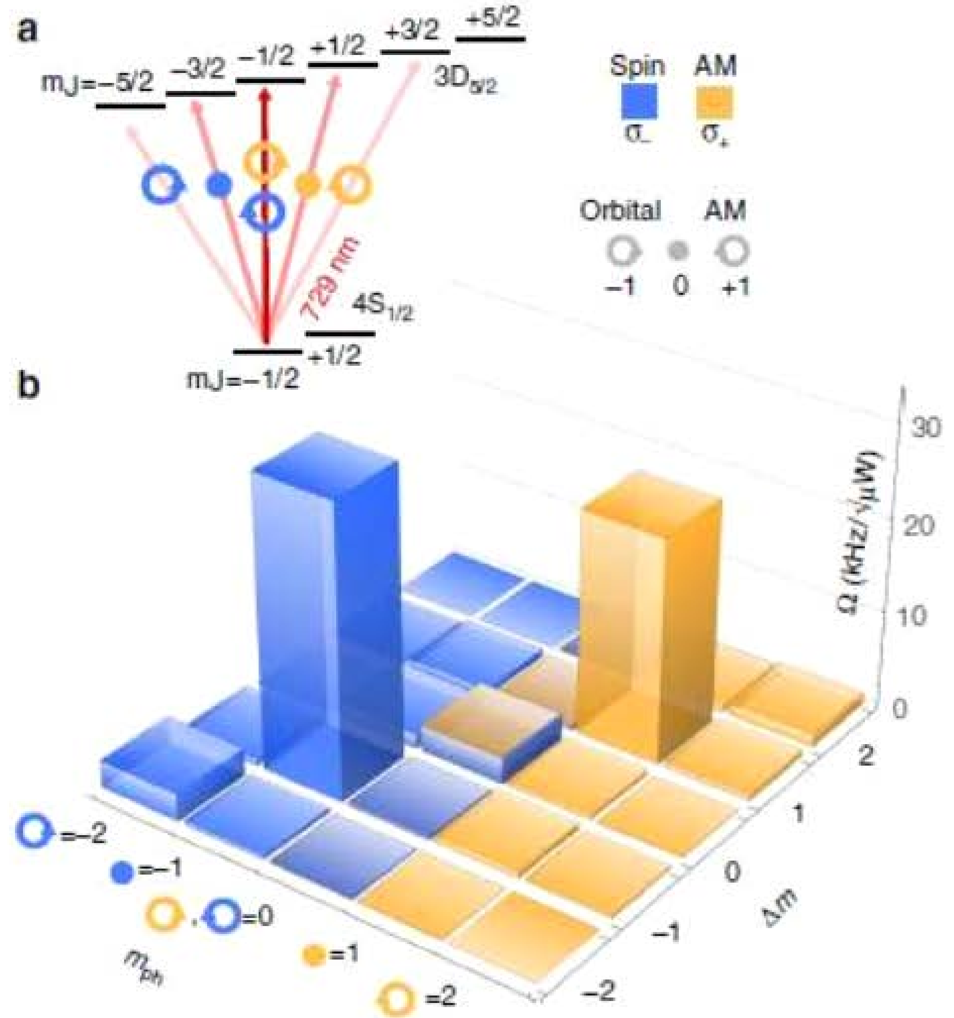
***E1* Transition does not occur?** **Influence to Nuclear Synthesis？**

nature COMMUNICATIONS
Received 2 Feb 2016 | Accepted 23 Aug 2016 | Published 3 Oct 2016
DOI: 10.1038/ncomms12998
OPEN

Transfer of optical orbital angular momentum to a bound electron

Christian T. Schmiegelow[1,†], Jonas Schulz[1], Henning Kaufmann[1], Thomas Ruster[1], Ulrich G. Poschinger[1] & Ferdinand Schmidt-Kaler[1]

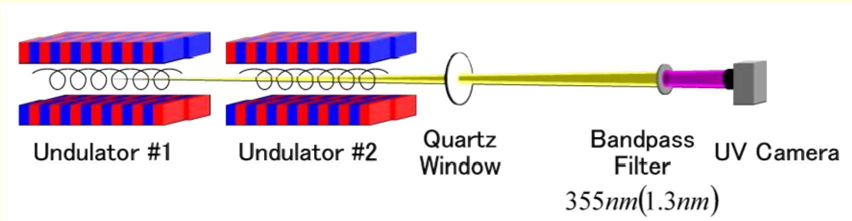




天体系での渦光生成

強磁場場中の電子からの輻射
渦光生成

M. Katoh　et al.., PRL 118, 094801 (17)



天体系での強磁場

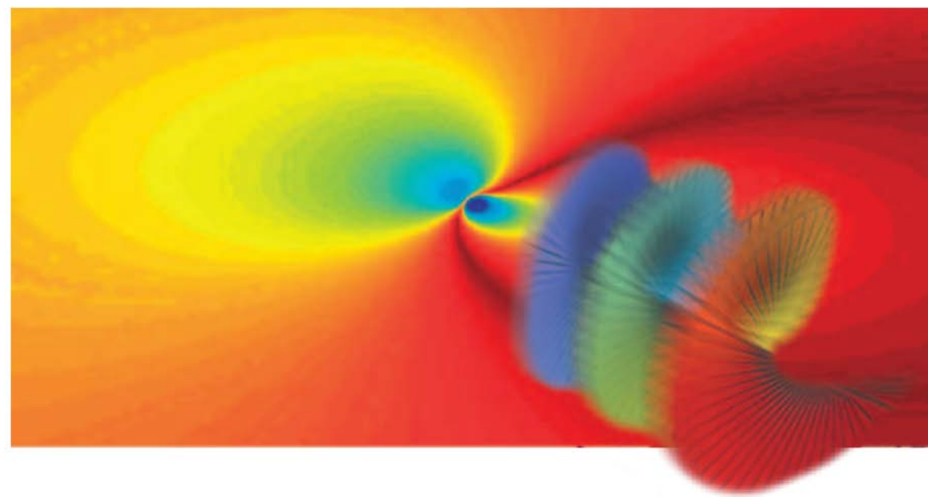
マグネター(表面磁場10^{14-15} G)

放出 γ線が渦？

1光子の波動関数？

回転ブラックホールでの渦光生成

Fabrizio Tamburini et al.



