Toward solving the sign problem in dense QCD matter via the path optimization method

Collaborator

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Y. Mori, K.K. and A. Ohnishi, Phys. Rev. D 96 (2017) 111501

Y. Mori, K.K. and A. Ohnishi, PTEP 2018 (2018) 023B04

K.K. Y. Mori and A. Ohnishi, Phys. Rev. D 99 (2019) 014033

K.K. Y. Mori and A. Ohnishi, Phys. Rev. D 99 (2019) 114005

Y. Mori, K.K. and A. Ohnishi, arXiv:1904.11140, to be published in PTEP

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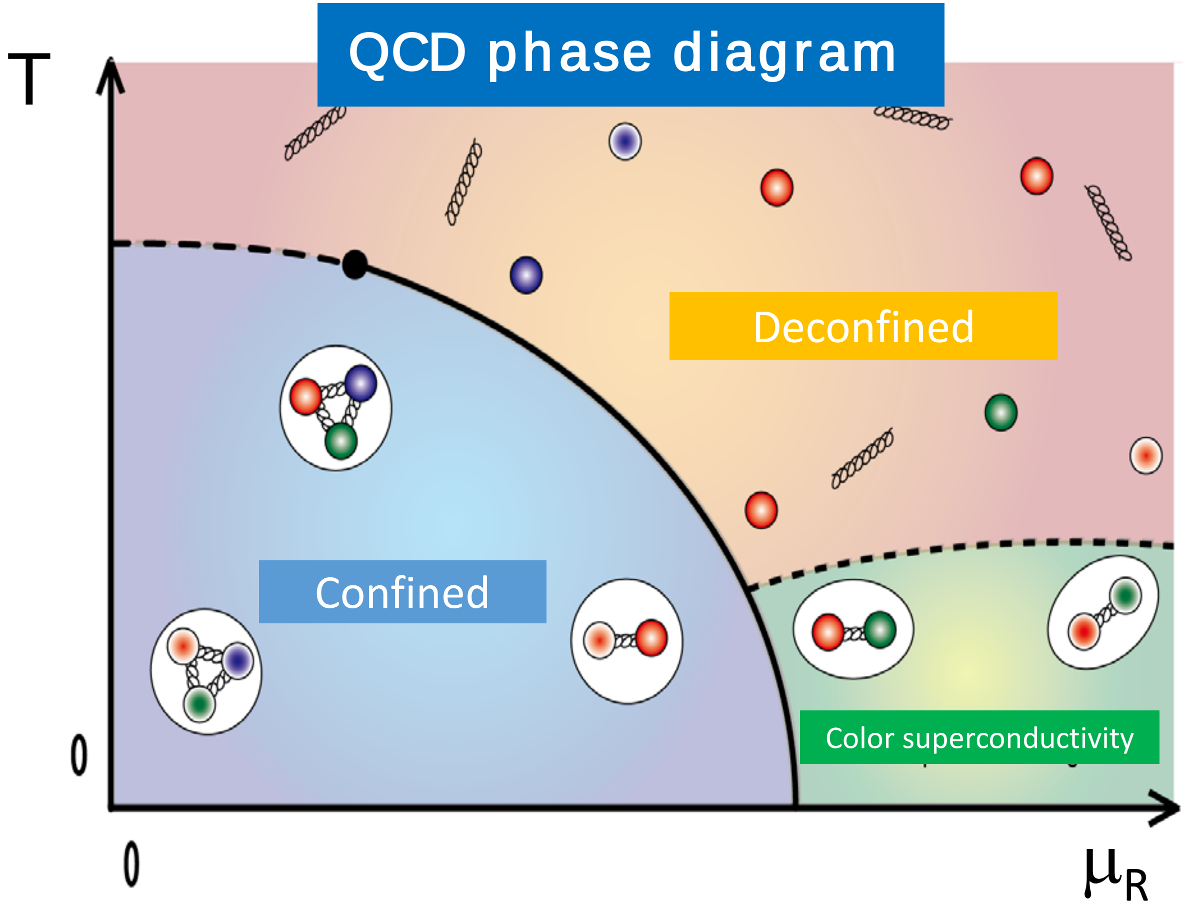
Collaborator Yuto Mori (Kyoto U.)

