Dense nuclear matter based on a chiral model with parity doublet structure

Masayasu Harada (Nagoya University)

@ Quarks and Compact Stars 2019

(September 27, 2019)

Based on

- T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019).
- T. Yamazaki and M. Harada, Phys. Rev. D 99, 034012 (2018).

Introduction

Origin of Mass ?

Origin of Mass of Hadrons ?

of Hadrons

Origin of Mass ?

of Us

of Hadrons

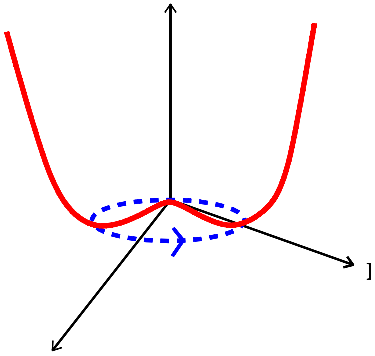
Origin of Mass ?

of Us

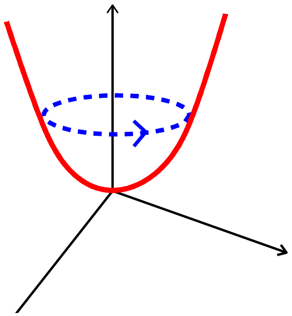
||

One of the Interesting problems of QCD

spontaneous chiral symmetry breaking



chiral symmetry broken phase at vacuum

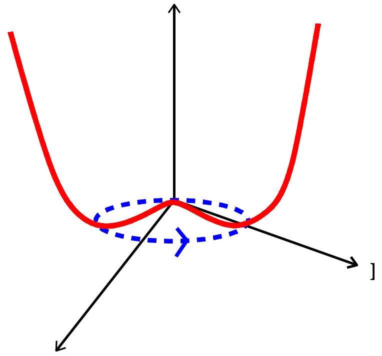


chiral symmetric phase at high T and/or density

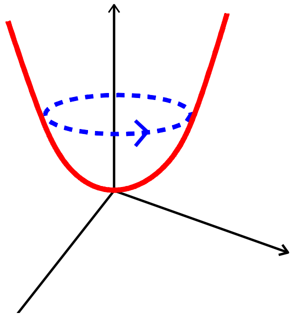
$\langle \bar{q}q \rangle \neq 0$ (chiral condensate)

$\langle \bar{q}q \rangle = 0$

spontaneous chiral symmetry breaking



chiral symmetry broken phase at vacuum



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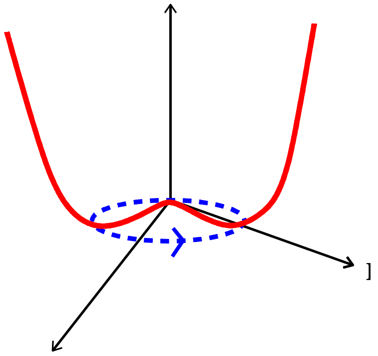
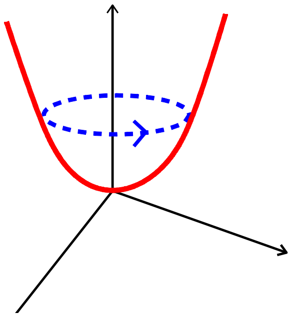
- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

spontaneous chiral symmetry breaking



chiral symmetry
broken phase at
vacuum



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- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of the nucleon ?

Parity Doublet models for nucleons

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of nucleon ?

- A Parity doublet model for light baryons
  - In [C.DeTar, T.Kunihiro, PRD39, 2805 (1989)], N*(1535) is regarded as the chiral partner to the N(939) having the chiral invariant mass.

$$\boldsymbol{m_N = m_0 + m_{\langle \bar{q}q \rangle}}$$

chiral invariant mass

spontaneous chiral symmetry breaking

- This model can be extended to include different excited nucleons.

- We constructed an extended parity doublet model including four light nucleons N(939), N(1440), N(1535) and N(1650).
- We showed that the chiral invariant masses are constrained by the saturation properties of nuclear matter and neutron star properties.
  - T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019).
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Outline

1. Introduction
2. An Extended Parity Doublet Model for Nucleons: Constraints to chiral invariant masses at vacuum
3. Constraints from Nuclear Matter and Neutron Star Properties
4. Summary

2. An Extended Parity Doublet Model for Nucleons: Constraints to chiral invariant masses at vacuum

T. Yamazaki and M. Harada, Phys. Rev. D 99, 034012 (2018).

chiral representation of baryons

- representation of quark under $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$

$$q \sim q_r + q_l \sim (2,1) \oplus (1,2)$$

- representation of baryon under $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$

$$\psi \sim q \otimes q \otimes q \sim \left[(2,1) \oplus (1,2)\right]^3$$
$$= 5\left[(2,1) \oplus (1,2)\right] \oplus 3\left[(3,2) \oplus (2,3)\right] \oplus \left[(4,1) \oplus (1,4)\right]$$

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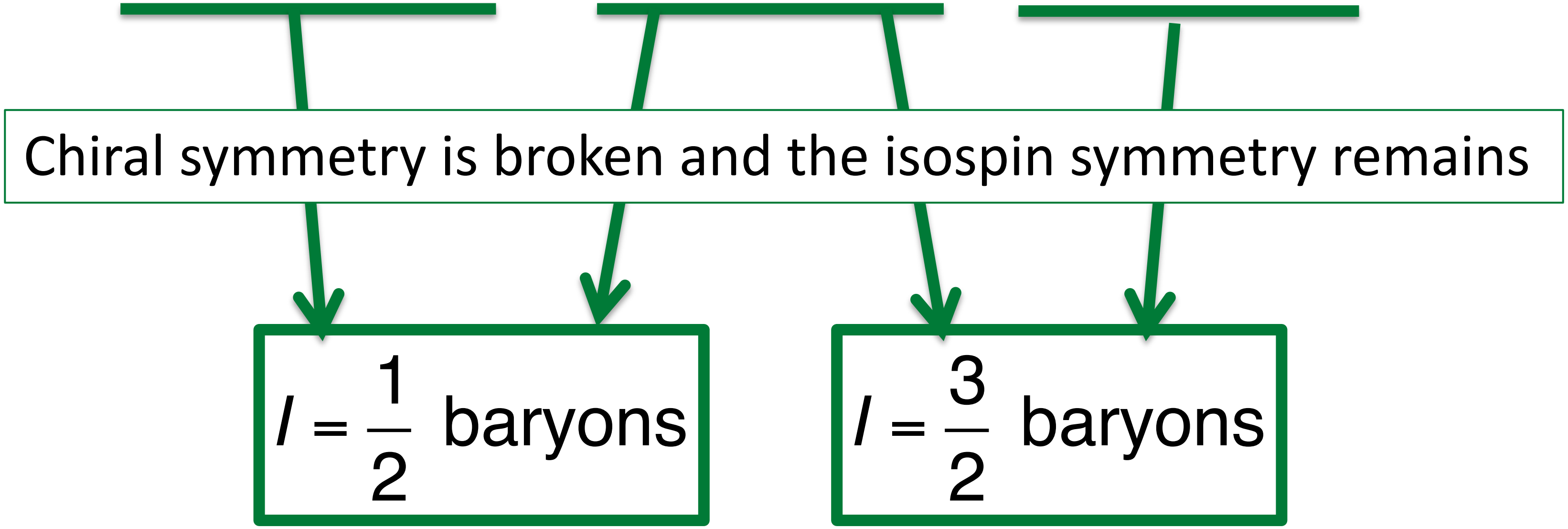


Chiral symmetry is broken and the isospin symmetry remains

$I = \frac{1}{2}$ baryons

$I = \frac{3}{2}$ baryons

Parity Doublet models with $[(2,1)\oplus(1,2)]$ nucleons

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)
D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)
S. Gallas, F. Giacosa, D. Rischke, PRD82, 014004 (2010)