自然界での生成　天文現象

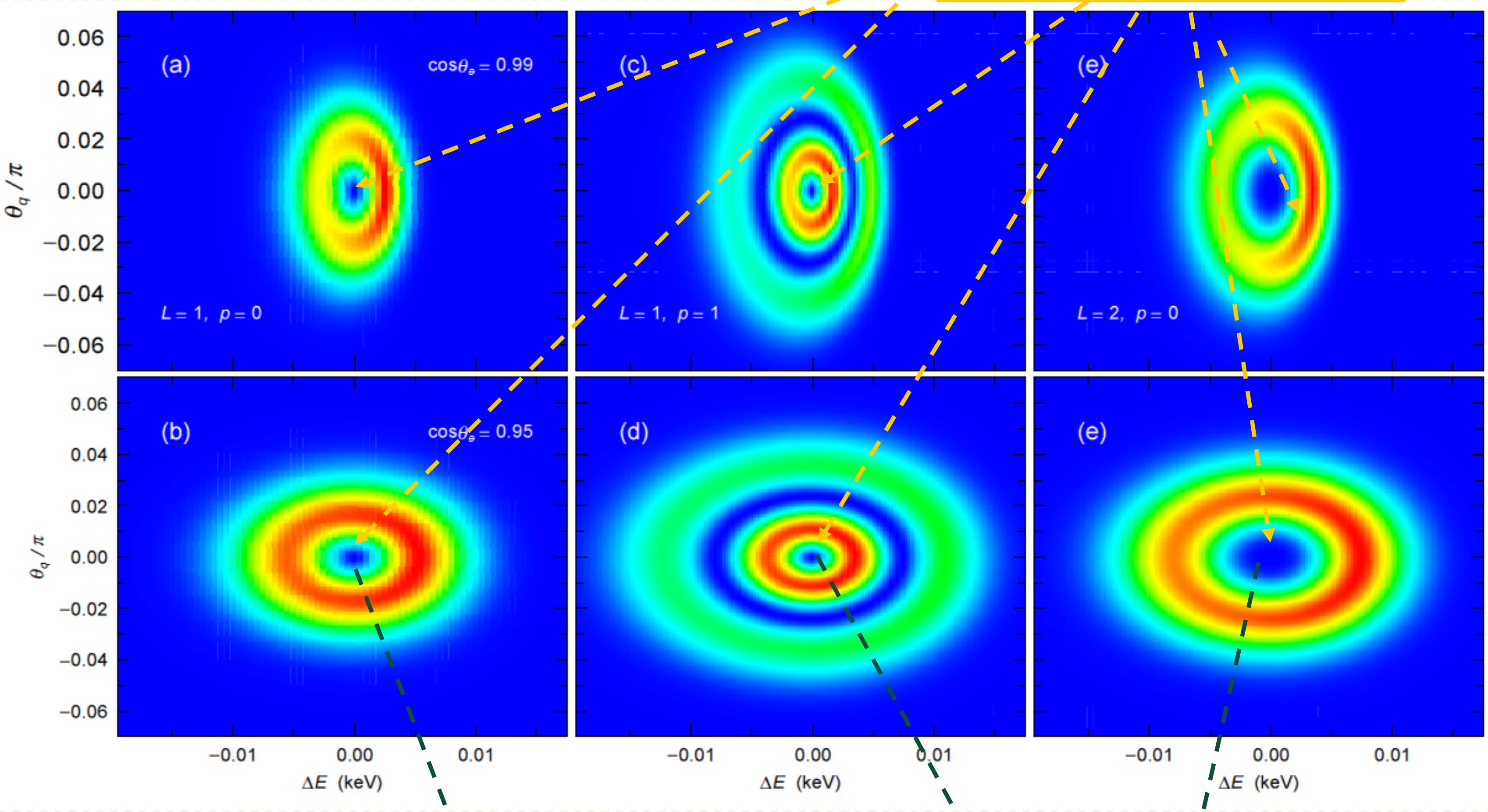
光吸収　角運動量選択則の変化

H原子：**A. Afanasev et al., PRA 88, 033841 (3)**

元素合成に影響？

角度エネルギー分布

原点　通常コンプトン



反応平面からの角度

エネルギーシフト

入射γ　平面波：原点(0,0)に散乱　　　渦γではゼロ強度

渦光：角運動量を持つ光

(1) 天体現象　　回転ブラックホールからの生成

重力による電磁流の急激な変化

(2) 強磁場中での電子からの輻射による生成

強磁場天体での光子生成 ⇒ 渦波(?)　……　ガンマ線

中性子星　　表面磁場　10^{12-13} G

マグネター　　　　10^{14-15}G

超新星爆発10秒後に生成される原始中性子星

光吸収　角運動量選択則の変化

H原子：A. Afanasev et al., PRA 88, 033841 (3)

軌道角運動量($L \gneqq 1$)＋スピン角運動量(S=1) = 全角運動量($J \gneqq 2$)

E1 遷移吸収が起きない?　　元素合成に影響？

Electron Wave Function

Dirac Eq.

$$\{\boldsymbol{\alpha}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) + \beta m_e c^2 - E\}\psi(\boldsymbol{r}) = \begin{pmatrix} m_e c^2 - E & \boldsymbol{\sigma}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) \\ \boldsymbol{\sigma}(-i\hbar\boldsymbol{\nabla}_r + e\boldsymbol{A}) & -m_e c^2 - E \end{pmatrix}\begin{pmatrix}\psi_U \\ \psi_L\end{pmatrix} = 0.$$

1粒子エネルギー: $E = \sqrt{p_z^2 + 2eB\hbar^2\left(n + \frac{L+|L|)}{2}\right) + m_e^2c^2} = \sqrt{p_z^2 + 2eBn_L + m_e^2c^2}$

L：軌道角運動量のz成分，n：xy平面の動径方向の節の数

波動関数　($L \geq 0$)