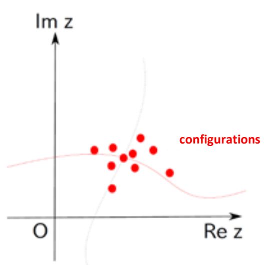
Goal: Understanding the QCD phase diagram



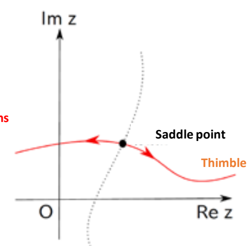
It shows which phase is realized at certain energy scale

Purpose of my (recent) works

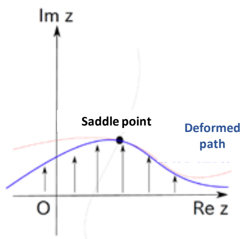
Control the sign problem



Complex Langevin



Lefschetz thimble



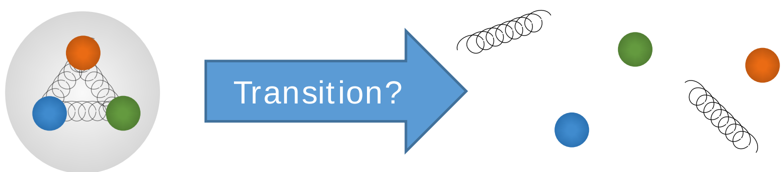
Path optimization

At present, we can not obtain any reliable lattice data at high density…

Understand the deconfinement transition

Confined phase

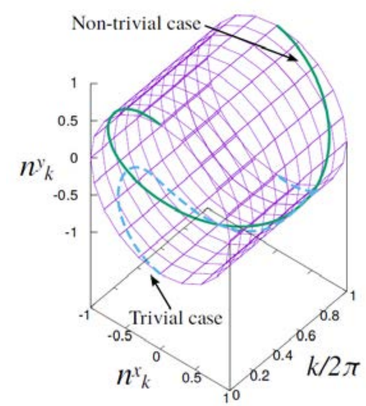
Deconfined phase



How to describe it in QCD

Topology?

(Uhlmann phase)



K.K., A. Ohnishi,
Phys. Lett. B750 (2015) 282

K.K., A. Ohnishi,
Phys. Rev D. 93 (2016) 116002

K.K., A. Ohnishi,
Phys. Lett. B772 (2017) 669

T.M.Doi and K.K., in progress

Purpose of my (recent) works

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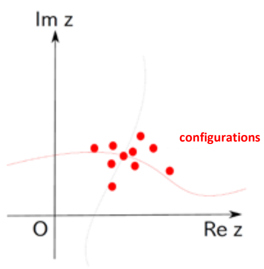
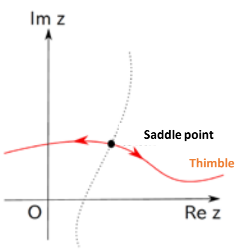
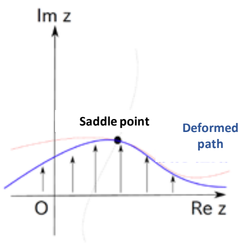






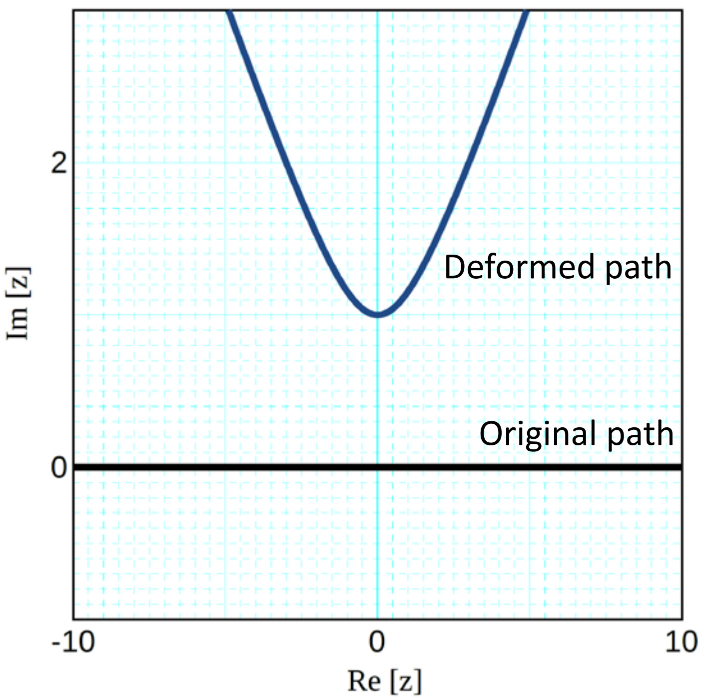
Complex Langevin | Lefschetz thimble | Path optimization