- An excited nucleon with negative parity such as N(1535) is regarded as the chiral partner to the N(939).
- N(939) and N(1535) have a chiral invariant mass:
  - ➢ $m_0[\bar{\psi}_1\gamma_5\psi_2 - \bar{\psi}_2\gamma_5\psi_1]$
- Spontaneous chiral symmetry breaking generates the mass difference between chiral partners.
  - ➢ $-g_1[\bar{\psi}_{1l}M\psi_{1r} + \bar{\psi}_{1r}M^\dagger\psi_{1l}] - g_2[\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}]$
    - $M = \sigma + i\,\vec{\tau}\cdot\vec{\pi}$ transforms $M \to g_L\, M\, g_R^\dagger$
    - $\langle M\rangle = \bar{\sigma} \neq 0$ causes the spontaneous chiral symmetry breaking.

$$m_\pm = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2\,\bar{\sigma}^2 + 4m_0^2} \mp (g_2 - g_1)\,\bar{\sigma}\right]$$

$m_+ = m(N(939))$
$m_- = m(N(1535))$

A model with [(1,2)⊕(2,1)] and [(2,3)⊕(3,2)] representations

T. Yamazaki and M.Harada, Phys. Rev. D99, 034012 (2018)

- We include two representations, $\psi \in [(1,2)\oplus(2,1)]$ and $\eta \in [(2,3)\oplus(3,2)]$ to study N(939), N(1440), N(1535), N(1650).
- There are 2 chiral invariant masses.
  - $-m_0^{(1)}[\bar{\psi}_1\gamma_5\psi_2 - \bar{\psi}_2\gamma_5\psi_1] - m_0^{(2)}[\bar{\eta}_1\gamma_5\eta_2 - \bar{\eta}_2\gamma_5\eta_1]$
- 6 Yukawa Interactions
  - $-g_1\left[\bar{\psi}_{1l}M\psi_{1r} + \bar{\psi}_{1r}M^\dagger\psi_{1l}\right] - g_2\left[\bar{\psi}_{2r}M\psi_{2l} + \bar{\psi}_{2l}M^\dagger\psi_{2r}\right]$ etc.
- We also have 4 terms with one derivative.
  - $a_1\left[\bar{\psi}_{1l}\gamma^\mu\partial_\mu M\psi_{2l} - \bar{\psi}_{1r}\gamma^\mu\partial_\mu M^\dagger\psi_{2r}\right]$ etc

Physical inputs

We first fix the values of the chiral invariant masses $m_0^{(1)}$ and $m_0^{(2)}$ to some constants, and use the following 10 physical inputs to determine 10 parameters (10 couplings).

Masses $m_{N(939)} = 939\mathrm{MeV}$ $m_{N(1440)} = 1430\mathrm{MeV}$
$m_{N(1535)} = 1535\mathrm{MeV}$ $m_{N(1650)} = 1650\mathrm{MeV}$

Decay widths

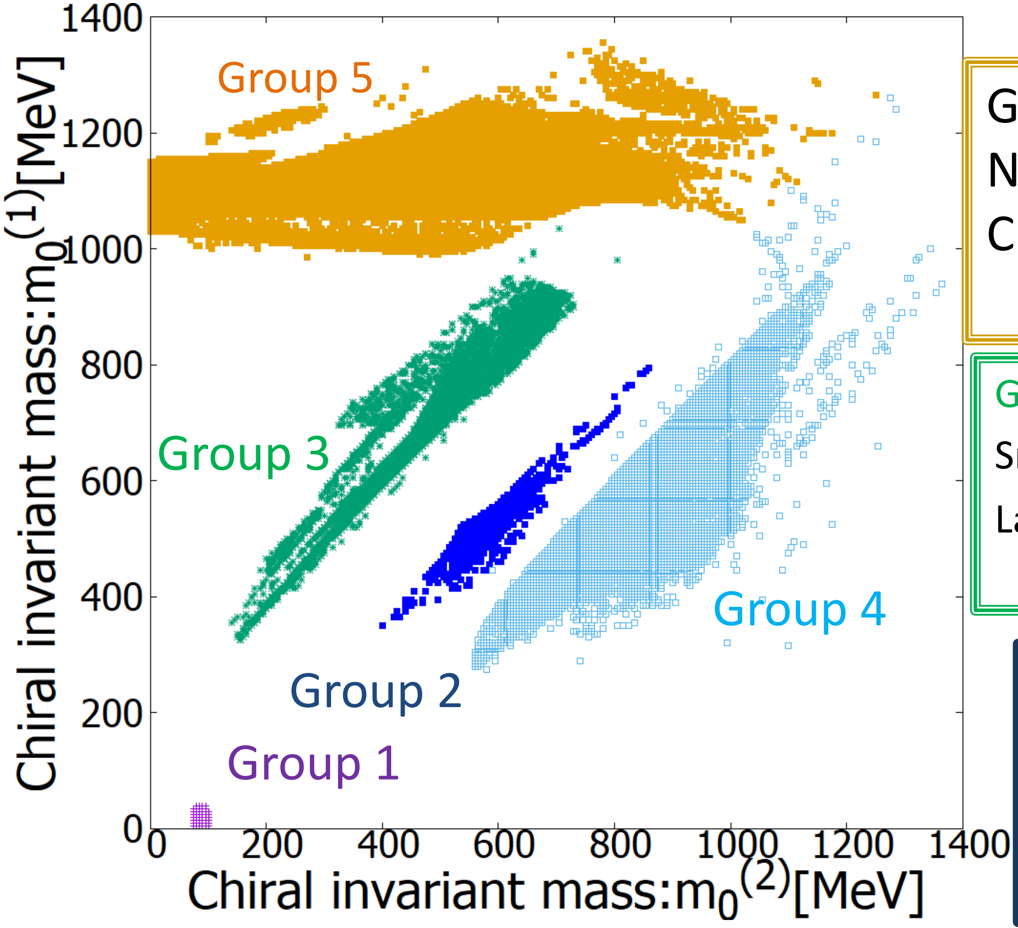
$$\Gamma(N(1440) \to N(939) + \pi) = 228\mathrm{MeV}$$
$$\Gamma(N(1535) \to N(939) + \pi) = 68\mathrm{MeV}$$
$$\Gamma(N(1650) \to N(939) + \pi) = 84\mathrm{MeV}$$
$$\Gamma(N(1650) \to N(1440) + \pi) = 22\mathrm{MeV}$$

Axial charges $g_A(N(939)) = 1.27$
$g_A(N(1650)) = 0.55$ (Lattice analysis [T.T.Takahashi, T.Kunihiro, PRD78 (2008)])

Constraint $-0.25 \leq g_A(N(1535)) \leq 0.25$

(Lattice analysis [T.T.Takahashi, T.Kunihiro, PRD78 (2008)] shows $g_A \sim O(0.1)$.

Chiral invariant masses & Chiral partner structure



5 Groups of solutions

Group 4, 5

N(939) is dominated by $[(1,2)\oplus(2,1)]$

Chiral partner to N(939)

= a mixture of N(1535) & N(1650)

Group 3

Small $m_0^{(2)}$: N(939) is dominated by (1,2)⊕(2,1).

Large $m_0^{(2)}$: N(939) is dominated by (2,3)⊕(3,2).

Chiral Partner ≈ N(1535)

Group 2

N(939) is dominated by (2,3)+(3,2)

Chiral partner to N(939)

= a mixture of 3 nucleons

Group 1

N(939) is dominated by (2,3)+(3,2) representation.