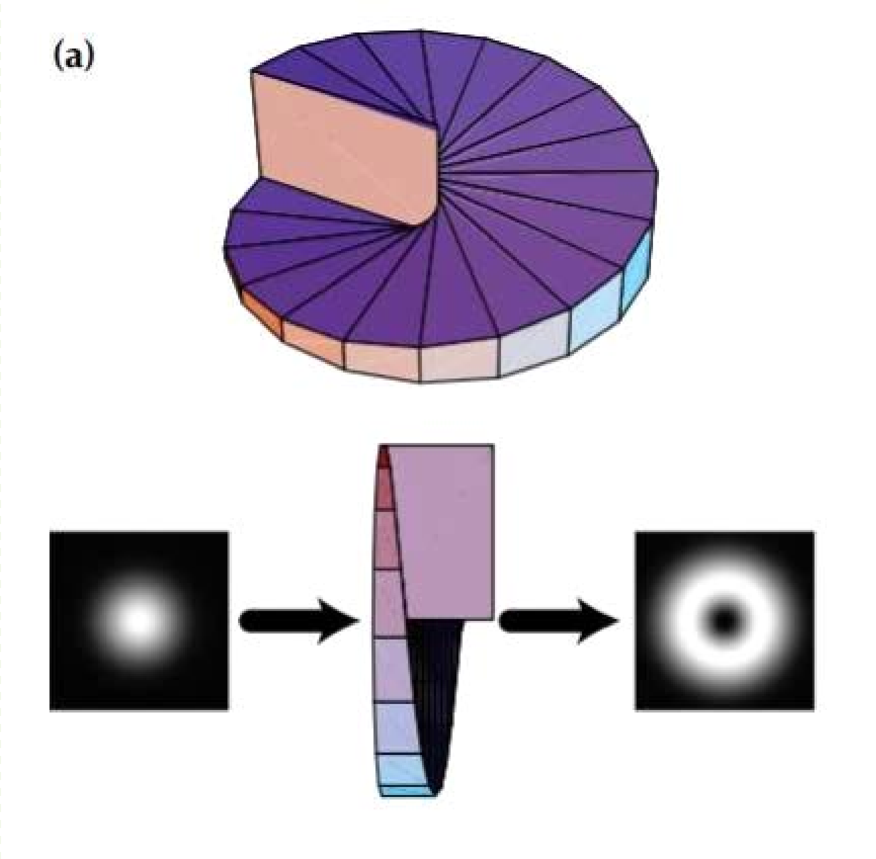
$\hbar = c = 1$

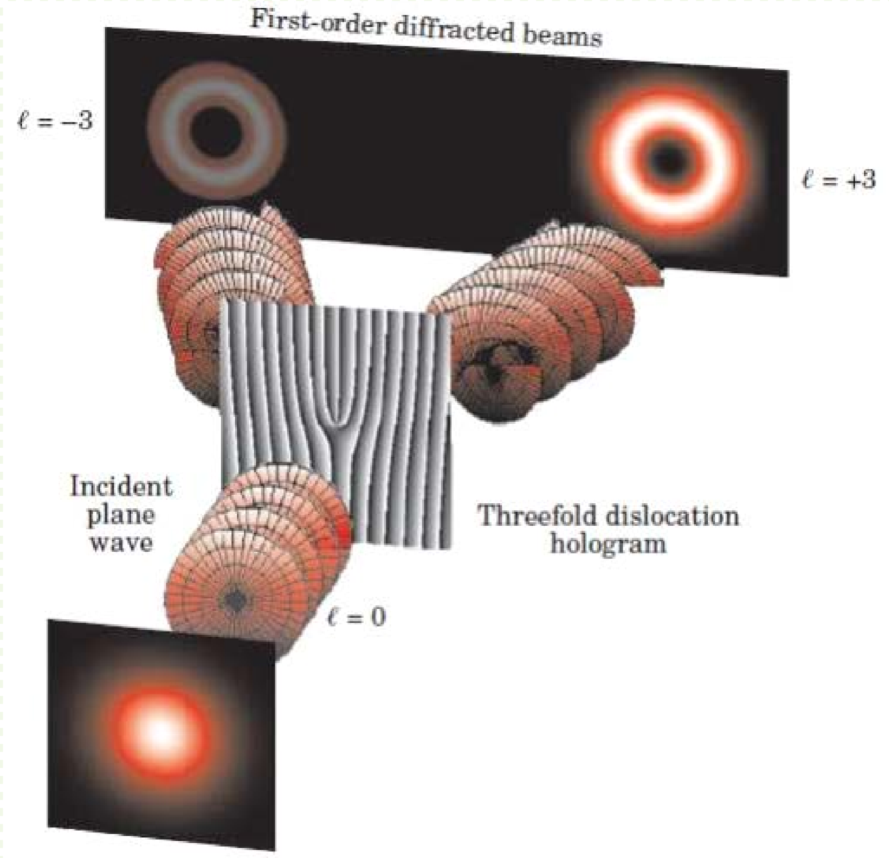
$|\lambda_1|^2 + |\lambda_2|^2 = 1$

$$\psi(\boldsymbol{r}) = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix} = \sqrt{\frac{E+m_e}{2E}}e^{ip_z z}\begin{pmatrix} \begin{bmatrix}\lambda_1 G_n^{L-1}(\boldsymbol{r}_T) \\ \lambda_2 G_n^{L}(\boldsymbol{r}_T)\end{bmatrix} \\ \dfrac{-\sqrt{2(n+|L|)}\sigma_y + p_z\sigma_z}{E+m_e}\begin{bmatrix}\lambda_1 G_n^{L-1}(\boldsymbol{r}_T) \\ \lambda_2 G_n^{L}(\boldsymbol{r}_T)\end{bmatrix}\end{pmatrix}$$