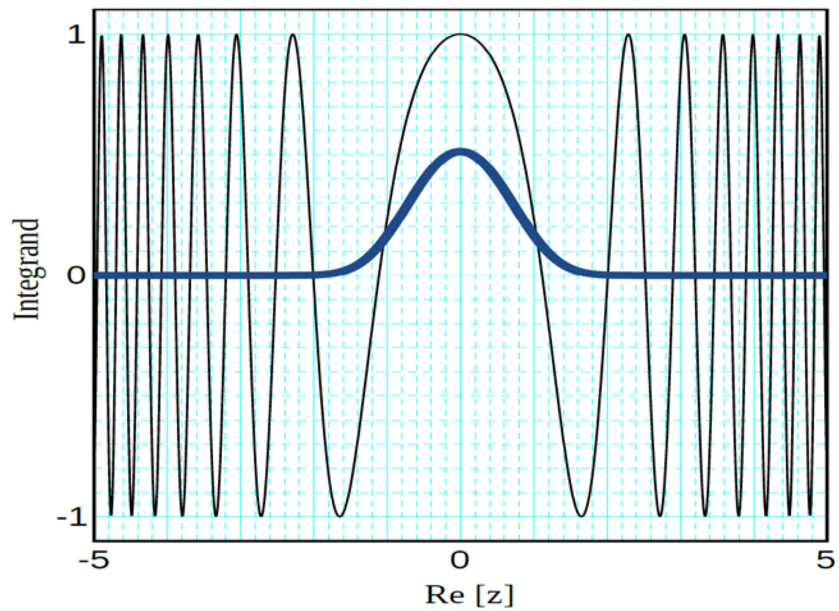
Today's topic!

Sign problem

Strong cancellation in the numerical integration process

Ex.: Airy integral $\mathrm{Ai}(a) = \int_{\mathbb{R}} \frac{\mathrm{d}x}{2\pi} \exp \mathrm{i} \left(\frac{x^3}{3} + ax \right)$





See also
Witten, AMS/IP Stud. Adv. Math. 50 (2011) 347

QCD grand partition function

Dirac operator

Real chemical potential

$$\mathcal{Z} = \int \mathcal{D}A \; \mathrm{Det}[\not{D}(A,\mu) + m] \; e^{-S_{YM}(A)}$$

Quark contribution $\in \mathbb{C}$

Gluon contribution $\in \mathbb{R}$

At finite real chemical potential,

QCD Boltzmann weight becomes **complex**

Phase reweighting

$$\langle O \rangle = \frac{\int O e^{-S} dx}{\int e^{-S} dx} = \frac{\int O \frac{e^{-S}}{|e^{-S}|} |e^{-S}| dx}{\int \frac{e^{-S}}{|e^{-S}|} |e^{-S}| dx} = \frac{\langle O e^{i\theta} \rangle_{pq}}{\langle e^{i\theta} \rangle_{pq}}$$

If $\langle e^{i\theta} \rangle_{pq}$ becomes smaller and smaller,

it becomes difficult to extract the expectation value with small error bar!

Therefore, the simple phase reweighting can not help us at high density

Recent progress

- Complex Langevin method

G. Parisi and Yong-shi Wu, Sci. Sin., 24, 483 (1981)
G. Parisi, Phys. Lett., B131, 393 395 (1983)

- Lefschetz thimble method

E. Witten, AMS/IP Stud. Adv. Math. 50, 347 446 (2011)
M. Cristoforetti, et al. Phys. Rev. D86, 074506 (2012)
H. Fujii, et al., JHEP, 1310, 147 (2013)

Point : Complexification of variables of integration

$$x_i \longrightarrow z_i \in \mathbb{C}$$

Recent progress

- Complex Langevin method
- Lefschetz thimble method

There is one more method with complexification!

- **Path optimization method**

Our method

Y. Mori, K.K. and A. Ohnishi, Phys. Rev. D 96 (2017) 111501

Strategy of the path optimization method

1. Prepare suitable cost function

   It reflects the seriousness of the sign problem

2. Modify the integral path in the complex domain

3. Select the better integral-path

Y. Mori, K.K. and A. Ohnishi, Phys. Rev. D 96 (2017) 111501

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Sign problem ⇨ Optimization problem

Y. Mori, K.K. and A. Ohnishi, Phys. Rev. D 96 (2017) 111501

Y. Mori, K.K. and A. Ohnishi, PTEP 2018 (2018) 023B04

Cost function

It reflects the seriousness of the sign problem

We use following form;

$$\mathcal{F}[z(t)] = \frac{1}{2}\int dt|e^{i\theta(t)} - e^{i\theta_0}|^2 \times |J(t)e^{-S(z(t))}|$$

Parametric variable

It aligns the phase at each t

Weight

If the cost function becomes small,

the average phase factor becomes large!