Chiral partner to n(939) = N(1440)

Prediction - Axial charges -

- In our model the following relation is satisfied:
  - $\sum_{i=1}^{4} g_A(N_i) = 0$

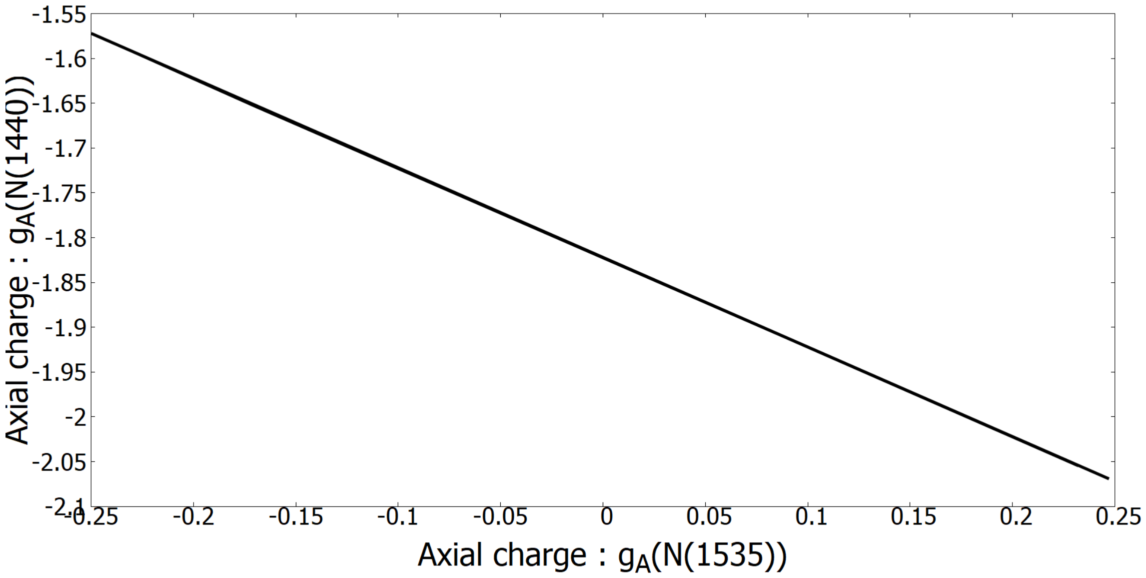
Constraint

$$-0.25 \leq g_A(N(1535)) \leq 0.25$$

Input $g_A(N(939)) = 1.27$
$g_A(N(1650)) = 0.55$

$$\Downarrow$$

$$-2.07 \leq g_A(N(1440)) \leq -1.57$$



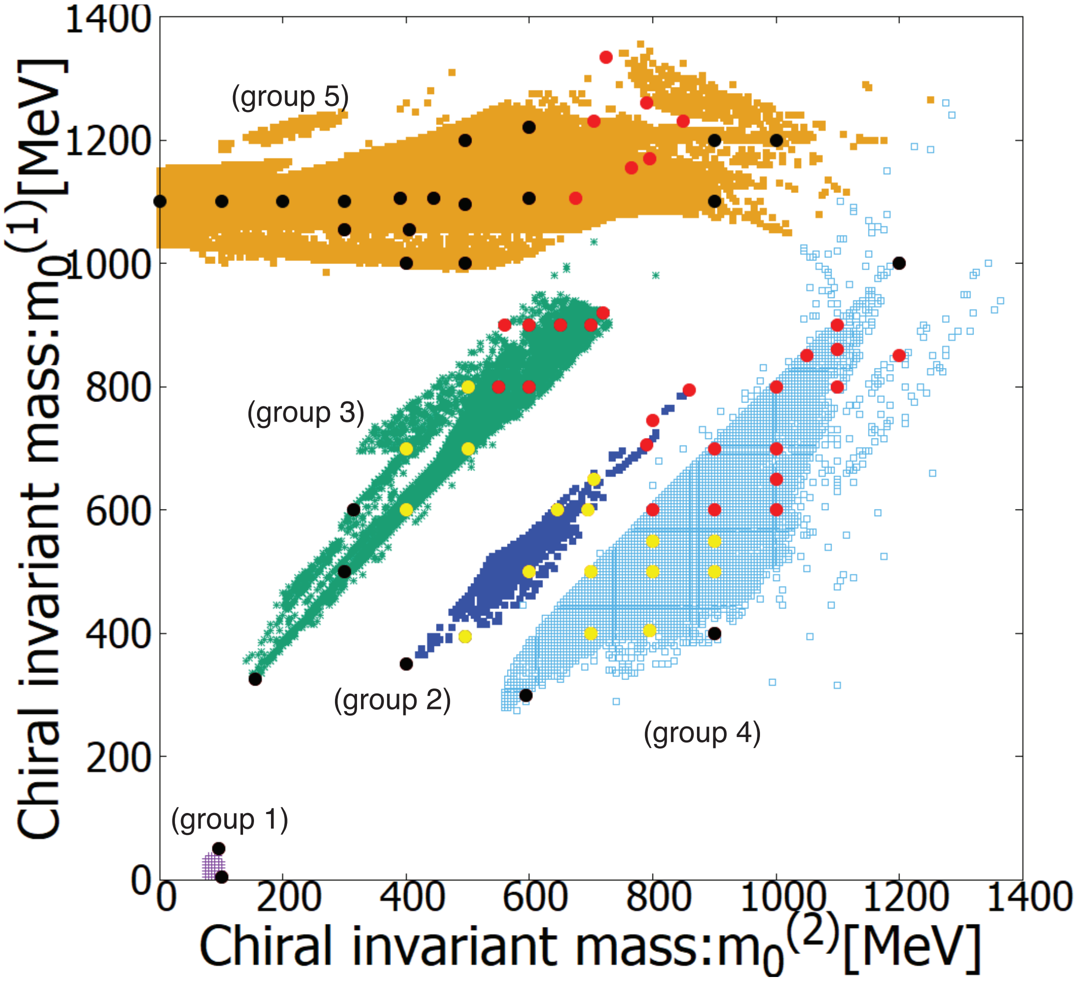
3. Constraints from Nuclear Matter and Neutron Star Properties

T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019)

Construction of Nuclear Matter

- We include the omega and rho mesons into our model using the hidden local symmetry.
- We calculate the thermodynamic potential in the nuclear medium in our model, using the mean field approximation.
- Then, we adjust model parameters to reproduce the following physical inputs for given values of the chiral invariant masses $m_0^{(1)}$ and $m_0^{(2)}$.
- Nuclear saturation density
  - $\rho(\mu_B^* = 923\mathrm{MeV}) = \rho_0 = 0.16\mathrm{fm}^{-3}$
- Binding energy at normal nuclear density
  - $\left[\frac{E}{A} - m(939)\right]_{\rho_0} = \left[\frac{\varepsilon}{\rho_B} - m(939)\right]_{\rho_0} = -16\mathrm{MeV}$
- Incompressibility
  - $K = 9\rho_0^2 \frac{\partial^2 (E/A)}{\partial \rho^2}\Big|_{\rho_0} = 9\rho_0 \frac{\partial \mu_B}{\partial \rho}\Big|_{\rho_0} = 240\mathrm{MeV}$
- Symmetry energy
  - $E_{\mathrm{sym}}(\rho_0) = 31$ MeV