全角運動量(z成分）J，軌道角運動量は$L = J$J　$\pm\frac{1}{2}$　が混ざる

How to produce Twisted Photons



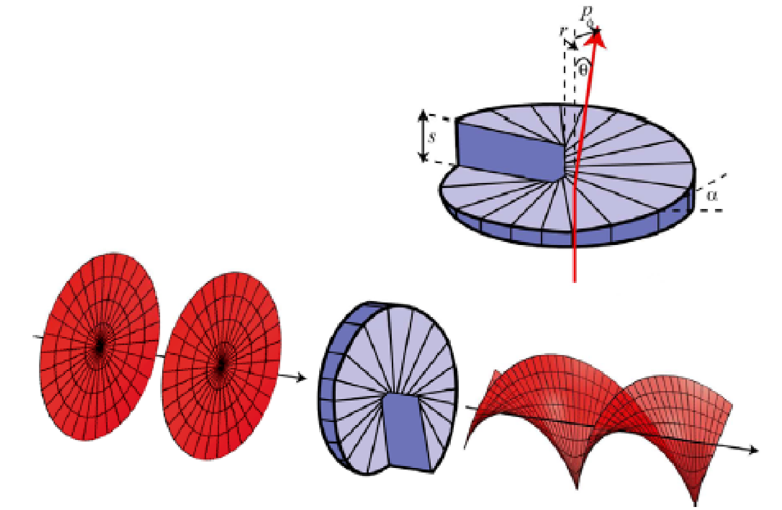


J. Courtial, K. O'Holleran, Eur. Phys. J. Special Topics 145, 35–47 (2007)

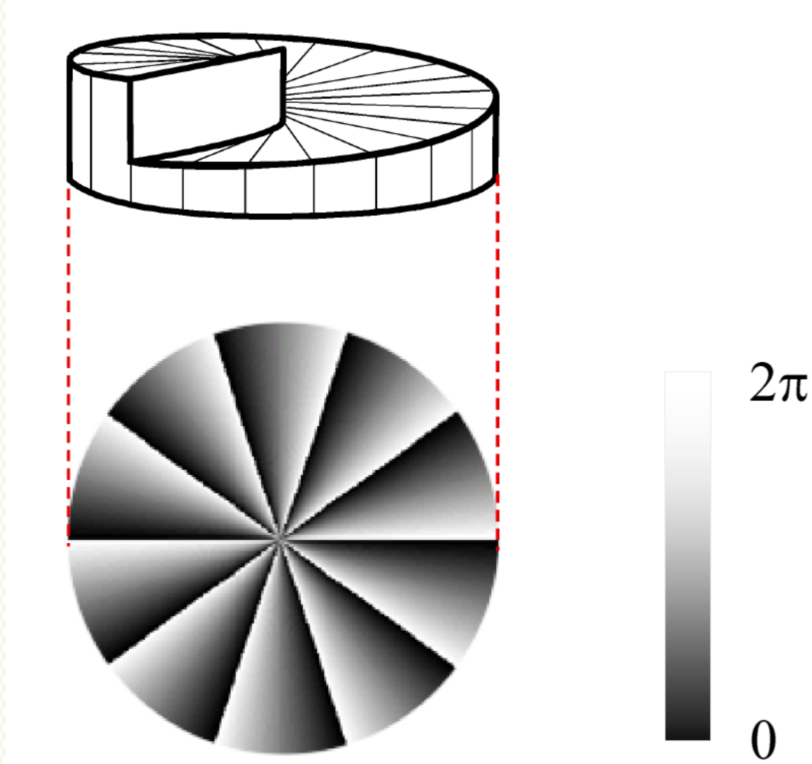
M. Padgett, J. Courtial, L. Allen, Physics Today (May, 2004), 35

Spiral Phase Plates with azimuthal dependence in thickness:

Gaussian beam is passed through optical media

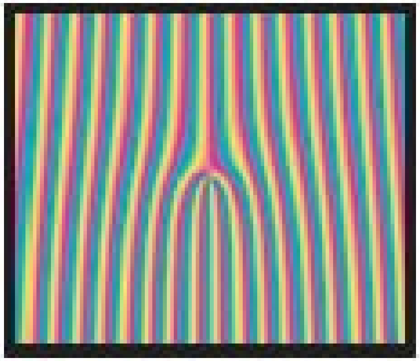


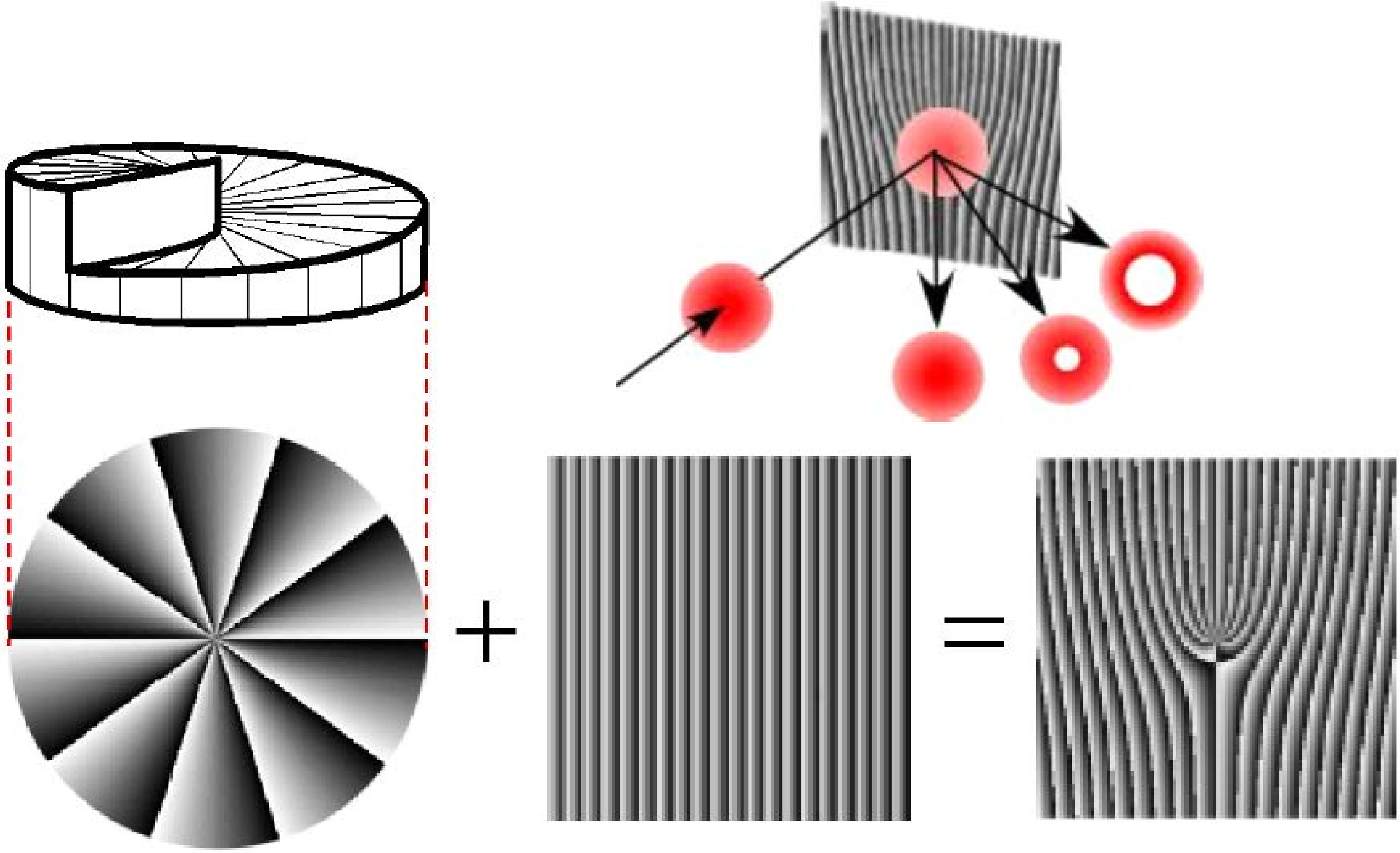
A spiral phase plate can generate a helically phased beam from a Gaussian. In this case $\ell = 0 \rightarrow \ell = 2$.