Y. Mori, K. K. and A. Ohnishi, Phys. Rev. D 96 (2017) 111501

Y. Mori, K. K. and A. Ohnishi, PTEP 2018 (2018) 023B04

Our task

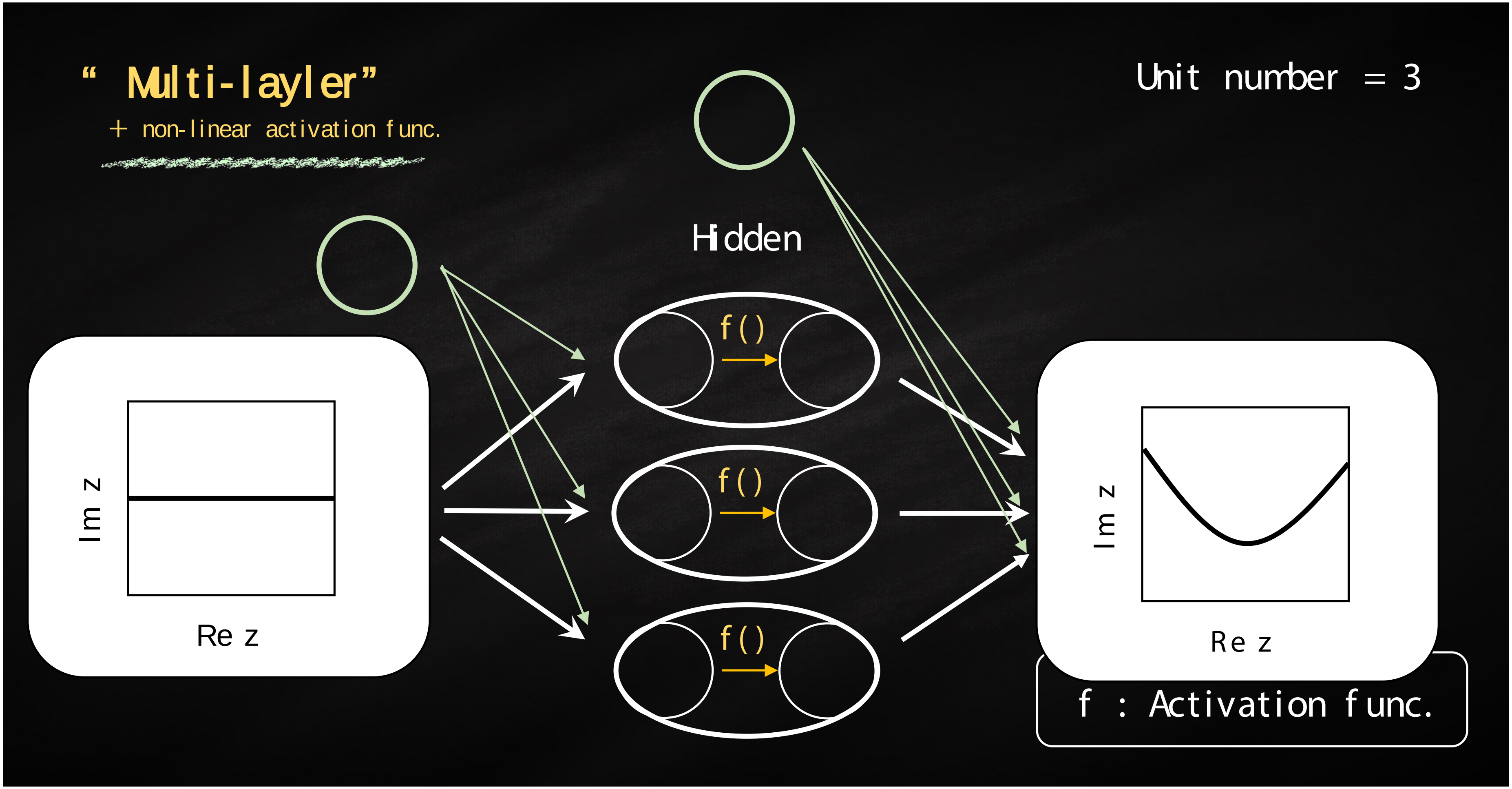
To find a good integral path via minimization of the cost function