Constraint to model parameters

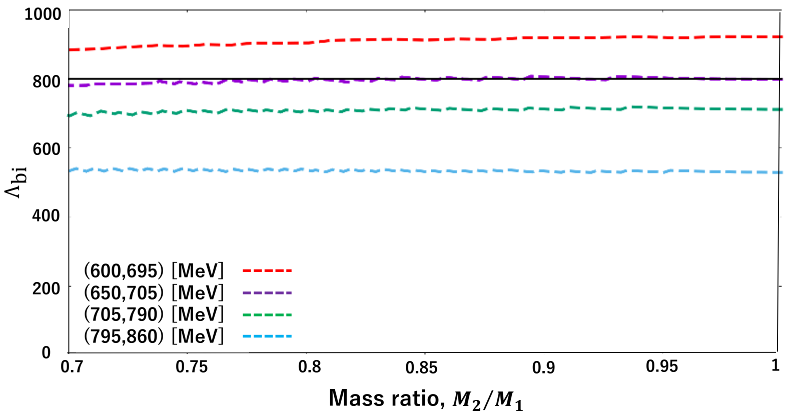


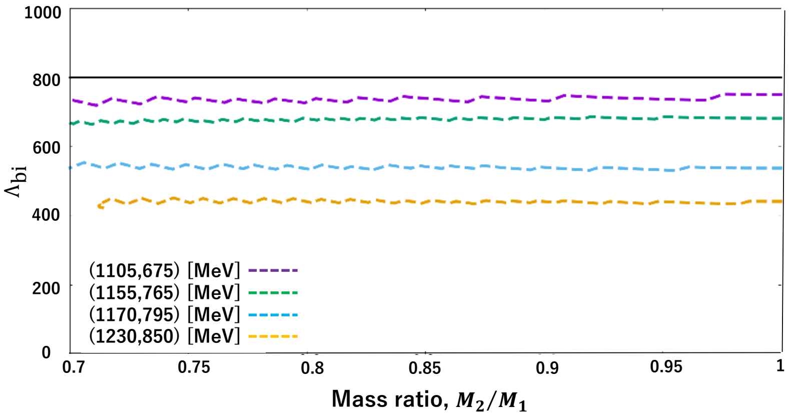
- We checked whether the saturation properties are satisfied for the parameter choices indicated by ● ● ● marks.
- We found that, for the parameter choices indicated by ● marks, the saturation properties are NOT satisfied.
- So, these parameter choices are excluded.
- In particular, please note that the parameter choices in Group 1 are all excluded.

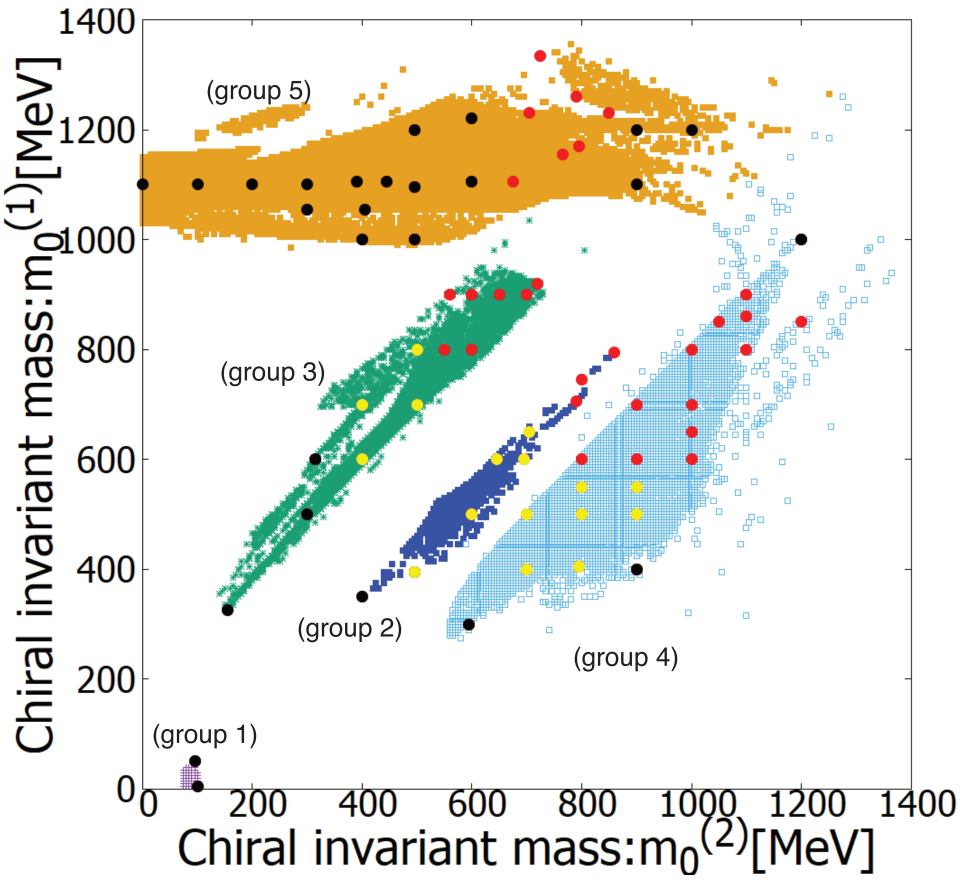
Constraint from Neutron Star Properties

- We obtained constraint to the chiral invariant masses from the tidal deformability of Neutron Stars.
  - Tidal deformability: $\tilde{\Lambda} \leq 800$ with $M_{\mathrm{chirp}} = 1.188\, M_{\odot}$