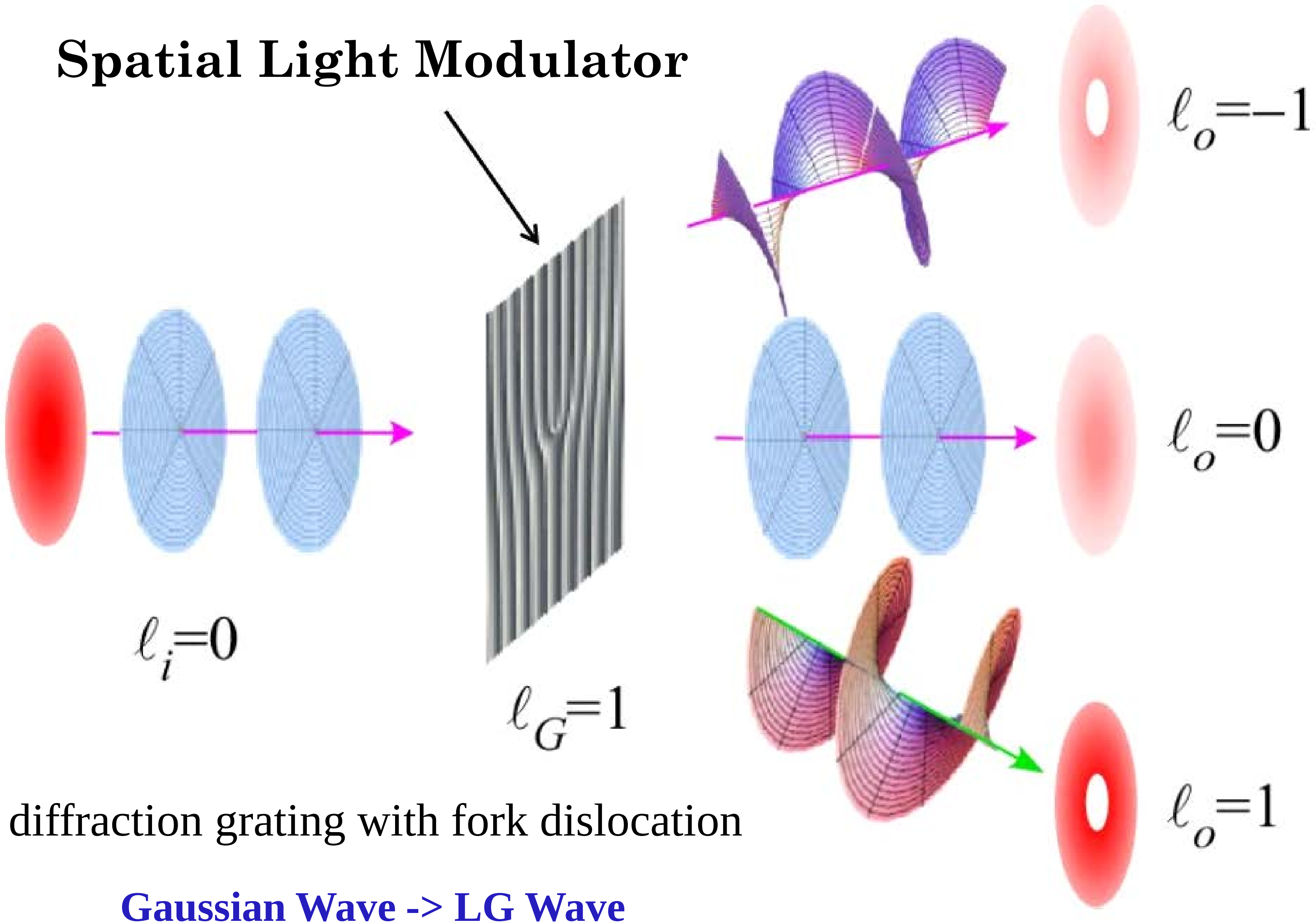


Spatial Light Modulator

V. Bazhenov, M. V. Vasnetsov, and M. S. Soskin,
JETP. Lett. 52, 429–431 (1990)







Gaussian Wave -> LG Wave

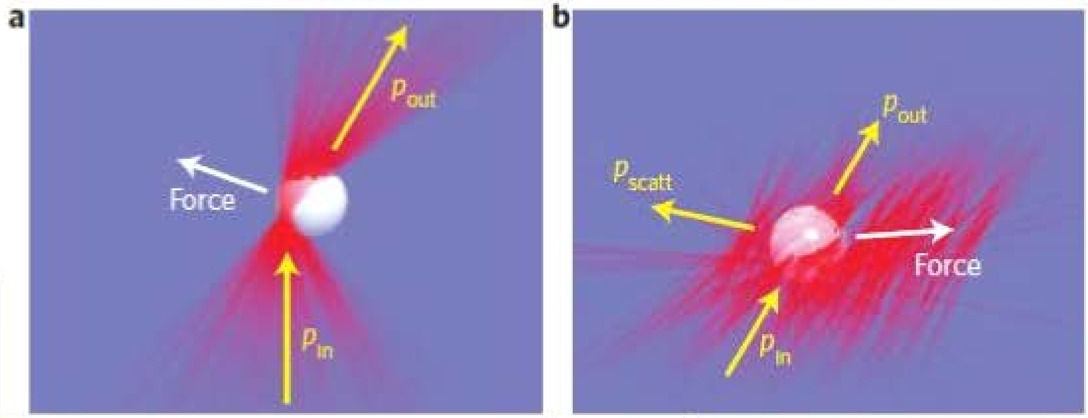
nature photonics
FOCUS | REVIEW ARTICLES
PUBLISHED ONLINE: 31 MAY 2011 | DOI: 10.1038/NPHOTON.2011.81