Optimization of the integral path is usually very difficult…

(We have so many degree of freedom in quantum field theory)

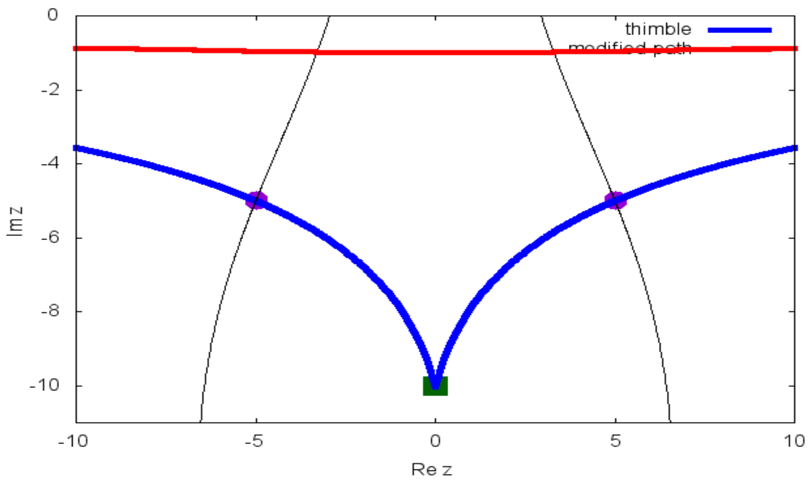
➡ Machine learning **is helpful and promising approach!**

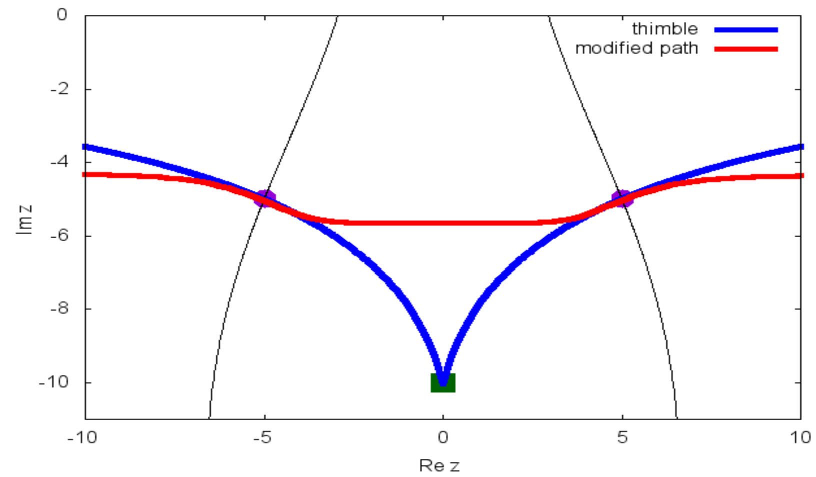
Explanation of Neural network: Neural network

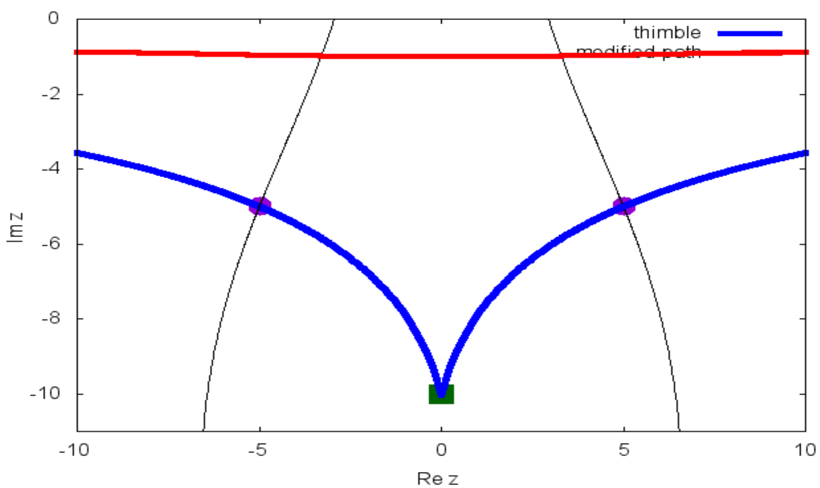


The real part of the integral path is input and then the imaginary part becomes output