- ● (yellow dots) are excluded, and ● (red dots) are allowed

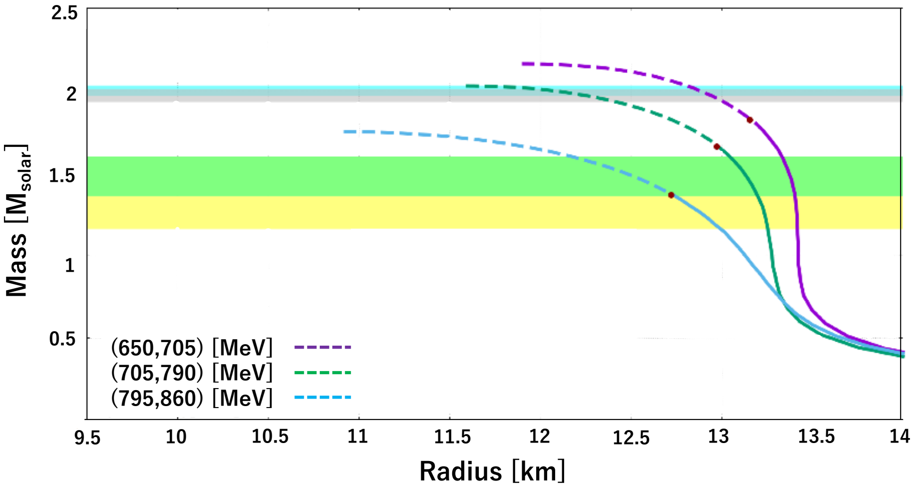




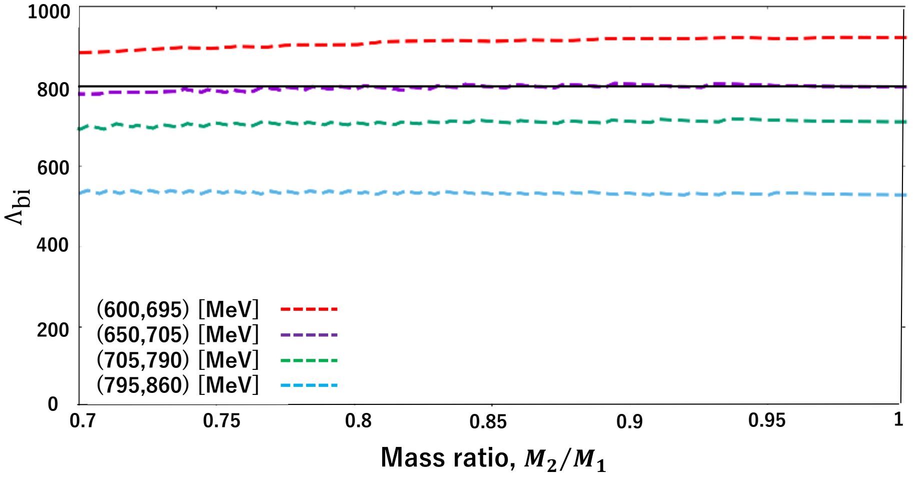


Mass-Radius Relation

- M-R relation



- Tidal deformability



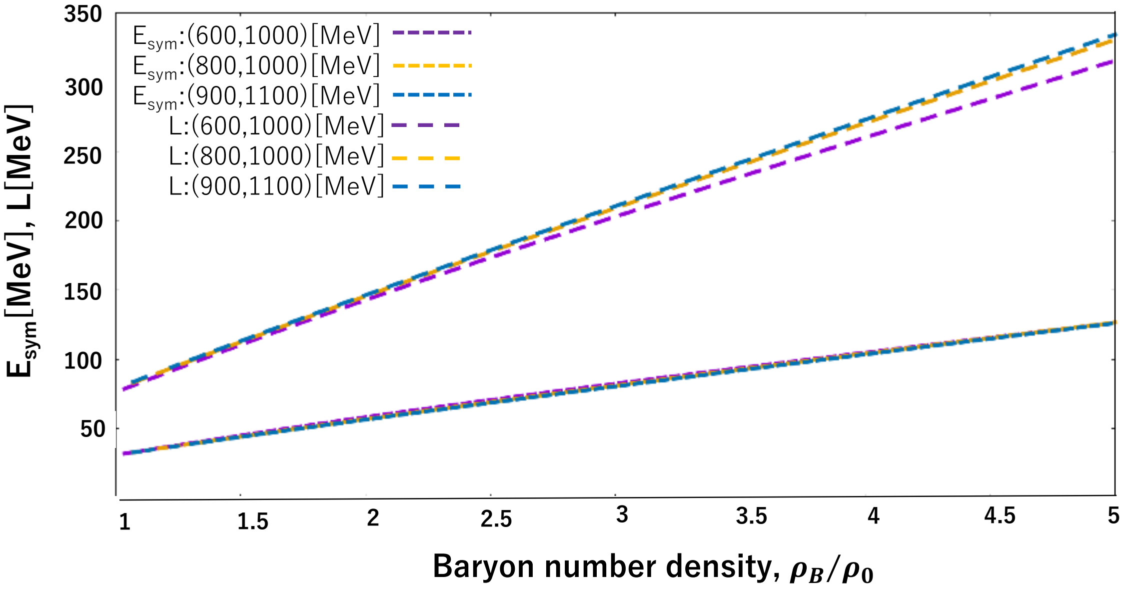
The reason why smaller chiral invariant masses are excluded.

- The attractive force mediated by sigma contribution is larger for smaller chiral invariant masses. $$m_\pm = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2\bar{\sigma}^2+4m_0^2}\mp(g_2-g_1)\bar{\sigma}\right]$$
- The repulsive force mediated by omega contribution is then larger for larger sigma contribution to satisfy the saturation properties of normal nuclear matter.
- The attractive force by the sigma contribution becomes smaller for larger density, while the repulsive force by the omega contribution becomes larger.
- The larger repulsive force makes the radius and tidal deformability larger.
- As a result, the smaller chiral invariant masses cause the larger tidal deformability.

Symmetry energy and Slope parameter

- We include ρ meson into the model and obtain the symmetry energy and the slope parameter.
  - $E_{\mathrm{sym}} = \frac{\rho_B}{8}\left(\frac{2\pi^2}{\sum_{N,j} k_{FN}^{(i)} E_{FN}^{(i)}} + \frac{g_{\rho NN}^2}{m_\rho^2}\right)$; $E_{FN}^{(i)} = \sqrt{\left(k_{FN}^{(i)}\right)^2 + \left(m_*^{(I)}\right)^2}$
  - $L = 3\rho_B\left[\frac{1}{8}\left(\frac{2\pi^2}{kE} + \frac{g_{\rho NN}^2}{m_\rho^2}\right) + \frac{\rho_B}{8}\frac{\pi^2}{2k^2}\left(-\frac{2\pi^2}{k^2E} - \frac{2\pi^2}{E^3}\right)\right]$

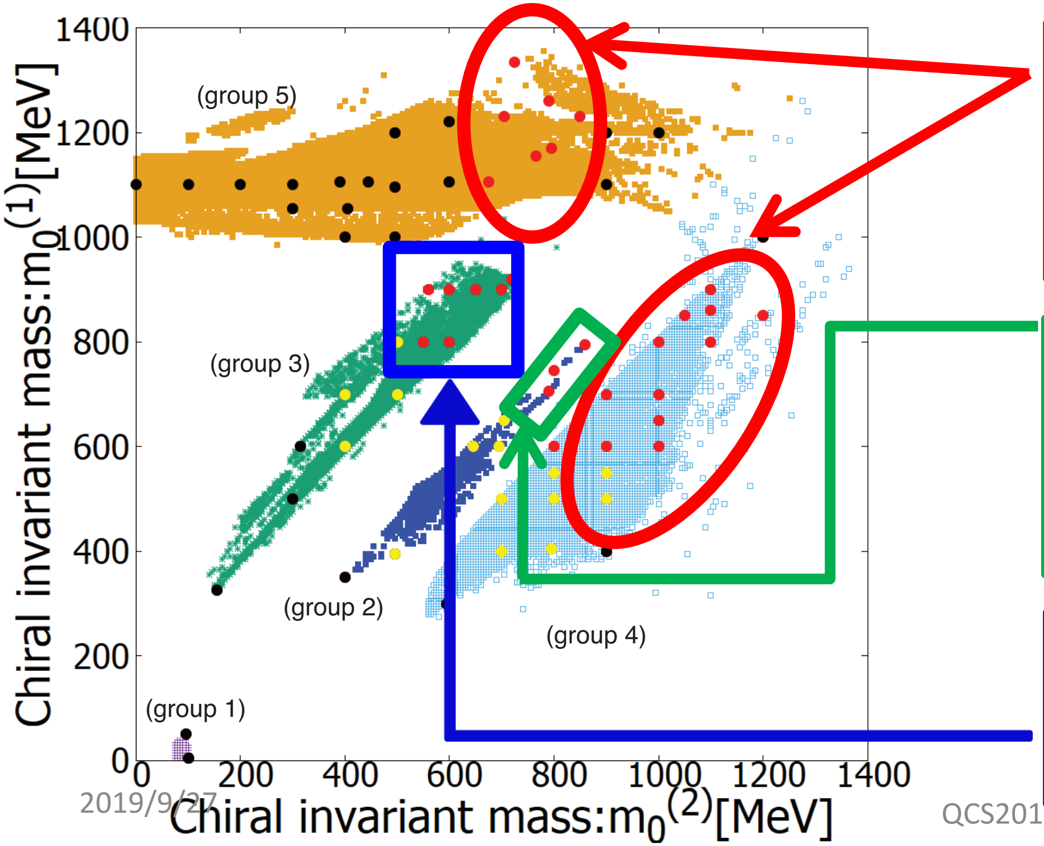
Predictions of the model



- The slope parameter has very little dependence on the chiral invariant mass.
- The symmetry energy does not depend on the choice of chiral invariant masses.
- Both increase linearly with density.