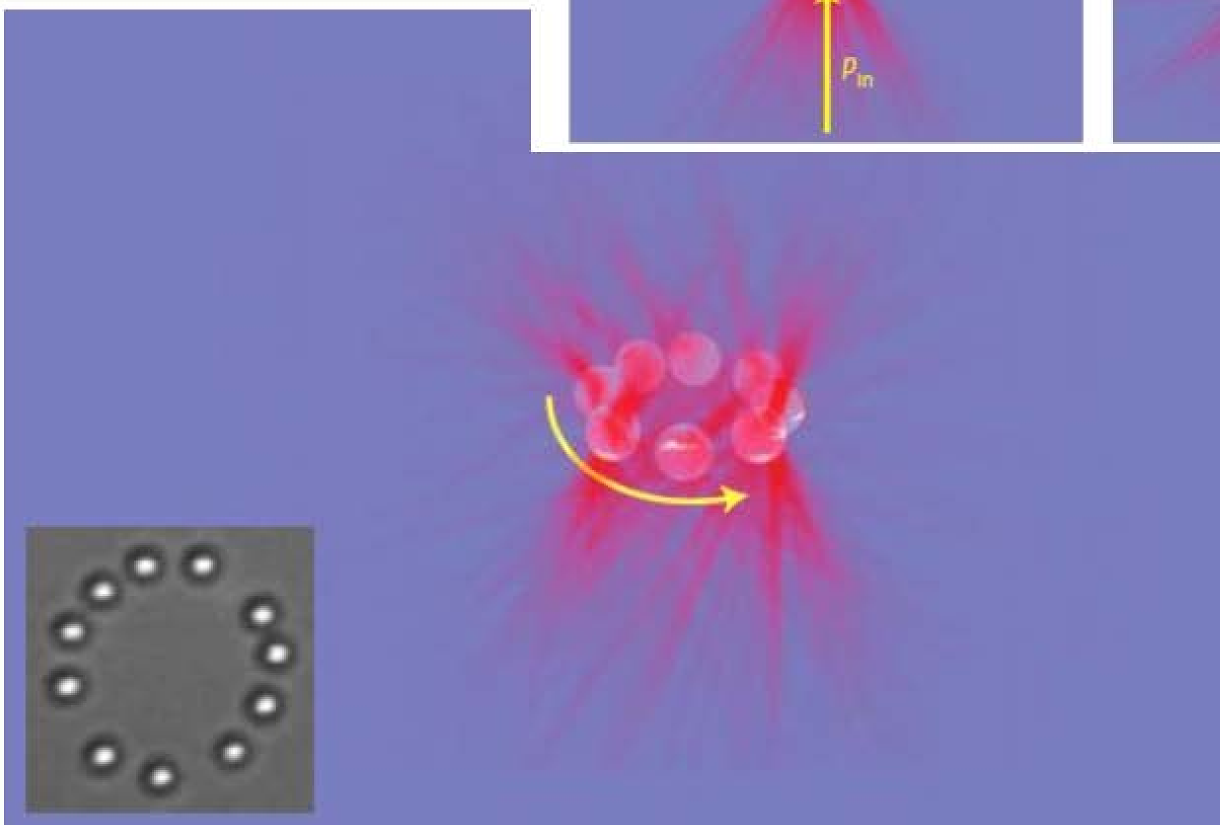
Tweezers with a twist

Miles Padgett* and Richard Bowman

Gradient Force

Scattering Force



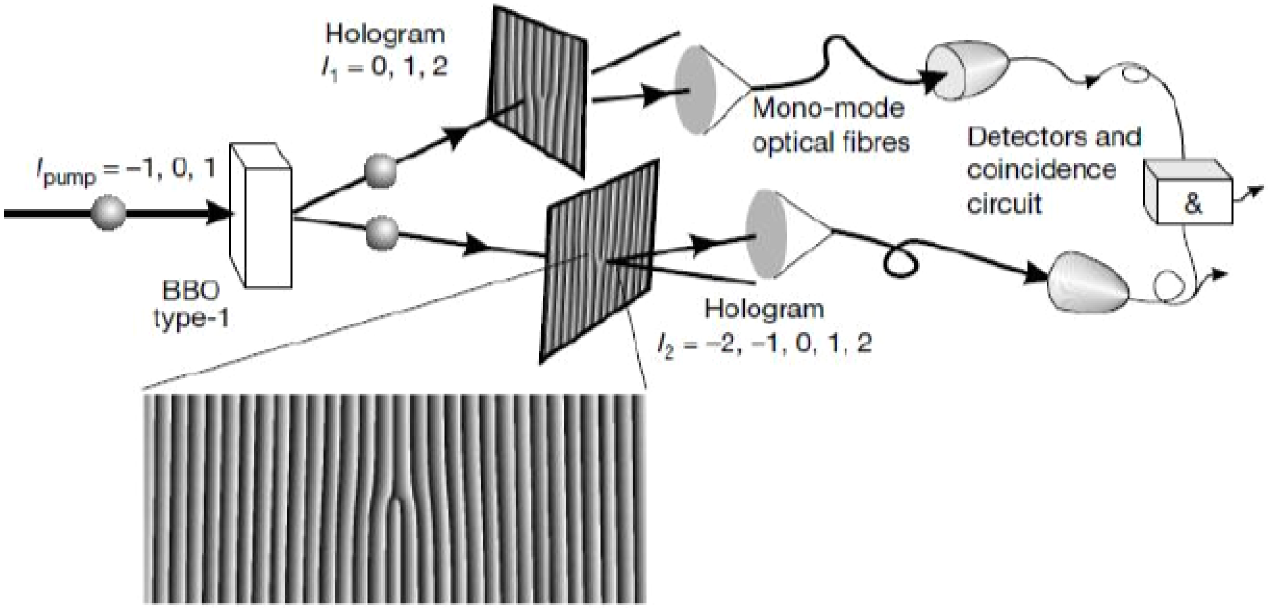


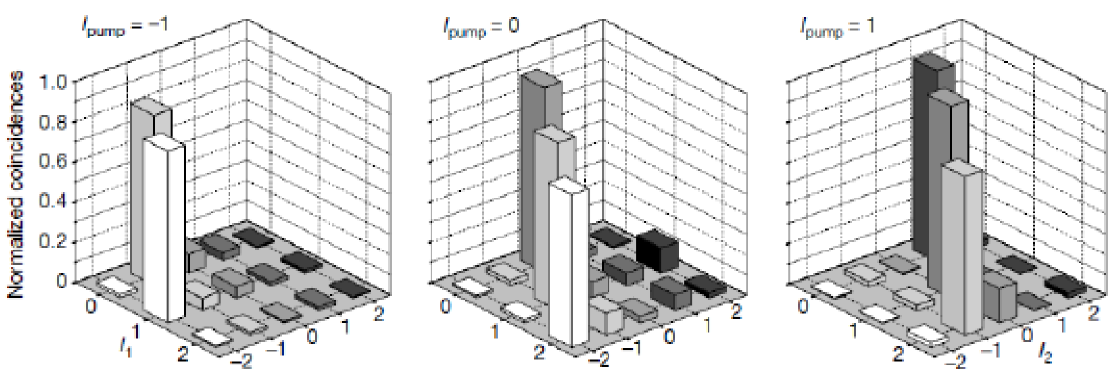
Trapped by Gradient Force and rotated by Scattering Force

Entanglement of the orbital angular momentum states of photons

Alois Mair*, **Alipasha Vaziri, Gregor Weihs & Anton Zeilinger**

NATURE | VOL 412 | 19 JULY 2001 | www.nature.com





$$\psi_a(\boldsymbol{r}) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} = \frac{E+m}{2E} \begin{bmatrix} \left(\frac{1+\sigma_z}{2} G_1 + \frac{1-\sigma_z}{2} G_2\right) \chi_s \\ \left(\frac{1+\sigma_z}{2} G_1 + \frac{1-\sigma_z}{2} G_2\right) \frac{\boldsymbol{p}\cdot\boldsymbol{\sigma}}{E+m} \chi_s, \end{bmatrix} e^{iL\phi + ip_z z}, \quad \boldsymbol{p} = (0, \sqrt{2n_L}, p_z)$$

$$\mathcal{M}(L_1, p_1; L_2, p_2) = \int dr r R_{p_2}^{L_2}(r^2) J_{L_1 - L_2}(q_T r) R_{p_1}^{L_1}(r^2)$$

$$\mathcal{M}_{22} = \mathcal{M}(L_i, p_i; L_f, p_f), \quad \mathcal{M}_{11} = \mathcal{M}(L_i - 1, p_i; L_f - 1, p_f),$$