- **Learning process in the simple one-dimensional integral**







Sign-problem serious integral

$$\mathcal{Z}_p = \int dx\ (x + i\alpha)^p e^{-\frac{x^2}{2}}$$

J. Nshimura and S. Shimasaki, Phys. Rev. D92 (2015) 011501

p = 50, α = 10

Y. Mori, K.K. and A. Ohnishi, PTEP 2018 (2018) 023B04

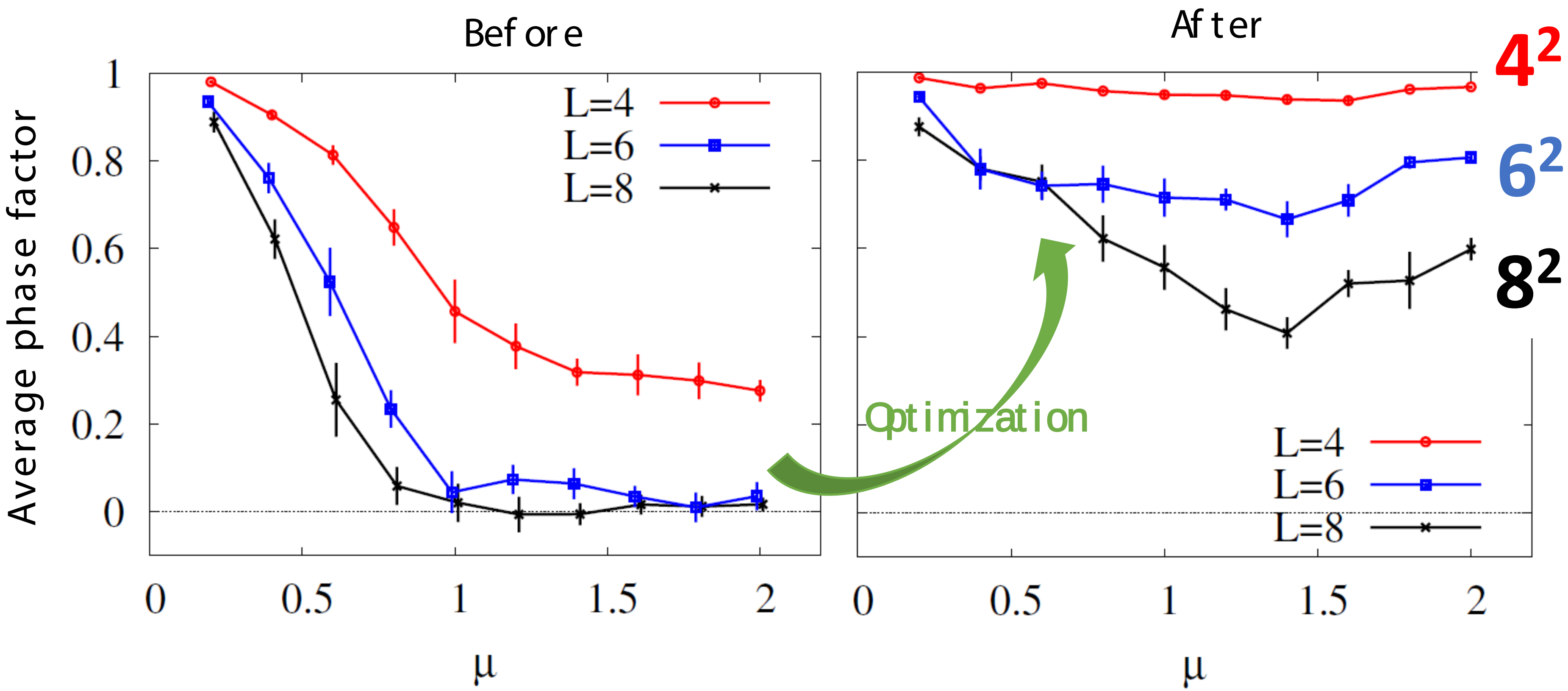
- **Two-dimensional complex λφ⁴ Theory**

$\varphi = \varphi_1 + i\varphi_2,\ \ \varphi_1, \varphi_2 \in \mathbb{R} \rightarrow z_1, z_2 \in \mathbb{C}$

Lattice action

$$S = \sum_{x \in \mathbb{L}^2} \left\{ \left(\varphi^\dagger(x+\hat{0})e^{+\mu} - \varphi^\dagger(x)\right)\left(e^{-\mu}\varphi(x+\hat{0}) - \varphi(x)\right) + \sum_{k=1}^{2} |\varphi(x+\hat{k}) - \varphi(x)|^2 + \frac{\kappa}{2}\varphi^\dagger(x)\varphi(x) + \frac{\lambda}{4}\left(\varphi^\dagger(x)\varphi(x)\right)^2 \right\}$$