4. Summary

- We constructed an extended parity doublet model including two representations, $\psi \in [(1,2)\oplus(2,1)]$ and $\eta \in [(2,3)\oplus(3,2)]$ to study N(939), N(1440), N(1535), N(1650).
- We use masses, decay widths and axial charges to constrain 2 chiral invariant masses, and found that possible combinations are categorized into 5 groups.
- We exclude some values of the chiral invariant masses by requiring the saturation properties of normal nuclear matter indicated by black dots.
- We further obtain more constraints from the tidal deformability determined by the observation of the gravitational waves from neutron star merger GW170817



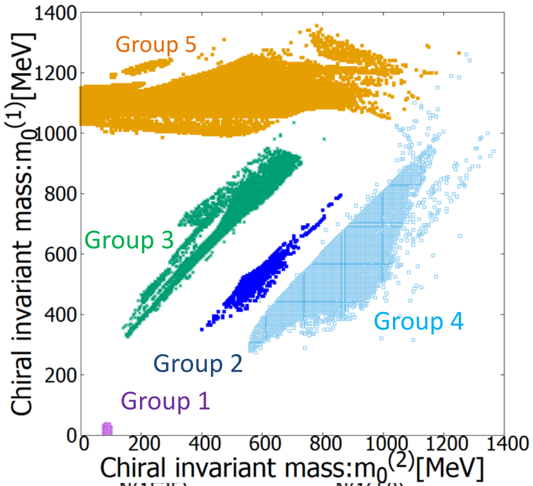
N(939) is dominated by$(1,2)\oplus(2,1)$, and the chiral partner to N(939) is a mixture of N(1535) and N(1650).

N(939) is dominated by$(2,3)\oplus(3,2)$, and the chiral partner to N(939) is a mixture of N(1440), N(1535) and N(1650)

N(939) is dominated by $(2,3)\oplus(3,2)$, and the chiral partner to N(939) is N(1535)

The End

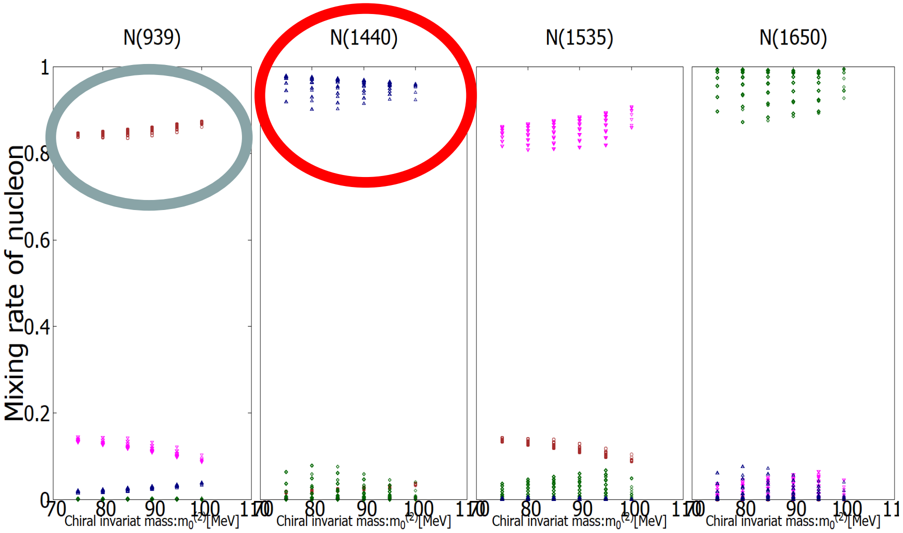
Mixing Rates of Nucleons



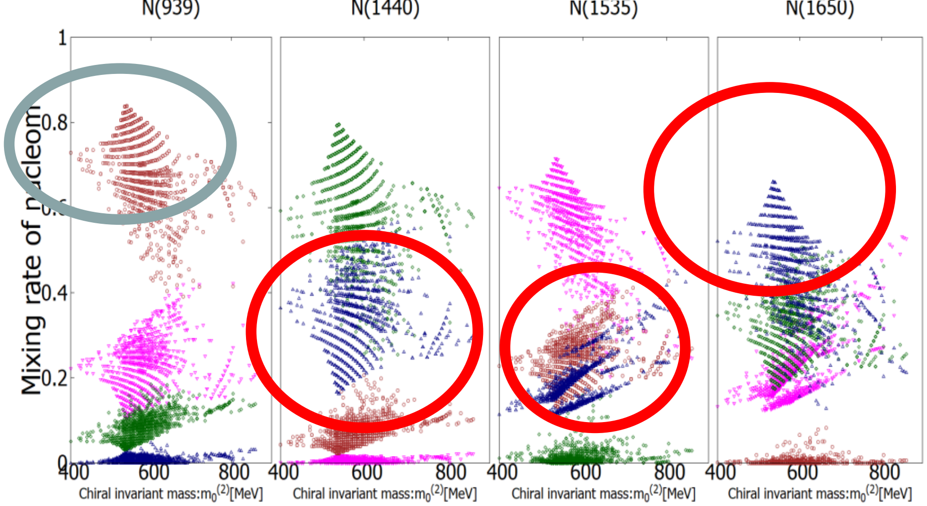
$(1,2)\oplus(2,1):\psi_1$ ▽ ψ_2 ◇

$(2,3)\oplus(3,2):\eta_1$ ○ η_2 △

Group 1



Group 2

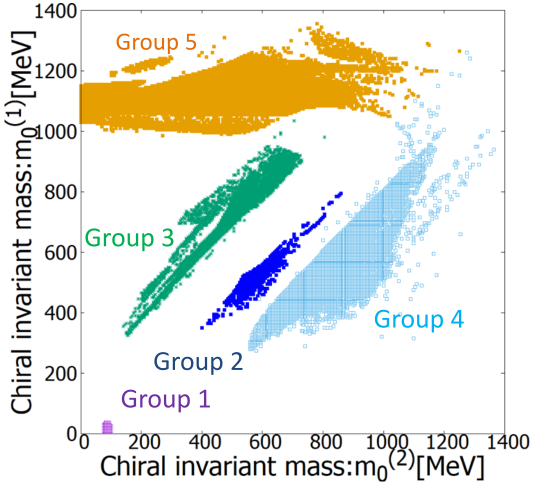


N(939) is dominated by[(2,3)⊕(3,2)]representation.

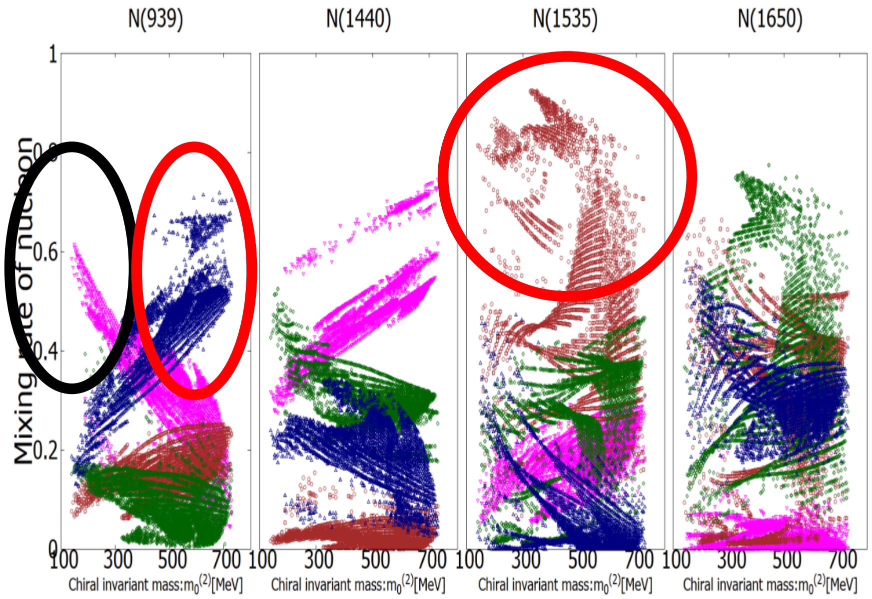
N(1440) is also dominated by [(2,3)⊕(3,2)] representation. [> 80 %]
⇒ Chiral partner to N(939) = N(1440)