$$\mathcal{M}_{21} = \mathcal{M}(L_i, p_i; L_f - 1, p_f), \quad \mathcal{M}_{12} = \mathcal{M}(L_i - 1, p_i; L_f, p_f).$$

$$\begin{aligned} & \int d^2\boldsymbol{r}_T \psi_f^\dagger(\boldsymbol{r}_T) \boldsymbol{\gamma} \psi_i(\boldsymbol{r}_T) A^\dagger(\boldsymbol{r}_T) \\ = \ & \sqrt{\frac{(E_f+m)(E_i+m)}{E_i E_f (|\boldsymbol{q}|^2 + q_z^2)}} \\ & \times \left\{ \frac{1-h}{2} \mathcal{M}_{12} q_z \begin{bmatrix} \frac{p_{iT}}{E_i+m} & \frac{ip_{iz}}{E_i+m} - \frac{ip_{fz}}{E_f+m} \\ 0 & \frac{p_{fT}}{E_i+m} \end{bmatrix} + \frac{1+h}{2} \mathcal{M}_{21} q_z \begin{bmatrix} \frac{p_{fT}}{E_f+m} & 0 \\ \frac{ip_{fz}}{E_f+m} - \frac{ip_{iz}}{E_i+m} & -\frac{p_{iT}}{E_i+m} \end{bmatrix} \right. \\ & \left. + q_T \mathcal{M}_{22} \begin{bmatrix} 0 & \frac{ip_{fT}}{E_f+m} \\ -\frac{ip_{iT}}{E_i+m} & \frac{p_{iz}}{E_i+m} + \frac{p_{fz}}{E_f+m} \end{bmatrix} + q_T \mathcal{M}_{11} \begin{bmatrix} \frac{p_{iz}}{E_i+m} + \frac{p_{fz}}{E_f+m} & -\frac{ip_{iT}}{E_i+m} \\ \frac{ip_{fT}}{E_f+m} & 0 \end{bmatrix} y \right\}. \end{aligned}$$

2-dim Harmonic Oscillator

$$\left[-\frac{1}{2}\left(\nabla_x^2+\nabla_y^2\right)+\frac{1}{2}\left(x^2+y^2\right)\right]G(\boldsymbol{r})=\left[-\frac{1}{2}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r}\frac{\partial^2}{\partial\phi^2}\right)+\frac{1}{2}r^2\right]G(\boldsymbol{r})=EG(\boldsymbol{r})$$

Def: Operators

$$a=\frac{e^{-i\phi}}{2}\left(r+\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\phi}\right),\quad a^\dagger=\frac{e^{i\phi}}{2}\left(r-\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\phi}\right),$$

$$b=\frac{e^{i\phi}}{2}\left(r+\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\phi}\right),\quad b^\dagger=\frac{e^{-i\phi}}{2}\left(r-\frac{\partial}{\partial r}+\frac{i}{r}\frac{\partial}{\partial\phi}\right),$$

Hamiltonian : $H=a^+a+b^+b+1$, OAM: $L_z=a^+a-b^+b$

$$aG_{L,n}=\sqrt{n+L}\,G_{L-1,n},\quad a^+\varphi_{L,n}=\sqrt{n+L+1}\,G_{L-1,n},$$

$$bG_{L,n}=-\sqrt{n}\,G_{L+1,n-1},\quad b^+\varphi_{L,n}=-\sqrt{n+1}\,G_{L-1,n+1}$$

§5-3 Emission Probability and Decay Width

Decay Width of Electron

$$\Gamma_e = \frac{e^2}{8\pi^2}\sum_{f,\alpha}\int \frac{dq_z dq_T q_T}{|\boldsymbol{q}|}\delta(E_i - E_f - |\boldsymbol{q}|)\left|\int d\boldsymbol{r}\overline{\psi}_f(\boldsymbol{r})\not{A}_\alpha^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

$$= \frac{\alpha_e}{2\pi}\sum_{f,\alpha}\int dq_z\left|\int d\boldsymbol{r}\overline{\psi}_f(\boldsymbol{r})\not{A}^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

Emission Probability

$$P_{if}^{(\alpha)} = \left|e\int d\boldsymbol{r}\overline{\psi}_n(\boldsymbol{r})\not{A}^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

§ 2-2 Bessel Wave

Gauge : A_0=0, $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$

z-Comp. of Momentum q_z

z -Comp. of Ang. Mom. κ

Eigen State

$$\boldsymbol{A}(\boldsymbol{r})=\boldsymbol{\epsilon}J_K(q_T r)e^{iK\phi}e^{ip_z z} \qquad \boldsymbol{\epsilon}_h=(1,\pm h)/\sqrt{2}$$

$$\boldsymbol{A}_{K,h}(\boldsymbol{r})=\frac{1}{\sqrt{q_0^2+q_z^2}}e^{iq_z z}\left[-i(1,ih)q_z\tilde{J}_{K-h},hq_T\tilde{J}_K\right]$$

Adding A_z for Gauge

$$\tilde{J}_L=J_L(q_T r)e^{iL\phi}, \quad K=J_i-J_f, \quad q_0^2=q_z^2+q_T^2=q_z^2+q_x^2+q_y^2, \quad q_0=E_i-E_f$$

κ : z-Comp of Ang, Momentum

$|q_z|\gg q_T$ ⇒ Bessel Wave (OAM: $\kappa - h$, Spi:h)

$|q_z|\ll q_T$ ⇒ **Cylindrical Wave** (OAM: κ , Polarized in z-Dir.)

Two Waves are connected in Lorentz Transformation

$\boldsymbol{A}(h{=}{+}1)$ & $\boldsymbol{A}(h{=}{-}1)$

are not orthogonal

$$\boldsymbol{A}^{(\pm)}\propto\boldsymbol{A}(h=+1)\pm\boldsymbol{A}(h=-1)$$

Orthogonal Wave-Functions

Production of Gamma-Ray Vortex

Inverse Compton Scattering low energy photons to ultrarelativistic electrons

U. D. Jentschura, V. G. Serbo
PRL 106, 013001 (2011)

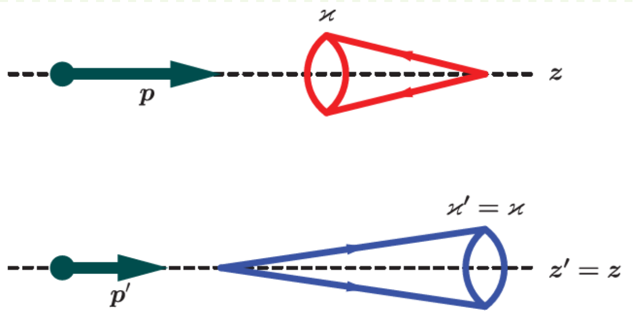
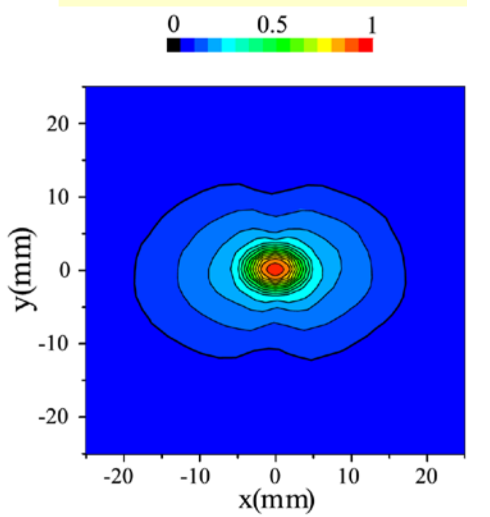


FIG. 2 (color). Initial (above) and final (below) states for the head-on Compton backscattering geometry of a twisted photon.