Parameters: $\kappa = 1.0,\ \lambda = 1.0$



K.K, Y. Mori and A. Ohnishi, Phys. Rev. D 99 (2019) 014033

K.K, Y. Mori and A. Ohnishi, Phys. Rev. D 99 (2019) 114005

- **QCD effective model**

Polyakov-loop extended Nambu—Jona-Lasinio (PNJL) model (2 flavor)

$$\mathcal{L} = \bar{q}(\not{D} + m_0)q - G[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + G_v(\bar{q}\gamma_\mu q)^2 + \mathcal{V}_{\rm g}(\Phi, \bar{\Phi})$$

K. Fukushima, Phys. Lett. B591 (2004) 277

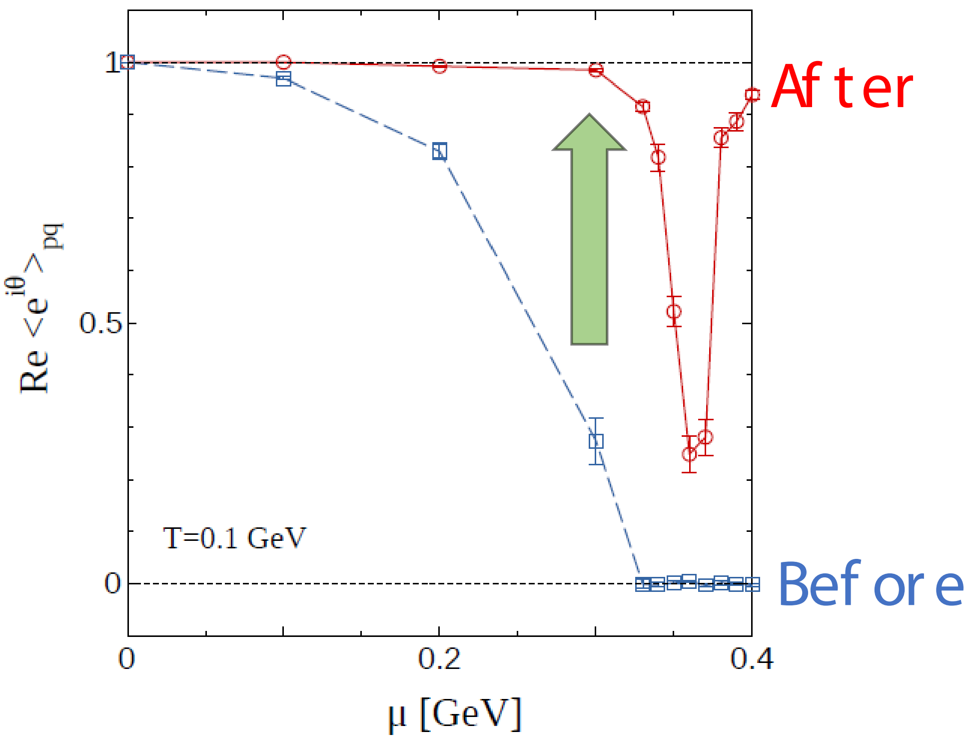
We assume inhomogeneous $(\sigma, \vec{\pi}, \omega_4)$ and (A_3, A_8) ➡ It is corresponds to the mean-field result in infinite volume limit

The sing problem is induced via the Polyakov-loop and also the repulsive vector-type interaction

(See also Y. Mori, K.K and A. Ohnishi, Phys. Lett. B781 (2018) 688)