N(1440) includes large amount of (1,2)+(2,1) representation. [[(2,3)⊕(3,2)]< 60%]
⇒ Chiral partner to N(939) = mixture of N(1440)＋N(1535)＋N(1650)

Mixing Rate of Nucleons NO.2



Group 3



(1,2)⊕(2,1) : ψ_1 ▽ ψ_2 ◇

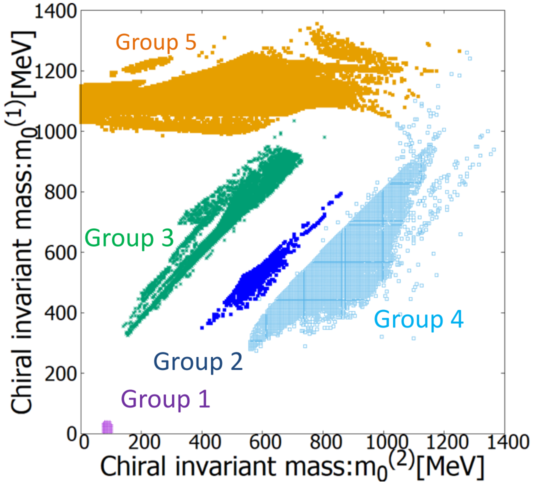
(2,3)⊕(3,2) : η_1 ○ η_2 △

- Small $m_0^{(2)}$: N(939) is dominated by (1,2)⊕(2,1).
- Large $m_0^{(2)}$: N(939) is dominated by (2,3)⊕(3,2).
  - Chiral Partner ≈ N(1535)

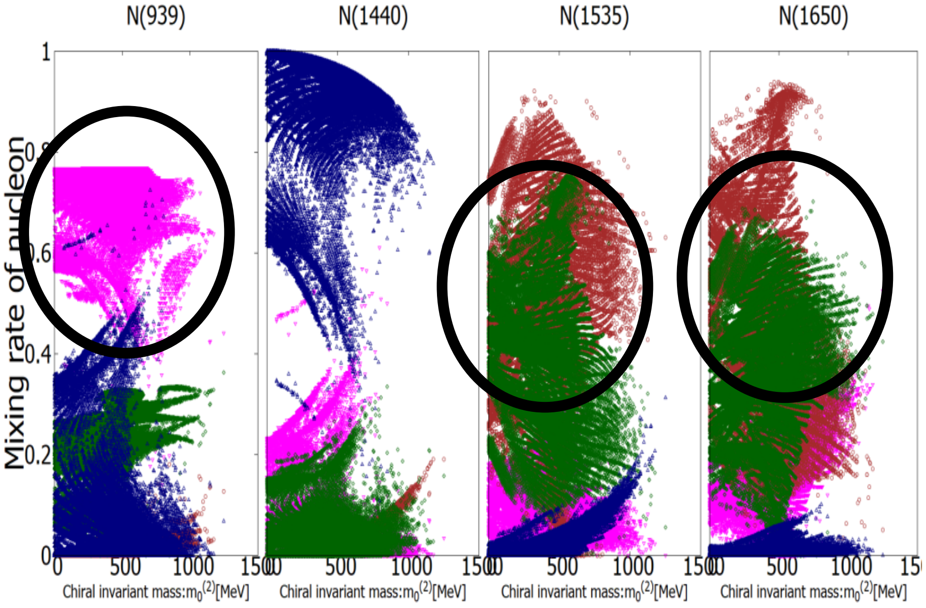
Mixing Rate of Nucleons NO.3

$(1,2)\oplus(2,1) : \psi_1$ ▽ ψ_2 ◇

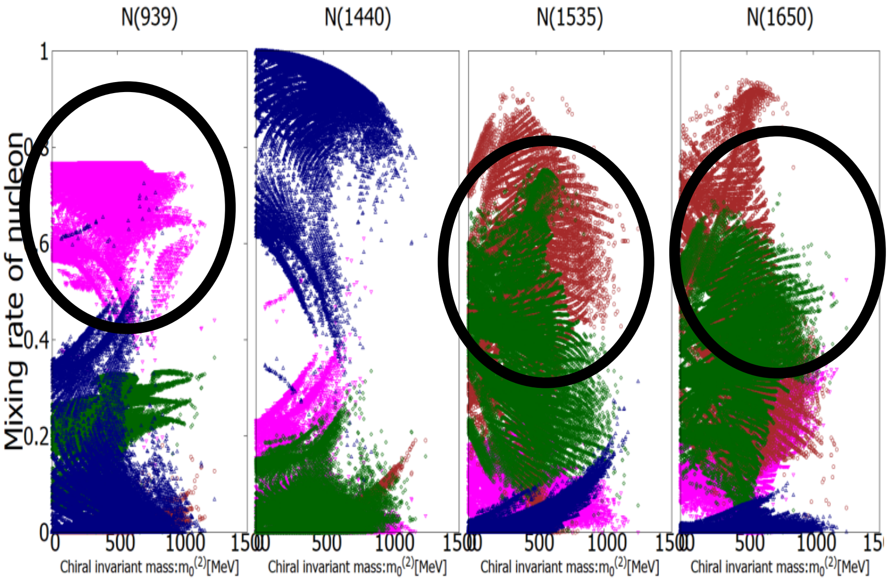
$(2,3)\oplus(3,2) : \eta_1$ ○ η_2 △



Group 4



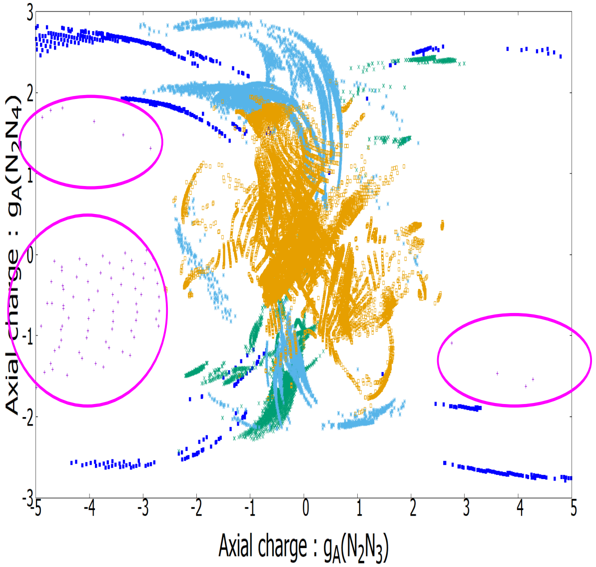
Group 5



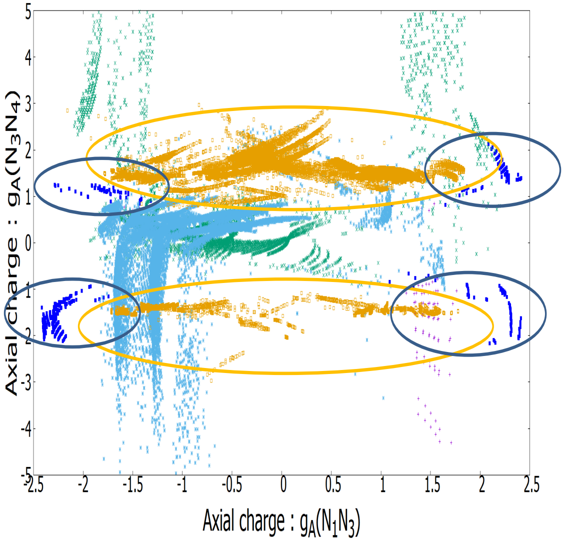
- N(939) is dominated by $(1,2)\oplus(2,1)$.
  - Chiral Partner ≈ a mixture of N(1535) and N(1650)

Transition Axial-Charges

- We calculated transition axial-charges:
  - $g_A(N_2(1440) - N_3(1535))$, $g_A(N_2(1440) - N_4(1650))$,
  - $g_A(N_1(939) - N_3(1535))$, $g_A(N_3(1535) - N_4(1650))$,
  - $g_A(N_1(939) - N_4(1650))$, $g_A(N_1(939) - N_2(1440))$,
- We find some features.