Initial Photon



U. D. Jentschura, V. G. Serbo
PRL 106, 013001 (2011)

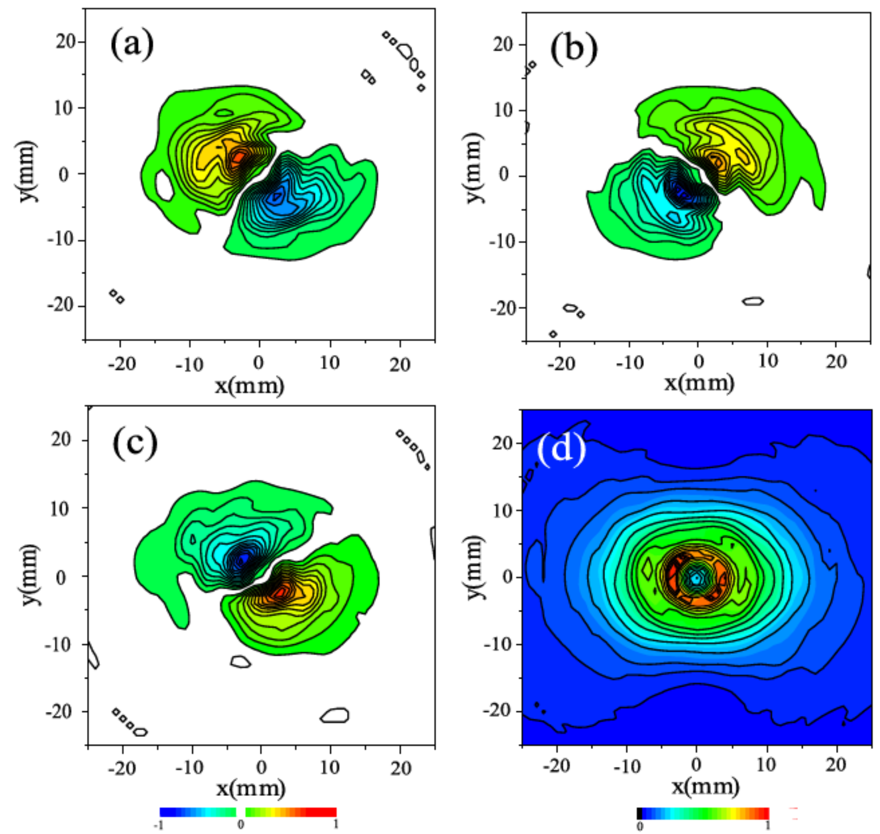


FIG. 4. y component of the electric field $\underline{E}$ of the radiation at different times. (a) $t = 0.2$, (b) $t = 0.4$, (c) $t = 0.6$ ps. (d) average intensity for $m = 1$, $\varepsilon = 1$.

§ 2-2 Bessel Wave

Gauge : $A_0=0$, $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$

$$\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{\epsilon} J_L(q_T r) e^{iL\phi} e^{ip_z z}.\qquad \boldsymbol{\epsilon} = \frac{1}{\sqrt{2}}(1, ih, 0)\qquad h=\pm 1 : \text{helicity}$$

Not Satisfying $\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$

Adding A_z

$$\boldsymbol{A}_{K,h}(\boldsymbol{r}) = \frac{1}{\sqrt{q_0^2+q_z^2}} e^{iq_z z}\left[i(1, ih) q_z J_{K-h}(q_T r) e^{i(K-h)\phi}, -h q_T J_K(q_T r) e^{iK\phi}\right]$$

$$K = J_i - J_f, q_0 = E_i - E_f, q_0^2 = q_z^2 + q_T^2 = q_z^2 + q_x^2 + q_y^2$$

K : z-Comp of Ang, Momentum

$|q_z| \gg q_T \Rightarrow$ Circular Polarized Bessel Wave （OAM: $K-h$, Spi:h）

$|q_z| \ll q_T \Rightarrow$ Linear Polarized **Cylindrical Wave** (OAM: K）

Two Waves are connected in Lorentz Transformation

Bessel Wave 2

$$\boldsymbol{A}_{K,h}(\boldsymbol{r}) = \frac{1}{\sqrt{|\boldsymbol{q}|^2 + q_z^2}} e^{iq_z z} \left[i(1, ih) q_z \tilde{J}_{K-h}, -h q_T \tilde{J}_K \right]$$

$$\tilde{J}_L = J_L(q_T r) e^{iL\phi}$$

$\boldsymbol{A}(\mathrm{h}{=}{+}1) \cdot \boldsymbol{A}(\mathrm{h}{=}{-}1) \neq 0$

$\boldsymbol{A}(h{=}{+}1)$, $\boldsymbol{A}(h{=}{-}1)$
are not orthogonal

$\boldsymbol{A}^{(1)} \propto \boldsymbol{A}(\boldsymbol{h} = +\mathbf{1}) - \boldsymbol{A}(\boldsymbol{h} = -\mathbf{1})$, $\boldsymbol{A}^{(2)} \propto \boldsymbol{A}(\boldsymbol{h} = +\mathbf{1}) + \boldsymbol{A}(\boldsymbol{h} = -\mathbf{1})$

are orthogonal

Orthogonal States

$$A^{(1)} = \frac{1}{2|\boldsymbol{q}|} e^{iq_z z} \left[-iq_z \left(\tilde{J}_{K+1} - \tilde{J}_{K-1} \right), -q_z \left(\tilde{J}_{K+1} + \tilde{J}_{K-1} \right), -2q_T \tilde{J}_K \right].$$

$$A^{(2)} = \frac{1}{2} e^{iq_z z} \left[i \left(\tilde{J}_{K+1} + \tilde{J}_{K-1} \right), \left(\tilde{J}_{K+1} - \tilde{J}_{K-1} \right), 0 \right]$$