T=100 MeV

$(G_v = 0.5G)$

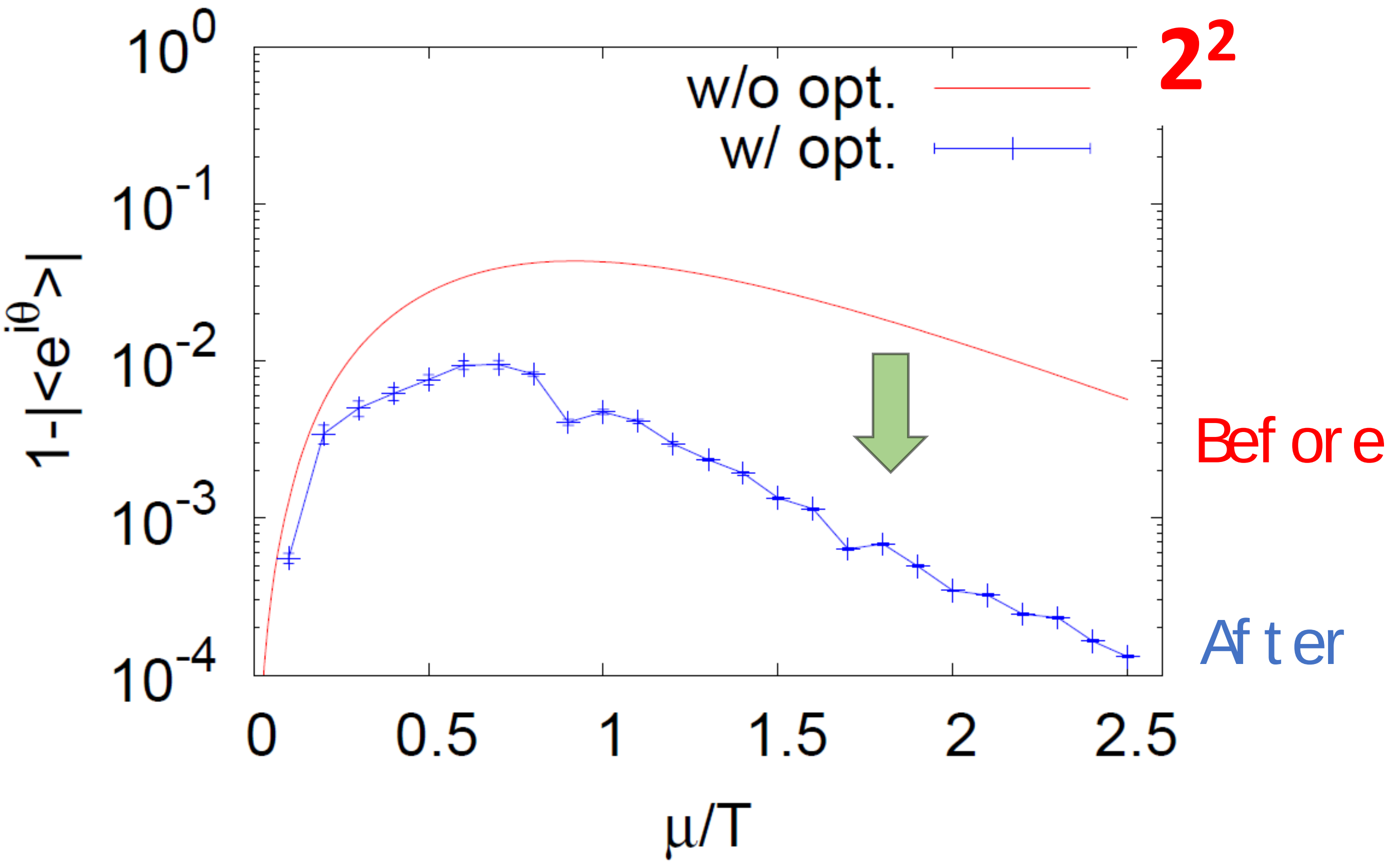


K. K., Y. Mori and A. Ohnishi, arXiv:1904.11140, to be published in PTEP

- **0+1 dim. lattice QCD**

$$\mathcal{Z} = \int \mathcal{D}\overline{\chi}\mathcal{D}\chi\mathcal{D}U\ e^{-S[\chi,\overline{\chi},U]} \qquad S = \frac{1}{2}\sum_{\tau=1}^{N_\tau}(\overline{\chi}_\tau e^{\mu}U_\tau\chi_{\tau+1} - \overline{\chi}_{\tau+1}e^{-\mu}U_\tau^{-1}\chi_\tau) + m\sum_\tau \overline{\chi}_\tau\chi_\tau$$

$$\mathcal{Z} = \int dU\,\det D(U) \qquad D(U) = X + e^{\mu/T}U(\theta) + e^{-\mu/T}U^{-1}(\theta)$$



Summary

We apply the path optimization method to the sign problem

Path optimization method

Cost function which reflects the seriousness of the sign problem

is minimized via the path modification (with machine learning)

In simple models and theories, we have checked effectiveness of the method

- Simple oscillating integral
- Complex $\lambda\phi^4$ theory
- PNJL model with and without the repulsive vector-type interaction
- 0+1 dimensional lattice QCD

We now take aim at 3+1 dim. lattice QCD! (but, the 1+1 dim. QCD is already not easy...)