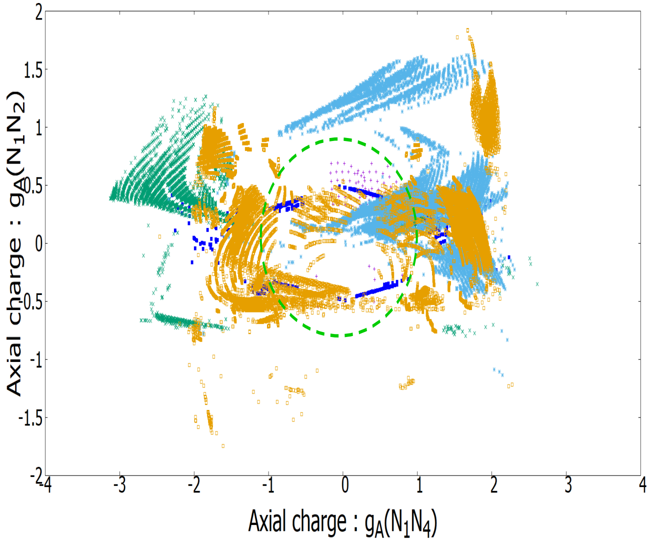


Predictions of Group 1 are separated from those from other groups.



$g_A(N_3N_4)$ [for Group 2 and Group 5]≈ 2

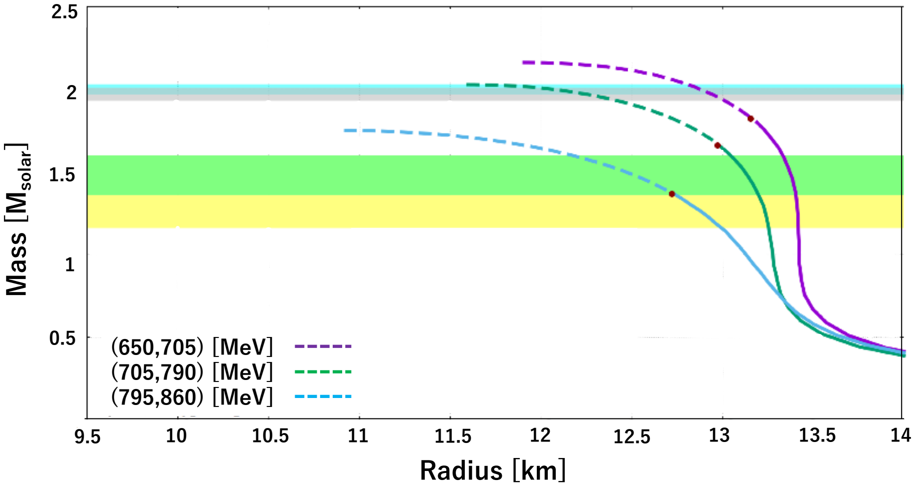
$g_A(N_1N_3)$ for Group 2 is large.

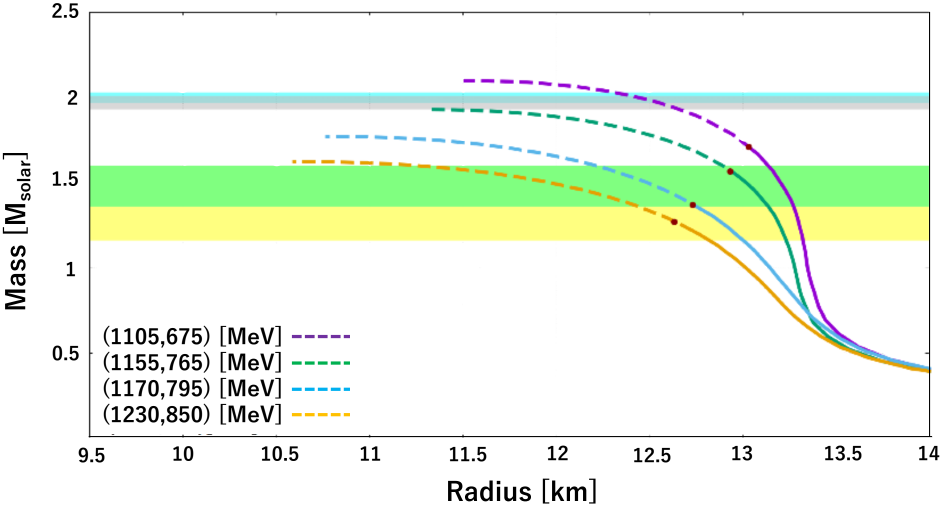


No Group 3 inside ◌

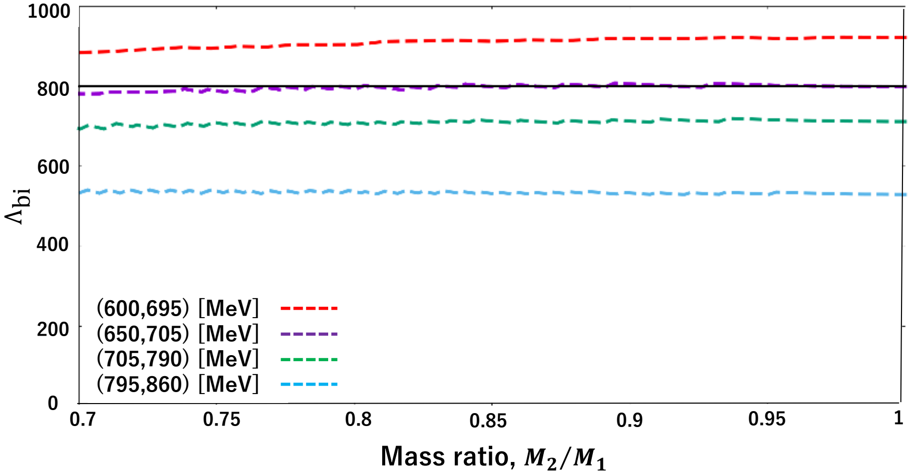
Neutron Star Properties

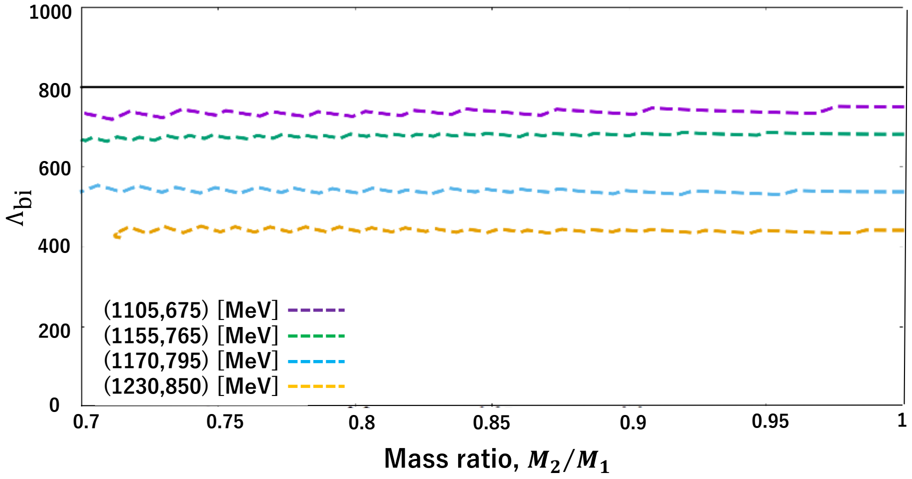
- M-R relation





- Tidal deformanility





Neutron Star Properties 2

- M-R relation
- Central density